Discretionary Policy in a Small Open Economy: Exchange Rate Regimes and Multiple Equilibria *

Christoph Himmels† — University of Cardiff

Tatiana Kirsanova‡ — University of Glasgow

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Abstract

We study a Markov-perfect monetary policy in an open New Keynesian economy with incomplete financial markets. We analyze inflation and exchange rate targeting regimes and demonstrate that both cases may result in multiple equilibria. These equilibria feature qualitatively and quantitatively different economic dynamics following the same shock. The model can help us to understand sudden changes in the interest rate and exchange rate volatility in 'tranquil' and 'volatile' times under a fully credible 'soft peg' of the nominal exchange rate.

Key Words: Small open economy, Incomplete financial markets, Discretionary Monetary policy, Multiple Equilibria

JEL References: E31, E52, E58, E61, C61,F4

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†Address: Department of Economics, University of Cardiff. E-mail: HimmelsC@cardiff.ac.uk.

‡Address: Department of Economics, University of Glasgow. E-mail: Tatiana.Kirsanova@glasgow.ac.uk

1 Introduction

Choosing the exchange rate regime for an open economy is one of the classic macroeconomic problems. The conventional idea behind an exchange rate peg is that it will anchor inflation expectations and increase trade through lower uncertainty and smaller adjustment costs. It may also encourage investment into long-term projects due to lower exchange rate risk/ transaction costs and therefore has a positive economic impact (see e.g. Cote (1994) and Prasad, Rogoff, Wei, and Kose (2003) for the potential benefits of fixed exchange rate regimes). However, being prone to speculative attacks hard pegs became less popular, especially after the Asian crisis of 1997. On the other hand recent evidence suggests that monetary authorities in many developing countries still see targeting the nominal exchange as a priority, despite that they officially claim to have floating regimes.[1] Developing and emerging countries like Indonesia, Malaysia, Thailand, South Korea, Turkey, Russia adopted *de jure* flexible exchange rate regimes, but *de facto* the exchange rate remained one of the most important if not the only target of their monetary policy in the post 1997 decade.[2]

Despite a relatively tranquil post-1997 decade in most developing and emerging countries, the exchange rate volatility under these 'soft pegs' varied over time. There is also a number of studies that document difficulties in explaining sudden changes in 'regimes' between periods of high and low volatilities.[3] Theoretical explanations for these different regimes include non-rational behaviour, non-linear decisions or heterogeneity of agents like

[1] See e.g Levy-Yeyati and Sturzenegger (2005) and Calvo and Reinhart (2002).

[2] See Rahmatsyah, Rajaguru, and Siregar (2002) for Thailand, Dogolnar (2002) for Turkey, Korea, Malaysia, Indonesia and Pakistan, and Arize, Osang, and Slottje (2000) for 13 developing countries. Furthermore, evidence by Reinhart and Rogoff (2004) suggests that partial exchange rate targeting is still the predominant monetary policy regime in many developing countries.

[3] See e.g. Engel and Hamilton (1990), Clarida et al. (2003) or Chen (2006) who apply Markov-switching models to explain these changes. These models have also been employed to describe exchange rate behaviour in floating regimes. However, their success is still a matter of debate see e.g. again Clarida et al. (2003) and Engel, Mark, and West (2007).

the presence of ‘noise traders’ (see Jeanne and Rose (2002) for an important example).

In this paper we offer a different theoretical approach which can contribute to the explanation of these empirical observations. Using a simple small open economy model in the spirit of Galí and Monacelli (2005) but with incomplete financial markets we demonstrate that *the way* monetary policy is conducted might be responsible for the existence of time periods with large difference in the volatility of all macroeconomic variables, including the exchange rate.

Specifically, we demonstrate that discretionary monetary policy may result in multiple equilibria, consistent with different sets of beliefs of the private sector and the policymaker. A discretionary policy maker takes current and future economic conditions into account, but can only commit to current behavior. The current economic condition is affected by the past behaviour of the rational private sector which is again based on a forecast of future economic conditions and future policy. As a consequence multiple equilibria may arise: A policy maker responds to a state that is at least partly determined by forecasts of his behaviour. Different sets of beliefs about the future policy generate different future courses for a policy maker to follow. Therefore, if the economy is hit by a shock, it can follow one of several adjustment paths, where the volatility along these paths is different, resulting in different welfare outcomes.[4]

Once multiplicity of discretionary equilibria is a reality, coordination on the best equilibrium may be difficult. Discretionary policy is sequential: each period a new policymaker arrives into office, observes the current state and makes prediction – which are consistent with those of the private sector under the RE assumption – about the policy of all future policymakers. The policy is time-consistent by construction, so if the current policymaker

[4]The existence of multiple equilibria under discretionary policy in non-linear models has been well established by King and Wolman (2004) and Albanesi et al. (2003). Blake and Kirsanova (2012) show that multiplicity can also occur in LQ RE models under discretionary policy.

perceives a particular – even inferior – policy of future policymakers, it will be optimal for the policymaker to implement the same policy in the current period. A unilateral deviation of the current policymaker from the perceived policy plan is not beneficial to this policymaker, and the resulting discretionary policy is a Nash equilibrium in the game of consequent policymakers, see e.g. (Oudiz and Sachs, 1985; Dennis and Kirsanova, 2017). In other words, the existence of multiple equilibria implies that consequent policymakers may fail to coordinate on the best equilibrium or, equivalently, fall into an expectation trap.[5]

This paper compares and contrasts inflation targeting with soft exchange rate targeting under discretionary policy. Multiplicity arises in both cases. Under conventional inflation targeting the model has three stable discretionary equilibria. If the economy is hit by a cost-push shock, in two of the three equilibria the monetary policy maker raises the interest rate and reduces demand. Although the terms of trade improve, their positive effect on marginal costs is dominated by the effect of lower demand. The private sector reduces its holdings of net foreign assets creating a persistent deficit. The deficit and all macroeconomic variables converge back to the steady state at a very slow rate. In contrast, in the second equilibrium the cost push shock is initially accommodated, and the large depreciation of the terms of trade causes the net foreign assets to accumulate. Once the level of assets is high, the interest rate is raised and inflation is brought back to its base. This delayed increase in the interest rate also brings net foreign assets – and all macroeconomic variables – back to the base line very quickly. In the presence of multiple equilibria, a coordination failure occurs: the agents can choose any of them and a sunspot decides which one will realise.

A similar coordination failure may also happen under soft exchange rate targeting,

[5]See Cooper and John (1988) for a detailed discussion about coordination failures and multiple equilibria.

either partial or strict. The policy maker introduces an additional positive weight – in otherwise standard policy objective – that punishes the volatility of the nominal exchange rate. There is still a conventional discretionary equilibrium in which the exchange rate remains on target. However, as it is acceptable – but costly – that the future exchange rate can deviate from the target, the economic agents may coordinate on the second admissible equilibrium, in which the exchange rate is volatile around the target.

Our model abstracts from many features that may characterize developing countries, e.g. capital controls or incomplete exchange rate pass-through. However, we show that the assumption of incomplete financial markets in combination with discretionary monetary policy is sufficient to generate 'expectation traps', i.e. multiple policy equilibria which are associated with different volatilities of all macroeconomic variables. Our results are not restricted to this simple model, but will also prevail in more detailed settings. Both assumptions – incomplete financial markets and discretionary monetary policy – are justifiable for developing countries: Such countries have restricted access to international financial markets and the ability of policymakers to precommit to future policies is generally weaker than in developed countries.

The remainder of the paper is organized as follows. The next Section presents the model. Section 3.1 discusses the policy equilibria for an inflation targeting regime and Section 3.2 discusses them under nominal exchange rate targeting. Section 4 concludes.

2 Model Highlights

We use a workhorse small open economy model, based on Galí and Monacelli (2005) and De Paoli (2009), with incomplete financial markets as in Benigno (2009). Specifically, there are two countries: the small open economy (Home) and the rest of the world (Foreign). The size of the Home economy is infinitely small relative to the size of the Foreign economy, therefore the economic performance and policy decisions do not have any impact on the rest of the world. Each economy is populated by infinity-living households and firms. Households' consume two goods, home- and foreign-produced, and their preferences reflect home bias in consumption. The law of one price holds. Firms are monopolistically competitive, and only use labor to produce differentiated tradable goods. Production takes place in two stages. First, there is a continuum of intermediate goods firms, which produce a differentiated input. In the second stage final goods producers combine these inputs into output and sell them to households in both countries. Monopolistic competition and sticky prices give a meaningful role for monetary policy. Each country has an independent fiscal authority, which finances spending by distortionary taxes and bonds. Home bonds are not tradable and in zero net supply, while foreign bonds are internationally tradable. Financial markets are incomplete, and the portfolio allocation is determined by transaction costs. All profits received by home country firms and financial intermediaries are rebated to home households. The Home country is subject to cost-push shocks. Full details of underlying microfoundations of the model are given in Appendix A. In the following we present only the linearized equations.[6]

2.1 Private Sector Equilibrium

The household optimization problem for the small open economy H yields a consumption Euler equation

$$\hat{c}_t = \mathbb{E}_t\hat{c}_{t+1} + \sigma\alpha\left(\mathbb{E}_t\hat{S}_{t+1} - \hat{S}_t\right) - \sigma\left(\hat{\imath}_t - \mathbb{E}_t\pi_{Ht+1}\right), \tag{1}$$

where $\hat{c}_t$ denotes consumption, $\hat{S}_t$ is the terms of trade (relative price of foreign producer price in terms of home producer price), $\hat{\pi}_{Ht}$ is Home producer price inflation and $\hat{\imath}_t$ is the

[6] We linearize the model around the unique zero-inflation efficient steady state, see Appendix A.

short term nominal interest rate.[7] Parameter σ is the inverse of the intertemporal elasticity of substitution and α is the degree of trade openness.

The firms' optimization problem gives the standard New Keynesian Phillips curve for the producer price inflation

$$\hat{\pi}_{Ht} = \hat{\upsilon}_t + \frac{\lambda}{\left(1+\frac{\tau^l}{\mu}\right)}\left(\alpha\hat{S}_t + \frac{1}{\sigma}\hat{c}_t + \left(\varsigma - \frac{\tau^l}{\mu}\right)\hat{y}_t + \frac{\tau^l}{\mu}\hat{g}_t\right) + \beta\mathbb{E}_t\hat{\pi}_{Ht+1}, \tag{2}$$

where $\hat{y}_t$ is output, $\hat{g}_t$ is government spending, financed by distortionary taxes, and $\hat{\upsilon}_t$ is an AR(1) Home cost-push shock. Parameter ς is the inverse of the Frisch elasticity of labor supply and $\mu = \frac{\epsilon}{\epsilon-1}$ is a monopolistic markup which is related to the elasticity of substitution between home goods ϵ. Parameter β denotes the household discount factor and the slope of the Phillips curve $\lambda = (1-\theta\beta)(1-\theta)/\theta$ is a function of the Calvo (1983) probability of price change θ. Finally, parameter τ^l is the steady state value of the labour income tax. The Home country maintains a balanced budget, so the effect of marginal cost on inflation is scaled by the factor $1/\left(1+\frac{\tau^l}{\mu}\right)$.

The aggregate resource constraint can be written as

$$\hat{y}_t = (1-\alpha)\frac{c}{y}\hat{c}_t + \eta\alpha\left((1-\alpha)\frac{c}{y} + \frac{c^*}{y}\right)\hat{S}_t + \alpha\frac{c^*}{y}\hat{c}_t^* + \frac{g}{y}\hat{g}_t. \tag{3}$$

where $\hat{c}_t^*$ is Foreign consumption. Parameter η is the elasticity of substitution between home and foreign goods.

For the other, large and effectively closed economy, the corresponding equations are

$$\hat{c}_t^* = \mathbb{E}_t\hat{c}_{t+1}^* - \sigma\left(\hat{\imath}_t^* - \mathbb{E}_t\pi_{Ft+1}^*\right), \tag{4}$$

$$\hat{y}_t^* = \frac{c^*}{y^*}\hat{c}_t^* + \frac{g^*}{y^*}\hat{g}_t^*, \tag{5}$$

[7] Here and below, hatted variables indicate that they have been linearized relative to their steady states. Steady state variables are denoted by letters without time subscript.

$$\hat{\pi}^*_{Ft} = \lambda \left(\frac{1}{\sigma} \hat{c}^*_t + \varsigma \hat{y}^*_t + \frac{\tau^{*l}}{\mu} \hat{\tau}^{*l}_t \right) + \beta \mathbb{E}_t \pi^*_{Ft+1}, \tag{6}$$

$$b^T_t = \varrho \hat{\imath}^*_t + \frac{1}{\beta} \left(b^T_{t-1} - \varrho \hat{\pi}^*_{Ft} + \frac{g^*}{y^*} \hat{g}^*_t - \frac{\tau^{l*}}{\mu} \left(\hat{\tau}^{l*}_t + \frac{1}{\sigma} \hat{c}^*_t + (\varsigma + 1) \hat{y}^*_t + \frac{\tau^{*l}}{\mu} \hat{\tau}^{*l}_t \right) \right), \tag{7}$$

where b^T_t is normalized total real Foreign debt issued by the Foreign government and held by Home and Foreign residents, parameter $\varrho = \frac{b^T_F}{y^*}$.

Finally, the model is closed with the risk premium equation

$$\hat{\imath}_t = \hat{\imath}^*_t + \mathbb{E}_t \pi_{Ht+1} - \mathbb{E}_t \hat{\pi}^*_{Ft+1} + \mathbb{E}_t \hat{S}_{t+1} - \hat{S}_t - \delta \beta \left(b_t + \kappa (1 - \alpha) \hat{S}_t \right), \tag{8}$$

the current account equation

$$\begin{aligned} 0 \;\; = \;\; & \left(\alpha \frac{c}{y} \frac{y}{y^*} (1 - \eta (1 - \alpha)) - \frac{c^*}{y^*} \alpha \eta - (1 - \beta) \kappa \right) S_t + \alpha \frac{c}{y} \frac{y}{y^*} \hat{c}_t + \beta b_t - b_{t-1} \\ & - \alpha \frac{c^*}{y^*} \hat{c}^*_t - \kappa \beta \hat{\imath}^*_t + \kappa \hat{\pi}^*_{Ft} \end{aligned} \tag{9}$$

and the definition of nominal exchange rate $\hat{E}_t$

$$\hat{E}_t = \hat{S}_t - \hat{P}_{Ht} + \hat{P}^*_{Ft}, \tag{10}$$

where δ is a Home portfolio adjustment cost parameter, $\kappa = \frac{b_F}{y^*}$ is a measure of Foreign debt held by non-residents in the steady state.

The system of ten equations (1)-(10) describes the private sector equilibrium and determines $\hat{c}_t$, $\hat{y}_t$, $\hat{\pi}_{Ht}$, b_t, b^T_t, $\hat{c}^*_t$, $\hat{y}^*_t$, $\hat{\pi}^*_{Ft}$, $\hat{E}_t$ and $\hat{S}_t$, given the policy variables $\hat{\imath}_t$, $\hat{\imath}^*_t$, $\hat{g}_t$, $\hat{g}^*_t$, $\hat{\tau}^{l*}_t$.

2.2 Small Open Economy Model in LQ Policy Framework

In the following we will only consider the dynamics of the small open economy and treat all variables of the large closed economy as exogenous shocks. Hence, we only work with the system of equations (1)-(3) and (8)-(10). Endogenous variables are $\hat{c}_t$, $\hat{y}_t$, $\hat{\pi}_{Ht}$, b_t, $\hat{E}_t$ and $\hat{S}_t$, the policy instrument is $\hat{\imath}_t$. The cost push shock $\hat{\upsilon}_t$ as well as government spending

$\hat{g}_t$ is treated as exogenous. System (4)-(7) determines other exogenous processes $\hat{c}_t^*$, $\hat{y}_t^*$, $\hat{\pi}_{Ft}^*, b_t^T, \hat{\imath}_t^*, \hat{g}_t^*, \hat{\tau}_t^{l*}$.

System (1)-(3), (8)-(10) can be represented in the following form, suitable for standard policy analysis in the linear-quadratic (LQ) framework. For convenience we introduce a new variable

$$u_t = \hat{c}_t - \sigma(1-\alpha)\hat{S}_t \tag{11}$$

which measures the excess consumption under incomplete financial markets, as $u_t \equiv 0$ under international risk sharing. Substituting out consumption and the interest rate (using equation (8)), the consumption Euler equation can be re-written as

$$u_t = \mathbb{E}_t u_{t+1} + \delta\sigma\beta\left(b_t + \kappa(1-\alpha)\hat{S}_t\right) - \sigma\left(\hat{\imath}_t^* - \hat{\pi}_{Ft+1}^*\right). \tag{12}$$

The Phillips curve becomes

$$\begin{aligned}\hat{\pi}_{Ht} &= \hat{\upsilon}_t + \frac{\lambda\left(1+(\varsigma-\Theta_H)\left((\sigma(1-\alpha)+\eta\alpha)(1-\alpha)\frac{c}{y}+\eta\alpha\frac{c^*}{y}\right)\right)}{\left(1+\frac{g}{y}\right)}\hat{S}_t \\ &+\frac{\lambda\left(\frac{1}{\sigma}+\left(\varsigma-\frac{g}{y}\right)(1-\alpha)\frac{c}{y}\right)}{\left(1+\frac{g}{y}\right)}u_t \\ &+\frac{\lambda\left(\left(\varsigma-\frac{g}{y}\right)\alpha\frac{c^*}{y}\hat{c}_t^*+\frac{g}{y}\left(\varsigma+1-\frac{g}{y}\right)\hat{g}_t\right)}{\left(1+\frac{g}{y}\right)} + \beta\mathbb{E}_t\hat{\pi}_{Ht+1}\end{aligned} \tag{13}$$

and the current account equation is given by

$$\begin{aligned}b_t &= \frac{1}{\beta}b_{t-1} + \frac{1}{\beta}\left((1-\beta)\kappa + \alpha\left(\frac{c^*}{y^*}\eta - (1-(\eta-\sigma)(1-\alpha))\frac{c}{y}\frac{y}{y^*}\right)\right)S_t \\ &-\frac{\alpha}{\beta}\frac{c}{y}\frac{y}{y^*}u_t + \frac{1}{\beta}\left(\alpha\frac{c^*}{y^*}\hat{c}_t^* + \kappa\left(\beta\hat{\imath}_t^* - \hat{\pi}_{Ft}^*\right)\right).\end{aligned} \tag{14}$$

In the following we will treat the terms of trade $\hat{S}_t$ as policy instrument, as it only enters contemporaneously in equations (12)-(14).[8] The three endogenous variables in system (12)-(14) are the foreign debt b_t, producer-price inflation $\hat{\pi}_{Ht}$ and excess consumption u_t. Once this system is solved, the interest rate needed to deliver the optimal policy can be found from (8), consumption can be recovered from equation (11) and output can be found from the aggregate demand equation (3).

2.3 Monetary Policy Regimes

2.3.1 Inflation Targeting

In this paper we assume that the following quadratic policy objective is delegated to the central bank by either society or legislation (see e.g. Kam et al. (2009)):

$$W_t^{IT} = \frac{1}{2}\mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \left(\pi_{H,s}^2 + \omega_y \hat{y}_s^2\right), \tag{15}$$

where β_m is monetary policymaker's discount factor. The above policy objective has been shown by Woodford (2003) to approximate the aggregate of individual utility functions in a closed economy model with complete financial markets. In our model, this approximation will not hold up to the second order, but is frequently considered in the literature as a likely objective given to the central bank. Where relevant, this policy objective also plays the role of 'social loss', so all welfare losses are computed using metric (15).[9]

2.3.2 Nominal Exchange Rate Targeting

Under nominal exchange rate targeting the central bank uses the following objective

$$W_t^{E} = \frac{1}{2}\mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \left((1-\omega_e)\left(\pi_{H,s}^2 + \omega_y \hat{y}_s^2\right) + \omega_e \hat{E}_s^2\right), \tag{16}$$

[8] This approach is common in the literature, see Clarida et al. (1999) where consumption is treated as policy instrument and the interest rate is later reinstated from the Euler equation. Our approach is similar. Once the solution is found, the interest rate can be reinstated from the risk premium equation.

[9] Appendix B provides the robustness analysis to this form of policy objective.

where we impose an additional weight ω_e on the stabilisation of the nominal exchange rate around its steady state value.

If $\omega_e = 0$ then the objective (16) reduces to the standard inflation targeting regime (15). If $\omega_e = 1$ then the objective (16) is equivalent to a strict exchange rate targeting regime:

$$W_t^{E1} = \frac{1}{2}\mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \hat{E}_s^2. \tag{17}$$

This targeting regime has some similarities with a fixed exchange rate regime. In particular, this regime assumes that the policy maker announces the target, perhaps within a corridor (which we do not model as binding in any way, so it does not affect expectations of the private sector) and implements policy to keep the exchange rate on target. The exchange rate, however, is allowed to deviate from the target, although such deviations are costly.[10]

2.3.3 Policy, Policy Instrument and Solution

The central bank manipulates the short term interest rate to affect the terms of trade $\hat{S}$ to minimise loss (15) (or (16)) subject to system(12)-(14). A discretionary solution can be written in the form of linear feedback rules

$$b_t = \bar{b}_y \bar{y}_t + \bar{b}_x \bar{x}_t, \tag{18}$$

$$u_t = \bar{u}_y \bar{y}_t + \bar{u}_x \bar{x}_t, \tag{19}$$

$$\hat{\pi}_{H,t} = \bar{\pi}_y \bar{y}_t + \bar{\pi}_x \bar{x}_t, \tag{20}$$

$$\hat{S}_t = \bar{s}_y \bar{y}_t + \bar{s}_x \bar{x}_t \tag{21}$$

where vector $\bar{y}_t$ denotes the vector of endogenous predetermined states, and vector $\bar{x}_t$ denotes the vector of exogenous predetermined states. In case of inflation targeting vector

[10] The peg is 'soft' as it is a result of optimisation policy and so is different from a 'hard' peg where the monetary policy maker is prepared to sell any quantity of reserves at a given price to keep the exchange rate *exactly* on target. A hard peg cannot be modeled within our framework of optimisation with a quadratic loss function because any regime with quadratic loss function allows (costly) deviations from the parity.

$\bar{y}_t = [b_{t-1}]$ is a scalar, while in case of nominal exchange rate targeting it is $\bar{y}_t = [p_{Ht-1}, b_{t-1}]$. Vector $\bar{x}_t$ contains the stochastic component of the solution $\bar{x}_t^{IT} = [\hat{v}_t]$ for all policy regimes we consider.[11] A representation in the form of (18)-(21)is conveniant to illustrate the multiplicity of discretionary equilibria.

2.4 Calibration

The share of government spending to GDP, g/y and g^*/y^*, is set to 0.20 for each country. We set $\omega = \frac{b_F}{b_F^T} = 0.0$ as the small open economy is unlikely to be a substantial non-resident holder of the large country's debt. We set $\varrho = \frac{b_F^T}{y^*} = 0.6 \times 4$ which reflects a 60% annual debt to output ratio. Calibration of parameters g/y, g^*/y^*, ϱ and ω yields the steady state tax level needed to service debt $\frac{\tau^l}{\mu} = \frac{g}{y}$ and $\frac{\tau^{*l}}{\mu} = \frac{g^*}{y^*} + \varrho\,(1-\beta)$ .Finally, we calibrate the adjustment cost parameter $\delta = 0.01$ following Benigno (2009). This implies a 10 basis point spread of the domestic interest rate over the foreign one. We assume that the policymaker's discount factor $\beta_m = \beta$.

The calibration of structural parameters is standard. The model frequency is quarterly. The household's discount factor β is set to 0.99 which gives the steady state interest rate of 4% and the Calvo parameter θ is set to 0.75 which implies the average length of fixed price contracts of about one year. Openness is set to $\alpha = 0.3$. The inverse of the intertemporal elasticity is calibrated $\sigma = 0.5$, based on evidence in Attanasio and Weber (1995). The elasticity between home goods $\epsilon = 11$ and the inverse of the Frisch elasticity of labour supply $\zeta = 3$ are calibrated consistently with most estimations of DSGE models (Liu and Mumtaz (2011), Justiniano and Preston (2010), Chen et al. (2013)). The intertemporal elasticity of substitution between domestic and foreign goods η is set to 1.5, see Albonico

[11] Coefficients $\bar{b}, \bar{u}, \bar{\pi}, \bar{s}$ can be found by solving the first order conditions. Our definition of discretion is conventional, see e.g Clarida et al. (1999). We present the general discretionary problem in LQ models in Appendix E.

et al. (2016) and Adolfson et al. (2008). Finally, following Galí and Monacelli (2005) cost push shocks in the domestic country and the rest of the world follow AR(1) processes with persistence parameter $\rho_v = 0.4$. The standard deviation of a cost push shock is 0.005. We only consider one shock in our welfare computations, as this does not affect any of our results. Adding more shocks simply rescales the loss numbers in a not-informative way.

3 Discretionary Policy and Expectations Traps

In this section we demonstrate that expectation traps are relevant for a monetary policymaker in a small open economy. We first consider standard inflation targeting policy. To stabilise inflation, the policymaker must choose a plan how quickly to bring the marginal costs back to their steady state level. The policymaker may be expected to stabilise marginal costs slower or faster, and multiple equilibria arise. Similarly, under soft exchange rate targeting, different speeds of stabilization of the nominal exchange rate is possible.

3.1 Inflation Targeting

Table 1 reports that the model exhibits three discretionary equilibria under inflation targeting, which we label A, B, and C.

Table 1 shows that equilibria A and B share certain characteristics while equilibrium C appears very different.[12] In particular, the feedback coefficients on inflation, terms of trade and the nominal interest rate in equilibrium C are all much larger in magnitude than those for equilibria A and B, suggesting greater volatility in a stochastic economy. In addition, while the nominal interest rate is lowered in response to higher foreign assets in equilibria A and B it is raised markedly in equilibrium C. These three equilibria produce qualitatively and quantitatively different economic dynamics, as shown in Figure

[12] The loss is measured in percentage of steady state consumption which needs to be sacrificed to eliminate stochastic volatility.

Table 1: Multiple Discretionary Equilibria under Inflation Targeting

Eqm.	Policy Reaction	Private sector Reaction	Implied response of interest rate	Speed of adjustment	Value function	Average Loss
	$[s_b]$	$\begin{bmatrix} u_b \\ \pi_b \end{bmatrix}$	$[i_b]$	$\lambda_{\max}$	$[V]$	$L, \%C$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A	-0.1106	$\begin{bmatrix} 0.1013 \\ 0.0008 \end{bmatrix}$	-0.0036	0.9534	8.9e-05	0.0166
B	-0.1258	$\begin{bmatrix} 0.0950 \\ -0.0867 \end{bmatrix}$	-0.0856	0.9505	0.0713	0.0715
C	-3.1888	$\begin{bmatrix} 0.0004 \\ -0.8100 \end{bmatrix}$	2.8586	0.0824	0.7127	0.4498

1 which plots the responses of key variables to a one-percent domestic mark up shock. To understand these results, and also to provide an intuition for the rise of mulitiplicity, we look closer at the transmission mechanism of shocks under optimal discretionary policy.

Monetary policy aims to stabilize inflation and does it via influencing the path for marginal cost. The forward representation of the Phillips curve (13) can be written as

$$\hat{\pi}_{H,t} = \lambda \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \hat{mc}_s + \frac{1}{1-\beta\rho_\upsilon} \hat{\upsilon}_t, \tag{22}$$

where the real marginal costs can be expressed as[13]

$$\hat{mc}_s = w_1 \hat{S}_t + w_2 \left(b_{t-1} - \beta b_t\right) + w_3 \varepsilon_{d,t}. \tag{23}$$

It is apparent that movements in mc_t and mc_{t+1} are highly substitutable in terms of their effect on π_{Ht} and that there are multiple paths for mc_t that will return inflation to target. These different paths for real marginal costs are associated with different monetary policies

[13]Parameters $w_1 > 0$, $w_2 > 0$, w_3 and the composite shock $\varepsilon_{d,t}$ are given in Appendix C.

and with different performances in terms of the loss. Equation (23) shows that monetary policy can affect mc_t through two distinct channels. The standard response to lower marginal cost is to tighten monetary policy. In our case this means an improvement in the terms of trade, as $w_1 > 0$ in equation (23). Households will respond by reducing consumption and selling foreign assets as the terms of trade drive the evolution of net foreign assets in equation (14). Alternatively, the policy maker can conduct expansionary monetary policy which implies a depreciation of the terms of trade, but also causes net foreign assets to accumulate.

The key for multiplicity is that the impact of $\hat{S}_t$ and b_t on $\hat{mc}_t$ is in opposite directions. Notice that a reduction in $\hat{S}_t$ causes a fall in b_t and that $\hat{S}_t$ and b_t have countervailing effects on $\hat{mc}_t$. As a consequence, the desirability of each policy from the perspective of the period-t policymaker turns on how future policymakers are expected to respond to movements in the stock of net foreign assets.

Consider the case where *future* policymakers are expected to lower the interest rate and depreciate the terms of trade in response to a rise in the stock of net foreign assets. Following a positive cost push shock $\hat{\upsilon}_t$, the *current* policy of raising the real interest rate and causing $\hat{S}_t$ and b_t to decline will successfully deliver lower real marginal costs and inflation because the boost in future real marginal costs caused by the decline in the stock of net foreign assets is offset by lower terms of trade in the future. Under this approach, monetary policy responds to the positive markup shock by contracting demand, lowering real marginal costs and inflation, and next by lowering interest rates and increasing the terms of trade as inflation declines allowing the economy to recover, producing an equilibrium. Alternatively, if *future* policymakers are expected to raise the interest rate and reduce the terms of trade in response to a higher stock of net foreign assets, then a *current* policy that lowers the real interest rate and raises the terms of trade

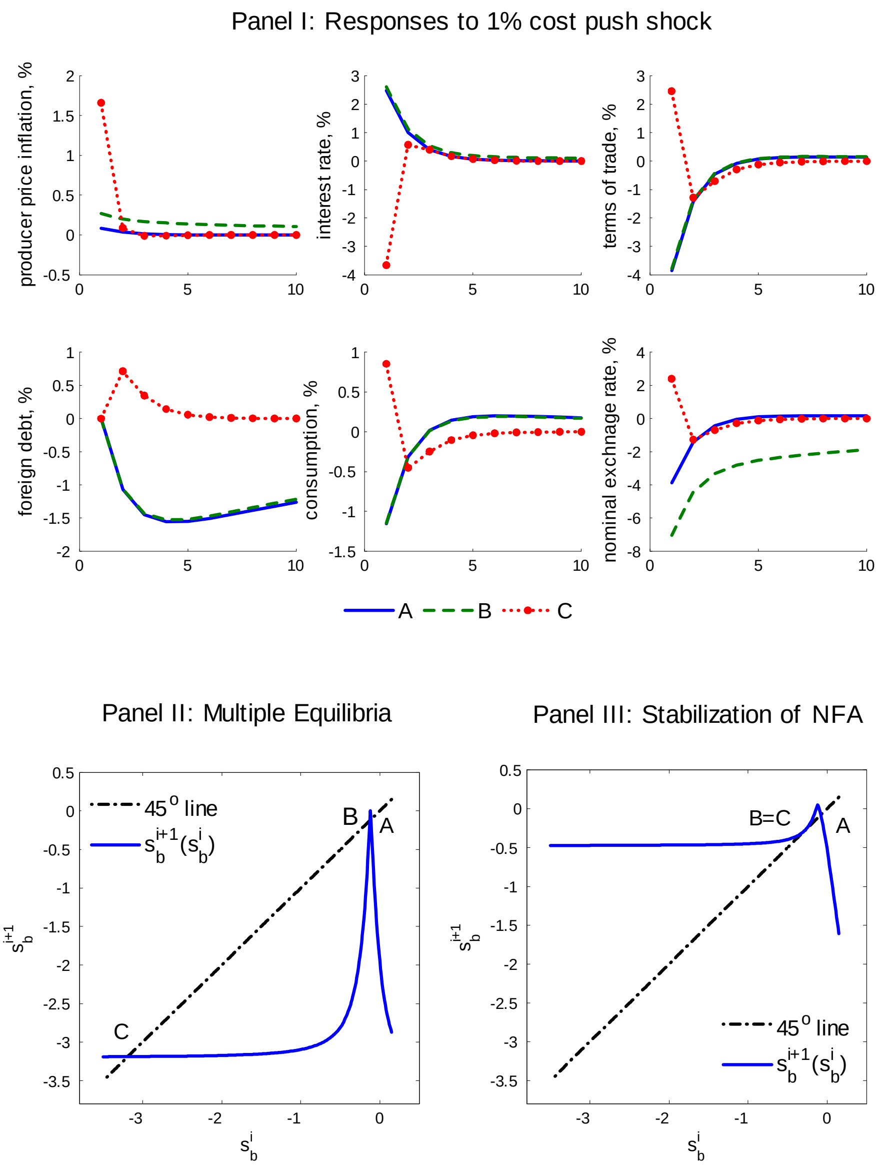

Figure 1: Inflation Targeting

can bring about a decline in inflation by stimulating the accumulation of foreign assets. This is despite the boost to $\hat{S}_t$ and mc_t today, because future policymakers respond to the higher foreign assets by tightening monetary policy, producing another equilibrium.

This is exactly what Panel I of Figure 1 illustrates. As anticipated, we find that the behaviour of the economy is notably different in equilibria A (and B) and C. In equilibrium C monetary policy accommodates cost-push shocks, but allows to bring the stock of assets back to the base line quickly. The tight monetary policy in equilibrium A (and B) results in a large reduction in the net foreign assets position with small but long-lasting effects on all macroeconomic variables.

These three equilibria are obtained numerically by searching for a fix-point in the policymaker's response to the stock of foreign assets and plotted in Panel II of Figure 1. Given a perceived policy reaction function (21) with feedback s_b^i we can compute the optimal reaction of the private sector (u_b^i, π_b^i) in (19)-(20) by solving for the rational expectations solution of system (12)–(14). The solution is unique for a wide range of specifications of the model. Given the reaction of the private sector (u_b^i, π_b^i), we can solve the maximisation problem of the central bank to find the best policy response, s_b^{i+1}. As the objective function is concave the solution is unique. In points where $s_b^{i+1} = s_b^i$ we have a discretionary equilibrium. There are three such points in Panel II of Figure 1. All three equilibria are private-sector learnable, but only equilibria A and C are jointly learnable, as discussed in Dennis and Kirsanova (2017). This implies that conventional methods of finding discretionary solution by backwards induction (Oudiz and Sachs, 1985) can only find equilibria A and C.

The discovered equilibria are robust to different calibrations of the model. Changes in σ, φ and b_H/y^* result in minor shifts of points of intersection in Panel II Figure 1, preserving qualitative differences in the dynamics of the different equilibria. The intermediation cost parameter δ does not affect equilibria in a significant way either. With δ tending to zero transaction costs fall and the degree of financial integration rises. All three equilibria survive for any $\delta > 0$.[14] However, there is a discontinuity at $\delta = 0$, where the problem is isomorphic to the one under international risk sharing, with private sector investing into state-contingent assets and one-period foreign bonds is one of them. A unique discretionary equilibrium with properties similar to those in equilibrium C can be obtained analytically (see Appendix D). Equilibria A and B correspond to a single solution with zero feedback of all control variables on net foreign assets, producing explosive dynamics of the economy, and are ruled out as discretionary equilibria by transversality conditions.[15]

Can the multiplicity be eliminated by delegating a particular policy objective to the policymaker? The answer is positive, and many such delegation schemes may exist. However, they do not necessarily result in higher overall welfare. For example, the intrinsic property of equilibrium C is that it implies a relatively fast adjustment of the endogenous predetermined state, foreign assets. Making the fast adjustment costly may help to eliminate this equilibrium. Panel III in Figure 1 illustrates a policy delegation where the policymaker minimises the loss with an additional penalty on changes in net foreign asset position

$$W_t^{IT,b} = \frac{1}{2}\mathbb{E}_t \sum_{s=t}^{\infty} \beta_m^{s-t} \left(\pi_{H,s}^2 + \omega_y \hat{y}_s^2 + \omega_b \left(b_t - b_{t-1} \right)^2 \right). \tag{24}$$

Panel III demonstrates that with $\omega_b > 0$ the line $s_b^{i+1}(s_b^i)$ shifts up, so points C and B eventually disappear, once ω_b is sufficiently large. However, this policy results in substantial welfare losses in the now equilibrium A. For our base line calibration, $\omega_b \simeq 32$ results in an average loss of 0.4549, which is slightly greater than loss of now eliminated equilibrium

[14] Checked numerically up to values $\delta =$ 1e-8.

[15] In this model we consider government debt. The results are unaffected by the debt ownership structure. Assuming that the foreign debt is privately-issued and its net supply is zero, would only affect exogenous processes.

C and substantially greater than the loss in equilibrium A with $\omega_b = 0$, see column (7) in Table 1.

Table 2: Effect of Discounting under Inflation Targeting

β_m	0.99	0.8	0.6	0.4	0.2	0.1	0.0
A	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166	0.0166
C	0.4498	0.4499	0.4502	0.4512	0.4566	0.4751	–

In contrast to the policy delegation scheme discussed in (24), the loss in the best equilibrium is unaffected if the monetary policymaker is impatient, $\beta_m < \beta$, see Table 2 which reports the losses in equilibria A and C for different discount factors β_m.[16] Once the policymaker discounts the future at a sufficiently high rate ($\beta_m \approx 0.08$ for the base line calibration), only the best equilibrium A survives.[17] Equilibrium C relies on the ability of the policymaker to delay stabilisation of inflation until future periods, which is ruled out by impatience. Table 2 demonstrates, however, that the loss in the worst equilibrium C increases with the degree of impatience, and an inability to choose the right discounting may result in a substantially worse outcome.

3.2 Exchange Rate Targeting

Unlike inflation targeting, discretionary policy *can be consistent* with keeping the nominal exchange rate on target at all times.[18] If the policy maker targets the nominal exchange

[16] This result is conditional on the form of the objective function. The above objective function contains only forward-looking terms. Had we for example kept the change in NFA as a target, the loss in both equilibria would change with discounting. However, for illustrative purposes we report the result for the standard inflation targeting regime. Here and below we use the 'social' objective with discount factor β to calculate the loss.

[17] Numerical simulations produce a picture which is qualitatively similar to Panel 3 in Figure III.

[18] Much of NOEM literature (see e.g. Galí and Monacelli (2005)) demonstrates that it is possible under interest rate rules.

rate, complete stabilization can be achieved. However, this equilibrium is not unique and the policymaker may not be able to achieve complete stabilisation of the nominal exchange rate.

The intuition for this result is similar to the one presented above. Exchange rate targeting requires to stabilise prices of foreign goods. We can rewrite

$$\begin{aligned}\hat{E}_t &= P_{F,t} - P_t^* = \hat{S}_t + P_{H,t} - P_t^* = \hat{S}_t + \pi_{H,t} + P_{H,t-1} - P_t^* \\ &= \hat{S}_t + \lambda \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \hat{mc}_s + \frac{1}{1-\beta\rho_\upsilon}\upsilon_t + P_{H,t-1} - P_t^*,\end{aligned}$$

which is similar to representation (22). Therefore, there is no surprise that we have several paths for marginal costs each of which eventually stabilises the nominal exchange rate. Figure 2 demonstrates the responses to a domestic cost push shock in two discretionary equilibria labelled A and C.[19] Responses are qualitatively similar to corresponding equilibria under inflation targeting plotted in Figure 1, Panel I. However, there are quantitative differences. In equilibrium A the policymaker reduces the terms of trade as little as needed to generate a small negative effect of marginal costs on inflation to match the increase in home price with the reduction in the terms of trade to keep the nominal exchange rate exactly on target. In this equilibrium inflation rises by much more than under inflation targeting in Figure 1, and this results in relatively large welfare losses, see Table 3.

In equilibrium C the policymaker lowers the interest rate sharply. The nominal exchange rate depreciates and is not kept on target. Nevertheless, this is a discretionary equilibrium consistent with the stabilization of the nominal exchange rate, as the policymaker is still able to keep the nominal exchange rate stable 'on average' in response to a positive cost push shock. As soon as the private sector expects that future policymakers

[19] To obtain these equilibria we used the standard way of backwards induction (Oudiz and Sachs 1985). It allows us to obtain equilibria which are jointly learnable, see Dennis and Kirsanova (2017). Iterations on policy reaction are not applicable, as we have more than one endogenous state variable. However, the discovered two equilibria are sufficient to illustrate multiplicity of equilibria.

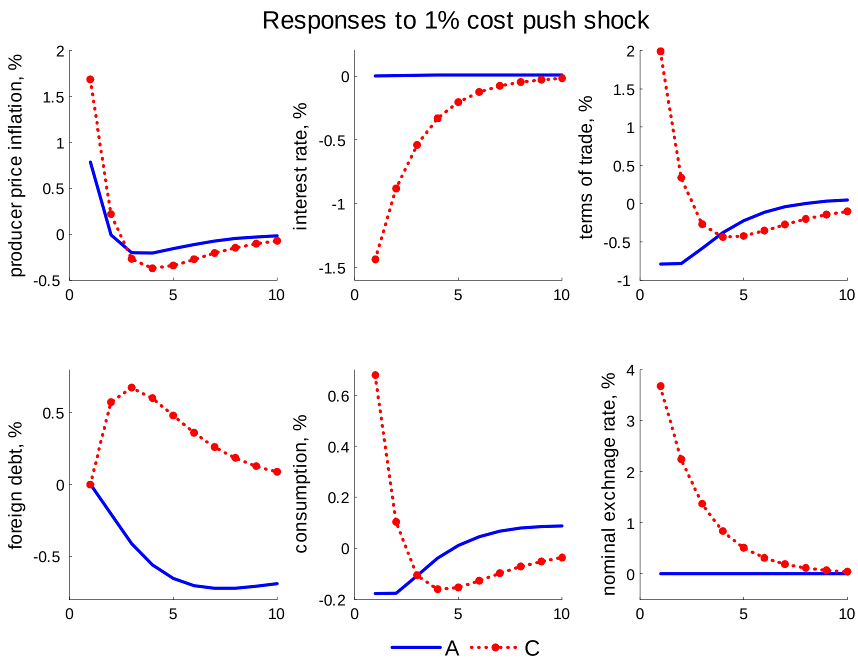

Figure 2: Soft Exchange Rate Targeting

would raise the interest rate and reduce the terms of trade in response to a higher stock of net foreign assets, the current policymaker lowers the interest rate, raises the terms of trade and therefore stimulates an accumulation of foreign assets. It, therefore, delivers the expected future policy response, and validates the expectations of the private sector. The stabilisation is costly, as the volatility of all variables is substantial. However, this strategy is consistent with the soft exchange rate target as it ensures the convergence back towards the target in the medium term, see Figure 2. In other words, as it is less costly to validate the expectations of the private sector than to accommodate them, the policymaker is trapped in this equilibrium. Similar to the inflation targeting regime the targeted variable is allowed to deviate from the target, and the current policymaker perceives that the private sector expects future policymakers to appreciate the nominal exchange rate, it will optimally choose to generate a depreciation of the nominal exchange rate today.

Table 3: Multiple Discretionary Equilibria under Strict Exchange Rate Targeting

Eqm.	Policy Reaction	Private sector Reaction	Implied response of interest rate	Speed of adjustment	Average Loss
	$\begin{bmatrix} s_p & s_d \end{bmatrix}$	$\begin{bmatrix} u_p & u_d \\ \pi_p & \pi_d \end{bmatrix}$	$\begin{bmatrix} i_p & i_d \end{bmatrix}$	$\lambda_{\max}$	$L, \%C$
(1)	(2)	(3)	(4)	(5)	(6)
A	$\begin{bmatrix} -0.6038 & -0.0409 \end{bmatrix}$	$\begin{bmatrix} -0.0462 & 0.1062 \\ -0.3962 & 0.0409 \end{bmatrix}$	$\begin{bmatrix} 0.0027 & -0.0097 \end{bmatrix}$	0.9532	0.1209
C	$\begin{bmatrix} 0.6537 & -2.7121 \end{bmatrix}$	$\begin{bmatrix} 0.0064 & -0.0053 \\ 0.0118 & -0.8257 \end{bmatrix}$	$\begin{bmatrix} -0.4247 & 1.0971 \end{bmatrix}$	0.6231	0.5437

Table 4: Multiplicity under Exchange Rate Targeting

ω_e	1.0	0.8	0.6	0.4	0.2	0.05	0.02	0.0
A	0.1209	0.1195	0.1171	0.1126	0.1006	0.0588	0.0328	0.0166
C	0.5437	0.5429	0.5417	0.5394	0.5336	0.5161	0.5087	0.4498

Multiplicity is also preserved in the more general form of exchange rate targeting using the hybrid objective (16). Table 4 reports losses for intermediate values of the relative weight on the exchange rate target $\omega_e \in (0, 1]$. If $\omega_e = 0$ the objective (16) coincides with inflation targeting objective (15). The losses in equilibria with $\omega_e > 0$ are higher than under inflation targeting.

Multiplicity, however, is eliminated if the policymaker is impatient. Equilibrium C arises as there is a possibility to stabilise the exchange rate tomorrow. Table 5 reports our results of reducing the policymaker's discount factor, β_m in the objective (17). Once it is sufficiently small ($\beta_m \lesssim 0.03$) the stable inferior equilibrium disappears.[20] The best

[20]This result was obtained numerically. Appendix F demonstrates that for $\beta_m = 0$ the nominal exchange

Table 5: Effect of Discounting under Exchange Rate Targeting

β_m	0.99	0.8	0.6	0.4	0.2	0.05	0.03	0.0
A	0.1209	0.1209	0.1209	0.1209	0.1209	0.1209	0.1209	0.1209
C	0.5437	0.5478	0.6038	0.8907	1.2740	1.6124	1.6608	–

equilibrium is invariant to β_m.

Despite it is commonly suggested that currency pegging is an efficient way to import low and stable inflation, it is apparent that in the case of a 'soft peg' the implied volatility of the nominal exchange rate and domestic inflation in the worst regime is higher than it is in the case of inflation targeting. This is not surprising: these are two 'second-best' scenarios, and there cannot be any *a priori* ranking between them.

4 Conclusion

This paper demonstrates how multiple equilibria can occur in a small open economy model with incomplete financial markets under discretionary monetary policy. As current policymakers cannot control the behaviour of future policymakers nor the expectation of the private sector coordination failures and expectation traps can occur. In our model policy makers need to decide if the economy should be stabilized today or at some point in the future.

We believe that the presented model is capable of explaining recent empirical evidence on exchange rate behaviour: there can be switches between policy regimes that are characterized by changes in the volatility of the nominal exchange rate. This can happen for a wide and realistic class of policy objectives, as long as the policy maker acts under discretion and there is at least one predetermined state variable in the system.

rate is always kept on target.

Although the presented model is highly stylised, and bringing it to the data is therefore beyond the scope of this paper, a sufficiently complex model with these features will retain multiplicity of equilibria and should be able to replicate the observed volatilities of key macroeconomic variables, in particular the nominal exchange rate.

References

Adolfson, M., S. Laseen, J. Linde, and M. Villani (2008). Empirical properties of Closed- and Open-economy DSGE Models of the Euro Area. *Macroeconomic Dynamics 12*(s1), 2–19.

Albanesi, S., V. V. Chari, and L. Christiano (2003). Expectation Traps and Monetary Policy. *Review of Economic Studies 70*, 715–741.

Albonico, A., A. Paccagnini, and P. Tirelli (2016). PIIGS in the Euro Area. An Empirical DSGE Model. Working Papers 331, University of Milano-Bicocca.

Arize, A., T. Osang, and D. Slottje (2000). Exchange-rate volatility and foreign trade: evidence from thirteen LDCŠs. *Journal of Business and Economic Statistics 18*, 10–17.

Attanasio, O. P. and G. Weber (1995). Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. *Journal of Political Economy 103*(6), 1121–1157.

Benigno, P. (2009). Price Stability with Imperfect Financial Integration. *Journal of Money, Credit and Banking 41*(s1), 121–149.

Blake, A. P. and T. Kirsanova (2012). Discretionary Policy and Multiple Equilibria in LQ RE Models. *Review of Economic Studies 79*, 1309–1339.

Calvo, G. (1983). Staggered Prices in a Utility-Maximising Framework. *Journal of Monetary Economics 12*, 383–398.

Calvo, G. A. and C. M. Reinhart (2002). Fear of Floating. *Quarterly Journal of Economics 117*(2), 379–408.

Chen, S. (2006). Revisiting the interest rate-exchange rate nexus: a Markov-switching approach. *Journal of Develpment Economics 79*(1), 208–224.

Chen, X., T. Kirsanova, and C. Leith (2013). An Empirical Assessment of Optimal Monetary Policy Delegation in the Euro Area? Paper presented at the Bundesbank/CEPR conference 'Inflation Developments after the Great Recession', Eltville 6-7 December 2013.

Clarida, R., L. Sarno, M. Taylor, and G. Valente (2003). The out-of-sample success of term structure models as exchange rate predictors: a step beyond. *Journal of International Economics 60*, 61–83.

Clarida, R. H., J. Galí, and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature 37*, 1661–1707.

Cooper, R. and A. John (1988). Coordination Failures in Keynesian Models. *Quarterly Journal of Economics 103*(3), 441–463.

Cote, A. (1994). Exchange Rate Volatility and Trade - A Survey. Working Paper 94-5, Bank of Canada.

De Paoli, B. (2009). Monetary Policy under Alternative Asset Market Structures: The Case of a Small Open Economy. *Journal of Money, Credit and Banking 41*(7), 1301–1330.

Dennis, R. and T. Kirsanova (2017). Equilibrium Coordination with Discretionary Policy Making. *Economic Journal*. Forthcoming.

Dogolnar, M. (2002). Estimating the impact of exchange rate volatility on exports: evidence from Asian countries. *Applied Economics Letters 9*, 859–863.

Engel, C. and J. Hamilton (1990). Long Swings in the Dollar: Are They in the Data and Do Markets Know It. *American Economics Review 80*(4), 689–713.

Engel, C., N. C. Mark, and K. D. West (2007). Exchange rate models are not as bad as you think. In *NBER Macroeconomics Annual 2007*, pp. 381–441.

Galí, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies 72*, 707–734.

Jeanne, O. and A. K. Rose (2002). Noise Trading and Exchange Rate Regime. *Quarterly Journal of Economics 117*(2), 535–569.

Justiniano, A. and B. Preston (2010). Monetary policy and uncertainty in an empirical small open-economy model. *Journal of Applied Econometrics 25*(1), 93–128.

Kam, T., K. Lees, and P. Liu (2009). Uncovering the Hit List for Small Inflation Targeters: A Bayesian Structural Analysis. *Journal of Money, Credit and Banking 41*(4), 583–618.

King, R. and A. Wolman (2004). Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria. *Quarterly Journal of Economics 199*(4), 1513–1553.

Levy-Yeyati, E. and F. Sturzenegger (2005). Classifiying Exchange Rate Regimes: Deeds vs. Words. *European Economic Review 49*(6), 1603–1635.

Liu, P. and H. Mumtaz (2011). Evolving Macroeconomic Dynamics in a Small Open Economy: An Estimated Markov Switching DSGE Model for the UK. *Journal of Money, Credit and Banking 43*(7), 1443–1474.

Oudiz, G. and J. Sachs (1985). International policy coordination in dynamic macroeconomic models. In W. H. Buiter and R. C. Marston (Eds.), *International Economic Policy Coordination*. Cambridge: Cambridge University Press.

Prasad, E. S., K. Rogoff, S.-J. Wei, and M. A. Kose (2003). Effects of Financial Globalization on Developing Countries: Some Empirical Evidence. International Monetary Fund Occasional Paper 220.

Rahmatsyah, T., G. Rajaguru, and R. Siregar (2002). Exchange-rate volatility, trade and 'fixing for life' in Thailand. *Japan and the World Economy 14*, 445–470.

Reinhart, C. M. and K. S. Rogoff (2004). The Modern History of Exchange Rate Arrangements: A Reinterpretation. *Quarterly Journal of Economics 119*(1), 1–48.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton, NJ.: Princeton University Press.