Foreign Exchange Rate and Financial Market Imperfections

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Abstract

The thesis discusses exchange rate dynamics in a small open economy Real Business Cycle model with financial frictions, aiming to investigate whether financial frictions in the global capacity to bear exchange rate risk had influences on Sterling real exchange rate dynamics between 1975 and 2016. In the model, international financial intermediaries as arbitrageurs face credit constraints and bear the risks caused by imbalances in the supply and demand of international bonds. The model has been estimated by using a simulation-based Indirect Inference approach, which provides a natural framework for testing the hypothesis implied by the model. The basic idea of Indirect Inference estimation is to search across model’s parameter space for the parameter set that the simulated data and the observed data look statistically the same from the vantage point of the chosen auxiliary model. The result shows that a comfortable non-rejection of the hypothesis that exchange rate dynamics are affected by financial forces at 5% significant level. It implies that financiers indeed require a risk premium to intermediate capital flows, and the uncovered interest parity fails to hold. Monte Carlo experiments support that the power of the Indirect Inference test to reject a false hypothesis is high; hence the results could be relied on. Empirical studies based on estimated model address that financial frictions will act as amplifiers of external shocks on the real exchange rate and other key UK macroeconomic variables. In addition, shocks to financial forces are the main driving forces behind large and sudden depreciations of the sterling exchange rates in the aftermath of the collapse of Lehman Brothers and the Brexit vote.
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Chapter 1

Introduction

Exchange rates are the core prices in both of the international macroeconomics and financial market. Understanding the exchange rate dynamics is still one of the most crucial questions for international economists, almost half a century after the collapse of the Bretton Woods fixed exchange rate system. “Why are exchange rates so volatile and so apparently disconnected from fundamentals?” (Obstfeld and Rogoff, 2000, p. 2). ‘Exchange Rate Disconnect Puzzle’ is one of six major puzzles in international macroeconomics. There are some other facts have emerged from the empirical analysis of exchange rate economics: failure of the uncovered interest parity and the related profitable carry trade, and the exposure of net debtor countries’ currencies to international financial shocks. These stylised facts stand at odds with the conventional general equilibrium models. This has given economists a new set of phenomena to explain; hence exchange rate economics is revitalized.

Recently the UK has been persistently running large current account deficits, which needs financing from abroad. As a net debtor country, Sterling is vulnerable to international financial shocks. There was a massive sterling depreciation at the end of 2007. Britain’s surprise decision to leave the European Union was followed by financial market tumult. Sterling dropped below $1.32, a 31 year low. Such plunges in Sterling during financial disruptions can not fully explained by macroeconomic fundamentals.
With the evolution of financial integration throughout the past few decades, international macro-finance becomes a new area of open economy macroeconomics that brings the theories of financial markets into the international macroeconomic context. Especially, the global financial crisis of 2007-2010 has emphasised the important role of the financial sector as the main transmission mechanism. There is a small set of studies\(^1\) that focused on exchange rate modelling in the presence of financial factors, and the thesis is contributed to this strand of the literature.

The central issue, in my view, is whether real exchange rate dynamics are determined by financial forces. And, if so, how do financial frictions help to generate currency risk premium and transmit financial shocks or external shocks into the real economy? What are the main driving forces of real exchange rate dynamics during financial disruptions, and implications for policies? This thesis attempts to address these questions by developing a theoretical framework of real exchange rate dynamics and the global financial markets in a small open economy UK, in which financial frictions take centre stage.

The theoretical framework highlights the main channel through how financial frictions affect behaviours of real exchange rates in the UK. Global financial intermediaries actively absorb imbalance caused by net foreign debt-based flows. However, financiers face binding credit constraints based on their limit risk bearing capacity and balance sheets. Thus, an endogenous risk premium, which compensates financiers for their currency risk-taking, has generated. Intuitively, a net debtor country’s currency should depreciate today and be expected to appreciate in the future to incentivise financier to intermediate capital flows. This mechanism also could help to explain the violation of uncovered interest parity and the empirical disconnect between exchange rates and macroeconomic fundamentals.

I consider a Real Business Cycle model of a small open economy adapted from Uribe and Schmitt-Grohe (2016), which I extend to include financial frictions in the intermediation

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\(^{1}\) Studies related to exchange rate and financial frictions include Gabaix and Maggiori (2016), Hau and Rey (2006), and Bruno and Shin (2014); Studies related to exchange rate and time-varying risk premium in the financial market include Alvarez et al. (2007), Farhi and Gabaix (2015).
process of international capital flow. Here, I name the model “currency risk premium model”. The design of the imperfect financial market is inspired by the spirit of Gabaix and Maggiori’s model (2016). Specifically, the model allows for a quantitative study of the three key factors of this analysis, real exchange rate, global financial frictions, and net foreign debt. Currency risk premium model accounts for the failure of uncovered interest parity through a currency risk-taking channel, while it helps to explain the exchange rate disconnect by introducing financial forces.

For the UK, there have been several well-documented shifts in monetary regime in the post-Bretton Woods period. The UK entered floating exchange rate regime in 1972; then it shifted to ‘income policy regime’, featured by a monetary policy that responded almost exclusively to the levels of output and unemployment; there followed ‘monetary targeting’, started at the end of 1979; the Sterling joined the Exchange Rate Mechanism of the European Monetary System (the ERM) in late 1990 and exited in September 1992, which is documented as the period of ‘exchange rate targeting’; finally, the whole period from 1992 is treated as ‘inflation targeting regime’. Numerous studies\(^2\) show that degree of nominal rigidities varies with changes in monetary regime. To avoid the issue of structure breaks, I choose a flexible price model rather than the model with nominal rigidities as an appropriate backdrop and focus on the real term behaviours of real exchange rate.

How best to evaluate the empirical performance of dynamic stochastic general equilibrium (DSGE) models is one of important, but unsolved issues in applied macroeconomics. Conventionally, the early version of the DSGE models, the Real Business Cycle models, are calibrated and evaluated by an informal comparison of the moments of the simulated variables with the moments of the observed data series. Le et al. (2011) argue that this kind of ‘matching moments’ method is a lack of formal standard statistical hypothesis provided by which closeness can be judged. Here I use a novel approach, called Indirect Inference, providing a classical statistical inferential framework for judging whether a DSGE model

\(^2\)Related studies include Meenagh et al. (2009).
with a particular set of parameters could have generated the behaviour found in a set of
observed data. Compared with the method of Likelihood Ratio test, Indirect Inference has
much more power, and it can be focused on the purposes you want the model for in a way
that the Likelihood Ratio test cannot (Le et al., 2015).

To answer the first research question whether financial forces have impacts on real
exchange dynamics in recent UK history, I set up a testable hypothesis that is examined
within the theoretical model described above. The model’s implied behaviour is formally
tested for its closeness to the UK experience through Indirect Inference method, which
applies a chosen statistical model (auxiliary model) to describe both the actual data and the
simulated data generated by the model. In this thesis, a cointegrated vector autoregressive
with exogenous variables (VARX) is used as the auxiliary model, and Indirect Inference test
is based on a function of the VARX estimates. The Wald statistic is chosen as the test statistic
to measure the statistical closeness of those estimates. Non-rejection of the null hypothesis
implies that the precisely specified mechanism and causal relationships embedded in the
model are accepted by the historical UK data.

The log-linearised currency risk premium model is estimated by Indirect Inference. This
estimation methodology is initially proposed by Goureroux et al. (1993), which provides a
natural framework for testing the hypothesis implied by the model. The basic idea of Indirect
Inference estimation is to search across model’s parameter space for the parameter set that
the simulated data and the observed data look statistically the same from the vantage point of
the chosen auxiliary model. Technically, the Wald statistic is minimised to find the optimal
choice of the set of parameter.

Empirically, I use the currency risk premium model to address several important issues.
First, financial frictions will act as amplifiers of external shocks on the real exchange rate and
other key UK macroeconomic variables. For instance, when a temporary decline in foreign
export demand, domestic consumption has to drop by more than it would without financial
frictions and real interest rate shoots up due to financier’s binding credit constraint. In order to compensate financial intermediaries for holding extra currency risk, real exchange rate depreciates more than it would in a world with the perfect global financial market.

Second, the presence of shocks arising in imperfect financial markets may give an alternative explanation for swings in exchange rates. Financial intermediary acts as a shock-absorber, however, it could itself become a source of shocks that drive the real exchange rate away from its fundamental level. I empirically investigate what are the main driving forces of sterling real exchange rate dynamics during financial disruptions: the global financial crisis of 2007 to 2010 and the Brexit vote. I use a variance decomposition method to quantify the sources of sterling exchange rate fluctuations in the reduced form of the currency risk premium model. Shocks to global intermediaries’ demand function, including shocks to the willingness of financiers to absorb sterling exchange rate risk and shocks to financiers’ balance sheet, explain most of the variations of the sterling exchange rate. I further examine the historical contribution of a variety of shocks to the sterling exchange rate. Shocks to financial forces made major contributions to sharp Sterling depreciations at the end of 2008 and after the Brexit vote.

Third, the structure of the currency risk premium model with explicit financial frictions provides a natural framework to explore the impact of policy responses to the changes in credit growth in a tractable manner. The authority could create the spread between the interest rate of the domestic bond and the policy rate, which is affected by both the currency risk premium and the regulation premium. Consequently, the policy could directly affect the foreign credit and the capital flow, which in turn influence the balance sheet of constrained global financial intermediaries. Since the macroprudential policy is countercyclical by design, a tightening of macroprudential measures would lower the interest rate on bonds and weaken the real exchange rate during cyclical booms, in turn, prevent large capital inflows, and credit expansion and currency appreciation from feeding on each other.
The thesis is structured as follows. In Chapter 2, I present the key facts regarding the UK’s net foreign debt, the international financial market and Sterling depreciation over the global financial crisis of 2007-2010 and the Brexit vote. Then, I survey the literature on exchange rate economics. In Chapter 3, a small open economy Real Business Cycle model with financial frictions is described in detail, particularly, the interest rate channel and the currency risk-taking channel have been emphasised. A starting calibration base on the UK economy is proposed. To highlight the role of financial friction, I compare impulse response functions generated from the calibrated currency risk premium model with corresponding impulse response functions generated from the model with the perfect financial market. Chapter 4 outlines the Indirect Inference Methodology. The hypothesis of financial forces driven exchange rate has been tested and the average risk-bearing capacity of global financiers during the sample period has been estimated by Indirect Inference. Chapter 5 empirically analyses sterling exchange rate dynamics during the financial disruption through variance decomposition and historical shock decomposition. Furthermore, macroprudential and fiscal policies have been proposed and followed by welfare evaluations. Chapter 6 concludes the thesis.
Chapter 2

A Literature Survey of Exchange Rate Economics

2.1 Empirical Evidence

In this section, I present the main empirical evidence that motivates this thesis. First, I detail facts related to the persistence of current account deficit and external imbalance observed in the United Kingdom after the 1980s. Second, I describe evidence on the global financial market. Finally, I review sterling movements in the global financial crisis of 2007-2010, and after the Brexit vote.

2.1.1 Current Account Deficit and External Imbalance

The United Kingdom fell into current account deficit in the middle of the 1980s, with an improvement of the current account in the middle of the 1990s, but a recent return to a fairly high deficit in the 2000s and 2010s. In 2015, the UK recorded the largest current account as a percentage of GDP deficit among the G7 economies. Figure 2.1 shows how the UK current account deficit remains high by historical and international standards. Current account
deficits imply that domestic expenditure is running ahead of national income, requiring net borrowing from overseas. Since the UK is traditionally a net debtor, then it needs to finance its deficit with continuing capital inflow from the rest of the world on its financial account.

There are several possible reasons for the UK’s persistent current account deficit. Firstly, the UK’s trade balance, which is a significant part of the current account, has been in deficit (imports higher than exports) since 1998. The UK has had a large amount of deficit in goods trade, since the process of de-industrialisation accelerated in the early 1980s. Although the level of total UK trade in goods as a proportion of total trade in goods and services has been gradually declining since 1986 and a deficit in goods is partly offset by a surplus in services, e.g. professional and management consulting services, it is not sufficient to overcome the total trade deficit. Secondly, there is a rapid growth in consumer spending and relatively low saving rate. Consumers have strong demand for imported goods. Thirdly, the deterioration in the current account balance has become more attributable to the decline in the primary investment income since 2011. This suggests that UK earnings on foreign assets dropped in value relative to the earnings of foreign investors in the UK. The report from ONS\(^1\) points out that income from the UK’s direct investments overseas had decreased, while payments to

\(^1\)UK Balance of Payments, the Pink Book: 2016 Website: https://www.ons.gov.uk/economy/nationalaccounts/balanceofpayments/bulletins/unitedkingdombalanceofpaymentsthepinkbook/2016
foreign investors in the UK had risen. As roughly 45% of the UK’s investment abroad is in Europe, UK’s direct investment earnings fell, and earnings on portfolio investment got worse during the European sovereign debt crisis.

Persistent UK’s trade deficits result in a build-up of net external debt, as residents borrow to fund spending in excess of income. Developments in UK’s external stock position can often be traced to the evolution of the current account. The international investment position (IIP) is a statement of the UK’s external balance sheet with the rest of the world. It records the holdings of (gross) UK assets by foreign residents and the holdings of (gross) foreign assets by UK residents at a specific point in time. There was a considerable growth of both UK assets and liabilities during the past two decades, except between the end of 2008 and the end of 2009 because of the world economic downturn brought on by the global financial crisis. Liabilities were always greater than assets in the amounts during this period, mainly reflecting the persistent current account deficit, which meant that the UK consistently ran a net liability position (i.e. where liabilities exceed assets).

Although the UK’s net liability position remained over the past two decades, its size has fluctuated. By looking at the long-run movement in cumulative financial flows in Figure 2.2, we can appreciate the interconnection between cumulative flows and the net IIP. The cumulative change in current account measured by the cumulative flows drives the changes to the UK’s net IIP over the long run, and the short run volatility of the net IIP (assets minus liabilities) is driven by changes in sterling exchange rate and asset prices. Specifically, exchange rate effects occur as the most of UK external assets are denominated in foreign currency, and to a lesser majority of external liabilities are denominated in sterling. This means that, all else being equal, a depreciation in the value of sterling will improve the UK’s net stock position. Between the end of 2007 and the end of 2008, there was a £129 billion fall in the UK’s net liability position, while the UK continued to borrow £160 billion from the rest of the world. The reason that the UK could improve its net IIP is mainly due to
sterling depreciation against major world currencies, which generated a positive £624 billion currency effect. Following the Brexit vote, the decrease in the value of sterling also had the effect of narrowing the current account deficit and boosting the net IIP.

Net IIP to GDP ratio is the key barometer of the financial condition and creditworthiness of a country. The scale and persistence of a net liability position of the UK indicate it is a net debtor to the rest of the world and may suggest an external vulnerability. Moreover, the Bank of England has highlighted the current account deficit as a potential risk, particularly if Brexit deters foreign investment.

2.1.2 Liquidity and Financier’s Risk Bearing Capacity

The global shifts in the supply and the demand of financial assets in different currencies trigger large-scale capital flows which mostly are intermediated by international financial institutions.
The 2016 Triennial Central Bank Survey of Foreign Exchange Market Activity documents foreign exchange trading continued to be dominated by other financial institutions, which roughly comprised 51 percent of turnover in 2016 (Figure 2.3). Those financial institutions include global investment banks, such as JP Morgan and Goldman Sachs, pension funds and active investment managers, such as BlackRock and PIMCO, macro and currency hedge funds such as Soros Fund Management. These intermediaries have the common feature that they actively participate in the currency markets and profit from imbalance on currency demand due to both trade and financial flows by bearing the resultant currency risk. Financial institutions usually take a long position in the current account deficit country (debtor country) and take a short position in the current account surplus country (credit country).

The UK has consistently run current account deficits and a net liability position over last two decades, such that there is an excess supply of sterling versus foreign currencies from...
the rest of the world. Hence the UK, as a net external debtor country, has borrowed from the international financial markets and is reliant on the willingness of investors to keep buying UK asset. Intuitively, the UK’s persistent net liability position implies that international financial institutions play an active investor role by holding Sterling and short selling other currencies.

Financial intermediaries, however, are subject to financial constraints that affect their ability to take positions, depending on their existing balance sheet risks and risk-bearing capacities. Here, we provide two examples of the UK to illustrate how financial intermediaries’ limited ability to take positions have impacts on capital flows. First, the Bank of England noted that in the run-up to the Brexit vote, there were signs that foreign liquidity inflow into the UK had slowed. Figure 2.4 provided by the Bank of England shows that foreign-owned gilt holding dropped by £4.4 billion in July 2016 – the second largest monthly fall by international investors in more than a year. Overseas investors cut back on UK gilts for the first time in six months, showing a short-term shift in investor sentiment towards UK assets following the Brexit vote. Roughly a quarter of outstanding UK government bonds, i.e. gilt, are held by foreign investors, so changes in overseas demand for gilts are considered important to the UK’s solvency. This is mainly due to a sharp increase in uncertainty that results in a damaging effect on sterling assets across the board and the further deterioration in overseas investor appetite for UK assets. Consequently, financial institutions who intermediate inward capital flows into the UK either reduced the amounts of sterling assets in their balance sheets or required more compensation for holding sterling assets to migrate the existing balance sheet risks.

Second, following the failure of Lehman Brothers, the global financial system came close to collapse in the autumn of 2008. Liquidity in some markets dried up due to increased volatility, tighter credit conditions and decreased financial institutions’ risk-bearing capacity. Thus, deteriorating capital market condition raised financing pressures on countries with large
external imbalances like the UK. In this environment, financial institutions’ willingness and ability to absorb an external imbalance by holding Sterling were severely affected. Therefore, the possible answer to the question at the beginning is that the UK’s external imbalance would be absorbed, at some premium, by international financial institutions. The UK’s net liability position implies that there were large amounts of sterling assets in financial intermediaries’ balance sheet.

2.1.3 Sterling Depreciation in Bad Times

The sterling is sensitive to any chaos that might occur in the financial markets. As we can see in Figure 2.5, there was a massive sterling depreciation around 2007-2008. Specifically, the sterling effective exchange rate – which measures the shift in the value of sterling relative to the currencies of UK’s major trading partners from the rest of the world – fell by more than a quarter between the third quarter of 2007 and the first quarter of 2009. These moves show a significant deviation from the decade of relative stability for Sterling which preceded the crisis.
Figure 2.5 Sterling Effective Exchange Rates During the Period of Global Financial Crisis

![Graph showing nominal and real effective exchange rates during the global financial crisis period. Source: J.P. Morgan.]

Figure 2.6 Sterling Effective Exchange Rates Following Brexit Vote

![Graph showing the impact of the Brexit vote on the sterling effective exchange rate. The graph highlights the sharp decline in the exchange rate on 23 June. Source: Financial Stability Report, Bank of England.]

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Britain’s surprise decision to leave the EU in 2016 was followed by financial market tumult that left the pound trading at a 31-year low (Figure 2.6). Since then the UK has been trading around 15% lower compared to the dollar and 12% lower compared to the euro than it was before the Brexit vote.

There are two common features of these two events. First, the UK continues to hold a large stock of external liabilities (Figure 2.2), with a significant proportion of those liabilities potentially vulnerable to refinancing risk. In other words, Britain is reliant on the willingness of overseas investors to keep buying UK asset. Second, during a financial disruption or a prolonged period of heightened uncertainty, international financial markets experienced tighter liquidity because of declined global financial institutions’ risk bearing capacity and increased their balance sheet risk. Therefore, international financial institutions could either continue to be deterred from holding Sterling or demand a currency premium.

To sum up, those stylised facts show that Sterling seems to depreciate dramatically in a financial disruption. Deterioration in investor appetite for UK assets—which could prompt more downward pressure on the exchange rate. Sterling, as an external debtor’s currency, is vulnerable to global financial shock.

### 2.2 Literature Review

Since the failure of the Bretton Woods System and the start of generalised floating exchange rates in 1973, exchange rate economics had been developed remarkably in order to explain large fluctuations in exchange rates. To this end, a huge theoretical and empirical literature on exchange rate economics had emerged over the period of the 1970s and 2000s. Unfortunately, those classical models are not able to account for a series of major puzzles in exchange rates: excess volatility and exchange rate disconnect, large excess returns of the carry trade, the uncovered interest parity puzzle. The global financial crisis in 2008, however, enlightened economists to consider the crucial role of financial intermediaries in the real
economy. Exchange rate economics is revitalised, and a growing body of research has recently emphasised interaction among financial forces, exchange rate dynamics and the real economy. This chapter contributes to that literature, focusing on the transmission of financial drivers affecting the sterling exchange rate into macroeconomic variables such as output and employment. In addition, it could be helpful to solve some puzzles in the exchange rate.

I am concerned to survey three strands of the literature on exchange rate economics in this section. The first strand is concerned with theories of foreign exchange rate determination as they have evolved during the pre-crisis period. This early modelling effort focused on the effects of macroeconomic fundamentals on exchange rates, which is the building block of the recent development of exchange rate economics. The second strand is concerned with classic exchange rate puzzles, which conventional macroeconomic determinants of exchange rates could not explain. The third develops a body of theory from the finance literature to emphasise the impacts of financial forces on exchange rate behaviours, and to tackle exchange rate puzzles.

2.2.1 Foreign Exchange Rate Determination

The Purchasing Power Parity and Real Exchange Rate

The purchasing power parity (PPP) is probably considered the oldest theory of exchange rate determination. The origins of PPP theory can be traced back to the writing of the Swedish economist Cassel (1918). Generally, there are two versions of the PPP. The absolute PPP postulates that the exchange rate between two currencies would equate the two relevant national price levels - the price of the same typical basket containing the same amounts of the same goods - if expressed in a common currency at that rate. Based on the relative PPP, the percentage variations in the exchange rate approximately equate the percentage variations in the ratio of the national price levels of the two countries. Most of the PPP literature, in any case, has focused on the relative PPP hypothesis rather than the absolute PPP, because
national price levels are generally in the form of price indices but not as absolute price levels. The PPP hypothesis is an empirical approximation of real exchange rate dynamics, which implies that the real exchange rate is time-invariant. Hence, a discussion of the real exchange rate is tantamount to a discussion of PPP.

In general, the empirical evidence suggests the failure of PPP in the short-run or medium-run. Dornbusch (1976)’s exchange rate overshooting model gives a possible explanation: the stickiness in nominal national price levels and wages, and sluggishness in the adjustment of goods market results in deviations of PPP in the short-run. Furthermore, most explanations of short-run exchange rate volatility point to nominal shocks such as short-term asset price bubble, changes in portfolio preferences, and monetary shocks, which buffet the nominal exchange rate and translate into real exchange rate variability (for example Obstfeld and Rogoff (1995); Chari et al. (2002); Bergin and Feenstra (2001); Benigno (2004)). Rogoff (1996) argued that if this were the complete story, one should anticipate considerable convergence to PPP over one to two years, as they can only happen during a time frame in which nominal prices and wages adjust to a shock. However, the empirical studies (for example Huizinga (1987); Dixon (1999); Chen and Engel (2004)) show instead that deviations from PPP- roughly three to five years - are much more persistent than that. In other words, the speed at which real exchange rates adjust to the PPP exchange rate is surprisingly slow. To address this persistence anomaly, Steinsson (2008) show that real shocks such as productivity shocks generate slightly more real exchange rate volatility than does the monetary shock. This finding implies that shocks might have highly persistent impacts on real exchange rates, which is consistent with the argument that real exchange rate swings mainly due to real shocks, as in Stockman (1980).

Moreover, the evidence on long-run PPP is still a matter of debate. The consensus in most empirical studies (for example Friedman and Schwartz, 1963) appeared to support the existence of a fairly stable real exchange rate in the period before the breakdown of the
Bretton Woods system. However, studies published mostly in the 1980s could not reject a random walk model for PPP deviations on modern floating rate data (for example, Meese and Rogoff, 1983). Particularly, real exchange rates, in some cases, exhibited significant long-run trends, particularly for countries in which real incomes have shown relatively significant trends. Some of the economists regard the failure of PPP in the long run as a real phenomenon and develop theoretical arguments to explain it. One way to account for the long-run deviation is to base on nontraded goods in a competitive world economy. Related literature includes Harrod (1933), Samuelson (1964), Balassa (1964), and Stockman and Dellas (1989). The mechanism is that: an increase in productivity in the traded goods sector will lead to a rise in wages in the whole economy; if firms in the non-traded goods sector would like to survive, they have to increase non-traded goods price relative to traded goods in order to offset relative lower productivity and the increased wage; price indexes such as CPI capture prices of both traded and non-traded goods, and since prices are positively correlated to wage and negatively to productivity, price index may vary across countries with different productivity. This is called Harrod-Balassa-Samuelson effect. Furthermore, some empirical studies documented by Richardson (1978), Krugman (1986) and Lapham (1992), suggested that similar traded goods in different countries are still influenced by changes in relative prices of non-traded goods, because most of the final goods may contain non-traded components.

On the other hand, Backus and Smith (1993) examined non-traded goods as a device to account for persistent deviations from PPP by studying the general equilibrium interconnections between real exchange rates and corresponding consumption ratios. Their main finding did not support a central role of nontraded goods in explaining the consumption and relative price evidence simultaneously.

Rogoff (1996) argued that real side shocks such as a technology shock causes a highly persistent real exchange rate, whereas the shocks that are original to aggregate demand such
as a financial shock or monetary shock leads to a slightly persistent effect on real exchange rate. Following that suggestion, I focus on effects of real shocks on the real exchange rate and attempt to capture permanent shifts in the fundamentals and permanent deviations from PPP. Furthermore, I introduce a financial shock to international banking institutions’ risk bearing capacity in order to get short-run fluctuations in real exchange rates from changes in liquidity. In addition, I use a flexible-price classical model instead of the sticky price model in order to eliminate the short-run real exchange rate volatility due to the Keynesian paradigm of stickiness in the adjustment of nominal wages and the price of goods.

**Other Theories of Foreign Exchange Rate Determination**

Economics is primarily concerned with the allocation of scarce resources to human wants, whose price is determined by the interaction of its supply and demand. Exchange rate economics, as one of the branches of economics, is no exception: the exchange rate is simply the price of foreign currency which clears the foreign exchange market. Hence theories of exchange rate determination can be divided into three groups—the traditional flow approach, the monetary approach, and the portfolio approach - in terms of variety in their different supply and demand for foreign exchange.

**The Traditional Flow Approach**

The traditional ‘flow approach’, also called the balance-of-payments view, sees demands for and supplies of foreign exchange as pure flows, deriving from imports and exports of goods, which in turn rely on the exchange rate.

Elasticity approach as a very early version of the flow approach was developed initially by Marshall (1923), Lerner (1936) and Metzler (1949). These studies exhibit the importance of the elasticities of demand for and supply of foreign exchange, and the demand for and supply of imports and exports. Here, the exchange rate as a relative price of imports and
exports clears a market with well-defined flow demand and supply curves, since there is no mechanism, in this case, to absorb the excess demand or supply of foreign exchange that a nonzero trade balance would generate. During the 1940s and 1950s, the Keynesian revolution and the rapid growth of international trade inspired the economists to rethink the behavioural linkages between exchange rates and balance of payments. In this environment, international capital flows as the component of the balance of payments were negligible due to relatively small proportion to the value of international trade. While the current account – and usually simply the trade balance – had been treated as the only endogenous component of the whole balance of payments in the most models of exchange rates and the balance of payments. Following that simplification, the absorption approach developed by Harberger (1950), Meade (1951), and Alexander (1952) emphasised that a devaluation of home currency through lowering the relative prices of domestic goods leads to an increased demand for home goods and enlarge the domestic output. On the other hand, a rise in real income would stimulate expenditures and have feedback effects on trade flows. Meade (1951) made a path-breaking contribution to the simultaneous analysis of internal and external balance in an open economy, specifically, he provided a framework to analyse the simultaneous relationship of the balance of payments to exchange rates and other macroeconomic variables. To some extent, the integrated elasticities-absorption model captures the short-run movements in the exchange rate.

In the early of 1960s, the evolution of the post-war world economy had stimulated interest in extending the Keynesian income-expenditure model by introducing capital flows into the analysis. In line with ‘flow approach’, capital flows are a further component of demand for and supply of foreign exchange. This gave rise to a series papers by Mundell (1961, 1962, 1963) and Fleming (1962) – and came to be known as the Mundell-Fleming model. The idea of the Mundell-Fleming framework of exchange rate determination was that net excess demand for foreign exchange, which is equal to the overall balance of payments, must be
zero in equilibrium under a perfect capital mobility. Then, it is possible to solve exchange rate and other endogenous variables by combining the balance of payments equilibrium condition with standard Keynesian model equilibrium conditions for the goods and money markets.

The ‘flow approach’ contains a fundamental shortcoming: it neglects stock adjustment. In particular, the Mundell-Fleming model had been criticised that "the capital account balance should be conceptualised not as an ongoing flow, but rather as a reflection of efforts to adjust asset stocks to the levels that economic agents desired" (Isard, 1995, p.102). In other words, the current account imbalance can be offset by capital flows across the capital account; finally, however, the current account and capital account should balance independently. To some extent, the traditional trade flow approach is inadequate in its specification of the determinants of the supplies of and demands for foreign exchange.

**The Asset Market Approach**

Following the collapse of the Bretton Woods system in the early 1970s, the theoretical literature on the ‘asset market’ view of exchange rates had been expanding. Johnson and Frenkel (1976) emphasised the difference between flow and stock equilibrium in the open-economy context, which becomes a hallmark of asset equilibrium models. Perfect capital mobility is one of the common assumptions of all asset-market models. Based on this assumption, “the exchange rate must adjust instantly to equilibrate the international demand for stocks of national assets” (Frankel, 1993, p. 86). That distinguishes the asset market approach from the traditional flow approach - the exchange rate adjusts to equilibrate the international demand for flows of national goods. There exist two distinct classes of asset equilibrium models: the monetary approach to the balance of payments, and the portfolio-balance approach. The monetary approach defines an exchange rate as the price of one country’s money in terms of that of another and attempts to model the determinant of that
price based on the relative supply of and demand for the two money (for example, Frenkel (1976,1978); Bilson (1978a,1978b)).

In common with the monetary models, the portfolio balance approach concerns on the relationships between the balance of payments flows and adjustments in asset stocks, and models the capital account in terms of the behaviour of the demands for and supplies for portfolio stocks. However, the portfolio balance approach regards domestic-currency financial assets as imperfect substitutes for foreign-currency financial assets, which is the main difference from the monetary approach. In general, imperfect capital substitutability has several implications. First, compositions of financial portfolios held by investors are different regarding valuation risks, and asset holders are not risk-neutral. Second, asset holders would require compensation for holding risky assets. Generally, risk premiums will alter over time in response to international swings through current account imbalances in the net financial assets wealth of different nations’ investors, since portfolio preferences of investors from different countries vary. Third, contrary to risk neutral asset holders who allocate their portfolio in proportions that are infinitely sensitive to expected rates of return, portfolio proportions under the assumption of imperfect substitutability between domestic and foreign assets are functions of expected rates of return. Fourth, a country’s net foreign assets may have impacts on exchange rates through their influence on the risk premiums that are required to clear financial markets. Uncovered interest parity - "the interest rate on a domestic bond is equal to the interest rate on a foreign bond plus the expected rate of appreciation of foreign currency" -, in this case, does not hold (Frankel, 1993, p. 86).

Based on the portfolio balance approach, the exchange rate is an important determinant of the current account of the balance of payments, while a country’s net foreign asset, which is defined as the cumulative change in its current account over time, in turn, affects the exchange rate through altering the level of wealth and asset demand. In essence, the portfolio
balance model is a dynamic model of exchange rate determination relied on the interactions between asset markets and current account (Sarno and Taylor, 2002).

This chapter contributes to the literature on the portfolio balance approach to exchange rate determination. The early theoretical literature\(^3\) includes Kouri (1976), and Branson and Henderson (1985). Especially, I develop the model suggested by Kouri (1976), who establishes a model to analyse the dynamic interaction between the exchange rate, exchange rate expectations and the balance of payments, and to determinate exchange rates in terms of the demand for and supply of assets denominated in different currencies, where assets are imperfect substitutes. In a similar spirit with Kouri’s work, I suppose that home-denominated bond and foreign-denominated bond are imperfect substitutes due to the valuation risk. It implies that the exchange rate adjustment based on valuation effects in the demand for and supply of bonds, and a risk premium exists in the uncovered interest parity condition. Moreover, I assume that domestic residents only borrow or lend in home currency - which is in line with the assumption of the ‘domestic small-country model’ (Frankel, 1993) - in order to emphasize the currency mismatch and identify a capital inflow (outflow) with a rise (fall) in the supply of foreign assets. The assumption implies that a fall in the supply of foreign-denominated assets in the international financial market would cause an increase in their price in terms to domestic currency.

 Compared with the monetary approach to the exchange rate, there are relatively few empirical studies conducted on the portfolio balance models, and existing empirical results have been mixed (for example, Obstfeld (1983); Frankel (1984); Kearney and MacDonald (1986); Hallwood and MacDonald (2000)). In this thesis, I attempt to improve on this record by providing a modern general equilibrium theory of portfolio balance model and empirically analysing it with UK data.

Exchange Rate Determination in DSGE Models

The macro-models of exchange rate determination of the 1970s, notably the Mundell-Fleming model, the monetary models and the portfolio balance model, are based on ad hoc assumptions about exchange rate expectations. Since the Lucas critique in 1976, dynamic general equilibrium (DGE) models have become a popular workhorse framework for macroeconomic analysis. In this new wave of research, open economy DGE models, which are based on the optimising behaviour of the microeconomic units, firms and households, are clearly a major accomplishment. Instead of approximating equilibrium using certain equivalence assumptions, the key feature of open economy DGE models is the use of microeconomic foundations, which allows for more rigorous and structured analysis of the origins and evolution of observed macro-variables than conventional models can provide.

The baseline model considered in this thesis is built on the open economy dynamic general equilibrium framework. To this end, this section surveys a collection of papers that present interesting features in this framework. The early studies in dynamic general equilibrium models with well-specified micro-foundations include Stockman (1980, 1987), Lucas (1982), and Backus, Kehoe and Kydland (1992, 1993, 1994), and most of them are based on an economic environment of flexible price and perfect competition among producers. More specifically, Lucas (1982) propose a neoclassical two-country rational expectations model with complete markets and a flexible exchange rate environment, where the fundamental determinants of the exchange rate are in line with those in the monetary model. The real exchange rate is determined by relative output levels in two countries and agents’ preferences. Backus et al. (1992) have extended real business cycle theory in a closed economy proposed by Kydland and Prescott (1982) to a competitive model of a two-country economy with a single homogeneous produced good and complete markets for state-contingent claims. They find large standard deviations and high degree of persistence of real exchange rates for eleven OECD countries.
Obstfeld and Rogoff (1995) is commonly recognised as the contribution that introduced monopolistic competition and sticky nominal prices into an open-economy dynamic general equilibrium model with rigorous micro-foundations. The Redux model provides intuitive forecasting about nominal exchange rate that sometimes varies from those of either conventional sticky-price overshooting monetary model or modern flexible-price intertemporal models. In general, the exchange rate is determined by the uncovered interest parity conditions. An unexpected domestic monetary expansion, since nominal prices are sticky, will induce a fall in interest rate and hence a depreciation of nominal exchange rate in the framework. Following the research wave of the ‘New Open Economy Model’\(^4\), subsequent studies in the open economy have devoted much more attention to including extensions in the form of more realistic nominal rigidities\(^5\), preferences, capital accumulation with adjustment costs, labour markets and financial structures, etc.

Furthermore, ‘stochastic’ part has been integrated into an open economy general equilibrium models, known as open economy dynamic stochastic general equilibrium (DSGE) models\(^6\). There exist unexpected shocks continually hit the economy, from demand and supply, stochastically disturb exchange rate and other closely related macro-variables, which are endogenously determined in the model. Open economy DSGE models provide a clear interpretation of shocks that are assumed to affect the economy and incorporate the expectation of agents into the modelling process, where exchange rate volatility is the relation of various stochastic shocks in the fundamentals.

\(^4\)A modelling framework that integrates imperfect competition and nominal rigidities into dynamic general equilibrium models has been labelled ‘neomonetarism’ by Kimball (1995) and the ‘new neoclassical synthesis’ by Goodfriend and King (1997).

\(^5\)In Obstfeld and Rogoff’s Redux model, firms simultaneously set prices one period in advance; one strand of the literature, including Kollmann (2001); Chari, Kehoe and McGrattan (2000, 2002); Gali and Monacelli (2005), captures price stickiness through staggered price-setting. Another strand of the literature - for example, Hau (2000); Obstfeld and Rogoff (2000) - rather consider nominal rigidities originated from sticky wages.

\(^6\)Small open-economy DSGE models include Gali and Monacelli (2005); Justiniano and Preston (2010); two-country DSGE models include Obstfeld and Rogoff (1998); Benigno and Thoenissen (2003); Devereux and Engel (2002).
There are two distinct assumptions in financial market completeness\textsuperscript{7} in open economy DSGE literature. The assumption of financial market completeness would imply that a strong positive correlation between the real exchange rates and relative consumption across countries. However, Backus and Smith (1993) argue that open economy models based on the complete market assumption fail to reproduce the key features of data. Especially, international risk sharing condition implied by the complete market assumption is in contrast to the empirical evidence that relative consumption across countries is not systematically correlated with and less volatile than its relative price, i.e. real exchange rate. Other empirical studies have also questioned the assumption of financial market completeness, including Kollmann (1995), and Benigno and Thoenissen (2008).

Alternatively, the consumption-real exchange rate anomaly has led economists to consider the assumption of incomplete international asset markets, under which there exists only a risk-free international bond in the global asset market, thereby breaking the link between the real exchange rate and relative consumption. The related literature on incomplete market assumption includes Schmitt-Grohé and Uribe (2004), Tuladhar (2003), Chari \textit{et al.} (2002), Corsetti \textit{et al.} (2008), and Rigobon \textit{et al.} (2011).

DSGE models that could in principle depend on completely different microeconomic foundations. On the one hand, models are based on monopolistic competition with nominal rigidities in the price and wage setting, on the other hand, real business cycle models with perfect competitive firms and no stickiness, for example, Meenagh \textit{et al.} (2010). Since the aim of this chapter is to examine the exchange rate risk premium and financial friction as essential parts of the transmission mechanism for generating exchange rate disconnect behaviour (see further literature discussion in Section 2.2.3), for simplicity, I restrict attention to a purely real model. Nominal rigidities and monopolistic competition are not considered in the baseline model.

\textsuperscript{7}The earlier and most prominent literature on exchange rate determination in complete asset market includes Lucas (1982), Pavlova and Rigobon (2007), Verdelhan (2010), Farhi and Gabaix (2016).
In next chapter, I present a small open economy, neoclassical, DSGE model with flexible prices that are taken by perfectly competitive firms. In addition, the model features an incomplete financial market structure.

2.2.2 Exchange Rate Puzzles

The Exchange Rate Disconnect Puzzle

Unfortunately, conventional models of exchange rate determination relied on macroeconomic fundamentals have not had much success in capturing the behaviour of exchange rates. Obstfeld and Rogoff (2000, p.373) point out that ‘the remarkably weak short-term feedback links between the exchange rate and virtually any macroeconomic aggregates’, and exchange rate disconnect is one of the most long-standing and challenging puzzles in the international macroeconomic literature.

Empirical regularities stand at odds with the existing international macro models. In particular, floating exchange rate exhibits a volatile random walk process, which is not linked to macroeconomic fundamentals such as outputs, interest rates and money supplies. Meese and Rogoff (1983) document structural international macro models - including a flexible-price monetary model, a sticky-price monetary model and a sticky-price hybrid model - failed to significantly outperform a random walk time series model in forecasting the behaviour of exchange rates out of sample at horizons of up to one year. Their results spurred vast studies in investigating the performance of various modified structural models - such as alternative specifications of portfolio-balance models (see Backus, 1984); models with nonlinearities (see Meese and Rose, 1991). Some of these studies found that structural models could beat the random walk model, but generally at the longer horizons and over different time periods. The success of these models has not been proved to be robust (Frankel and Rose, 1995).
Although numerous studies attempted to address the ‘Exchange Rate Disconnect Puzzle’, the puzzle has not been fully resolved. The existing literature could be allocated into two strands. The first strand is concerned with the transmission mechanism, which silences the influence of fluctuated exchange rate swings on prices and quantities. Nominal rigidity as the key part of the transmission mechanism to the real exchange rate has been documented in the literature (for example, Rogoff (1996); Chari et al. (2002)). Some other studies consider limitations of expenditure switching effect of the exchange rate (for example, Engel (1993); Parsley and Wei (2001)). More particularly, when nominal exchange rate changes do not fully pass through to traded goods prices, then relative prices of home produced goods and foreign produced goods do not change much for consumers, and it will result in weak substitutability between those two goods. Intuitively, the limited extent of expenditure switching conditional on the terms of trade would break the linkage of exchange rates through macro-variables. Devereux and Engel (2002, p.916) support this argument by providing "the presence of three factors –local currency pricing, heterogeneity international distribution of commodities, and ‘noise traders’ in foreign exchange markets" – can potentially generate higher exchange rate fluctuation than the fluctuation in other macroeconomic variables. This chapter, however, focuses on an entirely different perspective, which eliminates any effect of nominal rigidities and local currency pricing in the goods market, and attempts to emphasise the nature of shock process and the impact of financial friction on exchange rate behaviour in the international financial market.

The second strand of literature tackles the puzzle based on the driving force for exchange rates, which cannot simultaneously have a strong direct impact on contemporaneous macroeconomic variables such as output, interest rates, consumption. Engel and West (2005, p. 486) argue that the exchange rate as “the expected present discounted value of a linear combination of observable fundamentals and unobservable shocks” follows a near-random walk process when “at least one forcing variable (observable fundamental or unobservable shock) has
an autoregressive unit root”, and the discount factor is close to unity. Hence it may not be surprising that exchange rates are unpredictable. However, the exchange rate might help to predict the fundamentals. This chapter would give support to this strand of literature by emphasising that unobservable shocks such as a productivity shock have permanent or very persistent components, which are driving forces for the exchange rate.

This thesis is to offer a model of exchange rate disconnect and examine the features of the model empirically. Specifically, I emphasise that the enriched ‘expenditure switching’ role of exchange rates is the central channel for the transmission of financial driving forces for exchange rate into macro-variables.

The Uncovered Interest Parity Puzzle

Uncovered interest rate parity (UIP) is a no-arbitrage condition that states that the expected change of spot exchange rate equals to the interest rate differential, if investors have a rational expectation and are risk-neutral. In other words, an expected return on the foreign-currency bond expressed in units of the domestic currency relative to the return on the home-currency bond should be equal to 0. It can be summarised into the following equation,

\[ \gamma_t = R_t^* + E_t s_{t+1} - s_t - R_t. \] (2.2.1)

In this notation, \( R_t \) is the nominal interest rate on a riskless government bond held in domestic currency between periods \( t \) and \( t + 1 \), while \( R_t^* \) is the equivalent interest rate for foreign currency denominated bond. \( s_t \equiv \log S_t \) denotes the logarithm form of the nominal spot exchange rate, which is the price of the foreign currency in units of the domestic currency. A rise in \( S_t \) indicates a depreciation in the home currency. \( E_t s_{t+1} \) represents the rational expectation of \( s_{t+1} \) conditional on all information available to the market at time \( t \). \( R_t^* + E_t s_{t+1} - s_t \) measures the expected return on the foreign-currency bond converted into units of the domestic currency. In line with UIP, \( \gamma_t \), which is the expected excess return or
the foreign exchange risk premium, should be equal to zero. For example, if the domestic interest rate is one percent higher than the foreign interest rate for a one-period government bond, the domestic currency is expected to depreciate by exactly one percent after one period.

The classic forward premium puzzle, or the violation of the UIP proposed by Bilson (1981) and Fama (1984), who tested UIP with a regression:

\[
s_{t+1} - s_t = a + b(R_t - R^*_{t}) + u_{t+1}. \quad (2.2.2)
\]

Under UIP, the regression coefficients should be \( b = 1 \) and \( a = 0 \). However, a long history of empirical studies – including older surveys such as Hodrick (1987), Froot and Thaler (1990) and Engel (1996), and recent studies such as Burnside et al. (2006) - has found that the estimated coefficient \( b \) is significantly less than one, i.e. \( b < 1 \), and sometimes even \( b < 0 \). Intuitively, a 1% rise in interest rate differential does not translate one-for-one into expected currency depreciation. Furthermore, empirical results showed that higher interest rate currencies, sometimes, tend to appreciate relative to lower interest rate currencies. This is called the UIP puzzle or forward premium anomaly.

In addition, conventional models of exchange rates, such as Mundell-Fleming model or a series of monetary models (for example, Dornbusch (1976); Obstfeld and Rogoff (1995)), assume that UIP holds. Real exchange rate, in this case, depends only on the behaviour of current and expected real interest rates in the home and foreign countries. In particular, these models anticipate that a country has a higher than average relative interest rate, its currency should be stronger than average relative currency. Empirical evidence support this relationship, however, there is higher volatility or co-movement of exchange rate than rational expectations of expected interest differentials as the models suggest under UIP (Engel, 2016).

Existing literature attempted to account for the notable empirical regularities associated with the UIP puzzle could be divided into three categories. First, a vast literature focuses on a risk premium as a direct explanation for the deviation from the UIP. If the UIP does not
hold, then expected excess return or risk premium, i.e. $\gamma$ in Equation 2.2.1, would not be equal to zero, even not be constant. Risk premium arise from foreign exchange fluctuations. For example, the domestic agent bears foreign exchange risk by holding foreign-currency denominated bond, and vice versa.

To explain the UIP puzzle, recent risk-based studies have employed agent framework with various non-standard preferences and captured an impact of interest rate differential on currency risk premium. Backus et al. (2001) express the currency risk premium in terms of the different conditional variances of foreign and domestic stochastic discount factors when pricing kernels are lognormally distributed. Following that, Verdelhan (2010) show how the factors driving the currency risk premium can be related to macro-variables driving pricing kernels or stochastic discount factors. By introducing external habit persistence over consumption in the complete financial market, pricing kernels are driven by the surplus consumption ratio, which is defined as the percentage gap between consumption and habit, and consumption growth shocks in the model. Since the currency risk premium and interest rate differential are determined by pricing kernels, both of them are affected by the same forces which drive pricing kernels. Also, the model endogenously generates pro-cyclical real interest rate of risk-free bond and counter-cyclical risk-aversion. Therefore, when the gap between consumption and habit is small in the home country, the domestic interest rate is lower relative to foreign interest rate due to a precautionary impact, then home agents become very risk averse, underreact to a relatively high foreign interest rate due to foreign exchange risk, and expect positive currency excess returns. Expected risk premiums rise sharply with interest rate differentials; i.e. $\text{cov}(E_t\gamma_{t+1}, R_t^* - R_t) > 0$. Thus, this mechanism accounts for the UIP puzzle. Furthermore, another research (for example, Bansal and Shaliastovich (2013); Colacito and Croce (2013); Gourio et al. (2013)) develop the models based on a preference documented by Epstein and Zin (1989) to account for the UIP puzzle. A crucial feature of the preference is that it “permit risk attitudes to be disentangled from the degree of intertemporal
substitutability” (Epstein and Zin, 1989, p. 937). Lustig et al. (2011) employ asymmetric preferences and reproduced the UIP puzzle. Apart from those models with exotic preferences, Engel (2016) explain the deviation from UIP by highlighting a role of liquidity risk premium in the exchange rate-interest rate context. If a country’s assets are more valued for their liquidity, the country’s currency will appreciate.

The second strand of literature accounts for the UIP puzzle by abandoning the assumption that all agents are fully rational expectations. Related literature includes Gourinchas and Tornell (2004) and Burnside et al. (2011). Some literature in asset market has been employed to explain the puzzle. For example, Hong and Stein (1999) argue that market participants tend to overreact the available market information, combined with a momentum trading, rather than to form expectation rationally. It can help to understand why currency appreciates more than it would suggest by the UIP in the short-run when a country’s interest rate increases. A third possible explanation of deviation of the UIP focus on the phenomenon of delayed overshooting. Eichenbaum and Evans (1995) document that it takes approximately 8 to 12 quarters for a currency to depreciate from instantaneous appreciation due to a rise in interest rate. Bacchetta and Wincoop (2010) support the delayed adjustment argument by proposing an infrequent portfolio decisions model to account for the UIP puzzle.

To account for the failure of the UIP, this chapter develops a risk premium-based view of exchange rate determination inspired by the foreign exchange premium literature. Following a smaller literature8 that has analysed the crucial role of incomplete markets, I assume an incomplete international financial market, coupled with a standard utility function, which differs from most studies on currency risk premium. Moreover, the model in this thesis will focus on the effect of the net foreign asset on currency excess returns, which distinct from a pure interest rate differential channel. I will review the related literature on the net foreign asset and currency risk premium in Section 2.2.3).

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8Recent literature on incomplete markets includes Chari et al. 2002; Corsetti et al. 2008; Rigobon et al. 2011.
2.2.3 Exchange Rate Dynamics in the Context of Financial Forces/Conditions

Foreign Exchange Rate Risk Premium and Net Foreign Assets

The recent wave of financial globalisation has placed the spotlight on the link between the net foreign asset position and exchange rates. Gagnon (1996) and Lane and Milesi-Ferretti (1999) provide evidence of a significant relationship between net foreign assets and the real exchange rate among OECD countries. Furthermore, the challenge to link exchange rates to macroeconomic fundamentals received a major progress with the model of international financial adjustment proposed by Gourinchas and Rey (2007). They argue that conventional wisdom to the current account balance that is the result of forward-looking intertemporal saving decisions by agents is incomplete, and it should incorporate with capital gains and losses on the net foreign asset position. Two channels have been specified in the mechanism – the trade channel and the valuation channel. Intuitively, a depreciation in the domestic currency may improve trade balance through the trade channel, and raise the value of foreign currency denominated foreign asset relative to the value of domestic currency denominated foreign liabilities through the valuation channel. Della Corte et al. (2012) extended the model of Gourinchas and Rey (2007) empirically and suggested that exchange rates are determined and predictable by external imbalance.

This thesis relates to a small amount of literature on exchange rate risk premiums associated with external imbalance, rather than the literature on the exchange rate risk premium derived from an intertemporal consumer problem. Shimizu (2017) develop a simple two-period portfolio problem for the representative home agent, who chooses the amount of net foreign assets to maximise her expected utility of future wealth and faces risk arises from the future exchange rate variation. The empirical results support the argument that time-varying and persistent exchange rate risk premiums vary through changes in net foreign assets. Intuitively, risk-averse investors require a reward to hold more net foreign assets.

\footnote{Lustig and Verdelhan (2007); Engel (2014) reviewed this strand of literature extensively.}
which is in the form of a larger risk premium related to exchange rate variations. The theory of Gabaix and Maggiori (2016) relates net foreign assets to currency excess returns, which reproduces the link between external imbalances and currency risk premium. In particular, net debtor countries borrow from global financial intermediaries. Hence financiers take a long position in currencies of net debtors and require a risk premium due to a depreciation of those countries’ currencies in a bad time. The empirical finding in Corte et al. (2016) supports the theoretical prediction of Gabaix and Maggiori that net-debtor countries’ currencies tend to depreciate, while net-creditor countries’ currencies experience a currency appreciation when financial disruption happens. They also show that “net foreign asset positions capture information not identical to interest rate differential in the cross-section of currencies” (p.2164). Put it differently, net foreign asset position is viewed as an additional risk factor in the exchange rate risk premium, and its impact on currency excess returns distinct from a pure interest rate differential channel. This result is consistent with another empirical work of Habib and Stracca (2012).

In the similar spirit of the literature on exchange rate risk premium and net foreign asset positions, this thesis develops a risk-based view of exchange rate determination based on macroeconomic fundamentals and, especially, on net foreign asset positions.

**Exchange Rate and Financial Frictions**

This thesis relates to two streams of literature on exchange rate determination in the presence of financial frictions. The first stream focuses on the role of financial intermediation and financial constraints in DSGE models. The second stream of literature studies how frictions affect exchange rate behaviour.

First, I will briefly review this new generation of DSGE models that incorporate friction in financial intermediaries. In general, the standard DSGE models do not include financial intermediaries and the interaction of financial markets with the real economy. Therefore, the
impacts of financial market imperfections on macro-variables cannot be captured by those standard models. The origins of macroeconomics research with financial friction proposed by Bernanke and Gertler (1989) emphasise the fact that a temporary shock through financial transmission channel can have long-lasting persistent effects in a standard real business cycle model. Since the time of that survey, there has been a rapid growth of the literature on the role of financial friction in macroeconomics. Most of the earlier macroeconomics studies with financial frictions focused on credit market constraints faced by nonfinancial borrowers, while financial intermediaries are treated as a veil. This strand of literature\textsuperscript{10} includes Bernanke \textit{et al.} (1999) who develop the “financial accelerator” by assuming risk-averse household (lender) and risk-neutral entrepreneurs (borrower), and Kiyotaki and Moore (1997) who introduce a collateral constraint on borrowing due to incomplete contract. Other macroeconomic models (for example, Holmstrom and Tirole (1997); Carlstrom and Fuerst (1997); Christiano \textit{et al.} (2005); Brunnermeier and Sannikov (2014)) also feature financial market frictions by introducing an agency problem between lenders and borrowers.

The recent global financial crisis has featured a significant disruption of financial intermediation and fuelled interest in incorporating the linkages between frictions in financial intermediaries and the real economy. Gertler and Kiyotaki (2010, p. 4) endogenise financial market frictions by embodying “an agency problem that potentially constrains the ability of financial intermediaries to obtain funds from depositors”. The constraint works to introduce a wedge between the lending and borrowing rates. Financier’s ability to obtain funds from depositors and other financial institutions depends on the condition of financier’s balance sheet. In particular, this spread dramatically widens when there is a significant disruption of financial intermediation, which in turn pushes up the cost of credit that borrowers face. Thus, their framework emphasises the role of financial intermediaries’ borrowing constraints in transmitting and amplifying financial shocks to the real economy. Furthermore, Gertler and

\textsuperscript{10}More recent work includes Angeloni and Faia (2013), and He and Krishnamurthy (2011).
Karadi (2011) develop a quantitative DSGE model with endogenously determined constraints of financiers’ balance sheet to analyse the impacts of unconventional monetary policy.

Apart from those macroeconomics models with financial friction based on the closed economy framework, there have been a few attempts to incorporate financial frictions in the open economy environment. Notable contributions to the study of international financial friction include Caballero and Krishnamurthy (2002), who show that domestic firms are constrained in borrowing from foreign lenders due to the domestic country’s limited international collateral; Mendoza (2005), Jeanne and Korinek (2010), and Benigno et al. (2016), among others, who consider models with a stock or flow collateral constraint in which households’ ability of borrowing from abroad is limited by the value or price related to collateral; Kollmann (2013) who assume that a representative global financier intermediates between savers and borrowers in the two countries, and faces a capital requirement, which implies that the loan spread is a decreasing function of bank capital; Dedola et al. (2013) who develop an open economy version of Gertler and Karadi (2011)’s form of balance sheet constraints on financial intermediaries. Especially, most of those studies on open macroeconomics models with financial friction have placed the spotlight on the importance of financial frictions for the transmission of financial shock to the real economy.

Second, this thesis contributes to the literature on impacts of friction in financial market on exchange rate behaviour. The literature on this stream can be further divided into several branches based on various focus. One branch addresses informational frictions in the financial market. For example, Evans and Lyons (2012) argue that many macroeconomic fundamentals related information is dispersed and it takes time for financial market makers to assimilate that dispersed information completely. Consequently, there is a strong link between exchange rates and transaction flows which convey dispersed information that is known to the dealer.

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12Other open economy models with banks can be found in Correa et al. (2010); Davis (2010); Kollmann et al. (2011); Devereux and Sutherland (2011); Perri and Quadrini (2011); Lipinsky (2012); Kamber and Thoenissen (2013); Van Wincoop (2013).
One other branch focuses on frictions in access to funding market. Alvarez et al. (2009), who construct the model of exchange rates where the frictions, a form of endogenously segmented asset markets, only appear in the domestic money market.

Another branch is based on credit friction in global financial intermediaries. Bruno and Shin (2015) develop a partial equilibrium model of the global banking system where the international banking system bears and distributes the fundamental risk subjects to leverage and balance sheet constraints. Contrary to conventional macroeconomic models of exchange rate where the focus is on the current account, their framework emphasises the transmission channel behind the link between currency strength and a gradual accumulation of leverage in the banking sector. Gabaix and Maggiori (2016) build a modern micro-founded version of the portfolio balance model, where a global financier face a credit constraint in a similar spirit and formulation in Gertler and Kiyotaki (2010). The framework integrates macro effects with financial channels in exchange rate determination. More specifically, a representative global financial intermediary actively takes a risk in currency markets, however, her ability to take positions is limited on her risk-bearing capacities and existing balance sheet risks. Hence frictions in international intermediary play an important role in exchange rate determination in their work.

Following the spirit of the literature above, I propose a DSGE model of a small open economy with financial friction, where global financial intermediaries act as specialists that assist in channelling foreign denominated bonds (domestic denominated bonds) from foreign country (domestic country) to home country (foreign country) and absorb the currency mismatch. In particular, a representative global financier is unable to intermediate infinite capital flows arising from a non-zero trade balance, since the financier faces a credit constraint in a similar formulation in Gabaix and Maggiori (2016). In contrast to the perfect intermediation, the borrowing process is subject to an agency problem, which limits the size of the balance sheet of the financier and the arbitrage between risk-free bonds denominated in
different currencies. Therefore, the model in next chapter, where exchange rates are jointly determined by fundamental and financial forces, can endogenously produce a deviation from the uncovered interest parity and links it to the global intermediary’s risk bearing capacity. This allows me to quantitatively examine the effects of financial shocks and financial forces on exchange rate behaviours.

The thesis contributes to the literature in three aspects. First, it provides new insight on the exchange rate determination in the context of financial forces, especially the impacts of shocks to financial forces on sterling exchange rate dynamics. One innovation of the thesis is a small open economy Real Business Cycle model with Gabaix-Maggiori (2016) features in the international financial sector.

Second, the thesis contributes to a growing literature on resolving the exchange rate disconnect puzzle and the uncovered interest rate parity puzzle. This thesis shows that the exchange rate is disconnected from traditional macroeconomic fundamental and international financial intermediaries could be the source of financial shocks that distort exchange rates. Moreover, the thesis studies how financial forces affect currency risk premium, and accounts for the failure of the UIP.

The thesis also connects with the literature on financial frictions in an international context. A key insight is that financial frictions act as amplifiers of external shocks on the exchange rate and other key UK macroeconomic variables. To the best of my knowledge, I am the first in the literature to test whether financial frictions have impacts on exchange rate dynamics.
Chapter 3

An Open Economy DSGE Model with Currency Risk Premium

This chapter presents a formal analysis of real exchange rate behaviour under the assumption of an imperfect international financial market. To this purpose, I present a small open economy Real Business Cycle model adapted from Uribe and Schmitt-Grohe (2016), with the addition of financial frictions in the intermediation process of international capital flows based on Gabaix and Maggiori (2016).

Gabaix and Maggiori (2016) provide an analytically tractable two-period two countries general equilibrium model where the representative international financier is constrained to intermediate capital flows across countries. I build on their analysis and incorporate the intermediation friction in a small open economy Real Business Cycle model. This allows me to test whether financial frictions in the intermediation process of international capital flows had influences on Sterling real exchange rate dynamics against data between 1975 and 2016 in Chapter 4. Furthermore, I answer the question how much matter the constrained international intermediation of capital flows to the fiscal policy and macro-prudential policy in Chapter 5.
There are three features of the model. First, the economy is open as it is able to trade goods and services with the rest of the world in the frictionless goods market, however, is small compared to the rest of the world; that is, its economic behaviour has neglectable impacts on key macroeconomic variables of the world such as world interest rate, prices or income. Thus, I treat the world variables like world interest rate, foreign consumption demand as exogenous variables, and the economy is a price taker. Second, following Meenagh et al. (2010), nominal rigidities are not an essential part of the transmission mechanism for generating exchange rate dynamics. Therefore, I choose a Real Business Cycle model without the assumption of nominal rigidities as an appropriate backdrop against which to account for UK real exchange rate over the business cycle. Third, the economy can borrow assets from the rest of the world to smooth consumption, which is intermediated by global financiers. However, the international financial market is imperfect due to financiers’ limited risk-bearing capacity. Hence, uncovered interest parity does not hold in the model, and financiers require compensation for holding currency risk. In particular, I emphasise the role of capital flows in exchange rate determination as bonds denominated in different currencies are imperfectly substitutable.

The tractability of the model allows me to solve it in closed-form and emphasize two channels for exchange rate dynamics. The first channel is interest rate channel. Higher interest rate offers foreign lenders a higher return, and it attracts capital inflow and causes an expected depreciation in the exchange rate. Put it differently, to convince foreign lenders to supply fund when exchange rate depreciates, the interest rate would have to increase. The second channel is currency risk-taking channel. Global financiers are subject to credit constraints; hence they cannot take infinite positions to absorb imbalance between demand and supply of bonds denominated in different currencies arising from international trade. The model generates a currency risk premiums and relates it to financial forces, that is, global financiers’ risk-bearing capacity and balance sheets. In other words, global financier’s
liquidity is crucial in determining exchange rates, which could help to explain the empirical disconnect between exchange rates and macroeconomic fundamentals.

The currency risk premium model is presented in Section 3.1. Structural shocks are described in Section 3.2, and log-linearised behaviour equations are listed in Section 3.3. Furthermore, a baseline calibration is outlined in Section 3.4. A discussion of the model in a perfect financial market is shown in Section 3.5, followed by comparing its impulse response functions with corresponding impulse response functions generated from the calibrated currency risk premium model in Section 3.6. Finally, I conduct financiers’ risk-bearing capacity experiments in Section 3.7.

3.1 The Model in An Imperfect Financial Market

Consider an infinite periods world economy. Time is discrete and indexed by $t \in \{0, \infty\}$. The world economy is inhabited by a small open domestic economy and by the rest of the world. Goods are tradable among all countries, and there are a single industry and one broad type of consumption good traded at the global level. Both the domestic economy and the rest of the world can issue a one-period bond.

In the domestic country, there is one utility-maximising representative household, a representative profit-maximising non-financial firm operating in a perfectly competitive final goods market, and a government. Households are the owners of the firm. Both firms and households are price-takers.

In the global financial market, there is a risk-averse representative financier intermediates international financial transactions and requires a currency risk premium proportional to the size of their currency exposure (country’ net foreign debt position).
3.1.1 Representative Household Problem

The economy is populated by an infinite number of identical households with preferences described by the utility function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$  \hspace{1cm} (3.1.1)

where $C_t$ denotes consumption, $\beta \in (0, 1)$ is the subjective discount factor, the symbol $E_0$ denotes the expectation operator conditional on the information available at period 0. $U(\cdot)$ is a period constant relative risk aversion (CRRA) utility function\(^1\) which takes the following additively separable form,

$$U(C_t, N_t) = \omega_0 \varepsilon^r_t C_t^{1-\gamma_C} - (1 - \omega_0) \varepsilon^N_t N_t^{1+\gamma_N}$$ \hspace{1cm} (3.1.2)

Households enjoy utility from goods consumption, while they receive dis-utility from producing goods. Thus this utility depends positively on the consumption of goods, $C_t$, and negatively on labour supply $N_t$. $\gamma_C > 0$ is the Arrow-Pratt coefficient of relative risk aversion for consumption, and its reciprocal, $\frac{1}{\gamma_C}$, measures the inter-temporal substitution elasticity between consumption in two consecutive periods. $\gamma_N$, which is greater than 0, is the inverse of Frisch labour supply elasticity. $\omega_0(0 < \omega_0 < 1)$ is a preference weight of consumption in the utility function. $\varepsilon^r_t$ and $\varepsilon^N_t$ are preference shocks, which affect the inter-temporal and the intra-temporal decision of households, respectively. Both shocks are assumed to follow a first-order autoregressive process with an i.i.d. error term.

We assume that each period the representative household supplies $N_t$ hours to the labour market and earns consumer real wage ($w_t$), which is equal to the producer wage deflated by the consumer price index. Households finance their expenditure through labour income.

---

\(^1\)The CRRA utility function is often used in applied theory and empirical work due to its tractability and appealing implications, for example, the CRRA utility form implies stationary risk premium and interest rates even in the presence of long-run economic growth.
total profit income \((\Pi_t)\) received from the ownership of shares of domestic firms, and financial instruments in the form of risk-free bonds issued by the domestic government and the rest of the world. However, domestic households are not able to borrow directly from foreign countries. Instead they borrow credits, \(\widetilde{D}_{t+1}\), from the international intermediary who is willing to supply those credits at the rate of interest, \(\widetilde{r}_t\). To emphasize the currency mismatch that international financial intermediary has to absorb, I assume that home country only trades in its own currency bonds. A risk-free bond\(^2\) issued by the rest of the world is intermediated by a representative international financier.

Both financial instruments, \(D_{t+1}, \widetilde{D}_{t+1}\), with time subscripts \(t+1\) are the households’ debt positions with a unit price at \(t\), and require one plus the rate of interest agreed at time \(t\) in the following due period \((t+1)\). They use those funds to purchase consumption goods, \(C_t\), and pay back the principal and interest on its outstanding domestic and foreign debts, \((1 + r_{t-1})\) and \((1 + \widetilde{r}_{t-1})\), respectively. Also, households are taxed by a lump-sum transfer, \(T_t\); marginal tax rates are not included in the model explicitly and appear implicitly in the error term of the labour supply equation.

The period-by-period budget constraint of the representative household is given by

\[
C_t + D_t(1 + r_{t-1}) + \widetilde{D}_t(1 + \widetilde{r}_{t-1}) + T_t = w_tN_t + \Pi_t + D_{t+1} + \widetilde{D}_{t+1}. \tag{3.1.3}
\]

The household chooses processes \(\{C_t, N_t, D_{t+1}, \widetilde{D}_{t+1}, \lambda_t\}_{t=0}^\infty\) to maximise his utility (Equation 3.1.1 and Equation 3.1.2) subject to his budget constraint 3.1.3 and no-Ponzi constraints of the forms,

\[
\lim_{j \to \infty} E_t \frac{D_{t+1+j}}{\prod_{j=0}^j (1 + r_j)} \leq 0 \tag{3.1.4}
\]

\[
\lim_{j \to \infty} E_t \frac{\widetilde{D}_{t+1+j}}{\prod_{j=0}^j (1 + \widetilde{r}_j)} \leq 0, \tag{3.1.5}
\]

\(^2\)Risk-free here refers to paying one unit of foreign general consumption basket in all states of the world.
taking the processes \( \{r_t, w_t, \tilde{r}_t, \Pi_t, T_t\}_{t=0}^{\infty} \) and the initial conditions \( D_0(1 + r_{-1}) \) and \( \tilde{D}_0(1 + \tilde{r}_{-1}) \) as given. The conditions in Equation 3.1.4 and Equation 3.1.5 imply that debts do not grow faster than their corresponding interest rates.

The Lagrangian associated with household’s maximization problem in period 0 is given by

\[
\tilde{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t E_t \{ (\omega_0) e^r_t C_t^{1-\gamma_c} (1 - \omega_0) e^N_t N_t^{1+\gamma_N} + \tilde{\lambda}_t \Pi_t + \tilde{D}_{t+1} - C_t - D_t(1 + r_{-1}) - \tilde{D}_t(1 + \tilde{r}_{-1}) - T_t \},
\]

where \( \beta^t \tilde{\lambda}_t \) denotes the Lagrange multiplier associated with the sequential budget constraint 3.1.3. The first-order conditions corresponding to \( C_t, N_t, D_{t+1}, \tilde{D}_{t+1} \), and \( \tilde{\lambda}_t \), respectively, are

\[
(1 - \omega_0) e^N_t N_t^{\gamma_N} + \tilde{\lambda}_t w_t = 0 \tag{3.1.7}
\]

\[
\beta^t \tilde{\lambda}_t - E_t \beta^{t+1} \tilde{\lambda}_{t+1}(1 + r_t) = 0 \tag{3.1.8}
\]

\[
\beta^t \tilde{\lambda}_t - E_t \beta^{t+1} \tilde{\lambda}_{t+1}(1 + \tilde{r}_t) = 0 \tag{3.1.9}
\]

and

\[
w_t N_t + \Pi_t + D_{t+1} + \tilde{D}_{t+1} - C_t - D_t(1 + r_{-1}) - \tilde{D}_t(1 + \tilde{r}_{-1}) - T_t = 0. \tag{3.1.11}
\]

Household optimization implies that the constraints 3.1.4 and 3.1.5 hold with equality. The Euler Equation 3.1.12 could be obtained by combining optimality conditions 3.1.7 and 3.1.9,
describing inter-temporal substitution in consumption

\[
\frac{U_C(C_t, N_t)}{1 + r_t} = \beta E_t U_C(C_{t+1}, N_{t+1}). \tag{3.1.12}
\]

It states that the price of an extra unit of utility from consumption today is \(\frac{1}{1 + r_t}\) in terms of tomorrow’s expected marginal utility of consumption discounted by time preference.

Dividing optimality condition 3.1.8 by optimality condition 3.1.7 to eliminate \(\lambda_t\). This yields the intra-temporal condition,

\[
- \frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = w_t. \tag{3.1.13}
\]

This equates the marginal rate of substitution between leisure and consumption to their price ratio, the real wage. The left-hand side of expression 3.1.13 is the household’s labour supply schedule, which is increasing in hours worked, holding the level of consumption constant.\(^3\)

The optimality condition 3.1.9 for \(D_{t+1}\) yields

\[
\frac{1}{1 + r_t} = \beta \frac{\lambda_{t+1}}{\lambda_t}. \tag{3.1.14}
\]

Combing Equation 3.1.14 with the optimality condition 3.1.10 for \(D_{t+1}\) to eliminate \(\beta \frac{\lambda_{t+1}}{\lambda_t}\) yields a no-arbitrage condition,

\[
r_t = \bar{r}_t. \tag{3.1.15}
\]

It equates the real rate of return on the bonds issued by the domestic government to the real rate of interest on the bond supplied by the international financier. Hence the domestic households have no preference on either of the financing methods, and we can refer to a single asset return, \(r_t\).

\(^3\)A sufficient condition for \(- \frac{U_N(C_t, N_t)}{U_C(C_t, N_t)}\) to be increasing in \(N_t\), holding \(C_t\) constant, is \(U_{CN} < 0\), and the necessary and sufficient condition is \(\frac{U_{CN}}{U_N} > \frac{U_{CN}}{U_C}\).
This small open economy model assumes that the domestic country has a single, perfectly competitive final goods sector, producing a version of the final good that is distinct from the product of the foreign country. It is a single-industry version of the Armington model (Armington, 1969; see also Feenstra et al., 2014). The Armington assume that home and foreign goods are differentiated purely due to their origin of production. Households in home country consume a domestically traded good, and an imported good. The home consumption index includes only one type of good, $C_t$, is divided between home tradable, $C^d_t$, and foreign tradable goods consumption, $C^f_t$. Differentiated products of a given type bring utility to the household via a constant elasticity of substitution (CES) aggregator utility function,

$$C_t = \left[ \omega \left( C^d_t \right)^{\theta} + (1 - \omega) \left( e^{IM}_t \right)^{\theta} \right]^{\frac{1}{\theta}}$$  \hspace{1cm} (3.1.16)

where $\omega$ is the weight of domestically produced tradable goods, and $\theta > 0$ is the elasticity of substitution between home and foreign tradable goods. The value of $\omega$ is crucial since it describes the degree of home bias in preferences. $\omega > \frac{1}{2}$ implies a bias towards domestic produced tradable goods relative to imported goods from the rest of the world. Domestic produced goods and imported goods are perfect substitutes if $\theta$ approaches to infinity; those goods are perfect complements if $\theta$ approaches to zero. The degree of substitution between home-produced and imported goods may be affected by economic reasons, such as product quality or industry features, and also influenced by political variables and strategies. $e^{IM}_t$ is a random preference shock of home demand for foreign produced goods. The level of consumption $\tilde{C}_t$ chosen above must satisfy the expenditure constraint on consumption,

$$\tilde{C}_t = p^d_t C^d_t + Q_t C^f_t$$  \hspace{1cm} (3.1.17)

where $p^d_t$ denotes the domestic goods price level, $P^d_t$, relative to the general price level, $P_t$; $Q_t$ is the relative price of home and foreign countries’ consumption basket. It is unit free
measure of the price of the foreign consumption goods $P^F_t$ relative to the general price level in home country $P_t$ defined as $Q_t = \frac{S_t P^F_t}{P_t}$, where $S_t$ is the nominal exchange rate and is given in terms of domestic currency needed to buy a unit of foreign currency. Intuitively, an increase in $Q_t$ can be thought of as a real exchange rate depreciation, as it implies a real depreciation of domestic goods on the world market and a rise in the competitiveness of domestic exports. I treat the consumption bundle as the numeraire and, consequently, its price equals 1 in the domestic currency. Given that all prices in the budget constraint are expressed relative to the general price level, $P_t$. Hence, in terms of the domestic currency, the unit cost of imported goods, $C^f_t$, is $Q_t$.

The domestic household chooses processes $\{C^d_t, C^f_t\}_{t=0}^\infty$ to maximise composite utility index 3.1.16 subject to the constraint\(^4\) that

$$C_t \leq \tilde{C}_t,$$ (3.1.18)

taking as given the relative prices $\{p^d_t, Q_t\}_{t=0}^\infty$.

The Lagrangian for composite utility index maximization problem is

$$\mathcal{L}_t = \left[ \omega \frac{1}{\sigma} (C^d_t) \frac{\theta - 1}{\theta} + \frac{1}{\theta} \left( (1 - \omega) \frac{1}{\beta} \left( \epsilon^M_t \right)^{\frac{1}{\beta}} (C^f_t) \frac{\theta - 1}{\theta} - \Lambda_t \right) \right]^{\frac{\theta}{\theta - 1}} - \Lambda_t p^d_t C^d_t - Q_t C^f_t$$ (3.1.19)

and the first order conditions\(^5\) for $C^d_t$ and $C^f_t$ are:

$$\frac{\partial \mathcal{L}_t}{\partial C^d_t} = \frac{\theta}{\theta - 1} (C^d_t)^{\frac{\theta - 1}{\sigma}} (1 - \omega) \frac{1}{\beta} \left( \epsilon^M_t \right)^{\frac{1}{\beta}} (C^f_t)^{\frac{\theta - 1}{\theta}} - \Lambda_t p^d_t = 0$$ (3.1.20)

\(^4\)At the point of the maximum the constraint is binding, so that the consumption-equivalent utility, $C_t$ (the variable that appears in Equation 3.1.16), is equal to the amount spent on consumption goods, $C_t$ that the variables appears in household’s budget constraint 3.1.3).

\(^5\)Using the substitution

$$\left[ \omega \frac{1}{\sigma} (C^d_t) \frac{\theta - 1}{\theta} + \frac{1}{\theta} \left( (1 - \omega) \frac{1}{\beta} \left( \epsilon^M_t \right)^{\frac{1}{\beta}} (C^f_t) \frac{\theta - 1}{\theta} \right)^{\frac{\theta}{\theta - 1}} \right]^{\frac{\theta}{\theta - 1}} = \left[ \omega \frac{1}{\sigma} (C^d_t) \frac{\theta - 1}{\theta} + \frac{1}{\theta} \left( (1 - \omega) \frac{1}{\beta} \left( \epsilon^M_t \right)^{\frac{1}{\beta}} (C^f_t) \frac{\theta - 1}{\theta} \right)^{\frac{\theta}{\theta - 1}} \right]^{\frac{\theta}{\theta - 1}} = (C_t)^{\frac{1}{\theta}}.$$
\[
\frac{\partial L_t}{\partial C_t^{f}} = \frac{\theta}{\theta - 1} (C_t)_{\theta}^{\frac{1}{\theta}} (1 - \omega)_{\theta}^{\frac{1}{\theta}} \frac{\theta - 1}{\theta} (e^{IM}_t)_{\theta}^{\frac{1}{\theta}} (C_t^{f})_{\theta}^{\frac{1}{\theta}} - \Lambda_t Q_t = 0. \tag{3.1.21}
\]

At the maximum, \( C_t = C^*_t, \frac{\partial L_t}{\partial C_t^{f}} = \Lambda_t, \frac{\partial C_t}{\partial C_t^{f}} = 1, \) hence it follows that \( \Lambda_t = 1 \) when the constraint binds, implying that the change in the utility index is unity due to a unit increase in consumption.

Hence, the domestic demand for home goods is given by optimality condition 3.1.20

\[
C_t^{d} = \omega (p_d^t)^{-\theta} C_t, \tag{3.1.22}
\]

and the domestic demand for foreign produced goods (import equation) is given by optimality condition 3.1.21,

\[
C_t^{f} = IM_t = (1 - \omega) e^{IM}_t (Q_t)^{-\theta} C_t \tag{3.1.23}
\]

The domestic demand for home goods is positively affected by total consumption in the home country, \( C_t, \) and negatively by the price of domestic produced goods relative to the general price level \(^6\) \( p_d^t; \) while domestic import depends positively on the total home consumption of goods, \( C_t, \) and negatively on the real exchange rate, \( Q_t. \)

\[
(C_t)^{\frac{\theta - 1}{\theta}} = [\omega^{\frac{1}{\theta}} (C_t^{d})^{\frac{\theta - 1}{\theta}} + (1 - \omega)^{\frac{1}{\theta}} (e^{IM}_t)^{\frac{1}{\theta}} (C_t^{f})^{\frac{\theta - 1}{\theta}}]^{\frac{\theta}{\theta - 1}} \times \frac{\theta - 1}{\theta} \tag{3.1.24}
\]

Dividing 3.1.24 by \( (C_t)^{\frac{\theta - 1}{\theta}} \) to obtain \( 1 = \omega^{\frac{1}{\theta}} (C_t^{d})^{\frac{\theta - 1}{\theta}} + (1 - \omega)^{\frac{1}{\theta}} (e^{IM}_t)^{\frac{1}{\theta}} (C_t^{f})^{\frac{\theta - 1}{\theta}} ; \) substituting out \( C_t^{d} \) and \( C_t^{f}, \) by using Equation 3.1.22 and 3.1.23 above, gives \( 1 = \omega^{\frac{1}{\theta}} [\omega (p_d^t)^{-\theta}]^{\frac{\theta - 1}{\theta}} + (1 - \omega)^{\frac{1}{\theta}} (e^{IM}_t)^{\frac{1}{\theta}} [(1 - \omega) e^{IM}_t (Q_t)^{-\theta}]^{\frac{\theta - 1}{\theta}}. \) After rearranging it, I can obtain \( 1 = \omega (p_d^t)^{1-\theta} + (1 - \omega) e^{IM}_t (Q_t)^{1-\theta}, \) hence

\[
p_d^t = \left[ \frac{1 - (1 - \omega) e^{IM}_t (Q_t)^{1-\theta}}{\omega} \right]^{\frac{1}{1-\theta}}. \tag{3.1.25}
\]
3.1.2 Relationship with the Rest of the World

Given Equation 3.1.23 above, there exists a symmetric equation for the rest of the world which describes the foreign demand for domestic goods. Hence, this export equation for the home economy is

\[ EX_t = (1 - \omega^F)\varepsilon^{EX}_t (Q_t) \theta^F C^F_t \]  

(3.1.26)

where \( EX_t \) denotes the foreign demand for domestic goods (export from domestic country to the rest of the world). \( \omega^F, C^F_t \) and \( \theta^F \) are the foreign equivalents to home bias, total consumption of goods and the elasticity of marginal substitution between domestic and imported goods, respectively. \( \varepsilon^{EX}_t \) is random preference shock to the foreign demand for domestic goods. The volume of export demand goes up when total consumption of goods in the rest of the world, \( C^F_t \), increases. A depreciation of real exchange rate (a rise in \( Q \)) induces a rise in the competitiveness of domestic exports. Total consumption of goods in the rest of the world, \( C^F_t \), is treated as an exogenous variable given by a first-order autoregressive process,

\[ \ln C^F_t = \rho_{C^F} \ln C^F_{t-1} + \eta_{C^F,t}, \]  

(3.1.27)

where \( \eta_{C^F,t} \) is an independent and identically distributed innovation.

3.1.3 Representative Firm Problem

The output of the economy is assumed to depend on a production function that combines labour and capital inputs. Firms operate in perfectly competitive product and factor markets. A representative firm hires labour, purchases new capital goods to produce an homogeneous final good using production technology given by

\[ Y_t = A_t F(K_t, N_t) \]  

(3.1.28)
where $A_t$ is a random productivity shock variable and reflects the state of technology. $Y_t$ is an output of the economy. $F$ is an increasing and concave function, satisfying Inada-type conditions, i.e. the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0 and approaches 0 as capital (or labour) goes to infinity

$$\lim_{K \to 0} (F_K) = \lim_{N \to 0} (F_N) = \infty$$ (3.1.29)

$$\lim_{K \to \infty} (F_K) = \lim_{N \to \infty} (F_N) = 0.$$ (3.1.30)

Capital evolves according to the following law of motion

$$K_{t+1} = (1 - \delta)K_t + I_t$$ (3.1.31)

where $K_t$ is predetermined capital stock, $I_t$ is the firm’s investment, and $\delta$ measures the depreciation rate.

The Cobb-Douglas specification of the production function is widely used in the business-cycle literature, hence we adopt it for the bulk of our analysis,

$$F(K_t, N_t) = N_t^\alpha K_t^{1-\alpha}$$ (3.1.32)

where $\alpha (0 \leq \alpha \leq 1)$ is the output elasticity of labour. This specification implies a unitary elasticity of substitution between labour and capital, that is, a one percent increase in the labour price to rental price, $\frac{\partial F_Y(K_t, N_t)}{\partial F_K(K_t, N_t)}$, induces firms to increase the capital-labour ratio by one percent. Also, it describes a constant returns to scale and diminishing marginal products to labour and capital inputs.

Capital adjustment cost is a regular feature of business cycle model, as a property of most open economy models is that investment is excessively volatile in the absence of adjustment
costs (Schmitt-Grohé, 1998). Therefore, I assume that the change in the stock of capital comes at a cost, for the sake of dampening the volatility of an investment in response to variations in the productivity of domestic capital or in the foreign interest rate over the business cycle.

Suppose the representative firm faces convex adjustment costs to capital, for the sake of tractability, to take a quadratic form,

$$\Phi(. \mid .) = \kappa^2 (K_{t+1} - K_t)^2. \quad (3.1.33)$$

The function $\Phi(\cdot)$ is meant to capture capital adjustment costs and is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$ and $\Phi''(0) > 0$. The restrictions imposed on $\Phi(\cdot)$ and $\Phi'(\cdot)$ ensure that in the steady state, that is, when $K_{t+1} - K_t = 0$, adjustment costs are nil and the relative price of capital goods in terms of consumption goods is unity; $\kappa$ denotes a multiplicative constant affecting adjustment costs.

The firm maximises the present discounted value of profits,

$$\pi_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\bar{\lambda}_t}{\lambda_0} \left[ Y_t - (\bar{w}_t + \varepsilon^N_t)N_t - I_t - \frac{\kappa}{2} (K_{t+1} - K_t)^2 \right] \quad (3.1.34)$$

subject to constraints 3.1.28 and 3.1.31, through its choices of $\{N_t, I_t\}_{t=0}^{\infty}$, taking prices $\{\bar{w}_t, \bar{\lambda}_t\}_{t=0}^{\infty}$ and initial condition $K_0$ as given. Assume free entry into the industry and a large number of firms operating under perfect competition. $I_t$ and $Y_t$ can be eliminated from Equation 3.1.34 by using Equation 3.1.28-3.1.32, I could obtain the Lagrangian for the problem, which is

$$L_0 = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\bar{\lambda}_t}{\lambda_0} \left[ A_t N_t^{\alpha} K_t^{1-\alpha} - (\bar{w}_t + \varepsilon^N_t)N_t - (1 + \varepsilon^K_t)K_{t+1} + (1 - \delta)K_t - \frac{\kappa}{2} (K_{t+1} - K_t)^2 \right]. \quad (3.1.35)$$
Instead of choosing new investment at period \( t \), the firm choose processes \( \{N_t, K_{t+1}\}_t=0^- \). Since the representative household owns firms, he put cash flows (measured in goods) in terms of current utilities. The firm discounts future cash flows by the stochastic discount factor, \( \beta_t^{\frac{\lambda_t}{\lambda_0}} \), which is the value assigned by households to contingent payments of goods in period \( t \) in terms of units of goods in period 0. Specifically, one unit of profits returned to the household at time \( t \) generates \( U'(C_t) \) additional units of utility, which must be discounted back to the present period (which I take to be 0). \( \epsilon_t^{Nd} \) and \( \epsilon_t^K \) are the shocks to the net rental costs of labour and capital, respectively - these could capture the effect of excluded tax rates and other imposed regulations on firms’ inputs, for instance, the impact of depreciation allowances or national insurance.

The first order conditions with respect to \( N_t \) and \( K_{t+1} \) are as follows:

\[
\begin{align*}
\frac{\partial L_0}{\partial N_t} &= \alpha A_t N_t^{\alpha - 1} K_t^{1 - \alpha} - (\bar{w}_t + \epsilon_t^{Nd}) = 0; \\
\frac{\partial L_0}{\partial K_{t+1}} &= \beta_t^{\frac{\lambda_t}{\lambda_0}} [-1 - \epsilon_t^K] - \kappa (K_{t+1} - K_t) \\
&+ \beta_t^{1+} \frac{E_t^{\lambda_t+1}}{\lambda_0} [(1 - \alpha) E_t (A_{t+1} N_{t+1}^{\alpha} K_{t+1}^{-\alpha}) + (1 - \delta) + \kappa E_t (K_{t+2} - K_{t+1})] = 0.
\end{align*}
\]

Optimality condition 3.1.36 sets the marginal product of labour \( \alpha A_t N_t^{\alpha - 1} K_t^{1 - \alpha} \) equal to its price \( \bar{w}_t + \epsilon_t^{Nd} \) - the real unit cost of labour to the firm, \( \bar{w}_t \), and the stochastic cost shock term, \( \epsilon_t^{Nd} \). It can be rearranged to give the firm’s demand for labour condition,

\[
N_t = \alpha \frac{Y_t}{\bar{w}_t + \epsilon_t^{Nd}}.
\]

Here the rental rate of labour, \( \bar{w}_t \), is different from the real wage referred to the household problem, \( w_t \). The real wage in the household budget constraint is the nominal wage, \( Y_t \), relative to the general price level, \( P_t \) (treated as the numeraire throughout), which is the price
of the consumption bundle that includes both domestic and imported goods; that is

$$w_t = \frac{W_t}{P_t}. \quad (3.1.38)$$

However, the real rental price of labour paid by the domestic firm is the nominal wage relative to the unit value of domestically produced goods, $P^d_t$; that is

$$\tilde{w}_t = \frac{W_t}{P^d_t}. \quad (3.1.39)$$

Combining the domestic goods price level relative to the general price level, $p^d_t = \frac{P^d_t}{P_t}$, with Equation 3.1.39, the wedge between the real rental price of labour and the real wage of household can be expressed as

$$p^d_t = \frac{w_t}{\tilde{w}_t}. \quad (3.1.40)$$

Substituting out $p^d_t$ by using Equation 3.1.25, then I can obtain

$$\tilde{w}_t = \frac{w_t}{p^d_t} = \frac{w_t}{\left[1 - (1 - \omega)\frac{\beta^d}{\alpha} \left(Q_t\right)^{1-\theta} \right] \frac{1}{1-\sigma}}. \quad (3.1.41)$$

Eliminating $\frac{E_t\lambda_{t+1}}{\lambda_t}$ in optimality condition 3.1.36 by using Equation 3.1.9, then it can be written as

$$(1 + r_t)[1 + \epsilon^K_t + \kappa(K_{t+1} - K_t)]$$

$$= (1 - \alpha)E_t(A_{t+1}N^\alpha_{t+1}K_{t+1}^\alpha - \alpha_t + (1 - \delta) + \kappa E_t(K_{t+2} - K_{t+1})$$

where $1 + \kappa(K_{t+1} - K_t)$ represents the marginal costs of producing a unit of capital. It is equal to the relative shadow price of capital in terms of consumption goods, which is known as Tobin’s q. $\epsilon^K_t$ is the stochastic shock to capital demand. The left-hand side of Equation 3.1.42 is the return of investing $1 + \kappa(K_{t+1} - K_t)$ units of goods in bonds, and the right-hand side is the return associated with investing the same units of goods in
physical capital. More specifically, the additional unit of capital yields $A_{t+1} F_K(K_{t+1}, N_{t+1})$ (marginal product of capital) units of output next periods. Also, an extra unit of capital reduces tomorrow’s adjustment costs by $\kappa E_t(K_{t+2} - K_{t+1})$. Finally, the unit of capital can be sold next period at the price $1 - \delta$ due to the depreciation. Alternatively, the agent can invest $1 + \epsilon_t^K + \kappa(K_{t+1} - K_t)$ amounts of bonds in period $t$, which yields a gross return of $(1 + r_t)[1 + \kappa(K_{t+1} - K_t)]$ in period $t + 1$. At the optimum both strategies must obtain the same return.

Equation 3.1.42 can be rearranged to give a non-linear difference equation in capital,

$$K_{t+1} = \frac{1}{\kappa} + K_t + \frac{1 - \alpha}{\kappa(1 + r_t)} E_t Y_{t+1} + \frac{1 - \delta}{\kappa(1 + r_t)} \frac{E_t K_{t+2} - K_{t+1}}{1 + r_t} - \frac{1}{\kappa} \epsilon_t^K$$

(3.1.43)

This equation could be named as the demand for capital, and its non-linearity is caused by the quadratic capital adjustment costs that the firm faces.

### 3.1.4 International Financial Intermediary

Domestic households can freely trade domestic assets, i.e. $D_t$, however, they are constrained in their holdings of foreign assets. There is a unit mass of global financial firms in the international financial market, who can actively invest in bonds denominated in both of home currency and foreign currencies and are hence able to absorb any excess supply and demand of assets. Furthermore, in light of the insight of Gabaix and Maggiori (2016), financiers with no capital of their own face limited commitment constraints. It implies that global financial intermediaries require a currency risk premium due to the limited risk-bearing capacity, so uncovered interest parity is violated.

For simplicity, I assume that the financiers are owned by households from the rest of the world and the management of financial firms is a one-period job. At the end of each period, financiers pay their profits and losses out to the owners. The representative financier’s balance
sheet consists of $D_{t+1}$ domestic currency, and $-\frac{D_{t+1}}{Q_t}$ foreign currency, where $D_{t+1}$ is the value in domestic currency \(^7\) of domestic currency-denominated bonds the financier is long of, and $-\frac{D_{t+1}}{Q_t}$ the corresponding value in foreign currency of foreign currency-denominated bonds. The subscript $t+1$ expresses the maturity date of those financial instruments, which are issued at time $t.$

### International Financier’s Balance Sheets

<table>
<thead>
<tr>
<th>Assets (Credit to Domestic Households)</th>
<th>Liabilities (Debt to the rest of the world)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B/S$ at date $t$</td>
<td></td>
</tr>
<tr>
<td>$D_{t+1}$</td>
<td>$\frac{D_{t+1}}{Q_t}$</td>
</tr>
<tr>
<td>$B/S$ at date $t+1$</td>
<td></td>
</tr>
<tr>
<td>$(1 + \tilde{r}<em>t)D</em>{t+1}$</td>
<td>$\frac{D_{t+1}}{Q_t}(1 + r_f^t)$</td>
</tr>
<tr>
<td>Financier’s expected profit at date $t+1$</td>
<td>$[(1 + \tilde{r}<em>t) - (1 + r_f^t)\frac{Q</em>{t+1}}{Q_t}]D_{t+1}$</td>
</tr>
</tbody>
</table>

Table 3.1 International Financier’s Balance Sheets

Suppose that the expected value of his financial firms is generated by lending $D_{t+1}$ to domestic households at the interest rate $\tilde{r}_t$ and capturing corresponding funds $\frac{D_{t+1}}{Q_t}$, from the rest of the world at the world interest rate $r_f^t$. It is given by

$$V_t = E_t\{\beta \frac{\Lambda_{t+1}}{\lambda_t}[(1 + \tilde{r}_t) - (1 + r_f^t)\frac{Q_{t+1}}{Q_t}]D_{t+1}\}. \quad (3.1.44)$$

Since each representative financier manages the firm for one-period, expected value at period $t+1$ is discounted using the stochastic factor $\beta \frac{\Lambda_{t+1}}{\lambda_t}$, which is the value assigned by the financier to contingent payments of goods in period $t+1$ in terms of units of goods in period $t$. Since the financier pay back the principle to foreign countries one period later, the value of liability, $-\frac{D_{t+1}}{Q_t}$ should be adjusted with the expected relative price at the maturity date $t+1$,

---

\(^7\)In the absence of a nominal side to the model, the currency means a claim to the numeraire of the economy; domestic currency-denominated or foreign currency-denominated mean values expressed in units of general consumption baskets in each economy.
that is, expected real exchange rate, $E_t Q_{t+1}$. Table 3.1 shows the representative international financier’s balance sheet at period $t$ and period $t+1$.

In this small open economy model, the world interest rate, $r_f$, is treated as an exogenous variable, since the size of the domestic economy is too small to affect the rest of the world. I assume that $r_f$ follow a first-order autoregressive process,

$$r_f = \rho_f r_{f-1} + \eta_{r_f}.$$  (3.1.45)

In light with the assumptions of Gabaix and Maggiori (2016), the financiers’ borrowing process is subject to an agency friction that imposes a restriction on the size of the balance sheet of the financiers, which prevents perfect arbitrage between domestic-currency denominated bonds and foreign-currency denominated bonds. To take the role of limited financial risk-bearing capacity by the financiers, I assume that financiers can divert a portion $\varepsilon \Gamma t \Gamma g |\frac{D_t}{Q_t}|$ of the funds they intermediate in each period. Rational foreign lenders anticipate the incentives of the financier to divert funds and are willing to lend as long as the following constraint 3.1.46 holds. The representative financier faces a credit constraint of the form,

$$\frac{V_t}{Q_t} \geq \frac{D_{t+1}}{Q_t} |\varepsilon^\Gamma \Gamma |\frac{D_{t+1}}{Q_t}|.$$ (3.1.46)

The left-hand side of Equation 3.1.46 measures the intermediary value in foreign currency, while the right-hand side is the total divertable funds, which is convex in $D_t$. In addition, the value of the financier’s financial firm is linear in the position $D_t$, hence the constraint, Equation 3.1.46, always binds. The constraint limits the maximum position the financiers can take. The parameter $\Gamma$ ($\Gamma \geq 0$) captures the ability of financiers to bear risks, and governs the debt elasticity of the country interest rate. $\varepsilon^\Gamma$ is the financial shock which alters the financiers’ risk bearing capacity. The representative financier chooses processes $\{D_t\}_{t=0}^\infty$ to maximise

---

8In order to make economic sense the constraint must satisfy that $\varepsilon^\Gamma \Gamma |\frac{D_{t+1}}{Q_t}| \leq 1$. That is, the global intermediary cannot steal more than 100 percentage of the funds borrowed.
the expected value of his financial firms, Equation 3.1.44 subject to his limited commitment constraint 3.1.46, taking as given the processes \( \{ \tilde{r}_t, r_t^f, Q_t, \tilde{\lambda}_t \}_{t=0}^{\infty} \). \( V_t \) can be eliminated from Equation 3.1.44 by using Equation 3.1.46, I could obtain

\[
\tilde{D}_{t+1} = \frac{1}{\varepsilon_t^f \Gamma} \lambda_t E_t \{ \beta \frac{\lambda_t+1}{\lambda_t} [(1 + \tilde{r}_t)Q_t - (1 + r_t^f)Q_{t+1}] \}.
\]  (3.1.47)

Equation 3.1.47 shows the financier’ downward sloping demand for domestic currency. Alternatively, it shows the supply of foreign credit converted in home currency intermediated by the global financier. I will investigate this equation in more detail following, when the discount factor, \( \beta \frac{\lambda_t+1}{\lambda_t} \), is substituted out. Since marginal returns are equal across different types of bonds (see Equation 3.1.15), I can refer to a single asset return, \( r_t \). To eliminate \( \beta \frac{\lambda_t+1}{\lambda_t} \) and \( \tilde{r}_t \), I plug Equation 3.1.14 and Equation 3.1.15 into Equation 3.1.47, then it yields

\[
\tilde{D}_{t+1} = \frac{1}{\varepsilon_t^f \Gamma} [(1 + r_t^f)E_t Q_{t+1}].
\]  (3.1.48)

The global financial intermediaries’ demand for domestic bonds is increasing in the excess return of home bonds in comparison with the bonds issued by the rest of the world. The parameter \( \Gamma \) governs the size of the balance sheet of the global financial intermediaries and is hence an inverse measure of their risk-bearing capacity. Intuitively, an increase in the value of \( \Gamma \) leads to a decrease in the financier’s ability to carry the currency risk of their portfolio; in addition, their domestic asset demand curve becomes steeper due to the rise in the required compensation per unit of risk, and the global asset market tends to be more segmented. In particular, as the value of \( \Gamma \) goes to infinity, then the demand for domestic bonds, \( \tilde{D}_{t+1} \) goes to 0. In this case, the financiers are unable to take any position, put differently, they are unwilling to absorb any imbalance, for example, those caused by the trade flows. On the other hand, as the value of \( \Gamma \) goes to 0, then the financier is willing to trade (either borrow or lend) as much as possible in domestic currency-denominated and foreign currency-denominated
bonds given any non-zero expected excess return in the global financial market. In this situation, uncovered interest parity holds, that is, assets from different countries have the same expected rate of return when they are converted into the same currency.

Furthermore, Equation 3.1.48 implies the determination of real exchange rate. Rearrange Equation 3.1.48 to get

\[ Q_t = \frac{1 + r_f^t}{1 + r_t^t} E_t Q_{t+1} + \epsilon_t^\Gamma \bar{D}_{t+1}. \]  

(3.1.49)

The behaviour of exchange rate and currency risk premium are linked to home country’s external imbalances in a setting in which assets are imperfect substitutes. Currency risk premium, \( \epsilon_t^\Gamma \bar{D}_{t+1} \), is positively (negatively) correlated with home country’s net foreign debts (net foreign assets), since a net external debtor country’s currency generally depreciates in bad times and the financiers require compensations for holding that currency. In addition, the financiers’ risk bearing capacity has impacts on currency risk premium and in turn, real exchange rate; for instance, a global financial disruption, by decreasing international financial intermediaries’ risk bearing capacity, results in an immediate currency depreciation and a decrease in \( E_t Q_{t+1} \) in order to encourage the financiers to take positions and absorb imbalance.

In the model, there are two distinct channels- interest rate differentials channel and currency risk-taking channel through net foreign debt (asset) positions which affect currency excess returns. By providing a simple and tractable specification for the credit constrained problem, I emphasise that the financier’s demand function captures the feature of limits of arbitrage theory and the spirit of international financial intermediation.

### 3.1.5 Government

The government’s sources of income are tax revenue collected from households, and the issuance of new government bonds maturing one period ahead, \(-D_{t+1}\). Government’s spending consists of goods of consumption, \( G_t \), which is assumed to be non-productive and made up
strictly of welfare transfers, and interest payments on government debt agreed at a previous period, \(-r_{t-1}D_t\). The sequential budget constraint of the government is then given by

\[
T_t - D_{t+1} = G_t - D_t(1 + r_{t-1})
\]

(3.1.50)

where \(T_t\) is a lump-sum tax, capturing the revenue effects of all tax instruments that affect the household. Government spending \(G_t\) is treated as an exogenous variable given by the first-order autoregressive process,

\[
\ln G_t = \rho G \ln G_{t-1} + \eta_{G,t}.
\]

(3.1.51)

### 3.1.6 Market Clearing Conditions

According to Walras’ Law in general equilibrium theory, demand should be equal to supply in each market.

This leads to the following market clearing conditions in goods market for home country,

\[
Y_t = C_t + I_t + G_t + E_{t} - I_{M_t}
\]

(3.1.52)

In the volume terms, the equation above is saying that the supply of the goods is equated to the demand for consumption, investment, government consumption, and net exports. Relative goods prices to general price level move to ensure that market clearing also holds in value terms.

In addition, Walras’ Law imply that the overall assets and capital markets must clear,

\[
\Delta D^S_{t+1} + \Delta D^D_{t+1} = \Delta D^D_{t+1} + \Delta D^D_{t+1};
\]

(3.1.53)
\[ \Delta K_{t+1}^S = \Delta K_{t+1}^D. \]  

(3.1.54)

Changes in supply for assets (capital) on the left hand side is equated to its changes in demand on the right hand side by movement in the asset (capital) returns.

Labour market is cleared by wage price,

\[ N_t^S = N_t^D, \]  

(3.1.55)

where labour supply is equated to labour demand.

We consider the fundamental balance-of-payments identity in the open economy,

\[ \bar{D}_{t+1} - \bar{D}_t = r_{t-1} \bar{D}_t + IM_t Q_t - EX_t, \]  

(3.1.56)

It says that the change in the country’s net foreign debt position equals the repayment of foreign debt from the previous period and the net import. Alternatively, Equation 3.1.56 can be rewritten as the form of

\[ -(\bar{D}_{t+1} - \bar{D}_t) = EX_t - IM_t Q_t - r_{t-1} \bar{D}_t. \]  

(3.1.57)

The current account is defined as the sum of the trade balance and the net investment income on the country’s net foreign asset position.

The left-hand side of the expression 3.1.57 is the change of the country’s net foreign asset position, and the right-hand side of it measures the current account balance. In other words, a current account deficit (surplus) is associated with an increase (decrease) in the country’s external debt of equal magnitude.
3.2 Structural Shocks Summary

One of the aims of this real business cycle analysis is to determine the dynamic response of the small open economy to a variety of shocks. There are ten shocks included in the model. Three of them are obtained from exogenous variables, such as $C^F_t$, $G_t$ and $r^F_t$, while the rest of them are the residuals backed out of the structural model given the calibration. The standard autoregressive nature of the shocks provides the key propagation mechanism for these shocks in our model. Here, autoregressive-integrated-moving average (ARIMA) models proposed by Box et al. (1970) are used to capture shocks process. I assume stationary shock variables including structure residuals and exogenous variables, either level-stationary or trend-stationary, take the following ARIMA (1,0,0) form,

$$
\epsilon^i_t = \mu_i + b_i t + \rho_i \epsilon^i_{t-1} + \eta_{i,t}
$$

(3.2.1)

where $\eta_{i,t}$ represents independent identically distributed innovation with mean zero; $\epsilon^i_t$ is the shock, and superscript $i$ identifies the corresponding shock equation; $t$ defines the time trend; $\mu_i$, $b_i$, $\rho_i$ are the intercept, the coefficient of time trend, and the autoregressive coefficient, respectively.

In addition, I assume the logarithm of the Solow residual, $lnA_t$, is a random walk process with drift. Hence,

$$
lnA_t = \mu_A + lnA_{t-1} + \eta_{A,t}
$$

(3.2.2)

where $\mu_A$ represents a drift term in this unit root process, which captures the long-run rate of growth of technological change; $\eta_{A,t}$ is a serially uncorrelated innovation for productivity which, through the dynamic structure of economy, generates serially correlated behaviour in the economy’s main aggregates, such as output, consumption, capital, investment.

---

9ARIMA($p, d, q$) process, where $p$ and $q$ denote the orders of the autoregressive and moving average terms, respectively; $d$ indicates that the variable is integrated of order $d$. 
This ARIMA(0, 1, 0) representation of the Solow residual implies that technological change includes two components: a deterministic time trend and a stochastic trend, since, by recursively substitution, equation 3.2.2 can be rewritten as

$$\ln A_t = \mu_A t + \sum_{s=1}^{t} \eta_{A,t}$$

(3.2.3)

where $\mu_A t$ term is deterministic trend, and $\sum_{s=1}^{t} \eta_{A,t}$ term captures the stochastic trend, where all the past innovations enter in the permanent component.

As a result, this non-stationary productivity shock will drive output, consumption, capital, investment, export, and import to be non-stationary variables.

Since coefficients of $\mu_i$ in shocks processes and distributions of innovations are not known, limited information maximum likelihood method (LIML) are used to obtain model residuals. When the expectations enter the equations, expected variables, such as $E_t K_{t+1}$, $E_t C_{t+1}$, $E_t Q_{t+1}$, are estimated, using a robust instrumental variable method with the lagged endogenous data as instruments, proposed by McCallum (1976) and Wickens (1982)$^{10}$.

After obtaining the structure residuals, we regress them on a constant and a time trend to obtain the estimated de-trend residuals, $\hat{\varepsilon}_{i,t}$, then estimate AR coefficients, $\hat{\rho}_i$ and finally obtain approximated innovations $\hat{\eta}_{i,t}$. Similarly, an estimated technology innovation, $\hat{\eta}_{A,t}$, could be yielded by estimating following equation,

$$\Delta \ln A_t = \mu_A + \eta_{A,t}.$$  

(3.2.4)

These innovations are then used to simulate the models by bootstrapping method. Further discussion of the simulation is deferred to Chapter 4.

$^{10}$See Appendix A for the LIML methods in detail.
3.3 The Log-linearised Model

The competitive equilibrium of the model can be described by a system of non-linear stochastic difference equations, which can be expressed in an implicit form

\[ f(E_t y_{t+1}, y_t, \varepsilon_t, \eta_t) = 0 \]  \hspace{1cm} (3.3.1)

where \( f(.) \) is an \( k \times 1 \) vector value function. \( y_t \) is a set of variables, \( E_t y_{t+1} \) is the expectation of \( y_{t+1} \) formed by the model’s decision makers conditional on information available up to and including period \( t \),

\[ E_t y_{t+1} = E(y_{t+1} \mid y_t, y_{t-1}, \ldots, y_0). \]  \hspace{1cm} (3.3.2)

In this sense, expectations are said to be formed rationally. It is typically supposed that expectations are formed given full information regarding the decision makers’ environments, such as the collection of parameters associated with the model.

The components of \( y_t \) belong to one of three classification: exogenous variables, endogenous variables and predetermined variables. A set of exogenous variables, such as \( G_t, C^F_t, r^F_t \), which evolve over time independently of decision makers’ choice. While the evolutions of endogenous variables, such as \( r_t, Y_t, K_{t+1}, C_t, w_t, N_t, EX_t, IM_t, Q_t, D_t \) and \( D_t(Y_{t+1}) \) (or \( d_{t+1}^{11} \)), are affected by exogenous variables and taking as given values of the predetermined variables inherited in period \( t \). Predetermined variables include \( K_t \) and \( D_t \) (or \( d_t \)) which are carried from previous period. \( \varepsilon_t, \eta_t \) are vectors of structural errors on the equations and innovations, respectively. Note that \( \eta_t = \Phi(\varepsilon_t) \); that is, expectational errors arise from the realization of structural shocks.

In general, non-linear models cannot be solved in closed-form; hence solution techniques

\[ d_{t+1}^{11} = \frac{D_t}{Y_{t+1}}. \]
involve iterative numerical methods. Literally, it is easier to work with linear difference equations. Linear approximations are valuable, as they often serve as the foundation upon which non-linear representations are constructed. In addition, the logarithm is often a much more useful way to look at economic data. The present model, where all the stochastic equations in the system could be linear, need to be completed by accounting identities that are linear in levels. I normalise each equation on one of the endogenous variables and take nature logarithms for all variables in the model except variables that can take negative values or ratios, like \( r_t, r_f \) and \( d_t \).

Log-linearised representations of structural models are expressed as

\[
\begin{align*}
  r_t &= \ln \frac{1}{\beta} + \gamma_c (\ln E_t C_t + \ln G_t) + \ln e_t' \text{(Euler Equation)} \\
  \ln Y_t &= \alpha \ln N_t + (1 - \alpha) \ln K_t + \ln A_t \text{(Production Equation)} \\
  \ln N_t &= \ln \alpha + \ln Y_t - \ln \bar{\omega}_t + \ln e_t^{\text{N}} \text{(Labour Demand Equation)} \\
  \ln K_t &= \xi_1 \ln K_{t-1} + \xi_2 E_t \ln K_{t+1} + \xi_3 \ln Y_t - \xi_4 r_{t-1} + \ln e_t^K \text{(Capital Demand Equation)} \\
  \ln C_t &= \frac{\bar{\nu}}{C} \ln Y_t - \frac{\bar{K}}{C} E_t \ln K_{t+1} + \frac{1 - \delta}{C} \ln K_t - \frac{\bar{G}}{C} \ln G_t - \frac{\bar{E}_X}{C} \ln E_X_t + \frac{\bar{T}_{\text{M}}}{C} \ln IM_t \text{(Goods Market Condition)} \\
  \ln \bar{\omega}_t &= \gamma N \ln N_t + \frac{1 - \omega}{\omega} \ln Q_t + \gamma_c \ln C_t - \ln e_t^\omega \text{(Labour Supply Equation)} \\
  \ln E_X_t &= \ln (1 - \omega^F) + \theta F \ln Q_t + \ln C_f^F + \ln e_t^{EX} \text{(Export Equation)} \\
  \ln IM_t &= \ln (1 - \omega) + \ln C_t - \theta \ln Q_t + \ln e_t^{IM} \text{(Import Equation)} \\
  \tilde{d}_{t+1} &= (1 + r_{t-1}) \tilde{d}_t + \frac{\bar{IM}}{\bar{V}} (\ln IM_t + \ln Q_t) - \frac{\bar{E}_X}{\bar{V}} \ln E_X_t \text{(Evolution of Net Foreign Debts)} \\
  \ln Q_t &= \ln E_t Q_{t+1} + r_f - r_t + \Gamma d_{t+1} + \ln e_t^{F} \text{(Financiers’ Demand for Sterling Bonds)} \\
  \ln G_t &= \rho_G \ln G_{t-1} + \eta_{G,t} \text{(Government Spending Equation)} \\
  \ln C_{t-1}^F &= \rho_c F \ln C_{t-1}^F + \eta_c F, t \text{(Rest of the World Demand Equation)} \\
  r_f &= \rho_r r_{f,t-1} + \eta_{r_f,t} \text{(Rest of the World Real Interest Rate Equation)}. 
\end{align*}
\]
3.4 Calibration

In this section, I describe the set of parameter values used in the financial friction model to fit certain aspects of the UK data. More generally, the purpose of calibration is to evaluate the properties of the DSGE model by choosing numerical values for the parameters using extraneous information. The numerical values chosen are extraneous as they are based on either actual data or estimates from other empirical studies. The following interview of Sargent is very enlightening on the origins of calibration.

"The idea of calibration is to ignore some of the probabilistic implications of your model, but to retain others. Somehow, calibration was intended as a balanced response to professing that your model, though not correct, is still worthy as a vehicle for quantitative analysis." Sargent (Evans and Honkapohja, 2005, p.4).

Table 3.2 below outlines the initial set of parameter values used to evaluate the model’s performance. The parameters chosen is in line with the logic of the model and with the UK data. Most parameters of households and firms are taken from Meenagh et al. (2010).

On the household side, I calibrate the model using UK quarterly data assuming a subjective discount factor of 0.99, which is consistent with the standard in the DSGE literature. It implies annual steady-state real interest rate of around 4 percent, using that $\beta = \frac{1}{1+r}$.

More controversial is the assumption of Frisch labour supply elasticity, which is the elasticity of hours worked with respect to wages, holding marginal utility constant. This elasticity determines how employment, and hence output, responds to volatility in productivity. Micro-econometric estimates of the Frisch labour supply are around 0 to 0.5. However, macroeconomists tend to use much larger values, around 2 to 4, to calibrate DSGE models in order for matching the observed amount of volatility in aggregate hours worked over the business cycle. For example, an estimate of the Frisch labour supply in Euro area is around 2.38 reported from Smets and Wouters (2003). Peterman (2016) give some explanations for the gap between the micro and the macro Frisch elasticity. In general, higher the Frisch
elasticity of labour supply (smaller the inverse of the Frisch elasticity of labour supply) means labour is more willing to smooth labour hours as the wage rate changes. I calibrate a Frisch labour supply of 1 or an inverse of Frisch labour supply of 1. It implies that 1 percentage change in hours worked due to 1 percentage change in wages, holding constant the marginal utility of wealth (i.e., the multiplier on the budget constraint \( \lambda_t \)).

Grandelman and Hernandez-Murillo (2015) estimate the Arrow-Pratt coefficient of relative risk aversion at the country level, and document the estimates range between 0 and 3. According to their study, the average coefficient among developed countries is 0.92, and the estimated coefficient of UK is 1.03. Hence, I calibrate coefficient of relative risk aversion for consumption at 1.03, implying an intertemporal elasticity of substitution, i.e. \( \frac{1}{\gamma_C} \), of 0.97 between consumption in two consecutive periods. Generally speaking, higher the value of \( \gamma_C \) (lower intertemporal elasticity) means that consumption growth is less sensitive to changes in the real interest rate. For \( \gamma_C \) equal to 1.03, the growth rate of consumption responds 0.97 for one to changes in the real interest rate.

Following Meenagh et al. (2010), preference bias for domestic goods, \( \omega \), is set at 0.7, and likewise, the foreign equivalent, \( \omega^F \), is set at 0.7 by symmetry. It is consistent with the parameter of imported consumption share, \( 1 - \omega \), assigned by Adolfson et al. (2007). Intuitively, domestic consumers allocate 70 percent of weights on domestically produced goods relative to the imported goods.

The elasticity of substitution between goods from different countries - the Armington elasticity - is crucial in international macroeconomics, since it captures the strength of the relative demand response to relative international prices. I consider the UK as a small open economy country, and the rest of the world as a foreign country. Thus, the Armington elasticity in my model means the elasticity of substitution between home and foreign goods. A lower elasticity means less substitution between the two goods. A vast number of studies on empirical aggregate import equations have been surveyed by Goldstein and Khan (1985),
noting that for a typical country the import demand elasticity lies in or above the range of 0.5 to 1\(^{12}\). I assign the value of substitution elasticity in the UK to unity, \(\theta = 1\), which is in line with recent macro studies such as Heathcote and Perri (2002), Bergin (2006), and Meenagh et al. (2010). Intuitively, a one percent increase in the relative foreign to domestic price, \(Q_t\), leads to a one percent decrease in the quantity of imported goods, holding constant the quantity of domestic goods. Symmetrically, for the rest of the world, the Armington elasticity is set to unity, \(\theta^F = 1\). In other words, the sensitivity of export demand responds one for one to changes in the real exchange rate.

On the firm side, the quarterly depreciation rate of 0.025 is standard in the literature, and it implies annual depreciation rates of around 10 percent. Following Harrison and Oomen (2010)’s study, the capital share in the production function is set to 0.3, and therefore the elasticity of labour in the model is close to the value observed in the data in the environment of Cobb-Douglas technology and perfect competition, \(\alpha = 0.7\). The log-linear form of a non-linear difference capital demand equation is

\[
\ln K_{t+1} = \zeta_1 \ln K_t + \zeta_2 E_t \ln K_{t+2} + \zeta_3 E_t \ln Y_{t+1} - \zeta_4 r_t + \ln \epsilon_{K,t+1}^{13}
\]

There exists a relationship among coefficients in the capital demand equation, the sum of \(\zeta_1\), \(\zeta_2\), and \(\zeta_3\) is equal to 1.\(^{13}\) Meenagh et al. (2010) suggest a following calibration

\[
\ln K_t = 0.51 \ln K_{t-1} + 0.47 E_t \ln K_{t+1} + 0.02 \ln Y_t - 0.25 r_t + \ln \epsilon^K_t.
\]

On the international financial intermediary side, \(\Gamma\) is regarded as the financiers’ risk-bearing capacity. In Gabaix and Maggiori’s (2016) model, risk bearing capacity is driven by shocks

\(^{12}\)There exists another strand of literature tends to identify much more sensitive price responses. For example, Feenstra (1994); Harrison et al. (2010); Imbs and Mejean (2015).

\(^{13}\)The capital demand equation is linearised around the moving steady states of \(K, Y, r\), thus \(\zeta_1 = \frac{\kappa (1+\tau)\theta}{\kappa (2+\tau)\theta + (1-\alpha)\theta}, \ z_2 = \frac{\kappa \theta}{\kappa (2+\tau)\theta + (1-\alpha)\theta}, \ z_3 = \frac{(1-\alpha)\theta}{\kappa (2+\tau)\theta + (1-\alpha)\theta}, \ z_4 = \frac{(1-\alpha)\theta + \kappa (1-\delta)}{\kappa (2+\tau)\theta + (1-\alpha)\theta}.\)
to conditional foreign exchange volatility, and it is assumed to be a non-negative value, i.e. $0 \leq \Gamma < \infty$. In other words, $\Gamma$ captures the amount of capital available in the international financial market to bear risks. Corte, Riddiough and Sarno (2016) use the change in the foreign exchange index (VXY) designed by JP Morgan as a proxy for $\Gamma$. In this paper, I treat $\Gamma$ as a parameter that captures average risk-bearing capacity within specified sample range. It varies due to financial shocks in every period. Motivated by Cavallino (2016), I first set it at 1, and then conduct some numerical experiments on the value of $\Gamma$ to examine how the change of financier’s risk-bearing capacity affect exchange rate dynamics.

The log-linearised evolution of net foreign debts is

\[
\tilde{d}_{t+1} = (1 + \tilde{r}_{t-1})\tilde{d}_t + \frac{IM}{Y}(\ln IM_t + \ln Q_t) - \frac{EX}{Y}\ln EX_t
\]  

(3.4.3)

where the calibration of export-output ratio ($\frac{EX}{Y}$) and import-output ratio ($\frac{IM}{Y}$) are based on sample average of the UK data from 1975 Q1 to 2016 Q4.

The log-linearised market clearing condition in volume terms is

\[
\ln C_t = \frac{Y}{C}\ln Y_t - \frac{K}{C}\ln K_{t+1} + \frac{(1 - \delta)K}{C}\ln K_t - \frac{G}{C}\ln G_t - \frac{EX}{C}\ln EX_t + \frac{IM}{C}\ln IM_t
\]  

(3.4.4)

where the calibration of consumption-output ratio, government spending-output ratio are based on the UK data average. Please see Table 2.1 for detail. The value of $\frac{C}{Y}$ is inverse of consumption-output ratio ($\frac{C}{Y}$); $\frac{G}{Y}$, $\frac{EX}{Y}$, $\frac{IM}{Y}$ are calculated by government spending-output ratio ($\frac{G}{Y}$), export-output ratio ($\frac{EX}{Y}$) and import-output ratio ($\frac{IM}{Y}$), multiplying by the inverse of consumption-output ratio ($\frac{C}{Y}$), respectively. Capital output ratio ($\frac{K}{Y}$) is set at 2.66\(^{14}\).

\[\text{\textsuperscript{14}See Oulton and Wallis (2016) for detail.}\]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>a quarterly discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma_C$</td>
<td>CRRA coefficient for consumption(^{15})</td>
<td>1.03</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>the inverse of Frisch labour supply elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$\omega$</td>
<td>a bias towards domestic produced goods</td>
<td>0.7</td>
</tr>
<tr>
<td>$\omega^F$</td>
<td>foreign equivalent of $\omega$</td>
<td>0.7</td>
</tr>
<tr>
<td>$\theta$</td>
<td>elasticity of substitution between home and foreign goods</td>
<td>1</td>
</tr>
<tr>
<td>$\theta^F$</td>
<td>foreign equivalent of $\theta$</td>
<td>1</td>
</tr>
</tbody>
</table>

**Firms**

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>output elasticity of labour</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>a quarterly depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\zeta_1, \zeta_2, \zeta_3, \zeta_4$</td>
<td>capital equation coefficients</td>
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</table>

**Financiers**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\Gamma$</td>
<td>financiers’ risk bearing capacity</td>
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</tr>
</tbody>
</table>

**Calibration from UK data average (1975Q1-2016Q4)**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>consumption output ratio</td>
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</tr>
<tr>
<td>$K$</td>
<td>capital output ratio</td>
<td>2.688</td>
</tr>
<tr>
<td>$G$</td>
<td>government spending output ratio</td>
<td>0.21</td>
</tr>
<tr>
<td>$IM$</td>
<td>import output ratio</td>
<td>0.25</td>
</tr>
<tr>
<td>$EX$</td>
<td>export output ratio</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 3.2 Calibration

\(^{15}\)CRRA represents the coefficient of relative risk aversion. \(\frac{1}{\gamma_C}\) measures the inter-temporal substitution elasticity between consumption in two consecutive periods.
3.5 The Model with the Assumption of Uncovered Interest Parity

In the Section 3.1, I introduced an open economy DSGE model, emphasizing the role of financial frictions in determining the global financiers’ asset demand and in turn the exchange rate dynamics. In this section, I further discuss the model in a perfect financial market, where the competitive international financier, who intermediates an international bond to the domestic economy, is just a veil. Intuitively, arbitrage ensures that difference between the return on domestic and foreign assets is offset by an expected movement in the real exchange rate. In other words, the representative financier’s risk-bearing capacity $\Gamma$ is equal to 0, implying financiers are relaxed about risk-taking, thus uncovered interest parity holds and the exchange rate determination through currency risk-taking channel is blocked.

3.5.1 Representative Consumer

In line with the economy in Section 3.1, domestic households can trade goods with the rest of the world in a frictionless goods market. The international financial market is also frictionless, and thus it is no difference between borrowing a risk-free international bond directly from the rest of the world and through the global financial intermediaries. Here, I follow Meenagh et al. (2010) to model incomplete asset market structure. It is assumed that domestic households can hold two types of bonds $(D_{t+1}, D^f_{t+1})$ which are denominated in the units of the domestic currency and the foreign currency. A risk-free bond issued by the rest of the world is traded internationally, whose rate of return is exogenously determined abroad.

The domestic households finance their expenditures by borrowing from the domestic government or international financial market, apart from receiving wage payments and profits from firms. They can borrow credits, $D^f_{t+1}$, through the international bond market with the rate of interest, $r^f_t$, which is distinctive from the rate of interest, $\tilde{r}_t$, in financial friction.
market. I keep other set-ups, for example, the utility function of the household, the same as those in Section 3.1 in order to focus on features of the international financial market and make models comparable.

The period-by-period budget constraint of the representative household is given by

\[ C_t + D_t(1 + r_{t-1}) + Q_tD^f_t(1 + r^f_{t-1}) + T_t = w_tN_t + \Pi_t + D_{t+1} + Q_tD^f_{t+1} \]  

(3.5.1)

where \( Q_t \) is the relative price of home and foreign countries’ consumption basket. We treat the consumption bundle as the numeraire and, consequently, its price equals 1 in the domestic currency. Given that all prices in the budget constraint are expressed relative to the general price level, \( P_t \). The real foreign debt position \( D^f_t \) costs the amount of money that a unit of the foreign consumption basket \( C^f_t \) would cost \( P^f_t \). Hence, in terms of the domestic currency, the unit cost of the real foreign debt is \( Q_t = \frac{P^f_t}{P_t} \). The real domestic debt position \( D_{t+1} \) is likewise equivalent in value to a unit of the domestic consumption basket, the unit cost of the real home debt is \( \frac{P_t}{P_t} = 1 \).

The household chooses processes \( \{C_t, N_t, D_t, D^f_t, \lambda_t\}_{t=0}^{\infty} \) to maximise his utility (Equation 3.1.1 and 3.1.2 in Section 3.1.1) subject to his budget constraint 3.5.1 and no-Ponzi constraints, taking as given the processes \( \{r_t, w_t, r^f_t, \Pi_t, T_t, Q_t\}_{t=0}^{\infty} \) and the initial conditions \( D_0(1 + r_{-1}) \) and \( D^f_0(1 + r^f_{-1}) \).

The Lagrangian corresponding to the consumer’s maximization problem is

\[ L_0 = \sum_{t=0}^{\infty} \beta^t E_t \{ (\omega_0) \varepsilon_t C^1_t - (1 - \omega_0) \varepsilon_t N_t^{1 + \gamma_N} + \lambda_t [w_tN_t + \Pi_t + T_t(1 + r_{t-1})] \} \]

(3.5.2)

and the first-order conditions corresponding to \( C_t, N_t, D_t, D^f_t \), and \( \lambda_t \), respectively, are

\[ \omega_0 \varepsilon_t C^{-\gamma_c}_t - \lambda_t = 0 \]  

(3.5.3)
\[-(1 - \omega_0) e_t^N N_t^{\gamma_N} + \lambda_t w_t = 0 \quad (3.5.4)\]

\[\beta^t \lambda_t - E_t \beta^{t+1} \lambda_{t+1} (1 + r_t) = 0 \quad (3.5.5)\]

\[\beta^t \lambda_t Q_t - E_t \beta^{t+1} \lambda_{t+1} Q_{t+1} (1 + r^f_t) = 0 \quad (3.5.6)\]

and

\[w_t N_t + \Pi_t + D_{t+1} + Q_t D^f_{t+1} - C_t - D_t (1 + r_{t-1}) - Q_t D^f_t (1 + r^f_{t-1}) - T_t = 0. \quad (3.5.7)\]

The optimality conditions for $D_{t+1}$ in Equation 3.5.5 and $D^f_{t+1}$ in Equation 3.5.6 yield the real uncovered interest parity condition,

\[\frac{E_t Q_{t+1}}{Q_t} = \frac{1 + r_t}{1 + r^f_t} \quad (3.5.8)\]

It is saying that the difference in real interest rates between the home country and the rest of the world is equal to the expected change in real exchange rates between the countries. It implies that there is no opportunity to make a risk-free profit by using arbitrage techniques.

### 3.5.2 Representative Firms and Government

The problems of firms and the government are identical to those in Section 3.1.3 and 3.1.5.
3.5.3 Market Clearing Conditions

The overall bonds market clearing is required to close the model

\[ \Delta D_{t+1}^S + \Delta D_{t+1}^{fS} = \Delta D_{t+1}^D + \Delta D_{t+1}^{fD} \]  

(3.5.9)

where change in the supply of the bond on the left-hand side should be equal to its change in demand for the bond on the right-hand side.

The fundamental balance-of-payments identity in the open economy is

\[ (D_{t+1}^f - D_t^f)Q_t = r_t^{f-1}D_t^f Q_t + IM_t Q_t - EX_t. \]  

(3.5.10)

Alternatively,

\[ -(D_{t+1}^f - D_t^f) = \frac{EX_t}{Q_t} - IM_t - r_t^{f-1}D_t^f \]  

(3.5.11)

where the left-hand side of the expression is the change of the country’s net foreign asset position dominated in foreign currency, and the right-hand side of it measures the current account balance.

3.5.4 Log-linearised UIP Model

There are only two equations differed from the log-linearised currency risk premium model in Section 3.3. They are

\[ \ln Q_t = E_t \ln Q_{t+1} + r_t^f - r_t \]  

(3.5.12)

\[ d_{t+1}^{f} = (1 + r_{t-1}^f)d_t^f + \frac{TM}{Y} \ln IM_t - \frac{EX}{Y} (\ln EX_t - \ln Q_t) \]  

(3.5.13)

where \( d_t^f = \frac{D_t^f}{Y} \) is a ratio of foreign-currency denominated net foreign debts to real GDP at date T.
Equation 3.5.12 is the real uncovered interest parity condition and Equation 3.5.13 describes the evolution of net foreign debts.

### 3.5.5 Non-Stationarity and Terminal Conditions

In this small open economy model,

- domestic households borrow from the rest of the world in international financial markets with the world interest rate \( r^f_t \), which is treated as an exogenous variable, since the size of the domestic economy is too small to affect the rest of the world. There are no borrowing or lending constraints and adjustment costs in international financial markets;

- subjective discount factor, \( \beta \), is constant;

- and the international asset market is incomplete, since international financial market allows domestic households to smooth consumption over time by saving in a risk-free foreign bond, but does not allow them to smooth consumption across different states of nature.

Under this specification, the deterministic steady state of consumption depends on the assumed initial level of net external debt. In addition, up to first order, the equilibrium dynamics contain a random walk component in variables such as the trade balance, consumption, and net foreign debts. Hence this rational expectation model predicts that the steady-state levels of debt, consumption, and the trade balance depend on initial conditions, such as the initial level of debt itself. It implies that the steady state of the model is history dependent. In the model, the long run levels of endogenous variables will depend on the behaviour of the non-stationary driving variables as they have evolved stochastic trends. The relevant non-stationary variables in the model are productivity process (Solow Residual), \( A_{t-1} \) and net
foreign debt, $D_{t-1}^f$, both of which are functions of all previous shocks in the model through their unit roots. Their second moments are infinite and depend on time.

Here, terminal equilibrium conditions are imposed on this small open economy rational expectations of future variables model as a means of finding a unique solution among a continuum of potential solutions. As suggested by Minford et al. (1979), it is economically sensible to suppose that a convergent system will at some point in the future. This uses terminal conditions in a way similar to the transversality conditions that happen in infinite time horizon problems. Intuitively, the model will have reached an equilibrium solution by the terminal date. Imposing the terminal conditions on the expectations $E_tK_{t+1}, E_tC_{t+1}, E_tQ_{t+1}$ involves solving the equilibrium system at some notional future date $T$, as shocks have stopped, trended variables are growing at their constant rates, and stationary variables remain their long-run constant values.

3.5.6 Steady States

It is that at some undefined future date $T$ the ratio of net foreign debt to real output would, if there were such a path, stay at a constant rate from then on- i.e., $\frac{D_{T+i}^f}{Y_{T+i}} = \frac{D_T^f}{Y_T} (i = 1, 2, \ldots)$. Hence, the unique numerical solution path is picked among an infinite number of solutions, which is forced to be consistent with the constraints that place on the rational expectations. When solving the model, the balance of payments identity is scaled by real GDP. We can
obtain steady states of the model conditional on stochastic trends,

\[ r_T = \frac{1}{\beta} - 1 \]

\[ N_T = \left[ \frac{\omega_0}{1 - \omega_0} \right]^{\gamma} \alpha \left( \frac{K_T}{C_T} \right)^{1 - \alpha} \frac{1 - (1 - \omega)(Q_T)^{1 - \theta}}{\omega^{1 - \gamma} C_T^{\gamma}} \frac{1}{N_T - K_T} \]

\[ w_T = \alpha \left( \frac{K_T}{N_T} \right)^{1 - \alpha} \]

\[ 1 = \beta \left[ (1 - \alpha) \left( \frac{N_T}{K_T} \right)^{\alpha} + 1 - \delta \right] \]

\[ K_T \]

\[ C_T = 1 \]

\[ \frac{1}{\delta} \left( \frac{Y_T}{C_T} \right) - \frac{G_T}{C_T} - \frac{EX_T}{C_T} + \frac{IM_T}{C_T} - 1) \]

\[ Y_T = A_T N_T K_T^{1 - \alpha} \]

\[ Q_T = (1 - \omega)^{\delta} \left( \frac{IM_T}{C_T} \right)^{1 - \delta} \]

\[ EX_T = (1 - \omega^F)(Q_T)^{\theta F} C_T^{\theta F} \]

\[ r_T d_T + \frac{IM_T}{Y_T} - \frac{EX_T}{Y_T} \frac{1}{Q_T} = 0 \]

\[ r_T = r_T^f. \]

Non-stationary shock affects terminal condition. When the productivity and initial value of net foreign debts change, terminal points will change. The UIP model solution method is shown in Appendix A.

### 3.6 Comparing Impulse Response Functions Generated from Currency Risk Premium Model with Those from UIP Model

The focus of the present study is to assess the role of financial frictions on exchange rate dynamics and the transmission of external shocks in the real economy. To this end, I compare the impulse response functions generated from the currency risk premium model in Section

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3.1. where the global financiers have limited risk-bearing capacities, with those on the standard small open economy model with incomplete asset markets (the UIP model) shown in Section 3.5, where the assumption of the uncovered interest parity holds. Generally, an impulse response refers to the reaction of any dynamic system in response to some external changes. Mathematically, it can be described as $\frac{\partial y_{i,t+s}}{\partial \eta_t^i}$. It identifies the consequences of a one unit increase in the $j$th variable’s innovation at date $t$ ($\eta_t^j$) for the value of the $i$th variable at time $t + s$ ($y_{i,t+s}$), holding all other innovations at all dates constant.

To make those two models comparable, I calibrated them with the same parameters shown in Table 3.2. The impulse response functions are simulated with the first order approximation of the decision rules around the non-stochastic steady state. Starting off from the initial equilibrium, I will analyse the impulse responses to productivity, consumer preference, external demand and foreign interest rate shocks, respectively. All the variables, except for domestic and foreign real interest rates and net foreign debt to GDP ratio, are in the log scale. Thus the changes are expressed in proportion.

1. Productivity Shocks

The persistent nature of the productivity in the UK economy suggests that the stochastic process for the productivity shock is probably non-stationary, in the sense that a positive productivity shock produces an increasing path of productivity leading to a permanently higher long-run level. The calibrated model tries to identify the non-stationary type of productivity process.

Figure 3.1 shows the responses of the variables in two models to a positive total factor productivity (TFP) shock. Since the productivity shock process is highly persistent, the one-off shock has a long-lasting impact on the productivity level ($A_t$). The persistent rise in the TFP expands production ($Y_t$) frontier of the domestic economy and real income, which stimulates aggregate demand. In response to a 10% technology shock, consumption and import in the model of UIP increase by more than they would in the
currency risk premium model. This, in turn, leads to a larger rise in the accumulated net foreign debts. As can be seen in Figure 3.1, the net foreign debt position (the red dash line) based on the UIP model ends up with a new equilibrium, which is 2% higher than the original equilibrium level.

Financiers’ required compensation for holding currency risk eliminates the unit root in net foreign debts. In the currency risk premium model, the interest rate at which domestic agents borrow from the rest of the world increases with the net foreign debt positions rather than holds constant. Real exchange rate depreciates more than that in the UIP model through currency risk-taking channel. A weaker currency leads to fewer imports and more exports, which restores the external balance.
For the majority of the variables such as output, capital, real interest rate and the variables in the labour market, the impulse response functions are so similar that to the naked eye the graph appears to display just a single line.

2. Consumer Preference Shocks

A temporary shock $\eta^r_t$ to the disturbance term $\epsilon^r_t$ initially lifts the rate of return on domestic assets. On the one hand, an excess demand for domestic currency causes the real exchange rate appreciation, which has a positive impact on imports while a negative effect on export; on the other hand, a rise in the interest rate increases the borrowing costs, hence imposes a downward pressure on the demand for capital and investment.
As we can see in Figure 3.2, all variables in the UIP model but net foreign debt move in the same directions as the corresponding variables in the currency risk premium model, but react by more magnitudes, when a 1% consumer preference shock hits the economy. In particular, the real exchange rate in the UIP model appreciates triple times more than it in the currency risk premium model, because there is no credit constraint imposing in the global financial market in the UIP model.

One of the noticeable differences is given by the response of net foreign debt to GDP. In the UIP model, the transient shock has long-run effects on the net foreign debt position, while the introduction of financial frictions in the currency risk premium model prevents endogenous variables from wondering around an infinitely large region in response to a temporary interest rate shock, thus net foreign debts converge back to its original equilibrium when the shock dies out.

3. External Demand Shocks

A 20% negative external demand shock (export shock) generates a temporary fall in foreign export demand. This, in turn, deteriorates the trade balance and induces excess supply of the domestic currency. Figure 3.3 displays that the export in the UIP model plunges more than one time as compared with that in the currency risk premium model. When the uncovered interest parity holds (the red dash line), domestic households finance their present spending by borrowing from the rest of the world. As a result, imports and domestic consumption nearly remain at the same level, while the accumulated net foreign debt to GDP ratio reaches its new equilibrium that is almost 50% more against its original equilibrium. The real exchange rate only depreciates around 2% induced by an excess supply of currency, and the interest rate drops around 0.2%, which encourages the investment.

In response to the negative demand shock, the currency risk premium model (the solid black line with square), however, shows a distinguishable picture. When the credit
constraint that the representative global financier faces binds, domestic households are not able to smooth out the impacts of the temporary export shock by running up foreign debts. This implies that domestic consumption has to decline by more than it would without financial frictions (the UIP case), and domestic real interest rates shoot up to drive down domestic consumption. The real exchange rate as the relative price of foreign goods to domestic goods has to climb 10% further than it in the UIP model with the perfectly functioning financial market. Put it differently, the exchange rate depreciates more in order to compensate the financier for holding extra currency risk caused by the negative shock.

4. Foreign Interest Rate Shocks

Figure 3.4 exhibits a comparison of impulse responses implied by the currency risk premium model and the UIP model to an innovation of the foreign interest rate. An
unexpected 1% rise in the foreign interest rate leads to an excess supply of domestic currency, which induces real exchange rates depreciation. Because of the increase in the relative foreign price, current account starts to accumulate its surplus due to growth in exports and decline in imports. This leads to a modest expansion of output in the UIP model. A decline in the net foreign debt implies a net capital outflow, since the international bond has a higher rate of return and becomes attractive to domestic households.

In the currency risk premium model, the domestic interest rate surges by 0.5%, which narrows the interest rate differential caused by a 1% foreign interest rate shock. In turn, real exchange rate depreciates, exports increase, imports and net foreign debts
decrease by less magnitude as compared with those in the UIP model. This induces an initial fall in output.

### 3.7 Financiers’ Risk Bearing Capacity Experiments

In this section, I focus on the currency risk premium model. To illustrate the effect of international financiers’ risk-bearing capacity on the behaviour of exchange rates, I compare impulse responses functions with different values of parameters, $\Gamma$, and examine how key variables response to an external demand shock, domestic interest rate shock, and a foreign interest rate shock.

Parameter $\Gamma$ is altered by shocks to conditional foreign exchange rate volatility. Following the suggestions by Corte, Riddiough and Sano (2016), I use the change in the volatility indices for foreign exchange rate (VXY) as a proxy for $\Gamma$. Since $\Gamma$ is an inverse measure of financiers’ risk-bearing capacity, higher the value of $\Gamma$ means less willingness of international financial intermediaries to bear exchange rate risks and vice versa. Alternatively, larger the value of $\Gamma$ can be interpreted as less the amount of capital available in the international market to bear risks.

I consider two extreme values of parameter $\Gamma$ in the experiment. $\Gamma = 0.02$ represents the smallest quarterly change of VXY index between 1992 Q2 and 2016 Q4, while $\Gamma = 5.56$ is the largest quarterly change of VXY index and it happened during the global financial crisis.

1. A 10% Negative External Demand Shock

Figure 3.5 shows the impulse response for those two extreme values of $\Gamma$ to a 10% negative external demand shock. The dashed lines represent the IRFs for higher risk bearing capacity, $\Gamma = 0.02$, while the solid black lines with square correspond to lower risk-bearing capacity, $\Gamma = 5.56$. With a lower risk-bearing capacity, $\Gamma = 5.56$, an

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16G7 VXY index is designed by JP Morgan and tracks volatility of G7 currencies calculated based on currency 3-month at-the-money-forward option.
17The available data for G7 VXY index starts from 1992 Q2.
Figure 3.5 Response to -10% External Demand Shock

unexpected negative external demand shock will lead to a sharper rise in real interest rate, and around 3% more depreciation in real exchange rate, since financiers are less willing to intermediate capital flows and require higher compensation to bear the currency risk under the tighter global financial market environment. To incentivise financiers to absorb the imbalance, the real exchange rate has to depreciate immediately and be expected to appreciate in the future. To some extent, a relatively considerable real exchange rate depreciation offsets the negative impact of the drop in foreign export demand, and improve the current account. Hence the rise in net foreign debt is limited. Moreover, domestic consumption drops more in contrast to the case of $\Gamma = 0.02$, due to higher interest rate and cost of borrowing abroad.


Figure 3.6 summarises the impact of the real interest shock in the model with two different values of $\Gamma$. For a higher risk aversion environment, the global financiers’
downward sloping demand curve is relatively flatter than the other case; thus they require a higher risk premium to do their job. As a result, interest rate reacts 0.2% more on a 1% shock. The real exchange rate depreciates around 1% initially to improve international trade and to attract financiers to sustain the imbalance. Three quarters after the shock hitting, real exchange rate starts to appreciate due to increased interest rate, and then gradually converge back to its equilibrium as the temporary shock completely dies out. Net foreign debt to GDP ratio remains nearly constant.

A different picture appears when I consider the version of the model with the very low value of $\Gamma$. Real exchange rate moves in opposite directions as the shock hits the economy, that is, a rise in real interest rate induces an immediate 2% real exchange rate appreciation. Higher relative price of domestic goods discourages exports, while it stimulates imports. Therefore the current account deficit gradually increases. Since global financiers have higher risk-bearing capacity, they are more willing to take currency risks. In this situation, net foreign debts raise.

3. A 1% Positive Foreign Interest Rate Shock

The responses of a positive foreign interest rate shock in scenarios of two values of $\Gamma$ are displayed in Figure 3.7. A rise in foreign interest rate drives the increase in domestic interest rate and real exchange rate in both of the scenarios, however, real exchange rate depreciates 3% more in the case of $\Gamma = 0.02$.

The limited risk bearing capacity of global financiers suggests that international financial market is imperfect, that is, there exist different degrees of financial frictions. Foreign interest rate shock as a shock from the rest of the world affects the domestic economy through the global financial market. The degree of the global financial friction increases with the value of $\Gamma$. The domestic economy is less affected by the foreign interest rate shock, when $\Gamma$ is larger.
Figure 3.6 Response to 1% Consumer Preference Shock

Figure 3.7 Response to 1% Foreign Interest Rate Shock
Since financiers’ risk-bearing capacity plays an important role in both level and volatility of exchange rates, it is intuitive to consider comparative statics on $\Gamma$. To sum up, any shocks that cause a binding of the international financier’s credit constraint might lead to a sharp depreciation in the real exchange rate and an appreciation thereafter. The expected appreciation raises the relative return on home-currency denominated bonds in order to incentivise financiers to absorb shocks and ensure equilibrium in the asset market.

### 3.8 Concluding Remarks

This chapter has outlined the currency risk premium model used in empirical work in Chapter 4 and Chapter 5, and its properties have been analysed by comparing the impulse response functions with an open economy model without financial friction from varies structure shocks. The currency risk premium model is a workhorse that generates the impacts of global financial friction on the exchange rates in the context of a small open economy with flexible price and perfect competition. Therefore it is suitable for testing whether exchange rates are indeed affected by financial forces whose presence is controversial in the next chapter.
Chapter 4

Empirical Evaluation and Estimation by Indirect Inference Method

I introduce in this chapter a methodology to evaluate and estimate the model presented in Chapter 3. The focus is to test whether exchange rate behaviour can be affected by financial forces in an imperfect financial market and to estimate the average risk-bearing capacity of global financiers during 1975Q1 and 2016Q4. First of all, the philosophy of macroeconomic model evaluation will be discussed in Section 4.1. Secondly, the Indirect Inference methodology used to test the model will be introduced in Section 4.2. Thirdly, I confront the model with UK data, and the description of the data is presented in Section 4.3. I give results for an Indirect Inference test of the model given the starting calibration in Section 4.4, and then go on to estimate the model parameters using Indirect Inference estimation in Section 4.5 and utilise the estimated value to test alternative auxiliary models in order to determine the robustness of the results. The structural error properties under estimation are shown in Section 4.6. Finally, I design Monte Carlo experiments to detect the power of the Indirect Inference Wald test in Section 4.7.
4.1 Philosophy of Model Evaluation

Having calibrated the currency risk premium model from Chapter 3 the next step is to assess the ability of a model to mimic features of the actual economy. ‘Matching moments’ is one of the traditional ways to evaluate the fit of the calibrated version of the model to data. More specifically, population moments based on data simulated from the calibrated model is compared with historical sample moments of observed data series. Various moments such as variances, covariances, cross-correlation, and auto-correlation are generally chosen as main descriptions of the data. If the selected moments between the simulated data and the actual data are deemed to be reasonably close, then the overall fit of the model is good; hence the model is viewed as satisfactory.

Le et al. (2011) argue that the ‘matching moments’ procedure is distribution-free and is an informal approach to evaluate the performance of the model. There is a lack of formal standard statistical inference supplied by which closeness can be judged. In other words, this method is to test model with unknown acceptance and rejection regions. A similar argument has been made, "in the absence of statistical formality, communication regarding the results of an experiment is problematic. Judgements of ‘good’ or ‘bad’, or as Kydland and Prescott (1996, p.71) put it, judgements of whether ‘... the predictions of theory match the observations ...’, are necessarily subjective." by DeJong and Dave (2011, Chapter 6, p.34).

Some econometricians, for example, Hansen and Heckman (1996), criticise that calibrated DSGE models are not properly estimated and tested using statistical probability approach. Sims (1972) and Hansen and Sargent (1980) made the methodological contribution to remain in the tradition of the probability approach to econometrics by imposing theoretical discipline on the vector autoregressive (VAR) model. DSGE models featuring rational decision agents offered the source of this discipline, and all of the DSGE models can be expressed in the form of restricted VAR models. However, DSGE models are generally rejected when estimated and tested by the classical econometric methods, such as likelihood ratio tests.
As simplifications of reality, DSGE models are necessarily highly abstract and may suffer empirical shortcomings along certain aspects. As a result, "they are necessarily false, and statistical hypothesis testing will reject them" due to being partly misspecified or incompletely specified. "This does not imply, however, that nothing can be learned from such quantitative theoretical exercises" (Prescott, 1986, p.12). Too many good models were being rejected\(^1\) expressed a similar sentiment.

More recently, Bayesian methods have become a popular approach to the analysis of DSGEs in pursuit of empirical objectives. Bayesian methods incorporate the prior information about the structural parameters and uncertainty of the priors, in addition, permit the data to affect the final estimates. Furthermore, Bayesian procedures have been used to facilitate model comparison. However, Bayesian cannot judge models in the classical hypothesis testing sense, they treat all models as false and evaluate each model’s probability of being right instead. It is not precisely where the line is drawn between failure and success for the model. Moreover, criticism of this approach is the choice of the priors that is subjective. It could bias the results if priors are incorrect.

How best to evaluate the empirical performance of DSGE models is one of the crucial and unresolved issues in macroeconomics. Here I adopt a different approach, called indirect inference test, that restores the role of formal statistical tests of DSGEs and inherits the widely accepted foundation of economic testing methodology that I will call ‘Friedman utility’ of tests. Friedman (1994) argued that an economic model should be tested on ‘as if it is true’ rather than ‘literal truth’. Therefore, although DSGE models are inherently ‘false’ due to gross simplifications of or abstractions from reality, we should test DSGEs on their ability to mimic some essential features of the actual economy we designed them to explain - the ones we were interested in and concerned about.

\(^1\)In a recent interview Sargent remarked of the early days of testing DSGE models: "…after about five years of doing likelihood ratio tests on rational expectations models, I recall Bob Lucas and Ed Prescott both telling me that those tests were rejecting too many good models." Tom Sargent, interviewed by Evans and Honkapohja (2005).
In general, the model evaluation criterion can be summarised into two ways. First, it tries to answer how close the model gets to the data, all the data, for the set of variables. This category includes the criteria based on the size of mean square errors of the raw data, a comparison of the values of the likelihood function for the DSGE models. The likelihood ratio test is one of the methods of this type. Second, the tests results attempt to answer how close the model gets to the behaviour of a set of variables in particular aspects, for instance, over the business cycle. Indirect inference test is in this group. Le et al. (2016b) compare indirect inference test with likelihood ratio test. They find that indirect inference test has much greater power and it can be focused on features of interest that the likelihood ratio test cannot. More precise details of indirect inference procedure are given in the section 4.2.

4.2 Indirect Inference Method

Indirect inference method was first introduced into the econometrics literature by Smith (1993), and extended as a general methodology by Gourieroux et al. (1993)\(^2\). Indirect inference method is a simulation-based method for estimation and making inferences on the parameters of models. This method develops with the increasing computational power of computers, which makes it possible to generate substantial numerical simulations of large artificial datasets. The basic idea behind the indirect inference method is to match properties of the simulated data to those of the empirical data through a chosen statistical model (auxiliary model).

4.2.1 Indirect Inference Test

Indirect inference test provides a classical statistical inferential framework for judging whether a model with a particular set of parameters could have generated the behaviour found

in a set of actual data. Taking the parameters of the structural model and their distributions as given, the test is to compare the performance of the auxiliary model based on the actual data with its performance based on the data simulated from the macroeconomic model. The auxiliary model is employed to form a criterion function in indirect inference test. This criterion does not need to be an accurate description of the data generating process. "The auxiliary model serves as a window through which to view both the actual, observed data and the simulated data generated by the economic model: it selects aspects of the data upon which to focus the analysis" (Durlauf and Blume, 2008). Further discussion regarding choose an auxiliary model will be outlined in the following subsection ‘Choice of Auxiliary Model’.

Here the indirect inference testing procedure is given in brief:

1. Determine the residuals of the economic model conditional on the observed data and calibrated or estimated structural parameter set, and generate \( s \) sets\(^4\) of simulated data by bootstrapping.

2. Choose an auxiliary model and estimate\(^5\) it on both of all \( s \) simulated samples and the observed data.

3. Set up the null hypothesis and compute the Wald statistic (WS)\(^6\),

\[
WS = (\beta^a - \overline{\beta^s(\theta_0)})' \Omega^{-1} (\beta^a - \overline{\beta^s(\theta_0)}).
\]

where \( \beta^a \) is defined as the estimates of the true vector of descriptors in the auxiliary model derived from observed data; \( \overline{\beta^s(\theta_0)} = E(\beta^i(\hat{\theta}_0)) = \frac{1}{s} \sum_{i=1}^{s} \beta^i(\hat{\theta}_0) \) denotes the

\(^3\)The indirect inference testing procedure shown here is based on a group of seminal work at Cardiff. The reader is referred to Minford \textit{et al.} (2009), Le \textit{et al.} (2011), and to Meenagh \textit{et al.} (2012) and Le \textit{et al.} (2016a) for the application to non-stationary data.

\(^4\)In the empirical work carried out in this chapter, the number of bootstrap simulation has been set to \( s = 1000 \).

\(^5\)In this thesis, I use ordinary least squares (OLS) method to estimate an auxiliary model.

\(^6\)A Wald statistic is computed to determine whether functions of the parameters of the auxiliary model estimated on the actual data lie in some confidence interval implied by this sampling distribution.
sample average of estimates of the coefficients in auxiliary model based on \(s\) sets of simulated data from the macroeconomic model, taking calibrated or estimated vector of structural parameters, \(\theta_0\), as given; 

\[
\Omega = \text{cov}(\beta^i(\hat{\theta}_0) - \beta^s(\hat{\theta}_0)) = \frac{1}{s} \sum_{i=1}^{s} (\beta^i(\hat{\theta}_0) - \beta^s(\hat{\theta}_0))(\beta^i(\hat{\theta}_0) - \beta^s(\hat{\theta}_0))^T
\]

is the variance-covariance matrix of the distribution of simulated estimates \(\beta^i\).

I assume that there exists a particular value \(\theta_0\) such that \(s\) sets of simulated data derived from structural model, \(\{x_t(\theta_0)\}_{i=1}^{s}\), and observed data, \(\{y_t\}_{t=1}^{T}\) share the same distribution, where \(s = cT\) and \(c \geq 0\). The null hypothesis is \(H_0 : \theta_0 = \theta_0\).

Wald statistic is chosen to be the test statistic for evaluating the macroeconomic model and is based on the distribution of the distance between \(\beta^a\) and \(\beta^s(\hat{\theta}_0)\). In essence, it measures the gap between what the macroeconomic model says the data behaviour should be and what the observed data behaviour actually is. Therefore, the null hypothesis implies \(\beta^a = \beta^s(\hat{\theta}_0)\).

Non-rejection of the null hypothesis is taken to indicate that the dynamic behaviour of the structural macroeconomic model is not significantly different from that of the observed data. Rejection is taken to imply that the macroeconomic model is incorrectly specified.

4. Compare the test statistic with the critical value and obtain the conclusion. A test statistic is expressed as the percentile of the joint distribution in which the observed data based estimates, \(\beta^a\), fall. For the model to fit the data at the 95\% confident level, the Wald statistic for the observed data should be less than the 95th percentile of the Wald statistics from the simulated data. 

\[W_{S_i} = (\beta^i(\hat{\theta}_0) - \beta^s(\hat{\theta}_0))^\top \Omega^{-1}(\beta^i(\hat{\theta}_0) - \beta^s(\hat{\theta}_0)) \quad (4.2.2)\]

where \(\beta^i(\hat{\theta}_0)\) denotes estimates of the coefficients in auxiliary model based on \(s\) sets of simulated data; \(\beta^s(\hat{\theta}_0)\) are sorted into ascending order. Thus, the critical value at the 5\% significant level is the 95th percentile of the Wald statistics. The corresponding a Wald p-value is equal to \(\frac{100}{100} - \text{the Wald percentile}\).
same information as the Mahalanobis Distance based on the same joint distribution, normalised as a t-statistic\(^8\) to show Distance from 95% point

\[
MD_{\text{Norm}} = \left(\frac{\sqrt{2WS} - \sqrt{2k-1}}{\sqrt{2WS_{95}^{95th} - \sqrt{2k-1}}}\right) \times 1.645
\] 

(4.2.3)

where \(k\) is the length of \(\beta^a\) (the vector of auxiliary model parameters estimated using the observed data), \(\sqrt{2WS} - \sqrt{2k-1}\) is Mahalanobis Distance with a mean of 0, and standard deviation of unity, \(WS_{95}^{95th}\) is the Wald statistic for the 95th percentile of the simulated data. Mahalanobis Distance is scaled by 1.645 so that when \(WS = WS_{95}^{95th}\)

\(MD_{\text{Norm}}\) corresponds to the 95th percentile of the standard normal distribution.

**Bootstrap Simulation**

The structural residuals of each equation are backed out from the observed data and the DSGE model. The resulting structural residuals are treated as the error process in the model and together with exogenous variable processes, process the shocks perturbing the model. Instead of assuming shocks follow asymptotic distributions\(^9\), the shocks are bootstrapped by time vector to preserve any correlations between them.

Suppose the original sample shocks is a \(t \times n\) matrix, \(t\) denotes number of time periods and \(n\) is number of innovations in the macroeconomic model, I randomly draw a time vector from the original sample shocks, then I put the row vector back into the sample and draw another time vector, thus each time vector has equal probability of being drawn. By repeating this procedure \(t\) times, we can obtain another sample innovation with the same

---

\(^8\)Since the distribution of residuals of the structural model is not known, \(\beta^a - \beta(\theta_0)\) does not follow a normal distribution in the finite sample, but it is asymptotically normally distributed when the sample size is large. In turn, the Wald statistic can compare against an asymptotic chi-squared distribution. Thus, \(\sqrt{2WS}\) asymptotically follows a normal distribution.

\(^9\)In practice, we may not be sure about what distribution the shocks should follow. In a situation like this, it would be appropriate to use the bootstrapping, which samples directly from the empirical distribution of the shocks should follow.
size, whose distribution should be the same as the original sample shocks. To sum up, the bootstrapping procedure involves drawing uniformly from the set of shocks of the same period with replacement and using these pseudo-random innovations to generate $s$ pseudo-simulated datasets, each of which provides a set of estimated coefficients for the auxiliary model, $\beta_i(\hat{\theta}_0)$.

**Choice of Auxiliary Model**

In general, the solution to a log-linearised DSGE model takes the form of a restricted vector autoregressive and moving average (VARMA), or approximately, a vector autoregressive (VAR). Following Le et al. (2016b), I choose a cointegrated VAR with exogenous variables (VARX), as an auxiliary model. For simplicity, I consider VARX with one lag, VARX(1),

$$y_t = h + Ay_{t-1} + Bx_{t-1} + e_t$$ (4.2.4)

where $y_t$ is a vector of endogenous variables, $e_t$ is a vector of i.i.d. errors with zero means, $x_t$ represents a vector of non-stationary exogenous variables, $h$ is a vector of intercepts, $A$ and $B$ are corresponding coefficient matrices.

The non-stationary exogenous variables may consist of both observable and unobservable such as the Solow residual. I assume $x_t$ are driven by general ARIMA($p, 1, q$)\(^{10}\) processes,

$$a(L)\Delta x_t = c + b(L)e_t$$ (4.2.6)

\(^{10}\)Following Box et al. (1970), an ARIMA($p, 1, q$) model specifies $x_t$ as being integrated of order one and as having a representation of the form,

$$\Delta x_t = c + a_1\Delta x_{t-1} + \cdots + a_p\Delta x_{t-p} + e_t + b_1e_{t-1} + \cdots + b_qe_{t-q}$$ (4.2.5)

where $a_1, \ldots, a_p, b_1, \ldots, b_q$, and $c$ are constant parameters and where $e_t$ is serially uncorrelated.
where \( c \) is a vector of constant, \( L \) denotes the lag operator, and \( a(L), \) i.e. \( a(L) = \sum_{j=0}^{p} a_j L^j, \)
and \( b(L), \) i.e. \( b(L) = \sum_{i=0}^{q} b_i L^i \) are lag polynomials of order \( p \) and \( q, \) respectively; \( \epsilon_t \) is a vector of i.i.d. errors with zero means.

Suppose \( x_t \) and \( y_t \) are cointegrated and the linear relationship represents in the long-run is

\[
\begin{align*}
\bar{y}_t &= \Pi \bar{x}_t + g \\
\bar{x}_t &= ft + \Psi(1) \sum_{i=1}^{t} \epsilon_i + \Psi^r(L) \epsilon_t
\end{align*}
\]  

(4.2.7)  
(4.2.8)

where \( \Pi \) is interpreted as a long-run coefficient matrix; \( g \) is constant vector.

\[
\Psi(1) \overset{11}{=} \frac{\sum_{j=0}^{p} a_j}{\sum_{j=0}^{p} a_j}, \quad \Psi(L) - \Psi(1) = \Psi^r(L)(1 - L).
\]

Equation 4.2.8 can be decomposed into a deterministic trend part \( \bar{x}_t^D = ft \) and a stochastic trend part \( \bar{x}_t^S = \Psi(1) \sum_{i=1}^{t} \epsilon_i \) and a stationary component (or transitory component) \( \Psi^r(L) \epsilon_t, \) which is consistent with the Beveridge-Nelson (1981) decomposition for general ARIMA\((p, 1, q)\) processes.

Given a VARX\((1)\) and both of \( x_t \sim I(1) \) and \( y_t \sim I(1), \) there always exists an error correction representation of the form

\[
\Delta y_t = (A - I) \left[ y_{t-1} - \Pi x_{t-1} \right] + G \Delta y_{t-1} + H \Delta x_t + h + e_t
\]

(4.2.9)

where \( I \) is an identity matrix, \( G \) and \( H \) are functions of the \( A \) and \( B. \)

Equivalently, since \( \bar{y}_{t-1} - \Pi \bar{x}_{t-1} - g = 0, \)

\[
\Delta y_t = (A - I) \left[ (y_{t-1} - \bar{y}_{t-1}) - \Pi (x_{t-1} - \bar{x}_{t-1}) \right] + G \Delta y_{t-1} + H \Delta x_t + d + e_t.
\]

(4.2.10)

\( ^{11} \)Since \( \Psi(L) \) denotes an infinite-order polynomial in the lag operator,

\[
\Psi(L) = \psi_0 + \psi_1 L + \psi_2 L^2 + \ldots, \quad \Psi(1) = \psi_0 + \psi_1 + \psi_2 + \ldots
\]
Equation 4.2.10 can be rewritten as a cointegrated VARX(1), which acts as the auxiliary model

\[ y_t = \text{intercept} + A y_{t-1} + (I - A) \Pi x_{t-1} + ft + \zeta_t \]  \hspace{1cm} (4.2.11)

where the error term \( \zeta_t \) contains the suppressed lagged difference regressors, \( ft \) is included to capture a deterministic linear trend that affects both the endogenous and exogenous variables, \( x_{t-1} \) contains stochastic trend which must be present to control for the effect of past shocks of the model on the long-run path of endogenous and exogenous variables.

It is possible to estimate Equation 4.2.11 by ordinary least squares estimation method. Although there are other estimation methods that may achieve more accurate auxiliary model parameter estimates, OLS is simple and may be effective in the test procedure.

As I mentioned before, the auxiliary model acts as a ‘window’ through which to view both the observed data and simulated data. Gourieroux et al. (1993) show that a correct inference can be based on an ‘incorrectly’ specified auxiliary model\(^{12}\). Le et al. (2011) identify two types of Wald statistic - the ‘Full Wald’\(^{13}\) and the ‘Directed Wald’. I use the Directed Wald statistic that is derived from one aspect or some aspects of the model’s performance. Instead of including all variables in the DSGE model\(^{14}\), a group of endogenous variables have been selected and regarded as key or interests for evaluating the theory being tested.

As discussed in the Section 4.1, ‘too many good models were rejected’ is due to some misspecification in the DSGE model which prevents it from being the true data generating a process for the historical data. To some extent, the use of the Directed Wald can take into account just the key features and parameters of the model which may be well specified rather than all of the model’s features and parameters.

\(^{12}\)When the auxiliary model is correctly specified, the indirect inference is equivalent to maximum likelihood.

\(^{13}\)The ‘Full Wald’ criterion is based on the full joint distribution of VARX coefficients with the full covariance matrix; therefore it would include all the endogenous variables in the auxiliary model.

\(^{14}\)According to Le et al. (2016b), the power of the full Wald test increases with the number of endogenous variables and the lag orders of VARX, which leads to uniform rejections.
The open economy DSGE model with currency premium derived in Chapter 3 serves as an internally consistent backdrop for us to examine, with statistical formality, the causally identified theory that financial force drives the behaviour of exchange rates. In this case, the focus is on the financial friction hypothesis and on the behaviour of real exchange rate and domestic interest rate, conditional on net foreign debt to GDP ratio and productivity. Thus, I include \( Q_t, r_t \) as endogenous variables and \( d_{t-1} \) and \( A_{t-1} \) as exogenous variables in the auxiliary model to evaluate the structural model on this joint criterion. The empirical test results will be presented in the Section 4.4.

4.3 Taking the Model to UK Data

4.3.1 Description of UK Data

I investigate the behaviour of real exchange rates for the United Kingdom relative to the rest of the world. Here, I use the inverse of sterling real effective exchange rates to represent real exchange rates, \( Q_t \), which is defined as the relative foreign to the UK’s consumer price levels, expressed in common units. Intuitively, an increase in the real exchange rate indicates a depreciation. More specifically, sterling real effective exchange rates are calculated as geometric weighted averages of bilateral exchange rates adjusted by relative consumer prices. I obtain end-of-month series on sterling real effective exchange rate indices from Bank for International Settlements\(^{15} \), and convert them into quarterly series. Based on the bilateral trade with the UK, the sterling to euro, the sterling to dollar, and the sterling to Japanese yen bilateral exchange rates have been assigned the majority of the weights in calculating sterling real effective exchange rate indices.

In addition, I measure the UK’s external imbalances – the indebtedness of the UK to foreigners – using the net foreign debt position (the difference between the UK’s foreign

\(^{15}\)Please find the detail in http://www.bis.org/statistics/eer.htm
liabilities and foreign assets) relative to the size of the UK’s economy (GDP), which I denote $d_t$. Nominal net foreign debt is accumulated current account deficits (£m), taking a negative the Balance of Payments international investment position as a starting point. A positive ratio of nominal net foreign debts to nominal GDP implies the UK is a net debtor.

The data included in my study were obtained from the first quarter of 1975 to the last quarter of 2016, because the UK has had floating exchange rates among its major trading partners since the early 1970s. In order to capture the effects of the financial disruption on exchange rates, I include data during the turbulent periods from early 2008 until early 2013 due to the global financial crisis and the European debt crisis, and from early 2016 until late 2016 because of Brexit vote. In general, the UK has very low capital controls, little probability of default and deep markets in foreign exchange. Therefore, these factors narrow the possible explanations for exchange rate puzzles.

The majority of UK data are sourced from the UK office of National Statistics (ONS). Others from Bank of England (BoE), Federal Reserve Bank of St. Louis, Bank for International Settlements (BIS), International Monetary Fund (IMF).

4.3.2 Data Preparation

All data are seasonally adjusted and in constant prices unless specified otherwise. A full description of the data used is given in the Appendix B and Table B.1. I take nature logarithms of the unfiltered data except for ratios, such as interest rate and the ratio of net foreign debts to GDP, in order to be consistent with the log-linear model.

Seasonal Adjustment

Analysis of business cycle behaviour is typically conducted using quarterly data. Measurement of this frequency is not ideal, because it introduces the influence of seasonal fluctuations.
Figure 4.1 Actual Data Series
into the analysis; but on the other hand, aggregate to an annual frequency would entail an important loss of information regarding fluctuations observed at business cycle frequencies.

Systematic calendar related variation associated with the time of year, i.e. seasonal effects, could be removed through seasonal adjustment. This would facilitate comparisons between consecutive time periods. Most of the seasonally adjusted data in this study are collected from the Office for National Statistics (ONS) in the United Kingdom. The X-12-ARIMA \(^\text{16}\) has been chosen from the many available seasonal adjustment methods as the standard one for use.

**Why do not choose to remove trends from time series data?**

The concepts of ‘trend’ and ‘cycles’ in macroeconomic variables, such as output, are often treated separately. Theories of the business cycle model focus on explaining short-run swings and examine when macroeconomic policy might stabilise or exacerbate the fluctuations between boom and recession. As a result, solutions of a business cycle model are typically in terms of stationary versions of variables: the stochastic behaviour of the variables is in the form of temporary deviation from steady-state values, with eliminating trends from the model and actual data in a parallel fashion. Hodrick-Prescott (HP), Band Pass filters are some of the popular techniques used to the completion of a preliminary trend-removal step in most empirical applications.

However, it is difficult to convincingly establish the precision of the driving process that leads to trend behaviour. Harvey and Jaeger (1993) provide an analysis of spurious behaviour causing from H-P filtered data. Stochastic behaviour of a filtered macroeconomic time series data may vary systematically from its unfiltered counterpart along the dimension of original interest in the empirical analysis. Moreover, Cogley and Nason (1995), and Murray (2003)

\(^\text{16}\)Please see ‘Guide to Seasonal Adjustment with X-12-ARIMA’ on the ONS website for further detail of X-12-ARIMA.
explain spuriousness due to the application of the H-P and B-P filters to non-stationary data, as it is difficult to distinguish between difference-stationary and trend-stationary specifications.

Furthermore, although the dichotomy between trend and cycles seems natural for providing theories into macroeconomic movements over different time spans, it cannot shed light on macroeconomic insights if there are significant interactions between trend and cycles. Prescott (1986) illustrates that economic fluctuations are in response to changes in the long run growth prospects of the economy, and business cycles arise as an adjustment to new long-run growth paths.

These are some reasons why I use unfiltered data in this paper. First of all, the filters available do not seem appropriate and precise to decompose a non-stationary time series arbitrarily into a ‘long run potential trend’ component and swings around it. Since some transitional periods following a shock may be reasonably long in the model, and long cyclical swings might be mistakenly treated as a trend and removed by filters. Secondly, I would like to keep the features of non-stationarity and do not remove the stochastic trend. One of the important interests in this study is about how the behaviour of the stochastic trend, which arises from the unit root processes of the technology shock, transfers through the entire model. Stationarising the data may potentially distort some of the interactions of interests and the dynamic properties of the model in ways that are not easy to uncover.

4.4 Indirect Inference Calibration Test Results

Recall that the VARX(1) in 4.2.11 is the approximation to the reduced form of the structured model. Here, the VARX(1) has been specified in the form of 4.4.1, which serves as the unrestricted auxiliary model used throughout the test and estimation in the empirical work, being a parsimonious description of some key features of the DSGE model with currency
premium derived in Chapter 3.

\[
\begin{bmatrix}
Q_t \\
r_t
\end{bmatrix} =
\begin{bmatrix}
\beta_{11} & \beta_{12} \\
\beta_{21} & \beta_{22}
\end{bmatrix}
\begin{bmatrix}
Q_{t-1} \\
r_{t-1}
\end{bmatrix} +
\begin{bmatrix}
\beta_{13} & \beta_{14} & \beta_{15} & \beta_{16} \\
\beta_{23} & \beta_{24} & \beta_{25} & \beta_{26}
\end{bmatrix}
\begin{bmatrix}
\tilde{d}_{t-1} \\
\tilde{A}_{t-1} \\
t \\
\text{const}
\end{bmatrix} +
\begin{bmatrix}
\varsigma_1 \\
\varsigma_2
\end{bmatrix}
\] (4.4.1)

The coefficient vector \( \beta^s \) in Equation 4.2.1 used to construct the Direct Wald statistic includes OLS estimates of \( \beta_{11}, \beta_{12}, \beta_{13}, \beta_{21}, \beta_{22}, \beta_{23} \) and the variances of the fitted stationary residuals \( \varsigma_1 \), and \( \varsigma_2 \) based on each set of simulated data; the same coefficients make up \( \beta^a \) estimated on the observed data. The coefficients represent the dynamic properties found in the model and data, and the three variances of the residuals measure the volatility properties.

The main interests of the currency premium model are examining two channels of exchange rate dynamics, which are interest rate channel and currency risk-taking channel. Therefore, I attempt to answer whether the structure model can replicate the behaviour of real exchange rate and real interest rate conditional on net foreign debt and productivity. \( Q_t, r_t \) are chosen as key variables that are a small subset from among the full set.

Net foreign debt, \( \tilde{d}_{t-1} \) is included as an exogenous variable in the auxiliary model to capture the effect of net foreign debt on the behaviour of real exchange rate. In addition, productivity is measured by the Solow residual, which is backed out from the calibrated Cobb-Douglas production function on the assumption of constant returns to scale and fixed input shares. \( \tilde{A}_{t-1} \) is a key non-stationary exogenous variable and is included in the VARX to provide cointegration, since its stochastic movements have impacts on the long run solution path of the endogenous variables. Moreover, the trend term, \( t \), in the 4.4.1 captures the deterministic trend in the observed data and in the simulations.

I ask whether the model-implied OLS-estimated-VARX would generate the same OLS-estimated -VARX as the observed data. More specifically, this is a test of whether the DSGE
Table 4.1 Test Results with Calibration

<table>
<thead>
<tr>
<th>Auxiliary Models: VARX(1)</th>
<th>Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>endogenous</strong></td>
<td></td>
</tr>
<tr>
<td><strong>exogenous</strong></td>
<td></td>
</tr>
<tr>
<td>Included</td>
<td>Excluded</td>
</tr>
<tr>
<td>( Q_t, r_t ) ( d_{t-1} )</td>
<td>( A_{t-1} ), trend, const</td>
</tr>
<tr>
<td>( Q_t, r_t, Y_t ) ( d_{t-1} )</td>
<td>( A_{t-1} ), trend, const</td>
</tr>
<tr>
<td>( Q_t, r_t ) ( d_{t-1} )</td>
<td>( A_{t-1} ), trend, const</td>
</tr>
<tr>
<td>( Q_t, r_t, Y_t ) ( d_{t-1} )</td>
<td>( A_{t-1} ), trend, const</td>
</tr>
</tbody>
</table>

model can replicate the data features of real exchange rate and real interest rate jointly, in terms of their dynamics as well as their variance and covariance.

Table 4.1 shows the indirect inference test results for the currency premium model with the calibrated parameters. The first three columns list selected subsets of key variables in the auxiliary model. The fourth column states whether the residual variances have been included in the VARX(1). The test results, including ‘Wald percentile’ and the normalised ‘Mahalanobis Distance’, are presented in the last two columns.

More specifically, the Direct Wald test implies a rejection of the currency premium model with calibration at the 5% significance level for the VARX(1) auxiliary model described in 4.4.1, with endogenous variables - real exchange rate and real interest rate, and exogenous variables - lagged net foreign debt ratio and lagged productivity. The Wald percentile of the joint distribution of \( Q_t, r_t \) and \( d_{t-1} \) is 99.2, or the normalised Mahalanobis Distance measure implies a test statistic of 2.93. When the variances of endogenous variables are excluded from the auxiliary model, the Wald percentile for dynamics is 65.4, which implies that the

\(^{17}\)The calibrated parameters are listed in Table 3.2 in Section 3.4 Chapter 3.

\(^{18}\)The coefficients of exogenous variables listed in ‘Included’ column are included in the joint distribution of the \( \beta' \) and \( \beta'' \), while the coefficients of exogenous variables listed in ‘Excluded’ column are excluded.
observed data lies in 95% confidence interval implied by the sampling distribution of $Q_t$, $r_t$ and $d_{t-1}$.

The structural model with calibration was also tested using alternative auxiliary models, in which more endogenous variables are included. For instance, adding the output as an additional endogenous variable worsens the Wald percentile relative to the case of two endogenous variables. Although the tests statistic for dynamic aspect is still within the non-rejection region, it is close to the border. When the variances of those three endogenous variables are included in the auxiliary model, the observed data lies out even in 90% confidence interval.

The test results show that the structural model with the calibrated parameters does not perform well in generating the observed data. This may cause by either the inappropriate set of parameters or the failure of the structural model. Thus, it is only when the structural model with all coefficient values that are feasible within the structural model theory has been examined that the structural model has been properly tested. For this reason, indirect inference estimation is employed to find whether the structural model can be rejected in itself. If the structural model passes the test, the most satisfactory estimates of the model parameters could also be found by indirect inference estimation.

### 4.5 Indirect Inference Estimation

Indirect inference has been widely applied in the model estimation, for example, Gourieroux et al. (1993), Gourieroux and Monfort (1996) and Canova (2007). Similar to the test procedure presented in section 4.2.1, parameters of the auxiliary model can be estimated by both of the simulated data from the structure model and the observed data. The basic idea of indirect inference estimation is to choose the set of parameters of the structure model that "the observed data and the simulated data look statistically the same from the vantage point of the chosen window" - in other words, minimises the distance between a given criterion of the two
sets of estimated coefficients of the auxiliary model (Durlauf and Blume, 2008). Common choices of this criterion are the scores, impulse response function, or actual coefficients. Here I choose actual coefficients as the ‘descriptors’ of the data.

To find the optimal choice of the set of parameters, I calculate the minimum-value Wald statistic using a powerful algorithm based on "Simulated Annealing" in which search takes place over a wide range around the initial values, with optimising search accompanied by random jumps around the space" (Liu and Minford, 2014, p.414).

**The Estimates of the Currency Premium Model**

Using the simulated annealing method, the best fit set of coefficients have been discovered for the currency premium model. Table 4.2 shows the estimation results for the structural model. All parameters are allowed to change apart from quarterly discount factor ($\beta$), quarterly depreciation rate ($\delta$), and output elasticity of labour ($\alpha$) which are held fixed on theoretical grounds.

All of these coefficients have moved some way from their calibration values. On the household side, the estimated coefficient of relative risk aversion in the utility function for consumption ($\gamma_C$) has increased by 9%, implying that the consumption growth is less sensitive to changes in the real interest rate than that in calibration. It also implies an inter-temporal elasticity of substitution of 0.89 between consumption in two consecutive periods. The estimated coefficient of the inverse of Frisch labour supply elasticity ($\gamma_N$) is 35% higher than its calibrated value of 1, which implies that workers are less willing to smooth working hours than that with calibrated value as the wage rate alters. A 1% increase in wage rates leads to a 0.74% rise in hours worked, holding the marginal utility of wealth constant. The

---

19Simulated annealing is a method for finding a good solution to an optimization problem. The method models the physical process of heating a material and then slowly lowering the temperature to ensure that the defects are minimised globally. At each iteration of the simulated annealing algorithm, a new point is randomly generated. The distance between the new point and the current point, or the range of the search, is relied on a probability distribution with a scale proportional to the temperature. The algorithm avoids being caught in local minima and is able to explore globally for better solutions. See https://uk.mathworks.com/discovery/simulated-annealing.html
Table 4.2 Estimates of the Currency Premium Model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definitions</th>
<th>Estimation</th>
<th>Calibration</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>a quarterly discount factor</td>
<td>0.99</td>
<td>0.99</td>
<td>fixed</td>
</tr>
<tr>
<td>(\gamma_c)</td>
<td>CRRA coefficient for consumption</td>
<td>1.12</td>
<td>1.03</td>
<td>9</td>
</tr>
<tr>
<td>(\gamma_N)</td>
<td>the inverse of Frisch labour supply elasticity</td>
<td>1.35</td>
<td>1</td>
<td>35</td>
</tr>
<tr>
<td>(\omega)</td>
<td>a bias towards domestic produced goods</td>
<td>0.5</td>
<td>0.7</td>
<td>-29</td>
</tr>
<tr>
<td>(\omega^F)</td>
<td>foreign equivalent of (\omega)</td>
<td>0.16</td>
<td>0.7</td>
<td>-77</td>
</tr>
<tr>
<td>(\theta)</td>
<td>elasticity of substitution between home and foreign goods</td>
<td>2.74</td>
<td>1</td>
<td>174</td>
</tr>
<tr>
<td>(\theta^F)</td>
<td>foreign equivalent of (\theta)</td>
<td>1.83</td>
<td>1</td>
<td>83</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>output elasticity of labour</td>
<td>0.7</td>
<td>0.7</td>
<td>fixed</td>
</tr>
<tr>
<td>(\delta)</td>
<td>a quarterly depreciation rate</td>
<td>0.025</td>
<td>0.025</td>
<td>fixed</td>
</tr>
<tr>
<td>(\zeta_1)</td>
<td>capital equation coefficients</td>
<td>0.65</td>
<td>0.51</td>
<td>27</td>
</tr>
<tr>
<td>(\zeta_2)</td>
<td>capital equation coefficients</td>
<td>0.32</td>
<td>0.47</td>
<td>-32</td>
</tr>
<tr>
<td>(\zeta_3)</td>
<td>capital equation coefficients</td>
<td>0.02</td>
<td>0.02</td>
<td>0</td>
</tr>
<tr>
<td>(\zeta_4)</td>
<td>capital equation coefficients</td>
<td>0.72</td>
<td>0.25</td>
<td>188</td>
</tr>
<tr>
<td>(\Gamma)</td>
<td>financiers’ risk bearing capacity</td>
<td>0.3</td>
<td>1</td>
<td>-70</td>
</tr>
</tbody>
</table>

Note:
\(\omega^F\) (foreign equivalent of \(\omega\)) appears as an intercept constant in the linear Export Equation and does not enter other behaviour equations. Thus, the value of \(\omega^F\) would not have too much effects on the behaviour of the model. The best fit value of \(\omega^F\) is 0.16 without imposing the bounds. The model still passed the test when the search was limited to 30% either side of the calibrated value of \(\omega^F\).
domestic preference for domestic goods ($\omega$) has dropped by 29%. Domestic consumers allocate equal weights on domestically produced goods and foreign produced goods. Foreign equivalent ($\omega^F$) has declined by 77%, implying the consumers from the rest of the world put 16% weights on their own produced goods relative to the UK produced goods. The elasticity of imports ($\theta$) has jumped to 2.74, and the elasticity of exports ($\theta^F\omega$) has increased to 3.66. Intuitively, a 1% drop in the relative foreign to domestic price ($Q_t$) causes a 2.74% increase in the amount of imported goods from the rest of the world and a 3.66% decrease in the quantity of exported goods, holding constant the quantity of domestic goods. The Marshall-Lerner condition is satisfied since the sum of the elasticities of imports and exports with respect to a change in real exchange rate is greater than 1.

On the firm side, one of the capital equation coefficients, $\zeta_3$, which remains the same as the starting value. The long-run relationship among coefficients in the capital demand equation is also approximately satisfied, which is that $\zeta_1 + \zeta_2 + \zeta_3 = 1$. More specifically, the estimate $\zeta_1$ is 27% higher than the calibrated value of 0.51, implying higher adjustment costs, while the lower value of estimate $\zeta_2$ on the forward expectation of capital indicates a large discount rate at 0.32 for the firm, which is much higher than the discount rate for consumers.

On the international financial intermediary side, the global financier’s average risk-bearing capacity within 1975Q1 to 2016Q4 is estimated at 0.3, which implies that financial intermediaries require a premium to absorb imbalances caused by the international trade. The global financial market is imperfect and uncovered interest parity does not hold.

An estimated parameter set gets a lot closer to the data. With this estimation, the Direct Wald test suggests a strong non-rejection of the currency premium model at the 10% significant level for the VARX(1) auxiliary model described in 4.4.1, with dynamic of endogenous variables $Q_t$ and $r_t$ conditional on exogenous variables $A_{t-1}$ and $d_{t-1}$. The transformed Mahalanobis distance implies a t-statistic of 0.2, or a Wald percentile of 64.8
presented in Table 4.3. It can be seen that the currency premium model is more easily accepted when the auxiliary model 4.4.1 captures both of the dynamic and the volatility (the variance of the fitted residuals of VECM). The Wald statistic based on observed data lies at around 54.9th percentile of the distribution of simulated estimates $\beta^s$.

To check the robustness of test results, I add more endogenous variables to the existing auxiliary model. This should provide a more stringent test of the macroeconomic performance of the structure model and raise the power of the test by extending the features of the structural model that the auxiliary model seeks to match. Test results based on alternative auxiliary models are reported in Table 4.3.

In general, the structural model passes well for combining asset prices ($Q_t$ and $r_t$) with another endogenous variable, except for the employment. In the labour market, the addition of hours ($N_t$) leads to a rejection of the structural model at 99.9%, while asset prices together with real wage ($w_t$) are captured well jointly by the model with a Wald percentile of 77. In the goods market, adding the consumption, $C_t$, as an endogenous variable actually improves the Wald relative to the two endogenous variables ($Q_t$ and $r_t$) case. This can be used to explain the crucial statistical difference between joint moment-matching and single moment-matching. The currency premium model can also comfortably withstand the addition of the output ($Y_t$) or capital ($K_t$) to the auxiliary VARX(1) model without the transformed t statistic falling in the rejection region at the 5% even at 10% significant level.

Furthermore, I increase the number of endogenous variables to four. Real exchange rate and real interest rate match the data when combined with both of output and consumption, or both of output and capital within the 95% confidence interval. When the coefficients of lagged productivity and lagged net foreign debts are both included in the joint distribution of estimates, the simulated data from structural model seems to get closer to the actual data. The Wald percentile decreases by around 5. However, when the labour market is considered, the structural model performs badly with a Wald percentile of 100, implying a rejection of
Table 4.3 Test Results with Estimated Parameters

<table>
<thead>
<tr>
<th>Auxiliary Models</th>
<th>exogenous Variance</th>
<th>Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>included excluded</td>
<td>Wald M-dist</td>
</tr>
<tr>
<td><strong>VARX(1)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, No</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, No</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, No</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}$</td>
<td>$A_{t-1}$, trend, Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}, A_{t-1}$</td>
<td>trend, const Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}, A_{t-1}$</td>
<td>trend, const Yes</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}, A_{t-1}$</td>
<td>trend, const Yes</td>
</tr>
<tr>
<td><strong>VARX(2)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}, A_{t-1}$</td>
<td>trend, const No</td>
</tr>
<tr>
<td>$Q_t$</td>
<td>$d_{t-1}, A_{t-1}$</td>
<td>trend, const No</td>
</tr>
</tbody>
</table>
the model at 1% significant level. Moreover, the model is unable to jointly match a set of 5 endogenous variables - real exchange rate, real interest rate, output, capital and consumption.

It may be possible to raise the power of the Wald test further by increasing the order of the VARX. The test results based on VARX(2) is shown in Table 4.3. I estimate an auxiliary model for the real exchange rate, lagged productivity and lagged net foreign debts on all pseudo-samples to generate the joint distribution of that three VARX(2) parameters. The Wald statistic for the observed data lies in the 92.4th percentile of the Wald statistics from the simulated data. Hence, the currency premium model can accommodate $Q_t$, passing the test comfortably at 5% significance. However, the model is borderline rejected at 95% level of confidence on the basis of the selected set of VARX(2) parameters, when the real interest rate is included as an additional endogenous variable; though those VARX(2) parameters jointly lay inside the 99% confidence limits generated by the bootstrap process.

In summary, a small open economy DSGE model with currency premium performs well for the behaviour of real exchange rate, which gives the support for the currency premium hypothesis. The fact that the real exchange rate is captured jointly with the real interest rate, conditional on lagged net foreign debts is encouraging, as the two channels - interest rate channel and currency risk-taking channel - relates these variables tightly. The model is able to generate the joint patterns of the real exchange rate with other endogenous variables such as output, consumption, capital, and real wage. However, the model with estimated parameters is struggling to capture asset prices with employment jointly, and it fails the test when $N_t$ is included in the auxiliary model.

The Estimates of the Model with the Assumption of UIP

Also, the model with the assumption of UIP is estimated and tested by Indirect Inference method. The estimation results are shown on Table 4.4. With this estimation, the Direct Wald test suggests a non-rejection of the model with UIP at the 5% significant level for the
Table 4.4 Estimates of the Model with the Assumption of UIP

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definitions</th>
<th>Estimation</th>
<th>Calibration</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>a quarterly discount factor</td>
<td>0.97</td>
<td>0.97</td>
<td>fixed</td>
</tr>
<tr>
<td>$\gamma_C$</td>
<td>CRRA coefficient for consumption</td>
<td>1.48</td>
<td>1.03</td>
<td>-43</td>
</tr>
<tr>
<td>$\gamma_N$</td>
<td>the inverse of Frisch labour supply elasticity</td>
<td>0.99</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$\omega$</td>
<td>a bias towards domestic produced goods</td>
<td>0.90</td>
<td>0.7</td>
<td>28</td>
</tr>
<tr>
<td>$\omega^F$</td>
<td>foreign equivalent of $\omega$</td>
<td>0.71</td>
<td>0.7</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>elasticity of substitution between home and foreign goods</td>
<td>1.24</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>$\theta^F$</td>
<td>foreign equivalent of $\theta$</td>
<td>0.99</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>output elasticity of labour</td>
<td>0.7</td>
<td>0.7</td>
<td>fixed</td>
</tr>
<tr>
<td>$\delta$</td>
<td>a quarterly depreciation rate</td>
<td>0.0125</td>
<td>0.0125</td>
<td>fixed</td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>capital equation coefficients</td>
<td>0.54</td>
<td>0.51</td>
<td>6</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>capital equation coefficients</td>
<td>0.49</td>
<td>0.47</td>
<td>4</td>
</tr>
<tr>
<td>$\zeta_3$</td>
<td>capital equation coefficients</td>
<td>0.01</td>
<td>0.02</td>
<td>-50</td>
</tr>
<tr>
<td>$\zeta_4$</td>
<td>capital equation coefficients</td>
<td>0.24</td>
<td>0.25</td>
<td>-4</td>
</tr>
</tbody>
</table>

VARX(1) auxiliary model, with dynamic of endogenous variables – output, real exchange rate and real interest rate conditional on exogenous variables – lagged productivity, time trend and lagged net foreign debts to GDP ratio. The Wald statistic based on the observed data lies at around 85.98 the percentile of the distribution of simulated estimates $\beta^s$. The non-rejection result is consistent with many analyses (such as Meenagh et al., 2010) which have been made with an RBC open economy model of UK under UIP and they have all passed the Indirect Inference tests.

However, what the thesis does is to investigate the case where there is a financial friction. Given UK experience the idea of a financial friction in foreign lending appears plausible.

The full UIP model and the financial friction model are non-nested: they are alternative ways of modelling foreign relationships. It is quite possible both can match the data. This might suggest that there is some more general model that nests them both; for example, it
might be that sometimes there is UIP and sometimes there is friction. However, I do not investigate this here.

4.6 Error Properties

Many of the structural residuals in the currency premium model are serially correlated. These autocorrelated disturbances in a DSGE model are treated as exogenous shocks to the model’s specification. There are 11 shocks appeared in the currency premium model. Those shocks are not observable hence they are extracted from the structural errors based on the unfiltered data and estimated parameters.

Table 4.5 displays the results of two statistical tests - the Augmented Dickey-Fuller (ADF) test and the KPSS test - for the structural errors and also the autoregressive parameters that emerge from the estimation process. The ADF test evaluates the null hypothesis that a structural error $\varepsilon_{i,t}$ has a unit root, $\varepsilon_{i,t} \sim I(1)$, against the alternative hypothesis that it does not, $\varepsilon_{i,t} \sim I(0)$. The test results show that all of the time series reject the null hypothesis of unit root with various confidence levels, except the Solow residual and the error in government spending equation. The probability value for the Solow residual approximately equals to 1, implying a strong non-rejection of the null hypothesis, while the p-value for error in government spending implies borderline non-rejection at 10% significance. One of the problems of the ADF unit root test is the low power against alternatives that are close to being $I(1)$ (Elliott et al., 1996). In other words, unit root test does not perform well in distinguishing highly persistent stationary processes from the non-stationary process. Hence, I run the KPSS stationary test to examine the structural error again.

The KPSS stationary test, on the other hand, evaluates the null hypothesis that $\varepsilon_{i,t}$ is stationary versus the alternative hypothesis that $\varepsilon_{i,t} \sim I(1)$. The KPSS test for the Solow residual rejects the null hypothesis of the stationary process at 5% significant level, while the test for an error in government spending equation fails to reject the stationary. The KPSS test
Table 4.5 Stationarity of Shocks and Estimated AR(1) Parameters

<table>
<thead>
<tr>
<th>Shocks</th>
<th>ADF p-value(^a)</th>
<th>KPSS statistic(^b)</th>
<th>Conclusion</th>
<th>Coefficient (AR(1))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demand Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Preferences</td>
<td>0.0027***</td>
<td>0.3254</td>
<td>Stationary</td>
<td>0.9087</td>
</tr>
<tr>
<td>Factor Demand</td>
<td>0.0001***</td>
<td>0.3243</td>
<td>Stationary</td>
<td>0.8840</td>
</tr>
<tr>
<td>Export Demand</td>
<td>0.0156***</td>
<td>0.0934</td>
<td>Trend-Stationary</td>
<td>0.9288</td>
</tr>
<tr>
<td>Import Demand</td>
<td>0.0469**</td>
<td>0.0584</td>
<td>Trend-Stationary</td>
<td>0.9112</td>
</tr>
<tr>
<td>Government Demand</td>
<td>0.1205</td>
<td>0.2892</td>
<td>Stationary</td>
<td>0.9678</td>
</tr>
<tr>
<td><strong>Supply Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>0.999</td>
<td>1.5785***</td>
<td>Non-Stationary</td>
<td>0.0356</td>
</tr>
<tr>
<td>Wage Cost</td>
<td>0.0058***</td>
<td>0.1763</td>
<td>Stationary</td>
<td>0.9313</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>0.0021***</td>
<td>0.0635</td>
<td>Trend-Stationary</td>
<td>0.9191</td>
</tr>
<tr>
<td><strong>Shocks to the Rest of the World</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global Risk-Aversion</td>
<td>0.0253**</td>
<td>0.2833</td>
<td>Stationary</td>
<td>0.9614</td>
</tr>
<tr>
<td>Foreign Consumption</td>
<td>0.0866*</td>
<td>0.1718</td>
<td>Stationary</td>
<td>0.9846</td>
</tr>
<tr>
<td>Foreign Interest Rate</td>
<td>0.0021***</td>
<td>0.2598</td>
<td>Stationary</td>
<td>0.9254</td>
</tr>
</tbody>
</table>

Note:
\(^a\) For the Augmented Dickey-Fuller (ADF) test, p-value with ***, ** and * indicate a rejection of the unit root process at 10%, 5% and 1% significant level respectively.
\(^b\) For the KPSS test, due to Kwiatkowski *et al.* (1992), statistic with ***, ** and * indicate a rejection of the stationary process at 10%, 5% and 1% significant level respectively.
provides evidence in favour of modelling productivity shock as non-stationary in level and the other structural shocks as either stationary or trend-stationary in level.

The last column of Table 4.5 reports the estimated AR(1) parameters of structural error processes. The AR(1) coefficients for stationary or trend-stationary errors are estimated based on $\varepsilon_{i,t} = \mu_i + \rho_i \varepsilon_{i,t-1} + \eta_{i,t}$, while the AR(1) coefficient for the Solow residual relies on $\Delta \ln A_t = \mu_A + \rho_A \Delta \ln A_{t-1} + \eta_{A,t}$. From the table, we can find that AR(1) coefficients for foreign consumption, government demand and global risk aversion shocks are close to 1, which indicate that even though those errors are stationary, they show high persistence.

### 4.7 Power of the Indirect Inference Test

Le et al. (2016b) compare the power of the Indirect Inference Wald (IIW) test with the power of the Likelihood Ratio (LR) test based on Monte Carlo experiments on the widely used macroeconomic models.

20. LR test asks whether the data-based distribution from the unknown true model, generally VAR model, embodied in the data is able to generate the DSGE-model-restricted VAR coefficients, whereas the IIW test asks whether the DSGE-model-restricted distribution can generate the coefficients of the data-based auxiliary model. They argue that the re-estimation of the error process to bring the model back on track reduced the power of the LR test. Although a substantially false model will be rejected by both of the tests, the power of the IIW test is massively higher than of the LR test when the number of observation is finite, or the observed data is non-stationary. Here I focus on examining the power of the IIW test.

21. IIW test uses the structural model’s own restrictions to generate simulated samples while the LR test uses the actual data sample VAR estimates. With the LR test, each set of simulated data is created by redrawing the VAR innovations, and then it is used to re-estimate the VAR.
In the Section 4.5, I conduct the IIW test to evaluate an estimated structural model in a certain respect; the question then arises of whether it is a test of high quality, or how powerful the test is. The power of a hypothesis test is the probability of rejecting a false null hypothesis\(^{22}\). For a finite sample, the power of a test depends on a few factors, such as significant level and how wrong the null hypothesis is. Thus, having chosen the size of the test\(^{23}\), we can then ask how the rejection rate increases as the structural model becomes more and more false.

A consistent test rejects a false null hypothesis with probability approaching one as the sample size grows. In other words, as the sample size tends to infinity, the power of the test gets to unity. However, the number of observation used in the test in Section 4.5 is 168. Since for the finite sample, we do not exactly know what distribution the Wald statistic follows; it is difficult to calculate the size and the power of IIW test by straight algebra. In such situation, I have to rely on Monte Carlo simulation to estimate them. Therefore, I construct the following experiment where a large number of artificial data sets based on a True structural model and various False models are generated to investigate the power of the test under indirect inference.

1. Given the ‘True’\(^{24}\) structural model with actual data and the estimated parameters, obtain the structural residuals and innovations\(^{25}\), then generate 10,000 sets of shocks with mean, standard deviation, skewness and kurtosis the same as those innovations, and uses them to make 10,000 sets of artificial data called *True Data*.

2. Falsify structural and autocorrelation coefficients of the ‘True’ model by \(x\%\) in both directions in an alternating manner (odd-numbered parameters positive, even ones

---

\(^{22}\)The power of a test equals to 1 minus the probability of making Type II error. A Type II error occurs if we fail to reject the null hypothesis when the alternative hypothesis is true.

\(^{23}\)The size of a test is the probability of making Type I error, which is meant to be the same as the chosen significance level. A Type I error occurs if we reject the null hypothesis when the null hypothesis is true. For example, if we choose 5% significance level, it will reject the ‘True’ model with 5% of the time.

\(^{24}\)In this section, the ‘True’ model is the currency premium model with the estimated parameters.

\(^{25}\)See Appendix A for the LIML methods in detail.
negative); similarly, the second moments of the error processes (standard deviation) are altered by the same $+/ -x\%$. The various ‘False’ models have been constructed, which can be seen as the misspecified versions of the ‘True’ model. Then, generate a set of 10,000 samples from each ‘False’ model called False Data.

3. Following the procedure of the indirect inference test introduced in Section 4.2.1, estimate the auxiliary model VARX on all the False Data to get $\hat{\beta}_{false}$ and $\Omega_{false}$, then calculate the Wald statistic for each set of False Data based on Equation 4.7.1. Thus, we can obtain 10,000 number of Wald statistics and then construct the empirical distribution of the Wald statistics to get the 95 percentile.

$$W_S^{false}_i = (\hat{\beta}^{false}_i - \bar{\beta}^{false})'\Omega^{-1}_{false}(\hat{\beta}^{false}_i - \bar{\beta}^{false}). \tag{4.7.1}$$

4. Similarly, I estimate the auxiliary model on each set of True Data, and calculate the Wald of each of these using Equation 4.7.2.

$$W_S^{true}_i = (\hat{\beta}^{true}_i - \bar{\beta}^{false})'\Omega^{-1}_{false}(\hat{\beta}^{true}_i - \bar{\beta}^{false}). \tag{4.7.2}$$

5. Calculate how many of True Data from the ‘True’ model would reject the ‘False’ model on calculated distribution of the ‘False’ model with 95% confidence. The rejection rate for a given percentage degree $+/ -x$ of misspecification indicates the power of the test.

In the Monte Carlo experiment presented in Table 4.6, all parameters of the ‘True’ structural model is alternately falsified by 1%, 3%, 5%, 7%, 10%, 15% and 20%. I assume that the model users such as the policymaker could well tolerate a falseness in the structure of the model of up to 5%, which implies a 95% confidence level. Then a rejection rate at or above 50% to 70% range at this level of falseness could well provide some security in choosing a
Table 4.6 Monte Carlo Results

<table>
<thead>
<tr>
<th>Percent Misspecified</th>
<th>Rejection Rates at 95% Confidence Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2 variables</td>
</tr>
<tr>
<td></td>
<td>VARX(1)^a–6</td>
</tr>
<tr>
<td>True</td>
<td>5.00</td>
</tr>
<tr>
<td>1%</td>
<td>7.20</td>
</tr>
<tr>
<td>3%</td>
<td>9.40</td>
</tr>
<tr>
<td>5%</td>
<td>10.2</td>
</tr>
<tr>
<td>7%</td>
<td>14.0</td>
</tr>
<tr>
<td>10%</td>
<td>24.0</td>
</tr>
<tr>
<td>15%</td>
<td>50.8</td>
</tr>
<tr>
<td>20%</td>
<td>80.8</td>
</tr>
</tbody>
</table>

Notes:

^a^ It includes two key endogenous variables, real exchange rate and real interest rate with the variance matrix; conditional on exogenous variable lagged net foreign debts.

^b^ It includes three key endogenous variables, real exchange rate, real interest rate and output with the variance matrix; conditional on exogenous variable lagged net foreign debts.

^c^ It includes four key endogenous variables, real exchange rate, real interest rate, consumption and output with the variance matrix; conditional on exogenous variable lagged net foreign debts.

^d^ The order of the VARX with the same variables in [a] has been increased to 2.

currency premium model that passes the IIW test. As the discussion in the previous section, it is too ambitious to include all of 10 endogenous variables of the currency premium model in the auxiliary model VECM. Also, the model users care about whether the model may offer a good explanation of features of interest, for example, causal mechanism of real exchange rate dynamics in the currency premium model, but not of other features of less interest. Focusing on specific aspects of reality is a major strength of IIW test. Thus, results of power of the IIW test presented in Table 4.6 are based on auxiliary models with limited lags and selected endogenous variables.

From the Table 4.6, the results show that the rejection rates increase with the degree of falseness. Higher the rejection rate implies the greater the power of the IIW test. Comparing the rejection rates in column 2 and 4, we will find that when more of the endogenous variables of the structural model are included in the auxiliary models, the rejection rates increase with
the number of coefficients included in the test. It is consistent with the argument that more features of the structural model that the auxiliary model seeks to match are more likely rejected by the data. Furthermore, increasing the order of the VARX will raise the power of the IIW test as well 26 which can be confirmed by comparing the rejection rates of VARX(1) with those of VARX(2).

4.8 Concluding Remarks

This Chapter presented the procedure of the methodology of Indirect Inference testing and estimation. A small open economy DSGE model featuring a currency premium in an imperfect global financial market has been tested at the level of its simulated macroeconomic behaviour for its appropriateness to the UK experience between 1975 and 2016. The result from the test using the initial calibration, conclusively rejected the currency premium model. Then, the model has been estimated using the Indirect Inference method, which minimises the distance between a set of coefficients from the auxiliary model based on the model simulated data and observed data, and the test result of the model with estimated parameters implies a comfortable non-rejection of the model for this UK sample at the 5% significant level. Furthermore, the model performs well when a variety of endogenous variables are added to the auxiliary model VARX(1), explaining the output, physical capital, real wage and consumption in various combination. Monte Carlo experiments indicate that the Indirect Inference Wald test is a powerful test which could be relied on.

To sum up, the Indirect Inference test results show that the currency premium model is able to match the time series properties of the UK data jointly. Thus, in addition to a pure carry force due to the interest rate differential, the sterling behaviour can be affected by financial force in an imperfect global financial market.

26In Le et al. (2016b, p.19), they explain that “this additional power is related to the identifiability of the structural model. The more over-identified the model, the greater the power of the test. Adding an indexation lag has increased the number of over-identifying restrictions exploitable by the reduced form”.

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Chapter 5

Exchange Rate Dynamics During the Financial Disruption: A Empirical Analysis of Sterling Economics

Having established the estimated small open economy DSGE model that integrates the financial friction and fits the sterling exchange rate combined with other major UK macroeconomic data jointly, I now firstly display its impulse response functions to varies stochastic shocks, and go on to apply it to examine the role of global risk aversion shocks and transmission in the global financial crisis and the Brexit vote episodes in the UK, from 2006Q4 to 2016Q4. Finally, I focus on the use of macroprudential and fiscal policies for the estimated currency risk premium model.

5.1 Impulse Responses Analysis

In this section, I evaluate the estimated DSGE model by looking at the structural impulse response functions of the different shocks. The stochastic dynamics of the model are driven by some orthogonal structural shocks. According to the effects of shocks on various aspects
of the economy, those shocks could be divided into three groups, shocks on the domestic supply side (productivity, labour supply, and wage cost shocks) and domestic demand side (consumer preference, factor demand, external demand, demand for imports, government demand shocks) as well as shocks to the rest of the world (the rest of the world demand, foreign interest rate, and global financial shocks). Starting off from the initial equilibrium, I will analyse the impulse responses to productivity, labour supply, consumer preference, external demand, foreign interest rate and risk aversion shocks, respectively. All the variables, except for domestic and foreign real interest rates and net foreign debt to GDP ratio are in the log scale. Thus the changes are in proportion.

5.1.1 Supply Side Shocks

1. Productivity Shocks

Figure 5.1 shows the response of model variables to a positive total factor productivity (TFP) shock. Since the productivity shock process is a unit root process and highly persistent, the one-off shock has a long-lasting impact on the productivity level \( A_t \). The persistent rise in the TFP expands production \( Y_t \) frontier of the domestic economy and real income.

In the labour market, on the one hand side, the rise in productivity raises real wages through marginal productivity, because on the firm side marginal productivity is equal to the real wage in a fully flexible real business cycle model; on the other hand, the rising productivity lowers working hours through labour supply. Since income effect dominates substitution effect, there is a backward-bending long-run supply curve of labour (Hours Worked) in the absence of any labour market distortion such as unemployment benefits.

In the goods market, increased real income and real wages stimulate aggregate demand, including consumption, investment and imports. A stream of investments builds up the
capital stock in line, and capital stocks follow with some delay and increase gradually as a consequence of capital adjustment costs. An unexpected rise in real income leads to an increase in consumption larger than the increase in income itself, because households expect that future income to be higher than current income. The domestic country borrows from the rest of the world to finance its present spending. Therefore net foreign debt ratio climbs and interest rate surges to attract foreign credits. Real interest rate and net foreign debt ratio decline with the arrival of sufficient of funds to sustain the new level of consumption.

As the supply of the domestic goods is expanded, the relative price of these goods to the foreign price has to drop in order to restore external balance. In other words, the domestic economy experiences a real depreciation, which induces an increase in exports. However, the depreciation in real exchange rate causes a gradual rise in import prices. Thus it reduces the demand for imports. Finally, real exchange rate moves
to a new equilibrium that represents a real depreciation on the previous steady state, because output reaches a higher steady state level as well and must be sold on world markets by reducing its price. The effect of a productivity shock on real exchange rate here is different from the typical Balassa-Samuelson effect, since I assume that there is only one goods sector and the model does not allow to distinguish between tradable and non-tradable sectors.

2. Labour Supply Shocks

Figure 5.2 shows the response of the model variables to a 10 percentage labour supply shock, which takes the form of a decline in the disutility of allocating an extra hour to work. Thus, labour force participation increases, while real wages drop temporarily due to an excess supply of labour. Following by the rise of labour factors, domestic output expands; hence real income grows.

The impacts of this positive shock on aggregate demand and real exchange rates are qualitatively similar to those of a positive productivity shock. The real exchange rate depreciation creates contractionary pressures on domestic aggregate demand. The real interest rate initially decreases to a moderate downswing in domestic demand. However, it makes a U turn after around six periods and moves towards to the equilibrium.

Exports jump in the face of depreciation in the real exchange rate. Although a weak currency imposes a downward pressure on import demand, the increased consumption boosts the imports and seems to dominate the expenditure switching effect. However, the increase in exports is much more than that in imports. This ends up with the current account surplus for this shock. Net foreign assets accumulate and reach the peak at 6th period; then it gradually converges back to the long-run level as the shock dies out.
5.1.2 Demand Side Shocks

1. Consumer Preference Shocks

A 1% positive shock $\eta_t^r$ to the disturbance term $\varepsilon_t^r$ affects domestic households’ inter-temporal margin. Specifically, a positive shock increases the required return on domestic assets. At the same time, it also lifts the cost of capital and decreases the value of capital and investment, as shown in Figure 5.3. The decline in investment generates contraction in output. The influence on capital is about twice as large as on output. Employment is also reduced by domestic firms in face of lower production. The real wage increases in the very first quarter, but drops quickly afterwards due to reduced labour demand.

An increase in the real interest rate makes the domestic assets more attractive to the foreign investors, which drives up the demand of the sterling, and the real exchange rate.
in turn appreciates. The sterling appreciation lowers the prices of imports, which induce an increase the demand for foreign goods. The initial increase in the consumption reflects households in the face of a jump in imports. However, consumption contracts afterwards as a result of increased real interest rate. It means that consumption is now less valuable in utility terms. Thus households are willing to give up more units of consumption today. Then, the overall demand for imports falls with the decline in aggregate demand. On the other hand, the volume of exports plunges as a result of the strong currency. Thus, net foreign debts accumulate throughout first of five periods due to the increase in current account deficits, though gradually converging back to zero when imports rebound.

2. External Demand Shocks
In response to a 10% positive external demand shock (export shock) which generates a temporary rise in foreign export demand, the resulting excess demand of the domestic...
goods market generates the current account surplus. The decline in net foreign debt makes the domestic country more attractive to foreign lenders due to lower default risk. Thus there is an excess demand for home-currency denominated bonds compared with the foreign-currency denominated bonds. The international financial intermediary is willing to absorb such an imbalance by providing the domestic country with those domestic currencies and holding currencies from the rest of the world, at some premium. In other words, the financier is long of foreign bonds and short of the corresponding value in the domestic currency of domestic-denominated bonds. Since the rest of the world have borrowed from financiers, their currencies have high expected returns to incentivize international financial intermediaries to lend, which induces an immediate decrease in the real exchange rate (or currency appreciation), $Q_t$, of about 2% to increase financiers’ incentive to sustain the trade imbalance.

In turn, the appreciated real exchange rate raises the purchasing power of the domestic household’s income. The resulting positive income effect stimulates the imports by 6%. In order to clear goods market, the domestic representative household has to shoot up consumption expenditures by roughly 1% to offset excess supply from imports.

Because of sterling appreciation, real interest rate initially increases by 0.04%. Then it decreases modestly to offset excess demand of sterling, which in turn reduces the cost of capital and stimulates investment by small amounts. As a result, output expands by less than 0.5% from the export shock. Total worked hours increases by less than 1%, while real wage costs drop by around 0.2% due to a strong currency. The real wage rebounds after 5 quarters, which is caused by an expected depreciation. To sum up, it is an limited impact of external demand shock on the labour market. Households and firms do not change too much on their supply and demand behaviour. Hence, the change of bargained wage, and labour dynamics are limited.
Meanwhile, the expected depreciation in real exchange rate affects the domestic economy’s net foreign debts and thus net interest income, expressed in terms of the domestic goods. The domestic economy runs a current account surplus, and net foreign debt decreases over time, which reaches its lowest level in the fourth quarter, roughly 0.6% lower than that prior to the shock. Financiers are unable to take infinite positions in currencies whenever there is a positive expected excess return from doing so, because the shock-absorbing behaviour is a constraint to their limited risk-bearing capacity. As a result, accumulate current account surplus (or net foreign debt ratio) gradually shrinks after the 4th quarters and converges back to 0 in the long run. Dynamic adjustments to this positive export shock explained above can be shown in Figure 5.4.

Figure 5.4 Response to 10% Quarter External Demand Shock
5.1.3 Shocks to the Rest of the World

1. Foreign Interest Rate Shocks

Figure 5.5 exhibits the impulse response to the innovation of the foreign interest rate by 1%. The real exchange rates depreciate by around 1.5% due to excess supply of sterling, which stimulates exports to rise by 2.5% and decreases imports by approximately 4%. In order to clear the goods market, domestic real interest rates are only lifted by 0.5% to offset excess supply of sterling. This leads to a subtle decline in domestic consumption and investment; hence capital stock falls by 1%. A small contraction of output is caused by a fall in capital stock. In turn, labour demand and real wages drop initially. But because of the lower cost of labour factor, labour demand rebounds afterwards. The output then gradually recover and converge back to original equilibrium.
The current account starts to accumulate its surplus caused by growth in exports and decline in imports. In turn, net foreign debts decline. It takes around 1 year for net foreign debts to GDP ratio to reach its lowest rate at -2%. Thus, domestic bonds become more attractive to the foreign lender with lower currency risk. At the same time, domestic interest rate decreases and the real exchange rates are expected to depreciate.

Put it differently, higher expected excess return on foreign bonds encourages international financiers to be long of foreign-currencies denominated bonds from the rest of the world and to be short of domestic-currency denominated domestic bonds. Therefore, the real exchange rate has to appreciate to incentivize to absorb shocks and take currency risks. As we can see in Figure 5.5, real exchange rate appreciates after the foreign interest rate shock hitting the domestic economy for one year. Following by the real exchange rate appreciation, exports and imports rebound, in turn, net foreign debt to GDP makes a U turn and converges back to 0.

2. Financial Shocks

Shocks can also arise in the international financial market itself. Financiers act as shock absorbers; however, they are themselves the source of financial shocks that disturb the economy. I introduce financial shocks to the willingness of financiers to absorb currency risk. Figure 5.6 shows the impulse response to global risk aversion shocks. A positive risk aversion shock initially increases the value of parameter $\Gamma$, which induces a fall in financiers’ risk-bearing capacity. In other words, financial intermediaries become more risk averse, hence liquidity squeezes in the international financial market.

At the same time, a global risk aversion shock triggers an increase in real interest rate and a surge in the real exchange rate. Then, it follows by a shift in demand towards domestic goods. The domestic trade balance is improved, and net foreign debt declines.
In turn, the domestic-currency denominated bond becomes less risky and attractive to international financiers, which drives appreciation in real exchange rate through currency risk taking channel as net foreign debts hit the lowest level after one year. Finally, real exchange rates and net foreign debt to GDP ratio go back to their original equilibriums.

An increased interest rate lower the investment and consumption, in turn, output falls. This sends the economy into recession. The labour market is also affected by the financial shock. Both of real wages and hours worked drop immediately as the financial shock hits the economy. Labour demand recovers afterwards due to a low cost of labour.

The real exchange rate dynamics are consistent with the suggestion by Gabaix and Maggiori (2016). A financial disruption will reduce financiers risk-bearing capacity.
by raising the value of parameter $\Gamma$. Net external creditors experience a currency appreciation at the bad time, in contrast, net external debtors’ currency depreciates.

### 5.2 The Errors Driving the Episodes

The following figures show the shocks that are backed out of the currency premium model with estimated parameters and the observed data. In Figure 5.7, I display the shocks for the period of 2006Q4 to 2016Q4, covering the global financial crisis and Brexit vote.

It can be seen that financial shocks, that is, shocks to the willingness of global financiers to absorb exchange rate risks, started to climb triggered by the rising default rates on US subprime mortgages in the third quarter of 2007, and worldwide financial panic reached a peak followed by the collapse of Lehman Brothers in the third quarter of 2008. Then, the
British government bailed out several big banks, including Lloyds TSB, the Royal Bank of Scotland, and HBOS to maintain financial market confidence in the fourth quarter of 2008; hence the risk-averse shock faded.

The global financial shock had been transmitted through the real economy. The domestic demand contracted sharply due to the risk aversion and the tight credit conditions in the global financial market. As can be seen in Figure 5.7, factor demand shock plunged followed by the failure of Lehman Brothers and remained in negative territory for three-quarters, as the business confidence fell. A strong negative export demand shock, around -15%, hit the UK economy in the last quarter of 2008. The demand of the rest of the world had been affected by negative foreign consumption shocks during the crisis. There is a small negative effect on import demand in the late 2008 and an -10% import demand shock in the second quarter of 2009. On the other hand, a positive government spending shock hit after the banking crisis, reflecting the massive government bailout or stimulus bill that pumped into the market. Moreover, the negative shocks to real interest rates indicate that there was a downward pressure on short-term borrowing rates due to the zero lower bound on benchmark interest rates in the post-period of the financial crisis.

Similarly, the UK economy experienced a supply contraction during the global subprime mortgage crisis. In particular, negative productivity shocks hit the economy at the beginning of the crisis and did not fade until 2009. A severe productivity disruption happened in the fourth quarter of 2008. Following the decline in output, the employment and the real wage could be cut by corresponding negative shocks.

From Figure 5.7, we could also find that there was turbulence after the Brexit vote in many of these shocks. Britain voted to leave the European Union, triggering a global rush of capital flows and plunging the global financial markets into turmoil. Financier’s risk bearing capacity and risk aversion are driven by shocks to conditional foreign exchange volatility risk, which jumped after the EU referendum vote. The import demand shock dropped significantly, while
the export demand shock sharply climbed as consumers responded to a slump in the value of sterling. Shock to factor demand dropped to a negative territory due to the uncertainty of the UK economy after the Brexit. Consumer preference shock declined following the vote, as Bank of England cut interest rates to ward off Brexit recession. On the supply side, the productivity shock temporary remained in the positive territory; however, people were hit in the pocket as shocks to real wage stayed in the negative territory.

Overall, there was a wide set of shocks hitting the UK economy during the global financial crisis period and after the Brexit vote. The major shock is coming from the global financial market but in turn triggering domestic counterpart shocks.

5.3 A Stochastic Variance Decomposition of the Episodes

In this section, I attempt to answer the question of what are the main driving forces of sterling exchange rate during the episodes of financial disruptions by using a forecast error variance decomposition (or just variance decomposition for short) of such episodes.

Variance decomposition is a method to quantify how important each shock is in explaining the variation in each of the variables in the reduced form of the structural model. It is equal to the proportion of the variance of the forecast error of each variable caused by each shock at each time horizon.

\[
\phi_{i,j}(h) = \frac{\omega_{i,j}(h)}{\Omega_i(h)}
\]

(5.3.1)

where \(\phi_{i,j}(h)\) denotes the proportion of the variance of the forecast error of variable \(i\) caused by shock \(j\) at horizon \(h\); \(\Omega_i(h)\) represents the total forecast error variance of variable \(i\) at horizon \(h\) in reduced form of the structural model; \(\omega_i(h)\) stands for the forecast error variance
of variable $i$ due to specific shock $j$ at horizon $h$. In general, $\omega(h)$ \(^1\) will be equal to

$$\omega(h) = \sum_{k=0}^{h} C_{i,j}(k)^2; \quad (5.3.3)$$

$C_{i,j}(k)$ is the impulse response of variable $i$ to shock $j$ at horizon $k$.

Table 5.1 gives the variance decomposition of the sterling exchange rate, output, real interest rate and consumption covering the period of financial disruptions based on the currency premium model with estimated parameters and observed data. It can be seen that around 6.6% of the sterling exchange rate variation is due to the financial shock (here global risk aversion shock), which reduces the global financier’s risk bearing capacity. In addition, more than 50% of exchange rate movements are driven by shocks to international trade, i.e. the export shock and the import shock, which have impacts on the net foreign debts position in the financier’ balance sheet, in turn, affect the real exchange rate dynamics. Together, shocks to financial forces, both the financier’s risk bearing capacity and balance sheet, account for more than 72% of the error variance of sterling exchange rate during the period of financial disruptions. Furthermore, the bulk of the variation comes from supply shocks, such as the productivity shock, the wage cost shock and the labour supply shock, which together contribute to 17 percent of the variation. The results of the variance decomposition of the exchange rate emphasise the crucial role of currency risk-taking channel in explaining the variation of the sterling exchange rate in the imperfect financial market.

Shocks to financial forces explain more than a quarter of the variations in consumption, since costs of borrowing from the rest of the world to maintain the standard of consumption surge when there is a financial disruption. In addition, movements in consumption are

\[^1\] If the error $j$ is a non-stationary process, for example, the Solow residual, $\omega(h)$ will be equal to

$$\omega(h) = \frac{\sum_{s=1}^{n_{sim}} \omega^s_i(h)}{n_{sim}}; \quad (5.3.2)$$

$n_{sim}$ denotes the number of bootstrapping. $\omega^s_i(h)$ means the error variance of variable $i$ due to non-stationary shock $j$ based on the $s$th bootstrapping. I bootstrap the shocks in the episode by time vector.
Table 5.1 Variance Decomposition of the Reduced Form Shocks: 2006Q4-2016Q4

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Sterling Exchange Rate</th>
<th>Output</th>
<th>Real Interest Rate</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demand Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumer Preferences</td>
<td>1.100</td>
<td>19.64</td>
<td>25.21</td>
<td>24.92</td>
</tr>
<tr>
<td>Factor Demand</td>
<td>4.050</td>
<td>35.56</td>
<td>5.700</td>
<td>21.91</td>
</tr>
<tr>
<td>Export Demand</td>
<td>23.32</td>
<td>1.220</td>
<td>0.110</td>
<td>7.330</td>
</tr>
<tr>
<td>Import Demand</td>
<td>42.45</td>
<td>2.400</td>
<td>0.270</td>
<td>12.88</td>
</tr>
<tr>
<td>Government Demand</td>
<td>0.063</td>
<td>0.380</td>
<td>0.0037</td>
<td>0.620</td>
</tr>
<tr>
<td><strong>Supply Shocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>15.68</td>
<td>26.64</td>
<td>63.83</td>
<td>10.60</td>
</tr>
<tr>
<td>Wage Cost</td>
<td>0.078</td>
<td>0.810</td>
<td>0.013</td>
<td>0.750</td>
</tr>
<tr>
<td>Labour Supply</td>
<td>1.190</td>
<td>12.66</td>
<td>0.240</td>
<td>11.20</td>
</tr>
<tr>
<td><strong>Shocks to the Rest of the World</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Global Risk-Aversion</td>
<td>6.560</td>
<td>0.460</td>
<td>4.06</td>
<td>7.42</td>
</tr>
<tr>
<td>Foreign Consumption</td>
<td>4.670</td>
<td>0.160</td>
<td>0.0021</td>
<td>1.52</td>
</tr>
<tr>
<td>Foreign Interest Rate</td>
<td>0.860</td>
<td>0.070</td>
<td>0.560</td>
<td>0.840</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td><strong>Shocks to Financial Forces</strong></td>
<td>72.33</td>
<td>4.08</td>
<td>4.44</td>
<td>27.63</td>
</tr>
<tr>
<td>Supply Shocks</td>
<td>16.95</td>
<td>40.11</td>
<td>64.08</td>
<td>22.55</td>
</tr>
<tr>
<td>Demand Shocks</td>
<td>70.98</td>
<td>59.2</td>
<td>31.29</td>
<td>67.66</td>
</tr>
</tbody>
</table>

Note:

a). The values in the table are in the percentage level.

b). Shocks to Financial Forces include export demand shocks, import demand shocks and global risk aversion shocks.
primarily driven by supply shocks that affect the intra-temporal equations and another two shocks that influence the intertemporal Euler equations, i.e. the consumer preference shock which has impacts on both the consumption and investment and the factor demand shock which affects the investment, in turn, the consumption.

Table 5.1 also illustrates that shocks to financial forces explain a minor fraction of the total variations in both the level of output and interest rate, approximately 4%, whereas supply shocks, especially the productivity shock, play significant parts in generating the movements of them.

Therefore there is a distinct role for shocks to financial forces in such episodes of the financial disruption, and those shocks have important effects on the economy in this model, particularly the variation of sterling exchange rates.

5.4 Historical Decomposition of the Financial Disruption

In general, fluctuations in macroeconomic activity are explained in terms of the various shocks. In this section, I turn to investigate how the historical contribution of each of four groups of shocks - shocks to financial forces, the productivity shock, shocks to interest rates, other shocks - to the sterling exchange rate and output over the specific economic episodes of 2006Q4 - 2016Q4. This objective is achieved by applying the historical decomposition methodological framework to an estimated reduced form of the currency premium model, that is the structural vector autoregression (SVAR):

\[ y_t = A_0 y_t + A_1 y_{t-1} + \cdots + A_p y_{t-p} + \nu_t \]  

(5.4.1)

where \( A_i \) are the structural coefficients; \( y_t \) consists of endogenous variables; \( \nu_t \) represents the structural form error term, which is assumed to be identically independently distributed white noise;
The moving average representation of 5.4.1 is

\[ y_t = A^*(L)^{-1} u_t = \Psi(L)u_{t-s} = \sum_{s=0}^{\infty} \Psi_s u_{t-s} \tag{5.4.2} \]

where \( u_t \) denotes the reduced form shocks and \( u_t = (I - A_0)^{-1} v_t; A^*(L) = (I - A_0L - \cdots - A_p L^p)(I - A_0) \) and \( A^*_i \) describes the reduced-form coefficient matrices; the moving average matrix is given by \( \Psi(L) = A^*(L)^{-1} \).

In any case, \( y_{jt} \) may be decomposed as

\[ y^{(k)}_{jt} = \sum_{s=0}^{t-1} \psi_{jk,s} u_{k,t-s} + \sum_{s=t}^{\infty} \psi_{jk,s} u_{k,t-s} \tag{5.4.4} \]

where \( \psi_{jk,s} \) is the \((j,k)\)th element of \( \Psi_s \). The series \( y^{(k)}_{jt} \) indicates the contribution of the \( k \)th structural shock to the \( j \)th component series of \( y_t \), given information \( \sum_{s=t}^{\infty} \psi_{jk,s} u_{k,t-s} \), the ‘base projection’ of the vector \( y \). One may start the decomposition at any point \( t \) in the sample, and here I set 2006Q4 as the starting point of the decomposition.

The historical decomposition of the shocks to the sterling exchange rate is displayed in Figure 5.8. In the time of global turmoil, the UK suffered a large current account deficit due to an unanticipated great decline in world demand, which led to a significant increase in foreign borrowing, while borrowing costs shot up caused by the global risk aversion shock. As can be seen in Figure 5.8, the pound experienced a sharp depreciation as the global recession loomed at the end of 2008, and shocks to financial forces made a major contribution to the surge in the sterling exchange rate. Shocks to interest rates also contributed to the

\[ y^{(k)}_{jt} = \sum_{s=0}^{t-1} D_{jk,s} v_{k,t-s} + \sum_{s=t}^{\infty} D_{jk,s} v_{k,t-s} \tag{5.4.4}\]

where \( D_{jk,s} \) is the \((j,k)\)th element \( \Psi_s(I-A_0)^{-1} \).

In practice, I use Dynare package to solve the model and obtain the estimators \( \hat{\Psi}_s \). The corresponding series \( y^{(k)}_{jt} \) represents a historical decomposition of \( y_{jt} \).
pound depreciation, but with a small portion, because a deep recession triggered the Bank of England to slash interest rates, making the UK a far less attractive place for investors from abroad. On the other hand, a negative productivity shock put a limited pressure in appreciation.

Sterling depreciated to record level against top trading partners after the Brexit referendum vote at the third quarter of 2016. The departure from the European Union imposed an uncertainty on the UK’s future trade policy, fuelling the fear and a lack of confidence. Not surprisingly, shocks to financial forces played a dominant role in the pound depreciation. The foreign demand of Sterling dropped due to the uncertainty of the UK’s economy after the Brexit vote. Notice that the red dash line in Figure 5.8 describes the path for Sterling behaviour for the currency premium model in which all the structural shocks are considered, and the solid black line outlines the path for the model where the global risk aversion shock is excluded. By comparing those two paths for Sterling, we can find that the shock to the willingness of financiers to absorb exchange rate risk can produce the exchange rate disconnect properties and enlarge the volatility of sterling during a financial disruption.

Figure 5.9 shows how the estimated currency premium model suggests the shocks drove outputs of the UK in the episode of financial disruptions. Britain entered a recession in the third quarter of 2008. In particular, productivity shocks play a largely dampening role on output, and shocks to financial forces are by far the most crucial component of the negative shocks to the output. After the collapse of Lehman Brothers, Bank of England and other major central banks in the world stepped in the financial market. The base interest rates were globally cut to historically low levels, aiming to stimulate the economy. Shocks to interest rates made a positive contribution to the output over 2009 to 2011.

Although shocks to financial forces and interest rate differential channels imposed downward pressure on output, Britain’s economy continued growing in the three months after the EU referendum because of the strong fundamentals of the UK economy.
Figure 5.8 Shocks Decomposition of the Sterling Exchange Rate
Figure 5.9 Shocks Decomposition of Output
5.5 Implication of the Model for Policy

When the economy is vulnerable to fundamental shocks, countercyclical movements in the flexible exchange rate in the conventional economic theory have been seen as an expenditure switching mechanism that facilitates relative price adjustments to smooth out output and stabilise the economy. However, fluctuations in the real exchange rate are mainly driven by the shocks to financial forces, such as global risk aversion shock and shifts to balance sheet of the financiers, based on the view of this paper. The expenditure-switching mechanism is developed by being the key channel for the transmission of financial forces affecting the real exchange rate. Then, the volatility in the exchange rate itself, could force the real exchange away from its fundamental level and potentially be the source of, rather than the cure for, the whole economic instability.

When global bond markets are imperfect, shocks to financial forces generate boom-bust cycles in the domestic economy. For example, the global financial crisis of 2008-2010 has been preceded by periods when credit expansion and Sterling appreciation fed on each other. During this cyclical boom, the British economy had experienced substantial capital inflows from China and South-East Asia in order to balance their trade deficits caused by rapid consumption growth which induced consumers imported far more than exported. The dynamics of macroeconomic variables are reversed during the bust phase of the cycle. Gourinchas and Obstfeld (2012) argue that overvalued real exchange rates with large capital inflows during cyclical booms increase the financial vulnerabilities. This, in turn, potentially affects domestic welfare through their impacts on the terms of trade and output.

In this section, I will first propose a tightening of macroprudential policies to cope with expansionary appreciations during cyclical booms. Then, I introduce a fiscal policy which could smooth out consumption fluctuations and increase welfare.
5.5.1 Macro-prudential Policy

Macroprudential policies have re-emerged since the global financial crisis of 2008-2010, aiming to prevent risks from affecting the financial system more broadly. The procyclical financial markets are considered as the ‘original sin’. That is, during boom times, perceived risk declines; financiers’ risk bearing capacities increase, and foreign borrowing and leverage become mutually reinforcing. In essence, macroprudential policies could tackle procyclicality of financial markets and diminish the amplitude of the boom-bust cycles by design.

The existing literature and policy debate have suggested a list of instruments\(^4\) that could be applied to ” increase the resilience of the system and to moderate exuberance in the supply of credit to the economy, and especially to the financial system” (Bank of England, 2009, pp.3). Angelini et al. (2010) point out that macroprudential instruments, regardless of its specific form, would affect financial intermediaries.

In the currency risk premium model, I suppose that there exists a unit mass of global financial firms who channel funds from the rest of the world to domestic households who own non-financial firms resulting from trade flows. The international financiers’ limited commitment constraints imply that their demand for domestic bonds versus foreign bonds is positively depend on the interest rate on domestic bond.

Therefore, I consider a simple specification where the authority could set the interest rate on domestic bonds applicable to the international asset market in period \(t\) according to the macroprudential policy rule

\[
RP_t = \nu \left( \frac{D_t}{D_{t-1}} - 1 \right) \tag{5.5.1}
\]

where \(RP_t\) is the regulation premium, which is defined as an increasing function of the net foreign debt growth in the economy. It implies that the growth of net foreign debts or capital flows has been chosen as the policy objective. \(\nu\) is the adjusted coefficient.

\(^4\)The existing macroprudential tools include size dependent leverage limits, countercyclical capital requirements, caps on loan-to-value ratios, limits on interbank exposure.
In the presence of the macroprudential regulation, the international financiers’ demand for domestic bonds has been affected by the regulation premium. Thus, the lending cost for foreign borrowing becomes,

\[ r_t = r_f^t + \ln Q_{t+1} - \ln Q_t + \Gamma \tilde{d}_t - RP_t. \]  

When the foreign borrowing growth, tightening of macroprudential policies would lower the interest rate on domestic bonds which makes domestic bonds less attractive to the global financiers. This, in turn, reduces the capital inflow during boom periods, drives down the demand for domestic currency (real exchange rate depreciation) and lowers trade deficits.

### 5.5.2 Fiscal Policy

Since the government authority could manipulate his budget to exert influence on aggregate demand, conventional fiscal policies can be utilised to smooth out the extreme swings of the business cycle and stabilise the whole economy. Here, I model a contracyclical fiscal policy in terms of a simple and implementable rule in which the government sets the policy rate in response to the output gap,

\[ \ln G_t = \rho_G \ln G_{t-1} - \xi (\ln Y_{t-1} - \ln \bar{Y}) + \eta_{G,t} \]  

where \( \ln \bar{Y} \) denotes the economy’s long-term trend in output. \( \xi \) is the coefficient of output gap in the government fiscal tool.

In an overheated expansion with a positive output gap, a contractionary fiscal policy reduces government spending, while an expansionary policy increases government spending to stimulate the economy during a recession with a negative output gap.
Table 5.2 Stability Under Different Policy Rules

<table>
<thead>
<tr>
<th></th>
<th>Macro-prudential Policy&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Fiscal Policy&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Macro-prudential Policy+Fiscal Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp Welfare Cost (1)&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-30%</td>
<td>-16%</td>
<td>-46%</td>
</tr>
<tr>
<td>Variance(cons)</td>
<td>-33%</td>
<td>-11%</td>
<td>-42%</td>
</tr>
<tr>
<td>Variance(hours)</td>
<td>0%</td>
<td>-60%</td>
<td>-80%</td>
</tr>
<tr>
<td>Exp Welfare Cost (2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance(output)</td>
<td>-19%</td>
<td>-25%</td>
<td>-50%</td>
</tr>
</tbody>
</table>

Note:

a. Equal weights for each variance.
b. Optimal coefficient of policy rule $\nu = 0.001$.
c. Optimal coefficient of policy rule $\xi = 2.5$.

5.5.3 Policy Rules and Welfare Evaluations

I consider the welfare losses from responses to economic cycles through a macroprudential policy rule, a fiscal rule and a combination of those two rules, and compute the optimal degree of reaction. I take the variance of output and the variances of consumption and labour supply as the objectives. For simplicity, I assume the distortions created by macroprudential policy would be offset by lump-sum transfer.

The coefficients of policy rules \{\nu, \xi\} have been derived optimally by computing the values that minimize the total welfare cost of economic agents under all the structural shocks. Table 5.2 presents a comparative analysis of alternative policies in terms of two groups of variances. Numbers presented in the Table are percentage changes in welfare costs in terms of the variance of consumption, labour supply and output relative to the baseline economy. A smaller percentage change implies a smaller welfare loss, and hence indicates that the policy is more desirable from a welfare point of view.

We observe that the welfare loss decreases by around 46 percent of variances of consumption and labour supply and by about 50% of the variance of output under a combination

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<sup>5</sup>The macroprudential policy creates a wedge between the flexible price cost of capital and the prudential cost. Since the distortion is negligible and difficult to measure, I assume the cost of capital caused by the prudential policy could be offset by paying subsidy to firms.
Figure 5.10 Simulated Output Under Different Rules (Simulation 4)

of macroprudential policy rule and fiscal policy rule. In terms of the consumer’s utility, the reduction in welfare loss from using the macroprudential instrument is significant compared to the fiscal policy. In particular, macroprudential rule plays a significant role in smoothing out the consumption and decreases the volatility of consumption by 33 percent. However, the economy with the fiscal policy rule experiences much smaller fluctuation in output than the economy with the macroprudential instrument.

A thousand of bootstrapping simulations have been run for each policy rule. Then, I randomly choose two simulated samples of outputs under different policy rules and compare them with the sample without a policy rule, which are presented in Figure 5.10 and Figure 5.11. Clearly, the three policy rules inject stabilising action when the economy collapses or surges. In particular, the economy with macroprudential instruments smooths out output markedly, both counteracting the slump and moderating the boom.
5.6 Concluding Remarks

In this chapter, I conduct impulse response functions, variance decomposition and historical shock decomposition analyses to further examine the risk-taking channel in the estimated currency risk premium model. The main finding of this chapter is that shocks to financial forces play the crucial role in Sterling depreciation in the global financial crisis and Brexit vote. At last, I propose macroprudential and fiscal policies based on the baseline model and find that those policies indeed improve the welfare. Especially, macroprudential policies could countercyclically affect the interest rate of domestic bond and sustain foreign demand, therefore smoothing out the output.
Chapter 6

Conclusion

As the currency of the net-debtor country, Sterling experienced large and sudden depreciation in the aftermath of the failure of Lehman Brothers and the Brexit vote have received interest in both the real economy and in financial markets. According to the conventional views, short-term exchange rate movements should be consistent with the uncovered interest parity. However, such volatility in the exchange rate is difficult to support for UIP. The magnitude of the phenomena in question makes it important to gain a clear understanding of the driving forces behind the change in the exchange rate, particularly in light of their potential implications for policy and welfare analysis.

This thesis shows that the recent experience of sterling behaviours can partly be explained by the introduction of currency risk premium, which is associated with both macroeconomic fundamentals and funding liquidity conditions. A risk factor that captures exposure to external imbalances and risk-bearing conditions in financial market explains the bulk of currency excess returns. The economic intuition for this risk factor is that net debtor countries offer a currency risk premium to compensate financial intermediaries for using part of their limited risk-bearing capacity to finance countries’ negative external imbalances. This implies that currency risk premium is related to the evolution of net foreign debt positions and financial forces such as international financiers’ risk-bearing capacity.
I proposed a Real Business Cycle model of a small open economy where global financial markets are imperfect, and currency risk premiums are implied by the global financiers’ demand function. Two channels through exchange rate dynamics have been emphasised: the first is related to the interest rate differential, and the second is related to currency risk-taking. The currency risk premium model is tractable with closed-form solutions and able to applied to address a variety of issues in international macroeconomics, such as exchange rate disconnected puzzle and the failure of uncovered interest parity.

The currency risk premium model has two important features. First, financial frictions are introduced to feature limit of arbitrage in international capital markets. Specifically, global financial intermediaries as arbitrageurs face credit constraints and bear the risks caused by imbalances in the supply and demand of international bonds. Hence, there is no riskless arbitrage opportunities exist in global capital markets, and the uncovered interest parity fails to hold. Bonds denominated in domestic currency and foreign currencies are not perfect substitutes due to different risk characteristics. In comparison with impulse response functions generated from the model where global financiers are relaxed about risk-taking and the uncovered interest parity holds, the impulse response functions generated from currency risk premium model show that shocks to macroeconomic variables are amplified by the presence of credit constraints of international financial intermediaries. Especially, real exchange rates have to adjust further than they would in a world with perfect international capital markets.

Second, shocks that arise in the global financial sector itself are considered to characterise time-varying risks. Global financiers’ risk-bearing capacities are influenced by global risk aversion shocks, which alter the willingness of financial intermediaries to absorb exchange rate risks. When global risk aversion spikes, financiers become more risk-averse and less willing to bear exchange rate risks. As a result, liquidity squeezes in the international asset markets, and financiers require more compensation to intermediate capital flows. The UK,
as a net debtor country, is particularly vulnerable to the impairment of international asset market. A financial disruption, hit by global risk aversion shock, may lead financiers to reassess their ability to purchase sterling denominated bonds and thereby finance the UK trade deficit. All else equal, sterling exchange rates have to depreciate immediately and be expected to appreciate in the future to incentivise financiers to lend. This helps to explain a part of the story of sterling depreciation during the global financial crisis of 2007-2010.

The currency risk premium model serves as an internally consistent backdrop to examine, with statistical formality, whether the mechanism of financial frictions helps to explain real exchange rate dynamics. The model has been estimated using the Indirect Inference method, which minimises the distance between a set of coefficients from the auxiliary model based on the model simulated data and observed data. The VARX(1) serves as the auxiliary model used in estimation and model evaluation, being a parsimonious description of some key features of the currency risk premium model. The chosen auxiliary model ensures that the model is evaluated on the joint behaviour of real exchange rate and real interest rate, conditional on net foreign debt to GDP ratio and productivity.

The Indirect Inference estimation results show that financial forces did have influences on sterling exchange rate dynamics in the 1975-2016 period, giving support for the financial friction hypothesis. The estimated structure coefficient $\Gamma$, which captures financiers’ average risk-bearing capacity within the sample range, is 0.3. A non-zero value implies that financiers indeed require a risk premium to intermediate capital flows. The Indirect Inference test result based on the model with a set of estimated parameters suggests a comfortable non-rejection of the hypothesis that exchange rate dynamics are affected by financial forces at 5% significant level. Furthermore, the currency risk premium model performs strongly on the Wald statistic test when more endogenous variables are added to the auxiliary model, explaining output, consumption, physical capital and real wage in various combination.
Monte Carlo experiments support that the power of the Indirect Inference test to reject a false hypothesis is high; hence the results could be relied on.

To the best of my knowledge, this thesis is the first empirical study in the literature to estimate the coefficient financier’s risk bearing capacities ($\Gamma$). Thus, there is no prior information on this parameter and the Bayesian method is not applied here. Moreover, the thesis would like to test all aspects of the currency risk premium model when testing for the financial imperfections; hence Indirect Inference method is chosen.

To further examine sterling exchange rate dynamics over financial disruptions, I conduct an analysis of subsample between 2006 and 2016, covering the recent global financial crisis and the Brexit vote. Variance decompositions for the estimated currency risk premium model show that shocks to financial forces, including the global risk aversion shock and shocks to financiers’ balance sheet, account for the majority of the error variance of sterling exchange rate during the period of the subsample. In addition, the result of historical shock decomposition based on the estimated model proves that shocks to financial forces are the main driving forces behind large and sudden depreciations of the sterling exchange rates in the aftermath of the collapse of Lehman Brothers and the Brexit vote.

The welfare analysis based on the currency risk premium model shows that welfare costs from the macroprudential and fiscal policies are smaller than the baseline economy. One possible interesting extension would be to specify more explicitly the role of government in order to consider the possibility of foreign exchange rate interventions through the use of foreign reserves.

The currency risk premium model may be good to capture so called “sudden stops” in the emerging economies. A global risk averse shock worsens the financier’s risk bearing capacities ($\Gamma$ becomes large), which force deleveraging, and by implication debtor countries have to decrease their debt abruptly. The capital recipient emerging economies are vulnerable
to the ill-functioning financial markets. Thus, it could be a further study of the currency risk premium model.
References


Limited Information Maximum Likelihood Method

If we don’t know the shock process and error distribution, the LIML method can be applied. Under the LIML method, we only need to know the structural parameters. We then get residuals of the structural model from LIML. Assume that the form of a linearised structural model is:

\[ AE_t Y_{t+1} = BY_t + Z_t \]  \hspace{1cm} (A.0.1)

where \( Y_t \) denotes a matrix of endogenous variables, \( A \) and \( B \) are coefficient matrices, \( Z_t \) is a matrix of shocks.

Expectational variables \( E_t Y_{t+1} \) are estimated by using the robust instrumental variables method developed by McCallum (1976) and Wickens (1982). In practice, we estimate a VAR of all the expected variables and use this to calculate the expectations. For example,

\[ Y_t = CY_{t-1} + e_t \]  \hspace{1cm} (A.0.2)

we estimate the VAR model and obtain estimator \( \hat{C} \), then get the fitted value for \( \hat{Y}_{t+1} = \hat{C} Y_t \), which is the proxy value for \( E_t Y_{t+1} \).

Finally, we can calculate the residuals \( Z_t \) through subtracting \( BY_t \) from \( AE_t Y_{t+1} \).
The UIP Model Solution Method

The UIP model is a rational expectational model with forward-looking variables $E_t K_{t+1}$, $E_t C_{t+1}$, $E_t Q_{t+1}$. It is solved in the log-linearised form using a projection method along the lines of Fair and Taylor (1983). 'Solved' means reached points at which the 'forecast' from the model solution path for the endogenous variables is consistent with the guessed value used for the expectations in finding that solution, within some tolerance level. The expectations must satisfy the terminal conditions on the model at some terminal date $T$ (see Minford et al., 1979). These terminal conditions (long-run levels) depend on the behaviour of the non-stationary variables, which are productivity shocks and net foreign debts in the UIP model. Then, the long-run equilibrium system is solved at future data $T$, when shocks have ceased, trend-stationary variables are growing at balance growth rate and stationary variables have reached their long-run constant values.
Appendix B

Data

I use data over the period 1975Q1-2016Q4 on eleven UK macroeconomic variables: output, consumption, capital stock, export, import, total hours worked, real wages, real interest rates, real exchange rates, net foreign debt to GDP ratio, government spending. Two variables for the rest of the world: world consumption and foreign real interest rates. I convert all real variables to a per capita basis by dividing by an working-age population index. All variables are expressed in constant prices and seasonally adjusted, unless specified otherwise. Most of variables are in natural logs, except where variables have already been expressed in percentages, such as net foreign debt to output and interest rates.

This Appendix includes all definitions, sources of data, symbol keys and the detail of transformations of some data series used in the thesis.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable</th>
<th>Definition and Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Output</td>
<td>Gross Domestic Product; CVM¹; million pounds</td>
<td>ONS: AMBI</td>
</tr>
<tr>
<td>I</td>
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<td>Total gross fixed capital formation + changes in inventories; CVM</td>
<td>ONS: NPQT, CAFU</td>
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<td>(\bar{w}_n)</td>
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<td>Nominal wage divided by GDP deflator at market price</td>
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<td>UK working population index⁵, base period = 2010Q1</td>
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<td>Net foreign debts</td>
<td>Ratio of net foreign debts to nominal GDP¹¹</td>
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Table B.1 Data Description
Notes to Table B.1:
1. CVM represents chained volume measures.
2. Law of motion equation is $K_t = (1 - \delta) * K_{t-1} + I_t$. Here is the process of calculating capital stock, 
Step 1: start with the K/Y ratio (capital output ratio=2.69); 
Step 2: For a given year, I use initial output to calculate capital in first period $\frac{K}{Y} * Y_{1975Q1} = K_{1975Q1}$ (initial value); 
Step 3: Generating capital based on law of motion equation, $K_{1975Q2} = (1 - \delta) * K_{1975Q1} + I_{1975Q2}$.
3. Total employment (ONS code: MGRZ; units: thousands ); Total actual weekly hours worked (ONS code: YBUS, units:millions); Take the number of MGRZ, normalized so that its 2010Q1 value is 1, called it total employment index(MGRZ index); $N = \frac{YBUS}{MGRZ} * MGRZindex$.
Here is the website: http://www.bankofengland.co.uk/research/Pages/datasets/default.aspx.
5. Working population is the sum of total claimant count (ONS code: BCJD) and UK workforce jobs (ONS code: DYDC); take the number of working population, normalized so that its 2010Q1 value is 1, called it working population index.
6. Based on the bilateral trade with the UK, the sterling to euro, the sterling to dollar, and the sterling to Japanese yen bilateral exchange rates have been assigned majority of the weights in calculating sterling real effective exchange rate indices. Please find the detail in http://www.bis.org/statistics/eer.htm
7. According to the weights in sterling real effective exchange rate indices, the weighted average of nominal interest rate in Germany(0.62), US(0.23), Japan(0.15); Germany is a proxy for European Union.
8. FRED denotes Federal Reserve Bank of St. Louis; OECD stands for Organisation for
Economic Co-operation and Development, data website https://data.oecd.org/

9. The weights assigned for countries in $P^F$ is the same as the weights in $R^f$. 

10. One period ahead inflation (year-on-year change in $P^F$) based on the formula:

$$\text{Inflationrate} = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}}.$$

11. Nominal net foreign debt is accumulated current account deficits (millions of pounds), taking the Balance of Payments international investment position as a starting point (ONS code: HBQC at 1974). I converted annual data series to quarterly by quadratic-match-sum.
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