The External Finance Premium in the UK

A Small Open Economy DSGE Model with An Empirical
Indirect Inference Assessment

By

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A Thesis Submitted in Fulfilment of the Requirements for the
Degree of Doctor of Philosophy of Cardiff University

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February 2017
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Abstract

This thesis is planned to investigate to what extent external finance premium channels by amplifying the business cycle account for the economy turndown in the UK. It builds a DSGE model follows Smets and Woulters (2007), extends to incorporate with the Bernanke Gertler and Gilchrist (1999) financial accelerator mechanism and adjusts for an Armington (1969) version small open economy. We evaluation the model based on the calibration and re-estimate the model by Indirect Inference method using un-filtered nonstationary data in the period of 1975Q1 to 2015Q4. The overall performance of modelling fitting after estimation increases with the model significantly pass the Indirect Inference test. The estimation results are also robust to the period from 1992Q4 to 2015Q4 under the inflation targeting monetary policy regime. Although the model captures the counter cyclical feature of external finance premium proposed in most of the literatures, external finance premium shocks on the financial sector do not play a dominate role in explaining a recession. The main dominant effects of output fluctuations are still coming from the non-financial shocks, in particular, the non-stationary productivity shock and the labour supply shock.
Acknowledgements

Undertaking this PhD has been a truly life-changing experience for me and it would not have been possible to do without the support and guidance that I received from many people.

First and foremost, my gratitude goes to my first supervisor Professor Patrick Minford for giving me the opportunity to work on this exciting field of research and providing a full PhD scholarship. I am truly indebted and thankful for his excellence guidance, greatest patience, and tremendous academic support during the completion of this dissertation.

I am especially grateful to my second supervisor Dr David Meenagh for his highly valuable support, countless hours of discussion and extensive comments on many drafts throughout the stages of this thesis.

I also would like to express my gratitude to Dr Vo Phuong Mai Le for her concise comments and helpful suggestions.

I am thankful to Cardiff university business school for providing an exciting research environment and teaching opportunities during past years. I particularly thank Professor Trevor Boyns, Professor Helen Walker and Ms Elise Phillips for their understanding and support.

I wish to express appreciation to my colleagues and friends at Cardiff University Business School, including among others Dr Wei Yin, Dr Peng Zhou, Dr Yi Wang, Dr Yongdeng Xu, Dr Wenna Lü, Congyi Hu, Jiayi Huang, and Wenyan Zou. Sharing views and experiences with them gave me strength and encouragement.

I would also like to say a heartfelt thank you to my parents, Dr Chaoxun Zhu and Jinghua Li, parents-in-law, Hongtao Ji and Chang Liu for always believing in me and encouraging me and their generous support, both financially and personally.

Finally, I am truly indebted to my wife, Dr Sisi Ji, and our son, Oscar Siyuan Zhu with their selfless support and enduring love.

Without the support of my supervisors, colleagues, friends, and family this thesis would have been immeasurably harder to write and it is for this reason that it must be dedicated to them.
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Chapter 1: Introduction

The motivation behind this thesis is without doubts related to the recent financial crisis. The severity of the financial crisis and the Great Recession has prompted a reconsideration of its proximate origin in the financial sector. The initial economic crisis, which saw as a 7% peak to trough decline in output, has been followed by a prolonged period of low growth. Meanwhile, forecasts for GDP growth in the UK were revised downwards by the International Monetary Fund because of the severe effect of the global economic crisis on the UK financial sector. Unlike Eurozone, although there was much speculation of a ‘double dip’ recession that the UK economy’s recovery from the great recession had stalled during 2010s and early 2013, it turns out that the economy was under no such threat. Although the UK economy is now returning to sustained recovery, but there is some distance to make up after a sharp recession and a delayed return to growth. Figure 1 shows even after 12 consecutive quarters of growth from 2012 Q4 to 2015 Q4, the total UK output growth continued to be a ‘flat lining’ economy and remained below its pre-recession level.

Figure 1. UK Real GDP, Quarter on Quarter change

Source: Quarterly National Accounts, ONS, GDP (AMBI)
After the 2007 US financial crisis, a growing number of works focused their attention on the DSGE models incorporating with financial frictions. Until then, the prevailing literature framework to model financial frictions within dynamic models referred to Bernanke et al. (1999) financial accelerator. This kind of literature and its further extension such as Christiano et al. (2010) focused its attention mainly on the demand side of the credit market. Other authors have estimated DSGE models for the United Kingdom. Di Cecio and Nelson (2007) and Kamber and Millard (2012) use a ‘minimum distance’ estimation approach to estimate the Smets and Wouters (2007) model on UK data and Kamber and Millard (2012) also estimate a version of the Gertler et al. (2008) model, which extends the Smets and Wouters model to allow for search and matching frictions. More recently, Villa and Yang (2011) use Bayesian techniques to estimate a model on UK data that adds financial frictions to the Smets and Wouters model and Faccini et al. (2013) do the same for a model that adds labour market frictions. However, unlike the current model, these models are all ‘closed economy’ and so might not be thought of as the best models to use when considering the ‘open’ UK economy. To the best of my knowledge, there have been a few attempts to incorporate frictions in financial intermediaries in the open economy framework such as Harrison and Oomen (2010), Millard (2011) and Burgess et al. (2013) on UK data.

The most critique of the current state of New Keynesian DSGE models is that these models lack an appropriate financial sector so that the models failed to account for an important source of aggregate fluctuations. This thesis is planned to investigate to what extent external finance premium channels by amplifying the business cycle account for the economy turndown in the UK. Chapter 2 is a review of theoretical and empirical literatures try to embed financial frictions in the framework of DSGE macroeconomics models. In particular, the review starts from the work of Bernanke et al. (1999), under the assumption of the costly state
verification approach introduced by Townsend (1979). It demonstrated that the amplification and persistence mechanism introduced by financial friction have an important role in business cycles. The model generates a so-called "financial accelerator" effect since endogenous pro-cyclical movement in entrepreneurial net worth magnify investment and output fluctuations. The review then extends to a collateral constraint approach by imposing restrictions based on the need to collateralize the loan to cover inability to fulfil obligations under a financial contract. On the other hand, researchers such as Gertler and Karadi (2011), Gertler and Kyiotaki (2010), introduced an agency problem between banks and depositors that introduces endogenous constraints on intermediary leverage ratios that links the amounts of deposits to the net worth of the financial intermediary. We focus the attention on empirical contributions have as primary target to better understand the role played by financial frictions in the business cycle fluctuations.

In chapter 3, it builds a DSGE model follows SW (2007). It is then extended to incorporate with the financial accelerator mechanism as in BGG (1999) and an Armington (1969) open economy version. The wage and price setting follows Le et al (2011)’s hybrid model assuming that wage and price setters find themselves supplying labour and intermediate output partly in a competitive market with price and wage flexibility, and partly in a market with imperfect competition. The closest empirical exercise to the model framework is contained in Le et al (2012, 2013). Although this set-up does not explicitly model the banking system, the spread shock is suitable to capture the effect of financial tightening on entrepreneurs' borrowing capacity. A similar model framework (at least in the closed economy setup) has been adopted in numbers of studies (e.g. Gertler, Gilchrist and Natalucci (2007); Christensen and Dib (2008); De Graeve (2008); Queijo von Heideken (2009), Gelain, (2010), Le et al (2012, 2013)). The results from IRFs are not at odds with those found in other studies that financial frictions affected the economy through financial transmission channels, by amplifying the business cycle.
With respect the confirmation of the empirical relevance of the financial frictions, in chapter 4, I test the model with calibrated parameters, and re-estimate the structural parameters and assess the role of different shock, in particular, financial shocks as the drivers of the variability delivers. However, chapter 4 differs other studies in several aspects. First, in contrast to conventional estimation techniques, I evaluate and estimate the proposed model by using the method of Indirect Inference. Secondly, the model is estimated against non-stationary data. Thirdly, observed sample period is extended to 2015Q4. The result shows model with or without financial frictions are severely rejected using calibrated parameters, while the overall performance of modelling fitting improved under the Indirect Inference estimation. The studies of variance and historical shock decomposition suggest although external finance premium and entrepreneurs’ net worth shock are the main drivers of financial variable fluctuations, they do not play a dominate role of capturing macroeconomic dynamics as the expansion and collapse of the economic activity. This is a remarkable result which somewhat highlights Le et al (2012, 2013)’s conclusion that the financial crisis was most likely the result of non-stationary shocks impacting through the usual non-financial channels. Moreover, with the introduction of external finance premium shock, investment shock is relegated to account for a small fraction of the variance in nominal variables and entrepreneurs’ net worth. Finally, chapter 5 concludes and discusses future possible applications and extensions of existing works.
Chapter 2 DSGE models with financial Frictions: A Literature Review

2.1 Introduction

The DSGE model encompasses a broad class of macroeconomic models that spans the standard neoclassical growth model in King, Plosser, and Rebelo (1988). It then reaches a high level of sophistication with Christiano, Eichenbaum and Evans (2005) (henceforth CEE05), and Smets and Wouters (2003, 2007) (henceforth SW03, SW07). These DSGE models have become the workhorse framework that used not only for academic and policy analyses, but also for forecasting (Del Negro and Schorfheide, 2013). However, the failure of those DSGE models to predict the financial crisis 2007-2008 and aftermath Great Recession has rightly come under attack. The macroeconomic literature using DSGE models has modelled the financial sector mostly as a pass-through mechanism, not taking into account financial frictions and their role as amplifier of monetary policy decisions (Beck et al. 2014). The studies developed New Keynesian macroeconomic model that contains numerous real and nominal frictions, while assumptions of financial markets are smooth and perfect.

The recent turmoil has provided impetus that a significant disruption of financial frictions has turned out to be a relevant factor for economic fluctuations. In particular, Del Negro et al. (2013) showed that an extension with several financial frictions to the SW07 model helps to forecast the US economy during the great recession (from 2008Q3) (it features a sharp decline in output without forecasting a large drop in inflation), especially if the forecasts are conditioned on the available data on short-term interest rates and credit spreads. It motivates rapid growing theoretical and empirical literatures on DSGE model with financial frictions.

In this chapter, it sets by surveying theoretical and empirical literatures focusing on financial frictions in the framework of DSGE macroeconomics models. The
theoretical studies explicitly modelling financial frictions based on extension of existing models. These studies differentiate the role of financial frictions originate from different sectors (Bananke et al., 1999; Kiyotaki and Moore, 1997; Iacoviello, 2005; Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011) or the consequences of macroprudential policies (Angeloni and Faia, 2009; Curdia and Woodford, 2010; Le et al., 2014). The empirical papers use different data and evaluate and estimation methodologies to explore the role of financial frictions and to assess performance and implication of models. Some concluded the inclusion of financial frictions played dominates role of business cycle fluctuations (De Graeve, 2008), while conversely, other studies reach opposite conclusion (Meier and Muller, 2006; Le et al., 2012, 2013)). It should be noted Gertler and Kiyotaki (2010), Quadrini (2011), Beck et al. (2014), Brazdik (2010), Brunnermeier et al., (2012) have provided extensive surveys about macroeconomic implications of financial frictions.

The remainder of this paper is structured as follows. Section 2 reviews theoretical developments of DSGE models with financial frictions and their roles in the transmission shocks to real economy. Section 3 describes empirical assessment regarding to theoretical literatures. Section 3 describes recent DSGE model based on the UK data. Section 5 concludes.

2.2 Modelling frictions in financial market

Despite the large body of empirical literatures emphasized the importance of financing frictions and inherent instability of the financial system, the traditional macro models including the SW03, SW07 and CEE05 heavily rely on the Modigliani and Miller (1958) framework in which there is no role for financial sector. The great recession is then a reminder that financial frictions are one of the drivers of business cycle fluctuations. According to existing theoretical literature, two channels have been distinguished to account for the transmission of shocks originating in the financial sector to the real economy. A balance sheet channel depends on the financial friction imposed as credit constraints on non-financial
borrowers (demand side), while a banking lending channel depends on the credit constraint imposed on the side of financial intermediary (supply side). The two channels are always referred to the financial accelerator\(^1\).

### 2.2.1 External Finance Premium

Brazdik et al. (2011) provides a convincing description about the differences between the two approaches: external financial premium and the collateral constraints way of modelling financial frictions. The external finance premium of financial frictions grounds the key micro-foundation based the assumption of costly state verification framework of Townsend (1979) and Gale and Hellwig (1985) where the friction is due to information asymmetry about the future payoff of the project. Each entrepreneur purchases unfinished capital from the capital producers at the given price and transforms it into finished capital with a technology that is subject to idiosyncratic productivity shock which is not observable to outsiders and verifying it comes at a cost. The optimal contract between an entrepreneur and the households providing outside funding ensures that the entrepreneur doesn’t take advantage of the information asymmetry and minimizes the deadweight loss due to costly verification. Because monitoring a contract is costly, it drives an external finance premium between the cost of an entrepreneur to raise capital on financial market (lending rate) and the opportunity cost of an entrepreneur’s use of internal resources\(^2\), i.e. capital raised from profits.

The first version model originates from the seminal paper of Bernanke and Gertler (1989) that uses an overlapping generation model where agents live for only two periods and Carlstrom and Fuerst (1997) later embed the contract problem into the infinite horizon real business cycle framework. Following Bernanke and Gertler (1989) to analyze the dynamics of the model, it reveals that temporary shocks have a much stronger persistence through feedback effects of tightened financial frictions: supposing a negative productivity shock decreases the wage...

\(^1\) The term was originally introduced by Bernanke, Gertler and Gilchrist (1996) to challenge the Modigliani-Miller view of the irrelevance of financing for a firm’s or for a bank’s investment decision.

\(^2\) Lending rate is almost always positive and higher than risk free rate based on the data shown in De Graeve (2008), Abhijit(2002).
and current entrepreneurs’ net worth. This increases borrowing frictions to acquire household’s saving for the implementation of investment projects and leads to decreased investment in capital for the next period. The lower capital therefore reduces output and therefore the wage in the next period, which implies a lower net worth for the next generation of entrepreneurs. The next generation also invests less and the effect persists further. Bernanke and Gertler (1989) named this type of amplification a shock accelerator effect on investment income, and therefore further studies recognize this mechanism as the financial accelerator.

2.2.2 The Bernanke et al. (1999) model

Bernanke et al. (1999) (Henceforth BGG) extend the model of Bernanke and Gertler (1989) to present a complete dynamic New Keynesian framework with price stickiness that allows the possibility of credit relations between the households and the entrepreneurs. The model economy of BGG is populated by households, entrepreneurs, and retailers. In this context, the model mainly assumes households supply labour, consumption goods, and savings. Then households as net savers transfer resources to entrepreneurs through the financial intermediates. The entrepreneurs use the acquired funds to purchase physical capital used in the production of the intermediate goods. The intermediate goods are bought by the retailers and sold to the households. In addition, the government conducts both fiscal and monetary policy. The key assumption is to justify the existence of an external premium embedding an agency cost problem incorporating into a New Keynesian model. Following this approach, since financial intermediates are not able to control the debtor ex-ante, there exists an optimal contract between an entrepreneur and the financial intermediates ensuring that the entrepreneur doesn’t take advantage of the information asymmetry and minimizes the deadweight loss due to costly verification. Since the financial frictions in the model presented in Chapter 3 is to follow BGG’s setting up. The description will be discussed in Chapter 3 in detail. It should be noted that the role of financial intermediates is trivial in the original BGG model. The intermediates
are only a device to justify the existence of external risk premium.

Similar to Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), as in BGG, the net worth of entrepreneur is pro-cyclical and it then leads to a counter-cyclical external finance premium occurs. To show how a small shock can significantly affect the whole economy for a long time through financial accelerator mechanism, suppose in the event of a positive external finance premium shock, it will reduce demand for obtaining external funding due to an increasing cost of borrowing. Further deterioration of entrepreneur’s financial position leads to a further increase external finance premium and further reduction in demand for funding. The counter-cyclical external finance premium also imposes limits on the provision of funding for investment projects. It then consequently leads to a reduction of investment and future profits from investment. Because a decreasing in profits from investment, it also weakens the net worth of entrepreneurs and then strengthen the external premium.

BGG then use their framework to study the dynamic propagation of a monetary policy shock and they compare it to the standard new Keynesian framework without financial intermediaries. They find that an increase of the interest rate causes a reduction of the capital demand that consequently decreases the price of capital. The capital reduction weakens the net worth of the entrepreneurs enhancing the external premium. The investment goes down causing a decline of the total output.

The authors also contrast the immediate response to the monetary policy shock in the model with the delayed response in the data, it then shows the financial accelerator mechanism itself does not deliver the desired properties of the responses. They again that adding a delay in the investment process to correct this deficiency. The result leads to the presence of a financial accelerator may explain the extent and persistence of fluctuations, which are a response to monetary policy and demand and supply shocks. They also suggest possible extensions of their benchmark model such as nominal debt contracts, open economy model setup or involving roles for financial intermediates. In the following section, some of these extensions are discussed.
2.2.3 Extension based on BGG

During the last decade, the benchmark BGG New Keynesian model has been extended with two important features. The first extension is that the debt contract can be denominated in terms of the nominal interest rate in order to reflect the nature of debt contract in the US. This innovation considers the so-called ‘Fisher effect’ that describes the effect of debt deflation effect on nominal debt contract as mentioned in literatures on the Great Depression by Fisher (1933).

The second extension consists to introduce into a modified Taylor type rule under which the monetary authority adjusts short-term nominal interest rates. According to BGG, monetary policy is crucial in determining the quantitative importance of the financial accelerator. The greater the extent to which monetary policy can stabilize output, the smaller the role of the financial accelerator is, in amplifying and propagating business cycles in output or investment. In particular, since the financial crisis underlined the role of the financial stability of the credit market, central banks (at least advanced country) naturally have engaged in all sorts of unconventional monetary policies when greater monetary stimulus is required by cutting the policy rate to its effective lower bound.

The debate of whether or not unconventional policies (macroprudential policy against monetary policy) as an attempt to get around the borrowing constraints that play a central role; the pros and cons of pursuing an unconventional monetary policy are animated by a series of influential papers such as Angelini et al., 2010; Angeloni and Faia, 2009; Curdia and Woodford, 2010; Le et al., 2014 and Meh and Moran, 2010.

Curdia and Woodford (2010) modified a standard Taylor rule by introducing a contemporaneous response to the size of a credit spread. They found such spread

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3 Bernanke and Reinhart (2004) grouped unconventional monetary policies into three classes: (1) using communications policies to shape public expectations about the future course of interest rates; (2) increasing the size of the central bank’s balance sheet; and (3) changing the composition of the central bank’s balance sheet.
adjustment could reduce the distortions caused by a financial disturbance. This modification of the standard Taylor rule can also improve the economy’s response to certain variations in the size of debt-financed government transfers. 

Le et al. (2014) extended the work of Le et al. (2012) by augmenting an unconventional monetary policy and using cash as collateral. The model suspends a standard Taylor rule when the nominal interest rate hits the zero bound (0.25% annually in their setting) and replaces with exogenous lower bound. Their results suggested a Taylor rule for making monetary base (M0) respond to credit conditions could substantially enhance the economy’s stability. It combined with price-level and nominal GDP targeting rules for interest rates to stabilise the economy in further. The authors further argued that with these rules for monetary control, aggressive and distortionary regulation of banks’ balance sheets becomes redundant.

2.2.4 The model with collateral constraint

The BGG framework suffers from several limitations. One of the major criticism is that the external finance premium does not contain limits on the availability of the amount of borrowing. The second approach is then to introduce collaterals constraints incorporate with the financial accelerator into a model. It grounds its micro-foundation from the incomplete markets framework of Hart and Moore (1994), which the amount of credit issuance by lenders to entrepreneurs is limited due to collateral constraints.

This alternative approach originates firstly from the seminal paper of Kiyotaki and Moore (1997) (KM97, henceforth). The line of this research introduced financial frictions by introducing endogenous collateral constraints that limit the credit capacity of borrowers less than or equal to the value of their assets holdings. Agents are heterogeneous in terms of their rate of time preference, which divides

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4 The authors considered the KM97’s collateral idea was too extreme.
5 Kiyotaki and Moore (1997) distinguished two kinds of agents: a patient one, called the gatherer, which is a net saver and impatient one, called the farmer. The farmer can act as an entrepreneur who wishes to finance his own investment project acquiring external resources from the patient agent.
them into lenders and borrowers. The constraint in their setup has the form: 
\[ R_t b_t \leq q_{t+1} k_t \]. Where \( R_t \) is the nominal interest rate, \( b_t \) is the total amount borrowed, \( q_{t+1} \) is the capital (land) price in next period, and \( k_t \) is the capital (land) stock.

The financial sector intermediates between two groups and introduces frictions by requiring that borrowers provide collateral for their loans. The need for collateral is then motivated by the absence of contract enforcement in the economy and collateral constraint is set exogenously. Hence, this approach introduces frictions that affect directly the quantity of loans. The rest of the model is a standard model of real business cycles. They showed even small scale and short-term shocks to productivity or income distribution can leads to prolonged changes in production, consumption, and prices of capital that spread throughout the economy. This is because the assets of firms are used not only for the productions, but also as collaterals. Suppose a decrease of the land price because of a negative technological shock, could decrease the net worth of the farmer. Producers that become constrained by the credit limit are forced to reduce their demand for investment. The decline in investments causes a further decrease of the net worth and a contraction of the credit available induced by a reduction of the collateral value. The amplification of the shock is caused by a twofold effect: 1) the limited availability of credit and; 2) the role of assets price \( q_{t+1} \) further affects the collateral constraint. It should be noted that, different from the BGG model, the source of the financial accelerator effect and propagation of technology shocks is the interaction of asset prices for debt securitization and credit limits.

### 2.2.5 Extension and criticism of the model with collateral constraint

Iacoviello (2005) then extended the original model of KM97 by adding two features. First, the collateral used by entrepreneurs to obtain external funds is no longer the land; instead, they could be borrowing tied to the house stock owned by the entrepreneurs. Second, as Christiano et al. (2010) to BGG, it introduced
nominal debts against KM97 model. The paper augmented a New Keynesian general equilibrium model with endogenous collateral constraints and nominal debt that each households, entrepreneurs and banks would faces. The author finds a positive demand shock drives up consumer prices and asset prices, which relaxes the credit constraint (size of the loan) allowing agents to increase borrowing. In particular, Iacoviello and Neri (2010) estimated a model with collateral constraints on US data in order to study the role of housing market shocks on the economy. They find evidences that the real estate industry is a relevant source of the US business cycle. Other recent applications relying on this framework include Calza et al. (2013) who analysed the impact of mortgage market characteristics on monetary transmission. Gerali et al. (2009) embedded a banking sector into a medium scale DSGE model. It could be seen as an extension of the model proposed by Iacoviello (2005) in the previous section. Brzoza-Brzezina and Makarski (2010) used models with collateral constraints and monopolistic competition in the banking sector to examine the impact of financial frictions on monetary transmission and a credit crunch scenario. Marshall and Shea (2013) stated that the authors find that credit constraints act as a powerful ‘butterfly effect’ for the amplification and propagation of shocks. The amplification of shocks first reduces the price of collateral; second, restrict access to credit, which in turn reduce demand for the assets, further lowering its price. This financial accelerator effect helps to explain how relatively small shocks can result in large business cycle fluctuations. Although the collateral constraint approach of KM97 has some empirical advantages that the financial constraint of households can influence the constraint of entrepreneurs. However, in this framework, there is no endogenously determined financial premium, the borrower instead is rationed from the financial market if it reaches his maximum borrowing capacity determined by loan-to-value ratio. Another major criticism of KM97 and Iacoviello (2005) is that there is no assumption for uncertainty regarding the repayment of loans. Cordoba and Ripoll (2004) modified KM’s assumption to use more realistic version that the amplification of fluctuations in real economic cycles can be generated by a small
degree of smoothing and high utilization of assets to secure debt in the production function. However, they demonstrate the insignificance of the financial accelerator effect for the amplification of responses. They conclude that unless one has this right combination of parameters, usually collateral constraints can only generate relatively small amplification comparing with original models. Large amplification can only be obtained with the combination of a low elasticity of intertemporal substitution, a large (but not too close to unity) capital share and share of constrained agents. Brzoza-Brzezina et al. (2013) compared the impulse response functions from a model based on the BGG against a model based on collateral constraint. They find that the collateral constraint model failed to reproduce hump-shaped impulse response functions and they tend to generate volatilities for the price of capital and the rate of return on capital are not consistent with the data.

### 2.2.6 Credit Constraint on Banking Sector

In either BGG or KM model, the role of banks or financial intermediates were not specified, as financial contracts are arranged directly in the financial market under the known form of a contract for the acquisition of external funding. The literature introducing a bank or financial intermediate friction into DSGE models has been motivated mainly by the aim of explaining specific features of the financial crisis. The bank sector friction can be divided into two separate components: 1) the bank lending channel and 2) the bank capital channel. The idea of bank lending channel was manifested originally from Bernanke and Blinder (1988). The underlying idea behind the bank lending channel is that banks’ cost of funds increases in response to restrictive monetary policy. A tightening monetary policy on the one hand, is the standard effect of monetary policy that decreases money supply. On the other hand, it entails a change in the asset composition, leading to a stronger decline in credit supply.

Extending the idea of Bernanke and Blinder (1988), Goodfriend and McCallum
(2007) paved the way to contributions that tried to give to the banking system a greater role in the business cycle. Similar to external financial premium in BGG, it also emphasises the influence of the net worth or equity position of the financial intermediate on the credit conditions these agents face. The model features two opposite effects: a standard financial (bank) accelerator effect as in BGG against a banking attenuator effect. Banking attenuator effect refers to a sluggish and heterogeneous pass through of the change in the policy rate to the bank interest rate. This is because perfectly competitive banks are introduced to generate a variety of loans using a loan production function that employs both loan monitoring costs and collateral. Suppose there is an expansionary monetary policy shock to increase the consumption, due to the cash in advance constraints, it leads to an increase in households’ demand for bank deposits, which in turn increases banks’ demand for collateral and the price of issuing loans. The character of the production function also implies that the monitoring costs grow faster than the amount of loans. The higher costs of lending given by the increased spread dampen the demand for loans and discourage consumption. Hence, the overall effect of a monetary policy shock can be dampened by the presence of a banking system in the model.

Curdia and Woodford (2009) extended a standard NK model with a banking sector to consider financial intermediation. In their paper, intermediation exists among households and but not between households and firms. Due to different rates of patience, part of the households are borrowers while others are lenders. Borrowers have a higher marginal utility of consumption than lenders. Therefore, the optimality conditions of the model contain two discount factors and Consequently, the model produces two different interest rates. The spread between the interest rate available to lender and the interest rate that borrowers pay for the loan is time varying. The financial imperfection in their model takes the form of a wedge between borrowing and lending rates, which may be either due to the use of resources in intermediation, or due to the market power of intermediaries. An

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6 Bond price is inversely related to yield rate.
increase in the wedge, at the same average interest rate, decreases the lending rate and increases the borrowing rate. Gertler and Karadi (2011) then proposed a model with unconventional monetary policy. They embed financial intermediaries subject to endogenously determined financial constraints stemming from the agency problem. Specifically, after collecting household deposits, the bank can divert a fraction of the resources obtained from the market for their own purposes. This implies that the ability of a bank to attract deposits and to extend loans to firms is positively related to its current net worth and to its expected future earnings.

The bank capital models have been built upon Holmstrom and Tirole’s (1997) financial accelerator model. It assumes all bank lending is financed by capital, which provides the incentive for banks to monitor borrowers, and thereby overcome the moral-hazard problems present in borrowers’ investment decisions. Gerali et al. (2010) augmented a DSGE model with a monopolistically competitive banking sector. Banks are assumed to have the market power to set interest rates such that a spread between deposit and lending rates arises. Banks supply loans to the private sector using either deposit or bank capital and are subject to an exogenous leverage ratio. This implies that bank capital has a fundamental role in determining credit supply conditions. Since bank capital is accumulated through retained earnings, a shock negatively hitting the profitability of banks will impair their ability of raising new capital. As a result of their deteriorated financial position banks may reduce the amount of loans they are willing to supply, thus deepening the initial contraction. Several other studies have focused on bank capital requirement imposed by banking regulations (Dib, 2009; Van den Heuvel, 2002, 2008). Other more recent literatures are unanimous in concluding that banking sector shocks and investors’ sentiment explain the largest share of the contraction in the economic activity. (Gertler and Kiyotaki (2009); Gerali, Neri, Sessa and Signoretti (2010); Kollmann, Enders and Müller (2011)). Both the banking capital channel and the banking lending channel stress the importance of credit flow. Since financial intermediates are also responsible for
money supply by accepting deposits, they are key players in understanding the transmission mechanism of monetary policy.

### 2.3 Empirical evaluation of models with financial frictions

Despite the ample theoretical work based on the financial accelerator, more works are keen to evaluate the empirical relevance of the class of financial friction models. Among those, Christiano et al., (2003) presented a model with financial frictions by adding a banking sector. They proposed to evaluate the Friedman-Schwartz hypothesis\(^7\) and analysed the role of financial frictions during the Great Depression. They estimated a DSGE model with a financial accelerator but only calibrated the parameters related to the financial frictions. The model identifies an increase in preferences for holding money and a shift away from savings over the period. De Greave (2008) and Christensen and Dib (2008) emphasized the prominence role of financial accelerator mechanism. Christensen and Dib (2008) estimated the standard BGG model for the U.S. using maximum likelihood and find evidence in favour of the financial accelerator model. De Graeve (2008) estimated the external finance premium for the U.S. economy incorporating a financial accelerator into the SW03 model. Both results found that model incorporating financial frictions improves the empirical performance of an otherwise standard DSGE model. They find that increases in the external finance premium lead to significant and protracted declines in investment and output. Christiano et al. (2010) extended SW07 model augmented with a detailed description of the financial sector. The model presented and estimated to analysed the business cycle implications of financial frictions during the financial crisis on Euro Area and U.S. data. Christiano et al. (2010) featured both agency problems in entrepreneurs and financial intermediates. They further allowed producers to

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\(^7\) They suggested a more accommodative monetary policy could have greatly reduced the severity of the Great Depression
raise capital through nominal contracts. Besides the mechanism of propagation as in BGG, the study also emphasizes the role of financial intermediate sector as a source of shocks. The model proposed a competitive banking system in which the banks can decide the amount of deposits gather from the households and the amount entrepreneurial loans to issue that allow them to analyse the so-called ‘bank lending channel’ together the standard financial accelerator mechanism. The authors hope that the introduction of the banking sector provide a better fit of the data by the model. The financial shocks in the model are shown to play an important role of explaining business cycle fluctuations. They found that factors that pertain to monetary and financial sector, the frictions that motivate and shape finance and the shocks that hit the banking function are prime determinants of business cycle fluctuations. Besides that, the amplifying effect of the financial accelerator is similar as in BGG. Moreover, they found evidences that it is desirable for the monetary policy to target not only inflation and output gap, but also the variables related to the stock market to stabilize economic activity. Villa (2013) started with the SW07 model and compared it to two alternative frameworks. The first one is a SW model augmented with BGG; the second one is a SW07 model augmented with financial frictions originating in financial intermediation as in Gertler and Karadi (2011). All models are estimated with Euro Area quarterly data over the period 1980-2008. The analysis shows that the last version model outperforms the other models in terms of the predictive power of inflation pressure. In Brunnermeier and Sannikov’s (2011) finding, the net worth of the financial intermediary sector plays a key role. It stressed the fact that the distribution of wealth is an important determinant of economic activity in a setting where financial frictions limit the flow of funds. It makes a difference whether net worth is in the hands of more productive agents or less productive agents or financial intermediaries who facilitate credit ow from less productive to more productive agents. The key frictions are financial contracting frictions rather than price or wage rigidities that are the main drivers in New-Keynesian models. Despite the widespread perception that financial condition can contribute to economic downturns, the conclusion arising from estimated medium scale DSGE
models with financial frictions cast some doubts on the relevance of financial frictions.

Meier and Müller (2006) compared one model with a financial accelerator and the other model with increasing capital adjustment costs. They focused on the monetary policy transmission mechanism by matching the impulse response functions after a monetary policy shock. They argued that both models are able to replicate the characteristics of the observed data on investment. The authors consequently considered the external financing mechanism is not more important than the mechanism of costly investment for description of the properties of the transmission mechanism, and financial frictions do not play a very important role in the model.

Christiano et al. (2008) developed a large and richly-specified DSGE model that includes financial frictions. This model is then used to analyse the slowdown in economic activity that occurred in 2001. The model is estimated on both US and euro area data and time series for the model shocks are retrieved from the estimation procedure. These shocks suggest that the slowdowns in both the US and euro area were mainly driven by a combination of demand shocks and shocks to the business sector, whereas banking shocks affecting either the supply or demand of credit played only a minor role. Another interesting finding from this research is that, since interest rates are less volatile in the euro area, the European Central Bank was able to achieve the same degree of output stabilisation than the Federal Reserve with smaller changes in policy rates.

Brzoza-Brzezina and Kolasa (2013) compared three alternative DSGE models with Bayesian techniques. They consider as a benchmark SW07 NK model and compare it to a model characterized by an external finance premium and a model featuring a borrowing constraint as in Kiyotaki and Moore (1997). All models are estimated using U.S. quarterly data over the period 1973-2008. Evidence from marginal likelihoods shows that models with an external finance premium are more in line with the data than models with a collateral constraint, however a clear-cut improvement with respect to the benchmark NK model cannot be observed.
Brazdik et al. (2012) raised another criticism that since the premium is derived only from the current value of the net worth of capital producers, the model cannot capture the direct effect of expectations of future economic development at the current premium level. Models with a financial accelerator mechanism have only a limited ability to capture the increase in bankruptcy rates seen during economic bad times.

2.4 DSGE model for the UK

For UK data, all in all, there has been considerably less work done in terms of DSGE modelling than there has been for other economies, such as US and EU. Interest in DSGE modelling of the UK has been heightened in recent years with the induction of Bank of England Quarterly Model (BEQM)\(^8\). However, since the BEQM has no similarity in important respects with CEE model of the United States and SW03 model of the EU. It was difficult to use BEQM to compare the structure of the U.K. economy with that of other economies (DiCecio and Nelson, 2007).

The BEQM was then replaced since 2011 that the Monetary Policy Committee (MPC) has launched a new forecasting platform to help its quarterly economic forecasts, named Central Organising Model for Projection Analysis & Scenario Simulation (COMPASS). COMPASS is a medium to large-scale New Keynesian DSGE model\(^9\) built on the tradition of SW03, CEE05 and SW07 with similarities to those implemented in other central banks over recent years. COMPASS includes a suite of 50 forecasting models, covering a range of different frameworks and ways of thinking about the economy (Domit et al., 2016). A number of similar features about the real rigidities included in COMPASS, such habit formation and investment adjustment costs, and nominal prices and wages rigidities. DiCecio and Nelson (2007) estimated a DSGE model of CEE (2005).

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\(^8\) See Harrison et al., 2005 for a more detailed description.

\(^9\) See Burgess et al. (2013), Section 2 for a much fuller discussion.
on the U.K pre-crisis data. Their estimates suggested price stickiness is more important than wage stickiness as the major source of nominal rigidity in the U.K. Besides that, other features including international trade in imports and exports; and the presence of ‘rule of thumb’ households are also incorporated. The COMPASS model economy is populated by households, firms, a central bank, a government, while the rest-of-the-world economy are treated as exogenous. According to Fawcett et al. (2015), COMPASS is estimated with Bayesian maximum likelihood methods on UK data for 15 variables using 18 shocks. Among those shocks, a permanent labour augmented productivity shock shifts the stochastic trend of the model, reflecting a statistical assumption that GDP and the expenditure components of GDP are integrated of order one and cointegrated with each other. It should be noted COMPASS was introduced at the centre of a suite of models to organise the production of the MPC’s forecast. The suite of models contains many different models of varying types and classes with different purposes. It then translates existing models into the new platform.

Fawcett et al. (2015) evaluated the accuracy of real-time forecasts for inflation and GDP growth from COMPASS for the UK before, during and after the financial crisis. They found the accuracy of all forecasts fell during the financial crisis, and the deterioration was particularly marked for the GDP growth forecasts. They argued current DSGE models is not well suited to capturing the implications of large financial shocks that may have non-linear effects.

### 2.5 Conclusion

In this chapter, the review traces back the history of financial frictions within DSGE model, from the pioneering works of BGG and KM to the recent works of Gertler and Karadi (2011). The motivation behind the chapter is without doubts related to the recent financial crisis. In the last decade, we witness an explosion of models.

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of macro dynamic model that try to introduce financial frictions into the Real Business Cycle or New Keynesian framework. Financial frictions can be included by introducing an external financing premium that risky entrepreneurs, because of the uncertainty of the projects they undertake, have to pay when they borrow funds from the financial intermediaries. This friction originates from the problem of asymmetric information and costly state verification between the two types of agents. Financial frictions can also be included by imposing restrictions based on the need to collateralize the loan to cover inability to fulfil obligations under a financial contract. Nevertheless, the empirical studies do not always admit financial frictions as dominates to DSGE models. Studies either rejected the hypothesis of significant financial markets friction (Meier and Müller, 2006; Brzoza-Brzezina and Kolasa, 2013) or impeded financial shocks from other non-financial shocks as a key ingredient to model (Le et al 2012, 2013).
Chapter 3: A small open economy DSGE model with external finance premium

3.1 Introduction

The standard New Keynesian model assumes that financial markets work perfectly so that the interest rate set by central banks uniquely determines the cost of credit for borrowers. The recent financial crisis nevertheless has exposed the weakness of this simplifying assumption and revived interest in business cycle models with financial frictions. A growing number of literatures follow the trail set by seminal works developed in this field in the last two decades (among others, Bernanke, Gertler, and Gilchrist, 1999; Carlstrom and Fuerst, 1997; Kiyotaki and Moore, 1997; Christensen and Dib, 2008; De Graeve, 2008; Geltain, 2010). These studies regard the potential role of financial factor as a source of shocks itself and assumes that financial frictions work as a mechanism of transmission of macroeconomic shocks.

The closest empirical exercise to the framework of this study is contained in Le et al (2012, 2013) who borrows SW07 model but allow for more heterogeneity in price and wage behaviour and integrated in BGG financial accelerator mechanism. My contribution would be twofold. First, I would extend their framework by allowing the Armington (1969) substitution elasticity between domestic and foreign goods by adding a CES preference structure following the study of Meenagh et al (2005, 2010) and Minford (2015), to adjust for a small open economy model. Second, I would apply this modification to a variant of the Le et al (2013) model of EA and revisit it based on U.K. data. The rest of the paper is structured as follows. The following Section 3.2 lays out the structure of the model with adding features and relative equations. Section 3.3 discusses its calibration. The responses of macroeconomic and financial variables to a variety of shocks are illustrated and resentted in Section 3.4. Section 3.5 concludes.
3.2 The model economy

In this section, I describe a small open model economy populated by seven classes of agents: households, employment agencies, capital producers, entrepreneur/intermediate goods producers, final goods producer, a government and a central bank. The model features a continuum of infinite lived risk-averse households who consume one homogeneous consumption goods traded at the international level. Household consume a bundle of both home and import goods but with a preference bias towards the home good. The production of the home goods is also differentiated to domestic goods for home country and import goods for foreign country. Household supply labour partly to differentiated and sticky wage labour unions and partly in a perfectly competitive labour market without a union. There is also a continuum of risk-neutral entrepreneurs who use their own net worth and a debt contract with perfectly competitive financial intermediaries to finance the capital expenditure. Entrepreneurs use capital and hired labour from labour union as inputs to produce intermediate goods. Productivity of each entrepreneur is subject to an idiosyncratic shock which cannot be observed by financial intermediates. This presence creates financial frictions as in Bernanke, Gertler, and Gilchrist (1999) leads to a premium paid by entrepreneurs over the risk-free interest rate paid by banks to households’ deposits. Furthermore, a perfectly competitive sector of capital producers combines the existing capital from entrepreneurs with investment flows to produce the installed capital stock then rented to entrepreneurs one time to next. Perfectly competitive retailers sell the aggregated intermediate goods as a composite final good to the households. Final good is made up in a fixed proportion of intermediate goods sold partly in an imperfectly competitive market (sticky price) and in sold competitive market (flexible price). The aggregate output produced is then converted into consumption, investment, goods used up in capital utilization, and net export. In addition, the government finance their expenditures by collecting lump sum taxes from the households and the central bank conducting monetary policy according to a Taylor rule.
I extend a two country Armington (1969) version of the open economy DSGE model by adding financial frictions as modelled in Bernanke, Gertler and Gilchrist (1999). The main difference worth highlighting is that:

1. The core framework follows SW07 but essentially dropping an intertemporal Euler equation corresponding to household capital accumulation and adding three equations that characterize the financial frictions: a) the equation characterizing the contract selected by entrepreneur, b) the equation characterizing zero profits for the financial intermediaries and c) the law of motion of entrepreneurial net worth.

2. The set of stochastic shocks follows SW03 because in SW07 there is the so-called risk premium shock that represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households. However, in this model, the risk premium shock was not given a rigorous structural interpretation as it would have if it were specified as a feature of preferences shock as in SW03.

3. The wage and price setting follows Le et al (2011) proposed ‘a hybrid model\textsuperscript{11} assuming that wage and price setters find themselves supplying labour and intermediate output partly in a competitive market with price and wage flexibility, and partly in a market with imperfect competition.’

\section*{3.2.1 Households}

Each household \((j)\) chooses consumption, labour supply (hours) and savings to maximize a non-separable\textsuperscript{12} utility function with two arguments, consumption

\textsuperscript{11} In order to test against the original SW model, Meenagh et al (2009) assign a ‘New Classical’ model under the assumption of complete price and wage flexibility. Their results suggest that the observed demand shocks have too little persistence to capture the variability of real variables in the NK setting up, but they generate too much variability in nominal variables in the NC model. On the other hand, the observed supply shocks matter little for the NK but are about right in size and persistence for the real variables in the NC. The implication is that the flexibility of prices and wages may lie somewhere between New Keynesian and the NC models. They then proposed a hybrid model that a weighted average of the SWNK and SWNC with the weights respectively with certain fraction.

\textsuperscript{12} According to Merola (2014), the non-separable property of the utility function implies that consumption will also depend on expected employment growth. Therefore, when the inverse of elasticity of the intertemporal substitution is smaller than one \((\sigma_c, \sigma_l < 1)\), consumption and labour supply are complements.
and labour effort. The expected lifetime utility of a representative household is as follows

$$\max_{c_{t+s},L_{t+s}} \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}(j) - hC_{t+s-1})^{1-\sigma_c}}{1 - \sigma_c} \right) \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} L_{t+s}(j)^{1+\sigma_l} \right) \right\} (3.1)$$

where $\beta$ is the discount factor, $E_t$ is the rational expectation operator, $C_t(j)$ is consumption, $hC_{t+s-1}$ is set to capture the external aggregate consumption habit, $L_t(j)$ denotes the labour hours, $\sigma_c^{13}$ and $\sigma_l$ denote the inverse of the elasticity of inter-temporal substitution and labour supply, respectively. Each household $(j)$ faces the inter-temporal budget constraint of the form:

$$P_tC_t(j) + P_tL_t(j) + B_t(j) + S_tB^f_t(j) \leq \varepsilon^b R_{t-1}B_{t-1}(j) + R^f_{t-1}S_tB^f_{t-1}(j) + P_tY_t(j) \quad (3.2)$$

Households not only spend their total income $Y_t(j)$ on consumption but also hold their financial wealth in form of nominal domestic bonds $B_t(j)$, paying the risk-free gross nominal interest rate $R_t = (1 + r_t)$ in one period and nominal foreign bond $B^f_t(j)$ (denominated in foreign currency at $P^f_t$) paying the gross nominal foreign interest rate $R^f_t = (1 + r^f_t)$ in one period. $S_t$ is the nominal interest rate.

On the other hand, the households’ total income $Y_t(j)$ reads

$$P_tY_t(j) = W_t^h(j)L_t(j) + Div_t - T_t \quad (3.3)$$

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13 In general, according to SW07, only when $\sigma_c > 1$ (or elasticity of intertemporal substitution < 1), it implies that consumption and labour hours worked are complements in utility and consumption depends positively on current hours worked and negatively on expected growth in hours worked.
The household supplies perfectly elastic labour services $L_t(j)$, taking the form of nominal wage $W_t^h(j)$ as given. Also, $Div_t$ stands for the dividends from the entrepreneurs, and $T_t$ denotes lump sum tax transfer. Following Minford (2014), the foreign bond price is assumed to be cost at foreign consumption baskets $C_t^*$ at price $P_t^*$, the foreign CPI, or in terms of the domestic currency, $S_t P_t^*$. $\epsilon_t^b$ is a risk premium shock which represents a wedge between the policy rate controlled by the central bank and the interest rate faced by households. As such the risk premium shock was not given a rigorous structural interpretation as it would have if it were specified as a feature of preferences shock as in SW03 that explained by Chari et al. (2009). $\epsilon_t^b$ follows a AR(1) process with an IID Normal error term: 

$$\ln \epsilon_t^b = \rho_b \ln \epsilon_{t-1}^b + \eta_t^b.$$ 

Formally, the optimization problem can be summarized as:

$$\max_{(C_{t+s}, K_{t+s}, R_{t+s}B^f_{t+s})} E_0 \left\{ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}(j) - hC_{t+s-1})^{1-\sigma_c}}{1 - \sigma_c} \right) \exp \left( \frac{\sigma_c - 1}{1 + \sigma_c} L_{t+s}(j)^{1+\sigma_t} \right) \right\}$$

$$- \lambda_t [C_t(j) + I_t(j) + \frac{B_t(j)}{P_t} + \frac{S_t B_t^f(j)}{P_t} - \frac{\epsilon_t^b R_{t-1}}{P_t} - R_t \frac{S_t B_{t-1}^f(j)}{P_t}]$$

$$- \frac{W_t^h(j)}{P_t} L_t(j) + \frac{Div_t}{P_t} + \frac{T_t}{P_t}$$

The first order condition (FOC henceforth) by dropping $(j)$ therefore yields:

$$\partial C_t: \lambda_t = \left( \frac{\sigma_c - 1}{1 + \sigma_c} L_t^{1+\sigma_t} \right) (C_t - hC_{t-1})^{-\sigma_c}$$  \hspace{1cm} (3.4)

$$\partial L_t: \frac{\lambda_t W_t}{P_t} = \left[ \frac{1}{1-\sigma_c} (C_t - hC_{t-1})^{1-\sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1 + \sigma_c} L_t^{1+\sigma_t} \right) L_t^{\sigma_t}$$  \hspace{1cm} (3.5)

$$\partial B_t: \frac{\lambda_t}{\epsilon_t^b P_t} = E_t \left( \beta R_t \frac{\lambda_{t+1}}{P_{t+1}} \right)$$

$$=> E_t \left( \beta \frac{\lambda_{t+1}}{\lambda_t} R_t \frac{P_t}{P_{t+1}} \epsilon_t^b \right) = 1$$  \hspace{1cm} (3.6)
\[ \partial B_t^f : \lambda_t S_t = E_t \left( \beta R_t \frac{\lambda_t+1}{\lambda_t} S_{t+1} \right) \]

\[ = \Rightarrow E_t \left( \beta \frac{\lambda_t+1}{\lambda_t} R_t^f \frac{S_{t+1}P_t}{P_{t+1}} \right) = 1 \quad (3.7) \]

The optimality conditions with respect to consumption \((2.4)\) and domestic bond \((2.6)\) result in the consumption Euler equation:

\[ E_t \left( \beta \left( \frac{\sigma_c - 1}{1 + \sigma_t} L_{t+1}^{1+\sigma_t} \right) \left( C_{t+1} - h C_t \right)^{-\sigma_c} R_t \frac{P_t}{P_t - \varepsilon_t^b} \right) = 1 \quad (3.8) \]

The budget constraint is also equivalent\(^{14}\) to:

\[ C_t(j) + I_t(j) + b_t(j) + \frac{S_tP_t^*b_t^f(j)}{P_t} \leq \varepsilon_t^b R_{t-1} b_{t-1}(j) + R_t^f \frac{S_tP_t^*b_{t-1}^f(j)}{P_t} + Y_t(j) \quad (3.9) \]

where \(b_t(j), b_t^f(j)\) are real domestic bond and real foreign bond, respectively. It also assumes exports goods from domestic country have little impact on the rest of the world so that \(P_t^* \equiv P_t^f\), where \(P_t^f\) represents the foreign consumption goods price.

By replacing \(P_t^*\) with \(P_t^f\), the FOC (dropping \(j\)) also yields:

\[ \partial b_t : \lambda_t = E_t (\beta R_t \lambda_{t+1}) \]

\[ = \Rightarrow E_t \left( \beta \frac{\lambda_{t+1}}{\lambda_t} R_t \right) = 1 \quad (3.10) \]

\[ \partial b_t^f : \lambda_t S_t P_t^f = E_t \left( \beta R_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1} P_{t+1}^f \right) \]

\(^{14}\) The reason to express the domestic and foreign bond in real term is to derive the real exchange rate \(Q_t\) in the real uncovered interest parity equation.
\[ E_t \left( \beta \frac{\lambda_{t+1}}{\lambda_t} R_t^f \frac{S_{t+1}P_t}{S_t P_{t+1}} \frac{P_{t+1}^f}{P_t^f} \right) = 1 \] (3.11)

The optimality portfolio allocation between foreign and domestic bonds conditions for equation (3.10) and (3.11) implies optimal choice between real foreign and domestic bond can also yield the uncovered real interest parity condition (URIP):

\[
(1 + r_t) = E_t \frac{S_{t+1}P_t}{S_t P_{t+1}} \frac{P_{t+1}^f}{P_t^f} (1 + r_t^f) 
\] (3.12)

\[
=> (1 + r_t) = E_t \frac{Q_{t+1}}{Q_t} (1 + r_t^f) 
\] (3.13)

where \( Q_t = \frac{P_t^f}{P_t} S_t \) is equivalent to the real exchange rate that the foreign consumption goods price in domestic currency relative to the domestic general price level. In other words, the unit cost of the real foreign bond is \( Q_t \). This specifies that the returns on domestic and foreign bonds are equalised when measured in same currency. With the nominal exchange rate \( S_t \) can be fixed at unity, \( Q_t \) can also be treated as the import price relative to the domestic general price.

In the context of this small open economy model, one assumes complete financial markets at both domestic and foreign levels. We only consider effective returns on domestic bonds are affected by a time varying risk premium shock on bond holdings represented by \( \epsilon_t^b \). As it should be clear, the log linearized the URIP condition, \( r_t = E_t q_t - q_t + r_t^f \) is an exact equation. There is an implicit exogenous risk-premium shock in foreign bond holdings which is supposed to be constant. Similar assumptions can also be found in Minford (2016) and Meenagh et al (2013).
3.2.2 Foreign sector

According to Armington (1969), consumption bundle \( C_t \) are differentiated not only by inherent differences in their characteristics, but also by their place of production. However, for simplicity we assumed that goods do not enter in the production process but are only exchanged as final goods. The aggregate consumption as a bundle of domestic and foreign goods can be represented as a constant elasticity of substitution (CES) index given by:

\[
C_t = \left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-\frac{1}{\rho}}
\]  

(3.14a)

Preferences for the foreign countries (denoted with the asterisk) are defined in a similar fashion:

\[
C_t^* = \left[ \omega^f (C_t^d)^{-\rho^*} + (1 - \omega^f) \zeta_t^* (C_t^f)^{-\rho^*} \right]^{-\frac{1}{\rho^*}}
\]  

(3.15)

where \( C_t^d \) denotes the consumption of home goods, \( C_t^f \) denotes the consumption of imported goods, \( \omega^d \) denotes the weight of home goods in the consumption function, \( \omega^f \) denotes the foreign equivalents to home bias. The marginal substitution elasticity between home and foreign varieties of good is constant at \( \sigma = \frac{1}{1+\rho} > 0 \), \( \sigma^f = \frac{1}{1+\rho^*} > 0 \) respectively. \( \zeta_t, \zeta_t^* \) are preference errors.

The household chooses \( C_t^d \) and \( C_t^f \) to maximise equation (3.14) subject to:

\[
P_tC_t = P_t^d C_t^d + P_t^f C_t^f
\]  

(3.16)

where \( P_t^d \) is the price of domestically produced goods, \( P_t^f \) is the foreign price of imported goods in domestic currency and \( P_t \) is the aggregate CPI. The equation
can be alternatively expressed by numeraire $P_t$ to one unity:

$$C_t = p_t^d C_t^d + Q_t C_t^f$$  \hspace{1cm} (3.17)

where $p_t^d = \frac{p_t^d}{p_t}$ is the domestic price relative to the general CPI price level. As discussed above, $Q_t = \frac{p_t^f}{p_t} S_t$ is the real exchange rate that the foreign price in domestic currency relative to the domestic general CPI price level, with the nominal exchange rate $S_t$ can be fixed at unity.

The Lagrangian for the optimization problem then is:

$$L = \left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-1} \rho + \lambda (C_t - p_t^d C_t^d + Q_t C_t^f)$$  \hspace{1cm} (3.14a)

and the first order condition yields:

$$\partial C_t^d : \left[ \left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-1+\rho} \right]^{-1+\rho} \omega^d (C_t^d)^{-(1+\rho)} - \lambda p_t^d = 0$$

$$\Rightarrow \left[ \left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-1+\rho} \right]^{1+\rho} \omega^d (C_t^d)^{-(1+\rho)} = \lambda p_t^d$$  \hspace{1cm} (3.18a)

$$\partial C_t^f : \left[ \left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-1+\rho} \right]^{-1+\rho} \rho (1 - \omega^d) (C_t^f)^{-(1+\rho)} - \lambda Q_t$$

$$\Rightarrow \left[ \left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-1+\rho} \right]^{1+\rho} (1 - \omega^d) (C_t^f)^{-(1+\rho)} = \lambda Q_t$$  \hspace{1cm} (3.19a)

$$\partial C_t : \lambda = 1$$

By replacing $\left[ \omega^d (C_t^d)^{-\rho} + (1 - \omega^d) \zeta_t (C_t^f)^{-\rho} \right]^{-\rho}$ with $C_t$ Demands for domestically produced and imported consumption goods are rearranged to be given by:
\[ C_t^d = (\omega^d)\sigma (p_t^d)^{-\sigma} C_t \] (3.18b)

\[ C_t^f = ((1 - \omega^d)\xi_t)^\sigma (Q_t)^{-\sigma} C_t \] (3.19b)

The symmetric foreign demand for domestically produced goods relative to foreign consumption can also be specified:

\[ (C_t^d)^* = (\omega^f)^{\sigma_f} (p_t^d)^{-\sigma_f} C_t^* \] (3.20)

\[ C_t^{f*} = ((1 - \omega^f)\xi_t^*)^{\sigma_f} (Q_t^*)^{-\sigma_f} C_t^* \] (3.21)

The expression of the real exchange rate, \( Q_t \), can be obtained by plugging (3.18b) to (3.19b) back into (3.17):

\[ \omega^d (p_t^d)^{\rho\sigma} + (1 - \omega^d)\xi_t\sigma (Q_t)^{\rho\sigma} = 1 \] (3.22)

A simple transformation involves the linearization of (3.22) by means of a first-order Taylor series expansion around \( \rho = \sigma = 1 \) now reads:

\[ \ln p_t^d = -\left(\frac{1-\omega^d}{\omega^d}\right) \ln (Q_t) - \frac{1}{\rho} \left(\frac{1-\omega^d}{\omega^d}\right) \ln (\xi_t) + \text{constant} \] (3.23)

The import demand function from (3.19) reads:

\[ im_t = \ln C_t^f = \sigma \ln (1 - w) + \ln C_t - \sigma \ln Q_t + e_{im,t} \] (3.24)

where \( e_{im,t} = \sigma \ln \xi_t \). Since \( Q_t^* = \frac{p_t^f}{p_t^d} = \frac{p_t^d}{p_t^f} \), then \( \ln Q_t^* = \ln p_t^d - \ln Q_t \).

The export demand function from (3.20) reads:

\[ ex_t = \ln C_t^{d*} = \sigma^F \ln (1 - w^F) + \ln C_t^* + \sigma^F \frac{1}{w} \ln Q_t + \text{constant} + e_t^{ex} \] (3.25)

where \( e_t^{ex} = \sigma^F \ln \xi_t^* + \frac{1}{\rho} \sigma^F \frac{1-w}{w} \ln \xi_t \).
3.2.3 Capital producers

This subsection presents the setup of capital producers which determine the price of capital, which simplifies the optimisation problem of households compared to SW07. Capital producers operate in the competitive market and take prices as given. They choose the capital utilization rate that transforms physical capital into effective capital according to:

\[ K_t^z = z_t K_{t-1} \]

At the end of each period, they buy existing capital \( K_{t-1} \) from entrepreneurs and combine it with investment goods \( I_t \) to construct new capital \( K_t \), which is then sold to entrepreneurs. Following the set up in CEE (2005), capital goods producers are subject to quadratic investment adjustment costs specified as function \( S(I_t) \), and in steady state \( S = S' = 0, S''(.) > 0 \). With \( I_t \) purchased, they will produce \([1 - S(I_t)]I_t\) unit of investment goods. Adding the depreciated physical capital stock from the entrepreneurs, the new capital is given by:

\[ K_t(i) = (1 - \delta)K_{t-1}(i) + \epsilon_t^i \left[ 1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t(i) \quad (3.26) \]

where \( I_t(i) \) is investment, \( K_t(i) \) is capital holding, and \( \delta \in (0,1) \) is the rate of capital deprecation The function \( S(.) \) denotes the adjustment cost in investment and \( \delta \) denotes the depreciation rate. The capital production technology is also affected by an investment-specific shock \( \epsilon_t^i \) follows the stochastic process:

\[ \ln \epsilon_t^i = \rho_i \ln \epsilon_{t-1}^i + \eta_t^i. \]

The optimal problem of capital producer is to choose the level of investment and capital to maximize profits from the formation of new capital by dropping \( (i) \):

\[
\max_{I_{t+s},K_{t+s}} E_t \left[ \sum_{s=0}^{\infty} \beta^s \lambda_{t+s} M_{t+s} \right]
\]

\[
M_{t+s} = P_{t+s}^k \left[ (1 - \delta)K_{t+s-1} + \epsilon_t^i I_{t+s} \left( 1 - S\left(\frac{I_{t+s}}{I_{t+s-1}}\right) \right) \right]
- P_{t+s}^k (1 - \delta)K_{t+s-1} - I_{t+s}
\]
Subject to equation (3.26)

The profits $M_t$ are presented by the differences between the revenue from selling the capital $K_t$ at real price of capital $P_{t+1}^k$ and the costs of buying capital from entrepreneurs and investment $I_t$. Since the marginal rate of transformation from previously installed capital after depreciation to new capital is unity, the real price of new and used capital are the same.

The FOC to this optimization problem yields the following investment demand function:

$$1 = \epsilon_t P_t^k (1 - S \left( \frac{l_t}{l_{t-1}} \right) - S' \left( \frac{l_t}{l_{t-1}} \right)) - \beta E_t \left[ \frac{\lambda_t}{\lambda_t} P_{t+1}^k \epsilon_t^{t+1} S' \left( \frac{l_t}{l_{t-1}} \right) \left( \frac{l_t}{l_{t-1}} \right)^2 \right]$$ (3.27)

It relates the price of capital to investment adjustment cost and marginal adjustment cost. The presence of these two variables mitigates the response of investment to different shocks, which affects the price of capital.

### 3.2.4 Entrepreneurs (Intermediate goods producers)

The presence of financial friction alters the setup of intermediate goods producers compared to the SW07 model. The subsector describes the set-ups follow BGG by assuming that entrepreneurs act as the intermediate goods producer hire labour and purchase installed capital in a constant return to scale technology to produce the intermediate goods, and meanwhile use net worth and borrow funds from financial intermediaries to acquire the capital used in the production process. Entrepreneurs are risk neutral and face a constant probability of surviving to the next period. This ensures that the entrepreneurs’ net worth would never exceed the value of new capital acquisition.
An entrepreneur \((i)\) produces the intermediate good \(i\) according to the following production function:

\[
Y_t(i) = \varepsilon_t^a K_t^s(i)^\alpha (\gamma^t L_t(i))^{1-\alpha} - \gamma^t \Phi
\]  

(3.28)

where \(K_t^s(i)\) is capital services used in production, \(L_t(i)\) is aggregate labour input. The parameter \(\alpha\) captures the share of capital in production, \(\Phi\) denotes the fixed cost and \(\gamma^t\) represents the labour augmenting deterministic growth rate in the economy. \(\varepsilon_t^a\) is total factor productivity shock follows the ARIMA(1,1,0) process:

\[
\ln \varepsilon_t^a = \ln \varepsilon_{t-1}^a + \rho_a (\ln \varepsilon_{t-1}^a - \ln \varepsilon_{t-2}^a) + \eta_t^a
\]  

(3.29)

The entrepreneur firm\((i)\)’s profit is given by

\[
\sum_{s=0}^{\infty} \beta^s (P_{t+s}(i)Y_{t+s}(i) - W_{t+s}L_{t+s}(i) - R_{t+s}^k K_{t+s}(i))
\]  

(3.30)

it chooses optimal capital stock with \(R_k^k\), the nominal rental rate on effective capital and labour with \(W_t\), the aggregate nominal wage to minimise its cost. The solution to the cost-minimization problem determines subject to production function (3.28). The first order condition yields (dropping \((i)\)):

\[
\partial L_t(i): \quad W_t = MC_t \gamma^{(1-\alpha)t} (1 - \alpha) \varepsilon_t^a \left( \frac{L_t}{K_t^s(i)} \right)^{-\alpha}
\]  

(3.31)

\[
\partial K_t(i): \quad R_t^k = MC_t \gamma^{(1-\alpha)t} (\alpha \varepsilon_t^a) \left( \frac{L_t}{K_t^s(i)} \right)^{1-\alpha}
\]  

(3.32)

where \(MC_t\) is the marginal cost.

The above two equations can determine a capital to labour ratio across all producers:

\[
K_t^s = \frac{W_t}{R_t^k} \frac{\alpha}{1-\alpha} L_t
\]  

(3.34)

and the marginal cost for producing one extra unit is assumed to be the same for all firms and can also be derived as:
\[ MC_t = \frac{(R_t^k)^a (W_t)^{1-\alpha}}{\epsilon_t^a \alpha^a (1-\alpha)^{1-\alpha}} \]  

(3.35)

Equation (3.34) to (3.35) is identical to that in the SW07 economy.

Each entrepreneur also chooses the optimal capital utilization rate to solve the following maximizing problem:

\[ \max_{z_t(i)} R_t^k z_t K_{t-1} - \psi(z_t) K_{t-1} \]  

(3.36)

Where \( z_t \) is the real capital utilization rate, \( \psi(z_t(i)) \) denotes the cost of capital utilization per unit of physical capital\(^{15}\). The optimisation problem is presented by the following equilibrium condition:

\[ \partial z_t: R_t^k = \psi'(z_t) \]  

(3.37)

### 3.2.5 The External Finance Premium

Entrepreneurs then operating as the intermediate firms that buy the capital stock every period from the capital goods producers at price \( P_t^k \) determined by Tobin’s q, using both internal funds (that is, their own net worth) and external loans from the financial intermediaries. At the end of period \( t \), entrepreneur purchases new end-of-period stock of capital \( K_{t+1} \) (to use at time \( t+1 \)) from capital goods producers at the price \( P_t^k \). At time \( t+1 \), entrepreneur then receive the income from the marginal production of capital and gain from selling \((1 - \delta)K_{t+1}\) of capital to capital producer at price \( P_{t+1}^k \). On the other hand, at the end of period \( t \), each entrepreneur has a level of real net worth. The entrepreneurs combine their net worth, \( NW_{t+1} \) with a debt contract to purchase new, installed physical capital, \( K_{t+1} \), from the capital producer at \( P_t^k \). The amount of debt is \( P_t^k K_{t+1} - NW_{t+1} \).

\(^{15}\) In steady state, \( z = 1 \) and \( \psi(1) = 0 \).
In equilibrium, the optimal capital demand for entrepreneur is determined by the average expected marginal external financing cost at \( t + 1 \), by the given equation:

\[
E_t[CY_{t+1}] = E_t \left[ \frac{R_t^k + (1 - \delta)P_{t+1}^k}{P_t^k} \right]
\]  

(3.38)

where the average expected marginal external financing cost or the return on capital \( E_t[CY_{t+1}] \) should be equal to \( R_t^k \), the marginal productivity of capital or the rental rate of capital that determined by the capital producer; plus \( (1 - \delta)P_{t+1}^k \), the return to resell the undepreciated capital stock back to capital producers against the cost of acquiring the stock of capital \( P_t^k \) at \( t - 1 \). Equation (3.38) provides the linkage between the entrepreneur’s financial position and the cost of external funds, which in turn affects the demand for capital.

The marginal external financing cost \( CY_t \) to entrepreneurs depends on their financial conditions. According to BGG, basing on the costly state verification of Townsend (1979), due to asymmetry of information between entrepreneurs and financial intermediaries\(^{16}\), the agency problem makes the entrepreneurs external borrowing costs are more expensive than internal funds, and solve a financial contract that maximises the payoff to the firm subject to the lender earning the required rate of return. Following Townsend (1979), when the entrepreneurs costlessly observe their project’s ex-post return, external lenders incur an auditing cost to observe the realisation of project’s ex-post return. After observing its outcome, an entrepreneur decides whether to repay his debt or to default. If the entrepreneur pays in full, there is no need to verify the return, however if the entrepreneur defaults, external lenders then audit the loan and recover the outcome minus the monitoring costs. Accordingly, the gross return rate of capital is equal to a gross premium for external funds over the gross real opportunity costs, which

\(^{16}\) The costly state verification of Townsend (1979) arises from the standard information asymmetry problem where the borrower or entrepreneur has private information about their performance in contrast with the lender or bank which does not have any information. To obtain this information, the lender should pay a monitoring cost, which justifies an external finance premium for the borrower. On the other hand, the idiosyncratic shocks are only observed by entrepreneurs but not by financial intermediaries, hence lending involves agency costs. It reflects in a debt contract between these two parties.
is equivalent to the riskless real gross interest rate, \( R_t \frac{P_t}{P_{t+1}} \). According to zero profit condition for suppliers of funds, in this setting, the return rate of capital equates the premium for external funds over the real opportunity cost of investing in risk-free deposits:

\[
E_t[CY_{t+1}] = E_t \left[ EP_{t+1}(.)R_t \frac{P_t}{P_{t+1}} \right] \tag{3.39}
\]

with \( EP(.) < 0 \) and \( EP'(1) = 0 \)

The external finance premium, \( EP_t(.) \) indicates the intermediary would only lend to entrepreneurs if they can be compensated by the entrepreneurs’ default risk. The intermediary then would charge a premium over the cost of internal funds (risk free rate)\(^{17}\).

The external finance premium is then determined by the entrepreneurs’ leverage ratio, \( \frac{P_t K_{t+1}}{NW_{t+1}} \), and the elasticity with respect to the leverage ratio which depends on the structure of the financial contracts\(^ {18}\). Following Le et al (2012), the external finance premium also depends on an exogenous premium shock, \( \epsilon_{t}^{ep} \). that ‘can be thought of as a shock to the supply of credit: a change in the efficiency of the financial intermediary’s process, or a shock to the financial sector that alters the premium beyond what is dictated by the current economic and policy conditions.’

The external finance premium therefore is given by:

\[
E_t[EP_{t+1}(.)] = \epsilon_t^{ep} \left( \frac{P_t K_{t+1}}{NW_{t+1}} \right) \chi \tag{3.40}
\]

This shows the negative dependence of the premium on the amount of the net worth. Therefore, the higher the stake of entrepreneur in a project, the lower the premium would be required.

\[^{17}\text{Following Rannenberg (2013), the difference is also called the quasi profit margin since it does not account for the expected costs of bankruptcy that are borne by the entrepreneur through the loan rate agreed in debt contract.}\]

\[^{18}\text{The full derivation of the financial contract and the aggregation is shown in BGG, Appendix.}\]
After entrepreneurs have settled their debt to the lender in period $t + 1$, and the capital has been re-sold to capital producers, entrepreneurs’ net worth in period $t + 1$ is determined, which further affect the external finance premium through equation (3.40). The evolution of an entrepreneur’ net worth is defined as:

$$NW_{t+1} = \theta V_t$$

where $V_t$ denotes the value of entrepreneur’s equity. The probability that an entrepreneur will survive until the next period is denoted $\theta$. In other words, entrepreneur’s expected lifetime is $1/(1 - \theta)$. When an entrepreneur died from the economy with a probability of $1 - \theta$ during the current period, he will transfer his remaining value $(1 - \theta)V_t$ to new entering entrepreneurs. Following CMR and Christen and Dib (2008), with the assumption that there always exits entrepreneurs died from the economy, it ensures that entrepreneurs’ net worth do not accumulate enough to fully finance the new capital acquisition so that the entrepreneur has to go to the capital market to borrow funds prior to purchasing capital. Additional, the size of the entrepreneurial sector is constant with sufficient numbers of new arrivals replacing departed entrepreneurs.

The net worth of the entrepreneurs who survive is equal to the ex-post gross return on capital investment, $CY_t P^k_{t-1} K_t$ minus the cost of borrowing $E_t [CY_t (P^k_{t-1} K_t - NW_t)]$ at $t - 1$. Then, the aggregate entrepreneurial net worth evolves according to the following law of motion:

$$NW_{t+1} = \theta V_t = \varepsilon_{tw}^{nw} \theta (CY_t P^k_{t-1} K_t - E_{t-1} [CY_t (P^k_{t-1} K_t - NW_t)])$$

(3.42)

where $\varepsilon_{tw}$ represents a shock to the entrepreneurial equity value and follows the autoregressive process:

$$\ln \varepsilon_{t}^{nw} = \rho_{nw} \ln \varepsilon_{t-1}^{nw} + \eta_{t}^{nw}$$

(3.43)

---

In BGG setups, work hours provided by both households and entrepreneurs, therefore the evolution of an entrepreneur’ net worth is defined as: $NW_{t+1} = \varepsilon_{t}^{nw} \theta V_t + W^e$, where $W^e$ is the wage income received by new entrepreneurs. Based on the empirical evidence, the estimated value of $W^e$ is relatively small (BGG shows value equals to 0.01, Rannenberg (2013) shows the value equals to 0.008), in my model, I drop the $W^e$ from the equation line with Le et al. (2013).
Therefore, earning at time $t$ with a survival rate $\theta$ then becomes the net worth at next period $t + 1$. As in Gertler et al. (2007), equation (3.42) suggest unpredictable variation of $P_t^k$, play a key role since such variation provides the principle source of fluctuation in the equation (3.38). Suppose there is a decline in asset prices, this would deteriorate the borrowers’ balance sheet by decreasing the net worth leading to an increase in the external finance premium, and hence raise external financing cost. The increase in external financing cost, in turn, reduces the demand for capital and leads to further cuts in investment and output. The resulting slowdown in economic activity causes asset prices to fall further and deepens the economic downturn. This is the financial accelerator channel highlighted by BGG, and it tends to amplify the economic effects of any shock that has a pro-cyclical impact on economic activity. It also should be noted that the demand for capital of entrepreneur is determined by the return rate on capital from equation (3.39), the rental rate of capital from equation (3.32) and the dynamics of net worth from equation (3.42).

Entrepreneurial who close business at period $t$ consume their remaining resources. The amount of the consumption is given by:

$$C_t^e = (1 - \theta)V_t$$

(3.44)

Hence, the total amount of $(1 - \theta)V_t$ of equity from exiting entrepreneurs should remove from the market.

### 3.2.6 Final goods producer

The final good producer purchases intermediate goods $Y_t(i)$, aggregate them into a composite final good $Y_t$ and sold to consumers in a perfectly competitive market. The final good is produced according to the following Dixit-Stiglitz aggregator:

$$Y_t = \left( \int_0^1 Y_t(i) \frac{1}{1 + \lambda_{p,t}} dt \right)^{1 + \lambda_{p,t}}$$

(3.45)
Where $Y_t(i)$ denotes the goods used in the final goods production. $\lambda_{p,t}$ is the desired mark-up of prices over marginal costs at the intermediate goods level that follows the exogenous stochastic AR (1) process:

$$\ln(\lambda_{p,t}) = \rho_p \ln(\lambda_{p,t-1}) + \eta^p_t$$

(3.46)

Final goods producer maximises its profit as:

$$\max Y_tP_t - \int_0^1 Y_t(i) P(i)di$$

(3.47)

subject to equation (3.45).

The demand function of the intermediate goods then reads:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{1+\lambda_{p,t}} \lambda_{p,t} Y_t$$

(3.48)

Integrating the above equation and imposing the final goods production function, it shows the price of the final good, $P_t$ is a CES aggregate of the prices of the intermediate goods, $P_t(i)$

$$P_t = \left(\int_0^1 P_t(i) \frac{1}{\lambda_{p,t}} \lambda_{p,t} \right)^{1+\lambda_{p,t}}$$

(3.49)

Additionally, I follow Le et al (2011) assuming that final outputs are made up in a fixed fraction $v^p$ of intermediate goods $Y_t(i)$ from a monopoly market and others $Y_t(i)^{NC}$ with a fraction of $(1 - v^p)$ are from perfectly competitive market.

The hybrid final goods output therefore takes the form:

$$Y_{t}^{hybrid} = v_p \left(\int_0^1 Y_t(i) \frac{1}{\lambda_{p,t}} \lambda_{p,t} \right)^{1+\lambda_{p,t}} + (1 - v^p) \int_0^1 Y_t(i)^{NC} \, di$$

(3.50)

According to Calvo (1983), each period only a fraction of $(1 - \xi^p)$ entrepreneurs are allowed to re-optimize prices $P_t(i) = \tilde{P}_t(i)$. The remaining entrepreneurs with
a fraction of $\xi_p$ can only reset the price follow an adjustment mechanism with partial indexation. Non-re-optimized prices are partially indexed to past inflation, which gives rise to the backward-looking term adjusted according to the following indexation rule given by:

$$P_t(i) = \left(\frac{\pi_{t-1}}{\pi}\right)^{l_p} P_{t-1}(i)$$

(3.51)

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate and $\pi$ is the steady state inflation value. $l_p = 1$ means perfect indexation and $l_p = 0$ means no indexation. The optimisation problem for setting a new nominal price $\tilde{P}_t(i)$ is to maximize the expected discounted stream of future firm’s profits for all states of nature.

$$\max E_t \left[ \sum_{s=0}^{\infty} \beta^s \xi_p^{l_t} \xi_p s Y_{t+s}(i) (\tilde{P}_t(i)(\Pi_t) - MC_{t+s}) \right]$$

(3.52)

subject to intermediate goods demand function (3.49)

where $\Pi_{t,t+s} = \Pi_{k=1} s \left(\frac{\pi_{t+k-1}}{\pi^*}\right)^{l_p} \lambda_t$ is Lagrange multiplier associated with the budget constraint (equation (3.1) and (3.2)) in household optimization problem.

The first order condition with respect to $\tilde{P}_t(i)$ is then given by

$$E_t \left[ \sum_{s=0}^{\infty} \beta^s \xi_p^{l_t} \xi_p s \tilde{Y}_{t+s} \tilde{P}_t \Pi_{t,t+s} (1 + \lambda_{p,t+s}) - MC_{t+s} \right] = 0$$

(3.53)

where $\tilde{Y}_{t+s}$ is the demand in $t+s$ with the chosen optimal price $\tilde{P}_t$. The aggregate price index for intermediate goods $Y_t(i)$ sold in an imperfectly competitive market is then given by:

$$P_t = \left[ \xi_p (P_{t-1}(i) \left(\frac{\pi_{t-1}}{\pi_t}\right)^{l_p} + \left(1 - \xi_p\right)(\tilde{P}_t(i))^{l_p} \right]^{\lambda_{p,t}}$$

(3.54)

On the other hand, the aggregate price index for intermediate goods $Y_t(i)^{NC}$ sold in a perfectly competitive market is then derived when prices are flexible and the
price-mark-up shock is zero. It reduces to the condition that the price mark-up is constant:

\[ P_t^{NC} = MC_t \] (3.55)

Therefore, in the hybrid model the aggregate price equation is assumed to be a weighted average of the corresponding NK and NC equations as follows:

\[ P_t^{hybrid} = w^r P_t + (1 - w^r) P_t^{NC} \] (3.56)

### 3.2.7 Labour unions and labour packers

As in SW03 and SW07, the labour markets consist of labour unions, which allocate and differentiate homogenous labour supplied by households; and labour packers, who buy labour from the unions, package it into a Kimball (1995) composite aggregator and resell to entrepreneurs:

\[ L_t = \left( \int_0^1 L_t(i) \frac{1}{\frac{1}{1+\lambda_{w,t}} d i} \right)^{1+\lambda_{w,t}} \] (3.57)

where \( \lambda_{w,t} \) is the mark-up of real wages over the ratio of marginal disutility of labour to the marginal utility of consumption in a flexible economy which follows a AR(1) process:

\[ \ln(\lambda_{w,t}) = (1 - \rho_w) \ln(\lambda_w) + \rho_w \ln(\lambda_{w,t-1}) + \eta_t^P \] (3.58)

The representative labour aggregate combines household’s labour in the same proportion as entrepreneurs would choose, which ensures that its demand for household labour is the same as the sum of the firm’s demand for this type of labour.

The labour packer then minimises the cost by choose the optimal amount of labour services.

\[ \min L_t L(i) \int_0^1 W_t(i) L_t(i) d i \] (3.59)
subject to equation (3.58)

It then leads to the labour demand from FOC:

$$L_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t$$ (3.60)

and integrating the above equation can express the aggregate wage rate, $W_t$ that related to the individual wage $W_t(j)$:

$$W_t = \left( \int_0^1 W_t(i) \, di \right)^{\lambda_{w,t}}$$ (3.61)

Unions have market power over labour services and set wages that are subject to Calvo scheme which is similar to the price setup. Every period only $1 - \xi_w$ fraction of intermediate labour unions can optimally re-adjust wages, and $\xi_w$ fraction cannot. For those who cannot optimise their wages, the current wages are adjusted by Calvo pricing with partial indexation. The optimal wage rate set by the union maximizes the stream of future discounted wage incomes for all the time periods when the union is stuck with that wage in the future.

$$\max_{\tilde{W}_{t+s}(i)} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \lambda_{t+s} \beta^s \xi_w^s \tilde{L}_{t+s}(i) \left( \tilde{W}_{t+s}(i) \Pi_{t,t+s}^w - W_{t+s} \right) \right]$$ (3.62)

with $\Pi_{t,t+s}^w = \Pi_{k=1}^\xi Y \left( \frac{\pi_{t+k}}{\tilde{\pi}} \right)^{l_w}$

subject to the labour demand equation (3.60)\textsuperscript{20}. The FOC yields:

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \frac{\lambda_{t+s} P_t}{\lambda_t P_{t+s}} \beta^s \xi_w \tilde{L}_{t+s} \frac{1}{\lambda_{w,t+s}} \left( (1 + \lambda_{w,t+s}) W_{t+s} - \tilde{W}_t(i) \Pi_{t,t+s}^w \right) \right] = 0$$

\textsuperscript{20} Following the indexation scheme, $\tilde{L}_{t+s}(i) = \left[ \frac{W_{t+s}(i)}{W_{t+s}} \left( \frac{\pi_{t+s}}{\pi} \right)^{l_w} \right]^{-\lambda_{w,t+s}}$
The law of motion of the aggregate wage result in the following wage equation evolves:

\[ W_t = \left[ \xi_w \left( W_{t-1} \left( \frac{\pi_{t-1}}{\pi_t} \right)^{\lambda_w} \right) \right]^{1+\lambda_{w,t}} + (1 - \xi_w) \left( \overline{W}_t \right) \right]^{1/\lambda_{w,t}} \]  

(3.64)

Similar to hybrid price setting, Le et al (2012) assume that firms producing intermediate goods have a production function that combines in a fixed fraction \( w_w \) of labour in imperfect competition as well as other \( (1 - w_w) \) labour in competitive markets. Therefore, the labour used by intermediate firms becomes:

\[ L_t = v^w \left( \int_0^1 \frac{1}{L_t^{NK}(i)} \frac{1}{i^{1+\lambda_{w,t}}} \right)^{1+\lambda_{w,t}} + (1 - v^w) \int_0^1 L_t^{NC}(i) \, di \]  

(3.65)

so that

\[ W_t^{hybrid} = w^w W_t + (1 - w^w) W_t^{NC} \]  

(3.66)

where \( W_t^{hybrid} \) is proposed overall hybrid wage, \( W_t \) is set according to equation (3.64). If wages are perfectly flexible and mark up equates zero, the real wage \( W_t^{NC} \) is then equals to the marginal rate of substitution between consumption (equation 3.4) and leisure (equation 3.5).

### 3.2.8 Government policy

The fiscal authority is also set following SW07: government spending \( G_t \) and transfers to the households are fully financed by lump sum taxes, so that the government’s budget is balanced each period. \( G_t \) is assumed to have no direct effect on the utility of households. The government budget constraint is given by:

\[ P_t G_t + R_{t-1} B_t = T_t + B_{t+1} \]  

(3.67)
The monetary sector in this model also follows the original SW07 setup. It is operated under a monetary policy rule that specifies how the central bank reacts to deviations of inflation and output from steady state when it decides about policy interest rate.

\[
\frac{R_t}{\bar{R}} = \varepsilon_t^R \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho} \left[ \left( \frac{\pi_t}{\pi} \right)^{r_p} \left( \frac{Y_t}{Y^*_t} \right)^{r_y} \right]^{1-\rho} \left( \frac{Y_t}{Y_{t-1}} \overline{Y^*_t} \right)^{r_{\Delta y}}
\]

(3.68)

where \( \rho \) denotes the degree of interest rate smoothing, \( r_p, r_y \) and \( r_{\Delta y} \) determine the response to inflation, output and output change respectively. \( \bar{R} \) is the steady state value of gross nominal interest rate. \( \bar{\pi} \) is the steady state value of inflation. \( Y^*_t \) is the optional output. The monetary shock \( \varepsilon_t^R \) follows a AR(1) process:

\[
\ln \varepsilon_t^R = \rho_t \ln \varepsilon_{t-1}^R + \eta_t^R
\]

(3.69)

### 3.2.9 Net foreign assets

Financial intermediaries sell domestic bonds to households and lend to entrepreneurs. All the financial intermediaries are assumed to operate at perfectly competitive market. Under the zero-profit assumption, in equilibrium the intermediaries lend all the funds obtained from households to entrepreneurs. Regarding to foreign Bond Market, the evolution of net foreign assets position can be derived in the following way.

\[
S_t P_t^f b_t^f = R_{t-1}^f S_t P_{t-1}^f b_{t-1}^f + NX_t
\]

(3.70)

where \( NX_t \) is nominal domestic net exports in domestic currency. It can also be defined as the difference of nominal exports and nominal imports:

\[
NX_t = S_t \left( P_t^d \right)^* \left( C_t^d \right)^* - S_t P_t^f C_t^f
\]

(3.71)
Since the price of exports in domestic currency is given by \( S_t \left( P_t^d \right)^* = S_t \frac{P_t^d}{S_t} = P_t^d \),
the price of imports and foreign bonds in domestic currency is given by 
\( S_t \frac{P_t^f}{P_t} = Q_t P_t \)

The evolution of net foreign assets over GDP can be expressed by:

\[
Q_t P_t b_t^f = Q_t P_t R_{t-1}^f b_{t-1}^f + P_t^d (C_t^d)^* - Q_t P_t C_t^f
\]

\[
\Rightarrow b_t^f = R_{t-1}^f b_{t-1}^f + \frac{P_t^d}{Q_t P_t} (C_t^d)^* - C_t^f
\]

(3.72)

In addition, when defining the net foreign assets position \( b_t^f \) as a ratio of real foreign bonds value over the real GDP:

\[
\hat{b}_t^f = \frac{b_t^f}{Y_t}
\]

(3.73)

and expressing \((C_t^d)^*\) and \(C_t^f\) by using \(EX_t\) and \(IM_t\) respectively, it reads:

\[
\hat{b}_t^f = R_{t-1}^f a_{t-1}^f + \frac{P_t^d}{Q_t Y_t} EX_t - \frac{IM_t}{Y_t}
\]

\[
\Rightarrow \hat{b}_t^f = \left( 1 + r_{t-1}^f \right) \hat{b}_{t-1}^f + \frac{P_t^d}{Q_t Y_t} EX_t - \frac{IM_t}{Y_t}
\]

(3.74)

where \(R_{t-1}^f = \left( 1 + r_{t-1}^f \right)\) and \(P_t^d = \frac{P_t^d}{P_t}\).

In addition, according to Meenagh et al (2010) and Minford (2015), in order to ensure a balance growth equilibrium is reached, it requires the government will not run a trade surplus/deficit (in other words, lend to/borrow from abroad) forever. In some terminal time \( T \to \infty \) and real exchange rate \( Q_T \) is constant, the change in net foreign assets position as well as the ratio of net foreign assets position to GDP must equal to zero:

\[
\Delta \hat{b}_T^f = \Delta \left( \frac{b_T^f}{Y_T} \right) = 0
\]

(3.75)
3.2.10 Resource constraint

The resource constraint on final goods market can be obtained by combining household and government budget constraint and evolution of net foreign assets position with the zero-profit condition of the final goods producers and the employment agencies. The economy’s aggregate resource constraint reads:

\[ Y_t = C_t + I_t + G_t + \psi(z_t)K_{t-1} + C_t^e + EX_t - IM_t \]  

(3.76)

Since the exogenous government spending is set to be:

\[ G_t = \left(1 - \frac{1}{g_t}\right)Y_t \]  

(3.77)

where \( g_t \) is the follows a AR(1) process:

\[ \ln g_t = \rho_g \ln g_{t-1} + \eta^g_t \]  

(3.78)

The model is closed by imposing the following resource constraint finally reads:

\[ Y_t = (C_t + I_t + \psi(z_t)K_{t-1} + C_t^e + EX_t - IM_t)g_t \]  

(3.79)

3.2.11 Model without external finance premium

In order to be able to evaluate the importance of financial frictions, it is also considered to compare the results of the model with financial accelerator mechanism to the alternative specification of the model when accelerator mechanism switched off. The model then will assume that capital producer is owned by household, as in SW07.

The entrepreneurs can always obtain funds from household at cost of \( R_t \), and the leverage ratio then will no longer influence the external finance premium and so the variable of entrepreneurial net worth is not needed in the model anymore. Furthermore, it introduces a capital stock accumulation decision directly in the household’s intertemporal optimization problem. This modification implies that
the Tobin’s Q defined in the simple model:\(^{21}\):

\[
P_t^k = E_t \left[ \beta \frac{\lambda_{t+1}^k}{\lambda_t^k} (P_{t+1}^k (1 - \delta) + R_{t+1}^k z_t K_{t-1} - \psi(z_t)K_{t-1}) \right] \quad (3.80)
\]

or in log-linearized form:

\[
P_t^k = \frac{(1 - \delta)}{1 - \delta + R_t^k} E_t P_{t+1}^k + \frac{R_t^k}{1 - \delta + R_t^k} E_t r_{t+1}^k - (r_t - E_t \pi_{t+1}) \quad (3.81)
\]

This satisfies the standard equality condition of equation when external finance premium \(E P_{t+1}(.) = 0\):

\[
E_t C Y_{t+1} = E_t \left[ R_t \frac{P_t}{P_{t+1}} \right] \quad (3.82)
\]

Or in log-linearized form:

\[
E_t c y_{t+1} = r_t - E_t \pi_{t+1} \quad (3.83)
\]

### 3.3 Calibration

In this section, we confront the model described in Section 3.2 with UK data. The model is calibrated to UK data over the period from 1975Q1 to 2015 Q4 at a quarterly frequency. All data sources are described in appendix, Table 3.2. It should be noted that this exercise is to provide some intuition for the results, and guides the choice of several key ‘structural shocks’ that are incorporated in the model.

Before evaluating the log-linearized model, I first start with a set of structure parameters according to consensus values commonly used in the literature. There are two groups of parameters in calibration. The first group of parameters are important in determining the steady state of the model. For example, preference bias for the domestic and foreign produced goods or steady state inflation and

---

\(^{21}\) This expression of Tobin’s Q is identical to SW07 that defined as the ratio of two Lagrange multiplier associated with budget constraint and capital accumulation equation, respectively.
output growth. I use calibration values either directly from that study of UK data or used to matched the steady-state of our model in the reference. The second group of parameters govern the dynamics of the model and there is a general consensus for estimates of some parameters. For example, the coefficients of the monetary policy rule or parameters related to price and wage stickiness. I then make use of a range of estimates for the United States and the euro area using models with very similar structures to our own. Then these unconditional structural parameters of the model of interest are re-evaluated and re-estimated by Indirect Inference estimate by matching the properties of empirical data and simulated data using auxiliary model in Chapter 4.

Since model period corresponds to a quarter so the discount factor $\beta$ is set at 0.998 corresponding to a steady state annualized real interest rate of 4%\(^{22}\), as in the data. The quarterly capital depreciation rate $\delta$ is set equal to 0.025 following in SW07 to produce a 10% annual depreciation rate and the share of capital in the production function is set at 0.3. Share of fixed costs in production $(\Phi - 1)$ equals to 0.5. The degree of habit formation in consumption $h$ equals 0.7 indicated by SW07 and Adolfson et al. (2007). The value of the intertemporal elasticity of substitution $\sigma_c$ and elasticity of labour supply $\sigma_l$ are within the range of values indicated by SW03 for Euro area, equal 1.39 and 2.83 respectively\(^{23}\). The elasticity of capital adjustment cost is set equal to 5.47 and the elasticity of capital utilisation cost equals to 0.54, in line with Le et al (2012). Following SW07, the probability of a retailer being unable to re-optimize its price equals 0.67. This implies that the average duration of retail price for a certain variety is three quarters (i.e. $\frac{1}{1-\xi_p} = 3$), whilst setting a degree of inflation indexation follows the estimation results of SW equals to 0.43. The degree of wage stickiness equals to 0.70 ($\frac{1}{1-\xi_p} = 3.33$) and the degree of wage index indexation equals to 0.58.

\(^{22}\) $\bar{R} = \frac{1}{\beta} = 1.0101$, it is equivalent to 4% annually.

\(^{23}\) The inverse of intertemporal elasticity of substitution $\sigma_c$ is set to 0.72 in SW07 for US and 0.74 in SW03 for Euro area. This indicates $\sigma_c$ is set at $0.72 = 1.35$ for US and $0.74 = 1.39$ for EU. On the other hand, the $\sigma_l$ is $\frac{1}{0.54} = 1.83$ for US and $\frac{1}{0.842} = 2.38$ for EU. Here I take the starting calibration value for EU.
Proportion of sticky wages and proportion of sticky prices are set equal to 0.10 and 0.40 respectively, in line with the empirical evidence of Le et al (2012).

In the extension to the financial section setup, the survival rate of entrepreneurs $\theta$ is set equal to 0.99, in line with Le et al (2012). This implies the average duration of entrepreneurs is more than 6 years (i.e. 25 quarters: $\frac{1}{1-\theta} = 25$). This target is taken from Bernanke et al. (1999), which is close to the estimation after Christiano et al. (2010). Elasticity of the premium with respect to leverage then equals to 0.04.

The parameters pertaining to the foreign sector is set mainly relying on the empirical evidence of Minford (1984), Meenagh et al. (2010, 2012) and Minford (2015) based on UK data. Preference bias for the domestic, $\omega$ and foreign produced goods, $\omega^f$, are both set at 0.7. $\sigma$, the elasticity of marginal substitution between the domestic consumption bundle and the imported variety of goods is set equal to 1 assuming that the UK’s products compete but not sensitively with foreign alternatives, whereas the equivalent substitution elasticity in the foreign country $\sigma^f$ equals to 0.7. In the monetary policy rule, I assume the conventional coefficients of inflation, persistence and output gap are 1, 0.8 and 0.11, respectively.

The models are also calibrated to hit certain real and financial ratios based on empirical data for the UK. The real part of the economy is governed mainly by four parameters, which pin down four steady state proportions. The quarterly steady state inflation based on average value equals to 1.29, and quarterly steady-state output growth is 0.55. It is assumed to set to achieve a steady state government spending to output ratio of 0.20. The steady state values of components of output therefore are calculated as: the steady state value of the investment-to-output ratio, $\frac{I}{Y}$, equals to 0.18, the consumption-to-output ratio, $\frac{C}{Y}$ equals to 0.58, $\frac{EX}{Y}$ equals to 0.24 and $\frac{IM}{Y}$ equals to 0.25, respectively.
According to SW07 and de Walque et al. (2006), the curvature of Kimball (1995) aggregator $\epsilon_p / \epsilon_w$ is defined as the elasticity of the price/wage elasticity of demand with respect to relative price/wage at steady-state. Goods Market and labour market curvature of the Kimball (1995) Aggregator are all setting equal to 10 as in SW07.
Table 3.1: Structural parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fixed</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.998</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>1.39</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$h$</td>
<td>0.70</td>
<td>degree of External habit formation</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.70</td>
<td>degree of wage stickiness</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>1.83</td>
<td>Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\omega^w$</td>
<td>0.10</td>
<td>Proportion of sticky wages</td>
</tr>
<tr>
<td>$\tau_w$</td>
<td>0.58</td>
<td>Degree of wage indexation</td>
</tr>
<tr>
<td><strong>Producers</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>0.24</td>
<td>Degree of price indexation</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.54</td>
<td>Elasticity of capital utilization</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.50</td>
<td>1+Share of fixed costs in production</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>5.74</td>
<td>Steady state elasticity of capital adjustment</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Share of capital in production</td>
</tr>
<tr>
<td>$\omega^r$</td>
<td>0.40</td>
<td>Proportion of sticky prices</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.75</td>
<td>Degree of price stickiness</td>
</tr>
<tr>
<td><strong>Taylor rule</strong></td>
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<td></td>
</tr>
<tr>
<td>$r_p$</td>
<td>2.50</td>
<td>Response to inflation</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.60</td>
<td>Interest rate smoothing</td>
</tr>
<tr>
<td>$r_y$</td>
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<td>Response to output</td>
</tr>
<tr>
<td>$r_{\Delta y}$</td>
<td>0.22</td>
<td>Response to output change</td>
</tr>
<tr>
<td><strong>Financial frictions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>0.04</td>
<td>Elasticity of the premium with respect to leverage</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.99</td>
<td>Survival rate of entrepreneurs</td>
</tr>
</tbody>
</table>
Table 2 Steady state values in model economy

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^*_k$</td>
<td>0.04</td>
<td>Return rate of capital</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>0.55</td>
<td>Quarterly output growth</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>1.29</td>
<td>Quarterly inflation</td>
</tr>
<tr>
<td>$\frac{G}{\bar{Y}}$</td>
<td>0.20</td>
<td>Government spending to GDP ratio</td>
</tr>
<tr>
<td>$\frac{C}{\bar{Y}}$</td>
<td>0.58</td>
<td>Consumption to GDP ratio</td>
</tr>
<tr>
<td>$\frac{I}{\bar{Y}}$</td>
<td>0.18</td>
<td>Investment to GDP ratio</td>
</tr>
<tr>
<td>$\frac{EX}{\bar{Y}}$</td>
<td>0.24</td>
<td>Export to GDP ratio</td>
</tr>
<tr>
<td>$\frac{IM}{\bar{Y}}$</td>
<td>0.25</td>
<td>Import to GDP ratio</td>
</tr>
<tr>
<td>$\frac{C^e}{\bar{Y}}$</td>
<td>0.008</td>
<td>Net worth to GDP ratio</td>
</tr>
<tr>
<td>$e_p$</td>
<td>10</td>
<td>Goods market curvature of the Kimball aggregator</td>
</tr>
<tr>
<td>$e_w$</td>
<td>10</td>
<td>Labour market curvature of the Kimball aggregator</td>
</tr>
</tbody>
</table>
3.4 Impulse response functions for structural shocks

This section briefly examines the impulse response\(^{24}\) of the model economy to a set of shocks. The impulse responses of macroeconomic and financial variables to a 10 percent rise in a variety of structural shocks are plotted in Figure 3.1 through Figure 3.5 of Appendix. A 10 percent rise in a variable is denoted as 0.1 on the y-axis and the number of quarters elapsed since the shock begins are indicated on the x-axis. The solid line presents the full SWBGG model and the dash-circle line presents the SW model when external finance premium switched off.

The IRFs are built using calibrated coefficients and corresponding parameters of shock processes over the sample. The analysis therefore only helps to assess the validity of the model and highlights and helps to understand the key differences in the amplification and propagation mechanisms embedded in various setups. A detailed discussion would be provided in Chapter 4 where the IRFs then would be constructed base on re-estimated coefficients.

Figure 3.1 depicts the impulse response to a positive 10% non-stationary productivity shock. A non-stationary productivity shock has permeant impact on real variables including output, consumption investment etc., and leads to a decrease in prices due to the expansion in aggregate supply. Investment and consumption also increase due to the expansion in output. Since the monetary policy is operating, the nominal interest rate decreases as shown in Figure. In the SWBGG model there is a decrease of the external finance premium and hence there is a dampening of the investment response presented compared to that in the SW model. Figure 3.2 shows the impulse response to a positive 10% contractionary monetary policy shock. As seen in figure, through the standard transmission mechanism, nominal interest rises with output, consumption,

\(^{24}\) The plotted IRFs are calculated from the differences between the base run and simulated results after a one-off shock in the first simulation period. The base run results is the solution without any shocks so that it replicates the original data set.
investment, labour hours fall dramatically on impact. With the financial friction switching on, the transmission mechanism of the policy shock is enhanced. The mechanism is evident. When the shock hits the economy, since the net worth falls due to the declining return to capital rate, price of capital decline further in the SWBGG model, the external finance premium then increases, which reinforces a further contraction in capital and investment. Figure 3.3 plots the impulse response to a positive 10% investment specific shock. It is a demand shock that increases investment, while decreases the Tobin’s Q. Capital stock and the aggregate output increase and because investment grows. With the presence of financial frictions, it attenuates the fall in investment and output. The investment specific shock gives rise to a pro-cyclical external premium. Figure 3.4 plots the impulse response to a positive 10% external finance premium shock. A positive external finance premium shock increases the cost of funds borrowing. This should cause a decrease in capital stock and investment as shown in figure. Net worth declines due to the increase in external finance premium. The role of other shocks in the cyclicality in the premium can also be inferred from related studies (Christensen and Dib, 2008, De Graeve 2008, le et al., 2013, Villa, 2013, Cristina 2016, etc.).

3.5 Conclusion

The aim of this research is to discover how far the banking crises have been caused by financial shocks on the UK economy. To this start, this chapter lays out a new Keynesian DSGE model economy with the addition of the BGG financial accelerator mechanism, and small open economy setup to make the analysis more relevant to the UK. Dynamic properties also have been illustrated through the impulse response functions from a one-off policy shock. By using an alternative simple model when the financial friction setup is switched off as a comparison, in particular, we see the amplification response of investment to monetary policy shock and attenuation response to productivity and investment supply shock in the economy when the financial friction has been considered. However, this conclusion from the IRFs analysis may not be highly reliable empirically because
the calibration from other studies mainly based on US and EA data, and therefore it is worthy of further interrogation, to discover what it implies about the sources and nature of the crisis seriously. This would be done in empirical work in the following Chapter 4 for further evaluation and re-estimation.
## Appendix 3.A Data source and Figures

### Table 3.2 Data source, definition and derivation

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Source, (Code)</th>
<th>Definition and Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal interest rate</td>
<td>$R$</td>
<td>BoE, (IUQAAJNB)</td>
<td>Quarterly 3 month average sterling T-bill / 4</td>
</tr>
<tr>
<td>Output</td>
<td>$Y$</td>
<td>ONS, (ABMI)</td>
<td>Gross domestic product, SA,CP</td>
</tr>
<tr>
<td>Consumption</td>
<td>$C$</td>
<td>ONS, (ABJR)</td>
<td>Household final consumption expenditure, SA,CP</td>
</tr>
<tr>
<td>Investment</td>
<td>$I$</td>
<td>ONS, (NPQT+CPAU)</td>
<td>Total fixed capital formation + Changes in inventories, SA,CP</td>
</tr>
<tr>
<td>Price level</td>
<td>$\pi$</td>
<td>ONS, (CGBV)</td>
<td>Percentage change in GDP deflator, Quarterly</td>
</tr>
<tr>
<td>Labour hours</td>
<td>$L$</td>
<td>ONS, (MGRZ/YBUS)</td>
<td>Employment/Total actual weekly hours worked</td>
</tr>
<tr>
<td>Capital</td>
<td>$K$</td>
<td>N/A</td>
<td>Derived from investment Euler equation</td>
</tr>
<tr>
<td>Price of Capital</td>
<td>$p^k$</td>
<td>N/A</td>
<td>Derived from equation</td>
</tr>
<tr>
<td>Real wage</td>
<td>$W$</td>
<td>ONS, (ROYJ/YBUS)</td>
<td>Wage and Salaries/ Total actual weekly hours worked, divided by GDP deflator</td>
</tr>
<tr>
<td>Capital Rental rate</td>
<td>$R^k$</td>
<td>N/A</td>
<td>Derived from equation</td>
</tr>
<tr>
<td>External finance premium</td>
<td>$CY$</td>
<td>Reuters, DataStream</td>
<td>Difference between prime banking lending rate and bank official rate</td>
</tr>
<tr>
<td>Entrepreneur Net worth</td>
<td>$N$</td>
<td>Reuters DataStream</td>
<td>FTSE all share index, divided by the GDP deflator.</td>
</tr>
<tr>
<td>Export</td>
<td>$EX$</td>
<td>ONS, (IKBE)</td>
<td>Total exports, SA, CP</td>
</tr>
<tr>
<td>Import</td>
<td>$IM$</td>
<td>ONS,(IKBF)</td>
<td>Total imports, SA, CP</td>
</tr>
<tr>
<td>Real exchange rate</td>
<td>$Q$</td>
<td>BoE, (XUQABK67)</td>
<td>Inverse of quarterly average sterling effective exchange rate</td>
</tr>
<tr>
<td>Net foreign bond position</td>
<td>$b^f$</td>
<td>ONS, (AA6H)</td>
<td>Current account balance as per cent of GDP, SA</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------</td>
<td>-------------</td>
<td>---------------------------------------------</td>
</tr>
<tr>
<td>Foreign consumption</td>
<td>$c^f$</td>
<td>Reuters DataStream</td>
<td>Weighted average of consumption: US (0.6), Germany (0.19) and Japan (0.21)</td>
</tr>
<tr>
<td>Foreign price level</td>
<td>$p^f$</td>
<td>Reuters DataStream</td>
<td>Weighted average of CPI: US (0.6), Germany (0.19) and Japan (0.21)</td>
</tr>
<tr>
<td>Foreign interest rate</td>
<td>$r^f$</td>
<td>Reuters DataStream</td>
<td>Weighted average of interest rate: US (0.6), Germany (0.19) and Japan (0.21)</td>
</tr>
<tr>
<td>Total labour force</td>
<td></td>
<td>ONS, (BCJD+DYDC)</td>
<td>Total claim account + Work force jobs</td>
</tr>
</tbody>
</table>

† SA = Seasonal Adjusted, CP = Current Price, Total labour force is used to scale the data as per capita.
Figure 2.3.1 IRFs to Productivity shock
Figure 3.2 IRFs to monetary policy shock
Figure 4.3.3 IRFs to investment shock
Figure 5.3.4 IRFs to external finance premium shock

- Interest rate
- Investment
- Tobin's Q
- Consumption
- Output
- Labour hours
- Capital
- Premium
- Networth
- Export
- Import
- Real exchange
- Foreign assets/GDP ratio
Figure 6.3.5 IRFs to export demand shock
Appendix 3.B

Log-linearized model list

Resource constraint

\[ y_t = \frac{c}{y} c_t + \frac{l}{y} l_t + e g_t + \frac{k}{y} R K z_t + \frac{c^e}{y} c^e_t + \frac{x}{y} x_t - \frac{m}{y} m_t \]

\[ \frac{c}{y} = 1 - \frac{g}{y} - \frac{l}{y} - \frac{x}{y} + \frac{m}{y} \]

\[ \frac{l}{y} = (y - 1 - \delta) \frac{k}{y} \]

Consumption Euler Equation

\[ c_t = c_1 c_{t-1} + c_2 E_t c_{t+1} + c_3 (l_t - E_t l_{t+1}) - c_4 (r_t - E_t \pi_{t+1}) + e b_t \]

\[ c_1 = \frac{h}{1 + \gamma}; \quad c_2 = \frac{1}{1 + \gamma}; \quad c_3 = \frac{(\sigma_{c-1})(\theta_1 b + \theta_3 c)}{1 + \gamma}; \quad c_4 = \left( \frac{1 - h}{1 + \sigma_c} \right) \]

Investment Euler Equation

\[ i_t = \frac{1}{1 + \beta y (1 - \sigma_c)} \left[ l_{t-1} + \beta y (1 - \sigma_c) E_t \pi_{t+1} + \frac{1}{y \varphi} p^h_t \right] + e i_t \]

Aggregate production function

\[ y_t = \phi [a k_t^\sigma + (1 - \alpha) l_t + e a_t] \]

Relationship between effectively rented capital and capital

\[ k_t^\sigma = k_{t-1} + z_t \]

Degree of capital utilization

\[ z_t = \frac{1 - \psi}{\psi} r k_t \]

Capital accumulation equation

\[ k_t = \left( \frac{1 - \delta}{\gamma} \right) k_{t-1} + \left( 1 - \frac{1 - \delta}{\gamma} \right) i_t + \left( 1 - \frac{1 - \delta}{\gamma} \right) \left( 1 + \beta y (1 - \sigma_c) \right) y^2 \varphi e i_t \]

Hybrid Keynesian Phillips curve

\[ \pi_t^{NK} = \frac{\beta y (1 - \sigma_c)}{1 + \beta y (1 - \sigma_c) \psi} E_t \pi_{t+1} + \frac{1}{1 + \beta y (1 - \sigma_c) \psi} \pi_{t-1} - \frac{1}{1 + \beta y (1 - \sigma_c) \psi} \left( \left( 1 - \beta y (1 - \sigma_c) \right) \xi_p \right) \left( 1 - \xi_p \right) \left( 1 - \alpha \right) \psi_t - e a_t - e p_t \]

\[ (1 - \alpha) w_t - e a_t - e p_t \]
\[ \pi_t^{NC} = mc_t = (1 - \alpha)w_t + \alpha r_t^k - e_a_t \]

\[ \pi_t^{hybrid} = w^p \pi_t^{NK} + (1 - w^p)\pi_t^{NC} \]

Hybrid wage setting equation

\[ W_t^{NK} = \frac{\beta (1 - \sigma_c)}{1 + \beta (1 - \sigma_c) \lambda_p} E_t W_{t+1} + \frac{1}{1 + \beta (1 - \sigma_c) \lambda_p} w_{t-1} + \frac{\beta (1 - \sigma_c)}{1 + \beta (1 - \sigma_c) \lambda_p} E_t \pi_{t+1} - \frac{1 + \beta (1 - \sigma_c) \lambda_w}{1 + \beta (1 - \sigma_c) \lambda_w} \pi_t - \frac{\alpha}{1 - \gamma} (c_t - h \gamma c_{t-1}) + ew_t \]

\[ W_t^{NC} = \sigma_l l_t - \left( \frac{1}{1 - \beta} \right) (c_t - h \gamma c_{t-1}) - (\pi_t - \pi_{t-1}) + e w^*_t \]

\[ W_t^{hybrid} = w^w W_t^{NK} + (1 - w^w) W_t^{NC} \]

Labour demand (hours) equation

\[ l_t = -w_t + \left( 1 + \frac{1 - \psi}{\psi} \right) r k_t + k_{t-1} \]

Monetary policy Taylor rule

\[ r_t = \rho r_{t-1} + (1 - \rho) (r_p \pi_t + r_q y_t) + r \Delta \pi (y_t - y_{t-1}) + e r_t \]

External finance premium equation

\[ E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (q q_t + k_t - n w_t) + e p r_t \]

Arbitrage equation for the value of capital (Tobin’s Q):

\[ p^k_t = \frac{1 - \delta}{1 - \delta + R K} E_t p^k_{t+1} + \frac{R K}{1 - \delta + R K} E_t r k_{t+1} - E_t c y_{t+1} \]

The evolution of entrepreneur’s net worth

\[ n w_t = \theta n w_{t-1} + \frac{K}{n} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t + en w_t \]

Real uncovered interest rate parity

\[ q_t = E_t q_t + r^f_t - r_t \]

Export demand equation

\[ x_t = c^f_t + \frac{1}{\sigma} q_t + e x_t \]

Import demand equation
\[ m_t = c_t - \sigma q_t + e m_t \]

The evolution of net foreign assets position

\[ \hat{b}_t^f = (1 + \eta_t^f) \hat{b}_{t-1}^f + \frac{p_t^d x}{q_t y} e x_t + \frac{p_t^d x}{q_t y} q_t = \frac{m}{y} m_t \]

**Stochastic process**

Government spending shock

\[ e g_t = \rho_g e g_{t-1} + \sigma_g \eta_t^g + \eta_t^g \]

Risk premium shock

\[ e b_t = \rho_b e b_{t-1} + \eta_t^b \]

Productivity shock

\[ (e a_t - e a_{t-1}) = \rho_a (e a_t - e a_{t-1}) + \eta_t^a \]

Investment-specific shock

\[ e i_t = \rho_i e i_{t-1} + \eta_t^i \]

Monetary policy shock

\[ e r_t = \rho_r e r_{t-1} + \eta_t^r \]

Price mark-up shock

\[ e p_t = \rho_p e p_{t-1} + \eta_t^p \]

Wage mark-up shock

\[ e w_t = \rho_w e w_{t-1} + \eta_t^w \]

Labour supply shock

\[ e w^s_t = \rho_w e w^s_{t-1} + \eta_t^{ws} \]

External finance premium shock

\[ e pr_t = \rho_{pr} e pr_{t-1} + \eta_t^{pr} \]
Net worth shock

\[ enw_t = \rho_{nw}enw_{t-1} + \eta_t^{nw} \]

Export demand shock

\[ ex_t = \rho_x ex_{t-1} + \eta_t^x \]

Import demand shock

\[ em_t = \rho_m em_{t-1} + \eta_t^m \]

Exogenous foreign consumption process

\[ c_t^f = \rho_c c_{t-1}^f + \eta_t^c \]

Exogenous foreign interest rate process

\[ r_t^f = \rho_r r_{t-1}^f + \eta_t^r \]
Chapter 4 Evaluate and Estimate a DSGE model with financial frictions for the UK: An Indirect Inference method

4.1 Introduction

From the end of the early 1990s recession, the UK economy had experienced a steady growth in output, accompanied by low inflation and unemployment rate. However, the global financial crisis metamorphosed from the 2007 financial crisis in the US to many other advanced economies resulting in a so-called the ‘Great Recession’. In particular, in line with the US, the UK has recently experienced the worst recession since the Great Depression of the 1920s and 1930s. The ‘Great recession’ of 2008 and 2009 brought to an end the longest period of sustained, stable economic growth the UK has known with one of its sharpest contractions.

This paper targets two important challenges faced in DSGE model literatures. First, despite the advances of Bayesian techniques mentioned the DSGE model literature, while reported confidence sets for DSGE model parameters are often narrow, hence estimates of many important parameters tend to be fragile across empirical studies (Schorfheide 2008). Second, researchers usually use de-trending time series data before estimate the model. However, first differencing filter passes the higher frequency data behaviour and attenuates the lower frequency behaviour of the data, while moving average filter passes the lower frequency behaviour but blocks higher frequency behaviour, thus smooths the data. Eliminating or amplifying dynamics over certain frequency range can leaves potentially non-negligible influence of permanent shocks in the stationary detrended data. Time series exhibit either higher or lower frequency behaviour is difficult to reconcile with the model being estimated. This data frequency misspecification contaminates the estimation of shocks and thereby inference about the sources of business cycle fluctuations.

The contribution of this chapter is twofold. On the one hand, this appropriately complex medium-sized model of a small open economy incorporates important nominal and real rigidities as well as the financial frictions and foreign sector. This then allows us to describe the UK economy in a reasonable detail. On the other hand, we evaluation and estimate the model by Indirect Inference method using un-filtered nonstationary data in the period of 1975Q1 to 2015Q4.
We find the model with or without financial frictions are severely rejected using calibrated parameters. This indicates both models cannot explain the data behaviour. It is obviously that we attempt to evaluate the model based on unreasonable values for some parameters. When we therefore re-estimate structural parameters by Indirect Inference method, the overall performance of modelling fitting dramatically increases with the model significantly pass the Indirect Inference test. Moreover, the estimation results are also robust to the period from 1992 to 2015 under the inflation targeting monetary policy regime. We also document the effects of shocks to key macroeconomics variables and then assess the role of different shock to combat the financial crisis. We find that a) the non-financial shock, especially the productivity and labour supply shock are the primarily driver forces of real variables variability. b) The financial shocks also played an important role in the 2008 drop of the output. c) Financial shocks are an important source of financial variable fluctuations. d) With the existence of financial friction, demand shocks, in particular the investment shocks have been partially replaced by exogenous disturbances introduced by financial shocks. Investment shock is also relegated to account for a small fraction of the variance in nominal variables and entrepreneurs’ net worth.

The structure of this chapter is as follows. Section 2 describe the model evaluation and estimate procedure. I report the evaluation and estimation results in Section 3. In Section 4, I study the empirical performance by accessing IRFS and the relative importance of each shock and propagation of financial shocks. I also assess the robustness check for alternative monetary regime and nominal debt contractor. Section 5 concludes.

4.2 Indirect Inference

4.2.1 Why indirect inference?

Schorfheide (2008) investigates for instance the specification of the Phillips curve and find a wide range of estimated parameter among 43 surveys is because of differences in model specification, choice of observables and sample period, data definitions, and data detrending. He argued the fragility of estimates is partly due to lack of identification of key DSGE model parameters.

Over the past decades numerous econometric procedures for the analysis and estimates of DSGE models have been developed. Amongst those, Bayesian method is widely used. Blanchard (2016) criticized the standard method of estimation, which is a mix of calibration and Bayesian techniques, is also unconvincing. The problems of Bayesian approach are twofold. The first problem is that misspecification of part of the model affects estimation of the parameters in other parts of the model. The other problem comes a number of parameters are set a priori, through calibration. However, in many cases, the justification for the tight prior is weak, and what is estimated reflects more the prior of the researcher than the likelihood function.

Besides Bayesians techniques, researchers have used maximum likelihood (ML), generalized method of moments (GMM to estimate DSGE models. However, whether ML or GMM is being used, these estimators are relying on the same sample and theoretical information about first moments to identify DSGE model parameters. It is apparent that the assumption of a true model binds the identification problem to the issue of DSGE model misspecification. It is unsure that any parameters of a DSGE model can be identified when the model is mis-specified. A response to the identification problems is Indirect Inference (II) techniques. It is a methodology has been well explored in the classical literature but has received substantially less attention in the Bayesian paradigm. Smith (1993) and Gourieroux, Monfort, and Renault (1993) noted that II yields an estimator and specification tests whose asymptotic properties are standard even though the true likelihood of the DSGE model is not known. Although applications of Indirect Inference methods have appeared in diverse areas in economics, the approach has not been widely incorporated into standard econometric software packages. This is in part, due to the requirement that the package incorporates a flexible compute language. The II method used here is that originally proposed in Meenagh et al. (2009a) and subsequently refined by Le et al. (2011) using Monte Carlo experiments, these studies extend the II estimator by acknowledging that the DSGE model is false and found the power of the indirect inference tests are by far the greatest. Indirect inference technique is an intuitive and powerful way to organize estimation of deep parameters in complex models. Analysts often specify a model that relates parameters and exogenous variables of an economic model to some set of observable variables. In many situations, the economic model is too complicated to admit useful expressions for the probability distributions associated with the endogenous variables. Even

26 See Canova (2007) and DeJong and Dave (2007) provide a detailed overview
27 There is a programme package based on MATLAB is available for downloading from: http://patrickminford.net/Indirect/index.html. Le et al (2016) also provided a detailed user’s manual.
expressions for expectations of functions of the data may not exist. In such cases, fully efficient estimation procedures, such as ML, may not be applicable. In following, we would discuss the Indirect Inference in detail.

4.2.2 Introduction of Indirect Inference

The method of indirect inference is first proposed by Smith (1993) and further developed by Gourieroux, Monford and Renault (1993). It is widely known in the literature of estimation (e.g. Smith (1993); Gregory and Smith 1991, 1993; Gourieroux et al. 1993; Gourieroux and Montfort 1995; and, Canova 2005) and can be viewed as a generalization of the simulated method of moments. The II method is then extended to evaluate an already estimated or calibrated structural model.

The basic idea underlying indirect inference is to use an auxiliary model that is completely independent of the theoretical model to produce a description of the data against which the performance of the theory is evaluated indirectly to form a criterion function. The insight is then that the parameters of the auxiliary model can be estimated using both the observed data and data simulated from the structural model. The indirect inference estimator then acts as a minimum distance estimator that entails minimizing the difference between these two sets of estimates in a suitable metric. The parameters of the auxiliary model can be estimated by quasi-maximum likelihood.

When using indirect inference for evaluating a structural model, it can simulate the data from the macroeconomic model when given the parameters of the macroeconomic model and the distributions of the errors. Structural parameters are chosen so that when this model is simulated to generate estimates of the auxiliary model the results are similar to those obtained from the actual data. Consider an observed dataset taking values in $y$ of dimension $n$ assumed to have arisen from a structural model with the probability density function $f(y_t|\beta)$, where $\beta$ is the parameter vector of this model. Suppose that one can also specify a second statistical model that has a tractable probability density function. It defines the density function of this auxiliary model by $p(y_t|\theta)$, where $\theta$ denotes the parameter vector of this auxiliary model. The auxiliary model is defined to maximise a criterion function depending on the observed data and could be purely a data analytic model that does not offer any mechanistic explanation of how the observed data arose.
One then draws $s$ independent replicates of data simulated from the structural model, which denotes $x_t(\beta)$ and we assume a particular value of $\beta$ given by $\beta_0$ such that $\{x_t(\beta_0)\}_{s=1}^S$ and $\{y_t\}_{t=1}^T$ have the same distribution. However, for it to work well, two requirements need to be satisfied: (1) it is possible to simulate data from the structural model given the values of its parameters and (2) the auxiliary model captures important aspect of the data and is easy to estimate. The parameters of auxiliary model can be estimated using the observed data by maximizing the log of the likelihood function $p(y_t|\theta)$ to obtain parameter estimates $\theta_T$ given by:

$$\hat{\theta} = \arg \max_{\theta} \sum_{t=1}^T \log p(y_t|\theta) \quad (4.1)$$

where $\hat{\theta}$ serves to capture certain features of the observed data, in general it is an inconsistent estimator of $\beta$.

One then applies the estimation procedure to simulated paths. As explained above, using the structural model under $f(y_t|\beta)$, it is to simulate $S$ paths of length $T$ by drawing independently $S \times T$ times and generate pseudo observations, $\{x_t(\theta)\}_{s=1}^S$ by setting some initial values for the variables and the parameters. One in turn applies the estimation procedure and the likelihood function based on the simulation given by:

$$\tilde{\theta}(\beta) = \arg \max_{\theta} \sum_{s=1}^S \sum_{t=1}^T \log [p(x_t(\beta)|\theta)]$$

(4.2)

The indirect inference estimator will try to $\beta$ so that $\tilde{\theta}(\beta)$ is as close as possible to $\hat{\theta}$. It is based on the finding function and the simulated quasi-maximum likelihood estimator (SQMLE) of $\beta$ is given by:

$$b(\beta) = \arg\max_{\beta} \sum_{s=1}^S \sum_{t=1}^T \log [p(y_t|\tilde{\theta}(\beta))]$$

(4.3)

Since the value of $\beta$ produces a value of $\theta$ that maximises the likelihood function using the observed data. Suppose that the data really are generated under the parameter $\theta$, we then expect that the observed data and the simulated data are such that $\theta$ satisfies the sufficient condition:

$$\theta = \text{plim} \, \hat{\theta} = \text{plim} \, \tilde{\theta}(\beta) \quad (4.4)$$

This means the set of parameterised auxiliary model have to be rich enough to capture the essential feature of the data or distinguish the difference values of generative parameters. Therefore it needs at least as many auxiliary parameters as those of generative model and one
assume that the dimensionality of the auxiliary model parameter is at least as large as the dimensionality of the generative model parameter, i.e. \( \text{dim}(\theta) \geq \text{dim}(\beta) \).

### 4.2.3 The testing procedure

The application of model evaluation by indirect inference method is originally proposed in Meenagh et al. (2009a), and subsequently refined by Le et al. (2011) using Monte Carlo experiments and in Le et al (2014) for the application to non-stationary data. Therefore, for an exhaustive description of the testing procedure, I refer the reader to the original papers. The following is a brief procedure for the application to DSGE model by non-stationary data.

**Step 1: Calculate shock processes**

The residuals from structural model together with exogenous variable processes need to calculate to produce the shocks processes that drives the model. We then compute corresponding coefficients (persistence of shock process) and the innovation of shock process conditional on actual data and calibrated parameters to account for autoregressive behaviours.

**Step 2: Derive the simulated data by bootstrapping**

According to Meenagh and Minford (2012), in order to obtain the bootstraps, the innovations are first bootstrapped by time vector and add back to shock processes. Shock processes are then drawn in an overlapping manner and add into the model base run. For period \( t = 1 \), one vector of shocks is drawn and added into the model base run, given its initial lagged values; the model is solved for period 1 and this becomes the lagged variable vector for period \( t = 2 \). Then the second vector of shocks is drawn after replacement for period \( t = 2 \) and added into this solution for period 1; the model is then solved for period \( t = 2 \) and this in turn becomes the lagged variable vector for period 3. Hence, the process is repeated for onwards until the bootstrapping reached for a full sample size. The sequences of shock processes generate \( S \) bootstrap simulations and in this study, \( S \) is set equal to 1000.

The DSGE model presented is solved in log linearized equations using projection methods\(^{28}\) applied by Minford (1984, 1986) which bears a similarity to that of the extended path algorithm originally presented in Fair and Taylor (1983). The idea basically, was to solve for a terminal

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\(^{28}\) See Villaverde et al (2016) for a comparative survey of solution and estimation techniques for dynamic stochastic general equilibrium (DSGE) models.
condition far into the future data to reflect the equilibrium properties of the model as suggested by Minford et al. (1979). This implied the terminal conditions in a way analogous to the transversality conditions set in infinite time horizon problems (equation (3.75)), and implies that the model will have reached an equilibrium solution by the terminal date.

The method of solution first generates a base run that simulation results are set exactly equal to the actual data over the sample. This is to compute (Type II) residuals that reflect differences between the actual data and the value generated from Type II iterations (according to Fair and Taylor (1983)’s definition). After obtaining the 1000 different simulated scenarios (1000 simulations for selected variables are shown in Figure 4.1 in Appendix B by adding bootstrapped shocks from original data in the base run, it then computes the differences between the simulation data and original data to get the effects of these bootstrapped shocks. It then adds back the effects of deterministic trends (BGP) on the effects of the shocks and estimates the auxiliary model on all pseudo-samples. The full sample size of simulated data and the actual data has to be consistent.

**Step 3: Compute the Wald statistic**

Under the null hypothesis, the true economic model is the structural model with the given estimates. Deciding whether to reject or not reject the null hypothesis requires the estimation of the auxiliary model with simulated data. Here, a Wald test statistic is chosen to be the test statistic. One can apply the OLS estimates to the auxiliary model and compute both parameter vector from the actual data and the set of parameter vectors of pseudo samples and to obtain their distribution, from which one obtain corresponding estimated coefficient $\hat{\theta}$ and $\theta_s(\beta)$, respectively, where define $\theta(\beta)$ as the average value that is computed from:

$$\theta(\beta) = \frac{1}{1000} \sum_{s=1}^{1000} \theta_s(\beta)$$

(4.5)

The Wald statistic is to choose a suitable metric for measuring the distance between two set of parameters and the formula is specified as:

$$W = (\hat{\theta} - \theta(\beta))' \Omega(\beta)^{-1} (\hat{\theta} - \theta(\beta))$$

(4.6)

where $\Omega(\beta)$ the variance and covariance matrix of $(\theta_s(\beta) - \theta(\beta))$. This process measures the distance that the actual estimated paremeters are from the average of the simulated ones. The following step is to access the combinations of all estimated coefficient the model can fit. For the model to fit the data at the 95% confidence level, it requires the Wald statistic for the actual data to be less than the 95% confidence level of the Wald statistics from the simulated data.
One can present a straightforward statistic by either a P-value\textsuperscript{29} or transforming the Wald result a normalised t-statistic\textsuperscript{30}.

Figure 4.1 The steps for estimating structural parameters of a DSGE model in II estimate

\[ \hat{\theta} = \theta(\beta) \]

Minimize \[ (\hat{\theta} - \bar{\theta}(\bar{\beta}))'\Omega(\beta)^{-1}(\hat{\theta} - \bar{\theta}(\bar{\beta})) \]

Optimization iterations until Wald is minimized

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\textsuperscript{29} The P-value = (100 – Wald percentile)/100

\textsuperscript{30} The transformed Mahalanobis distance can be computed as:

\[ T = 1.645 \frac{\sqrt{W^2} - \sqrt{W^9}}{\sqrt{W^9} - \sqrt{W^2}} \]

where \( W^2 \) is the Wald statistic on the actual data and \( W^9 \) is the Wald statistic for the 95\% of the simulated data. If the null hypothesis has not been rejected by the data, the transformed Mahalanobis distance should be less than 1.645. The way is normalised following Le et al. (2012), and Meenagh and Le (2013), so that the resulting t-statistic is 1.645 at the 95\% point the distribution, and thus anything falling beyond would lead to the rejection of the model.
4.2.4 Indirect Inference Estimation

Estimation by indirect inference is the optimal choice of parameters for the macroeconomic model so that the distance of those two estimates of the parameters of the auxiliary model is minimized. In order to find the minimised distance of those two estimates of the coefficients in the quadratic form (4.2), I use the algorithm based on Simulated Annealing\textsuperscript{31} (SA) in which search takes place over a wide range around the initial values, with optimising search accompanied by random jumps around the parameter space\textsuperscript{32}. The use of SA attempts to imply the II estimation into practice. It exploits an analogy between the way in which a metal freezes into a minimum energy crystalline structure and search for a minimum in a more general system. Such process in SA can be considered as the way of finding the minimum Wald statistics implied by the observed and simulated data. At each step, the SA heuristic considers some neighbouring states of the current states, and decides between moving the system to other states or staying in states. These probabilities ultimately lead the system to move to states of lower Wald statistics. Typically, this iteration until the quadratic form is minimised, or until a given computation budget has been exhausted.

SA’s major advantage over other methods is its ability to avoid becoming trapped at local minima. It then loops over the testing procedure to search for the global minima of Wald statistic. Under the SA algorithm, an initial choice of parameter vector is chosen, and the Wald at that point is evaluated by running through steps 1-3 above. The algorithm then moves randomly to try a new point in the parameter space. When a new point in the parameter space is found to have a smaller Wald than any point preceding it in the sequence, it is chosen to be the current point from which the search for the minimum proceeds. The algorithm can also move to points which have a larger Wald, although the probability of this happening decreases with the number of points at which Wald statistics have previously been evaluated. Eventually, after a certain number of best points are found, the search is once again widened by increasing the acceptance probability. There are many different available stopping rules for the algorithm. In this study, the bounds are set to be within 30% of the initial calibrated parameters, and the maximum number of iterations is set to be equal to 1000.

The steps for estimating structural parameters of a DSGE model in II estimation is shown in Figure 4.1. It should be noted that II estimate is the point esitmate, by definition a point estimate

\textsuperscript{31} The estimation is mainly based on Matlab code file ‘run_CalcWald_SA’ but only change the up and lower bounds of each coefficient. The file can be provided on request.

\textsuperscript{32} The state in an II estimation procedure can be considered as the set of structural parameters.
of a parameter gives a certain value as an ‘best estimate’ of the unknown population parameter which can be regarded as a sensible value. Theoretically, if the algorithm works perfectly, one can build confidence interval of the point estimate by calculating the corresponding standard error. The standard error of an point estimation \( \beta \) is its standard deviation of the the sampling distribution of point estimators:

\[
\sigma_{\beta} = \frac{\sigma}{\sqrt{N}}
\]

(4.7)

where \( \sigma \) is population standard deviation. In doing this, we can estimate the model in many times and find similar sets of structural parameters and then calculate the standard deviation. However, in practice, this is not sensible. The problem arisen that one might find sets of structural parameters are virtually different. Most of them are contradictory to the economy theory of the model. Hence if one takes the average of these sets, it would not pass.

The primary goal of carrying out II evaluation and estimation in this exercise is that one test the model unconditionally against the data and re-estimate to find a certain set of structural parameters to ensure it to fit as closely as possible. On the other hand, in contrast to classical ‘frequentist’ FIML, Bayesian ML and conventional interval estimations, as in discussed in Le et al (2012), both and indirect estimators are consistent and asymptotically normal in estimation, but the testing power of indirect inference in small samples is much stronger than that of direct inference, as found in Monte Carlo experiments. Therefore, it should also give more reliable results from estimation in small samples if we use the II procedure both to estimate the model on our available small samples and to test its specification.

### 4.2.5 Why Non-stationary and how to handle

Since the influential paper of Nelson and Plosser (1982), a large body of time series empirical works on unit roots and co-integration indicate that most of macroeconomics time series are non-stationary. In literature, it observes two types of non-stationary processes: (i) trend stationary where non-stationarity is deterministic, or in other words, processes are stationary around a trend; (ii) difference stationary where non-stationarity is stochastic that follows a unit roots process.

---

33 Refer as ‘Full information maximum likelihood’
According to Wickens (1982), non-stationary data has two main implications for modelling. First, it enables us to distinguish between temporary and permanent shocks. In the existence of stationary or trend stationary process, endogenous variables have short memories and shocks have temporary impact. Variables return to their steady trend after the shock. In the existence of a unit root process, the time series or endogenous variables have long memories and shocks have permanent effect. The variables do not return to their former path following a random disturbance. With permanent shock, endogenous variables sharing with the same BGP are transmitted by levels of permanent shock and the level of the variables then shift permanently. The former can be interpreted as the business cycle effect ‘cyclical component ‘and the latter is the long run growth path effect.

Indeed, the most traditional practices are mapping the data to stationary by detrending time series data, although the problem has been recognized in literatures. Researchers use linear (or higher order polynomial) detrending when assuming deterministic trends for model or first differencing data when assuming stochastic trends for model. However, transformations as input in the estimation process do not isolate fluctuations with the required periodicity (Canova, 1998). The former approach is not proper when the data generating process includes stochastic trends, while the later approach tends to magnify the high frequency noise component in data. Eliminating or amplifying dynamics over certain frequency range can leaves potentially non-negligible influence of permanent shocks in the stationary detrended data.

Alternatively, researchers apply Hodrick-Prescott (HP) filter or similar band pass (BP) filter to the data based on decomposition of economic time series into trend and cyclical components, see Baxter and King (1999) and King and Rebelo (1993). However, the use of the HP filter or BP filter also have been subject to heavy criticism. The main problems with HP filter are first the spurious effect it can produce to generate cycles may not exist when applied to detrending time series and second it markedly distorts key business cycle stylised facts between the cyclical components of the variables of interest because its two-sided moving average filter can alter the timing of the data information. For example, the HP filtering transforms the forward-looking properties of the model, and seriously defects in the estimation of a DSGE model where both the expectations structure and the impulse response functions are usually matters of considerable interest (Meenagh et al 2012). It can also significantly bias the estimated dynamic parameters (Doorn,2006).

---

34 Gorodnichenko and Ng (2010) summarize a table to show how trends are treated in some notable papers.
Given ambiguity of the validity, a growing literature has criticised the nature of detrending data before DSGE model estimation. Andrle (2008) criticizes that the detrending data in DSGE model may be unable to explain co-movements of filtered time series because permanent shocks inducing dynamics usually have large influence on the business cycle and models using detrended data are less likely to capture the true business cycle dynamics. Canova (2014) compares several univariate filtering devices and finds that different approaches yield significantly different estimates of parameters. Approaches can potentially extract the cyclical component rely on assumptions about trend processes that can cause mismeasurement of cyclical components and bias the estimation of deep parameters. Other criticisms can also be found in Ferroni (2011); Canova and Ferroni (2011); Gorodnichenko and Ng (2010).

The method applied in this chapter follows Meenagh et al (2012) that extend the work of Le et al (2011) to evaluation a model when ‘the data are nonstationary but not made stationary’. When the authors apply II mechanism on non-stationary data they reduce data to stationarity. This is done by assuming the endogenous variables are co-integrated with a set of exogenous non-stationary variables, so that the residuals are stationary. They assume the relationships can be written as a Vector Error Correction (VECM) model or Vector Auto Regression with Exogenous variable model (VARX) as the auxiliary model used to present the solution of log-linearized model.
4.2.6 The choice of auxiliary model

As discussed, the state-space representation of log-linearized DSGE model in general has a restricted VARMA representation for the endogenous variables. It then can be approximately rewritten by a finite order reduced from VAR model. Hence, it follows that a VAR can be the natural auxiliary model to use for evaluating how closely a DSGE model fits the data whichever of the measures above are chosen for the comparison, and the data can be represented by an unrestricted VAR (Le et al 2016). The advantage of using auxiliary model over the others is that since the auxiliary model can be a mis-specified one and typically not even generative, but is easily fit to the data alone. As long as the model is identified with a restricted VAR, the structural restrictions of the DSGE model are then reflected in the data simulated from the model and will be consistent with the VAR, whereas the auxiliary model can be then estimated unrestrictedly both on those simulated data and on the original data.

Following Meenagh et al (2012), a VECM specification can be used as an auxiliary model if the shocks or exogenous processes are non-stationary. Non-stationary exogenous processes will drive one or more structural equations have non-stationary residuals. Since these shock processes are backed from actual data and calibrated parameters, and if we treat these processes as observable variables then the number of cointegrating vectors will be less than the number of endogenous variables. This allows one to represent the solution of the estimated model as a VECM in which the nonstationary residuals appear as observable variables, and to use an unrestricted version of this VECM as the auxiliary model.

As shown in Figure 4.2, the structural model presented in chapter 2 implies that the set of variables that we consider for our empirical analysis has several common trends or a balanced growth path. This suggests that we can obtain an approximation of the model if we generate a VECM.

As in Meenagh et al. (2013) and Le et al. (2015), the VECM model is an approximation of the reduced form of DSGE model and can be represented as a cointegrated VARX model. We suppose that the structural model can be written in log linearized form as a function given by:

\[ A(L)y_t = B(L)E_t y_{t+1} + C(L)x_t + D(L)e_t \]  

(4.8)

It assumes exogenous variables \( x_t \) are non-stationary and follows a unit root process:

\[ \Delta x_t = a(L)\Delta x_{t-1} + d + c(L)e_t \]  

(4.9)
where \( y_t \) have a \( px1 \) vector of endogenous variables and \( x_t \) have a \( qx1 \) vector of exogenous variables. \( E_t y_{t+1} \) has a \( r\times1 \) expected future endogenous variables. \( e_t \) and \( \epsilon_t \) are vectors of i.i.d error process with zero means and covariance matrix \( \Sigma \). \( L \) donotes the lag operator and \( A(L) (B(L) \ etc.) \) is a matrix polynomial functions in the lag operator of order \( h \) that have roots of the determinantal polynomial lies outside the complex unit circle. \( y_t \) is also assumed to be non-stationary since it is linearly dependent on \( x_t \). If \( y_t \) and the variables are non-stationary and potentially co-integrated, the levels form of the VAR may not be the most useful representation since it does not contain the co-integration relations explicitly.

The general solution of \( y_t \) is given by:

\[
y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \tag{4.10}
\]

where \( f \) is a vector of constant and polynomial function in lag operator. Since \( y_t \) and \( x_t \) are both non-stationary, the solution has the \( p \) cointegrating relationship that:

\[
y_t = [I - G(1)^{-1}[H(1)x_t + f] = \Pi x_t + g \tag{4.11}
\]

The \( p \times p \) matrix \( \Pi \) has rank \( 0 \leq r < p \) , where \( r \) is the number of linearly independent cointegrating vectors. Trends can be easily modelled within a DSGE model in various ways. There are two types of exogenous processes in the model: drifting and autoregressive processes. A generic exogenous variable \( X \) can be decomposed into two components:

\[
\tilde{x}_t = \tilde{x}^d_t \tilde{x}^s_t \tag{4.12}
\]

In long run, the solution is given by,

\[
\tilde{y}_t = \Pi \tilde{x}_t + g \tag{4.13}
\]

\[
\tilde{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \tag{4.14}
\]

\[
\xi_t = \sum_{s=0}^{t-1} \epsilon_{t-s} \tag{4.15}
\]

Where \( \tilde{y}_t \) and \( \tilde{x}_t \) are the long run solution to \( y_t \) and \( x_t \) respectively. The solution of \( \tilde{x}_t \) can be decomposed into a deterministic trend \( \tilde{x}^d_t = [1 - a(1)]^{-1} dt \) and a stochastic trend \( \tilde{x}^s_t = [1 - a(1)]^{-1} c(1)\xi_t \).

In that case, it may be advantageous to reparametrize the equation (4.12) by subtracting \( y_{t-1} \) on both sides to obtain:

\[
\Delta y_t = P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)e_t - [I - G(1)](y_{t-1} - \Pi x_{t-1})
\]
\[ P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t - [I - G(1)](y_{t-1} - \Pi x_{t-1}) \quad (4.16) \]
\[ \omega_t = M(L)e_t + N(L)e_t \quad (4.17) \]

where \( \omega_t \) is a mixed Moving Average process. The above VECM approximately consists of Vector Autoregression with exogenous variables (VARX) representation of the form:

\[ \Delta y_t = -K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \quad (4.18) \]

where \( \zeta_t \) is i.i.d with zero mean.

Since \( \bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t] \) and \( \bar{y}_t = \Pi \bar{x}_t + g \)

The form of the VARX can also be rewritten as:

\[ \Delta y_t = K[(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \quad (4.19) \]

where time trend the deterministic trend in \( \bar{x}_t \).

\[ y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + t + q_t \quad (4.20) \]

According to Le et al. (2015), either (4.18) or (4.19) can serve as the auxiliary model. Throughout this paper, I follow Le et al (2015), Minford (2015) to use equation (4.19) which distinguishes between the effect of the trend component and the temporary deviation of \( x_t \) from trend. The advantage is that the estimation of the parameters of the VARX can be carried out by classical OLS methods. Meenagh et al. (2012) also proved that this procedure is extremely accurate using Monte Carlo experiments.

### 4.2.7 The Property of Auxiliary Model

The II test criterion is determined by the difference between empirical auxiliary Wald statistic from observed data and simulated auxiliary Wald statistic from simulated data as shown in equation (4.6). Those parameters (\( \theta \)) of an auxiliary model can be not an accurate description of the data-generating process, but they can be estimated easily by conventional estimation methods. Therefore, there is no simple rule to identify the best auxiliary model, while asymptotically different models make no difference. As in Minford et al. (2016), there could be many choices for the auxiliary model or ‘data descriptors’ for the criterion function, as defined. Apart from VAR or VARX model, the impulse response function match that widely
used as a model evaluation method or for estimating the structural parameters of DSGE models (Schorfheide, 2002, Christiano et al., 2005), can also serve as a data descriptor. With VAR or VARX coefficients used as data descriptors, the estimated VAR/VARX parameters are to describe the dynamic property of the data, while the variance of the errors is to capture the data volatility. With IRFs functions served as the data descriptions, the IRF function can be transferred as a nonlinear combination of VAR coefficients and the error covariance matrix\(^{35}\).

Le et al. (2016) argues that since the most DSGE model are over-identified, hence with addition of more VAR (such as adding more variables in VAR or raising the order of the VAR), it would increase the power of test. However, increasing the power in means it would also reduce the chances of finding a tractable model that would pass the test, therefore there was a trade-off for users between power and tractability. According to empirical results (Le et al 2011, 2015, 2016), when including a broader set of endogenous variables in auxiliary model, it usually results to a strong rejection. Le et al. (2015) pointed out that the power of the full Wald test increases as more endogenous variables is added and as the lag order is raised, leading to uniform rejections. Meenagh et al (2012) also argued it usually led to a rejection when a model appears to share with too many elaborate structures\(^{36}\).

The auxiliary model used in this paper is a VARX(1) and is choose to describe main interest of three key macro variables data behaviour, and once one find that the structural model is rejected by a VARX(1), we do not proceed a more stringent test based on a higher order VARX. For example, if we start to look for Directed Wald statistics\(^{37}\) involving three subsets of all variables, Y, Q and R. For instance, a VARX (1) with three endogenous variables can therefore reads:

\[
\begin{bmatrix}
Y_t \\
\tau_t \\
q_t
\end{bmatrix} = B
\begin{bmatrix}
Y_{t-1} \\
\tau_{t-1} \\
q_{t-1}
\end{bmatrix} + C
\begin{bmatrix}
T \\
e^{yt} \\
b_{t-1}^F \\
e_{yt} \\
e_{rt} \\
e_{qt}
\end{bmatrix}
\]

(4.21)

where $B = \begin{bmatrix}
b_{yy} & b_{yr} & b_{yq} \\
b_{ry} & b_{rr} & b_{rq} \\
b_{qy} & b_{qr} & b_{qq}
\end{bmatrix}$

\(^{35}\) Minford et al. (2016) show that the error $e_t$ from a VAR model can be write as a: \(e_t = B\nu_t\), where $\nu_t$ is the structural innovations. $B$ denotes the error covariance matrix.

\(^{36}\) They point out models such as SW and CEE have many nominal rigidities in the goods and labour markets and real rigidities such as habit formation in consumption, investment adjustment costs, and variable capital utilization.

\(^{37}\) Le et al. (2011) proposed directed Wald tests where the information used in evaluating a DSGE model was deliberately reduced to cover essential features of the data. The directed Wald is then focused a small subset of variables or aspects of their behaviour.
where \( t = 1, \ldots, T \) denotes the time. Besides the lagged endogenous variables, the VARX (1) also includes lagged productivity trend \((e^{YT})\), time trend \(T\) and the lagged level of net foreign assets \((b^f_{t-1})\). \(C\) captures the effect of exogenous variables that considered as the driving factors of non-stationarity. The parameter vector \(\theta\) used for calculating the Wald statistics would contain all coefficients in \(B\) matrix and the variance of three fitted errors:

\[
\theta = [b_{yy}, b_{yr}, b_{yr}, b_{ry}, b_{ry}, b_{ry}, b_{ry}, b_{ry}, b_{ry}, b_{ry}, b_{ry}, b_{ry}, \text{var}(e_{yt}), \text{var}(e_{rt}), \text{var}(e_{qt})]'
\]  

(4.22)

We check whether the model can replicate the behaviour of three endogenous variables jointly. In other words, the model will pass the test if it can match at least twelve parameters distribution jointly. Now if the number of chosen variable increases to four, it turns out we have to match at least twenty parameters in \(\theta\). The testing power would therefore dramatically increase if one extra variable were included, and the model is usually severely rejected.

4.3 Empirical Results

4.3.1 Indirect Inference Test result based on Calibration

In order to implement II estimation, calibration value of the parameters can be predefined as starting values. We follow the testing process discussed above with the hypothesis that the calibrated model replicates the actual data. Table 4.1 reports the Wald statistic and normalised Transformed Mahalanobis Distance (TMD) of the II test results. It should be noted that in this study, instead of using the asymptotic distribution of the Wald statistic, we follow use an empirical estimate of its small sample distribution obtained by bootstrap explained in Section 4.2. It indicates that estimated Wald statistics do not follow a Chi squared distribution, in other word, a Wald statistic reported less than 90 does not necessarily mean to pass the test. We then use the TMD t statistic as reference for assessment.

Not surprisingly, it is turns out hypothesises are severely rejected. For example, the TMD statistic for variables subset are 3.68 (for \(Y, \pi, R\)), 3.21 (for \(Y, Q, R\)), and 4.32 (for \(Y, \text{EFP}, R\)) respectively. This indicates by using calibrated parameters, the model with or without financial frictions cannot explain the data behaviour. It is obviously that we attempt to evaluate the model based on unreasonable values for some parameters. For selected variables subsets, Table 4.2 also tells whether each estimated coefficients of auxiliary model based on the actual date lies

---

38 In this case, the vector \(\theta\) does not include the parameter matrix \(C\). It turns out to be 21 parameters if \(C\) included.
between the 95% the up and lower bound of coefficients from simulation. In most cases, four out of twelve are falling out of the bound. However, the TMD here is used as the guidance not only to tell how bad the model performs to fits the data but also to access how far the model deviates away from non-rejection. The estimation process is then to search for a vector of structural coefficients within chosen bounds\textsuperscript{39} that minimises the Wald statistic given the chosen auxiliary model.

\textsuperscript{39} The whole process is conducted as follows: we first simulate 1000 vector of parameters within 30% bound by using SA in MATLAB, and then aggregate these vectors in a large vector by sequences. FORTRAN reads these vectors in order and repeat the testing process until it find the minimized Wald statics and corresponding TMD.
Table 4.1 Wald test results based on Calibration

<table>
<thead>
<tr>
<th>VARX(1)</th>
<th>Subsets</th>
<th>SW</th>
<th>SWBGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Y, \pi, R$</td>
<td>31.23</td>
<td>49.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.31)</td>
<td>(3.68)</td>
</tr>
<tr>
<td>2</td>
<td>$Y, R, EFP$</td>
<td>-</td>
<td>53.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.08)</td>
</tr>
<tr>
<td>3</td>
<td>$Y, Q, EFP$</td>
<td>-</td>
<td>78.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(5.98)</td>
</tr>
<tr>
<td>4</td>
<td>$Y, Q, R$</td>
<td>32.70</td>
<td>38.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.51)</td>
<td>(3.21)</td>
</tr>
<tr>
<td>5</td>
<td>$NW, EFP, R$</td>
<td>-</td>
<td>44.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(4.32)</td>
</tr>
<tr>
<td>6</td>
<td>$Y, EFP, NW$</td>
<td>-</td>
<td>29.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.20)</td>
</tr>
<tr>
<td>7</td>
<td>$R, EFP, Q$</td>
<td>-</td>
<td>96.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(7.42)</td>
</tr>
</tbody>
</table>

†TMD t statistics are reported in parenthesis

Table 4.2 VARX parameters and Bootstrap Bounds for output, inflation and interest rate (SWBGG based on calibration)

<table>
<thead>
<tr>
<th>$Y \pi R$</th>
<th>Estimated</th>
<th>95% Lower Bound</th>
<th>95% Upper Bound</th>
<th>IN/OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{yy}$</td>
<td>0.988</td>
<td>0.587</td>
<td>0.968</td>
<td>OUT</td>
</tr>
<tr>
<td>$b_{\pi\pi}$</td>
<td>-0.248</td>
<td>-0.067</td>
<td>0.074</td>
<td>OUT</td>
</tr>
<tr>
<td>$b_{yr}$</td>
<td>-0.519</td>
<td>-0.680</td>
<td>0.200</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{\pi y}$</td>
<td>-0.005</td>
<td>-0.117</td>
<td>0.116</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{\pi\pi}$</td>
<td>0.935</td>
<td>0.963</td>
<td>1.006</td>
<td>OUT</td>
</tr>
<tr>
<td>$b_{\pi r}$</td>
<td>0.074</td>
<td>-0.308</td>
<td>0.301</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{r y}$</td>
<td>0.007</td>
<td>-0.068</td>
<td>0.063</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{r\pi}$</td>
<td>0.0127</td>
<td>-0.027</td>
<td>0.028</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{rr}$</td>
<td>0.900</td>
<td>0.624</td>
<td>0.976</td>
<td>IN</td>
</tr>
<tr>
<td>$Var(e_y)$</td>
<td>$5.629\times 10^{-5}$</td>
<td>$4.294\times 10^{-4}$</td>
<td>0.004</td>
<td>OUT</td>
</tr>
<tr>
<td>$Var(e_{\pi})$</td>
<td>$5.758\times 10^{-6}$</td>
<td>$3.695\times 10^{-4}$</td>
<td>$4.289\times 10^{-4}$</td>
<td>OUT</td>
</tr>
<tr>
<td>$Var(e_{r})$</td>
<td>$4.894\times 10^{-6}$</td>
<td>$7.015\times 10^{-5}$</td>
<td>$7.843\times 10^{-4}$</td>
<td>OUT</td>
</tr>
</tbody>
</table>
4.3.2 Indirect inference Estimation Results

Following the method discussed above, we then apply the II estimate on two models to empirically assess the difference across the two models and evaluate the contribution of financial frictions. Table 4.3 provides the results of II estimates for two models, column 4 reports the SWBGG model results, column 5 contains the SW model results and column 3 states calibrated parameters for comparison. It should be noted that the discount factor $\beta$, depreciation rate $\delta$ and entrepreneurs’ survival rate $\theta$ are set to be fixed.

We first compare the II estimates of the parameters for SWBGG model with the calibration. For nominal rigidities parameters, the Calvo parameter for price $\xi_p$ is estimated to be 0.7108, slightly lower than the starting value. Wage inflation follows a similar path to price inflation. The Calvo parameter for wage $\xi_w$ increases to 0.8186 from 0.70. Degree of price indexation $\iota_p$ decreases to 0.1969, still lower than the Degree of wage indexation $\iota_w$ that estimated to be 0.4817. This indicates that wage inflation is more persistent than price inflation. The proportion of sticky wages that encodes the weight of Neoc Keynesian price is estimated to 0.3764 while the proportion of sticky price is equal to 0.1082. Both of parameters are readily increased. The Share of capital in production adjusts to 0.2404 after the estimation, which decreases by nearly 30%. For real rigidities parameters, the parameter $h$ that encodes external habits in consumption decreases to 0.5763. The intertemporal elasticity of substitution $\sigma_c$ decreases to 1.2985. The parameter governing the elasticity of steady state investment adjustment ($\varphi$) drops to 5.6431, while the parameter governing the degree of capital utilisation increases from 0.54 to 0.6315. Overall, monetary policy is estimated to be more responsive to inflation and correspondingly less responsive to real output fluctuations and less auto-correlated. The responsiveness of interest rates to inflation $r_p$ increases from 2.50 to 2.6764. In contrast, the policymaker's reaction coefficient to output $r_y$ and output change $r_{\Delta y}$decrease to 0.0642 and 0.2070 respectively compared with the calibrated value. Moreover, interest rate smoothing parameter $\rho$ also reduces to 0.5646.

Turning to the second comparison in column 4 (estimates of SWBGG model) and column 5(estimates of SW model), the results reveal the degree of Calvo price and Calvo wage stickiness are higher in SW model, both with a higher proportion with the hybrid setting. It is striking that the steady state elasticity of investment adjust cost in SW model (6.8227) is significantly higher than in SWBGG model (5.6431). On the other hand, one plus the share of fixed cost in production also increased to 1.9422 in SW model, compared to that of in SWBGG
model is 1.7541. It seems that the parameter relating to the degree of real frictions is higher in SW model due to the absence of financial friction. Elasticity of capital utilization in SW model falls to 0.4431 revealing that the capital utilisation is costlier in SWBGG model (0.6315). The Taylor rule response to inflation is lower in the SWBGG model, while the other policy coefficients are higher.
### Table 4.3 Structural parameter estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Calibration</th>
<th>II estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>SW+BGG</td>
<td>SW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SW</td>
<td></td>
</tr>
<tr>
<td>Fixed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Survival rate of entrepreneurs</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Households</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.39</td>
<td>1.2985</td>
</tr>
<tr>
<td>$h$</td>
<td>degree of External habit formation</td>
<td>0.70</td>
<td>0.5763</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>Frisch elasticity of labour supply</td>
<td>1.83</td>
<td>3.1109</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>degree of wage stickiness</td>
<td>0.70</td>
<td>0.8186</td>
</tr>
<tr>
<td>$\iota_w$</td>
<td>Degree of wage indexation</td>
<td>0.58</td>
<td>0.4817</td>
</tr>
<tr>
<td>$\omega^w$</td>
<td>Proportion of sticky wages</td>
<td>0.40</td>
<td>0.3764</td>
</tr>
<tr>
<td>Producers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>Degree of price stickiness</td>
<td>0.75</td>
<td>0.7108</td>
</tr>
<tr>
<td>$\iota_p$</td>
<td>Degree of price indexation</td>
<td>0.24</td>
<td>0.1969</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Elasticity of capital utilization</td>
<td>0.54</td>
<td>0.6315</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1+Share of fixed costs in production</td>
<td>1.50</td>
<td>1.7541</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Steady state elasticity of investment adjustment cost</td>
<td>5.74</td>
<td>5.6431</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of capital in production</td>
<td>0.33</td>
<td>0.2759</td>
</tr>
<tr>
<td>$\omega^r$</td>
<td>Proportion of sticky prices</td>
<td>0.10</td>
<td>0.1082</td>
</tr>
<tr>
<td>Taylor rule</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_p$</td>
<td>Response to inflation</td>
<td>2.50</td>
<td>2.6764</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Interest rate smoothing</td>
<td>0.60</td>
<td>0.5646</td>
</tr>
<tr>
<td>$r_y$</td>
<td>Response to output</td>
<td>0.08</td>
<td>0.0642</td>
</tr>
<tr>
<td>$r_{\Delta y}$</td>
<td>Response to output change</td>
<td>0.22</td>
<td>0.2070</td>
</tr>
<tr>
<td>Financial frictions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>Elasticity of the premium with respect to leverage</td>
<td>0.04</td>
<td>0.0477</td>
</tr>
</tbody>
</table>
4.3.3 Shock process

For each calculated shock process, it conducts two different types of stationarity test: The Augmented Dickey-Fuller (ADF) test and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test so as to test both the null hypothesis of a unit root and that of stationarity. The results are reported in Table 4.4. It shows that under both the ADF and KPSS test, the null hypothesis of a unit root cannot be rejected at 5% level for productivity shock. It rules out the specification in which productivity shock is deterministic trend stationary and concludes that the productivity shock appears to be integrated process of order one I (1), which contains a stochastic trend.

On the other hand, for labour supply shock, net worth shock and exogenous foreign consumption, although the null hypothesis of a unit root cannot be rejected at conventional 5% level by the ADF test, the KPSS statistics suggest it cannot reject the null of stationarity at 10% level. Here, I assume these exogenous processes are stationary or trend stationary in line with the setup in Le et al (2011) and Le et al (2012). For the rest of the shock processes, the P-value from ADF test and KPSS statistic suggest relatively strong rejections of the unit-root. The results provide solid evidence for the existence of stationarity. Although some empirical works suggest different specification of the law of motion for the exogenous shocks can somewhat help to fit the model\textsuperscript{40}, however in this paper, other than the productivity shock, the other exogenous shocks are assumed to exhibit AR (1) dynamics or AR (1) dynamics with a deterministic trend. Clearly, AR (1) persistent values suggest that differences among these shocks are sizable. As discussed above, an important implication of the deterministic components of the stochastic processes is that they generate the balanced growth path (BGP) of the model. In practice, after simulating the model from original data in the base run, it computes the differences between the simulation data and original data to get the effects of these shocks, either stationary or non-stationary. It then adds in the BGP on the effects of the shocks, whereas in the version of the model, deterministic components and then BGP are fixed.

\textsuperscript{40} Smets and Wouters (2007) use an ARMA mark-up shock to improve model fit. Del Negro and Schorfheide (2009) let their government spending shock follow a higher-order autoregressive process.
Table 4.4 Testing the Null Hypothesis of Non-stationarity

<table>
<thead>
<tr>
<th>Shock</th>
<th>Process</th>
<th>Constant</th>
<th>Trend</th>
<th>AR(1)</th>
<th>KSPP Statistic</th>
<th>ADF P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Spending</td>
<td>Trend</td>
<td>3.102**</td>
<td>0.0008**</td>
<td>0.571**</td>
<td>0.059+++</td>
<td>0.009</td>
</tr>
<tr>
<td>Preference</td>
<td>Stationary</td>
<td>-0.0003</td>
<td>-0.091</td>
<td>0.210+++</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>Stationary</td>
<td>0.333**</td>
<td>0.327**</td>
<td>0.364+++</td>
<td>0.015</td>
<td></td>
</tr>
<tr>
<td>Taylor rule</td>
<td>Stationary</td>
<td>-0.411</td>
<td>0.387</td>
<td>0.439++</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>Non Stationary</td>
<td>0.002**</td>
<td>0.094**</td>
<td>1.531</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>Price mark-up</td>
<td>Stationary</td>
<td>0.0005</td>
<td>0.076*</td>
<td>0.254+++</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Wage mark-up</td>
<td>Stationary</td>
<td>-0.270**</td>
<td>0.078</td>
<td>0.150+++</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Labour hours</td>
<td>Stationary</td>
<td>-25.079**</td>
<td>0.915**</td>
<td>0.257+++</td>
<td>0.172</td>
<td></td>
</tr>
<tr>
<td>External premium</td>
<td>Stationary</td>
<td>-0.088**</td>
<td>0.621**</td>
<td>0.387++</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>Net worth</td>
<td>Stationary</td>
<td>0.039**</td>
<td>0.622*</td>
<td>0.525+</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>Export</td>
<td>Trend</td>
<td>-4.837**</td>
<td>0.0028**</td>
<td>0.862**</td>
<td>0.183+</td>
<td>0.044</td>
</tr>
<tr>
<td>Import</td>
<td>Trend</td>
<td>-1.885**</td>
<td>0.0046**</td>
<td>0.833**</td>
<td>0.158+</td>
<td>0.020</td>
</tr>
<tr>
<td>Foreign Consumption</td>
<td>Trend</td>
<td>14.505**</td>
<td>0.0128**</td>
<td>0.984**</td>
<td>0.246+</td>
<td>0.155</td>
</tr>
<tr>
<td>Foreign Interest rate</td>
<td>Stationary</td>
<td>0.024**</td>
<td>0.915**</td>
<td>0.154++</td>
<td>0.097</td>
<td></td>
</tr>
</tbody>
</table>

† ‡ **Significant at the 5% level; **significant at the 10% level

†† For KSPP Statistic: +++ Significant at the 1% level; ++significant at the 5% level; +significant at the 10% level;

††† Productivity shock follows a ARIMA(1,1,0) process.
Figure 4.2 Residual calculated from log linearized behaviour model using estimated parameter.
4.3.4 Indirect Inference Test Result Based on Estimation

The structural parameters are re-estimated by Indirect Inference, and then the minimised Wald and corresponding TMD t statistics for different chosen variable subsets are shown in Table 4.5. For SWBGG model, the auxiliary model with output, inflation and interest is significantly not rejected with a Wald percentile of 21.17 and t statistics of 1.01. Individual VARX (1) coefficients of the model are generally within the 95% bounds generated from simulation. Although the variance of output error and the variance of interest rate error are outside the bound. By assessing the coefficients distribution plotting, Figure 4.14 illustrates that these variances just have little derivation from the lower bound, and then it does not affect the model parameters jointly to pass the Wald. Moreover, with other different subset of variances, the auxiliary VECM model also comfortably jointly pass the Wald and TMD t statistics. It should be noted that for subset interest rate, EFP and real exchange rate, the t statistic of 1.88 is slight over the 1.64 pass criteria, which indicates a rejection of the model. However, considering on the one hand, it poses extras challenge for adding a financial friction and an open economy setting; on the other hand, the result is not rejected by 10% critical level. The overall performance of modelling fitting by estimated structural parameters is fairly acceptable (Distribution of individual VARX (1) coefficients are plotted in Figure 4.14).
Table 4.5 Direct Wald Test Results Based on II estimation

<table>
<thead>
<tr>
<th>Subsets</th>
<th>SW</th>
<th>SWBGG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y, \pi, R$</td>
<td>20.24</td>
<td>21.17</td>
</tr>
<tr>
<td></td>
<td>(1.19)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>$Y, R, EFP$</td>
<td>-</td>
<td>19.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.02)</td>
</tr>
<tr>
<td>$Y, Q, EFP$</td>
<td>-</td>
<td>24.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.49)</td>
</tr>
<tr>
<td>$Y, Q, R$</td>
<td>17.71</td>
<td>29.73</td>
</tr>
<tr>
<td></td>
<td>(0.91)</td>
<td>(1.55)</td>
</tr>
<tr>
<td>$NW, EFP, Q$</td>
<td>-</td>
<td>28.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.73)</td>
</tr>
<tr>
<td>$Y, EFP, NW$</td>
<td>-</td>
<td>13.41</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.23)</td>
</tr>
<tr>
<td>$R, EFP, Q$</td>
<td>-</td>
<td>29.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.88)</td>
</tr>
</tbody>
</table>

†TMD t statistics are reported in parenthesis

Table 4.6 VECM parameters and Bootstrap Bounds for output, inflation and interest rate (SWBGG)

<table>
<thead>
<tr>
<th>$Y \pi R$</th>
<th>Estimated</th>
<th>95% Lower Bound</th>
<th>95% Upper Bound</th>
<th>IN/OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{yy}$</td>
<td>0.9970</td>
<td>0.5472</td>
<td>1.0649</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{y\pi}$</td>
<td>-0.1330</td>
<td>-1.0480</td>
<td>1.0707</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{y\pi}$</td>
<td>-0.3281</td>
<td>-0.9472</td>
<td>0.4837</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{\pi y}$</td>
<td>0.0138</td>
<td>-0.0138</td>
<td>0.1216</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{\pi\pi}$</td>
<td>0.2560</td>
<td>-0.0111</td>
<td>0.3984</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{\pi r}$</td>
<td>0.0749</td>
<td>0.0055</td>
<td>0.2623</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{r y}$</td>
<td>0.0068</td>
<td>-0.0838</td>
<td>0.1266</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{r r}$</td>
<td>0.0272</td>
<td>-0.3890</td>
<td>0.2850</td>
<td>IN</td>
</tr>
<tr>
<td>$b_{r r}$</td>
<td>0.8863</td>
<td>0.5984</td>
<td>0.9990</td>
<td>IN</td>
</tr>
<tr>
<td>$\text{Var}(e_y)$</td>
<td>0.0000556</td>
<td>0.000175</td>
<td>0.0073</td>
<td>OUT</td>
</tr>
<tr>
<td>$\text{Var}(e_\pi)$</td>
<td>0.0007228</td>
<td>4.26×10⁻⁵</td>
<td>0.0012</td>
<td>IN</td>
</tr>
<tr>
<td>$\text{Var}(e_r)$</td>
<td>4.80×10⁻⁶</td>
<td>5.66×10⁻⁵</td>
<td>0.0006</td>
<td>OUT</td>
</tr>
</tbody>
</table>
4.4 Empirical analysis

In this section, I first contrast overall dynamic with those of the SWBGG model and the SW model with financial friction shutting off\(^{41}\). The impulse responses of macroeconomic and financial variables to a 10 percent rise in a variety of structural shocks are plotted in Figure 4.3 through Figure 4.8 of Appendix 4.A. 10 percent rise in a variable is denoted as 0.1 on the y-axis and the number of quarters elapsed since the shock begins are indicated on the x-axis. The solid line presents the full SWBGG model and the dash-circle line presents the SW model when external finance premium switched off. Overall, the quantitative dynamics regarding the macroeconomic variables are mostly similar across the two models. However, qualitative differences emerge through the modification with respect to net worth and the finance premium. It also should be noted since the parameter estimates differ between the two models reported in table 4.3, the comparisons among the IRFs is qualitative rather than quantitative.

We also use variance and historical decomposition of the observable variables in order to understand the role of each shock as drivers of endogenous variables to quantify the role of different shocks. In particular, we analyse the importance of financial shocks, before and during the financial crisis period. The variance decomposition is computed based on 2006Q1-2014Q4 that focuses on the financial crisis and post-crisis period. The historical decomposition is built using the II estimated coefficients and corresponding parameters of shock processes over the sample 1975Q1-2014Q4.

For variance decomposition, we proceed a similar way as in historical decomposition but bootstrap each actual structural shock. After obtaining the difference between the actual and simulated data for each endogenous variable, we calculate its corresponding variance. The variance then measures the contribution of each type of shock to the overall forecast error variance, while the overall forecast error variance is the sum of variance computed from each structural shocks’ simulation. It is noted that with bootstrapped actual shocks, is not surprising that the variance decomposition is rather different from other empirical studies (maybe obtained from DYNARE). Because in this exercise, we use the actual errors and re-estimated auto-regression parameters from observed data\(^{42}\).

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\(^{41}\) The open economy set up are included in both models.

\(^{42}\) See Meenagh et al (2008) for detailed discussion
For historical decomposition, given the starting values of all the endogenous variables, we run a simulation in which one historical shock affect the economy while the other shocks are set equal to zero in all periods. This simulation shows us a proportion of movements in each endogenous variable caused by the chosen shock. The simulated model variables at each point in time can be represented as a function of initial values plus each structural shock of the model. The difference between the actual and simulated data can then be attributed to shock originating from the endogenous variables. We can repeat this exercise for the rest of the shocks in order to apportion all movements in the endogenous variables between them all. Therefore, the sum of each shocks to each variable then would be treated as the overall effect. By using bar plot, one can easily compare the relative contribution of individual shocks to each variable.

### 4.4.1 Impulse response function

**Productivity shock**

I first consider the impacts of a non-stationary positive productivity shock. As can be seen from figure 4.3, the whole structure of the interest rates coherently decreases through the horizon. Due to a positive persistent parameter in ARIMA (1,1,0) in the de-trended shock process (i.e. a non-stationary process), macroeconomic variables including output, consumption, investment and the stock of physical capital react positively to the realization of productivity progress and this permanent effect becomes more persistent afterwards lasting over 30 quarters. Inflation falls as a result of increasing in the supply of goods because products are produced with a superior technology as well as the decreasing in marginal cost while monetary policy does not respond enough to offset the negative impact. The demand for physical capital is stimulated and, as a consequence, its price (Tobin’s Q) increases, which in turn pushes up entrepreneurs’ net worth. A negative effect on financial premium then is expected to be accompanied by a positive effect on entrepreneurial net worth, which helps to further increases investment. Labour hour is negatively affected due to higher productivity and the presences of nominal price rigidities, consumption habit formation, and adjustment cost owing to SW07. To foreign variables, because output is higher, it must be sold on world markets by lowering its price. This would then lead to devaluation of the real exchange rate (a rise in Q) as shown

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43 This is because when the output supplied increases, because prices adjust slowly, aggregate demand does not immediately match this increase so employment fall.
in the figure. Moreover, real exchange rate devaluation which improves domestic goods’ competitiveness also leads to an increasing export and a decreasing import results to an increasing in net exports, which consequently leads to an accumulated net foreign asset. The responses to a productivity shock present the empirical evidences are in line with those observed in Le et al. (2012). In SW model, a productivity shock causes an ‘overshooting’ of investment and output in response to SWBGG. This results contrast with BGG but in line with Christensen and Dib (2008) and De Graeve (2008), in which favourable productivity shock smooths the reaction of investment and output to a model with financial frictions.

Monetary policy shock

Figure 4.4 depicts the IRFs of the baseline SWBGG model and the alternative simple SW model to a 10% monetary policy shock which increase nominal interest rate. The standard interest rate channel of monetary policy transmission suggests such monetary contraction discourage the investment and consumption, and therefore reduces output. It is striking that the bottom effect on consumption occurring in the second quarter reflecting the impact of habit formation on the dynamics of consumption. Demand downward shift then pressures feed through changes in output gap to inflation, which also lower the marginal cost and inflation.

Aggregate demand falls together with the falls on demand for all inputs, which also lower labour demand that bring down the real wage. The deflation also causes higher entrepreneur’s borrowing rate of capital and lower the real return to investment. The fall in real capital price and investment demand are due to high expected financing cost in the future, which reduces the net worth as a consequence and the external premium turns out to be counter-cyclical. Higher nominal and real interest rate appreciates the British pound that reduces real exchange rate. This helps to increase imports as domestic currency becomes less attractive, while exports fall with real exchange rates. However, from T=2 onwards, nominal interest rate starts to increase and the continuing dropping in consumption sufficiently reduce the values of imports. Since exports start to converge back, imports reach the bottom at T=3, and start to converge back from T=4. With exports falls and converge back, imports fall further, along with aggregate demand, trade balance goes into surplus, therefore net foreign bond start to accumulate for 15 quarters.

When comparing two models, the accelerating mechanism of the external finance premium is clearly represented demonstrated in the response of a marginal dampening investment in SWBGG model relative to the base SW model. The logic behind the amplification effect is that
in SWBGG framework, unexpected increase in nominal interest rate lifts the cost of raising capital and reduces capital price which leads to a decrease of the net worth which makes the entrepreneur riskier. The financial intermediaries translate into higher premium and it further depresses investments, generating the extra response displayed in the figure. This confirms the theoretical prescription and empirical finding in most of literatures. (Christensen and Dib (2008), CMM (2010) that suggest stronger amplification of monetary policy shock in the real economy when the financial friction has been considered.)

**External premium shock**

In Figure 4.5, the responses from a 10% standard deviation increase in the external finance premium shock are depicted. As in SWBGG model, the increase in the cost of capital funding borrowing drives Tobin’s Q down immediately, which leads to a further drop in initial declining in investment. On the other hand, the decrease in net worth further exacerbates the balance sheet effect. The implication of balance sheet effect is that lower net worth decreases entrepreneurs' stake in financing capital expenditures and so pushes them to borrow at a higher premium over the risk-free rate. Investment then falls further due to the effects of the external finance premium friction.

The nominal interest rate slightly increases on impact at first period however the impact is short-lived. The monetary policy rapidly drops starting from T=2. The interest rate differential relative to abroad is rapidly widen through URIP channel, leads to a rise of Q. Therefore, a positive premium shock immediately depreciates the real exchange rate which makes export become more competitiveness. The export then increases with import reduces. The net foreign assets position increases overall. The variables in SW model have no response because the financial frictions are switched off.

**Investment specific shock**

I then analyse the response of a 10% investment specific shock depicted in figure 4.6 and the responses of variables are similar for both models as expected. In response to a positive shock, the price of capital decreases which leads to a rise in investment and hence output and labour hours in both models. In SW model, investment, output and labour hours increase before returning gradually to their steady states. In SWBGG model, the response of real variables to this shock are smaller than those from the SW model. According to Christensen and Dib (2008), this is due to the cost of buying new capital falls. This then decreases the return on capital and the net worth, as shown in the figure. Therefore, it consequently increase the external finance...
premium and then in turn raise the cost of purchasing capital to fund the investment. Therefore, the investment would decrease further to the steady state value, and so as the output. It confirms the results described in De Graeve (2008) and Christensen and Dib (2008) but sharply contrast to the conclusion from BGG.\textsuperscript{44} The response of investment is attenuated in SWBGG model where it enjoys a nearly 0.2 margin when investment hit the bottom. Such response to investment shock may also be coming from the presence of investment adjustment cost according to De Graeve (2008).\textsuperscript{45} Under the presence of investment adjustment, when investment rises, in order to minimize costs associated with changing their investment flow, investment will be positive for a protracted period of time. This brings up the premium because entrepreneurs’ borrowing needs also increase due to high investments. The implications of including finance premium for interest rate, inflation and output are found to be relatively minor comparing to investment and capital. It induces an increasing in nominal interest rate and CPI inflation then consumption decreases. Real exchange rate decreases leading to a decline of export and increase of import. This causes a trade balance deficit so as to decrease the net foreign assets position in medium term. The net foreign assets position also contributes to a smaller response under SWBGG.

**Export demand shock**

Figure 4.7 depicts the responses from a 10% standard deviation increase of an export demand shock that increase export demand on impact. In this experiment, the effect from an export demand shock comes with two folds. First, it acts as an aggregate demand shock through the expenditure switching effect. It then stimulates output, inflation and nominal interest rates but crowds out investment. It also crowds out consumption, since the AR (1) parameter of shock is persistent enough. Second, the effect is through the real uncovered interest rate parity. Due to the excess demand, real exchange rate is required to appreciate to dampen exports and thus return the economy back to steady state. In response to the currency depreciation and the decrease in the inverse of real exchange rate $Q$ (increase in the exchange rate), the central bank then raises the nominal interest rate to compensate for the currency depreciation. Import has a consequence increase since $Q$ decreases, however the magnitude of increase is small related to export. It will eventually result in an improvement in net foreign

\textsuperscript{44} In BGG, investment shocks reduce the premium and therefore boost investment relative to a model without financial frictions. This implies a investment shock should lead to a counter-cyclical external finance premium

\textsuperscript{45} De Graeve (2008) explains that in BGG’s study it assumes a capital adjustment cost, while in Christensen and Dib (2008) and De Graeve’s model following SW03 setup, it includes with investment adjustment cost which implies a more gradual response of investment.
The presence of the external finance premium of SWBGG model amplifies the decline in capital price and investment, due to its negative impact on the net worth, but the responses of output, consumption, export and import are almost identical in the models with and without external finance premium.

**Wage mark-up shock**

Figure 4.8 plots the responses of variables to a positive 10% standard deviation wage mark-up shock. The wage mark-up shock leads to an increase in the cost of production, it then exerts a contractionary effect on output. Although a rise in inflation accompanied by a rise in nominal interest rate, increase in inflation is higher than that in nominal interest rate. This leads to a decrease in real interest rate than in turn causes an increase in external finance premium and a decrease in net worth. This effect acts in the direction of attenuating the impact of the wage mark-up shock. The responses of foreign sector variables are similar to that to a positive monetary shock. Since both nominal and real interest rate increases, it then appreciates domestic current and real exchange rate declines. It then follows a fall in exports and a rise in imports.

**4.4.2 Variance decomposition**

Table 4.9 and 4.10 report the variance decompositions of model shocks without and with the presence of the financial friction respectively for output, inflation, nominal interest rate, consumption and real exchange rate the based on the model estimation. First of all, output variance decomposition is heavily influenced by the supply shocks, in which productivity shock contributes 29.8% and labour supply shock contributes 23.4%, in the SW model. The contribution of investment shock is also strong with a 20.4% contribution. Government spending, preference, monetary policy and price mark-up shocks are all have a small but significant contribution to the output variance at the business cycle frequency. Wage mark-up shock is too weak to explain a significant proportion of the output fluctuation. The export and import demand shocks originating in the open economy setting do have a significant impact, counting as 6.4% and 5.0% respectively.

In the SWBGG model when financial shocks are introduced, EFP shock together net worth shock explain a sizable fraction for explaining output movement. These shocks owing to
financial friction account for around 22%. The contributions from other shocks expect for the investment shock are quite similar with those in the SW model. Productivity (27.7%) and labour supply (26.6%) are still the main driving forces of output fluctuations. However, the contribution from investment shock sharply decreases to 3.1%. According to the results, the impact of investment shock has transmitted to the financial disturbances.

The result found here is at odd with De Graeve (2008), Christiano, Motto and Rostagno (2010) who reported the financial risk shock is the dominants. However, it is also somewhat different from the empirical finding from Meier and Miller (2006), Gelain (2010) arguing that financial shocks are not the main driver. Le et al (2012) that argues that financial shocks only account for 9% of the output variance, although the study is based on U.S data. With similar model structural, my finding suggests financial disturbances give a 22% contribution of UK output variation which serve as the third largest impact. Furthermore, although investment-specific shock plays the third largest role in SW model, although it is also not a dominants driver account for other variable fluctuations. This is somewhat in line with Justiniano and Preston (2010) that argued the investment shocks act as the main driver of fluctuations.

The variance decomposition of exchange rate is different for both models. Foreign disturbances (export and import demand shock) make the most contribution (40% in SW model and 35% in SWBGG model) of explaining the real exchange fluctuation. Productivity and labour supply shock are also the key driver that contribute around 20% in both models. However, in SW model, government spending, preferences, monetary policy shocks are play a relevant role, while impact from these shocks is reduced in SWBGG model. With the introductory of financial disturbances, EFP shocks make a 13% contribution and net worth contributes a 4.1%, which is quite significant. Other studies, as in de Walque, et al (2005), the UIP shock explains around 60% to 70% of the exchange rate variance. However, since the UIP shock is assumed to be constant backing to equation (3. 13), such empirical results are not suitable for comparing.

Turning to nominal variables, the variance decomposition of inflation is quite similar for both models. Productivity and price mark-up shock are indicated by far the two-major source for explaining in inflation. Foreign shocks make up 5 to 6 % of the variance. In SWBGG model, financial shocks only explain less than 2% of inflation. Other than that, labour supply shocks compete with monetary policy shocks, technology shocks, and preference shocks to explain the bulk of movements in inflation.
However, financial shocks (especially EFP shock) play a virtual role for variability of interest rate. They situate with a significant impact of 20% in SWBGG model, although monetary policy plays in a dominate role. The impact of productivity and labour supply shock are also strong as they have a contribute of around at least 15% in both models. The impact of price mark-up is situated at 5.1% in SWBGG model, while the contribution doubled when the financial friction switched off.

Last but not the least, EFP, net worth and investment shock are the main source of external finance premium variability. The rest of the fluctuations that explained by labour supply shock and monetary policy shock are limited, while productivity shock contributes the impact of 7%. All in all, the impact of wage mark-up shock is virtually not trivial.

### 4.4.3 Historical decomposition

In discussing the historical decomposition, it concentrated on three variables: output, interest rate and inflation. Figure 4.9 to 4.12 depicts the historical decomposition for selected variables. The historical decomposition of the shocks to output is shown in Figure 4.9, as shown, in the recession of the early 1990s productivity shocks (in turquoise), labour supply shocks (in brown) are the most significant component of negative shocks to output, and interest rate (in yellow) shock plays a negative role in explaining the output; while the external finance premium (EFP) shock (in dark grey) makes a positive contribution to output. This implies this recession period is not due to any shocks in financial market, but the traditional ‘macro’ shocks, such as productivity and labour supply shocks. Moreover, investment specific shocks (in light grey) also make a positive contribution during the early 1990s recession and almost a positive contribution over 1998 to 2007.

During the financial crisis period after 2008, productivity shock contributes negatively to output in the crisis period until 2013Q2. It then makes a positive contribution in recovery the economy. Labour supply shocks however contribute positively to output before the crisis, but their contribution turns negative after the 2008, then return positive from 2013 onwards to the end of the period. It is noted during recession of the early 1990s, the recovery period of late 1990s to 2000, EFP shock make a material positive contribution, before it plays a dampening role during the financial crisis. The EFP shock accounts for a significant portion of drop in output from 2008Q2, and continues the negative effects on output onwards. Entrepreneurial
new worth shock (in sandy brown) also contributes negatively to output, whereas their contribution is limited. The investment shock turns to have a significant negative impact from 2008 to 2015, including the recovery time from 2014. The negative contribution from investment shock is intuitive since the negative impact from EPF shock drive investment then falls further due to the effects of the external finance premium friction. Compared with the previous recession, interest rate shock partially offsetting the downturn with its positive contribution due to a slashing and continuing interest rate cuts. Government spending shock (in dark blue) makes virtually little contribution to not only the downturn both for early 1990s recession and the financial crisis period, but also the whole cyclical fluctuations. Turning to open economy shocks, the effects from export demand and import demand (in dark green) shocks usually offset each other for most of the period.

By comparing the two periods of recession discussed above, the main dominant effect is still coming from the non-financial shocks. With the introduction of financial shocks, they have material negative contributions in the recent cession, however they do contribution positively in the early 1990s recession. It is striking that the investment shock has an almost identical impact on output as the EPF shock has. Therefore, it seems that financial shocks, especially EFP shock partially involves to relegated the role of investment specific shock.

Figure 4.11 explains the contribution of the shocks to the nominal interest movement. The monetary policy rule shock was clearly but not surprisingly, had a major impact on interest rate over the horizon, in particular, before 2000. EPF shock also give a important role for explaining the movement. In the recent crisis, it together with monetary policy shock, had a negative contribution. Such negative impact then partially offset by labour supply shock and price mark-up shock. Besides that, government spending shock was also contributing to movement of interest rate and the impacts are negative over the periods. Figure 4.12 shows the price mark-up and productivity shock has the largest effect on the movement of inflation. The productivity shock in the sense has a negative effect on output and a positive effect on inflation. Price mark-up shocks come to the second largest shock in the sense that usually have a positive effect on the movement of inflation. Monetary policy also contributes a positive effect on the movement of inflation during and after the crisis time. Not surprisingly, the three shocks were situated for explaining movement. Besides, labour supply and EPF shocks also have sizeable effects.
4.4.4 Robustness Check

In this section, we present a suite of robustness checks to evaluate alternative sample periods and the strength of misspecification of debt contract.

**Post 1992 sample re-evaluation**

Our baseline estimation period spans from 1975Q1 through to 2015Q4 includes different monetary regimes. First, monetary targeting in the late 1970s and early 1980s, second, the sterling’s exit from the Exchange Rate Mechanism (ERM), and third the adoption of inflation targeting in October 1992 as the objective of monetary policy. To check for the robustness of the results, we consider re-evaluating the model with the post-1992Q3 sample by using the estimated structural parameter. I then investigate whether previous estimates results are sensitive to the chosen sample. In particular, I am interested to assess whether model can still fit the set of nominal interest rate with other variables. The test is based on the subset of output, inflation, real exchange rate and EPF shown in Table 4.7.

For subset $Y, \pi, R$, although the TMD t statistics (reported 1.34) is larger than that of in full sample (1.01), it is lower than 1.645 and still inside the 95% non-rejection critical level. For subset $Y, Q, R$, the t statistics is reported as 1.94 implying that this model cannot replicate the dynamic properties of a combination of output, inverse of real exchange rate and nominal interest rate in VARX (1) estimates. However, such result is close to a non-rejection level. Furthermore, for $Y, EFP, R$, the model is nevertheless significantly rejected.

The above results imply that, the estimated coefficients can be used to capture some key dynamic features of the non-financial data. In particular, model performances of output, inflation and nominal interest are not sensitive to the changing of monetary regimes or at least the monetary regime after 1992Q3. However, when considering a combination of finance and non-finance variables, such as EFP with output and interest rate, one has to re-estimate the model to fit the data better.
Table 4.7 Test result based on II estimation, full sample against sub-sample

<table>
<thead>
<tr>
<th>VARX(1)</th>
<th>Subsets</th>
<th>Full Sample</th>
<th>Sub Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Y, \pi, R$</td>
<td>21.17</td>
<td>22.77 (1.01) (1.34)</td>
</tr>
<tr>
<td>2</td>
<td>$Y, Q, R$</td>
<td>29.73</td>
<td>29.73 (1.55) (1.94)</td>
</tr>
<tr>
<td>3</td>
<td>$Y, EFP, R$</td>
<td>28.96</td>
<td>35.57 (1.73) (2.33)</td>
</tr>
</tbody>
</table>

†TMD t statistics are reported in parenthesis

Fisher debt deflation effect

As discussed in Section 2.2.3, more recent papers suggest that the inflation response to shocks can have powerful debt deflation effects when debt contracts are denominated in nominal terms. Christiano et al (2010) and von Heideken (2009) suggest that when debt contracts are denominated in nominal terms there are two factors which impact the cost of entrepreneurs’ borrowing; first the cost of borrowing fluctuates with the flow of entrepreneurial earnings and through capital gains and losses on entrepreneurial assets. This is the standard channel highlighted in BGG which tends to magnify the economic impact of a shock that affects economic activity. But they also highlight a second mechanism where entrepreneurs’ obligation to pay debt varies because inflation can ex post alter the real burden of debt. This second effect is referred to as a ‘Fisher debt deflation’ impact. Christiano et al. (2010) suggested that the Fisher debt deflation effect and pure accelerator mechanisms tend to reinforce each other, in the case if certain shocks move the price level and output in the same directions, but tend to be offsetting each other, in the case if shocks move the price level and output in opposite directions. Furthermore, for estimation result, they reject models without a Fisher deflation effect in favour of models that include them.

Recall the equation (3.39): $E_t[CY_{t+1}] = E_t\left[EP_{t+1}(\cdot)R_t\frac{P_t}{P_{t+1}}\right]$, with ‘Fisher debt deflation effect’, now the debt contract is re-specified in terms of the nominal interest rate $R_t$:

$$E_t[CY_{t+1}] = E_t[EP_{t+1}(\cdot)R_t]$$ (4.23)
The version of the model with ‘Fisher debt deflation effect’ retains all the channels of transmission, which are embodied in the model discussed in Chapter 3. Together with equation (3.40), the external finance premium now reads:

\[ E_t[EP_{t+1}(.)] = \frac{E_t[CY_{t+1}]}{R_t} = \varepsilon_t^{ep} \left( \frac{P_t^k K_{t+1}}{NW_{t+1}} \right)^{\chi} \]  

(4.24)

We can also derive the log linearized equation as:

\[ E_t c y_{t+1} - r_t = \chi(qq_t + k_t - n w_t) + e p r_t \]  

(4.25)

Figure 4.18 illustrates a productivity shock drives output and inflation in the different directions, since the response of capital price is larger in the SWBGG Model than it is when it includes the Fisher deflation effect (red dotted line). The responses of capital, investment and output in SWBGG are also comparatively larger. On the other hand, when a monetary policy shock hits the economy shown in Figure 4.19, it raises the nominal interest while decreases the output and inflation in a same direction. The response of capital and then investment and output is smaller than that of the model with fisher deflation effect. The IRFs is then in line with the conclusion of Christiano et al. (2010). However, when we continue to test this alternative model using the estimated coefficients from baseline SWBGG model (when fisher effect shut down), it does not surprisingly result to a strong rejection (although subset \( Y, \pi, R \) is close to a non-rejection critical level). Therefore, in order to match the dynamic properties of overserved variables by coping an extra Fisher deflation effect, we should re-estimate the model.

\begin{table}[h]
\centering
\begin{tabular}{llll}
\hline
VARX(1) & Subsets & SWBGG & SWBGG \\
& & & Fisher deflation effect \\
\hline
1 & \( Y, \pi, R \) & 1.01 & 1.80 \\
2 & \( Y, Q, R \) & 1.55 & 3.01 \\
3 & \( Y, EFP, R \) & 1.73 & 3.40 \\
\hline
\end{tabular}
\caption{TMD t-Test Result Based On II Estimation, Nominal Debt Deflation Effect}
\end{table}
4.5 Concluding remarks

In this chapter, we evaluate a medium sized DSGE model incorporating with financial friction and open economy setting that described in Chapter 3, to re-examine whether the calibrated structural parameter can fit the observed data. Instead of detrending the series, we estimate and evaluate the model using non-stationary data by indirect Inference method. We find models with and without the existence of financial frictions do not pass the Indirect Inference test using original calibrated parameters. We therefore search for set of parameters that fit the data best under the criteria of test, and use the set of parameters as the estimated one. The exploration of the drivers of the variability delivers the finding that financial shocks play a mixed role. In particular, they are the main driver of the variances in financial variables, investment. They play a fairly significant role as the driver of nominal interest rate and investment. However, they play less important roles as drivers of fluctuations for other variables including output, consumption, and inflation. The result also affirms the irrelevance of the investment-specific technology shock as a dominate driver of the variability when financial shocks are involved.
## Appendix 4.A Table and Figures

### Table 4.9 Variance decomposition of SW model for post-financial crisis episode: 2006Q1 to 2015Q4

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Interest rate</th>
<th>Inflation</th>
<th>Consumption</th>
<th>Output</th>
<th>Ex. rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Residual</td>
<td>1.1</td>
<td>0.9</td>
<td>0.7</td>
<td>3.8</td>
<td>2.4</td>
</tr>
<tr>
<td>Preference</td>
<td>1.2</td>
<td>0.3</td>
<td>18.5</td>
<td>1.9</td>
<td>3.7</td>
</tr>
<tr>
<td>Investment</td>
<td>0.9</td>
<td>4.3</td>
<td>10.3</td>
<td>20.4</td>
<td>2.8</td>
</tr>
<tr>
<td>Taylor rule</td>
<td>43.5</td>
<td>2.8</td>
<td>7.8</td>
<td>5.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Productivity</td>
<td>18.1</td>
<td>41.9</td>
<td>17.4</td>
<td>29.8</td>
<td>26.3</td>
</tr>
<tr>
<td>Price mark up</td>
<td>10.3</td>
<td>35.7</td>
<td>3.9</td>
<td>2.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Wage mark up</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Labour supply</td>
<td>13.5</td>
<td>8.0</td>
<td>25.5</td>
<td>23.4</td>
<td>16.6</td>
</tr>
<tr>
<td>Export</td>
<td>6.4</td>
<td>3.2</td>
<td>8.6</td>
<td>5.8</td>
<td>21.0</td>
</tr>
<tr>
<td>Import</td>
<td>5.0</td>
<td>2.9</td>
<td>7.3</td>
<td>6.7</td>
<td>19.8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>
Table 4.10 Variance decomposition of SWBGG model for post-financial crisis episode: 2006Q1 to 2015Q4

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Variable</th>
<th>Interest rate</th>
<th>Inflation</th>
<th>EFP</th>
<th>Consumption</th>
<th>Output</th>
<th>Ex. rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Residual</td>
<td>2.3</td>
<td>1.0</td>
<td>0.1</td>
<td>1.6</td>
<td>2.9</td>
<td>1.7</td>
<td></td>
</tr>
<tr>
<td>Preference</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>13.3</td>
<td>1.6</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Investment</td>
<td>3.8</td>
<td>3.1</td>
<td>11.5</td>
<td>2.7</td>
<td>3.1</td>
<td>2.1</td>
<td></td>
</tr>
<tr>
<td>Taylor rule</td>
<td>32.3</td>
<td>2.2</td>
<td>0.6</td>
<td>6.6</td>
<td>4.8</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>15.6</td>
<td>39.1</td>
<td>6.9</td>
<td>20.6</td>
<td>27.7</td>
<td>21.3</td>
<td></td>
</tr>
<tr>
<td>Price mark up</td>
<td>5.1</td>
<td>37.5</td>
<td>0.3</td>
<td>4.6</td>
<td>3.1</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Wage mark up</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Labour supply</td>
<td>15.6</td>
<td>7.8</td>
<td>2.6</td>
<td>30.1</td>
<td>26.9</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td>External premium</td>
<td>17.8</td>
<td>1.6</td>
<td>62.3</td>
<td>5.6</td>
<td>15.4</td>
<td>13.0</td>
<td></td>
</tr>
<tr>
<td>Net worth</td>
<td>2.1</td>
<td>0.1</td>
<td>13.9</td>
<td>1.2</td>
<td>6.1</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>Export</td>
<td>3.4</td>
<td>3.8</td>
<td>0.9</td>
<td>7.1</td>
<td>4.1</td>
<td>17.2</td>
<td></td>
</tr>
<tr>
<td>Import</td>
<td>2.0</td>
<td>3.7</td>
<td>0.5</td>
<td>6.6</td>
<td>4.3</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.3 IRFs to a productivity shock
Figure 4.4 IRFs to a monetary policy shock
Figure 4.5 IRFs to an investment specific shock
Figure 4.6 IRFs to an external finance premium shock
Figure 4.7 IRFs to an wage markup shock
Figure 4.8 IRFs to an export demand shock
Figure 4.9  Historical shock decomposition of Output

- government spending residual
- preference
- productivity
- price markup
- premium
- networth
- output
- investment
- wage markup
- taylor rule
- export
- labour supply
- import
Figure 4.10  Historical shock decomposition Consumption
Figure 4.11  Historical shock decomposition Interest rate
Figure 4.12  Historical shock decomposition Inflation
Figure 4.13  Distribution of VARX (1) parameters of output, inflation and interest rate for SWBGG model based on calibration
Figure 4.14  Distribution of VARX (1) parameters of output, inflation and interest rate for SWBGG model based on II estimation
Figure 4.15  Distribution of VARX (1) parameters of Interest rate, external financial premium rate and exchange rate for SWBGG model based on II estimation
Figure 4.16  Simulations plot for selected endogenous variables
Figure 4.17  Actual data observed from 1975Q1 to 2015Q4
Figure 7.4.18 IRFs for a productivity shock with and without Fisher deflation effect
Figure 4.19  IRFs for a monetary shock with and without Fisher deflation effect
Appendix 4.B How to derive terminal condition

The model is solved by the projection method following Fair and Taylor (1983) and Minford et al. (1984, 1986). The following is the process to find the terminal condition for forward looking variables used in Fortran. This method is based on Minford et al. (1979) that terminal conditions can be obtained from an equilibrium analysis of the model. Over a finite time period at terminal date $T$, we assume that $Y_{T-2} = Y_{T-1} = E_{T-1}Y_T = \bar{Y}$.

$$
\begin{bmatrix}
\bar{r} \\
\bar{p} \\
\bar{k} \\
\bar{\tilde{c}} \\
\bar{\bar{w}} \\
\bar{\bar{c}} \\
\bar{\bar{e}} \\
\bar{\bar{m}} \\
\bar{\bar{q}}
\end{bmatrix}
$$

where $\bar{Y}$ is a vector of terminal values of all endogenous variables of the model.

Recall the log-linearized model presented in Appendix, the steady state gives the following system equations.

$$
\bar{r} = \frac{1}{e_\gamma} (c_1 \bar{c} + c_2 \bar{\tilde{c}} - c_3 \bar{p}) + c_3 (\bar{I} - \bar{l}) - \bar{c} + e b_\gamma
$$

$$
\bar{I} = \frac{1}{1 + \beta Y^{(1-\sigma_c)}} [\bar{I} + \beta Y^{(1-\sigma_c)} \bar{I} + \frac{1}{\gamma \sigma} \bar{p}_k] + e I_t
$$

$$
\bar{p}_k = \frac{1-\delta}{1-\delta + \rho k} \bar{p}_k + \frac{\beta \gamma}{1-\delta + \rho k} \bar{r} - c \gamma
$$

$$
\bar{k} = \left(1 - \frac{\delta}{\gamma}\right) \bar{k} + \left(1 - \frac{\delta}{\gamma}\right) \bar{I} + \left(1 - \frac{\delta}{\gamma}\right) \left(1 + \beta Y^{(1-\sigma_c)} \gamma \sigma \phi\right) e I_t
$$

$$
\bar{p} = \frac{1}{r_p} \left(\bar{r} - \rho \bar{r} + r_y (\bar{y} - \bar{y}) + e r_t\right) - r_y \bar{y}
$$

$$
\bar{w} = \frac{w^w}{1 + \beta \gamma^{(1-\sigma_c)}} \bar{w} + \frac{1}{1 + \beta \gamma^{(1-\sigma_c)}} \bar{w} + \frac{\beta \gamma^{(1-\sigma_c)}}{1 + \beta \gamma^{(1-\sigma_c)}} \bar{w} - \frac{1 + \beta \gamma^{(1-\sigma_c)\bar{w}}}{1 + \beta \gamma^{(1-\sigma_c)}} \bar{w} + \frac{I_w}{1 + \beta \gamma^{(1-\sigma_c)}}\bar{w} -
$$

$$
\left(\frac{1 - \beta \gamma^{(1-\sigma_c)\xi_w}}{\xi_w (1 + (\psi - \phi) \xi_w)}\bar{w} - \sigma_\bar{I} \bar{I} - \left(1 - \frac{\gamma}{1 - \gamma}\right) \bar{c} - \begin{pmatrix} \beta \gamma \phi \\ \gamma \end{pmatrix} + e w_t\right) + \left(1 - w^w\right) \left(\sigma I - \left(1 - \frac{\gamma}{1 - \gamma}\right) \bar{c} - \begin{pmatrix} \beta \gamma \phi \\ \gamma \end{pmatrix} -
$$

$$
(\bar{p} - \bar{p}) + e w_t\right)
$$

$$
\bar{c} = \frac{\bar{c}}{\bar{c}} \left(\bar{y} - \bar{I} - \bar{k} \bar{g} - \bar{c}_e - \frac{\bar{c}}{\gamma} \bar{x} + \frac{\bar{c}}{\gamma} \bar{c} - e g_t\right)
$$

$$
\bar{y} = \phi [a k \tilde{c} + (1 - \alpha) \bar{I} + e a_t]
$$
\[ I = -\bar{w} + \left( 1 + \frac{1}{\psi} \right) \bar{r}k + \bar{k} \]

\[
\bar{r}k = w^p \left( \frac{1}{a(1+\beta y^{(1-\sigma c)p})(1-\xi_p)} \right) \bar{R} - \frac{\beta y^{(1-\sigma c)p}}{1+\beta y^{(1-\sigma c)p}} \bar{r}k + \frac{\xi_p}{1+\beta y^{(1-\sigma c)p}} \bar{\pi} + \frac{1}{1+\beta y^{(1-\sigma c)p}} \left( (1-\xi_p \bar{w} - ea_t) - ep_t \right) + (1 - w^p)(-\frac{1-a}{a} \bar{w} + en_{t-1})
\]

\[
\bar{cy} = \chi(p^k + \bar{k} - \bar{nw}) + \bar{r} - \bar{\pi} + epr_t
\]

\[
\bar{nw} = \theta \bar{nw} + \frac{k}{nw}(\bar{cy} - \bar{cy}) + \bar{cy} + enw_t
\]

\[
\bar{x} = \bar{c}^T + \sigma \bar{q} + ex_t
\]

\[
\bar{m} = \bar{c} - \sigma \bar{q} + em_t
\]

\[
\bar{q} = \frac{1}{x} \left( -\frac{x}{y} \bar{x} + \frac{m}{y} \bar{m} - \bar{r}^T \cdot b_{f_{t-1}} \right)
\]

We express the system equations as Matrix form reads:

\[ \bar{Y} = A\bar{Y} + BZ_t \]

where \( \bar{Y} = \begin{bmatrix} \bar{r} \\ \bar{I} \\ \bar{p}^k \\ \bar{k} \\ \bar{\pi} \\ \bar{w} \\ \bar{c} \\ \bar{y} \\ \bar{r}k \\ \bar{cy} \\ \bar{nw} \\ \bar{x} \\ \bar{m} \\ \bar{q} \end{bmatrix} \) and \( Z_t = \begin{bmatrix} eb_t \\ et_t \\ ew_t \\ ew^r_t \\ ea_t \\ eg_t \\ ep_t \\ er_t \\ epr_t \\ enw_t \\ ex_t \\ em_t \\ b_{f_{t-1}} \end{bmatrix} \)
\[ A = \begin{bmatrix}
0 & 0 & \frac{1}{y} \beta y^{1-\sigma_c} & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \frac{1}{y} \beta y^{1-\sigma_c} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1-\delta}{1-\delta+\rho^K} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & (1-\frac{1-\delta}{y}) & 0 & (1-\frac{1-\delta}{y}) & 0 & 0 & 0 \\
\frac{1}{\tau_p} (\frac{1}{(1-\rho)} - \rho) & 0 & 0 & 0 & 0 & 0 & -\frac{\tau_p}{\tau} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \phi \alpha & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & \chi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 \\
\end{bmatrix} \]

\[ a_c = \frac{1}{c_4} (c_1 + c_2 - c_4 - 1) \]

\[ f_\pi = w^w \left( \frac{\beta y^{1-\sigma_c} - 1 - \beta y^{1-\sigma_c} \xi w - \xi w}{1 + \beta y^{1-\sigma_c}} \right) \]

\[ f_w = w^w \left( 1 - \frac{1}{1 + \beta y^{1-\sigma_c}} \left( \frac{1-\beta y^{1-\sigma_c} \xi w}{\xi w(1+\phi\rho-1)\xi w} \right) \right) \]

\[ f_c = w^w \left( \frac{1}{1 + \beta y^{1-\sigma_c}} \left( \frac{1-\beta y^{1-\sigma_c} \xi w}{\xi w(1+\phi\rho-1)\xi w} \right) \right) \]

\[ f_i = w^w \left( \frac{1}{1 + \beta y^{1-\sigma_c}} \left( \frac{1-\beta y^{1-\sigma_c} \xi w}{\xi w(1+\phi\rho-1)\xi w} \right) \right) \]
$$j_{\pi} = \frac{wp(1 - \beta y^{(1-\sigma_c)\omega})}{\alpha(1 + \beta y^{(1-\sigma_c)\omega})\left(1 - \beta y^{(1-\sigma_c)\rho}(1-\xi_p)\right)\left(1 - \beta y^{(1-\sigma_c)\epsilon_p}(1-\xi_p)\right)}$$

$$j_{rk} = \frac{wp(1 - \beta y^{(1-\sigma_c)\omega})}{\alpha(1 + \beta y^{(1-\sigma_c)\omega})\left(1 - \beta y^{(1-\sigma_c)\rho}(1-\xi_p)\right)\left(1 - \beta y^{(1-\sigma_c)\epsilon_p}(1-\xi_p)\right)} \left(\frac{1}{1 + \beta y^{(1-\sigma_c)\rho}(1-\xi_p)}\right)$$

$$j_w = (1 - \alpha) \frac{wp(1 - \beta y^{(1-\sigma_c)\omega})}{\alpha(1 + \beta y^{(1-\sigma_c)\omega})\left(1 - \beta y^{(1-\sigma_c)\rho}(1-\xi_p)\right)\left(1 - \beta y^{(1-\sigma_c)\epsilon_p}(1-\xi_p)\right)} - (1 - \alpha) \frac{(1-w^p)}{\alpha}$$

$$B = \begin{bmatrix}
\frac{1}{\epsilon_p} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & o_k & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}$$

$$o_k = \left(1 - \frac{1 - \delta}{\rho}\right)(1 + \beta y^{(1-\sigma_c)})^2 \varphi$$

$$m_w = -\frac{wp(1 - \beta y^{(1-\sigma_c)\omega})}{\alpha(1 + \beta y^{(1-\sigma_c)\omega})\left(1 - \beta y^{(1-\sigma_c)\rho}(1-\xi_p)\right)\left(1 - \beta y^{(1-\sigma_c)\epsilon_p}(1-\xi_p)\right)}$$
\[ n_{rk} = \frac{w^w}{1+\beta y^{(1-\sigma c)/p}} \left( \frac{(1-\beta y^{(1-\sigma c)/p})(1-\xi_p)}{\xi_p(1+\theta_p-1)e_p} \right) + \frac{(1-w^w)}{\alpha} \]

\[ t_{rk} = -\frac{w^w}{1+\beta y^{(1-\sigma c)/p}} \left( \frac{(1-\beta y^{(1-\sigma c)/p})(1-\xi_p)}{\xi_p(1+\theta_p-1)e_p} \right) \]

This gives, after rearranging the equation:

\[ I\bar{Y} - A\bar{Y} = BZ_t \Rightarrow \bar{Y} = \left( \frac{1}{I-A} \right) BZ_t \]

It verifies at terminal date \( T \), all endogenous variable values can be expressed as a function of values of error processes. In the model, productivity \( ea_t \) and foreign net assets over GDP ratio \( bf_{t-1} \) are considered as the two unit-root processes drive the non-stationarity property, while other error processes would be zero at \( T \). Hence, we only interested that how endogenous variables are affected by \( ea_t \) and \( bf_{t-1} \). We then write the expression in Fortran for solving the model.
Chapter 5: Conclusion

The motivation behind my research is related to the recent banking crisis. DSGE models with sticky prices and wages are not just attractive from a theoretical perspective, but they are also emerging as useful tools for forecasting and quantitative policy analysis in macroeconomics (Del Negro et al, 2005). However, these macroeconomic models have come under severe criticism for failing to predict the crisis. It is then important during the time of disruption of the financial markets, to know about the dynamic properties of the variables pertaining to those markets and discover how far the banking crisis have been caused by financial shocks.

At its core, this thesis follows the approach of the financial accelerator mechanism à la BGG (1999) reviewed in Chapter 2 and attempts to quantify the role of such frictions in business cycle fluctuations by estimating a DSGE model with using an Indirect Inference approach against the non-stationary data in Chapter 3 and 4. I base my analysis on a New Keynesian DSGE which closely follows the structure of the model developed by SW (2003, 2007), with the addition of the financial accelerator mechanism. The model is adjusted for Armington (1969) version small open economy with certain features according to Meenagh et al (2007) and Le et al (2009), and the model will adopt the setting from Le et al (2011) model that introduced the price and wage setting equation with a hybrid model: a weighted average of the New Classical flexible price and wage and New Keynesian nominal rigidity with Calvo (1983) mechanism. Further work in Chapter 4 lies in terms of fulfilling Kydland and Prescott (1982)’s original promise of integrating growth and business cycle theory. We evaluate and re-estimated model that can be successfully taken to non-detrended data which match both growth and business cycle features of the data at the same time following the approach developed by Meenagh et al (2012), and further used and assessed by Le et al (2012, 2013) and Minford (2015). It then highlights the role of the external finance external premium and its contribution to the last financial crisis.

Overall, the model captures the counter cyclical feature of external finance premium proposed in most of the literatures. The IRF shows an increase of the external finance premium shock would lead to an increase in the cost of capital borrowing. It then drives a reduction of capital price immediately which consequently decreases the investment. The increasing in investment causes a decline of the total output. The capital reduction on the other hand, weakens the net worth of the entrepreneurs enhancing the external premium. This exacerbates the balance sheet effect. Moreover, the role of the financial accelerator is highly procyclical because it amplifies the positive effect of the cut of the interest rate and it worsens the outcome of a contractionary
monetary policy. Although it does not impede financial frictions from remaining an ingredient to model, External finance premium shocks on the financial sector do not play a dominate role in explaining a recession. Financial friction shocks only account for less than 22% of output fluctuation. This is a much less effect than that found in previous studies such as Christensen and Dib (2008) and De Graeve (2008). The main dominant effects are still coming from the non-financial shocks, in particular, the non-stationary productivity shock and the labour supply shock. The result is in line with the finding from le et al (2012, 2013) that crises or financial crises mostly result from non-financial shocks, while financial shocks will add an extra layer of recession. The authors further proved that by simulation, even without the banking sector the financial crisis can still be created by non-stationary shocks.

Despite the progress made until serval challenges and extensions are still worth to investigate. The first novelty that could be added by banking sector friction in this framework. Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) explicitly model the financial intermediary as a source of financial frictions due to the agency problem between financial intermediaries and depositors that limits the amount of credit. Differing from the costly state verification problem as in BGG (1999), these model frameworks proposed the financial accelerator as an agency problem that introduces endogenous constraints on the entrepreneur’s leverage ratios. Gertler and Karadi (2011) also considered shocks to the net worth of private banks, who face BGG-like financial frictions in raising funds.

Secondly, in our contributions, we do not focus our attention on the problem of the zero lower bound of the interest rate. Since after the crisis and great recession, the notion of an effective lower bound on policy interest rates has become a concrete concern for monetary policy and the unconventional monetary policy is usually used in a context of liquidity trap. Fortunately, contributions like the ones proposed by Le et al., 2014 are evidences of this new promising direction. Their results suggested a Taylor rule for making monetary base respond to credit conditions could substantially enhance the economy’s stability. It combined with price-level and nominal GDP targeting rules for interest rates to stabilise the economy in further. The authors argued that with these rules for monetary control, aggressive and distortionary regulation of banks’ balance sheets as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) becomes redundant. The third novelty could be extending the model to a two or multi-country symmetric model frameworks such as de Walque et al (2005) or Le et al (2013). In general, we hope that exotic elements now will became standard features of the next generation of DSGE models.
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