Search for the isotropic stochastic background using data from Advanced LIGO’s second observing run

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The stochastic gravitational-wave background is a superposition of sources that are either too weak or too numerous to detect individually. In this study, we present the results from a cross-correlation analysis on data from Advanced LIGO’s second observing run (O2), which we combine with the results of the first observing run (O1). We do not find evidence for a stochastic background, so we place upper limits on the normalized energy density in gravitational waves at the 95% credible level of \( \Omega_{GW} < 6.0 \times 10^{-8} \) for a frequency-independent (flat) background and \( \Omega_{GW} < 4.8 \times 10^{-8} \) at 25 Hz for a background of compact binary coalescences. The upper limit improves over the O1 result by a factor of 2.8. Additionally, we place upper limits on the energy density in an isotropic background of scalar- and vector-polarized gravitational waves, and we discuss the implications of these results for models of compact binaries and cosmic string backgrounds. Finally, we present a conservative estimate of the correlated broadband noise due to the magnetic Schumann resonances in O2, based on magnetometer measurements at both the LIGO Hanford and LIGO Livingston observatories. We find that correlated noise is well below the O2 sensitivity.

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I. INTRODUCTION

A superposition of gravitational waves from many astrophysical and cosmological sources creates a stochastic gravitational-wave background (SGWB). Sources which may contribute to the stochastic background include compact binary coalescences [1–8], core collapse supernovae [9–14], neutron stars [15–24], stellar core collapse [25,26], cosmic strings [27–31], primordial black holes [32–34], superradiance of axion clouds around black holes [35–38], and gravitational waves produced during inflation [39–47]. A particularly promising source is the stochastic background from compact binary coalescences, especially in light of the detections of one binary neutron star and ten binary black hole mergers [48–55] by the Advanced LIGO detector, installed in the Laser Interferometer Gravitational-wave Observatory (LIGO) [56], and by Advanced Virgo [57] so far. Measurements of the rate of binary black hole and binary neutron star mergers imply that the stochastic background may be large enough to detect with the Advanced LIGO-Virgo detector network [58,59]. The stochastic background is expected to be dominated by compact binaries at redshifts inaccessible to direct searches for gravitational-wave events [60]. Additionally, a detection of the stochastic background would enable a model-independent test of general relativity by discriminating the polarization of gravitational waves [61,62]. Because general relativity predicts only two tensor polarizations for gravitational waves, any detection of alternative polarizations would imply a modification to our current understanding of gravity [63–65]. For recent reviews on relevant data analysis methods, see Refs. [66,67].

In this paper, we present a search for an isotropic stochastic background using data from Advanced LIGO’s second observing run (O2). As in previous LIGO and Virgo analyses, this search is based on cross-correlating the strain data between pairs of gravitational-wave detectors [68,69]. We first review the stochastic search methodology and then describe the data and data quality cuts. As we do not find evidence for the stochastic background, we place upper limits on the possible amplitude of an isotropic stochastic background as well as limits on the presence of alternative gravitational-wave polarizations. Upper limits on anisotropic stochastic backgrounds are given in a publication that is a companion to this one [70]. We then give updated forecasts of the sensitivities of future stochastic searches and discuss the implications of our current results for the detection of the stochastic background from compact binaries and cosmic strings. Finally, we present estimates of the correlated noise in the LIGO detectors due to magnetic Schumann resonances [71] and discuss mitigation strategies that are being pursued for future observing runs.

II. METHOD

The isotropic stochastic background can be described in terms of the energy density per logarithmic frequency interval

\[
\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d \rho_{GW}}{df},
\]

(1)
where $d\rho_{GW}$ is the energy density in gravitational waves in the frequency interval from $f$ to $f + df$ and $\rho_c = 3H_0^2c^2/(8\pi G)$ is the critical energy density required for a spatially flat universe. Throughout this work, we will use the value of the Hubble constant measured by the Planck satellite, $H_0 = 67.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [72].

We use the optimal search for a stationary, Gaussian, unpolarized, and isotropic stochastic background, which is the cross-correlation search [66,67,73,74] (however, see Ref. [75]). For two detectors, we define a cross-correlation statistic $\hat{C}(f)$ in every frequency bin

$$\hat{C}(f) = \frac{2}{T} \text{Re}\{\tilde{s}_1(f)\tilde{s}_2(f)^*\},$$

(2)

where $\tilde{s}_i$ is the Fourier transform of the strain time series in detector $i = \{1, 2\}$, $T$ is the segment duration used to compute the Fourier transform, and $S_0(f)$ is the spectral shape for an $\Omega_{GW} = \text{const}$ background given by

$$S_0(f) = \frac{3H_0^2}{10\pi^2f^3}.$$

(3)

The quantity $\gamma_{T}(f)$ is the normalized overlap reduction function for tensor (T) polarizations [73], which encodes the geometry of the detectors and acts as a transfer function between strain cross-power and $\Omega_{GW}(f)$. Equation (2) has been normalized so that the expectation value of $\hat{C}(f)$ is equal to the energy density in each frequency bin

$$\langle \hat{C}(f) \rangle = \Omega_{GW}(f).$$

(4)

In the limit where the gravitational-wave strain amplitude is small compared to instrumental noise, the variance of $\hat{C}(f)$ is approximately given by

$$\sigma^2(f) \approx \frac{1}{2T\Delta f} \frac{P_1(f)P_2(f)}{\gamma_{T}(f)^2S_0^2(f)},$$

(5)

where $P_{1,2}(f)$ are the one-sided noise power spectral densities of the two detectors and $\Delta f$ is the frequency resolution, which we take to be $1/32$ Hz.

An optimal estimator can be constructed for a model of any spectral shape by taking a weighted combination of the cross-correlation statistics across different frequency bins $f_k$,

$$\Omega_{GW}(f) = \sum_k \omega_k \hat{C}(f_k)\sigma^{-2}(f_k).$$

(6)

where the optimal weights for spectral shape $\Omega_{GW}(f)$ are given by

$$w(f) = \frac{\Omega_{GW}(f)}{\Omega_{GW}(f)}.$$

(7)

The broadband estimators are normalized so that $\langle \Omega_{GW}(f) \rangle = \Omega_{GW}(f)$. By appropriate choices of the weights $w(f)$, one may construct an optimal search for stochastic backgrounds with arbitrary spectral shapes, or for stochastic backgrounds with scalar and vector polarizations.

Many models of the stochastic background can be approximated as a power laws [74,76],

$$\Omega_{GW}(f) = \Omega_{GW}(f_\text{ref})^\alpha,$$

(8)

with a spectral index $\alpha$ and an amplitude $\Omega_{GW}(f_\text{ref})$ at a reference frequency $f_\text{ref}$. As in the search in Advanced LIGO’s first observing run (O1) [68], we will take $f_\text{ref} = 25$ Hz, which is a convenient choice in the most sensitive part of the frequency band. While we will seek to generically constrain both $\Omega_{GW}(f)$ and $\alpha$ from the data, we will also investigate several specific spectral indices predicted for different gravitational-wave sources. In the frequency band probed by Advanced LIGO, the stochastic background from compact binaries is well approximated by a power law with $\alpha = 2/3$ [77]. Slow roll inflation and cosmic string models can be described with $\alpha = 0$ [78]. Finally, following previous analyses [68], we use $\alpha = 3$ as an approximate value to stand in for a variety of astrophysical models with positive slopes, such as unresolved supernovae [11–14].

### III. DATA

We analyze data from Advanced LIGO’s second observing run, which took place from 16:00:00 UTC on November 30, 2016 to 22:00:00 UTC on August 25, 2017. We cross-correlate the strain data measured by the two Advanced LIGO detectors, located in Hanford, Washington, and Livingston, Louisiana, in the United States [56]. Linearly coupled noise has been removed from the strain time series at Hanford and Livingston using Wiener filtering [79,80]; see also Refs. [81–83]. By comparing coherence spectra and narrowband estimators formed with and without Wiener filtering, we additionally verified that this noise subtraction scheme does not introduce correlated artifacts into the Hanford and Livingston data.

Virgo does not have a significant impact on the sensitivity of the stochastic search in O2 because of the larger detector noise, the fact that less than one month of coincident integration time is available, and that fact that the overlap reduction function is smaller for the Hanford-Virgo and Livingston-Virgo pairs than for Hanford-Livingston. Therefore, we do not include Virgo data in the O2 analysis.

The raw strain data are recorded at 16,384 Hz. We first downsample the strain time series to 4096 Hz and apply a 16th-order high-pass Butterworth filter with knee frequency of 11 Hz to avoid spectral leakage from the noise power spectrum below 20 Hz. Next, we apply a Fourier
transform to segments with a duration of 192 s, using 50% overlapping Hann windows, and then we coarse grain six frequency bins to obtain a frequency resolution of 1/32 Hz. As in Ref. [68], we observe in the band 20–1726 Hz. The maximum frequency of 1726 Hz is chosen to avoid aliasing effects after downsampling the data.

Next, we apply a series of data quality cuts that remove non-Gaussian features of the data. We remove times when the detectors are known to be unsuitable for science results [84] and times associated with known gravitational-wave events [55]. We also remove times where the noise is nonstationary, following the procedure described in the supplement of Ref. [69] (see also Ref. [68]). These cuts remove 16% of the coincident time, which is in principle suitable for data analysis, leading to a coincident live time of 99 days.

In the frequency domain, we remove narrowband coherent lines that are determined to have instrumental or environmental causes, using the methods described in Ref. [85] and times associated with known gravitational-wave events [55]. We also remove times where the noise is nonstationary, following the procedure described in the supplement of Ref. [69] (see also Ref. [68]). These cuts remove 16% of the coincident time, which is in principle suitable for data analysis, leading to a coincident live time of 99 days.

In Fig. 1, we plot the observed cross-correlation spectrum $\hat{C}(f)$ and uncertainty $\sigma(f)$ obtained from Advanced LIGO’s O2 run. We only plot the spectrum up to 100 Hz to focus on the most sensitive part of the frequency band. These data are also publicly available on the webpage [86] and can be used to search for stochastic backgrounds of any spectral shape.

We perform several tests that the cross-correlation spectrum is consistent with uncorrelated Gaussian noise. The $\chi^2$ per degree of freedom for the observed spectrum is 0.94. The loudest individual frequency bin is 51.53 Hz, and can be used to search for stochastic backgrounds of any spectral shape.

In Table I, we list the broadband point estimates and $1\sigma$ uncertainties obtained from the O2 data when assuming power laws with $\alpha = 0, 2/3,$ and 3. Given the uncertainties, uncorrelated Gaussian noise would produce point estimates at least this large with probability 30%, 22%, and 21%, respectively. We conclude there is not sufficient evidence to claim detection of the stochastic background.

V. UPPER LIMITS ON ISOTROPIC STOCHASTIC BACKGROUND

Since we do not find evidence for the stochastic background, we place upper limits on the amplitude $\Omega_{\text{ref}}$. We use the parameter estimation framework described in Refs. [61,62,76], applied to the cross-correlation spectrum obtained by combining the results from O1 given in Ref. [68] with those from O2 which are described above (please see the Supplemental Material [87] for more details). We present results assuming two priors, one which is uniform in $\Omega_{\text{ref}}$ and one which is uniform in log $\Omega_{\text{ref}}$. We additionally marginalize over detector calibration uncertainties [88]. In O2, we assume 2.6% and 3.85% amplitude uncertainties in Hanford and Livingston, respectively [89,90]. In O1, the calibration uncertainty for Hanford was 4.8% and for Livingston was 5.4% [89]. Phase calibration uncertainty is negligible.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\hat{\Omega}_{\text{ref}}$ (O2)</th>
<th>$\hat{\Omega}_{\text{ref}}$ (O1)</th>
<th>O2 sensitive band</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(2.2 \pm 2.2) \times 10^{-8}$</td>
<td>$(4.4 \pm 6.0) \times 10^{-8}$</td>
<td>20–81.9 Hz</td>
</tr>
<tr>
<td>2/3</td>
<td>$(2.0 \pm 1.6) \times 10^{-8}$</td>
<td>$(3.5 \pm 4.4) \times 10^{-8}$</td>
<td>20–95.2 Hz</td>
</tr>
<tr>
<td>3</td>
<td>$(3.5 \pm 2.8) \times 10^{-9}$</td>
<td>$(3.7 \pm 6.6) \times 10^{-9}$</td>
<td>20–301 Hz</td>
</tr>
</tbody>
</table>
limit $\Omega_{\text{ref}} < 1.1 \times 10^{-7}$. The prior for $\alpha$ is described in the Supplemental Material [87].

VI. IMPLICATIONS FOR COMPACT BINARY BACKGROUND

In Fig. 3, we show the prediction of the astrophysical stochastic background from binary black holes (BBHs) and binary neutron stars (BNSs), along with its statistical uncertainty due to Poisson uncertainties in the local binary merger rate. We plot the upper limit allowed from adding the background from neutron star–black hole (NSBH) binaries as a dotted line. We use the same binary formation and evolution scenario to compute the stochastic background from BBH and BNS as in Ref. [59], but we have updated the mass distributions and rates to be consistent with the most recent results given in Refs. [55,91]. For NSBHs, we use the same evolution with redshift as BNSs. As in Refs. [54], for BBHs, we include inspiral, merger, and ringdown contributions computed in Ref. [92], while for NSBH and BNSs, we use only the inspiral part of the waveform. For the BBH mass distribution, we assume a power law in the primary mass $p(m_1) \propto m_1^{-2.3}$ with the secondary mass drawn from a uniform distribution, subject to the constraints $5 M_\odot \leq m_2 \leq m_1 \leq 50 M_\odot$. In Ref. [55], rate estimates were computed by two pipelines, PyCBC [93] and GstLAL [94]. We use the merger rate measured by GstLAL, $R_{\text{local}} = 56_{-27}^{+44}$ Gpc$^{-3}$ yr$^{-1}$ [55], because it gives a more conservative (smaller) rate estimate. Using the methods described in Ref. [59], the inferred amplitude of the stochastic background is $\Omega_{\text{BBH}}(25 \text{ Hz}) = 5.3_{-2.5}^{+4.2} \times 10^{-10}$.

For the BNS mass distribution, following the analysis in Ref. [55], we take each component mass to be drawn from a Gaussian distribution with a mean of $1.33 M_\odot$ and a standard deviation of 0.09 $M_\odot$. We use the GstLAL rate of $R_{\text{local}} = 920_{-790}^{+2220}$ Gpc$^{-3}$ yr$^{-1}$ [55]. From these inputs, we predict $\Omega_{\text{BNS}}(25 \text{ Hz}) = 3.6_{-3.1}^{+2.4} \times 10^{-10}$. Combining the BBH and BNS results yields a prediction for the total SGWB of $\Omega_{\text{BBH+BNS}}(25 \text{ Hz}) = 8.9_{-5.6}^{+12.6} \times 10^{-10}$. This value is about a factor of 2 smaller than the one in Ref. [59], due in part to the decrease in the rate measured after analyzing O1 and O2 data with the best available sensitivity and data analysis techniques.

For NSBH, we assume a delta function mass distribution, where the neutron star has a mass of 1.4 $M_\odot$ and the black hole has a mass of 10 $M_\odot$, and we take the upper limit on the rate from GstLAL [55]. The upper limit from NSBH is $\Omega_{\text{NSBH}}(25 \text{ Hz}) = 9.1 \times 10^{-10}$. We show the sum of the upper limit of $\Omega_{\text{NSBH}}(f)$, with the 90% upper limit on $\Omega_{\text{BBH+BNS}}(f)$, as a dotted line in Fig. 3.

We also show the power-law–integrated (PI) curves [96] of the O1 and O2 isotropic background searches. A power-law stochastic background that is tangent to a PI curve is detectable with SNR = 2 by the given search. We additionally show a projected PI curve based on operating
We will consider two analytic models of cosmic string loop distributions [104,105]. The former [104] gives the distribution of string loops of given size at fixed time, under the assumption that the momentum dependence of the loop production function is weak. The latter [105] is based on a different numerical simulation [106] and gives the distribution of non–self intersecting loops at a given time [107].

The corresponding limits found by combining O1 and O2 data are $G\mu/c^2 \leq 1.1 \times 10^{-6}$ for the model of Ref. [104] and $G\mu/c^2 \leq 2.1 \times 10^{-14}$ for the model of Ref. [105]. The Advanced LIGO constraints are stronger for the model of Ref. [105] because the predicted spectrum is larger at 100 Hz for that model. This can be compared with the pulsar timing limits, $G\mu/c^2 \leq 1.6 \times 10^{-11}$ and $G\mu/c^2 \leq 6.2 \times 10^{-12}$, respectively [108].

VIII. TEST OF GENERAL RELATIVITY

Alternative theories of gravity generically predict the presence of vector or scalar gravitational-wave polarizations in addition to the standard tensor polarizations allowed in general relativity. Detection of the stochastic background would allow for direct measurement of its polarization content, enabling new tests of general relativity [61,62].

When allowing for the presence of alternative gravitational-wave polarizations, the expectation value of the cross-correlation statistic becomes

$$
\langle \hat{C}(f) \rangle = \sum_A \beta_A(f) \Omega_{GW}^A = \sum_A \beta_A(f) \Omega_{GW}^A \left( \frac{f}{f_{\text{ref}}} \right)^\alpha ,
$$

where $\beta_A = \gamma_A(f)/\gamma_T(f)$ and $A$ labels the polarization, $A = \{T, V, S\}$. The functions $\gamma_T(f)$, $\gamma_V(f)$, and $\gamma_S(f)$ are the overlap reduction functions for tensor, vector, and scalar polarizations [61]. Because these overlap reduction functions are distinct, the spectral shape of $\hat{C}(f)$ enables us to infer the polarization content of the stochastic background. While we use the notation $\Omega_{GW}^A(f)$ in analogy with the general relativity (GR) case, in a general modification of gravity, the quantities $\Omega_{GW}^T(f)$, $\Omega_{GW}^V(f)$, and $\Omega_{GW}^S(f)$ are best understood as a measurement of the two-point correlation statistics of different components of the stochastic background rather than energy densities [109].

Following Refs. [61,62], we compute two Bayesian odds: odds $\mathcal{O}^\text{N}$ for the presence of a stochastic signal of any polarization(s) vs Gaussian noise and odds $\mathcal{O}^\text{NGR}$ between a hypothesis allowing for vector and scalar modes and a hypothesis restricting to standard tensor polarizations. Using the combined O1 and O2 measurements, we find $\log \mathcal{O}^\text{N} = -0.64$ and $\log \mathcal{O}^\text{NGR} = -0.45$, consistent with Gaussian noise. Given the nondetection of any generic stochastic background, we use Eq. (9) to place improved upper limits on the tensor, vector, and scalar background amplitudes, after marginalizing over all three spectral indices, using the priors described in the Supplemental

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**FIG. 3.** Sensitivity curves for O1, combined O1 + O2, and design sensitivity. A power law stochastic background which lies tangent to one of these curves is detectable with 2σ significance. We have used the Advanced LIGO design sensitivity given in Ref. [95], which incorporates improved measurements of coating thermal noise. Design sensitivity assumes that the LIGO noise curve is determined by fundamental noise sources only. The purple line is the median total stochastic background, combining BBHs and BNSs, using the model described in Ref. [59] with updated mass distributions and rates from Refs. [55,91], and the gray box is the Poisson error region. The dotted gray line is the sum of the upper limit for the BBH + BNS backgrounds with the upper limit on the NSBH background.

Advanced LIGO and Advanced Virgo at design sensitivity for 2 years, with 50% network duty cycle. By design sensitivity, we refer to a noise curve which is determined by fundamental noise sources only. We use the Advanced LIGO design sensitivity projection given in Ref. [95], which incorporates improved measurements of coating thermal noise relative to the one assumed in Ref. [58]. This updated curve introduces additional broadband noise at low frequencies relative to previous estimates. As a result, the updated design-sensitivity PI curve is less sensitive than the one shown in Ref. [58].

VII. IMPLICATIONS FOR COSMIC STRING MODELS

Cosmic strings [97,98] are linear topological defects which are expected to be generically produced within the context of grand unified theories [99]. The dynamics of a cosmic string network is driven by the formation of loops and the emission of gravitational waves [100,101]. One may therefore use the stochastic background in order to constrain the parameters of a cosmic string network.

We will focus on Nambu-Goto strings [102,103], for which the string thickness is zero and the intercommutation probability equals unity. Gravitational waves will allow us to constrain the string tension $G\mu/c^2$, where $\mu$ denotes the mass per unit length. This dimensionless parameter is the single quantity that characterizes a Nambu-Goto string network.
IX. ESTIMATE OF CORRELATED MAGNETIC NOISE

Coherent noise between gravitational-wave interferometers may be introduced by terrestrial sources such as Schumann resonances, which are global electromagnetic modes of the cavity formed by the Earth’s surface and ionosphere [71]. These fields have very long coherence lengths [110] and can magnetically couple to the gravitational-wave channel and lead to broadband noise that is coherent between different gravitational-wave detectors. As the detectors become more sensitive, eventually this source of correlated noise may become visible to the cross-correlation search and, if not treated carefully, will bias the analysis by appearing as an apparent stochastic background. Unlike the lines and combs discussed in Ref. [85], we cannot simply remove affected frequency bins from the analysis because Schumann noise is broadband.

Here, we estimate the level of correlated electromagnetic noise (from Schumann resonances or other sources) in O2 following Refs. [68,81,112]. We first measure the cross-power spectral density $M_{12}(f)$ between two Bartington Model MAG-03MC magnetometers [113] installed at Hanford and Livingston. We then estimate the transfer function $T_i(f)$ ($i = \{1,2\}$) between the magnetometer channel and the gravitational-wave channel at each site, as described in Ref. [114]. Finally, we combine these results to produce an estimate for the amount of correlated magnetic noise, which we express in terms of an effective gravitational-wave energy density $\Omega_{\text{mag}}(f)$.

$$\Omega_{\text{mag}}(f) = \frac{|T_1(f)||T_2(f)|\text{Re}[M_{12}(f)]}{\gamma(f)S_0(f)}.$$  \hfill (10)

We show $\Omega_{\text{mag}}(f)$ in Fig. 4, alongside the measured O1 +O2 PI curve and the projected design-sensitivity PI curve. The trend for the magnetic noise lies significantly below the O1+O2 PI curve, indicating that correlated magnetic noise is more than an order of magnitude below the sensitivity curve in O2, although it may be an issue for future runs.

Material [87]. These limits are shown in Table III, again for both choices of amplitude prior.

TABLE III. Upper limits on different polarizations. To obtain the upper limits, we assume a log uniform and a uniform prior on the amplitude $\Omega_{\text{ref}}$ for each polarization, using combined O1 and O2 data. We assume the presence of a tensor, vector, and scalar backgrounds and then marginalize over the spectral indices and two amplitudes for the three different polarization modes, as described in the main text.

<table>
<thead>
<tr>
<th>Polarization</th>
<th>Uniform prior</th>
<th>Log-uniform prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensor</td>
<td>$8.2 \times 10^{-8}$</td>
<td>$3.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>Vector</td>
<td>$1.2 \times 10^{-7}$</td>
<td>$2.9 \times 10^{-8}$</td>
</tr>
<tr>
<td>Scalar</td>
<td>$4.2 \times 10^{-7}$</td>
<td>$6.1 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

Experimental improvements can mitigate this risk by further reducing the coupling of correlated noise. From O1 to O2, for instance, the magnetic coupling was reduced by approximately an order of magnitude, as indicated by the dotted and dot-dashed curves in Fig. 4. Additionally, work is ongoing to develop Wiener filtering to subtract Schumann noise [110,112,115] and to develop a parameter estimation framework to measure or place upper limits on the level of magnetic contamination [116]. This work will take advantage of low noise LEMI-120 magnetometers [117] that were recently installed at both LIGO sites, as described in the Supplemental Material [87].

X. CONCLUSIONS

We have presented the results of a cross-correlation search for the isotropic stochastic background using data from Advanced LIGO’s first and second observing runs. While we did not find evidence for the stochastic background, we obtain the most sensitive upper limits to date in the approximately 20–100 Hz frequency band. We have also placed improved upper limits on the existence of a stochastic background from vector and scalar-polarized gravitational waves.

While the upper limits on the SGWB presented in this work are the strongest direct limits in the frequency band of current ground-based gravitational-wave detectors, other
observations place stronger constraints in other frequency bands. The NANOGrav Collaboration has reported the 95% upper limit of $\Omega_{GW} < 7.4 \times 10^{-10}$ at a frequency of 1 yr$^{-1}$ after marginalizing over uncertainty in the Solar system ephemeris [118]. Combining data from the Planck satellite and the BICEP2/Keck array constrains the tensor-to-scalar ratio from the cosmic microwave background to be $r < 0.064$ at 95% confidence at comoving scales of $k = 0.002$ Mpc$^{-1}$, corresponding to a gravitational-wave frequency of $f_{0.002} = (2\pi)^{-1}ck = 3.1 \times 10^{-18}$ Hz [119], assuming the single field slow roll consistency condition. Using Eq. (4) of Ref. [108], this can be converted into the constraint $\Omega_{GW}(f) \leq 3.2 \times 10^{-16} \times \left(f/f_{0.05}\right)^{-7/8}[16/9 + f_{eq}^2/(2f^2)]$, where $f_{eq}$ is the frequency of a gravitational wave of which the wavelength was the size of the Universe at matter-radiation equality and $f_{0.05}$ is the pivot scale. Combining constraints at different frequency ranges can probe models which span many orders of magnitude in frequency [108,119].

While we have targeted an isotropic, stationary, and Gaussian background, other search techniques can probe backgrounds that violate one or more of these assumptions. Upper limits on an anisotropic gravitational-wave background from O1 were presented in Ref. [120]. Furthermore, non-Gaussian searches targeting the compact binary stochastic background are currently being developed [121–124]. A successful detection of the stochastic background by any of these approaches would offer a new probe of the gravitational-wave sky.

The supporting data for this paper are openly available via the LIGO Document Control Center (DCC) [86].

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[75] Strictly speaking, the optimal search would also include the detector autocorrelation in the likelihood, effectively describing subtraction of the noise power spectrum. However, in practice, the Advanced LIGO noise spectrum is not known well enough for this approach to be effective.
[84] More precisely, we require that both detectors are in observing mode and that no Category 1 vetoes are applied [125].
[107] These models are dubbed model $M = 2$ and model $M = 3$ in Ref. [31]. We do not discuss model $M = 1$ of Ref. [31], which assumes that all loops are formed with the same relative size, since such a hypothesis is not supported by any numerical simulation of Nambu-Goto string networks.
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