An Analytical Approach for Improving Patient-centric Delivery of Dialysis Services

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Abstract

In this paper, we report on the development of an analytical model and a decision support tool for meeting the complex challenge of scheduling dialysis patients. The tool has two optimization objectives: First, waiting times for the start of the dialysis after the patients' arrivals must be minimized. Second, the minimization of lateness after the scheduled finish time, which is relevant for transport services, are pursued. We model the problem as a mathematical program considering clinical pathways, a limited number of nurses managing the patients, and dialysis stations. Furthermore, information about patients' drop-off and pick-up time windows at/from the dialysis unit are considered. We develop a platform in Microsoft Excel and implement the analytical model using an Open Source optimization solver. A case study from a dialysis unit in the UK shows that a user can compute a schedule efficiently and the results provide useful information for patients, caregivers, clinicians and transport services.

Introduction

Recent research has reported an increasing prevalence of chronic kidney disease worldwide[1]. Patients who are in the end stage of chronic kidney disease (ESRD) rely on dialysis treatments to survive. Hemodialysis is a common approach to manage the condition. It is typically performed in a clinical setting three times a week for several hours, where the patient is connected to a machine via a vascular access. Patients seeking treatment in dialysis units have individual characteristics which can be distinguished on a variety of metrics such as blood flow rate, dialysate flow rate and composition, volume of fluid to be removed and size of dialyzer. Regular laboratory tests of the patient's blood help the physicians to determine a suitable treatment plan. Patients may have many preferences, including the desire for short treatment times and preferred starting times during the day. Dialysis facilities, on the contrary, pursue planning the treatment efficiently by optimizing resource utilization for the best patient outcomes. Patient scheduling which is done in dialysis units world-wide, can be defined as assigning patients to scarce resources and time slots to maximize some objective[2].

In this paper, we develop an analytical approach to deliver a patient-centric scheduling platform for dialysis patients. Individual patients' characteristics are taken into account, such as the patient's clinical pathway, alongside the consideration of the availability of scarce resources that are required. We formulate the scheduling problem as a mathematical program which includes the scheduling decisions around the patients' availability time windows. Using Microsoft Excel, a manager of a dialysis unit can specify parameters in our tool. The manager selects the waiting time objective and obtains feasible solutions based on the objective function and constraints which are implemented using the Open Source solver back-end of our tool. In doing so, patient schedules can be obtained effectively and efficiently such that waiting-times for patients can be minimized. Finally, the optimal schedules can be shared with patients, caregivers, clinicians and transport services.

The remainder of this paper is structured as follows. In the next section, we provide an overview of related patient scheduling work followed by the presentation of the analytical model, demonstration of the platform and discussion, and conclusions.

Related Work

Patient scheduling is the process of assigning individual patients and/or patients' activities to time and/or healthcare resources[2] on the operational decision level. In contrast, appointment scheduling defines a blueprint of patients' appointments on a tactical level. While some reviews focus exclusively on patient scheduling[2,3], appointment scheduling problems have been reviewed by several authors[4–7]. In this section, we position our paper in the relevant patient scheduling literature and focus on patient-related objectives as shown in Table 1.

Minimize	
penalties	8–14
waiting time of (prioritized) patients	9–11, 15–30
welfare loss	31
number of night treatments	32
quality of life proxies	33
Maximize	
# patients to be scheduled	19, 34–38
patients' satisfaction / preferences	17, 18, 22, 32, 39

Table 1: Patient-related objectives

The table reveals that most of the research focus has been on one single patient-related objective which is the minimization of patient waiting times which is similar to the objective of the collaborating dialysis unit. The difference of our work is, however, that we consider patient waiting times which occur before the start of the treatment and after the end of the treatment.

The analytical approach proposed in this paper can be categorized into and differentiated from the literature on patient scheduling as follows. One relevant paper focuses on hemodialysis scheduling but on a tactical decision level[32]. The authors schedule patients' treatments across several days and not within a day as we do. Furthermore, we provide a decision support tool that allows managers to accommodate patient availability and schedule patients more efficiently. In another relevant paper, therapy jobs are scheduled hospital-wide[15]. The difference to our work is, again, that we provide a decision support tool that is based on an Open Source solver as compared to a commercial solver. Using Microsoft Excel and Open Source software increases the usability in the National Health Service because most of the computers have Microsoft Excel pre-installed, users are familiar with it and the Open Source package can be downloaded free and installed as a plugin. Another difference is that dialysis stations are considered as a scarce resource and patients' clinical pathways consist of a setup, dialysis and a finish activity as our next section will reveal.

An Analytical Model

In what follows, we will introduce the parameters for our analytical model, the decision variables, objective function and constraints. Finally, an example schedule is given.

Parameters

Planning horizon, patients and activities

Let $\mathcal{T} := \{1, 2, \ldots, T\}$ be the set of 15-minute slots with planning horizon T. In practice, we start the day at 7:00am and finish it at 11:00pm so that the planning horizon comes up to $T = 64$ periods enumerated using the set $\mathcal{T} := \{1, 2, \ldots, 64\}$. Dialysis patients are denoted by set $\mathcal{P}$. $\mathcal{A}$ denotes the set of all clinical activities to be scheduled and $\mathcal{A}_p \subset \mathcal{A}$ denotes the subset of activities for patient $p \in \mathcal{P}$.

Hospital resources, capacity and demand

Nurses and stations have a capacity R_t^{nurse} and R_t^{station}, respectively, in period $t \in \mathcal{T}$. For example, $R_1^{\text{nurse}} = 1$ means that 1 nurse is available between 7:00am and 7:15am. The demand of activity $i \in \mathcal{A}$ on nurses and stations is denoted by r_i^{nurse} and r_i^{station}, respectively. This parameter will be used in the clinical pathways which are introduced next.

Clinical pathways

In our model, clinical pathways represent standardized, typically evidence-based health care processes as defined by van De Klundert et al.[40]. For more definitions and a literature review on clinical pathway modelling, see Aspland et al.[41]. We depict the clinical pathway of a patient as an activity-on-node graph in which the set of nodes represents the clinical activities. Weighted arcs represent minimum time lags between clinical activities[3] and we write the activities' capacity requirements below the nodes, see the legend in Figure 1.

Consider, for example, patients' set of dialysis activities which are the setup, dialysis and the finish activity, depicted by σ_p, δ_p, and ϕ_p, respectively shown in Figure 1. $r^{\text{nurse}}_{\sigma_1} = 1$ means that the patients' setup activity requires 1 nurse who is busy with the patient setting her up on the dialysis machine.

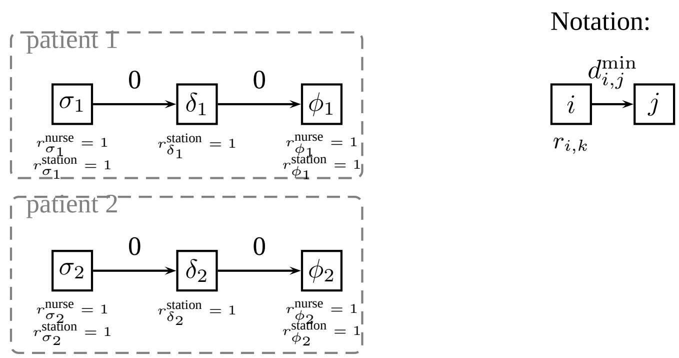

Figure 1: Clinical pathways for two dialysis patients

Let $\mathcal{E}$ denote the set of all minimum time lags between clinical activities. A minimum time lag $(i, j) \in \mathcal{E}$ of weight $d^{\min}_{i,j} \in \mathbb{Z}_{\geq 0}$ stipulates that activity j has to be scheduled at least $d^{\min}_{i,j}$ periods later than activity i. Given the graph $(\mathcal{A}, \mathcal{E})$, the earliest and latest start of activities, denoted by E_i and L_i, respectively can be calculated using longest path methods (see, for example,[42]). Let $\mathcal{W}_i := \{E_i, E_i + 1, \ldots, L_i\}$ denote the time window of activity i. Once we have calculated the latest time slot L_{ϕ_p} in which the dialysis activities can be scheduled, the index of the last period can be calculated by $T = \max L_i$. Table 2 provides an overview of all parameters and decision variables. The latter will be introduced next.

Table 2: Sets, indices, constants and decision variables

Parameter	Description
$\mathcal{A}$	Set of activities
$\mathcal{A}_p$	Set of activities corresponding to patient $p \in \mathcal{P}$
δ_p	Dialysis activity of patient p which excludes the setup and finish. They are modelled as separate activities.
$d^{\min}_{i,j}$	Minimum time lag for precedence relation $(i, j) \in \mathcal{E}$
$\mathcal{E}$	Set of precedence relations
E_i	Earliest period to schedule activity $i \in \mathcal{A}$
L_i	Latest period to schedule activity $i \in \mathcal{A}$
$\mathcal{P}$	Set of patients
ϕ_p	Dialysis finish activity of patient $p \in \mathcal{P}$
p_i	Duration of activity i
r^{nurse}_i	Nurse demand by activity $i \in \mathcal{A}$
r^{station}_i	Station demand by activity $i \in \mathcal{A}$
R^{nurse}_t	Nurse capacity in period $t \in \mathcal{T}$ (e.g. 1 nurse available in period 1)
R^{station}_t	Station capacity in period $t \in \mathcal{T}$ (e.g. 1 station available in period 1)

σ_p	Dialysis setup activity of patient $p \in \mathcal{P}$
$\mathcal{T}$	Set of periods
$\mathcal{W}_i$	Set of consecutive periods to schedule activity $i \in \mathcal{A}$
Decision variable	**Description**
$x_{i,t}$	1, if activity i starts in period $t \in \mathcal{W}_i$, 0 otherwise

Decision variables

We use binary activity-to-period assignment variables, a concept which has been used successfully in other scheduling literature, see[43]. Accordingly,

$$x_{i,t} = \begin{cases} 1, & \text{if clinical activity } i \in \mathcal{A} \text{ starts in period } t \in \mathcal{W}_i \\ 0, & \text{otherwise.} \end{cases}$$

Objective function

Having introduced all necessary parameters and decision variables, the objective functions are given by Equations (1) and (2).

$$\text{Minimize } z = \max_{p \in \mathcal{P}} \sum_{t \in \mathcal{W}_{\sigma_p}} t \cdot x_{\sigma_p,t} \tag{1}$$

$$\text{Minimize } z = \max_{p \in \mathcal{P}} \sum_{t \in \mathcal{W}_{\phi_p}} t \cdot x_{\phi_p,t} \tag{2}$$

Objective function (1) minimizes the maximum waiting time for patients to start the dialysis session as follows: Each patient p has a time window to start the session which is defined for the individual's dialysis start activity σ_p (see the patients' clinical pathways shown in Figure 1). Now, one decision variable $x_{\sigma_p,t}$ is equal to 1, we multiply the time point t which leads to the scheduled start times across all patients. Now, the maximum value of this start time vector is minimized. Similarly, for objective function (2), we minimize the maximum scheduled finish time of the treatments.

Constraints

In what follows, we add constraints to our model which we break down by clinical pathways, nurse and machine constraints.

Clinical pathways, nurse and machine constraints

Constraints (3) use the information from the clinical pathways defined earlier and ensure that minimum time lags between all consecutive activities are guaranteed.

$$\sum_{t \in \mathcal{W}_j} t \cdot x_{j,t} \geq \sum_{t \in \mathcal{W}_i} t \cdot x_{i,t} + d_{i,j}^{\min} \qquad \forall p \in \mathcal{P}, (i,j) \in \mathcal{E}_p \tag{3}$$

Nurse constraints (4) ensure that the demand for nurses does not exceed the nurse capacity.

$$\sum_{i \in \mathcal{A}: t \in \mathcal{W}_i} r_i^{\text{nurse}} \sum_{\tau=\max(E_i, t-p_i+1)}^{\min(L_i, t)} x_{i,\tau} \leq R_t^{\text{nurse}} \qquad \forall t \in \mathcal{T} \tag{4}$$

Constraints (5) ensure that the demand for the stations does not exceed the station capacity.

$$\sum_{i \in \mathcal{A}: t \in \mathcal{W}_i} r_i^{\text{station}} \sum_{\tau=\max(E_i, t-p_i+1)}^{\min(L_i, t)} x_{i,\tau} \leq R_t^{\text{station}} \qquad \forall t \in \mathcal{T} \tag{5}$$

Constraints (6) ensure that each activity is scheduled exactly once.

$$\sum_{t \in \mathcal{W}_i} x_{i,t} = 1 \qquad \forall i \in \mathcal{A} \tag{6}$$

Variable definitions and their domains are given by (7).

$$x_{i,t} \in \{0, 1\} \qquad \forall i \in \mathcal{A}, t \in \mathcal{W}_i \tag{7}$$

Example

Table 3 shows a station and nurse allocation example based on the clinical pathways defined in Figure 1. "–" means that the variables are not defined in these periods because they are outside the activities' time windows. We assume that the processing times of the patients' start activities are 1, the finish activities require two time periods and the durations of the dialysis (δ_p) take 4 and 1 period for patients 1 and 2 respectively. Naturally, the durations are longer in reality as the next section will reveal.

Table 3: A station and nurse allocation example

$t \in \mathcal{T}$	1	2	3	4	5	6	7
$x_{\sigma_1,t}$	1	0	0	–	–	–	–
$x_{\delta_1,t}$	–	1	0	0	–	–	–
$x_{\phi_1,t}$	–	–	–	–	–	1	0
$x_{\sigma_2,t}$	0	1	0	–	–	–	–
$x_{\delta_2,t}$	–	0	1	0	–	–	–
$x_{\phi_2,t}$	–	–	–	1	0	0	–
$\sum_{i \in \mathcal{A}: t \in \mathcal{W}_i} r_i^{\text{nurse}} \sum_{\tau=\max(E_i, t-p_i+1)}^{\min(L_i, t)} x_{i,\tau}$	1	1	0	1	1	1	1
$\sum_{i \in \mathcal{A}: t \in \mathcal{W}_i} r_i^{\text{station}} \sum_{\tau=\max(E_i, t-p_i+1)}^{\min(L_i, t)} x_{i,\tau}$	1	2	2	2	2	2	1

The example reveals that the first activity is scheduled in period $t = 1$ which means that the nurse is allocated in the same period. In the next period ($t = 2$) the second patient's first activity (σ_2, see Figure 1) is scheduled which allocates the nurse in the second period ($t = 2$) as can be seen in the second to last row. In period $t = 3$, the nurse is not assigned to tasks that involve the connection and disconnection of patients to/from machines.

The last row shows the demand profile for the capacity requirement from the dialysis stations. As can be seen, one station is allocated in period $t = 1$, followed by an allocation of two stations in periods $t = 2, 3, \ldots, 6$. As only one patient is on the station at period $t = 7$, the demand profile goes down to 1.

Decision Support Tool

We created a decision support tool in Microsoft Excel. The tool is broken down into a parameters tab which is shown in Figure 2, the solver tab shown in Figure 3, and a solution tab shown in Figure 4.

Figure 2 shows the user interface where the manager can input each patient, along with their treatment duration and arrival time on the left-hand side. Sometimes, a station may be unavailable because of maintenance operations going on for the dialysis machine. Accordingly, the user can parametrize the station availability by using the “station availability table” on the right-hand side. The nurses’ availability and the cleaning time for the machines can also be inputted.

Patient Name:	Patient ID:	Treatment Duration(hours):	Ready Time (in hours after 7am):
Omar Scruggs	1	2	0
Fiona Swingle	2	3	0
Sharee Bax	3	3	0
Debrah Walford	4	3.5	0
Valentine Manor	5	3.5	0.25
Carly Ploss	6	3.5	0.25
Michale Wertz	7	3.5	0.25
Shantay Wasser	8	3.5	0.25
Ariane Grauer	9	3.5	0.5
Kristian Hankerson	10	3.5	0.5
Kerstin Niver	11	4	0.5
Rayna Penland	12	4	0.5
Winston Topper	13	4	0.75
Emilio Porcelli	14	4	0.75
Dorine Aman	15	4	0.75
Arden Aiken	16	4	0.75
Dong Eakins	17	4	1
Gloria Ptacek	18	4	1
Tami Pompa	19	4	1
Zofia Newby	20	4	1
Audrea Whited	21	4	1.25
Rosaria Bacher	22	4	1.25
Chere Eye	23	4	1.25

Station Availability:	Start:	End:
1	07:00	23:00
2	07:00	23:00
3	07:00	23:00
4	07:00	23:00
5	07:00	23:00
6	07:00	23:00
7	07:00	23:00
8	07:00	23:00
9	07:00	23:00
10	07:00	23:00
11	07:00	23:00
12	07:00	23:00

Cleaning time of machine:	0.5

Nurses Availability:	Shift Start:	Shift End:	Break start	Break end
1	07:00	15:00	12:00	12:30
2	07:00	15:00	12:30	13:00
3	07:00	15:00	13:00	13:30
4	15:00	23:00	19:00	19:30
5	15:00	23:00	19:30	20:00
6	15:00	23:00	18:30	19:00

Figure 2: Parameters for scheduling the patients in the Dialysis Unit. All patients’ names are synthetic.

In Figure 3, the user can choose which objective function they wish to optimize when solving the scheduling problem. As mentioned in the modelling section, we have two patient lateness objectives, the first being the time they wait after arrival before starting their treatment, and the latter being the time a relative, Welsh ambulance or a private taxi service has to wait for the treatment to finish. Once the user selects the objective function, OpenSolver[44] will use the inputted values and constraints to create a schedule which is then displayed to the clinician using metrics such as activity start times and lateness.

Click on your required objective function to create your schedule:

Minimise Maximum Patient Lateness (Before Treatment)

Minimise Maximum Patient Lateness (After Treatment)

Patient	1	2	3	4	5	6	7	8	9
Ready Time	0	0	0	0	0.25	0.25	0.25	0.25	0.5
Weight	1	1	1	1	1	1	1	1	1
Session length(hours)	2	3	3	3.5	3.5	3.5	3.5	3.5	3.5
Session length + Cleaning Time	2.5	3.5	3.5	4	4	4	4	4	4
Pick up time(after time 0)	2	3	3	3.5	3.75	3.75	3.75	3.75	4
Allowance	2	3	3	3.5	3.5	3.5	3.5	3.5	3.5
Slack(t=0)	0	0	0	0	0	0	0	0	0

Position	(S1) 1	(S1) 2	(S1) 3	(S2) 1	(S2) 2	(S2) 3	(S3) 1	(S3) 2	(S3) 3
Patient	9	20	7	15	13	26	19	22	17
Ready Time	0.5	1	0.25	0.75	0.75	1.5	1	1.25	1
Session Length	3.5	4	3.5	4	4	4	4	4	4
Session Length +Cleaning Time	4	4.5	4	4.5	4.5	4.5	4.5	4.5	4.5
Pick up time (after)	4	5	3.75	4.75	4.75	5.5	5	5.25	5
Allowance	3.5	4	3.5	4	4	4	4	4	4
Completion	4.5	9	13	5.25	9.75	14.25	5.5	10	14.5
	Station 1			Station 2			Station 3		
Flow Time	4.5	9	13	5.25	9.75	14.25	5.5	10	14.5
Lateness calculation 1	0.5	4	9.25	0.5	5	8.75	0.5	4.75	9.5
Tardiness	0.5	4	9.25	0.5	5	8.75	0.5	4.75	9.5
Flow Time	4	8	12.75	4.5	9	12.75	4.5	8.75	13.5

Figure 3: Solving the problem of scheduling the patients in the Dialysis Unit

Figure 4 shows the output of the tool in a Gantt chart. For each station, we can see which patients have been scheduled at time t. The black bars relate to the patients' start activity σ_p and finish activity ϕ_p (introduced in Figure 1). Recall that these activities require nurses in addition to the stations. The coloured bars relate to activity δ_p, where only the station is required for the actual dialysis activity δ_p.

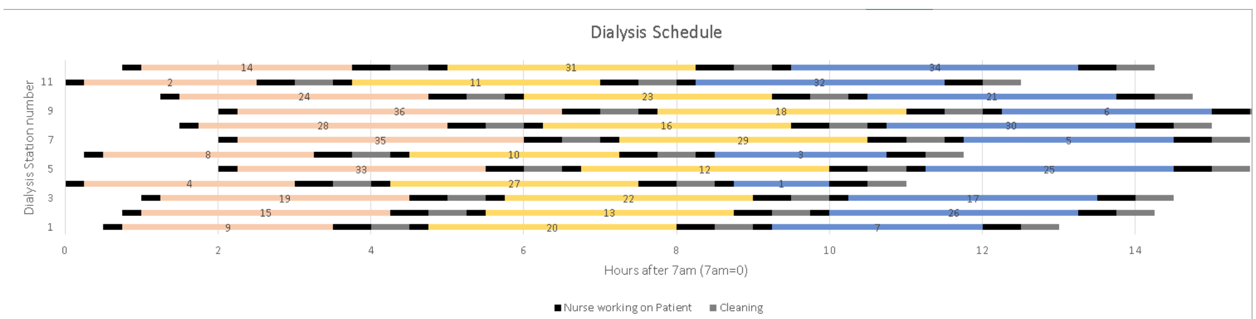

Figure 4: Solution of scheduling the patients in the Dialysis Unit

Discussion

In several workshops we discussed the usability and acceptability of the platform with managers from the National Health Service. One major feedback to improve the usability of the platform was that we have to link our tool with the clinical information system which operates on a separate database. However, using an ODBC connection, we will be able to link Microsoft Excel with the patient data in the collaborating dialysis unit.

The usage of our platform will have several implications and benefits for different stakeholders: Using our tool, patients can be communicated an expected dialysis start time which may reduce perceived waiting time[45]. Nurses will be able to plan their breaks during the day which is very difficult at the moment because of the workload imposed by the unfavorable schedules: Currently, nurses struggle to take breaks during midday because patients who started the treatments in the morning finish at midday and need to be taken care of. In parallel, new patients who arrive at midday have to be setup on the machines and delays may impose overtime at the end of the nurses' shifts. Another benefit of our structured approach is that transport services and patients' caregivers can be informed about the scheduled dialysis finish times. Using information about the patients' home addresses, transport services may aggregate trips for patients and provide more efficient services.

The mathematical model has been formulated as a deterministic problem which means that uncertainty is not taken into account at this stage. Uncertainty may happen in patients' no shows, late arrivals and uncertain dialysis durations. Using information from the past, however, machine learning algorithms may be used to accurately predict no-shows[46] which then can be incorporated into our tool. Alternatively, a rolling-horizon procedure may be used to take into account variation during the execution of the schedule.[3,47,48]

Conclusions

In this paper we have presented an analytical model and a decision support tool for the problem of scheduling dialysis patients in a dialysis unit in the UK. One objective is to minimize the maximum waiting time for patients to start the dialysis session. The second objective minimizes the maximum scheduled finish time of all treatments. In doing so, the model avoids nurses' overtime at the end of the day. Using data from a hospital we demonstrated the effectiveness of our approach and showed the solution output using a Gantt chart. This helps the dialysis unit to find out the optimal sequence of patients on the different dialysis stations.

Future work will include further patient related objective functions including patient preferences, clinician guidelines and targets, and also taking into account resource-related measures such as utilization maximization. Furthermore, we will evaluate the importance of each of the different objectives and incorporate the result into a multi-criteria optimization approach. Also, our aim is to quantify the effectiveness of the approach in practice.

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