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Implications of Binary Black Hole Detections on the Merger Rates of Double Neutron Stars and Neutron Star–Black Holes

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Abstract

We show that the inferred merger rate and chirp masses of binary black holes (BBHs) detected by advanced LIGO (aLIGO) can be used to constrain the rate of double neutron star (DNS) and neutron star–black hole (NSBH) mergers in the universe. We explicitly demonstrate this by considering a set of publicly available population synthesis models of Dominik et al. and show that if all the BBH mergers, GW150914, LVT151012, GW151226, and GW170104, observed by aLIGO arise from isolated binary evolution, the predicted DNS merger rate may be constrained to be 2.3–471.0 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ and that of NSBH mergers will be constrained to 0.2–48.5 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$. The DNS merger rates are not constrained much, but the NSBH rates are tightened by a factor of ∼4 as compared to their previous rates. Note that these constrained DNS and NSBH rates are extremely model-dependent and are compared to the unconstrained values 2.3–472.5 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ and 0.2–218 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, respectively, using the same models of Dominik et al. (2012a). These rate estimates may have implications for short Gamma Ray Burst progenitor models assuming they are powered (solely) by DNS or NSBH mergers. While these results are based on a set of open access population synthesis models, which may not necessarily be the representative ones, the proposed method is very general and can be applied to any number of models, thereby yielding more realistic constraints on the DNS and NSBH merger rates from the inferred BBH merger rate and chirp mass.

Key words: binaries: close – gravitational waves

1. Introduction

The first three binary black hole (BBH) detections by the aLIGO detectors (Abbott et al. 2016c, 2016d, 2017) have fundamentally impacted our understanding of the astrophysics and underlying physics (Abbott et al. 2016a, 2016b, 2016e, 2016f). With these detections, the rate of BBH mergers is constrained between 12 and 213 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ (Abbott et al. 2017). It is expected that, as aLIGO and Virgo reach their design sensitivity, we will detect double neutron star (DNS) and neutron star–black hole (NSBH) mergers too. In fact, the LIGO Scientific Collaboration had predicted DNS and NSBH merger rates to be 10–10,000 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ and 0.5–1000 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, respectively, by extrapolating the rates from the observed binary pulsars in the Milky Way Galaxy, way before their first observing run had begun (see Abadie et al. 2010 and references therein). After the non-detection of DNS and NSBH coalescences in the first observing run, they placed upper bounds on the DNS and NSBH merger rates to be 12,600 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$ and 3600 $\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$, respectively (Abbott et al. 2016g).

Many population synthesis and Monte Carlo simulations have also predicted the rates of DNS and NSBH systems in our universe (Tutukov & Yungelson 1993; Belczynski et al. 2002; Hurley et al. 2002; Nelemans 2003; Voss & Tauris 2003; Belczynski et al. 2008; Dominik et al. 2012, 2013). However, these rates are highly uncertain and span three orders of magnitude, depending on the assumptions that go into these simulations (Dominik et al. 2012). While certain population synthesis models do not have the capability to predict the rates of all three compact binary populations (DNS, NSBH, and BBH), some of them can do so for a given set of input

parameters (Belczynski et al. 2002, 2008; Gültekin et al. 2004; Grindlay et al. 2006; O'Leary et al. 2006; Ivanova et al. 2008; Miller & Lauburg 2009; Sadowski et al. 2008; Downing et al. 2010). Typically, such simulations consider many different models that capture different input parameters to model the galaxy and physics associated with the binary evolution and obtain the rates of all three binary populations. The input parameters may include metallicity of the stellar environment, mass loss due to stellar winds, details of mass transfer episodes in the binary evolution, kick imparted by the supernova explosion, chemical homogeneity of the surroundings, and age of galactic globular clusters if binaries are formed in dense clusters via dynamical interactions. It is interesting to note that different input physics and formation channels lead to very different merger rates of compact binaries containing NSs. For instance, models involving chemically homogeneous evolution (Mandel & de Mink 2016) or dynamical interactions (Gültekin et al. 2004; Grindlay et al. 2006; O'Leary et al. 2006; Ivanova et al. 2008; Sadowski et al. 2008; Miller & Lauburg 2009; Downing et al. 2010) tend to produce far fewer DNS and NSBH mergers as compared to the scenarios where binaries form in isolation (Dominik et al. 2012).

In this work, we argue that we should be able to combine the results of those population synthesis models that can predict the rates of all three compact binary mergers (DNS, NSBH, and BBH) with the *inferred* BBH merger rates by aLIGO and chirp masses[5] of the detected BBHs, leading to a considerable reduction in the uncertainty in the predicted DNS and NSBH

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[5] Chirp mass is the best measured parameter by gravitational wave observations during the inspiral phase of a binary and defined as $\mathcal{M} = (m_1 m_2)^{3/5}/(m_1 + m_2)^{1/5}$, where m_1 and m_2 are the component masses.

merger rates. Intuitively, the inferred BBH merger rates and their properties can constrain the uncertain physics that go into the population synthesis models, thereby narrowing down the range of DNS and NSBH merger rates. Indeed, this method implicitly assumes that the set of models we use to carry out this study is representative of the actual binaries that exist in nature. Our rate estimates can hence be refined when more accurate and more representative models are available. We note that several other methods have been proposed in the past to compare and constrain various astrophysical binary formation models using the compact binary merger rates and their observed parameters (Mandel & O'Shaughnessy 2010; Mandel et al. 2010, 2015; Messenger & Veitch 2013; O'Shaughnessy 2013; Dominik et al. 2015; Stevenson et al. 2015; Zevin et al. 2017). Similar studies have also been done using supernovae rates (O'Shaughnessy et al. 2008) and DNS populations (O'Shaughnessy et al. 2005).

The remainder of the paper is organized as follows. In Section 2, we briefly describe the binary formation models of Dominik et al. (2012), which predict merger rates for all the three binary populations. Section 3 explains our method of constraining DNS/NSBH merger rates while making use of BBH detections from aLIGO. Finally, Section 4 discusses the implications of our findings.

2. Isolated Binary Formation Models

Dominik et al. (2012, hereafter Dominik2012) proposed formation models for "isolated" compact binaries (DNS, NSBH, and BBH) merging in a Hubble time and calculated their merger rates using the StarTrack population synthesis code (Belczynski et al. 2002, 2008). Moreover, these models are publically available and provide the distribution of chirp masses of the compact binary and other properties of the mergers.[6] Because the details of the common envelope, the maximum mass of the NS, the physics of supernova explosions forming compact objects, and wind mass-loss rates of the progenitor stars are still very uncertain, Dominik2012 provided a set of population synthesis models while changing the associated parameters and input physics one at a time. There are 16 models: 1 standard (S) and 15 variations (V1-V15), and all of them are summarized in Table 1 in Dominik2012. In the standard model, the maximum NS mass is assumed to be $2.5\,M_\odot$, the rapid supernova engine (Fryer et al. 2012), and a physically motivated common envelope binding energy (Xu & Li 2010) have been used while the natal kicks in core-collapse supernovae are drawn from a Maxwellian distribution with $\sigma = 265\,\mathrm{km\,s^{-1}}$ (Hobbs et al. 2005).

Additionally, Dominik2012 considered two scenarios for the Hertzsprung gap of the donor stars. In the first scenario, they ignore the core-envelope boundary issue and calculated common envelope energetics as normal. This is the *optimistic* Hertzsprung gap evolution model (submodel A). In the other scenario, they assume that when a binary, in which the donor star is on the Hertzsprung gap, enters into a common envelope phase, it merges prematurely. This reduces the number of merging compact binaries and hence their rates. This is the *pessimistic* Hertzsprung gap evolution model (submodel B). Dominik2012 also considered two fiducial stellar populations with different metallicity: solar ($Z = Z_\odot$) and sub-solar ($Z = 0.1\,Z_\odot$). Consequently, they have models for four

scenarios: (1) submodel A, solar, (2) submodel B, solar, (3) submodel A, sub-solar, and (4) submodel B, sub-solar. Hence, in total, there are 64 formation models. The rates and chirp mass estimates for DNS, NSBH, and BBH mergers predicted by all these 64 models are given in Tables 2–3 and 6–9 in Dominik2012. If we assume these 64 models to be the representative ones in estimating the merger rates of compact binaries, we find that DNSs will have merger rates in 0.6–$774\,\mathrm{Gpc^{-3}\,yr^{-1}}$ while the merger rates for NSBHs will be 0–$330\,\mathrm{Gpc^{-3}\,yr^{-1}}$.

However, in reality, the universe has a distribution of metallicities that is also a function of redshift: low-metallicity at high redshift and vice-versa. The metallicity of the stellar environment affects the merger rates of compact binaries especially those containing BHs with lower metallicity leading to higher binary merger rates (Belczynski et al. 2010). Therefore, the above DNS and NSBH merger rates predicted by Dominik2012 using all 64 models may not be realistic as they are derived from only two metallicities. To overcome this issue, we instead use rate estimates for DNS, NSBH, and BBH systems as a 50%–50% contribution from both low- and high-metallicity environments (Belczynski et al. 2010; Stevenson et al. 2015). We take the average of solar and sub-solar metallicity rates as a rough approximation, predicted by an underlying model, i.e., submodel A and submodel B. In this way, we now have only 32 models (16 submodel A and 16 submodel B models) with DNS and NSBH merger rates to be 2.3–$472.5\,\mathrm{Gpc^{-3}\,yr^{-1}}$ and 0.2–$218\,\mathrm{Gpc^{-3}\,yr^{-1}}$, respectively, as compared to 0.6–$774\,\mathrm{Gpc^{-3}\,yr^{-1}}$ and 0–$330\,\mathrm{Gpc^{-3}\,yr^{-1}}$. Similarly, one can combine the chirp mass estimates from solar and sub-solar models by taking the minimum (maximum) of minimum (maximum) chirp masses predicted by solar and sub-solar models. We will use these combined rates and chirp masses from solar and sub-solar scenarios to rule out the binary formation models.

We note that later on, Dominik et al. (2015) incorporated the effect of cosmological evolution of binaries in their StarTrack code and computed merger rates for DNS, NSBH, and BBH systems. They considered two scenarios for metallicity evolution: (1) high end: a distribution of metallicities with median $1.5\,Z_\odot$ at redshift $z = 0$, and (2) low end: 50% of the stars form in galaxies at $z \sim 0$ with $Z = 0.2\,Z_\odot$ whereas the other 50% have metallicity $Z = 1.5\,Z_\odot$. Further, in de Mink & Belczynski (2015), it has been noted that the treatment for tidal locking of stars in Dominik2012 was incorrect, which led to an over-estimation of compact binary merger rates. We are, however, not considering these models in our analysis.

Soon after the detection of GW150914, Belczynski et al. (2016) updated their StarTrack code and proposed a new set of models to interpret the origin of GW150914 from the evolution of isolated binaries. Although the proposed models of Belczynski et al. (2016) are more realistic, the authors have not made the predicted merger rates for DNS and NSBH systems public. Similarly, while using the rapid binary population synthesis code COMPAS, Stevenson et al. (2017) demonstrated that the origin of the first three BBHs can be explained by classical isolated binary evolution in a low-metallicity environment. Additionally, these models do not provide any estimate for DNS and NSBH merger rates. Therefore, in our analysis, we do not consider these models to constrain the DNS and NSBH merger rates.

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[6] http://www.syntheticuniverse.org

Table 1
The Isolated Binary Formation Models that Survived
after the BBH Rates Constraint

Scenario	Models
submodel A	V1, V4, V8, V15
submodel B	S, V1, V2, V3, V5, V6, V7, V9, V10, V11, V12, V13, V14, V15

Table 2
90% Credible Bounds on the Chirp Mass for 3 GW detections, GW150914
(Abbott et al. 2016d), GW151226 (Abbott et al. 2016c), and GW170104
(Abbott et al. 2017), and 1 Candidate LVT151012 (Abbott et al. 2016b)

GW Event	Chirp Mass Range ($M_\odot$)
GW150914	26.6–29.9
LVT151012	14.0–16.5
GW151226	8.6–9.2
GW170104	18.4–23.5

Very recently, Belczynski et al. (2017) proposed another suite of population synthesis models to explain the origin of GW170104-like BBHs, as there could be a misalignment between the orbital angular momentum of the binary and BH spins (Abbott et al. 2017), assuming that the spin magnitudes are *not* inherently small (Farr et al. 2017). These models suggest that all of the BBHs detected by aLIGO so far can be formed from isolated binary evolution with BHs having small natal spins. Unlike Belczynski et al. (2016), Belczynski et al. (2017) provide the merger rates of DNS and NSBH systems along with that of BBHs. Our constraints on DNS and NSBH merger rates will be unaffected even if we include these latest models in our analysis.

Chruslinska et al. (2017) recently demonstrated that the StarTrack code used in Dominik2012 does not incorporate enough physics regarding the natal kick velocities of NSs in DNSs and hence fails to reproduce orbital period and eccentricities of the observed galactic DNSs. While improving upon the natal kick distribution of NSs in the DNS population, Chruslinska et al. (2017) also provide galactic merger rates for DNSs. We verify that the DNSs merger rates predicted by Chruslinska et al. (2017) fall within the bounds provided in this paper.

In the next section, we show how we can tighten the DNS and NSBH merger rate estimates with the help of known BBH mergers.

3. Methodology

After the third aLIGO detection, GW170104, the BBH merger rates were revised to 12–213 Gpc^{-3} yr^{-1} (Abbott et al. 2017). Moreover, after the end of the first observing run, aLIGO has also placed an upper bound on the DNS and NSBH merger rates of 12600 Gpc^{-3}yr^{-1} and 3600 Gpc^{-3}yr^{-1}, respectively (Abbott et al. 2016g). In this paper, we present an independent method of constraining the DNS and NSBH merger rates from the BBH detections by aLIGO.

We rule out those isolated binary formation models that predict BBH merger rates outside the range inferred by aLIGO (12–213 Gpc^{-3} yr^{-1}). In this way, we rule out 14 models, and the remaining 18 models are given in Table 1. For DNS binaries, the lowest merger rate is predicted by both submodel A and submodel B scenarios of the V1 model (2.3 Gpc^{-3} yr^{-1}), whereas the highest DNS merger rate is given by the submodel A scenario of the V15 model (471 Gpc^{-3} yr^{-1}). Note that we use the following conversion to translate the rates given in Dominik2012 in the units of Gpc^{-3} yr^{-1},

$$1 \text{ MWEG}^{-1} \text{ Myr}^{-1} = 10 \text{ Gpc}^{-3} \text{ yr}^{-1}, \quad (1)$$

where MWEG stands for Milky Way Equivalent Galaxy. The resulting bound on DNS merger rates, 2.3–471 Gpc^{-3} yr^{-1}, is not much different than the previous rates. Similarly, the

Table 3
The Isolated Binary Formation Models that Survived
after the BBH Chirp Mass Constraint

Scenario	Models
submodel A	S, V1, V2, V3, V4, V5, V6, V7, V8, V9, V10, V11, V12, V13, V14, V15
submodel B	V1, V15

Table 4
The Isolated Binary Formation Models that Survived
after Both BBH Rates and Chirp Mass Constraints

Scenario	Models
submodel A	V1, V4, V8, V15
submodel B	V1, V15

surviving models place a bound on the predicted NSBH merger rate to be 0.2–170 Gpc^{-3} yr^{-1}, which has tightened by a factor of ~1.3.

We can also rule out certain binary formation models on the basis of chirp mass measurements of detected BBHs. In Table 2, we provide the 90% credible bound on each of the GW events, and we see that the lowest and highest observed chirp masses are 8.6 $M_\odot$ and 29.9 $M_\odot$, respectively. As there are uncertainties in the chirp mass measurement, we take a rather conservative approach in considering the lowest and highest measured chirp masses to rule out the isolated binary formation models. We use the 90% upper limit for GW151226 (9.2 $M_\odot$) as our lowest measured chirp mass and the 90% lower limit for GW150914 (26.6 $M_\odot$) as the highest measured chirp mass. Therefore, we rule out models whose lowest estimated chirp mass >9.2 $M_\odot$ or highest estimated chirp mass <26.6 $M_\odot$. In this way, we rule out 14 models, and the remaining ones are listed in Table 3. We find that the surviving models in Table 3 predict the DNS and NSBH merger rates to be 2.3–472.5 Gpc^{-3} yr^{-1} and 0.2–218 Gpc^{-3} yr^{-1}, respectively. This implies that ruling out the binary formation models on the basis of observed BBH chirp masses does not constrain DNS and NSBH merger rates. Note that the predicted merger rate for NSBHs would have been tighter (0.2–94.5 Gpc^{-3} yr^{-1}), if we had chosen 8.6 $M_\odot$ and 29.9 $M_\odot$ as our lowest and highest observed chirp masses, respectively, instead.

Finally, if we apply both of the constraints, i.e., BBH rates and chirp masses, only 6 out of 32 models survive, and they are listed in Table 4. We find that the lowest merger rate for DNS systems (2.3 Gpc^{-3} yr^{-1}) is given by both the submodel A and submodel B scenarios of the V1 model, whereas the highest rate (471 Gpc^{-3} yr^{-1}) is predicted by the submodel A scenario of the V15 model. Therefore, the final bound on the DNS merger rate after both of the constraints are applied is 2.3–471

Gpc^{-3}yr^{-1}. Similarly, the final bound on the NSBH merger rates turns out to be 0.2–48.5 Gpc^{-3}yr^{-1}. Our method has tightened the DNS merger rates slightly whereas that of NSBHs are tightened by a factor of ~4. The predicted NSBH merger rates would have been 0.2–3.6 Gpc^{-3} yr^{-1} , tightened by a factor of ~60, if we had chosen 8.6 $M_\odot$ and 29.9 $M_\odot$ as our lowest and highest observed chirp masses, respectively.

4. Discussions

In this paper, we have shown that it is possible to constrain DNS and NSBH merger rates from the BBH detections by aLIGO. Many of the population synthesis models predict merger rates and other properties (such as chirp mass) for all three types of binary populations (DNS, NSBH, and BBH), assuming similar physics and input parameters. We argue that those models that predict BBH merger rates and chirp masses outside the range inferred by aLIGO will be ruled out. Consequently, the predicted DNS/NSBH merger rates inferred from the remaining models are better constrained. As a demonstration, we applied this method on publicly available models of Dominik2012. Assuming all of the BBHs, GW150914, LVT151012, GW151226, and GW170104, observed by aLIGO are formed from isolated binary evolution, we find that the DNS merger rate will be constrained to 2.3–471 Gpc^{-3} yr^{-1}, whereas the NSBH mergers will have rates in the range 0.2–48.5 Gpc^{-3} yr^{-1}. Note that the DNS and NSBH merger rates predicted by the full set of models of Dominik2012 were 2.3–472.5 Gpc^{-3} yr^{-1} and 0.2–218 Gpc^{-3} yr^{-1}, respectively, before such constraints were applied. Therefore, our method has tightened the NSBH merger rates by a factor of ~4, whereas the DNS merger rates are marginally constrained. While the results presented here are limited to a specific formation channel, the method proposed in this paper can be applied to any number of population synthesis models.

Our underlying assumption of all the detected BBHs being formed via isolated binary evolution provides the highest possible upper bound on the DNS and NSBH merger rates. This is because if the BBHs would have formed via some other channel, e.g., dynamical formation, the predicted merger rates for DNSs/NSBHs would be even smaller for those models.

Note that a fundamental assumption of this Letter is that BBHs and DNSs/NSBHs form from the same types of progenitors, namely isolated massive binary systems and the same underlying physical mechanism. In reality, these two populations can originate from multiple progenitors, some of which could be common to both and others distinct to a specific system. Hence, it is possible that the merger rates of these systems are not quite related. Future studies would need to consider multiple formation scenarios while constraining rates of different populations.

It is interesting to note that the new rates we quote here based on the BBH detections are much stronger than the upper limits from the actual DNS/NSBH searches (Abbott et al. 2016g) by aLIGO, though the methods are very different. Our new rate estimates may have implications for SGRB progenitor models assuming they are powered (solely) by DNS or NSBH mergers. For instance, after accounting for the beaming effect, the rate of observed SGRBs, 3–30 Gpc^{-3} yr^{-1} (Coward et al. 2012), cannot be accounted for solely by a population of NSBH mergers, whereas the DNS merger rates may account for SGRBs with an appropriate beaming

correction. The beaming angle for SGRBs is given as

$$\cos \theta_j = 1 - \frac{R_{\mathrm{SGRB}}}{R_{\mathrm{merger}}}, \qquad (2)$$

where R_{SGRB} is the SGRB rate and R_{merger} is the progenitor merger rate. Therefore, with the new predicted merger rates for DNSs (NSBHs), one can place a bound on the beaming angle to be 6.5°–107.7° (20.3°–67.6°) assuming SGRBs are solely produced by DNS (NSBH) mergers. On the other hand, if we consider NSBH merger rates from pessimistic chirp mass constraints, i.e., 0.2–3.6 Gpc^{-3} yr^{-1}, one can place only a lower bound on the beaming angle of 80.4°, assuming SGRBs are solely produced by NSBH mergers. Considering this lower bound on beaming angle, it seems unrealistic for NSBH mergers to be the only progenitors of SGRBs.

Any future BBH detections will further tighten the DNS/NSBH merger rates, as they will not only place a tighter bound on the current BBH merger rates but will also improve observed chirp mass distribution. For example, let us consider that at some time in the future, aLIGO detects a BBH merger of chirp mass $<5\,M_\odot$ and BBH merger rates are updated to be 25–150 Gpc^{-3} yr^{-1}. Then both submodel A and submodel B scenarios of the V1 model will be ruled out and the resulting DNS and NSBH merger rates will be 128–471 Gpc^{-3} yr^{-1} and 1.5–48.5 Gpc^{-3} yr^{-1}, respectively, which would tighten at the lower-end side.

As mentioned earlier, the models of Dominik2012 used in this paper are not the representative ones and have many shortcomings as compared to the models proposed recently (Belczynski et al. 2016, 2017; Chruslinska et al. 2017; Stevenson et al. 2017). Though these recent models are more realistic, there are still many uncertainties involved that need to be addressed. For example, these uncertainties include initial conditions (de Mink & Belczynski 2015), modeling of massive stellar evolution, chemically homogeneous evolution, rotation of stars, magnetic fields and their effect on stellar wind strength (Petit et al. 2017), mass transfer efficiency and mass-loss modes, common envelope binding energy, metallicity specific star formation rate as a function of redshift, and cosmological effects. The improvements in these binary formation models not only bring down the a priori uncertainties in the predicted DNS/NSBH merger rate but can be further constrained using the method proposed in this paper.

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