

# Currency Board and Full Exchange Intervention in HK

Small Open Economy DSGE with Indirect Inference Assessment

by

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# Abstract

This thesis studies the economy of Hong Kong through the lens of a small open economy DSGE model with a currency board exchange rate commitment. It assumes flexible prices and a banking system that provides credit to entrepreneurial household-firms, with both collateral and cost of verification. We estimate and evaluate the model by Indirect Inference over the sample period of 1994Q1-2018Q3; we find that it matches the data behaviour, as represented by a VAR. We also evaluate a second version of the model in which there is a housing collateral constraint on consumers as in Iacoviello and Neri (2010), and widely used in Hong Kong modelling. However, this version is rejected by the Hong Kong data. In addition, we find out that the housing market has no role in the economy as the housing demand shock accounts for nearly zero, even in the estimated collateral model. We examined the economy's volatility using bootstrapping of the model innovations, under both the estimated currency board model and a standard alternative regime with floating exchange rate and a Taylor rule; we found that Hong Kong welfare is higher in the currency board, as it substantially reduces output and inflation volatility.



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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Literature Review</b>	<b>7</b>
2.1	Hong Kong's monetary system in DSGE models . . . . .	7
2.1.1	DSGE models and Hong Kong . . . . .	7
2.1.2	Banks and the Financial Accelerator . . . . .	9
2.2	Housing as Collateral in Hong Kong . . . . .	9
2.3	Alternative Regime Choice of Monetary Mechanism for Hong Kong. . . . .	11
<b>3</b>	<b>A Small Open Economy DSGE for Currency Board: Case of Hong Kong with Full Foreign Exchange Intervention.</b>	<b>17</b>
3.1	Introduction . . . . .	17
3.2	Model Setups . . . . .	19
3.2.1	Households . . . . .	20
3.2.2	Entrepreneurs . . . . .	23
3.2.3	Housing Producer . . . . .	27
3.2.4	Imports and Exports . . . . .	28
3.2.5	Monetary Operation and Currency Board . . . . .	30
3.2.6	Government . . . . .	31
3.2.7	Balance of Payment . . . . .	32

---

3.2.8	Market Clearing Conditions and Identities . . . . .	33
3.3	Log-linearisation . . . . .	34
3.4	Calibration . . . . .	39
3.5	Impulse Response . . . . .	41
3.6	Conclusion . . . . .	45
Appendix		
3.A	IRFs Figures . . . . .	45
<b>4</b>	<b>Estimation and Testing by Indirect Inference.</b>	<b>53</b>
4.1	Introduction. . . . .	53
4.2	Indirect Inference. . . . .	55
4.2.1	Background. . . . .	55
4.2.2	Indirect Inference Tesitng and Estimation Process. . . . .	56
4.2.3	Choice of Auxiliary Model. . . . .	59
4.2.4	Property of Auxiliary Model. . . . .	63
4.3	Data Description and Preparation. . . . .	64
4.3.1	Data Source. . . . .	64
4.3.2	Why Unfiltered Data. . . . .	66
4.4	Empirical Results by Indirect Inference. . . . .	67
4.4.1	Indirect Inference Estimation and Test Result. . . . .	67
4.4.2	Power of the Test . . . . .	72
4.4.3	Residual and Shock Property . . . . .	74
4.5	Empirical Analysis . . . . .	82
4.5.1	Impulse Responses . . . . .	82
4.5.2	Stochastic Variance Decomposition . . . . .	94



---

4.5.3	Historical Decomposition . . . . .	98
4.6	Conclusion . . . . .	100
Appendix		
4.A	Data Source . . . . .	101
4.B	Impulse Response from Estimated Model . . . . .	103
4.C	Historical Decomposition . . . . .	109
4.D	VARX(1) Parameter Distribution and Fit . . . . .	111
<b>5</b>	<b>Model with Housing Collateral Constraint</b>	<b>113</b>
5.1	Introduction . . . . .	113
5.2	Model Setups . . . . .	115
5.3	Log-linearisation . . . . .	123
5.4	Calibration and Data . . . . .	129
5.5	Indirect Inference Test and Estimation . . . . .	132
5.6	Residual and Shock Property . . . . .	137
5.7	Impulse Responses . . . . .	145
5.8	Stability and Welfare Evaluation . . . . .	154
5.9	Conclusion . . . . .	157
Appendix		
5.A	Data Source in Collateral Model . . . . .	159
5.B	Impulse Response from Estimated Collateral Model . . . . .	161
<b>6</b>	<b>Conclusion</b>	<b>165</b>
	<b>References</b>	<b>167</b>



# List of Figures

1.1	Hong Kong Exchange Rate in 1983 . . . . .	2
1.2	Hong Kong Monetary Growth . . . . .	3
1.3	Hong Kong and US Interest Rate . . . . .	4
3.1	The Balance Sheet of Hong Kong Economy and Currency Board . . . . .	30
3.2	Historical Fiscal Budget Balance in Hong Kong. . . . .	32
3.3	Impulse Response to Non-stationary Technology Shock . . . . .	46
3.4	Impulse Response to Consumption Preference Shock . . . . .	47
3.5	Impulse Response to House Demand Shock . . . . .	48
3.6	Impulse Response to Fiscal Shock . . . . .	49
3.7	Impulse Response to Export Demand Shock . . . . .	50
3.8	Impulse Response to Foreign Interest Shock . . . . .	51
4.1	Residuals from the Structure Model by Estimation . . . . .	76
4.2	Shocks from the Structure Model by Estimation . . . . .	79
4.3	Full Estimated Impulse Response to Consumption Shock . . . . .	84
4.4	Diagram of the dynamics to Consumption Shock . . . . .	85
4.5	Full Estimated Impulse Response to Export Shock . . . . .	87
4.6	Diagram of the dynamics to Export Shock . . . . .	88
4.7	Full Estimated Impulse Response to Foreign Interest Shock . . . . .	90

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4.8	Diagram of the dynamics to Foreign Interest Rate Shock . . . . .	91
4.9	Historical Decomposition of Output . . . . .	99
4.10	Historical Decomposition of Interest Rate . . . . .	100
4.11	Estimated Response to 1% Consumption Preference Shock . . . . .	103
4.12	Estimated Response to 1% Housing Demand Shock . . . . .	104
4.13	Estimated Response to 1% Non-Stationary Productivity Shock . . . . .	105
4.14	Estimated Response to 1% Government Spending Shock . . . . .	106
4.15	Estimated Response to 1% Foreign Consumption Shock . . . . .	107
4.16	Estimated Response to 1% Foreign Interest Rate Shock . . . . .	108
4.17	Historical Decomposition of Consumption . . . . .	109
4.18	Historical Decomposition of Inflation . . . . .	109
4.19	Historical Decomposition of Real Exchange Rate . . . . .	110
4.20	VARX(1) Parameter Distribution and Fit . . . . .	111
5.1	Residuals from the Collateral Model by Estimation . . . . .	139
5.2	Shocks from the Collateral Model by Estimation . . . . .	142
5.3	IRFs from Collateral and Base Model by Preference Shock . . . . .	145
5.4	IRFs from Collateral and Base Model by Export Shock . . . . .	148
5.5	IRFs from Collateral and Base Model by Foreign Interest Shock . . . . .	151
5.6	IRFs to Monetary Shock in Floating Exchange Rate . . . . .	156
5.7	Collateral Model Estimated Response to Government Spending Shock . . . . .	161
5.8	Collateral Model Estimated Response to Housing Demand Shock . . . . .	162
5.9	Collateral Model Estimated Response to Non-Stationary Productivity Shock	163

# List of Tables

3.1	List of Variables in log-linearisation . . . . .	34
3.2	Calibration Parameters . . . . .	40
3.3	Steady State Ratio . . . . .	41
4.1	Indirect Inference Estimates of the Structure Model . . . . .	68
4.2	VARX Parameters and Bootstrap Simulation Bound . . . . .	69
4.3	Power of Indirect Inference Test . . . . .	73
4.4	Residual Stationarity Test and AR(1) Coefficients . . . . .	75
4.5	Variance Decomposition (Long Run) . . . . .	96
4.6	Variance Decomposition (Short Run) . . . . .	97
4.7	Data Source or Derivation . . . . .	101
5.1	Calibration Parameters in Collateral Model . . . . .	130
5.2	Steady State Ratio in Collateral Model . . . . .	131
5.3	Indirect Inference Estimates of the Structure Model with Housing Collateral	134
5.4	Comparison of Testing Result on Estimation between Base Model and Collateral Model . . . . .	134
5.5	Residual Stationarity Test and AR(1) Coefficients in Collateral Model . .	138
5.6	Stability and Welfare Loss Under Two Exchange Rate Regime . . . . .	157
5.7	Data Source or Derivation in Collateral Model . . . . .	159



# Chapter 1

## Introduction

After 1972 when the Bretton Woods system collapsed, a majority of countries turned to floating exchange rates. Under this mainstream regime of floating exchange rates, monetary policy has been usually modelled as a Taylor rule, a rule setting interest rates to target inflation and real GDP. By the international trilemma, whereby an economy cannot have an independent monetary policy, free capital flows and a fixed exchange rate at the same time, the Taylor rule can work well under floating exchange rates and maintain free capital flows. However, plainly it could not operate under a currency board, the extreme case of pegged exchange rate, where the exchange rate is set to be fixed according to a basket of currencies.

In a pegged exchange rate regime, the central bank usually holds a large amount of foreign exchange reserves to manage a stable exchange rate. In practice, a pegged exchange rate regime can be regarded as managed floating within a range. Economies with pegged exchange rate, like China and Singapore, still have central banks which have the power to set monetary policy by their domestic targets and conditions. In contrast, an economy with a currency board does not have a central bank. It cannot have independent monetary policy: the board must simply keep exchange rate fixed. Thus, currency boards must operate in the foreign exchange market to supply money,

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instead of doing open market operations in the bonds market.

Hong Kong is a typical and conventional currency board system, with the currency strictly linked to the US dollar. Before 1983, Hong Kong dollar had silver standard and sterling standard, but both were pure floating exchange rates. In 1982, the Sino-British joint declaration resulted in a sharp depreciation in the Hong Kong dollar, in a sequence of speculative attacks, after a drop-in confidence about the future. The Hong Kong dollar experienced a 'Black Saturday' in 1983. To maintain the stability of the economy, the currency and financial markets, the Hong Kong authority turned to a currency board and fixed exchange rate.

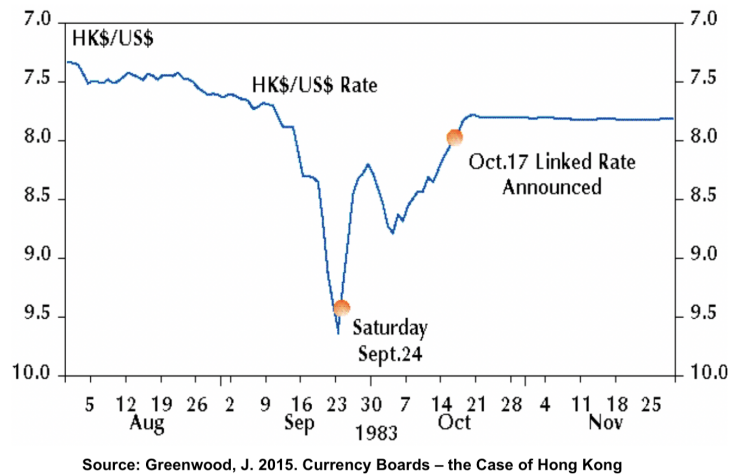


Figure 1.1: Hong Kong Exchange Rate in 1983

For every Hong Kong dollar issued in the economy, the Hong Kong Monetary Authority must hold equivalent US dollar. This 100% back up foreign exchange reserve had ensured the stability. As stated in Greenwood (2015), the currency board arrangement restored the monetary stability in Hong Kong. The fluctuation in the money growth became controlled within a range since 1983, except for the period of financial crisis. The success of the currency board in Hong Kong is from three reasons. First, the authority imposes



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no capital controls and the Hong Kong dollar is fully convertible. This is the base stone because it guarantees that public and private sector can buy and sell Hong Kong dollar as they like, which ensures the confidence about the currency. Secondly, the choice of the anchor currency. It has been widely argued since 1983, which currency to be linked to. US dollar was an attractive anchor currency both technically and economically. First, over 50% of the world's foreign exchange reserves are in US dollars. Second, the US dollar is fully convertible, and it is the dominant trading currency, while the US was Hong Kong's largest trade partner at that time. Finally, as argued by Greenwood (2008), US inflation rate was low, and its business cycle dominated the global business cycle.

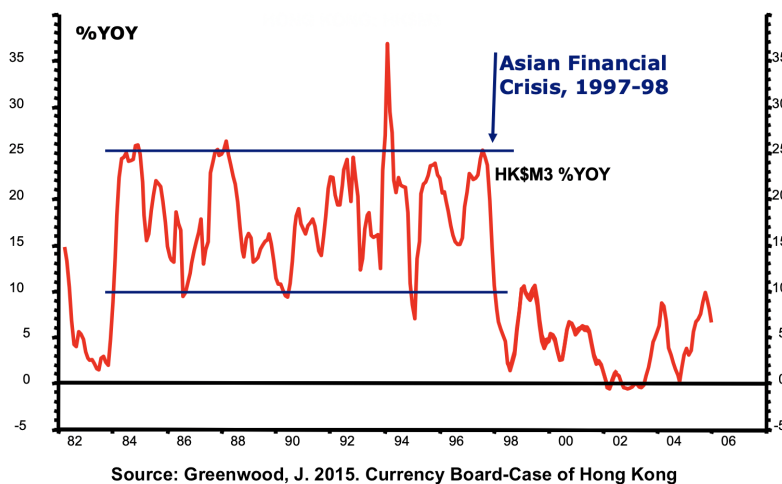


Figure 1.2: Hong Kong Monetary Growth

In Figure 1.3, apart from money growth, prices and inflation, the Hong Kong interest rate will typically equal the US interest rate, since the HK dollar is simply a fixed conversion of the US dollar. The only exception was in the 1997 Asian Financial Crisis, when the interest rate rose sharply on fears the Hong Kong dollar would be devalued.

After the crisis, during which the rate was held, the normal equality resumed.

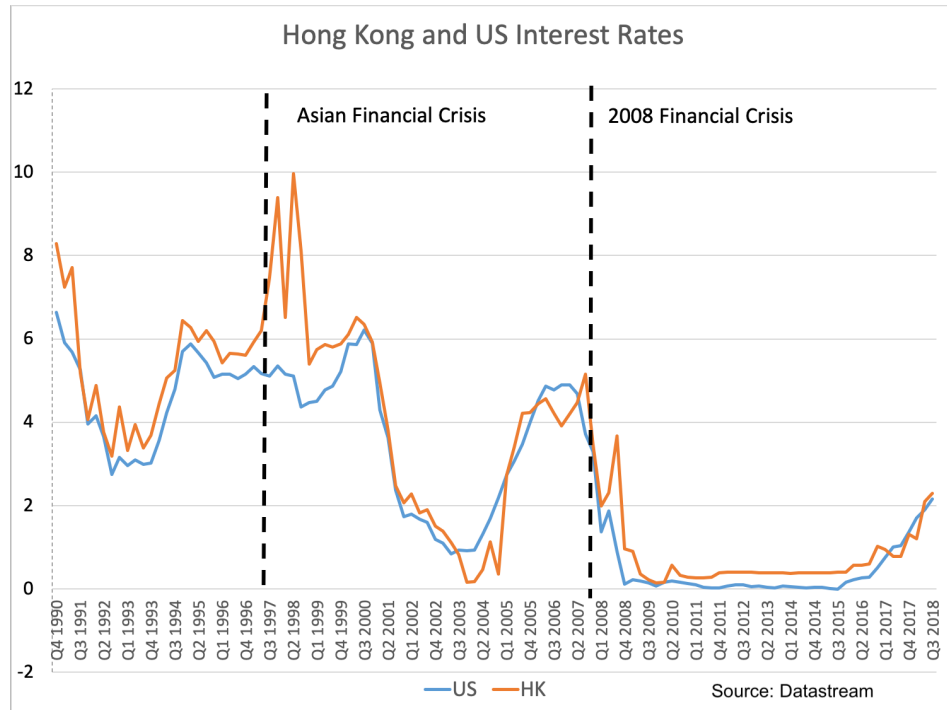


Figure 1.3: Hong Kong and US Interest Rate

One of the major well-being issue in Hong Kong is housing, the extraordinarily high price in housing market makes residents more and more unaffordable to purchase. Like other economies, housing market in Hong Kong is highly correlated with the overall economic performance, but demand and supply structure has its unique characteristics. In the demand side, households income expands slower than the housing price, residents tends to demand more small unit which they can cover more with the income. SAR government regulates the market through the loan-to-value ratio, but which is different from type to type. First payment ratio in commercial resident house is higher than in public house, and tax is lower for the small unit, which further encourages residents to purchase small unit. Apart from the commercial housing market, the public sector also provides large amount of public rental houses. Compare to the demand of housing,

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supply of housing in Hong Kong faces more complex problem. First, the shortage of the land limits the development and then the supply in the long run. Second, government puts a cap in total land auction for building new units, leading to the inelasticity of the supply in the short run. Since the large gap between demand and supply, housing price is pushed even higher while some residents seek to rent a house. Although rental market is in boom, home ownership in Hong Kong is considered to be high relatively, over 50% in recent years.

This thesis aims to answer the following research questions: 1. How does the Hong Kong economy work under the currency board? I construct a small open economy DSGE model to answer this question. 2. How much impact on the economy from the housing market? 3. Should Hong Kong abandon the currency board for an independent monetary policy with floating exchange rates?

The thesis begins with reviews of the related literatures in monetary and exchange rate regimes, financial frictions and housing collateral, as well as those empirical studies of Hong Kong. In the following chapter 3, I set out a small open economy DSGE model with a currency board. In Chapter 4 I discuss my empirical testing and estimation methods based on indirect inference, and relate it to other methods, including Bayesian. The next chapter contains the empirical results, and examine the model behaviour. The model passes the test and fits the data well. I consider in the next chapter whether the model should be extended to include housing collateral, as some have argued.; I find the data reject this extension however. Finally, I carry out a simulation analysis to evaluate welfare outcomes under alternative monetary policy regimes; I find the currency board regime dominates the alternatives. Chapter 6 concludes.



## Chapter 2

# Literature Review

In this thesis I consider three research questions: 1. How does the HK economy work under its currency board? 2. How much importance the housing market has to the economy? 3. Should HK switch to another monetary regime and abandon currency board? In this chapter I reviews previous work related to these areas.

### 2.1 Hong Kong's monetary system in DSGE models

#### 2.1.1 DSGE models and Hong Kong

DSGE models of Hong Kong largely focus, within a New Keynesian framework, on wage and price rigidity, wage dynamics, the stock market and the housing market. I will discuss these last two aspects in the next two sections.

Porter and Vitek (2008) study the impact of minimum wages in a New Keynesian model, separating households into skilled and unskilled; skilled labour has flexible wages and unskilled labour faces a regulated labour market with minimum wages. They argued that

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prices are flexible, compared to other economies. A Minimum wage could enhance the volatility of the economy, and should be introduced in a way that protects price flexibility. They suggest indexation to wage inflation or unit labour costs, as this benefits labour market flexibility.

Cheng and Ho (2009) build a standard New Keynesian DSGE model with price setting firms and labour unions. They find that prices and wages are relatively flexible. Compare to other economies, Hong Kong has much more flexibility.

These two papers are also supported by Paetz (2010), who in a similar New Keynesian DSGE framework, generates the same result that wage and price are relatively flexible in Hong Kong. In addition, it also checks the frequency of adjustment in price and wage, as well as their mark-ups. Wage is almost flexible, but price duration is two years on average. However, variations in both price and wage mark-ups seem to be very short-lived, so that any deviation caused by shocks would die out very quickly. I follow these results by assuming price and wage flexibility in my DSGE model of Hong Kong.

However, these papers have different views about the impact of shocks. Porter and Vitek (2008), as well as Cheng and Ho (2009) argue that Hong Kong is highly driven by external shocks. But Paetz (2010) concludes by the historical decomposition that domestic shocks are the driving force, particularly the domestic interest rate, which is driven by the preference shock, rather than the foreign interest rate shock. Alba et. al (2011) comparing the behaviour of economies in East Asia, find for Hong Kong that its currency board implies the largest response of output comes from foreign output shocks. Funke et.al (2011) introduce the stock market into their New Keynesian model of Hong Kong, suggesting that there is a large wealth effect from the stock market. Although Greenwood (2008) emphasizes that the key in the success of currency board in Hong Kong is the commitment and the ability to hold large and enough foreign backing assets, i.e. foreign reserves, to support the Hong Kong dollar, these New Keynesian

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models past literatures either only feature fixed exchange rate, or have different types of Taylor rules.

### **2.1.2 Banks and the Financial Accelerator**

My DSGE model of Hong Kong contains a banking set-up that follows Bernanke and Gertler (1989)'s financial accelerator mechanism; in this the bank sets a contract with an external finance premium, designed to incentivise the borrower to avoid falsely declaring bankruptcy, which will cost the bank in verification checks. The resulting optimal debt contract implies that the borrower's falling net worth drives up the external premium—the 'accelerator' effect of a cycle downturn. Le et al (2016) introduce collateral into the model and the external finance premium, so that quantitative easing affects the premium by increasing the use of money as cheap collateral, much as in a search model from Williamson (2013). Firms demand money for collateral purposes, to reduce their financing costs.

## **2.2 Housing as Collateral in Hong Kong**

The housing market has been put into the centre of business cycle dynamics by Iacoviello (2010). For Hong Kong Greenwood and Hercowitz (1991) find data indicating that the residential capital stock value is larger than the business capital stock value. Davis and Heathcote (2001) estimate that in the US the market value of residential property stock is approximately the same size as annual GDP.

For the anchor economy of Hong Kong, the US, Iacoviello (2010) introduces some facts about the housing and macro economy. The same as other papers, housing wealth is an important component of national wealth, it could be supported by the US data that the

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housing wealth is larger than the GDP, for which the ratio is around 1.5. Moreover, the co-movement of housing to other variables. Housing wealth has the same direction of movement to aggregate consumption, housing investment and other prices.

The 2008 global financial crisis originated mainly from the debt default and value drop in housing collateral held by the banks. Iacoviello (2004) and Iacoviello and Minetti (2008) and Iacoviello and Neri (2010) develop this housing collateral model to explain US and European behaviour in the financial crisis. Other related studies include Liu et.al (2011), Garriga et.al (2012), Yopez (2018), Michaud and Rothert (2014) and Walentin (2014).

For Hong Kong, the Currency Board and linked exchange rate regime makes the housing in Hong Kong behave differently from other economies. There is also the extreme pressure on land; meanwhile the Hong Kong authority uses loan-to-value ratio as a macro-prudential policy to control the housing price. Chan et.al (2001), Fund and Cheng (2015) Gerlach, Peng (2005) and Raymond (1996) all emphasize the importance of the housing market in Hong Kong, stating that housing market is highly correlated with other factors in the economy. Funke and Paetz (2013) build the model from the base of Iacoviello's version in the housing market, together with collateral channel. They focus on the impact of different shocks to the housing market, saying that housing has large substitution effect to the tradable goods and the collateral channel is the main source for the big impact of housing preference shocks to the consumption volatility. A further paper by Funke and Paetz (2012) discusses the effect of policy of loan to value ratio to the housing. It is concluded that the high loan to value ratio as a macro prudential policy for Hong Kong is effective in taming property price booms. Rabanal (2018) argues similarly that without the loan to value ratio policy, housing price would be 10.5% higher and households' credit to GDP ratio will be 14% higher. The loan to value ratio cap is relatively more effective in maintaining leverage stability while stamp duty taxes are more effective in controlling house prices. Against all this, for China, Gai



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et.al (2020) find, using indirect inference, that the housing collateral model is rejected by the data while the model without collateral fits the data well.

### **2.3 Alternative Regime Choice of Monetary Mechanism for Hong Kong.**

Various authors have explored the issue of whether a different monetary regime from the currency board would have improved macro stability. For switching to a floating exchange rate and Taylor rule regime, Gerlach-Kristen (2006) applied a structural VAR model to Hong Kong and compare the performance of the currency board system and the alternative regime of the interest reaction Taylor rule form.

This paper begins with 6 equations for output gap, CPI inflation, property price inflation, nominal interest rate, nominal effective exchange rate and import price inflation. Because the level of each domestic variable is non-stationary, each equation initially has lagged change of dependent variable and lagged level term of itself, as well as other variables with change and level lagged term, as suggested by theory. In addition, three foreign variables are modelled as exogenous AR (2) processes. Firstly, the authors find that only the change in US rate is significant in determining the change in the HK rate, while HK output and inflation are not; this fits the currency board feature. The nominal effective exchange equation has the change in the interest rate on the right hand side, suggesting that a higher domestic interest rate leads to capital inflow and then an appreciation of the effective exchange rate. The paper then re-estimates the above 9 equations, including 3 foreign variables, as a structural VAR, by three-stage least squares. Simulating this benchmark currency board model with foreign shocks only, the authors show that the interest rate is very close to the actual one, while inflation is lower before but higher after the crisis. The behaviour of the nominal effective exchange rate is more stable than

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in in the data sample.

To simulate the alternative choice of Taylor rule, the authors simply replace the interest rate equation by the standard Taylor rule equation in Taylor (1993), while keeping the rest of the model the same as estimated. Simulating this model on the over the same period, they find that the interest rate would have been higher than in the actual currency board case, but would have dropped a lot during the Asian Financial Crisis, due to the recession. The simulated nominal effective exchange rate is close to the currency board path but would have sharply depreciated during the crisis, owing to the lower interest rate. Their welfare loss function suggests that the Taylor rule performs better than the currency board. However, there is not much difference between them on an alternative loss function with only the output gap. The fundamental problem with this analysis is that it violates the Lucas Critique because it assumes parameters are the same in different regimes, whereas plainly the VAR parameters would all be affected by the regime change.

This discussion of alternative choice of monetary regime can be partially supported by another early paper, discussed by Kwan and Liu (1999). They build a VAR model with output and price, dividing the full sample into two parts, free-floating years from 1975 to 1983 and currency board years from 1983 to 1995. Comparing actual data with the simulation of free-floating sample by the periods after 1983, the density functions under floating rate and currency board do not differ much but a significant difference is that the volatility under floating is slightly larger. With the aim to stabilise the economy, there is no benefit for Hong Kong to deviate from the currency board to the floating rate.

McNelis and Lim (2012) build a typical New Keynesian DSGE model to address the question of the behaviour of Hong Kong economy under linked exchange rate and floating exchange rate, where the linked exchange rate is currency board and floating exchange

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rate is Taylor rule or inflation targeting. In general equilibrium, representative households maximise the discount expected lifetime utility subject to their budget constraint by choosing the consumption, labour, bonds and foreign assets.

In this model, authority sets the tax on labour income and consumption. This is close to the real world as Hong Kong set a low tax rate in order to attract investment activity and limit the government spending. In addition, they also assume that the foreign assets return, and the risk premium follow an autoregressive stochastic process. In the production side, firms produce the output with a single function of labour only. The intuition behind it is that they believe Hong Kong economy is highly labour intensive and for simplicity, it would be better to have labour only. A controversial fact is that this model ignores the productivity in the production function. Although it can be explained that the productivity shock is less important than other shocks, like the premium shock. But the productivity factor still cannot be omitted as it would have more or less impact to the Hong Kong economy through some innovation and R&D development, even if it is not direct. Actually, without estimation and testing, we do not know the contribution from technology shock. In order to compare the currency board and the floating exchange rate, they give different rules to the monetary target. In the fixed exchange rate, the domestic interest rate equals to foreign interest rate and the risk premium, while standard Taylor rule for the floating exchange rate. As a standard New Keynesian setting, the monopolistically competitive firm would have a Calvo Price setting activity, a fraction of firms would keep the previous price and the rest could reset the price to the optimal domestic goods price. This setting follows both backward looking and forward looking, which is from the extension of Calvo (1983).

Taking the above critical review of this paper, the final Bayesian estimation gives a quite similar result to the previous DSGE literature reviewed above. The output variation is driven by export demand and foreign price shocks through the foreign assets holding,

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while the inflation response is mostly to shocks in foreign prices and interest rates. For the alternative floating exchange rate with Taylor rule, this paper draws the conclusion that inflation volatility would decrease slightly, but at a cost of a relatively large increase in the volatility of the interest rate and consumption. Therefore, Hong Kong should stay in the currency board and fixed exchange rate. Nevertheless, we are not given any measure of how close this DSGE model is to Hong Kong's data behaviour. Hsiao et.al (2012) discuss the benefit of integration with mainland China. They compare the output growth in HK with the counterfactual growth if there were no structural change<sup>1</sup>, where the counterfactual growth with no change is estimated and simulated by the factor approach, so that the treatment effect from the structural change in the difference between the real GDP growth and the counterfactual growth. It is pointed out that the political integration in 1997, the handover to mainland, has no significant effect on the HK economy, in terms of the growth. However, the economic integration in 2003, the Closer Economic Partnership Agreement (CEPA), raises the growth by 4%. This paper clears up some doubt on the political effect on HK economy and provides empirical evidence on the economic integration with the mainland, which would support the discussion of possibility on linking to RMB or a money union.

The work using the structural VAR models generally has the different but weak conclusion that, while Hong Kong would be slightly better with floating and a Taylor rule, it should keep the currency board. However, the structural VAR models violate the Lucas Critique and so cannot be considered reliable. A more serious problem in both VAR and DSGE which taking about the alternative Taylor rule, is the definition of shocks. Those papers use Hong Kong data to generate the historical shocks, which is not correct because Hong Kong actually never has Taylor rule. By applying the Hong Kong data,

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<sup>1</sup>We do not think the CEPA Agreement should have affected the business cycle behaviour of the HK economy, even if it raised the trend growth rate. However, in future work it would be useful to investigate whether there is evidence of structural change as a result of CEPA, and estimate the model with sub-periods.

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those shocks in Taylor rule would only reflect the responses from international condition, which violates Taylor rule.

In addressing this regime question I will develop below a DSGE model that matches Hong Kong data and so both meets the Lucas critique and can be considered empirically reliable.



## Chapter 3

# A Small Open Economy DSGE for Currency Board: Case of Hong Kong with Full Foreign Exchange Intervention.

### 3.1 Introduction

This chapter is to build a small open economy Dynamic Stochastic General Equilibrium model, studying the monetary transmission in the currency board and fixed exchange rate. There are full literatures in the floating exchange rate and Taylor rule, but those empirical studies are based on the fact of independent monetary policy. Independent monetary policy is not applicable for most economies who have fixed exchange rate, especially those small economy with high level of capital mobility, because of Impossible Trilemma. The high level of capital mobility and fixed exchange rate ensure that the

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domestic economy must follow the monetary policy from another economy, most likely the anchor country of which the domestic currency is linked to. These facts indicate that those research on fixed exchange rate and currency board with a Taylor rule are mis-specified, because the story of the Taylor rule is the interest rate as a tool to reach the domestic target level like output and inflation. A currency board model with Taylor rule actually says that this economy has independent monetary policy, which violates the impossible trilemma.

A widely applied setting for the currency board is the fixed exchange rate, which is usually displayed in the uncovered interest rate parity. The story of the standard uncovered interest rate parity (UIP) is that the domestic interest rate is higher than the foreign interest rate, because it is going to compensate the expected depreciation. This standard UIP is well-specified for floating exchange rate and most literatures further make it to real UIP with real interest rates. However, UIP in fixed exchange rate becomes the equality of domestic and foreign nominal interest rate, while some would have a risk premium to close the model. Apart from the interest rate, another key feature is the full foreign exchange intervention. The monetary authority in currency board must have at least 100% foreign reserve back up to the money supply, the authority uses the foreign exchange reserve to decrease or increase the money supply to meet the money demand. Foreign exchange intervention is the core in the currency board arrangement to ensure the balance in money market, any unbalance in the money market will lead to the fluctuation of the exchange rate. Following Section 3.2 and 3.3 shows the structure of the model, section 3.4 is for the calibration and section 3.5 discusses the impulse responses from some shocks.



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## 3.2 Model Setups

The economy is populated by households and entrepreneurs. Households buy consumption goods both from home and foreign country, work for the entrepreneurs and consume housing. The entrepreneur produces consumption goods by using capital and labour. Housing producer produces housing to the households by using the housing investment with the housing depreciation. The financial intermediate banking sector is borrowed from Bernanke et.al (1999), and modified by the Le et.al (2016) who introduce the money into the financial accelerator model. The other settings are close to Smets and Wouters (2007) but no rigidity in the model, this is because Hong Kong has high degree of freedom in the capital flow and movements, from which I would prefer there are flexible price and wage.

The open economy settings with export and import are borrowed from Armington (1969) for substitution elasticity between home goods and foreign goods, as well as the Constant Elasticity substitution in Minford and Meenagh (2019). From the currency board system, there is no Taylor rule in this model but a full foreign exchange intervention to model the behaviour of the monetary system.

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### 3.2.1 Households

The representative patient household maximises the expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\gamma_t^c \log C_t^c + \gamma_t^h \log H_t - \frac{N_t^{1+\eta}}{1+\eta}] \quad (3.1)$$

Where households' utility is from current consumption  $C_t^c$ , housing  $H_t$ <sup>1</sup> and disutility from working  $N_t$ . Here are inverse elasticity of labour  $\eta$ , consumption shock  $\gamma_t^c$ , housing demand shock  $\gamma_t^h$ . These two shocks follow AR(1) process with i.i.d normal distribution.

This maximisation problem is subject to households' budget constraint:

$$\begin{aligned} P_t C_t^c + P_t I_t^k + P_t^h [H_t - (1 - \delta^h) H_{t-1}] + D_t + B_t + S_t B_t^f = W_t N_t + R_t^k K_{t-1} + (1 + R_{t-1}^d) D_{t-1} \\ + (1 + R_{t-1}) B_{t-1} + (1 + R_{t-1}^f) S_t \phi_{t-1} B_{t-1}^f + T_t \end{aligned} \quad (3.2)$$

and capital accumulation function with investment adjustment cost:

$$K_t = (1 - \delta^k) K_{t-1} + [1 - S(\frac{I_t^k}{I_{t-1}^k})] I_t^k \quad (3.3)$$

$\kappa^k$  is a parameter measures the adjusting investment cost where the cost is  $S(\frac{I_t^k}{I_{t-1}^k}) = \frac{\kappa^k}{2} (\frac{I_t^k}{I_{t-1}^k} - 1)^2$ , while  $S(1) = S'_k(1) = 0, S''_k(1) = \kappa^k$ .

For every period, households buy consumption goods, make investment decisions and purchase new housing with a relative housing price  $q_t^h = \frac{P_t^h}{P_t}$ , make deposit  $D_t$  and purchase domestic and foreign bonds. At the same time, households receive wage  $w_t$

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<sup>1</sup>Here we have housing stock in utility, because housing service is a constant portion of housing stock, and home ownership in Hong Kong is high. More, following Iacoviello (2005), we assume holding of housing stock accumulates overtime.

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from working, return from physical capital rent, return from deposit, domestic bonds and foreign bonds with their rates  $R_{t-1}^d$ ,  $R_{t-1}$ ,  $R_{t-1}^f$  respectively, where money is not interest-bearing.  $T_t$  is the lump-sum transfer. To ensure there is a well-defined steady state, this model follows Schmitt-Grohe and Uribe (2003), as well as Adolfson et.al (2007) that there is a risk premium which depends on the ratio of net foreign assets position.  $S$  is the nominal exchange rate and to be set fixed for a currency board.

$$\phi_t = \exp[-\phi_\alpha(Z_t - \bar{Z})] \quad (3.4)$$

where  $\phi$  is the elasticity of country risk premium,  $Z_t$  is total foreign assets position including the foreign bonds held in the public and those foreign reserve held in the monetary authority, where  $Z_t = B_t^f + F_t$ .

By choosing  $C_t^c, H_t, I_t^k, K_t, N_t, D_t, B_t, B_t^f$ , FOCs of households are as following:

$$C_t^c : \lambda_t^c = \frac{\gamma_t^c}{P_t C_t^c} \quad (3.5)$$

$$I_t^k : q_t^k [1 - S(\frac{I_t^k}{I_{t-1}^k}) - S'(\frac{I_t^k}{I_{t-1}^k}) \frac{I_t^k}{I_{t-1}^k}] + \beta_c E_t [\frac{\lambda_{t+1}^c}{\lambda_t^c} q_{t+1}^k S'(\frac{I_{t+1}^k}{I_t^k}) (\frac{I_{t+1}^k}{I_t^k})^2] = 1 \quad (3.6)$$

$$K_t : q_t^k = \beta_c E_t \frac{\lambda_{t+1}^c}{\lambda_t^c} [(1 - \delta^k) q_{t+1}^k + R_{t+1}^k] \quad (3.7)$$

$$H_t : \frac{\gamma_t^h}{H_t} = \lambda_t^c P_t^h - \beta_c E_t \lambda_{t+1}^c P_{t+1}^h (1 - \delta^h) \quad (3.8)$$

$$N_t : N_t^\eta = \lambda_t^c W_t \quad (3.9)$$

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$$D_t : \lambda_t^c = \beta_c E_t \lambda_{t+1}^c (1 + R_t^d) \quad (3.10)$$

$$B_t : \lambda_t^c = \beta_c E_t \lambda_{t+1}^c (1 + R_t) \quad (3.11)$$

$$B_t^f : \lambda_t^c = \beta_c E_t \lambda_{t+1}^c (1 + R_t^f) \phi_t \frac{S_{t+1}}{S_t} \quad (3.12)$$

By taking the no arbitrage condition between domestic bonds and deposit  $R_t^d = R_t$ , the Euler equation for consumption can be given by combining (5) and (11):

$$\frac{\gamma_t^c}{C_t^c} = \beta_c E_t \frac{\gamma_{t+1}^c}{C_{t+1}^c} \frac{(1 + R_t)}{\pi_{t+1}} \quad (3.13)$$

The optimal condition for housing is from (5) and (8):

$$\frac{\gamma_t^h}{H_t} = \frac{\gamma_t^c}{C_t^c} q_t^h - \beta_c E_t \frac{\gamma_{t+1}^c}{C_{t+1}^c} q_{t+1}^h (1 - \delta^h) \quad (3.14)$$

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Given (5) and (9), the intratemporal condition yields. This condition gives that marginal substitution between consumption and leisure is equal to the real wage.

$$N_t^\eta C_t^c = \frac{W_t}{P_t} \quad (3.15)$$

The international no arbitrage condition can be taken from (11) and (12):

$$E_t\left(\frac{1 + R_t}{\pi_{t+1}}\right) = E_t\left(\frac{(1 + R_t^f)\phi_t}{\pi_{t+1}}\right)\frac{S_{t+1}}{S_t} \quad (3.16)$$

$\phi_t$  is the country risk premium discussed in equation (4) which depends on the net foreign assets position and a risk premium shock. On one hand, it is to explain the fact the lenders would require higher return with those countries in higher debt position. On the other hand, it is to avoid misspecification and singularity problem in closing the model.

The UIP in log-linearised:

$$\hat{r}_t = \hat{r}_t^f + \Delta S_{t+1} - \phi \hat{z}_t$$

As the Hong Kong has fixed exchange rate,  $\Delta S_{t+1} = 0$ , the UIP is:

$$\hat{r}_t = \hat{r}_t^f - \phi \hat{z}_t$$

### 3.2.2 Entrepreneurs

Entrepreneurs behave as the final goods producer who hire labour and rent capital from households, apply the funds from bank and net worth from themselves to acquire capital. Entrepreneurs are risk neutral and have a constant survival rate to the next period, so that entrepreneurs will always need external funds to finance its cost of capital requirement. The set up here has one special state that there exists a perfect competition

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market in the domestic goods market, which is for the fully flexible economic environment in Hong Kong. This is because Hong Kong is a really small economy that there is no firm is able to set the price. The rest settings of this sector and the external finance premium follow the BGG framework extended by Le et.al (2016) and Gilchrist et.al (2009).

Entrepreneurs maximise the profit from producing goods with the profit function by choosing how much labour to hire and how much capital to operate with rental rate:

$$P_t^d Y_t - W_t N_t - R_t^k K_{t-1} \quad (3.17)$$

Where  $P_t$  is the general price level,  $N_t$  is labour and  $K_{t-1}$  is capital. The corresponding nominal wage and rental rate are  $W_t$  and  $R_t^k$ .

Subject to the following production technology:

$$Y_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \quad (3.18)$$

Here  $A_t$  is the technology process follows ARIMA(1,1,0) process, the log-linearised equation:

$$\hat{a}_t - a_{t-1} = \rho_a (a_{t-1} - a_{t-2}) + \varepsilon_t^a \quad (3.19)$$

First order condition of entrepreneur sector are: Marginal production of labour and labour demand:

$$\frac{W_t}{P_t^d} = (1 - \alpha) A_t K_{t-1}^\alpha N_t^{-\alpha} \quad (3.20)$$

Marginal production of capital and capital demand:

$$\frac{R_t^k}{P_t^d} = \alpha A_t K_{t-1}^{\alpha-1} N_t^{1-\alpha} \quad (3.21)$$

---

Additionally, entrepreneurs need external funds to finance its cost of renting capital. The external finance premium framework is fully borrowed from Gilchrist et.al (2009) and Le et. al (2013), with an extension to include the money in the external finance premium in Le et.al (2016). Every period, it needs to finance its cost in capital with external funds to cover part of the capital price, the Tobin's  $q$ ,  $q_t^k$ . Entrepreneur take the net worth  $NW_t$ , together with the external funds, with a loan contract from the financial intermediate to cover the cost of capital, the total amount of funds is  $q_t^k K_t - NW_t$ . The loan contract with financial intermediate faces an idiosyncratic shock on the expected return on capital. When the shock hit a thresholds value, the firm would repay the loan while when the value below the thresholds, firm defaults. The optimal loan contract takes a form to ensure the expected return on bank lending equals to the bank's cost of lending. Here the optimal choice for this loan contract would give the condition for the external finance premium and credit rate just like those in Bernanke et. al (1999), Gilchrist (2009) and Le et.al (2016) that:

Log linearised external finance premium:

$$E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (q_t^k + k_t - n w_t) \quad (3.22)$$

where the left hand indicates the return of capital equals the real opportunity cost of risk-free deposit with a premium on it,  $c y_t$  is borrowing rate or the credit rate; while the right hand includes the leverage ratio and positive  $\chi$  measures the elasticity of premium to the leverage ratio,  $n w_t$  is entrepreneur net worth given by a fixed survival rate firms' net worth from past plus the total return on capital, minus the expected return or cost on the external financing:

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Log-linearised net worth evolution:

$$nw_t = \nu nw_{t-1} + \frac{K}{NW}(cy_t - E_{t-1}cy_t) + E_{t-1}cy_t \quad (3.23)$$

where  $\nu$  is the survival rate which is assumed to be fixed and  $\frac{K}{NW}$  is the steady state capital to net worth ratio. As those who cannot survive would consume their net worth, the entrepreneur consumption in each period would equals to  $(1 - \nu)$  of the total net worth, which follows linearised equation:

$$c_t^e = nw_t \quad (3.24)$$

Following Le et.al (2016), here introduces the money as collateral in the loan contract. Firms hold some cash on the balance sheet, which can be recovered at no value loss and no verification cost. Same like in the paper, here it assumes only firm holds money where households only deposit it to the bank <sup>2</sup>. The difference is that in Le et.al (2016), it focuses on the quantitative easing effect and thus the model contains tool of money and treasury bills, so the open market operation. However, in this thesis, there is no open market operation in the domestic bonds but the foreign exchange intervention through the foreign reserves. Similarly, the modified credit premium equation in this thesis is:

$$E_tcy_{t+1} - (r_t - E_t\pi_{t+1}) = \chi(q_t^k + k_t - nw_t) - \mu m_t^d \quad (3.25)$$

The money demand is from the firm's balance sheet that firm holds money as collateral to its borrowing to finance the cost of capital, in the form as money to capital demand

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<sup>2</sup>Le et.al (2016) show the approval in the appendix 1 that with bankruptcy and bank contract decision, the rise in the money would decrease the required return on capital and the credit premium as well



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ratio together with the firm's net worth:

$$m_t^d = (1 + \xi)k_t - \xi nw_t \quad (3.26)$$

where  $\xi$  is the net worth to money ratio in steady state.

### 3.2.3 Housing Producer

Housing producer is to maximise its profit by choosing the level of  $I_t^h$ , following the Smets and Wouters (2007), Christiano et.al (2005) the set up for the capital producer, the housing producer behaves similar and the maximising problem is:

$$\max E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^e [q_t^h (H_t - (1 - \delta^h)H_{t-1}) - I_t^h] \quad (3.27)$$

subject to the law of motion in housing:

$$H_t = (1 - \delta^h)H_{t-1} + [1 - \frac{\kappa^h}{2}(\frac{I_t^h}{I_{t-1}^h} - 1)^2]I_t^h \quad (3.28)$$

This dynamic profit maximisation problem can be solved with the real price of housing

$$q_t^h = \frac{P_t^h}{P_t}:$$

$$q_t^h [1 - S(\frac{I_t^h}{I_{t-1}^h}) - S'(\frac{I_t^h}{I_{t-1}^h})\frac{I_t^h}{I_{t-1}^h}] + \beta_e E_t [\frac{\lambda_{t+1}^e}{\lambda_t^e} S'(\frac{I_t^h}{I_{t-1}^h})(\frac{I_t^h}{I_{t-1}^h})^2] = 1 \quad (3.29)$$

$\kappa^h$  is a parameter measures the adjusting investment cost where the cost is  $S(\frac{I_t^h}{I_{t-1}^h}) = \frac{\kappa^h}{2}(\frac{I_t^h}{I_{t-1}^h} - 1)^2$ , while  $S(1) = S'_h(1) = 0, S''_h(1) = \kappa^h$ .

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### 3.2.4 Imports and Exports

With the spirit of small open economy in Armington (1969), Gali and Monacelli (2005), and Minford and Meenagh (2019), the total consumption index  $C_t$  is a CES function of domestic consumption goods  $C_t^d$  and foreign imported consumption goods  $IM_t$

$$C_t = [\omega^{\frac{1}{\theta}} (C_t^d)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (IM_t)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \quad (3.30)$$

and the bundle of the total consumption should satisfy the expenditure constraint of domestic consumption and imported consumption:

$$C_t = \frac{P_t^d}{P_t} C_t^d + Q_t IM_t \quad (3.31)$$

Where  $\omega$  is the home bias preference towards domestic goods and  $\theta$  measures the elasticity of substitution between domestic and foreign goods, while  $Q_t$  denotes the real exchange rate  $\frac{SP_t^f}{P_t}$ .  $S$  is the nominal exchange rate it is set to be fixed by currency board,  $P_t^d$  is the domestic goods price,  $P_t^f$  is the foreign price. The optimal allocation of the domestic demand for domestic goods and imported goods can be found by the following composite utility index maximisation:

$$\mathcal{L} = [\omega^{\frac{1}{\theta}} (C_t^d)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (IM_t)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} + \Lambda_t (C_t - \frac{P_t^d}{P_t} C_t^d + Q_t IM_t) \quad (3.32)$$

by choosing  $C_t^d, IM_t$ , optimal conditions are:

$$C_t^d = \omega \left( \frac{P_t^d}{P_t} \right)^{-\theta} C_t \quad (3.33)$$

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And domestic demand for foreign goods, which is hence the import demand:

$$IM_t = (1 - \omega) \left( \frac{SP_t^f}{P_t} \right)^{-\theta} C_t \quad (3.34)$$

Consumer price index (CPI):

$$P_t = [\omega (P_t^d)^{1-\theta} + (1 - \omega) (SP_t^f)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (3.35)$$

Symmetrically, the export demand, or the foreign demand for domestic goods can be given as:

$$EX_t = (1 - \omega^f) \left( \frac{P_t}{SP_t^f} \right)^{\theta^f} C_t^f \quad (3.36)$$

$\omega^f, \theta^f$  are home bias preference and elasticity of substitution in foreign economy. By assuming the small open economy, this model treats foreign variables  $\{C_t^f, R_t^f, \pi_t^f\}$  follows AR(1) process, and i.i.d innovation  $\varepsilon_{cf,t}, \varepsilon_{rf,t}, \varepsilon_{\pi^f,t}$  respectively with the definition of foreign policy shock, export demand shock and foreign price shock.

### 3.2.5 Monetary Operation and Currency Board

As the banking sector and currency board sector are the main to explain the monetary system for currency board in Hong Kong, the Figure 1 is to explain the full mechanism.

Currency Board	
Assets	Liabilities
Reserve	Reserve Certificates
Note Issuing Banks	
Assets	Liabilities
Reserve Certificates	Notes and Coins (M0)
Domestic Banks	
Assets	Liabilities
Loans	Deposits
Non-Bank Public and Government	
Assets	Liabilities
Net Foreign Assets (NFA)	$\sum S$
Deposit	Loans
$K = \sum I$	
M0	
Rest of the World	
Assets	Liabilities
$\sum NX$	NFA
	Reserve

Figure 3.1: The Balance Sheet of Hong Kong Economy and Currency Board

There are two main channels of overseas monetary transmission. First, as net foreign

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assets fall with current account deficits, the risk-premium on the HK dollar rises pushing up interest rates. Second, at this interest rate HK entrepreneurs can acquire the money they demand, by borrowing from abroad: equivalently excess money demand creates a slight rise in interest rates, causing money to flow in via private capital flows. This raises the reserves within total net foreign assets, increasing private foreign liabilities; money supply rises in line with reserves, meeting the money demand. Hence money demand in HK is automatically supplied via the balance of payments, in this currency board system just like in any fixed exchange rate regime.

We can summarise this second channel in the following equation:

$$S_t F_t = M_t^s = M_t^d \quad (3.37)$$

where  $S_t = \bar{S}$  is fixed.

The natural or automatic mechanism in Hong Kong monetary system is not by adjusting the interest rate, but by buying foreign reserve and printing money. It means that there is no Taylor style interest rate and targeting rule in this model. Instead, the monetary authority fully applies the foreign exchange intervention. Any excess demand for domestic money would cause an increase in the money supply via the foreign exchange reserve by the foreign exchange market.

### 3.2.6 Government

In Hong Kong, the government is required to keep fiscal surplus. However, the past data in Figure 2 shows that the government is possible to have government deficit, especially in the financial crisis. Moreover, there is no monetary policy independence in the fixed exchange rate and currency board, which means that government fiscal policy is the only

tool that the authority may have impact on the economy. Therefore, this model keeps the ability that government can borrow to finance its spending.

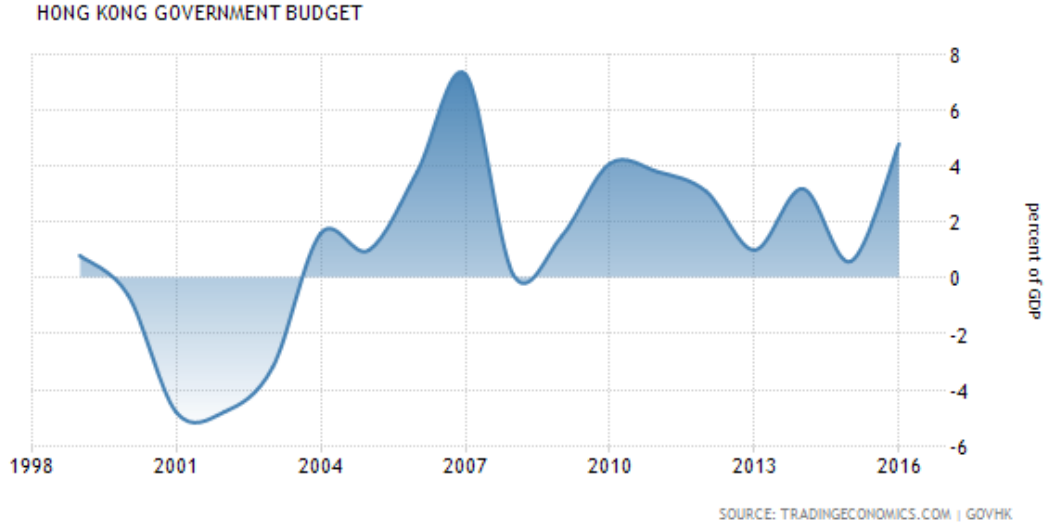


Figure 3.2: Historical Fiscal Budget Balance in Hong Kong.

Government budget:

$$G_t + (1 + r_{t-1})B_{t-1} = B_t + T_t \quad (3.38)$$

$G_t$  follows AR(1) process and allows government spending shock  $\varepsilon_{g,t}$ .

### 3.2.7 Balance of Payment

Balance of payment with foreign exchange reserve

$$Z_t = (1 + r_{t-1}^f)Z_{t-1} + \frac{EX}{Q_t} - IM_t \quad (3.39)$$

$Q_t$  for real exchange rate.

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### 3.2.8 Market Clearing Conditions and Identities

Total foreign assets:

$$Z_t = B_t^f + F_t \quad (3.40)$$

Goods market:

$$Y_t = C_t^c + C_t^e + I_t^k + I_t^h + G_t + EX_t - IM_t \quad (3.41)$$

Gross inflation:  $\pi_t = \frac{P_t}{P_{t-1}}$ .

Relative price of house:  $q_t^h = \frac{P_t^h}{P_t}$ ;

### 3.3 Log-linearisation

The full non-linear stochastic model can be transferred to the linear form using log-linearisation method around steady state, taking the note that lower capital means  $\hat{x}_t \approx \frac{X_t - \bar{X}}{\bar{X}} \approx \log X_t - \log \bar{X}$ .  $\bar{X}$  is the steady state value of  $X_t$ .

Table 3.1: List of Variables in log-linearisation

Variable	Definition	Variable	Definition
$c^c$	Households Consumption	$\pi$	Domestic Inflation
$\gamma^c$	Consumption Preference Shock	$i^k$	Capital Investment
$q^k$	Tobin's q	$r^k$	Real Capital Rent
$h$	Housing Stock	$q^h$	Housing Price
$\gamma^h$	Housing Demand Shock	$n$	Labour
$w$	Real Wage	$nw$	Net Worth
$Y$	Output	$A$	Technology
$k$	Capital	$z$	Total Net Foreign Assets
$cy$	Borrowing Rate	$c^e$	Entrepreneur Consumption
$m^s$	Money Supply	$m^d$	Money Demand
$ex$	Export	$im$	Import
$q$	Real Exchange Rate	$\pi^f$	Foreign Inflation
$\pi^d$	Home Inflation	$p^d$	Domestic Price
$g$	Government Spending	$b^f$	Foreign Bonds (Assets)
$f$	Foreign Exchange Reserve	$c^f$	Foreign Consumption
$i^h$	Housing Investment		



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## i. Households

Euler Equation:

$$\hat{c}_t^c = E_t c_{t+1}^c - (\hat{r}_t - E_t \pi_{t+1}) + \hat{\gamma}_t^c$$

Investment:

$$\hat{i}_t^k = \frac{1}{1 + \beta_c} \hat{i}_{t-1}^k + \frac{\beta_c}{1 + \beta_c} E_t \hat{i}_{t+1}^k + \frac{1}{\kappa^k (1 + \beta_c)} \hat{q}_t^k$$

Capital tobin's q:

$$\hat{q}_t^k = \beta_c (1 - \delta^k) E_t \hat{q}_{t+1}^k + [1 - \beta_c (1 - \delta^k)] E_t r_{t+1}^k - (\hat{r}_t - E_t \pi_{t+1})$$

House Demand:

$$[1 - \beta_c (1 - \delta^h)] (\hat{\gamma}_t^h - \hat{h}_t) = \hat{q}_t^h - \hat{c}_t^c - \beta_c (1 - \delta^h) E_t (\hat{q}_{t+1}^h - \hat{c}_{t+1}^c) + \beta_c (1 - \delta^h) \hat{\gamma}_t^c$$

Labour Supply and real wage

$$\eta \hat{n}_t + \hat{c}_t^c + \hat{\gamma}_t^c = \hat{w}_t - \hat{p}_t$$

UIP with Risk Premium:

$$\hat{r}_t = \hat{r}_t^f - \phi_a \hat{z}_t$$

Capital Accumulation:

$$\hat{k}_t = (1 - \delta^k) \hat{k}_{t-1} + \delta^k \hat{i}_t^k$$

## ii. Entrepreneurs

Production Function:

$$\hat{Y}_t = \hat{A}_t + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t$$

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Labour Demand:

$$\hat{A}_t - \alpha \hat{n}_t + \alpha k_{t-1}^{\hat{}} = \hat{w}_t - \hat{p}_t^d$$

Capital Demand and real capital rental rate:

$$\hat{A}_t + (1 - \alpha) \hat{n}_t + (\alpha - 1) k_{t-1}^{\hat{}} = \hat{r}_t^k - \hat{p}_t^d$$

Credit premium:

$$E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (q_t^k + \hat{k}_t - n \hat{w}_t) - \mu \hat{m}_t^d$$

Net worth evolution

$$n \hat{w}_t = \frac{\bar{K}}{\bar{NW}} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t + \nu n \hat{w}_{t-1}$$

Money demand from entrepreneur:

$$\hat{m}_t^d = (1 + \xi) \hat{k}_t - \xi n \hat{w}_t$$

Entrepreneur consumption:

$$\hat{c}_t^e = n \hat{w}_t$$

### iii. Housing Producer

Housing Price, Housing Supply:

$$\hat{i}_t^h = \frac{1}{\kappa^h (1 + \beta_c)} \hat{q}_t^h + \frac{\beta_c}{1 + \beta_c} E_t \hat{i}_{t+1}^h + \frac{1}{1 + \beta_c} \hat{i}_{t-1}^h$$

Housing Accumulation:

$$\hat{h}_t = (1 - \delta^h) \hat{h}_{t-1} + \delta^h \hat{i}_t^h$$

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## v. Monetary Operation

Foreign Reserve intervention and Currency board balance sheet:

$$\hat{f}_t = \hat{m}_t^s$$

## vii. Marketing Clearing

Goods Market:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{c}_t + \frac{\bar{I}^k}{\bar{Y}}\hat{i}_t^k + \frac{\bar{I}^h}{\bar{Y}}\hat{i}_t^h + \frac{\bar{G}}{\bar{Y}}\hat{g}_t + \frac{\bar{E}X}{\bar{Y}}e\hat{x}_t - \frac{\bar{I}\bar{M}}{\bar{Y}}i\hat{m}_t$$

Money Market:

$$\hat{m}_t^d = \hat{m}_t^s$$

## viii. Trade

Balance of payment with foreign reserve:

$$\hat{z}_t = \bar{r}r_{t-1}^f + (1 + \bar{r})z_{t-1} + \frac{\bar{E}X}{\bar{Z}}(e\hat{x}_t - \hat{q}_t) - \frac{\bar{I}\bar{M}}{\bar{Z}}(i\hat{m}_t)$$

$$i\hat{m}_t = -\theta\hat{q}_t + \hat{c}_t \text{ (Import Demand)}$$

$$e\hat{x}_t = \theta^f\hat{q}_t + \hat{c}_t^f \text{ (Export Demand)}$$

Real exchange rate:

$$\hat{q}_t = \hat{p}_t^f - \hat{p}_t^d$$

## ix. Some Identity

CPI and CPI inflation:

$$\hat{p}_t = \omega\hat{p}_t^d + (1 - \omega)\hat{p}_t^f$$

---


$$\pi_t = p_t - p_{t-1}$$

$$\pi_t^f = p_t^f - p_{t-1}^f$$

$$z_t = \frac{\bar{B}^f}{\bar{Z}} \hat{b}_t^f + \frac{\bar{F}}{\bar{Z}} \hat{f}_t$$

## x. Structure Shocks Process

Preference shock to consumption:

$$\gamma_t^c = \rho_c \gamma_{t-1}^c + \varepsilon_{c,t}$$

House demand shock:

$$\gamma_t^h = \rho_h \gamma_{t-1}^h + \varepsilon_{h,t}$$

Technology shock:

$$A_t - A_{t-1} = \rho_a (A_{t-1} - A_{t-2}) + \varepsilon_{A,t}$$

Hong Kong is a small open economy and can be treated as no effect to the rest of the world, world shock  $\varepsilon_{r^f,t}, \varepsilon_{\pi^f,t}$

$$r_t^f = \rho_{r^f} r_{t-1}^f + \varepsilon_{r^f,t}$$

$$\pi_t^f = \rho_{\pi^f} \pi_{t-1}^f + \varepsilon_{\pi^f,t}$$

Foreign consumption innovation and export demand shock  $\varepsilon_{c^f,t}$

$$c_t^f = \rho_{c^f} c_{t-1}^f + \varepsilon_{c^f,t}$$

---

Government spending shock  $\varepsilon_{g,t}$ :

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

### 3.4 Calibration

In order to simulate the model to see the behaviour of the economy, all parameters are calibrated according to the literatures like Smets and Wouters (2007), Bernanke et.al (1999), Funke and Paetz (2011) and Le et.al (2014) or the data. The parameter value is in summary of Table 1 and here give some descriptions. The inverse elasticity of labour supply  $\eta$  is set to be 3. The households discount factor  $\beta_c$  is 0.9929 , which gives the with the corresponding quarterly steady state interest 0.72%.

Output elasticity of capital  $\alpha$  is standard to be 0.3 Capital depreciation rate  $\delta^k$  for 0.025, while the housing depreciation  $\delta^h$  is 0.01. In the bundle of consumption goods, the home bias  $\omega$  for 0.4 and elasticity between domestic goods and imported goods  $\theta$  for 1. Symmetrically, the foreign home bias  $\omega^f$  and foreign elasticity between foreign domestic goods and exported goods from home country  $\theta^f$  are 0.4 and 1 respectively. Capital adjustment cost parameters in physical capital and housing  $\kappa^k$  and  $\kappa^h$  are 6.

For goods market, the consumption to output ratio  $\frac{C}{Y}$  is 0.6367, house investment to output ratio  $\frac{I^h}{Y}$  is 0.1148, the capital investment to output ratio  $\frac{I^k}{Y}$  is 0.1471, the government spending to output ratio  $\frac{G}{Y}$  is 0.1051, the export to output ratio  $\frac{EX}{Y}$  is 1.6803 and import to output ratio  $\frac{IM}{Y}$  is 1.6571. It is worth to mention here that those export and import to output ratio are the special case in Hong Kong. This is because Hong Kong acts like an international port and domestic economy does not produce the re-export goods. The domestic export accounts for 5% of total export as indicated from the data in trade section of Hong Kong Census Department.

Table 3.2: Calibration Parameters

Parameter	Definition	Value
$\beta_c$	Households Discount Factor	0.9929
$\eta$	inverse Elasticity of Labour Supply	3
$\alpha$	Capital Share	0.3
$\delta^k$	Capital Depreciation	0.025
$\delta^h$	Housing Depreciation	0.01
$\omega$	Domestic Home Bias	0.4
$\theta$	Elasticity between Domestic and Imported goods in Home	1
$\omega^f$	Foreign Home Bias	0.4
$\theta^f$	Elasticity between Domestic and Imported goods in Foreign	1
$\kappa^k$	Capital Investment Adjustment Cost	6
$\kappa^h$	Housing Investment Adjustment Cost	6
$\chi$	Feedback from Leverage to Finance Premium	0.05
$\mu$	Feedback from Money to Finance Premium	0.7
$\xi$	Response of Net Worth to Money ratio	0.2
$\phi$	Risk Elasticity to Total Foreign Assets	0.04

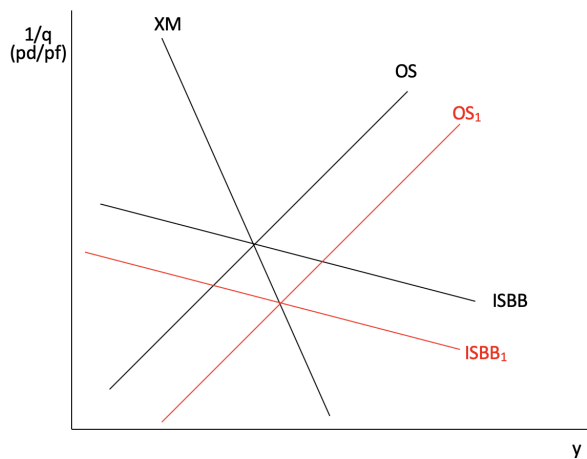
Table 3.3: Steady State Ratio

Ratio	Definition	Value
$\frac{\bar{C}^c}{\bar{Y}}$	Consumption to Output ratio	0.6367
$\frac{\bar{C}^e}{\bar{Y}}$	Entrep. Consumption to Output ratio	0.008
$\frac{\bar{I}^h}{\bar{Y}}$	Housing investment to Output ratio	0.1148
$\frac{\bar{I}^k}{\bar{Y}}$	Capital investment to Output ratio	0.1471
$\frac{\bar{G}}{\bar{Y}}$	Government Spending to Output ratio	0.1051
$\frac{\bar{EX}}{\bar{Y}}$	Total export to Output ratio	1.6803
$\frac{\bar{IM}}{\bar{Y}}$	Total import to Output ratio	1.6571
$\frac{\bar{F}}{\bar{Z}}$	Foreign Exchange Reserve to Total Foreign Assets ratio	0.1
$\frac{\bar{B}^f}{\bar{Z}}$	Foreign Bonds to Total Foreign Assets ratio	0.9
$\frac{\bar{K}}{\bar{NW}}$	Capital to Net Worth ratio	1.5
$\nu$	Survival Rate	0.9
$\bar{r}$	Steady state interest rate	0.72%

### 3.5 Impulse Response

This section gives a simple evaluation of the impulse responses from some structure shocks with the calibrated model, all shocks are stationary except for the technology or productivity shock, which is set to be non-stationary. Those impulse responses are from figures at the end of this section, positively with the standard deviation to be 0.01. As they are simulated from the calibrated model, the evaluation here is for examination of the mechanism of the model set ups. It could help to see if the working path of those variables matches the proposed theory and research questions. To help with the understanding of the channels working with the economy and model, there is a diagram for better explains.

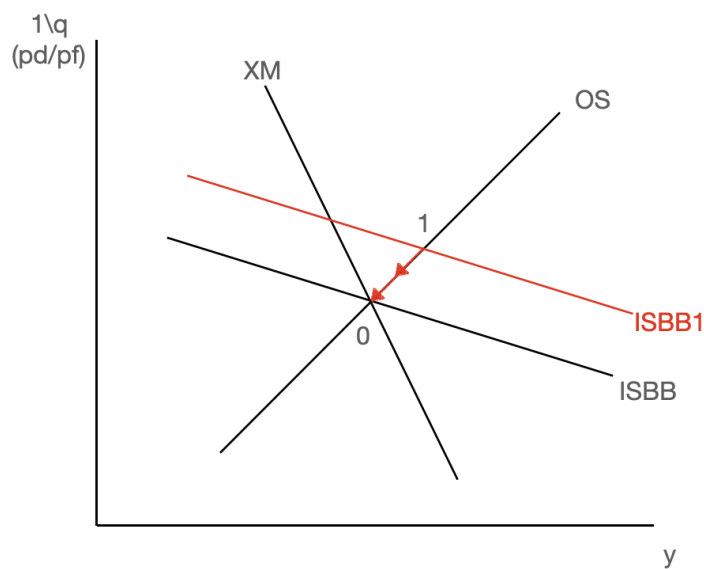
Figure 3.3 first displays the IRFs to non-stationary technology shocks. Output, household consumption, both capital investment and housing investment rise due to the permanent effect from the technology and output expansion. Due to the rising in the output, entrepreneur demands more capital as well as more money as required from the bank to be the collateral from the loan. The increase in the money demand pushes the authority to supply more money via the fully foreign exchange intervention, which gives the increase in the foreign reserve holding in the authority, following a decrease in the credit premium and borrowing rate. This positive productivity shock increases output, capital, money and foreign reserve. At the same time, credit premium, which can be seen as a less tight credit condition. In the diagram below, rising in the technology shifts the OS curve to the right, giving more output and lower domestic price. This temporary equilibrium gives a current account deficit and a decline in the net foreign assets, which further shifts the ISBB curve to the left, intersecting with XM and OS curve at the new equilibrium. In the meanwhile, the decline in the NFA makes interest rate keep rising via the risk premium. Finally, the technology shock raises the output, money and foreign reserve, while having decrease in inflation, domestic price and credit premium.



Technology Impact

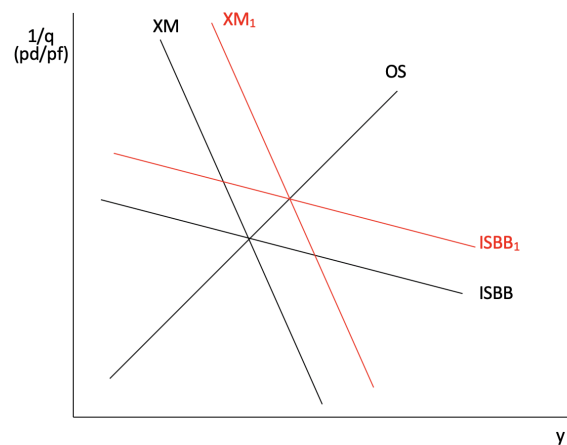


Figure 3.6 deals with the response from the fiscal shock. This temporary fiscal expansion shifts the ISBB to the right to ISBB1, which raises the output as well as the domestic price. OS curve measures the long run relationship and it is determined mainly by productivity, a temporary fiscal shock does not have effect on supply curve. As now OS and ISBB1 intersect to the right of the XM curve, a current account deficit occurs with a lower net foreign assets. The lower net foreign pushes ISBB1 along the OS curve to the intersection of XM and OS, where the equilibrium is. Overall, the fiscal expansion under currency board arrangement would have a rise in the output and domestic price, as well as interest rate, then reversed later.



Fiscal Expansion

The rest 2 are from the outside of the economy, the foreign demand shock which is treated as the export shock and foreign interest shock, as shown in Figure 3.7 and 3.8. A higher export demand would cause more output and thus more capital demand. Again, the firm would demand more money as collateral to borrow more from the bank. Rising in money demand results in the excess demand for money and the monetary authority needs to supply more money via the foreign exchange intervention, so that the foreign reserve is also increased in the authority. Simultaneously, the increase in the net foreign assets from higher export cuts the domestic interest rate down via the uncovered interest parity by the term of risk premium. Credit premium and borrowing rate decrease as there is more funds and credit available in the economy. In the diagram below, the higher export shifts the XM curve to the right. With a higher domestic price and current account surplus, the net foreign assets and output increase, which shifts the ISBB curve and OS curve to the right. The impact from foreign demand shock or export demand shock is that output, domestic price, foreign reserve and money increase while the interest rate, credit premium and borrowing rate decrease.



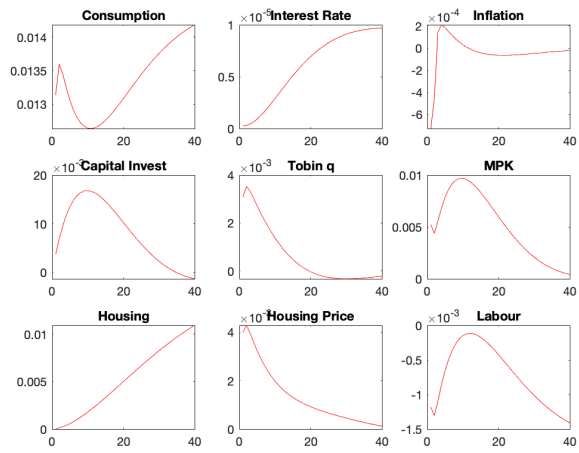
Export Demand

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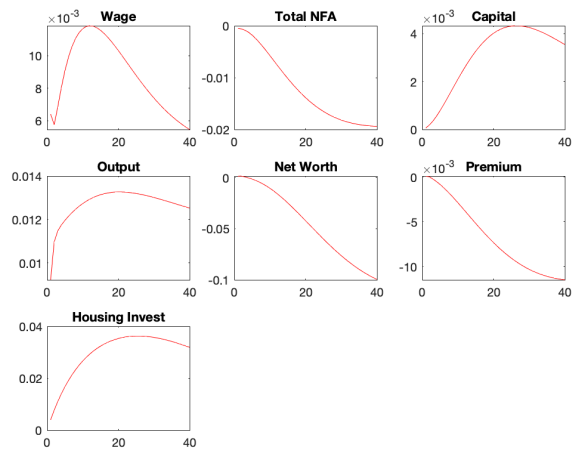
## 3.6 Conclusion

This small open economy model is trying to give a measure how the economy works with a currency board, which uses fully foreign exchange intervention to adjust money, especially the working to the domestic credit condition. At the beginning, this chapter displays the detail of the model in each sector. The model behaves with the financial accelerator which includes the money in the credit premium. Dynamic behaviour is presented by the impulse response function through varieties of shocks. The core channel in the currency board model is the money, money plays an important role when Hong Kong dollar is linked to the US dollar at a fixed rate so that balance in the domestic money market ensure the exchange rate to be fixed. More credit is available when there is a rising in the output, as the more money demand results in the more money supply via foreign exchange intervention. Overall, the credit and money condition is improved when the entire economy is better. The model behaves well to match the theory and ISBB-XM-OS mechanism, which is modified from IS-LM-BB. Next step is estimating and testing the model by Indirect Inference, to see if the calibrated and estimated model can pass the test and fit the Hong Kong data well.

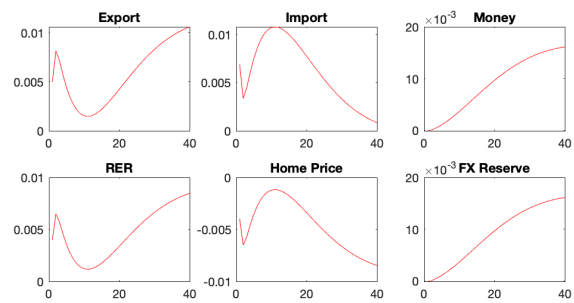
### 3.A IRFs Figures



(a)

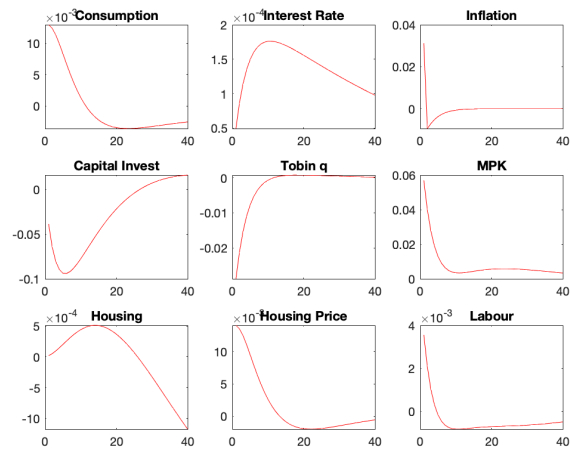


(b)

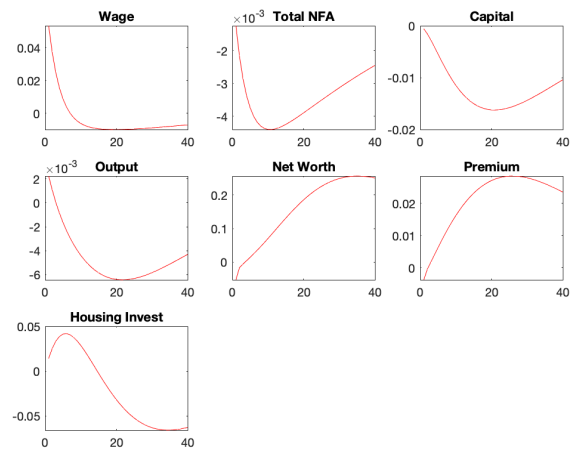


(c)

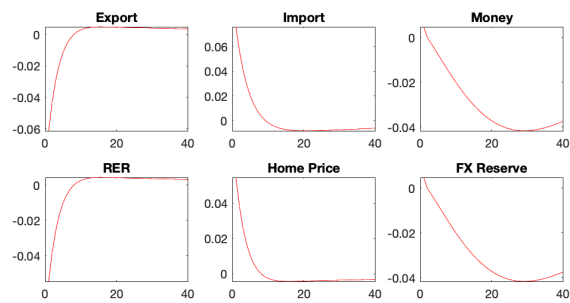
Figure 3.3: Impulse Response to Non-stationary Technology Shock



(a)

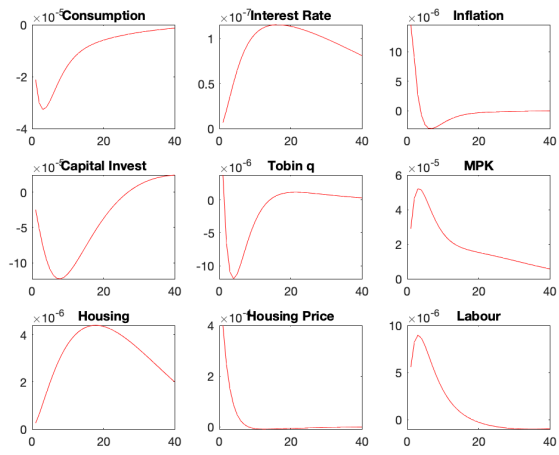


(b)

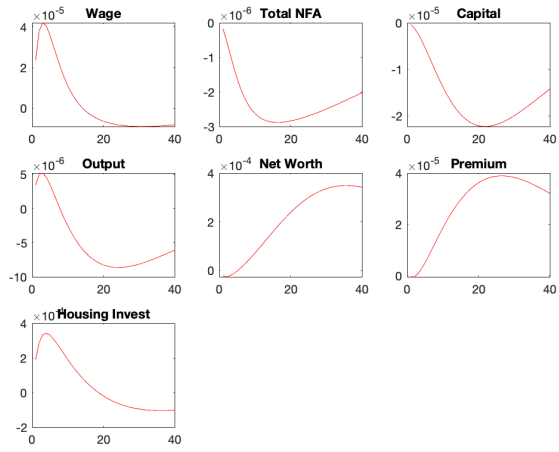


(c)

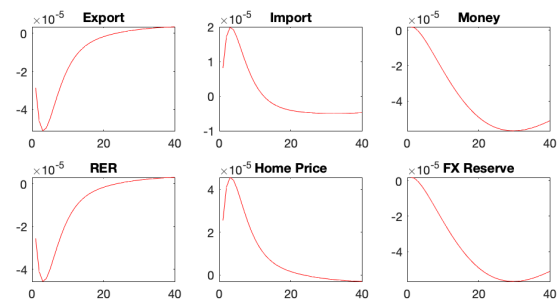
Figure 3.4: Impulse Response to Consumption Preference Shock



(a)

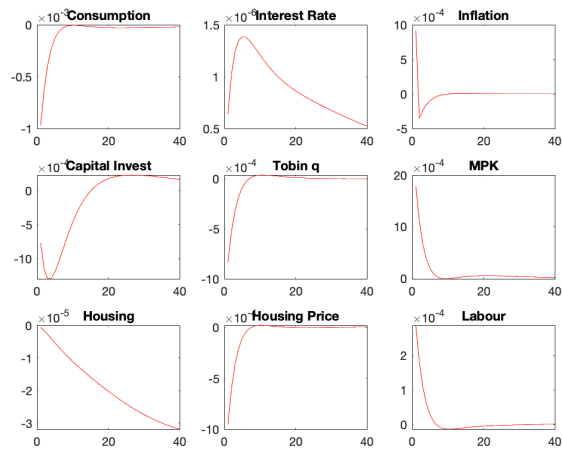


(b)

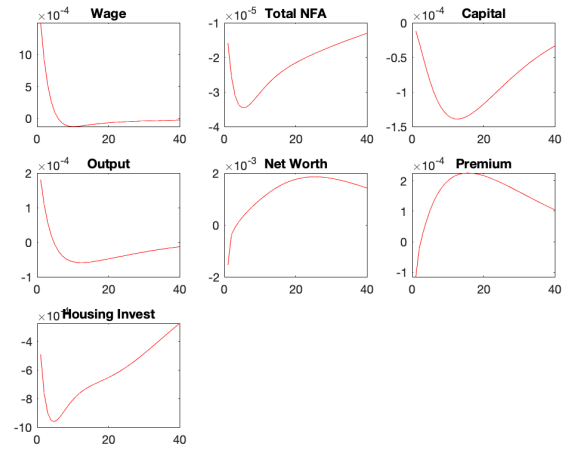


(c)

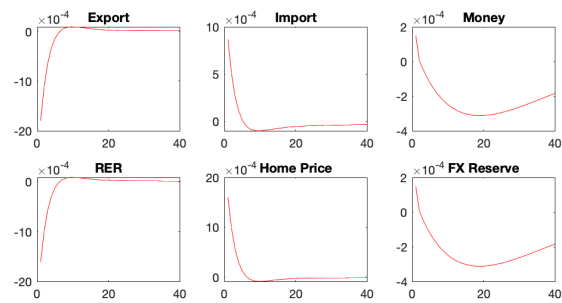
Figure 3.5: Impulse Response to House Demand Shock



(a)

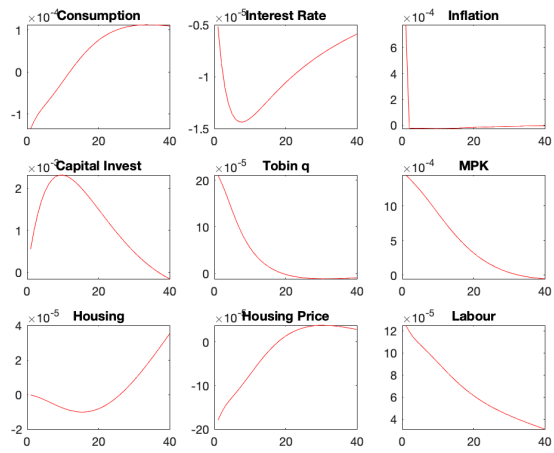


(b)

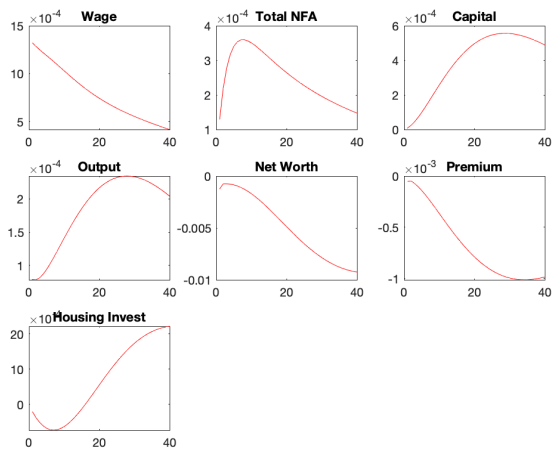


(c)

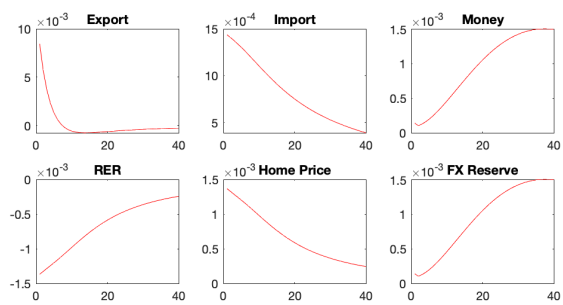
Figure 3.6: Impulse Response to Fiscal Shock



(a)



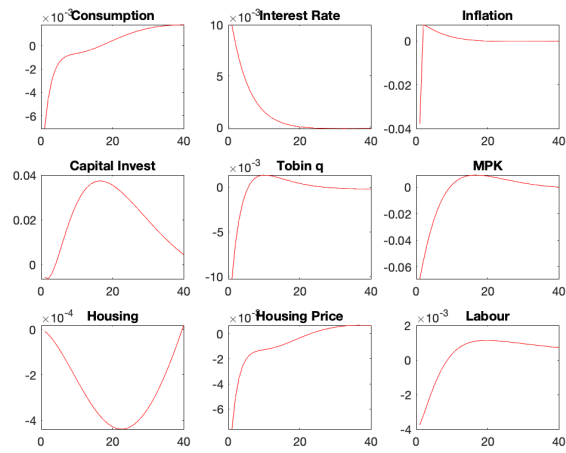
(b)



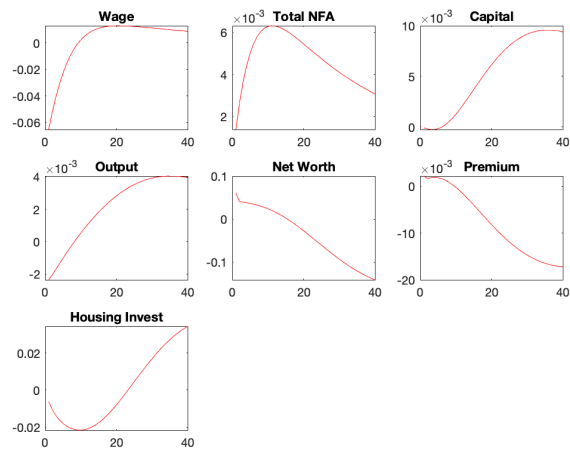
(c)

Figure 3.7: Impulse Response to Export Demand Shock

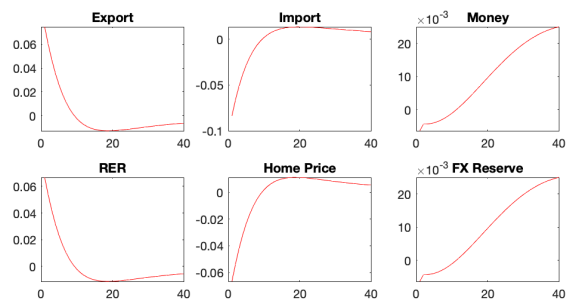




(a)



(b)



(c)

Figure 3.8: Impulse Response to Foreign Interest Shock



## Chapter 4

# Estimation and Testing by Indirect Inference.

### 4.1 Introduction.

The micro-foundation DSGE model is a big step in macroeconomics modelling, especially for the decision of policy makers. The basic DSGE was developed from the Real Business Cycle model when the Ramsey model was to be tested if it could explain the business cycle without adopting the imperfectly flexible prices. Calibration, applied in this attempt, is then to be a welcomed method in the DSGE model. Kydland and Prescott (1982) have indicated that traditional econometric method to evaluate the performance of the RBC model would easily make it being rejected, for the fact that it is highly stylised and non-linear. The reason why choosing calibration not estimation is pointed out by Sargent, interviewed by Evans and Honkapoja (2005) that too many good model would be rejected that the Maximum Likelihood takes all the information, not just the key features and parameters, being well specified. However, calibration is

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to be welcomed if the model is partially miss-specified or incompletely specified.

Apart from the calibration, the next issue is to determine how well the calibrated model is to explain the data or how close they are. The Bayesian estimation fills the gap that DSGE model is hardly to be estimated and tested. It could incorporate the prior information and some uncertainty to give more freedom to the estimation.

As described in Gourieroux et. al (1993), Canova (2005) and Le et. al (2010), the indirect inference is a simulated based method. It provides a comparison of the joint behaviour with the variables and parameters from the auxiliary model which is estimated on both simulated data and observed data. The choice of the auxiliary model should be able to look at the observed data and model simulated data. Le et. al (2016) point out that VAR model is quite often applied in the indirect inference, but the Impulse Response Function could also be fine. Generally, the auxiliary model includes moments, cross moments and key Impulse Responses.

This chapter would introduce the method of indirect inference and then the evaluation of the model by this method, both estimation and testing. Following the indirect inference evaluation, the rest will discuss the impulse response, stochastic variance decomposition and historical decomposition with the estimated model.

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## 4.2 Indirect Inference.

### 4.2.1 Background.

Recently, most widely used in the DSGE model estimation is the Bayesian method. The purpose of Bayesian method is to extend the sample on the observation of variables with the likelihood function. It relies on the prior information about the parameter, where calibration can be treated as an extreme case of Bayesian because the calibration parameters are taken as the correct information to measure the dynamic behaviour of the model, while the maximum likelihood is another extreme case that it is based on the data evidence only. Bayesian method is somehow between these two that it take the prior distribution and then the posterior distribution. One issue with Bayesian method is the universal problem in DSGE method, the prior information. Although Bayesian is the attempt try to give the solution caused by the calibration, it is still relatively rely on the prior information too much.

As in Hensen and Heckman (1996), Bayesian estimation is not much better than other method like maximum likelihood, when the priors are not fully informed. If we know all the true information, Bayesian is logical to apply in the estimation. However, most macroeconomics research models tell us that those priors are not fully known, or some of them are not really correct with the sample of the economy of interest. It is to say that we must test many theories applied in the sample economy to build up those informations. According to Meenagh et.al (2019), Indirect Inference is based on the point that testing an economic model is not about whether it is true or not. Instead, it should be tested that if it can match the data of interest. If the model is to be seen as true but rejected, it is assumed to be a misspecification. Besides the testing and estimation, the power of the test in Indirect Inference is a judge to justify the usefulness.

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Le et. al (2016) indicate that for the stationary data, Indirect Inference is more likely to reject the false model and the reject rate is higher when the model deviates from the true model. The test is much more powerful when applied to the non-stationary data mainly because the IIW uses model-restricted variance matrix that changes with changing falseness. The advantage of using Indirect Inference estimation is that it is much less biased than maximum likelihood for small samples and Bayesian estimation would be biased for the priors when they are incompletely specified. After all, we say the Indirect Inference would have much less bias than maximum likelihood, while the power is much higher than ML.

#### **4.2.2 Indirect Inference Testing and Estimation Process.**

The testing process would follow Le et.al (2012, 2016) that the structure model parameter could be from the calibrated model or other estimation method. Then the null is that model is true and if the DSGE model in research is true, the chosen moment of the auxiliary estimates from the simulation will not significantly differ from those derived from the observed data. The simulation is to bootstrap the DSGE model with shocks for many times and generate a large number of pseudo-samples, which represent the set of the estimated coefficient for the auxiliary model. The test would then compare the auxiliary coefficients estimated on the observed data with the distribution of auxiliary coefficient estimates from the sets of the simulated data. Then use the Wald statistics based on the difference between those two.

Together with Minford et. al (2009) who applied this method to calibrated model of UK with stationary data and Le et. al (2014) who test the model of crisis in China with non-stationary data, a brief process follows Le et.al (2016) can be set as:

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*Step 1. Calculate the errors of the model conditional on the observed data and calibrated parameters.*

Estimate the structural errors of the DSGE macroeconomic model, given the parameter and the observed data. The number of independent structural errors is taken to be less than or equal to the number of endogenous variables. The errors are not assumed to be normally distributed. When the equations contain no expectations the errors can simply be backed out of the equation and the data. When there are expectations which is quite often in the DSGE model, it is necessary to use the robust instrumental variables methods of McCallum (1976) and Wickens (1982), with the lagged endogenous data as instruments. Thus effectively the auxiliary model applied is VAR. An alternative method for expectations estimation is the exact method, here the model itself is to project the expectations and because these depend on the extracted residuals there is iteration between the two elements until convergence.

*Step 2. Bootstrapping the model with implied shock by many time to get a set of coefficients and simulated data.*

With those structure error get from the step 1, the simulated data can be bootstrapped. In the DSGE model, those structure shocks are assumed to be the autoregressive process. Most of those shock would be a stationary process like AR(1) with a high persistence, but in this thesis model, the technology shock is taken as a nonstationary process. To deal with this issue, the process of the technology is empirically set as first order difference stationary. To get N simulations, the randomly and independently drawing procedure is repeated for each sample by the projection method in Minford et.al (1983, 1986).

*Step 3. Get the Indirect Inference Wald Statistics*

To determine rejecting a model or not, we need estimated the auxiliary model, here the VAR, both by the actual data and simulated sample. By estimating the VAR, the estimates of actual data  $\alpha_T$  and estimates of simulation  $\alpha_S(\theta_S)$  can be obtained. Next is

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to calculate the asymptotic distribution of  $(\alpha_T - \overline{\alpha_S(\theta_S)})$ , where  $\overline{\alpha_S(\theta_S)}$  is the average of all repeated simulation samples, and the variance-covariance matrix  $\Omega$  which is estimated from bootstrapping  $\alpha_S(\theta_S)$ , the Wald Statistics is then calculated as:

$$WS = (\alpha_T - \overline{\alpha_S(\theta_S)})' \Omega (\theta_S - \overline{\theta_S})^{-1} (\alpha_T - \overline{\alpha_S(\theta_S)}) \quad (4.1)$$

In order to evaluate the fitness of model at a 95% confidence level, the Wald statistics from actual data is required to be less than the 95th percentile of the one from the simulated data. Wald statistics from simulated data follow  $\chi^2$  distribution with degree of freedom  $k - 1$  where  $k$  is the number of parameter. For convenience, the Wald can be transformed into t statistics as a transformed Mahalanobis distance by:

$$T = \left( \frac{\sqrt{2W^\alpha} - \sqrt{2k - 1}}{\sqrt{2W^{0.95}} - \sqrt{2k - 1}} \right) 1.65 \quad (4.2)$$

Where  $W^\alpha$  is the Wald statistic on the actual data and  $W^{0.95}$  is the Wald statistic on the 95th percentile of the simulated data. For a model to pass the test, T is required to be less than 1.65.

Besides the testing, indirect inference could also do the estimation to find the 'best' set of coefficients. The estimation process is actually based on the testing, the main method is to find the smallest Wald and coefficients that could minimising the distance between the actual data and the simulated data. The Wald statistics mentioned in the testing process measures the distance between the actual data and simulated data. Therefore to estimate the model, it needs to minimise this distance or the corresponding Wald for the actual data. In practice, the Simulated Annealing algorithm which finds a global minima is chosen in the estimation. According to Kirkpatrick et.al (1983) and Cerny (1985), Simulated Annealing takes a possible solution into the iteration and define a



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wide range. In each iteration, the algorithm heuristic considers some neighbouring state of the current state and probabilistically decides between moving to the neighbour state or staying at the current state. This process is repeated until the final reach to a state which is good enough to the function. The advantage of the Simulated Annealing is that it would have a global minima rather than local minima; it is a method based on the Monte-Carlo iteration which is a random optimisation algorithm. Together with the Indirect Inference, the estimation chooses the initial vector of the parameters into the steps above in the testing process. Next to converse the data from actual data and simulated data into the auxiliary statistics. Then minimise the Wald statistics which measure the distance between the empirical auxiliary model and simulated auxiliary model, optimising the Wald until the Wald is minimised. Given the result in Meenagh et.al (2012), both Bayesian Maximum Likelihood and Indirect Inference estimation are consistent and asymptotically normal, but Indirect inference has stronger power in testing in the small sample. Indirect Inference is more likely to reject the model when the model deviates more from the 'true'. Another method is the random search, given the range of parameters. For a specified range of all parameters, the algorithm searches randomly within the range to find sets of parameters that could pass the test. This method can find many sets that could pass the test and it is the researcher's decision to choose which is 'best' for the research interests.

### **4.2.3 Choice of Auxiliary Model.**

After log-linearisation, the DSGE model can have a restricted VARMA representation, or approximately VAR, of the endogenous variables. Naturally in practice, VAR is an easy auxiliary model applied in estimating the model to measure the fitness of the model to the actual data. As the model in this thesis would use unfiltered data and contains the non-stationary technology shock, the vector error correction model ( VECM) specification

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would be used instead of VAR. Following Meenagh et.al (2012), this is because that the non-stationary technology process can make some structure equations have non-stationary residuals. In the estimation and testing process where those residuals will be backed from the actual one and simulated one, the number of cointegrating vectors will be less than the number of endogenous variables if these processes are treated as unobservable variables. In other words, if those processes are treated as observable variables then there will be as many cointegrating relations as the number of endogenous variables. Therefore, the VECM can be a representation as the solution of the estimated model where non-stationary residual appears as observable variable, and unrestricted VECM can be used as the auxiliary model. As in Meenagh et.al (2012) and Le et.al (2015), the log-linearised DSGE model with  $p \times 1$  vector of endogenous variables  $y_t$ , a  $r \times 1$  vector of expected future endogenous variables  $E_t y_{t+1}$ , a  $q \times 1$  vector of non-stationary variables  $x_t$  and a vector of error  $e_t$  which follow identically independently distribution, can be in a form as:

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t \quad (4.3)$$

$$\Delta x_t = a(L)\Delta x_{t-1} + d + b(L)z_{t-1} + c(L)\epsilon_t \quad (4.4)$$

Where  $x_t$  is a vector of non-stationary processes, of which may contain a dependency on the lag of  $z_t$ , a stationary exogenous variable;  $\epsilon_t$  an error vector follows i.i.d with zero mean. In the function property,  $A(L), B(L), C(L), D(L)$  as well as the lower capital function like  $a(L)$  is a polynomial matrix function with lag order and the roots of the determinant polynomial is outside of the unit cycle.  $y_t$  is also a unit root process as it depends on  $x_t$ . The general solution of  $y_t$  can be a form of:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \quad (4.5)$$

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Of which  $f$  is a vector of the constants. In the null hypothesis that the model is 'true', the equilibrium solution for the endogenous variables that has  $p$  cointegrating relationship is the form as:

$$y_t = [I - G(1)]^{-1}[H(1)x_t + f] = \Pi x_t + g \quad (4.6)$$

where  $\Pi$  is a  $p \times p$  matrix with rank lies between  $0 \leq r < p$ . It is also worth noticing that in the short run,  $y_t$  is a function of deviation from the equilibrium with the error correction term  $\eta_t$ :

$$y_t - (\Pi x_t + g) = \eta_t \quad (4.7)$$

The long run solution of the model can be represented as:

$$\bar{y}_t = (\Pi \bar{x}_t + g) \quad (4.8)$$

$$\bar{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \quad (4.9)$$

$$\xi_t = \sum_{s=0}^{t-1} \varepsilon_{t-s} \quad (4.10)$$

$\bar{y}_t$  and  $\bar{x}_t$  are the long run equilibrium solution of  $y_t$  and  $x_t$ . Therefore the long run solution  $\bar{x}_t$  can be decomposed into a deterministic trend part  $\bar{x}_t^D = [1 - a(1)]^{-1}dt$  and a stochastic part  $\bar{x}_t^S = [1 - a(1)]^{-1}c(1)\xi_t$ , saying that  $\bar{x}_t = \bar{x}_t^D + \bar{x}_t^S$ . Because the long run dynamic of endogenous variables also depend on  $x_t$ , those endogenous variables

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is dependent on these two parts as well. It can be seen that solution of endogenous variables have a trend and the deviation from the trend. Therefore, the solution of the endogenous variables can be written as a trend and a VARMA in deviation from the trend. In this case, it can be a form as a cointegrated VECM with the mixed moving average error term by abstracting  $y_{t-1}$ :

$$\Delta y_t = P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f - [I - G(1)](y_{t-1} - \Pi x_{t-1}) + \omega_t \quad (4.11)$$

$$\omega_t = M(L)e_t + N(L)\varepsilon_t \quad (4.12)$$

Or being an approximate form of VAR with exogenous variable (VARX):

$$\Delta y_t = K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \quad (4.13)$$

$\zeta_t$  follows i.i.d zero mean process. Because  $\bar{x}_t = x_{t-1}^- + [1 - a(1)]^{-1}[d + \epsilon_t]$  and  $\bar{y}_t = (\Pi \bar{x}_t + g)$ , this approximate VECM can be rewritten as:

$$\Delta y_t = K[(y_{t-1} - y_{t-1}^-] - \Pi(x_{t-1} - x_{t-1}^-)] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \quad (4.14)$$

This equation (14) is being used as the auxiliary model following Le et.al (2015). In theory and practice, more endogenous variables included in the auxiliary model and higher lag order used, the testing and estimation would be more precise, saying having stronger power. However, a stronger power means more likely a model being rejected. if three variables included in the auxiliary model, it is to test twelve parameters jointly. But if one more variable added into the auxiliary model, twenty parameters are going

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to be tested jointly, which means the criteria is much more strict. Thus, here is the trade off between testing power and tractability. Empirically, the number of variables included in the auxiliary model is usually three and the lag order frequently applied is one, trying to avoid that 'too many good model being rejected'. The evaluation process would check if the model could pass the test with three central variables or those in the interest.

#### 4.2.4 Property of Auxiliary Model.

In order to capture the Wald statistics, we need to get the estimates  $\theta$  and estimates from simulations  $\theta_0$  by the auxiliary VARX(1), which represents the behaviour of those variables of interests. In this case, VARX(1), which contains three endogenous variables, interest rate, inflation and output ( $Y$ ,  $R$ ,  $\pi$ ) and exogenous can be in the form as:

$$\begin{bmatrix} r_t \\ \pi_t \\ y_t \end{bmatrix} = A \begin{bmatrix} r_{t-1} \\ \pi_{t-1} \\ y_{t-1} \end{bmatrix} + B \begin{bmatrix} e^{YT} \\ \text{constant} \\ T \end{bmatrix} + \begin{bmatrix} e_t^r \\ e_t^\pi \\ e_t^y \end{bmatrix} \quad (4.15)$$

where

$$A = \begin{bmatrix} a_{rr} & a_{r\pi} & a_{ry} \\ a_{\pi r} & a_{\pi\pi} & a_{\pi y} \\ a_{yr} & a_{y\pi} & a_{yy} \end{bmatrix} \quad (4.16)$$

This VARX(1) includes three exogenous, a non-stationary productivity residuals from structure model  $e^{YT}$ , the constant contains  $1 \times T$  ones, and the time trend  $T$ . Our VARX(1) fits the data very well, in line with the usual strong descriptive capacity of VARs. It has an average  $R^2$  of 0.7715. across the three variables. The parameter vector  $\theta$  for Wald statistics calculation includes all estimates in  $A$  and the variance of fitted error in the VARX(1):

$$\theta = [a_{rr}, a_{r\pi}, a_{ry}, a_{\pi r}, a_{\pi\pi}, a_{\pi y}, a_{yr}, a_{y\pi}, a_{yy}, \text{var}(e_t^r), \text{var}(e_t^\pi), \text{var}(e_t^y)]' \quad (4.17)$$

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## 4.3 Data Description and Preparation.

### 4.3.1 Data Source.

Hong Kong has an extraordinary complex history both politically and economically in the past decades. The current linked exchange rate system was established in 1983 as a tool to stable the value of the Hong Kong dollar which is seen as a success thereafter. During the periods from 1983 to 1997, there was a concern about the return of Hong Kong to China. By abstracting the non-economic effect to Hong Kong, it is reasonable to select the data range after 1997. For more prudential, the data in this these are taken from the first quarter of 1994 to the third quarter of 2018, which includes the Global Financial Crisis. In general, Hong Kong has extremely high openness and freedom in economics, nearly no capital control and free trade with the rest of world.

Most HK data are captured from the Datastream which is then taken from either the Census and Statistics Department of Hong Kong government or directly from the Hong Kong Monetary Authority, while some are from the World Bank and US Bureau of Economics Analysis, as well as the International Monetary Fund. All the data are in the constant price or those in current price in the original source will be transferred into constant price using the consumer price level. Detail of the data source is referred to the appendix in this chapter, but here would give a brief description. Since most macro model would use the quarterly data to conduct the analysis of the business cycle property, this frequency would naturally introduce the seasonal effect. To remove the seasonal effect, the original data needs to be seasonal adjusted. The most widely used method is US Census Bureau's software package X12-ARIMA.

The capital price is derived from the investment Euler equation; Capital stock and housing stock are derived from capital accumulation and housing accumulation respectively.

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Capital rental is derived from the marginal product of capital in entrepreneur production choices, by definition. All those in HKD value would be transformed into real term by CPI<sup>1</sup> if they are not in constant price, and then into per capita by the total employment (Source code: HKEMPTOTP).

It is worth noticing about the data in the export and import in Hong Kong. Unlike other open economy, Hong Kong most time acts like the financial centre and international port. It would import and re-import, as well as export and re-export. The concern about the trade statistics is whether to exclude the re-export from the total export. By questioning the Census and Statistics Department of Hong Kong, it has two major concern to not exclude export but have the total export and import. First, The share of domestic exports in total exports has been on a secular decline, from around 70% in the eighties to a very low level of only about 1% in the recent years. For analytical purpose, domestic exports have little significance in helping to understand and analyse the latest trends of the overall economy. Second, the C&SD has decided that domestic exports and re-exports will no longer be published in this report as from the January 2018 issue, but the analysis on total exports and imports will remain unchanged. However, it is reasonable to exclude the re-export and re-import as those are produced physically in Hong Kong. Since the actual domestic export and import are declining dramatically, they cannot display the trend of trade activity in Hong Kong. For analysis purpose, here the trade data are generated by 5% of total amount, where 5% is the historical average domestic produced trade amount as a share of total.

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<sup>1</sup>CPI components include both home goods and foreign goods, it represents the general price level with the households in the economy. In Hong Kong, CPI and GDP deflator are highly correlated with correlation 0.7380

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### 4.3.2 Why Unfiltered Data.

It is very common to use filtered data in estimating the DSGE model or other time series economic models. The detrending method treats the macroeconomic variables as a combination of potential growth and short run cycle. Like the output, the traditional practice would detrend output by making it into two parts, the trend and cycle, or called the potential output and output gap. Nelson and Plosser (1982) believe most macroeconomic variables do not have deterministic trend, but unit root. Therefore, those data need to be detrended to be stationary. At the beginning, the business cycle theory is to explain the cyclical behaviour of the economy, while the stochastic process is driven by the shock which push the variables to deviate from the steady state. Thus, it is naturally to remove the trend from the data as it is not the focus of the research. To smooth the data, the popular technique of separating the trend and cycle components are Hodrick-Prescott (1997) and Band-Pass filter. However, there are two concerns about the non-stationary data and filtering method.

First, the non-stationary data itself would contain some information from the trend. By making the data stationary, the dynamic property of model in the stochastic trend which is from the unit root productivity process may be eliminated. Second, the problem in filtering method. The differencing method which removes the time trend does not fit into the data by making too much noise, because transformations as input in the estimation process do not meet the actual requirement for periods, saying Canova (1998). Similarly, the linear detrending is also not quite appropriate for the data having stochastic trend. Hordrick-Prescott (HP) filter, treating the economic variable neither constant nor stochastic moving, but having trend, is seriously criticised. Harvey and Jaeger (1993), Cogley and Nason (1995), as well as Murray (2003) explain the spurious effect from HP filter. As HP decomposes a series by removing trend and taking cycle, it may enlarge the behaviour of the cyclical components or even make the non-exist cycle.



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More, the detrending process may distort the timing property, especially for the DSGE model with rational expectation which would include the lag of variables.

Following Meenagh et.al (2012), this DSGE model will generate the non-stationary data because the productivity process is assumed to have unit root. Instead of making the actual data stationary by the popular HP filter, the choice of auxiliary model is the VECM or VARX to be the solution of DSGE. As described in the previous section, this can be done in the Indirect Inference by cointegrating the endogenous variables with non-stationary exogenous process.

## **4.4 Empirical Results by Indirect Inference.**

### **4.4.1 Indirect Inference Estimation and Test Result.**

Following the method in 4.2.2, the Indirect Inference estimation is to find structure parameters that could minimise the distance between the simulated data and actual data. Given this basic idea of the estimation, the process will search randomly in the range specified from the starting value of structure parameters, here starting value is set to be the calibration. I also test the calibrated model and the estimated model, the comparison of these two can be found at the bottom of the estimation result table. The fit of the VARX(1) estimates is shown in Table 4.2.

Table 4.1: Indirect Inference Estimates of the Structure Model

Parameter	Definition	Calibration	Estimation
$\alpha$	Capital Share in Production	0.3	0.3443
$\eta$	Inverse Elasticity of Labour Supply	3	5.0880
$\delta^k$	Capital Depreciation	0.025	0.0177
$\delta^h$	Housing Depreciation	0.01	0.011
$\omega$	Domestic Home Bias	0.4	0.1822
$\theta$	Elasticity between Domestic and Imported goods in Home	1	1.5340
$\omega^f$	Foreign Home Bias	0.4	0.1809
$\theta^f$	Elasticity between Domestic and Imported goods in Foreign	1	1.2499
$\kappa^k$	Capital Investment Adjustment Cost	6	6.4153
$\kappa^h$	Housing Investment Adjustment Cost	6	11.3376
$\chi$	Feedback from Leverage to Finance Premium	0.05	0.0287
$\mu$	Feedback from Money to Finance Premium	0.7	0.8971
	Variable in the VARX(1)	Trans-W	p-value
Calibration	$Y, r, \pi$ (Output, Interest rate, Inflation)	2.694	0.006
Estimation	$Y, r, \pi$ (Output, Interest rate, Inflation)	1.0924	0.122

Table 4.2: VARX Parameters and Bootstrap Simulation Bound

Parameter	Estimate	95%Lower Bond	95%Upper Bound	IN or OUT
$a_{rr}$	0.8628	0.5761	0.8859	IN
$a_{r\pi}$	0.0052	-1.5168	1.1420	IN
$a_{ry}$	0.0074	-1.3703	1.1689	IN
$a_{\pi r}$	-0.1055	-0.0270	0.0247	OUT
$a_{\pi\pi}$	-0.1104	-0.1331	0.3292	IN
$a_{\pi y}$	0.1982	-0.0371	0.0526	OUT
$a_{yr}$	-0.7549	-0.0352	0.0462	OUT
$a_{y\pi}$	-0.1441	-0.6759	0.0629	IN
$a_{yy}$	0.9610	0.5439	1.2219	IN
$var(e_t^r)$	0.0004%	0.0002%	0.0006%	IN
$var(e_t^\pi)$	0.0100%	0.0178%	0.0457%	OUT
$var(e_t^y)$	0.0166%	0.0111%	0.0297%	IN

1. To test if the model can replicate the behaviour of data, we take those 12 parameters jointly into consideration. The overall p-value of 0.122 reflects the joint distribution of all these parameters

The estimation results of structure parameters from Indirect Inference are shown in table 4.1. In the households side, the only structure parameter is the inverse elasticity of labour supply  $\eta$ , the value has increased by 69.6% from 3 to 5.0880. This implements that households are less concern on smoothing the labour working so that when the wage moves, households would decide to change the supply of the labour more.

Turning to the supply side and production, the capital share in the production  $\alpha$  is 14.8% more than the calibrated value of 0.3. This is to say that in Hong Kong, capital

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is more applied in producing the general goods. However, the value is still less than 0.5, which means the Hong Kong economy is still labour oriented. Unlike other literatures, the depreciation of the capital and housing is no longer fixed here. But the housing depreciation  $\delta^h$  is almost the same as the calibrated value 0.01, 0.011 by estimation, saying that housing actually depreciates really little. With respect to capital depreciation  $\delta^k$ , the value decreases from 0.025 to 0.0177, by 29.2% falling. In other words, compared to calibration, capital is less depreciated by estimation. Two interesting parameters which are quite different from other literatures are the capital and housing investment adjustment cost,  $\kappa^k$  and  $\kappa^h$ . Most other researchers, who study the economy other than Hong Kong, would find them most likely to be less than 1. Based on the calibration of them being 6 in the literatures of Hong Kong, the estimation finds them are 6.4153 for capital adjustment cost  $\kappa^k$  and 11.3375 for housing investment adjustment cost  $\kappa^h$ . Regarding to the large amount of difference, the calibration is not well specified in housing investment. Both capital investment and housing investment are more costly to be converted into capital and housing, while housing investment will cost more to be converted into housing relative to the capital investment.

The rest two parts of parameters are the open economy and financial premium. The home goods bias of Hong Kong  $\omega$  moves from 0.4 to 0.01822, indicating that Hong Kong consumers actually prefer more on import goods. This is a tricky one because Hong Kong has extraordinary large amount of the export and import, however most of them are re-exported to other destinations. But this parameter value can be interpreted in another way: it can meet the reality that Hong Kong actually does import a lot from the rest of world to satisfy the large domestic demand. The corresponding home bias for rest of world  $\omega^f$  changes from 0.4 to 0.1809, saying that the rest of world in this model would prefer less on domestic goods. Furthermore, the rest of world data in this model is taken from the US data, the value of 0.1809 is an appropriate one to replicate the behaviour that US imports a lot from other economies. The elasticity between domestic

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goods and imported goods in home economy  $\theta$  rises to 1.5340 and the corresponding one in foreign economy  $\theta^f$  increases to 1.2499. So that 1% increase in the real exchange rate  $Q$  would make the imports drop by 1.5340% and the exports rise by 1.2499%. Feedback from leverage to finance premium  $\chi$  decreases by 42.6% from calibration 0.05 to 0.0287 and feedback from money to finance premium increases by 28.2%. Compare to the calibration which is from the US data in Le et.al (2016), feedback to premium from the leverage would be lower but feedback from the money is much more in Hong Kong.

After the Indirect Inference estimation, it can be seen that the set of the central variables, output, interest rate, inflation  $(Y, r, \pi)$  is statistically significant and not rejected by indirect inference test. Given the critical value of transformed statistics 1.645 with 5% significance level, it tests against it for statistic being 1.0924 which is lower than 1.645, while the p-value of 0.122 which is larger than 5% and it can help to prove that the model can pass the test. Compare to the calibration test result 2.694 and p-value of 0.006, this estimation has a great improve from the calibration. This is to say that calibrated parameters perform badly in fitting the data, while the estimated model can explain the behaviour of the data quite well.

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#### 4.4.2 Power of the Test

Given the result in Indirect Inference estimation and those tests on the estimation, we can see that the model can explain the behaviour of the economy and fit the Hong Kong data well by passing the test. Another question is how powerful is the test on this model? How likely is the model to be rejected if the model is somehow falsified? Le et al. (2016) compare the indirect inference and direct inference, while the performance of the direct inference is represented by the likelihood ratio. They found that the indirect inference is much more powerful than direct inference by likelihood ratio, although direct inference does reject the false model to some extent. In addition, they check the non-stationary as well, not surprising, indirect inference performs even better as it is more likely reject the model when it is falsified. Following their process, the power of the test is conducted by the following steps:

*Step 1. Generate simulations from true model*

Given the model with estimation parameters who can pass the indirect inference test as the 'true' model, I get the residuals and innovations from the structure 'true' model. Then I generate 1000 simulations from the true model with Monte Carlo method by its residuals and innovations.

*Step 2. Falsify true model*

Falsifying the true model by mis-specifying the estimated parameters with the same direction in alternating way: odd order number parameter mis-specified by  $(1+x\%)$  and even order number parameter mis-specified by  $(1-x\%)$ , while the standard deviation of the structure residual processes are altered by the same way.

*Step 3. Generate simulations from true model*

Given the simulations from the 'true' model as the 'true' data, testing the false model with the simulation data from the true model 1000 times, while the testing procedure is

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the same as in the indirect inference test. The power is then measured on the frequency that how many times the false model is rejected by the true model simulation data (probability that Transformed Wald statistics is bigger than 1.645). By default, the true model should report the power is 5% at 95% confidence level.

All Monte Carlo experiments are reported in Table 4.3, 1%, 3%, 5%, 7%, 10%, 15% and 20% false models are tested here to give some examples. It can be seen that indirect inference test of this model is highly reliable as the test is very powerful, more than 50% of the experiments are rejected if the model deviates from the 'true' by 5% and the probability goes to 100% if the false rate increases to 15%. The three variables VAR represents would be an appropriate choice of the auxiliary model, as it can generate a fair result of the power test. In the power test, I would expect there is a strong power but it should not be too much. That is, if I was to increase the number of variables in the VAR, or increase the order of the VAR, the power would be increased. A too powerful VAR represent will make us drop into the trap that too many good models are rejected. Since the variables of interests and the number of variables I choose have seen strong power, I would not test more variables.

Table 4.3: Power of Indirect Inference Test

		VAR represent: 3 variable-VAR(1) ( $Y, r, \pi$ )						
False Rate	True	1%	3%	5%	7%	10%	15%	20%
Power	5%	8%	29.7%	66.2%	89.6%	97%	100%	100%

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### 4.4.3 Residual and Shock Property

This section discusses the properties of these structure shock processes in the model. Those shock are calculated from the estimated coefficients by Indirect Inference estimation, and then fit into the non-stationary actual data. In order to test whether the shock is well specified in the form of AR(1) process or ARIMA (1,1,0) process for the productivity, the test of stationarity is taken for each shock by two method, the ADF test and KPSS test. Table 4.4 provides the result of unit root tests and their AR(1) coefficients. It worths noticing that sometime the test results from two types may have different interpretation, so it will rely on a personal judgement to decide the stationarity. Figure 4.1 displays graphs for the structure residuals from the estimated model, while Figure 4.2 shows those shocks from estimated model.

ADF test has the null hypothesis of unit root (non-stationary), while KPSS has the null hypothesis of stationary. The application of two types of test, not only one, is for the consideration of prudence. From the column two of ADF p-value, it is clear that housing demand shock, government spending shock, foreign consumption shock ( or can be treated as export demand shock) and foreign inflation shock are all rejected even at 1% significance level, while consumption preference shock and foreign interest rate shock are sure to be rejected at 5% which is the rate not that strict but more commonly applied. The productivity shock process is tested to be non-stationary under ADF with the p-value of 0.9485. In KPSS test, productivity is rejected against stationary clearly. In addition, the estimation of level productivity is 1.0004, which is another prove that productivity should be treated as non-stationary.

It can be seen all other processes look like a stationary process in Figure 4.1. Overall, for consumption preference, foreign consumption (or export shock in this model), foreign inflation and foreign interest rate, the ADF test of unit root is rejected and the KPSS



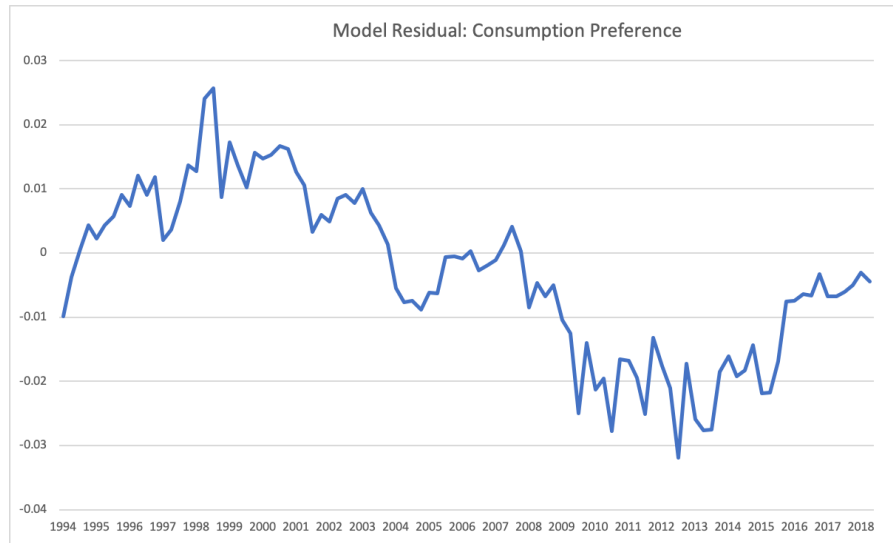
of stationary is not rejected both at 5% significance level. So we can conclude these processes are all stationary. The housing demand can reject the unit root in ADF but it also rejects stationary in KPSS, while government spending cannot reject unit root in ADF but it cannot reject stationary as well. According to their AR(1) coefficients that they are both less than 1, I would treat them as stationary.

Table 4.4: Residual Stationarity Test and AR(1) Coefficients

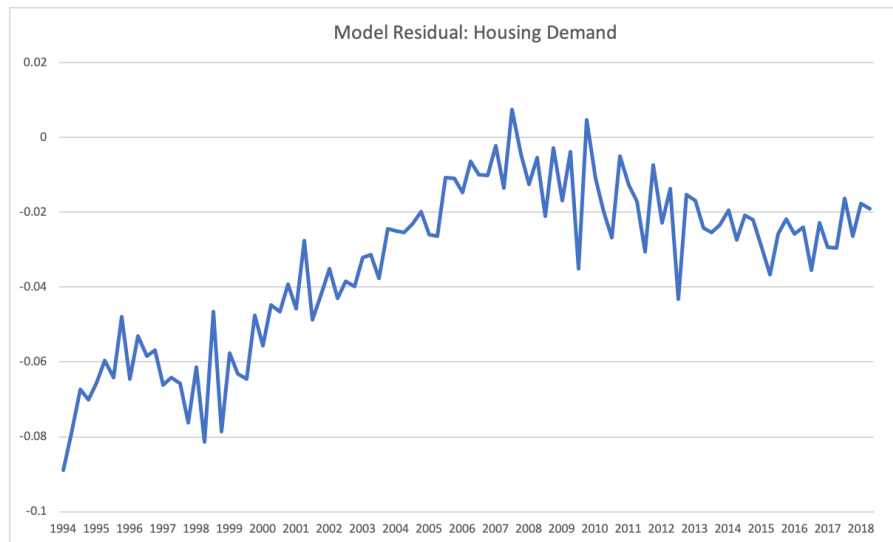
Shock	Stationarity Test		Conclusion	AR(1)
	ADF p-value	KPSS stats		
Consumption Preference	0.0364**	0.1431*	Trend Stationary	0.9207
Housing Demand	0.0092	0.281274***	Trend Stationary	0.9256
Productivity	0.9485***	1.1235***	Non-stationary	0.1804†
Government Spending	0.4239***	0.2953	Stationary	0.9852
Foreign Consumption	0.0056	0.1316	Stationary	0.8693
Foreign Inflation	0.001	0.4254*	Stationary	0.6868
Foreign Interest Rate	0.0094	0.0873	Stationary	0.8624

1. KPSS \*, \*\*\* indicates rejection of stationary at 10% and 1% respectively.
2. ADF p-value \*\*, \*\*\* indicates do not reject unit root at 1% and 10% respectively.
3. † The AR(1) coefficient of productivity is for the first order differenced one.

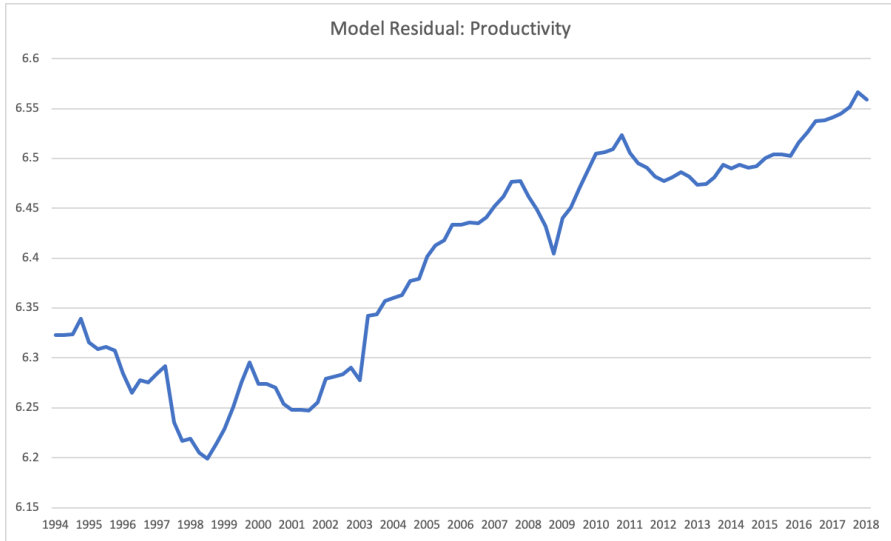
Figure 4.1: Residuals from the Structure Model by Estimation



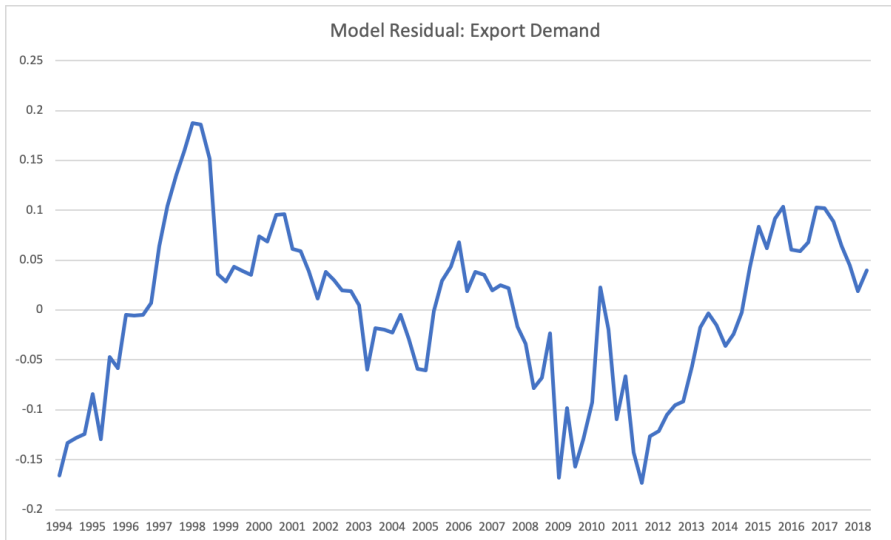
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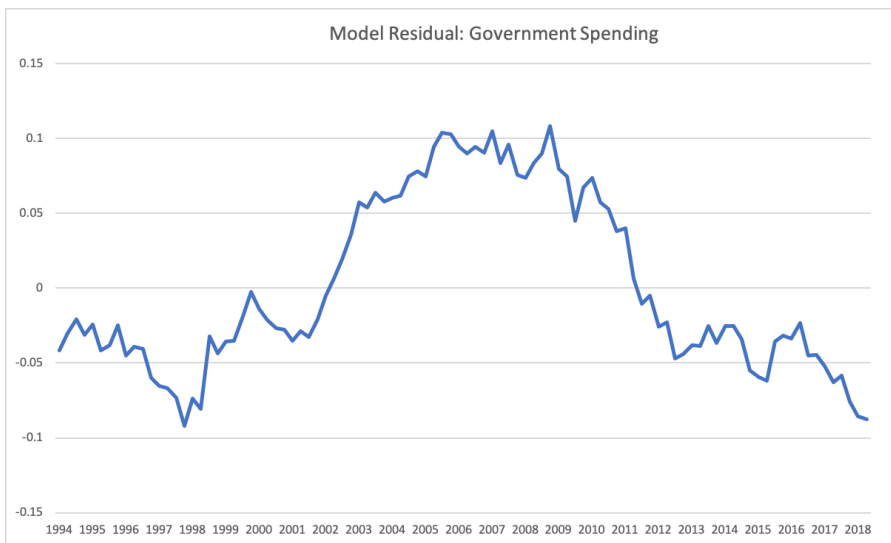
(b)



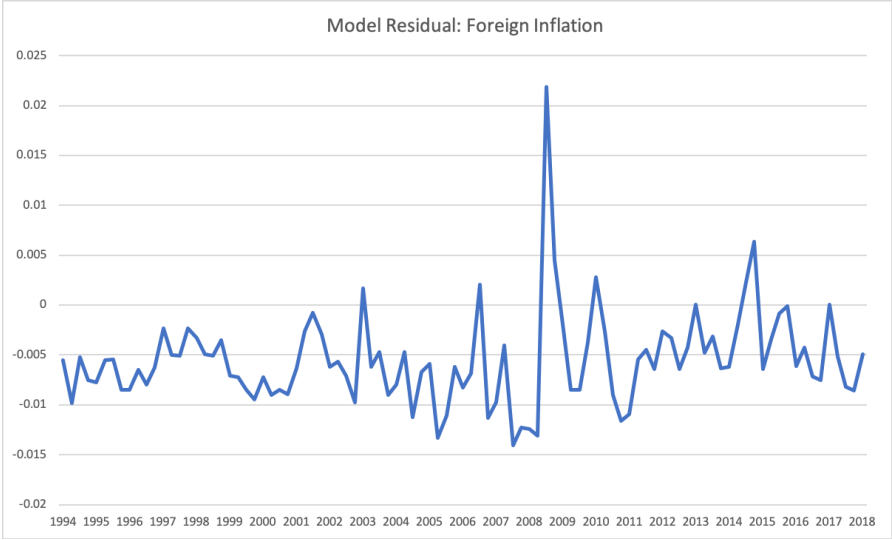
(c)



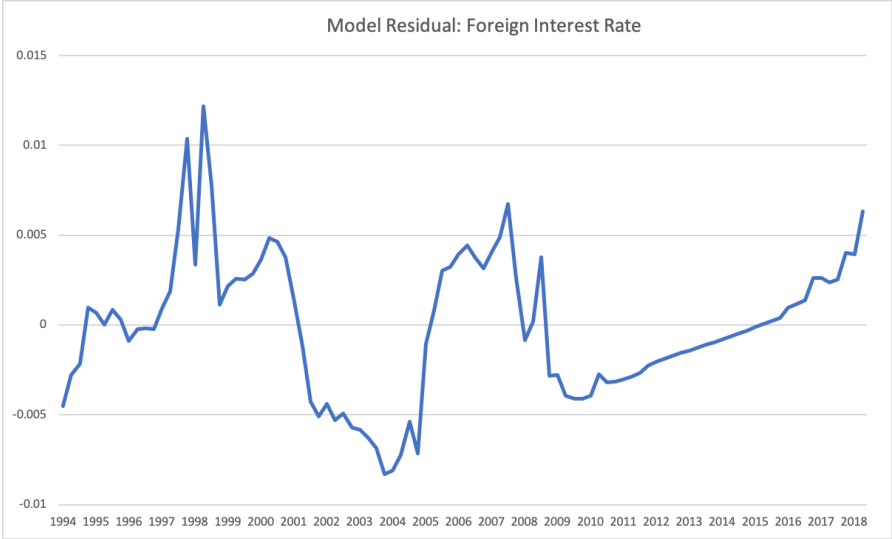
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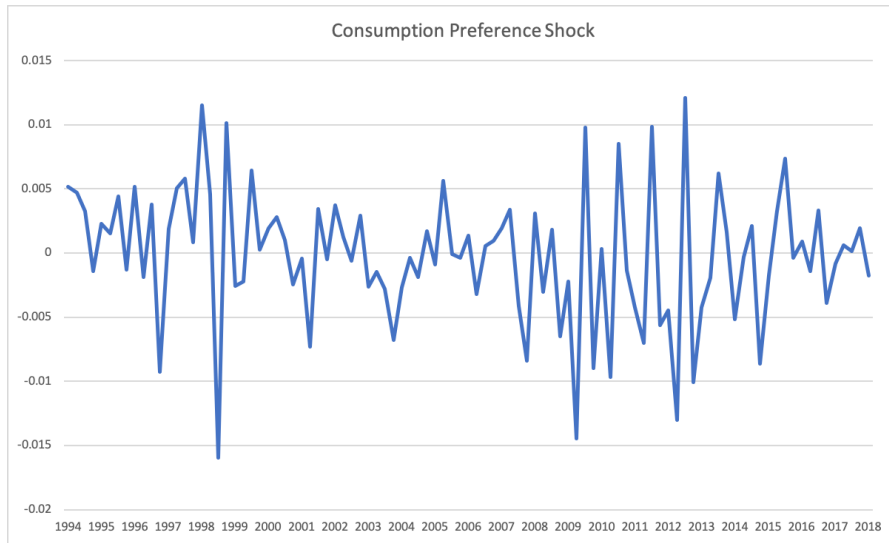


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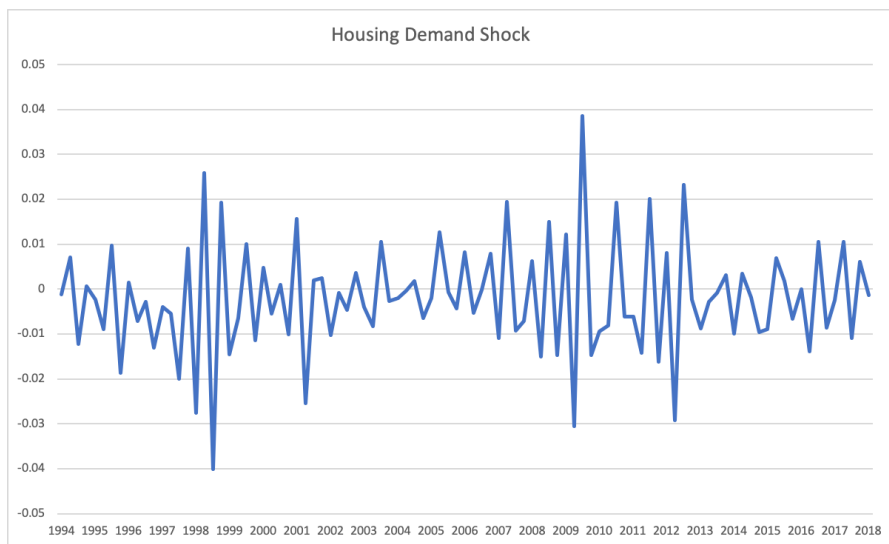


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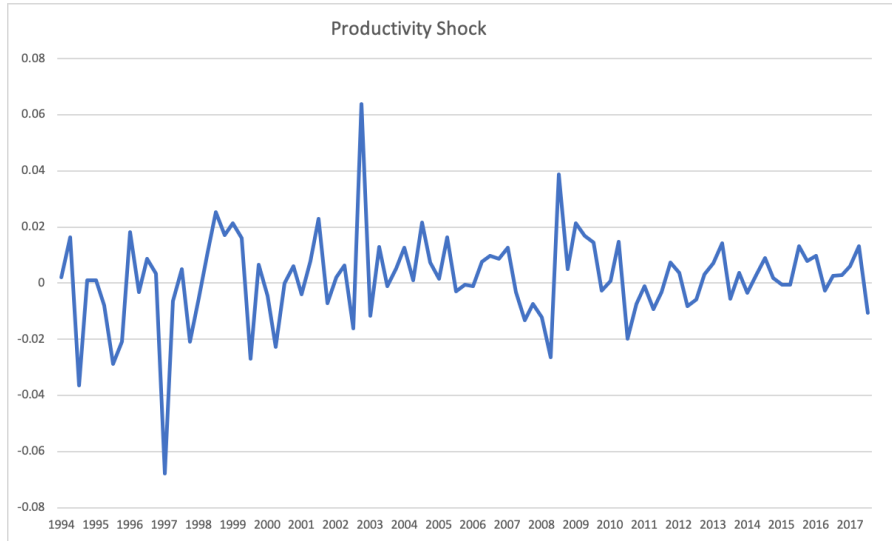
Figure 4.2: Shocks from the Structure Model by Estimation



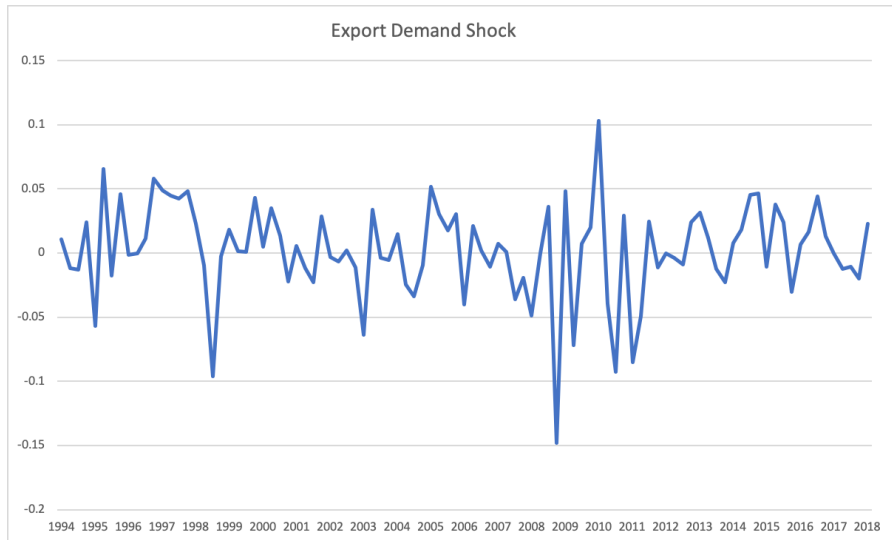
(a)



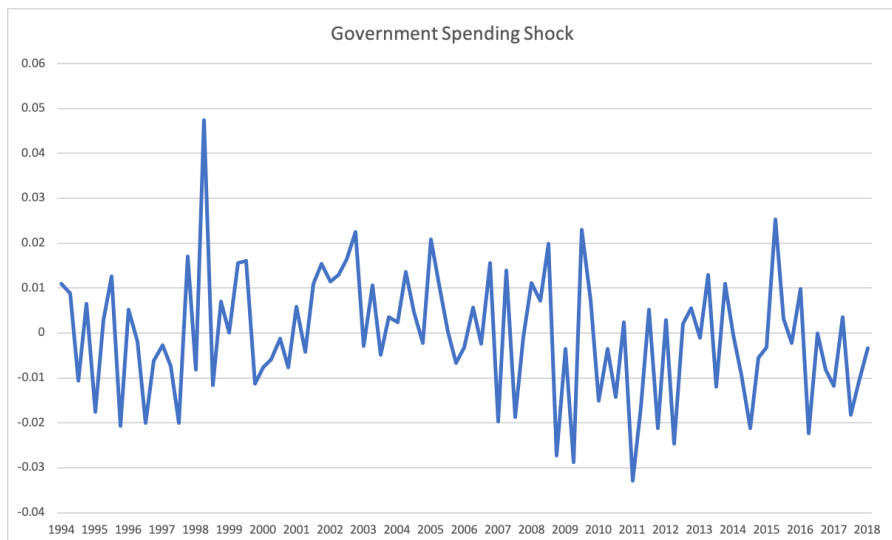
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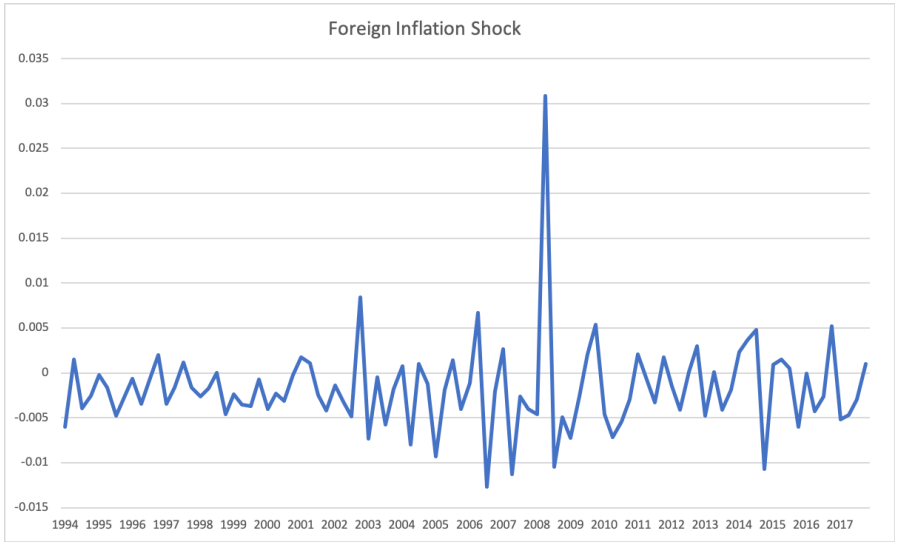
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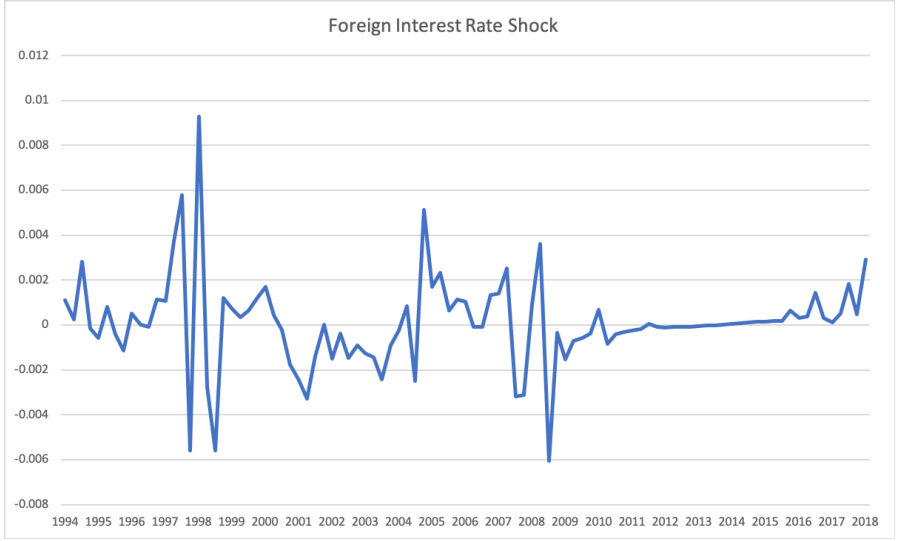
(d)



(e)



(f)



(g)

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## 4.5 Empirical Analysis

This section firstly discusses the Impulse Response of those endogenous variables of interest, to each structure shock, from the model with estimated parameters. The dynamic movements for all shock are very similar to those in the calibrated model, the detail will be discussed in the first part. After, the variance decomposition and historical decomposition for those endogenous variables of interest are the focus in the following parts.

### 4.5.1 Impulse Responses

#### *Consumer Preference Shock.*

A 1% consumption demand shock, also as known as the consumption preference shock, hit the consumption in households to increase it by approximately 6%. The increase of the consumption is met by the more output produced by firms, so that firms are going to hire more labour in the economy, with a rise in the wage. The decline in the capital and investment is because that the interest rate rises with the consumption preference shock, followed by the increased cost in investment and capital. Later, the real exchange rate decreases by the goods marketing clearing condition and the home price increases. Accordingly, the export decreases and import increases because home price is relatively higher than foreign price. Then, the net foreign assets in the domestic economy decline because the current account deficit caused by the drop in net export.

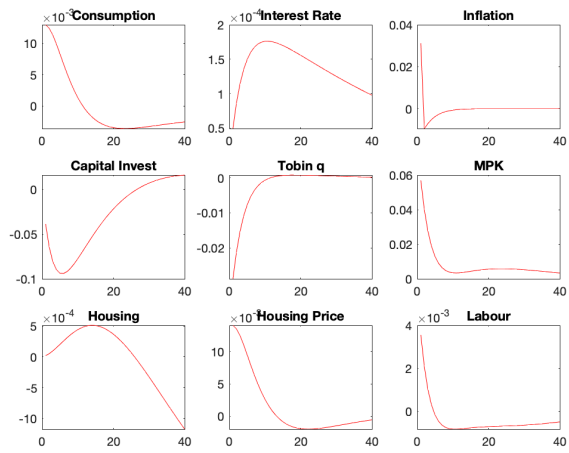
As shown in the diagram in Figure 4.4, the consumption demand shock initially shifts the ISBB curve to the right. The new temporary intersection of ISBB' and OS curve gives more output and lower real exchange rate. After, we can see the intersection of the ISBB' and OS is to the right of the XM curve, which means there is a current account deficit, as shown in the Figure 4.3 that the Total NFA declines. The decline in the



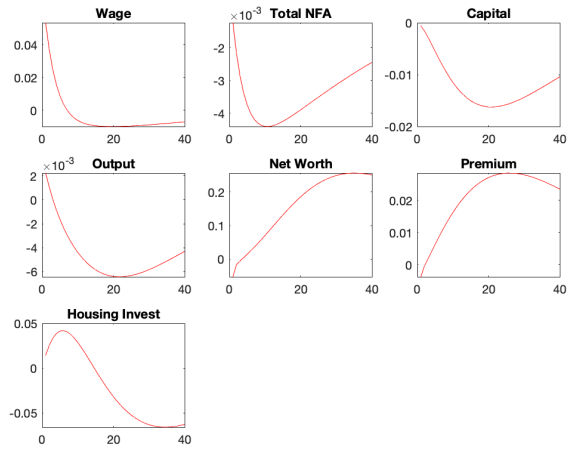
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NFA will shift the ISBB' to its original place, ISBB. That is to say that the decline in NFA drives the output and real exchange rate back to its equilibrium, the steady state in the IRF. It is worth noticing that the current account deficit also leads to a tightened monetary condition, together with an increased interest rate. Another point is that firms would borrow less because of the less demand for capital, which means it is another reason for the decline in the money and foreign reserve. Because there is not that much money demand in the economy, the borrowing rate and credit premium increase but not immediately.

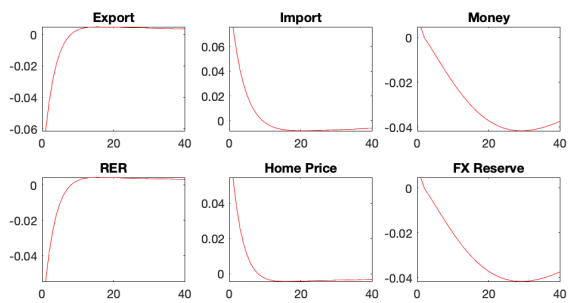
Overall, the consumption demand shock leads to the rise in the output by the reduction in the investment and capital, which is from the increase in the cost of those. The expansion in the supply side makes firms hire more labour from households, but need less money to finance acquiring capital because the higher cost, the credit premium increases not only the demand for loan decline, but also the aggregate monetary condition is tightened. The real exchange rate decreases by the initial shock, followed by the current account deficit. More, the decrease in the real exchange rate can have an increase in the domestic price relatively to the foreign price, which can match the fact that output rises under this shock that firms are more willing to produce with the increasing domestic price. Later, the net foreign assets decrease to drive the output and real exchange rate back to the steady state.



(a)



(b)



(c)

Figure 4.3: Full Estimated Impulse Response to Consumption Shock

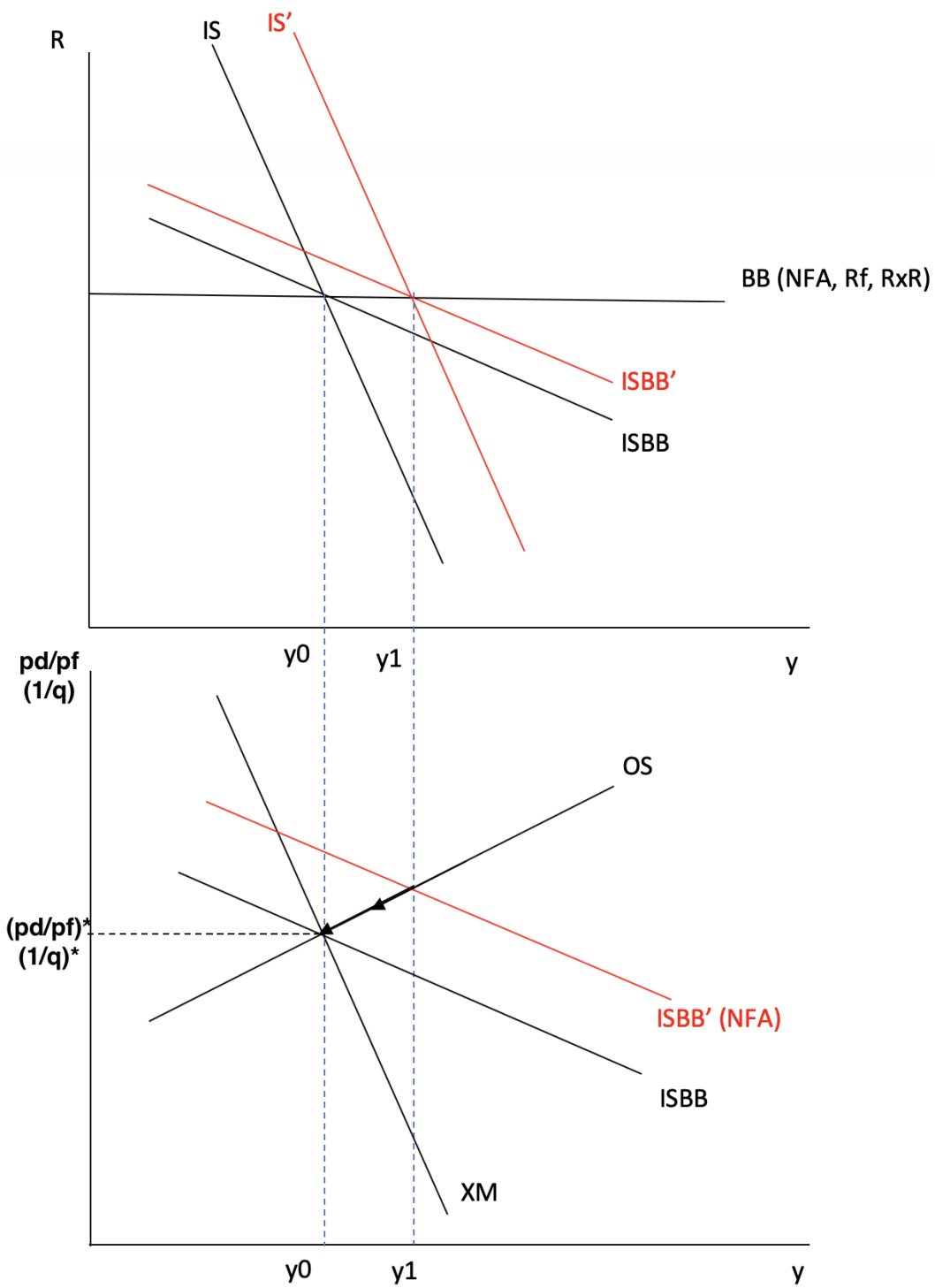


Figure 4.4: Diagram of the dynamics to Consumption Shock

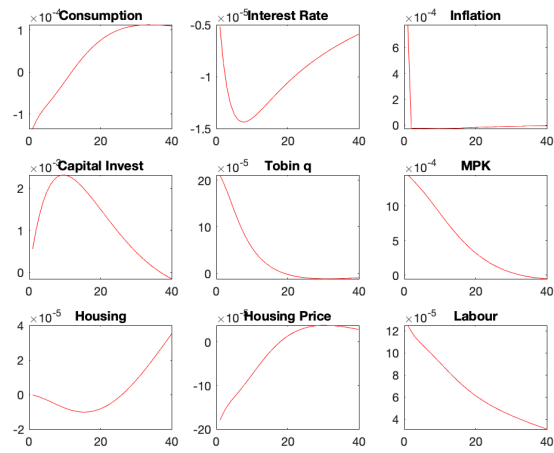
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***Export Demand Shock.***

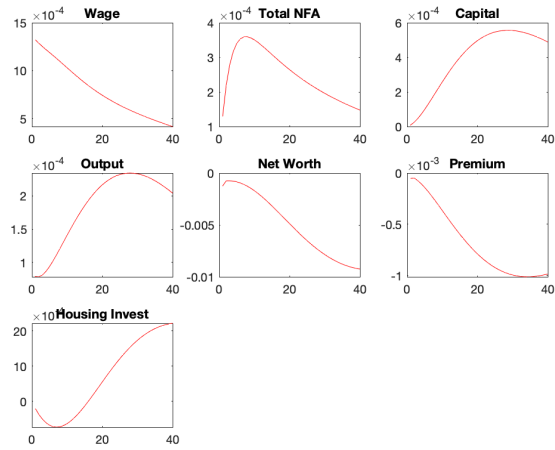
The initial export demand shock drives up the foreign demand for the domestic goods, export increases. In the monetary condition, the interest rate decreases as the money supply increases through the foreign exchange intervention, with a same increase in the foreign reserve. The monetary condition is loosen because the firm is willing to produce more as the domestic price is higher, by the decrease in the real exchange rate. The rise in the output means there is more demand for capital, firm will need more funds to finance its cost on capital. Thus, the credit premium is decreased due to increasing demand for loans. In households, they are going supply more labour since firms need more labour to meet the increasing production activity, the wage rises as well.

As in Figure 4.6, the initial shock drives up the demand for export, which directly shifts the XM to the right. This would have a current account surplus in the domestic economy and the new intersection of OS and ISBB is to the left of the new XM curve. At this temporary point, output increases not only because the increasing demand for export from foreign economy, but also the real exchange rate decreases by the market clearing condition. A higher domestic price level would encourage firms to produce more to get more profits and meet the rising demand for goods to export. Then the net foreign assets increase from the current account surplus, which would shift BB curve down so domestic interest rate would decrease via the risk premium uncovered interest rate parity, as well as the ISBB curve being shifted to the right.

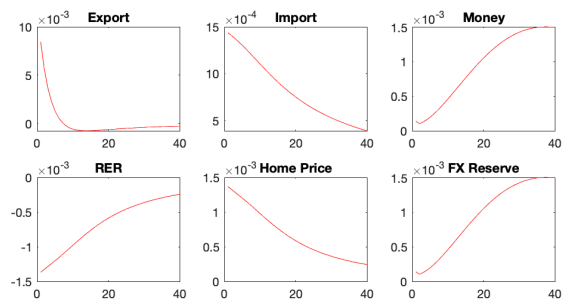
The export demand shock initially makes an increase in the output, capital, investment. The increase the production supply drives up the demand for labour, followed by a rise in the wage. The monetary condition is loosen because the firms need more funds to finance its cost on capital to produce more, through the foreign exchange intervention with a rise in foreign exchange reserve.



(a)



(b)



(c)

Figure 4.5: Full Estimated Impulse Response to Export Shock

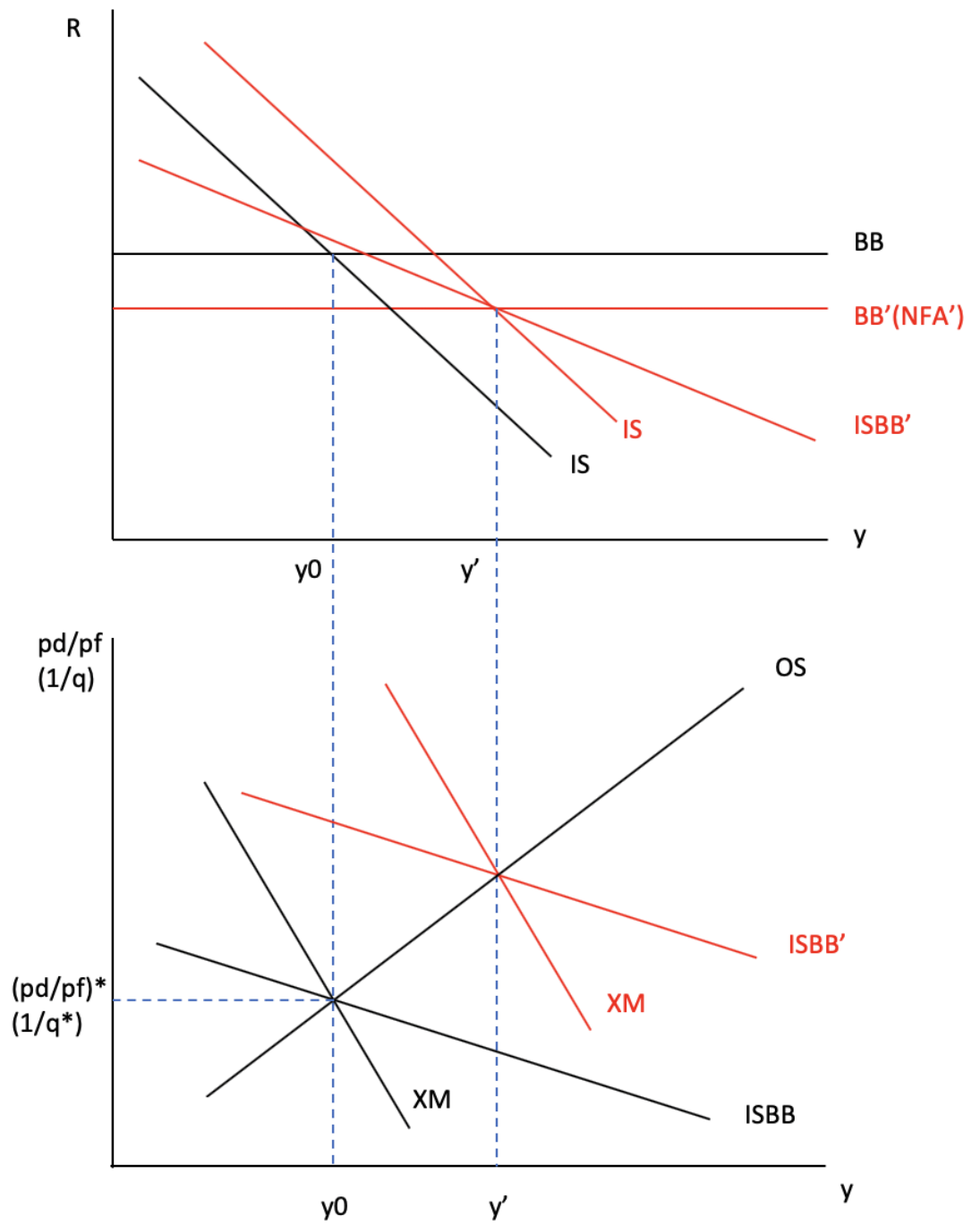


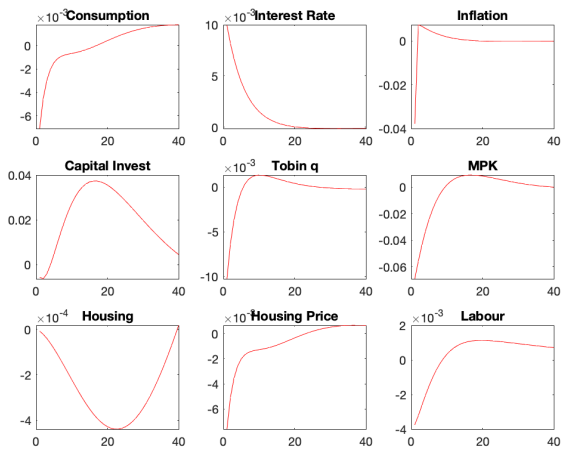
Figure 4.6: Diagram of the dynamics to Export Shock

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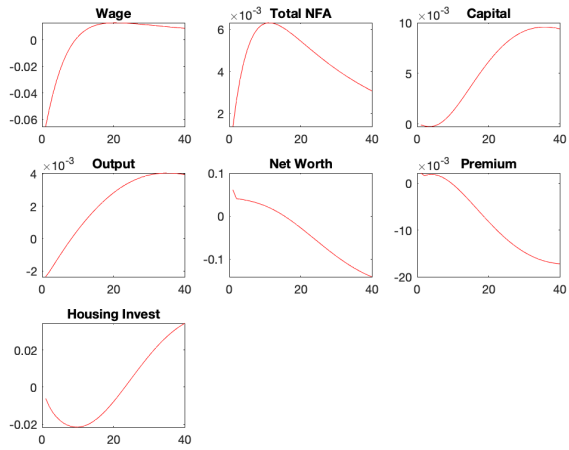
### *Foreign Interest Rate Shock.*

The foreign interest rate shock firstly drives up the domestic interest rate through the Uncovered Interest rate Parity and the BB curve in Figure 4.8. The tightened monetary condition causes less funds available for firms, with the capital and investment decrease following the shock. The effect afterwards is that output starts to decrease and the labour decreases as well, together with a reduction in the real wage after the initial shock. The real exchange rate increases from the initial shock, which is followed by a current account surplus. The surplus can also be seen in the diagram that the temporary equilibrium caused by the upwards shifting of the BB curve is to the left the XM curve. Following the current account surplus, the net foreign assets is accumulated more. So that more NFA drives output and real exchange rate to their steady state by 6 quarters.

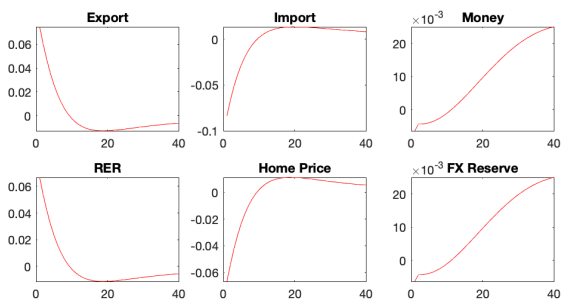
It can be seen both in Figure 4.7, the IRFs and Figure 4.8, the diagram, that the foreign interest shock affects the domestic economy through the domestic interest rate. The domestic interest rate increases by 1% and moves back to steady state after 20 quarters. The increased funding cost means a contraction in the capital and investment. Output begins to decline by the reduction in capital and investment, with a following decline in the labour. Together with the increase in the real exchange rate, the domestic economy sees a temporary current account surplus with more accumulated NFA. Finally, the rise in the NFA makes output and real exchange rate moves to their equilibrium much quicker than the domestic interest rate. Regarding to the monetary condition, the less demand for capital means there is less demand for money in the aggregate economy. The authority then must decrease the money supply by foreign exchange intervention.



(a)



(b)



(c)

Figure 4.7: Full Estimated Impulse Response to Foreign Interest Shock



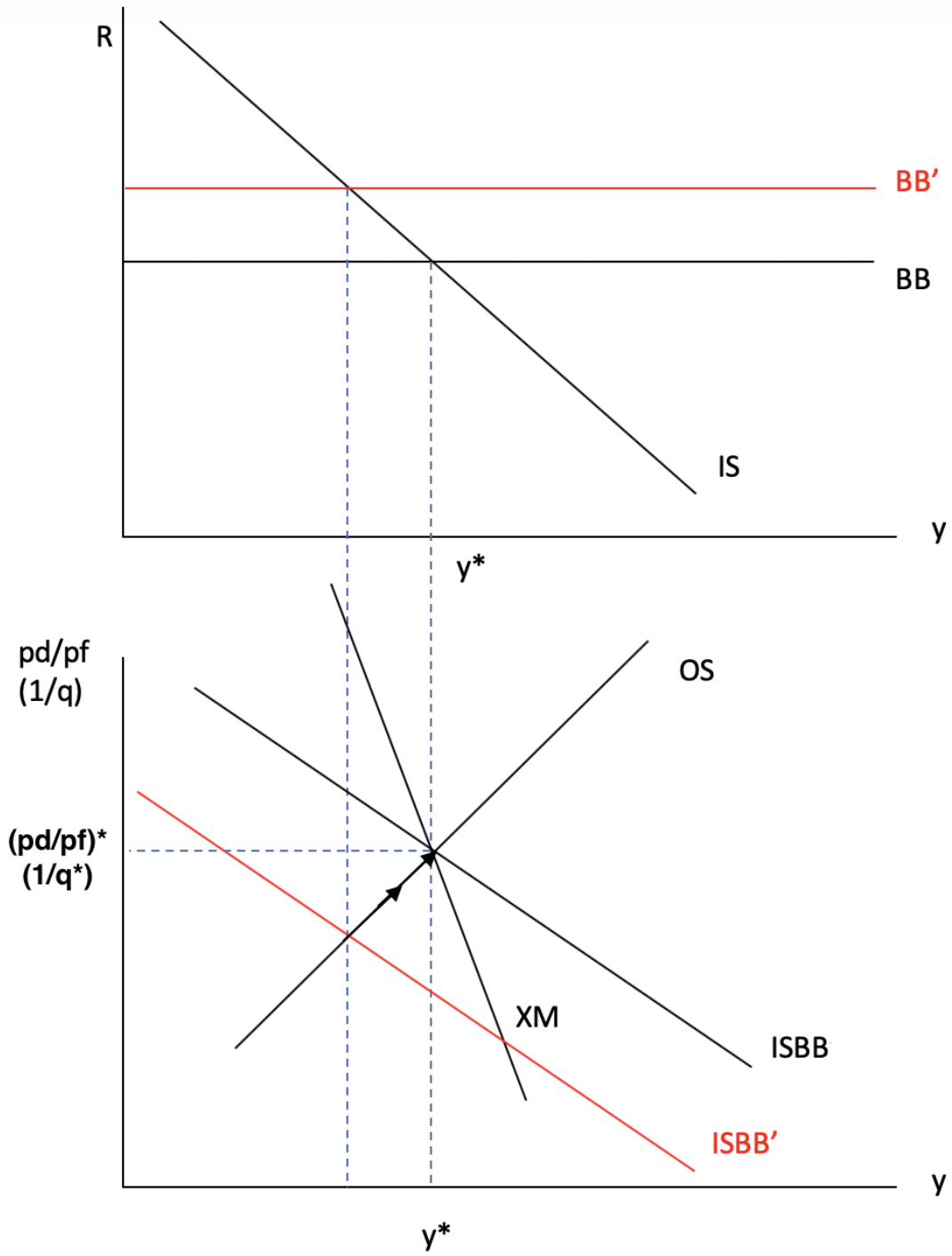


Figure 4.8: Diagram of the dynamics to Foreign Interest Rate Shock

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### *Housing Demand Shock.*

As shown in Figure 4.13 in appendix B, housing demand shock directly increases the preference of the households to purchase more housing, the housing investment then increases due to the higher demand for housing. This change results in that firms are going to produce more to meet the demand for more housing investment to produce more housing. Sequentially, the demand for capital and capital investment increase as well by firms. Because labour is also the input in producing the general goods, labour and wage increase but less than the rise in capital and investment. A more obvious response to the housing demand shock is the rise in the housing and a similar increase in the inflation. This phenomenon actually can reflect the fact that Hong Kong housing price inflation is more volatile than the general inflation. However, this model is not able to tell exactly why this happens. One proper answer might be that the increase in the housing price leads to the increase in the housing rent and housing rent is a large proportion in the consumer price index basket.

It worth noticing that this model, which has a very simple housing sector, generates different transmission mechanism as those literatures studying the housing. The consumption is decreased by the housing demand shock, means housing wealth and consumption are not significantly correlated. As the effect is not as in Iacoviello and Neri (2010), who study the US economy, this model supports the finding that housing wealth does not have spillover on the consumption. This is because now the households are not required to use the housing as collateral, which means housing is a substitution goods to consumption. Overall, output and its input, investment, housing price, inflation all increase by the housing demand shock. It can be seen from the appendix that the effect from this shock is relative small that other shocks, but need more time to go back to steady state. The long run movement has exception on housing price and inflation as housing price is more volatile that inflation, which can also be found in the actual data.

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### ***Non-stationary Productivity Shock.***

As in Figure 4.14, the permanent productivity shock makes a rise in the output. In order to produce more, firms would demand more capital and then more funds to finance its cost in capital. The loan market is expanded with more demand for loans in firms, following by a loosen monetary condition. The monetary authority uses its foreign exchange intervention to supply more money to meet the higher demand for money in the economy, as well as keeping the nominal exchange rate fixed under currency board arrangement. This central bank balance sheet is expanded as well with more foreign reserve held by the monetary authority. Consequently, the credit premium decline not only the loan market is expanded, but also the aggregate monetary condition is loosen. In the international trade, a rise in the real exchange rate leads to the fact that domestic goods price is actually lower than the foreign price, driving up export and current account surplus. A current account surplus accumulates more net foreign assets.

Overall, the productivity shock has positive effect on output and capital. The expanded supply side leads to an expanded loan market and loosen monetary condition. Furthermore, the rise in the real exchange rate drives up export and net foreign assets accumulation. This permanent shock results in the response quite persistent, all those last more than 50 periods but not for interest rate.

### ***Government Spending Shock.***

This fiscal expansion generates similar effect as from the productivity shock. It is clear in Figure 4.15 that output rises by the fiscal expansion, followed by the increase in labour. The rising interest rate hits the investment negatively and capital decreases following the decrease in the capital investment. Just like the open economy fiscal expansion theory, more government spending can crowd out the export. It can be seen in the IRF that real exchange rate decreases with a relative higher domestic price, which then makes domestic goods more expensive and export declines. The current account deficit

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accumulates less net foreign assets and interest rate increases further.

#### 4.5.2 Stochastic Variance Decomposition

Variance decomposition is trying to verify the contribution of each shock to those variables of interests. Because of the non-stationary technology shock, the variance decomposition here cannot be computed as the sum of squared impulse response function. Instead the variance decomposition, which is based on the estimated model with the corresponding parameter estimated in the exogenous stochastic process, is computed by the following steps: 1. Get the residuals and innovations from the estimated model. 2. Bootstrapping each shock to get the simulations of endogenous variables, other shock will be treated as zero when bootstrapping every single shock. 3. Compute the variance of simulations. In this way, the variance can be a measurement of the contribution of each shock. Then the proportion will be the ratio of contribution from each shock to the overall variance, where the overall variance is the sum of all variance from those structure shocks by bootstrapping.

Table 4.4 shows the variance of decomposition in detail, here we focus on the output, interest rate, inflation, consumption and real exchange rate with different time scale. Not surprisingly, output is highly influenced by the technology with around 90% of the contribution in the fluctuation is from the productivity shock, in all time scales. Apart from that, the consumption preference shock would take the role of the second largest contribution with around 4% in short run and 8% in long run. Government spending, export demand and foreign interest rate have little impact on the output, but foreign inflation would contribute to the fluctuation in output about 3.75% in the long run. From the short run with 1 year to long run with 25 years, the proportion from productivity would decrease a little and contribution from consumption preference shock is slightly increased.

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Another interesting point is the variance decomposition on the domestic interest rate. From the table, we can see that almost all fluctuation in the domestic interest rate is due to the foreign interest rate shock, while only consumption preference shock can contribute by 0.01% in short run and 0.5% in long run. The explanation is that the currency board runs the fixed exchange rate, the Hong Kong case links its currency to US dollar at a fixed rate. This arrangement would make the domestic economy have no choice, but strictly follow the interest rate of its anchor economy. Because by theory, any difference in the interest rate would have pressure on the exchange rate to move. When it comes to the inflation, all shocks are significant except for government spending, housing demand shock and export demand. The most important one to inflation is consumption preference shock, from 75% in 1 year to 61.99% in 25 years long.

Consumption would mostly be affected by the consumption preference shock and then the productivity. Although consumption preference shock contributes to the consumption the most, but it becomes less important from short run to long run while productivity gains more importance and takes the leading role. Then, the key variable of relative price, the real exchange rate. Only government spending and housing demand are not significant at all, the rest shocks would have different stories in short run and long run. Firstly, consumption preference explains the majority and the proportion is steady between 76% to 85% in both short run and long run. Secondly, productivity has significant increase in the contribution in long run, while foreign interest rate take the second important role in fluctuation of real exchange rate but it becomes less significant in the long run.

Table 4.5: Variance Decomposition (Long Run)

Variable	Preference	Housing	Productivity	Export	Government	Foreign Inflation	Foreign Interest
<i>25 Years</i>							
Output	7.58%	0.00%	88.31%	0.06%	0.02%	3.75%	0.28%
Interest Rate	0.69%	0.00%	0.03%	0.00%	0.00%	0.09%	99.19%
Inflation	61.99%	0.00%	0.69%	0.83%	0.06%	31.35%	5.09%
Consumption	39.48%	0.00%	56.39%	0.05%	0.07%	2.73%	1.28%
RxR	76.02%	0.02%	7.28%	0.80%	0.08%	10.35%	5.45%
<i>15 Years</i>							
Output	7.12%	0.00%	89.29%	0.07%	0.02%	3.22%	0.28%
Interest Rate	0.37%	0.00%	0.01%	0.00%	0.00%	0.05%	99.57%
Inflation	63.92%	0.00%	0.65%	0.68%	0.05%	30.53%	4.18%
Consumption	49.04%	0.00%	47.47%	0.04%	0.06%	1.89%	1.50%
RxR	82.05%	0.02%	3.03%	0.74%	0.07%	9.00%	5.08%

Table 4.6: Variance Decomposition (Short Run)

Variable	Preference	Housing	Productivity	Export	Government	Foreign	Foreign
					Inflation	Inflation	Interest
<b>5 Years</b>							
Output	4.58%	0.00%	93.30%	0.15%	0.02%	1.52%	0.42%
Interest Rate	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	99.89%
Inflation	63.04%	0.00%	0.58%	0.64%	0.05%	30.85%	4.84%
Consumption	64.32%	0.00%	29.79%	0.02%	0.07%	2.78%	3.01%
RxR	83.52%	0.01%	0.92%	0.80%	0.08%	8.62%	4.23%
<b>1 Year</b>							
Output	5.23%	0.00%	93.42%	0.19%	0.03%	0.64%	0.49%
Interest Rate	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	99.99%
Inflation	75.88%	0.00%	0.41%	0.58%	0.04%	17.15%	5.95%
Consumption	67.87%	0.00%	23.94%	0.01%	0.04%	3.42%	4.72%
RxR	84.02%	0.00%	0.94%	0.67%	0.06%	6.79%	7.53%

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### 4.5.3 Historical Decomposition

In this section, I turn to investigate how much historical contribution to output, interest rate by each shock. Similar to variance decomposition, I apply the actual residual and innovations from estimated structure model to bootstrap the simulation. The difference is that I use the full sample of one shock and set all other shocks to zero when I evaluate the contribution for that shock. Step 1, get residuals and innovations from the estimated structure model. Step 2, starting with the initial value of each variable, accumulate the variable by adding the shock on it at each period, so that the value of the contribution at each time is based on the previous contribution plus the shock from this period. Step 3, repeat the process in step 2 for all shock and the total effect is the sum of all at each period.

From figure 4.9, similar to variance decomposition, we can see that productivity contributes to the fluctuation in output the most. Output is significantly determined by the productivity in Hong Kong that the Asian Financial Crisis draws an overwhelming hit to the productivity and then to the output. Although the output recovered afterwards it suffers another small hit in the 2008 Global Financial crisis. It worth noticing that in the long recovering periods, there are some decline in 2002 and 2003 which is because of the SARS. Hong Kong had the second largest amount of confirmed cases with SARS in the world, which significantly and negatively affected the economic activities.

The interest rate increases a lot during the Asian Financial Crisis , which is similar in Figure 4.10. In order to response to large amount of hedge funds and speculative attack to Hong Kong dollar, the Hong Kong Monetary Authority increased the overnight HIBOR. But later, the authority realised that the excessive high interest rate would have extraordinary negative impact to the stock market. Therefore, they turned to use the foreign exchange intervention only to cool down the money market, instead



of the high interest rate, trying to reduce the expectation on HK dollar depreciation. This operation has been seen as a successful case to support the exchange rate and stock market. Another time point is the 2008 Global Financial Crisis, Hong Kong interest rate decreased a lot because of the expansionary monetary policy in the US. To maintain the fixed exchange rate and currency board arrangement, Hong Kong monetary authority had to decrease the interest rate as well. The same as in the variance decomposition, foreign interest rate shock is the dominant one in the historical fluctuation of Hong Kong interest rate.

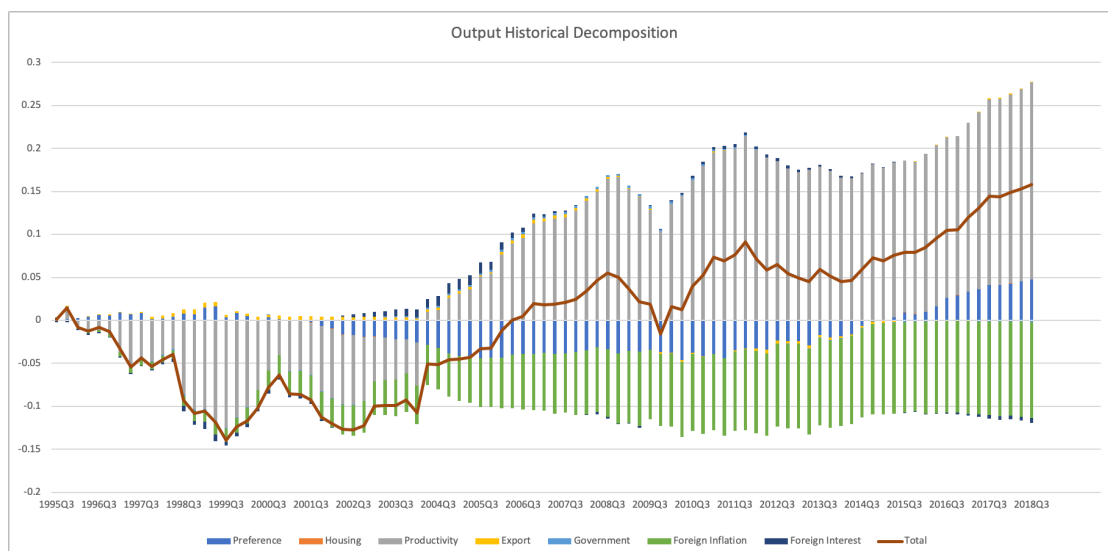


Figure 4.9: Historical Decomposition of Output

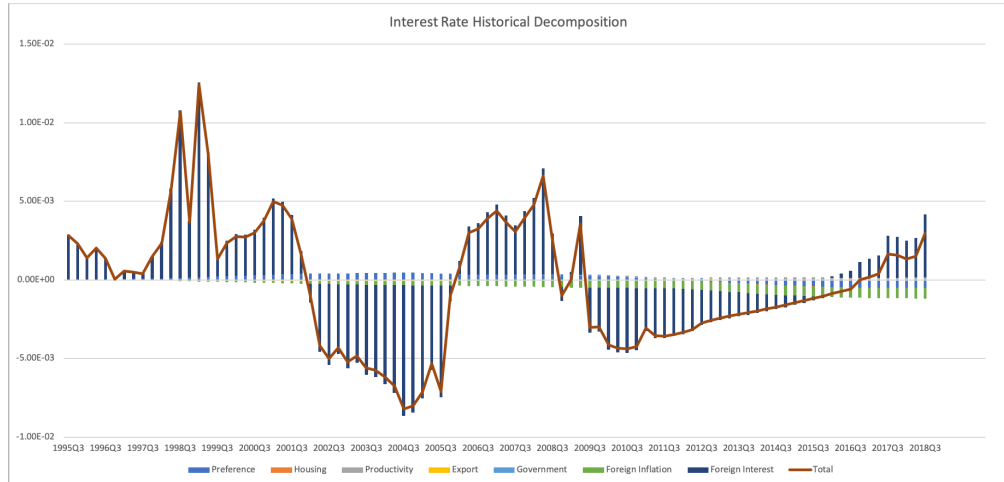


Figure 4.10: Historical Decomposition of Interest Rate

## 4.6 Conclusion

This chapter evaluates the model discussed in the previous chapter, the calibrated model with full exchange intervention cannot pass the indirect inference test. However, it is not an evidence that the model fail to match the Hong Kong data because calibrated parameters are borrowed from past literatures. Therefore, it is necessary to estimate the full exchange intervention model by indirect inference to search for a set of parameters that could fit the Hong Kong data, as well as matching the theories. The impulse responses from estimation are diagnostics to prove and display the working and transmission of the model, while the test result from the estimated model proves the model can fit the Hong Kong data well. Stochastic variance decomposition shows that productivity is the driven force of output, interest rate fluctuation is from foreign interest rate, which is consistent with the currency board and fixed exchange rare regime, while consumption preference shock contributes the most to inflation, consumption and real exchange rate. Historical decomposition generates similar results in variance decomposition and it can replicate the historical data behaviour.

## 4.A Data Source

Table 4.7: Data Source or Derivation

Variable	Notation	Code or Source	Definition
$C^c$	Households Consumption	HKCNPFR	Private consumption expenditure, HKD, CP, SA
$Y$	GDP output	HKGDP	GDP, HKD, CP, SA
$I^k$	Capital Investment	HKGFCF	Gross fixed capital information.exclude Housing investment, HKD, CP, SA
$I^h$	Housing Investment	HKCONIESC	Gross fixed capital information-Construction, HKD, CP, SA
$G$	Government Spending	HKCNGOV	Government consumption expenditure, HKD, CP, SA
$EX$	Export	HKEXNGS	Export, HKD, CP, SA
$IM$	Import	HKIMNGS	Import, HKD, CP, SA
$K$	Capital	/	Derived from equation
$H$	Housing	/	Derived from equation, scaled with the actual market size in 2017
$M^s$	Money Supply	HKXMON0	M0, HKD, CP, SA
$Z$	Total NFA	HKXNFA	Net foreign Assets owned by public and government, USD, CoP, NSA
$B^f$	Private NFA	/	Derived from Total NFA by excluding the government NFA, USD, CoP, SA
$CY$	Borrowing Rate	HKQ60P	Bank lending rate/4, Quarterly
$NW$	Net Worth	HNGKNGI	Hang Seng share index/GDP deflator, SA, Price index

Variable	Notation	Code or Source	Definition
$P^d$	Home Price	HKGDPIPDE	GDP deflator, Price index
$\pi^d$	Home Inflation	/	Quarterly percentage change in GDP deflator
$P$	CPI	HKCPI	Consumer Price Index, SA, Price index. 2017=100
$\pi$	CPI Inflation	/	Quarterly percentage change in CPI
$P^f$	Foreign Price	USQCP009F	US Consumer Price Index all item, SA, Price index, 2017=100
$\pi^f$	Foreign Inflation	/	Quarterly percentage change in US CPI
$r$	Domestic Interest Rate	HKMA	Three month HIBOR/4, Quarterly
$r^f$	Foreign Interest Rate	USGBILL3	Three month US Treasury bill rate/4, Quarterly
$C^f$	Foreign Consumption	US BEA	US GDP, CoP, SA
<i>Premium</i>	Credit Premium	/	Difference between bank lending rate and HIBOR
$P^h$	Housing Price	HKBPPCN	Residential property price, Price index, SA
$q^k$	Capital Price	/	Derived from investment Euler equation
$R^k$	Capital Rent	/	Derived from equation
$N$	Labour Force	HKEMPTOTP	Total employment, SA
$W$	Wage	HKXWCMF.F	Manufacturer unit wage cost, SA. 2017=100
$Q$	Real Exchange Rate	HKBISRNR	Inverse of real effective exchange rate, SA. 2017=100

† CP=Constant Price, CoP=Current Price, SA=Seasonal Adjusted, NSA=Not Seasonal Adjusted

‡ HKMA=Hong Kong Monetary Authority, US BEA=US Bureau of Economic Analysis

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## 4.B Impulse Response from Estimated Model

Figure 4.11: Estimated Response to 1% Consumption Preference Shock

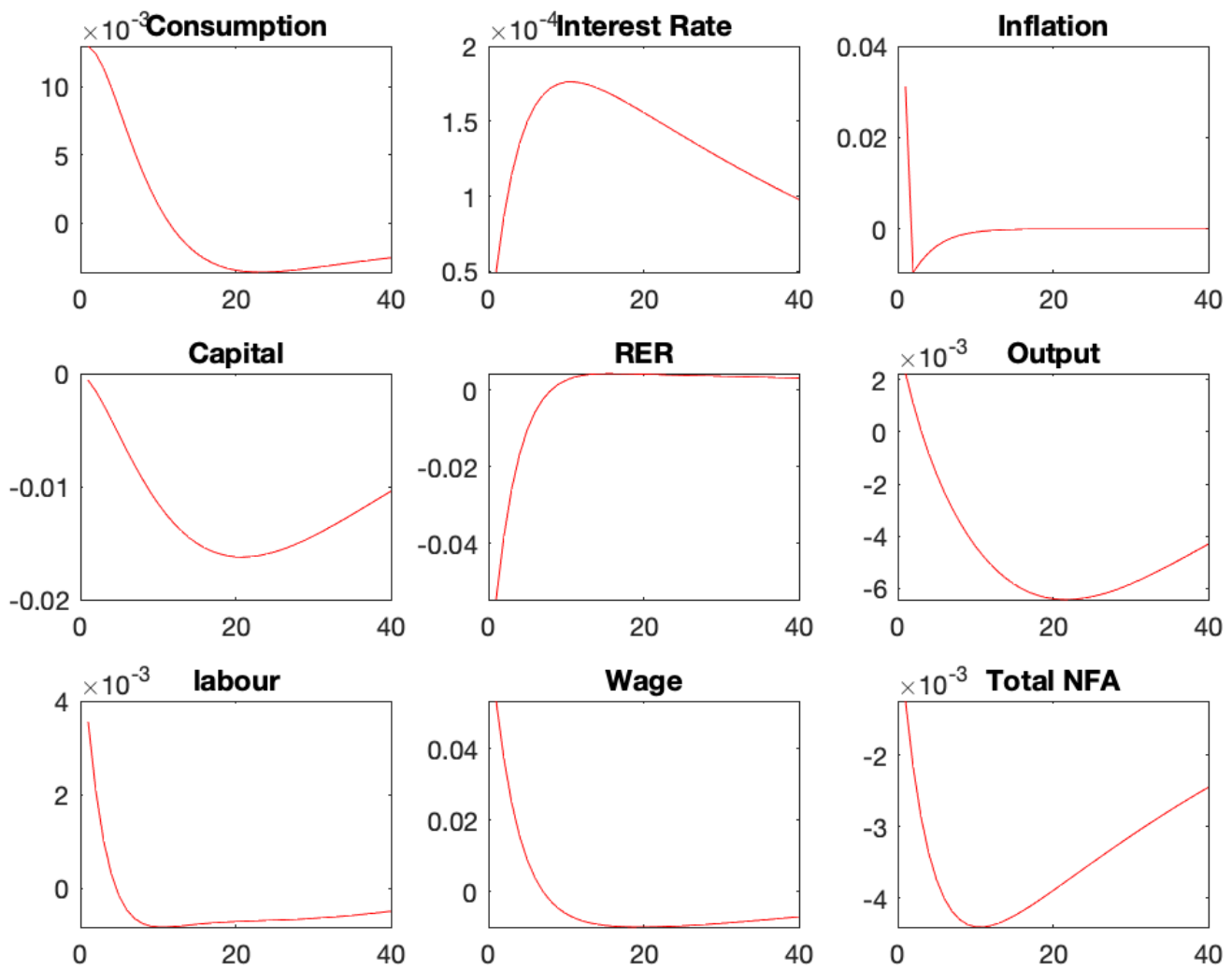


Figure 4.12: Estimated Response to 1% Housing Demand Shock

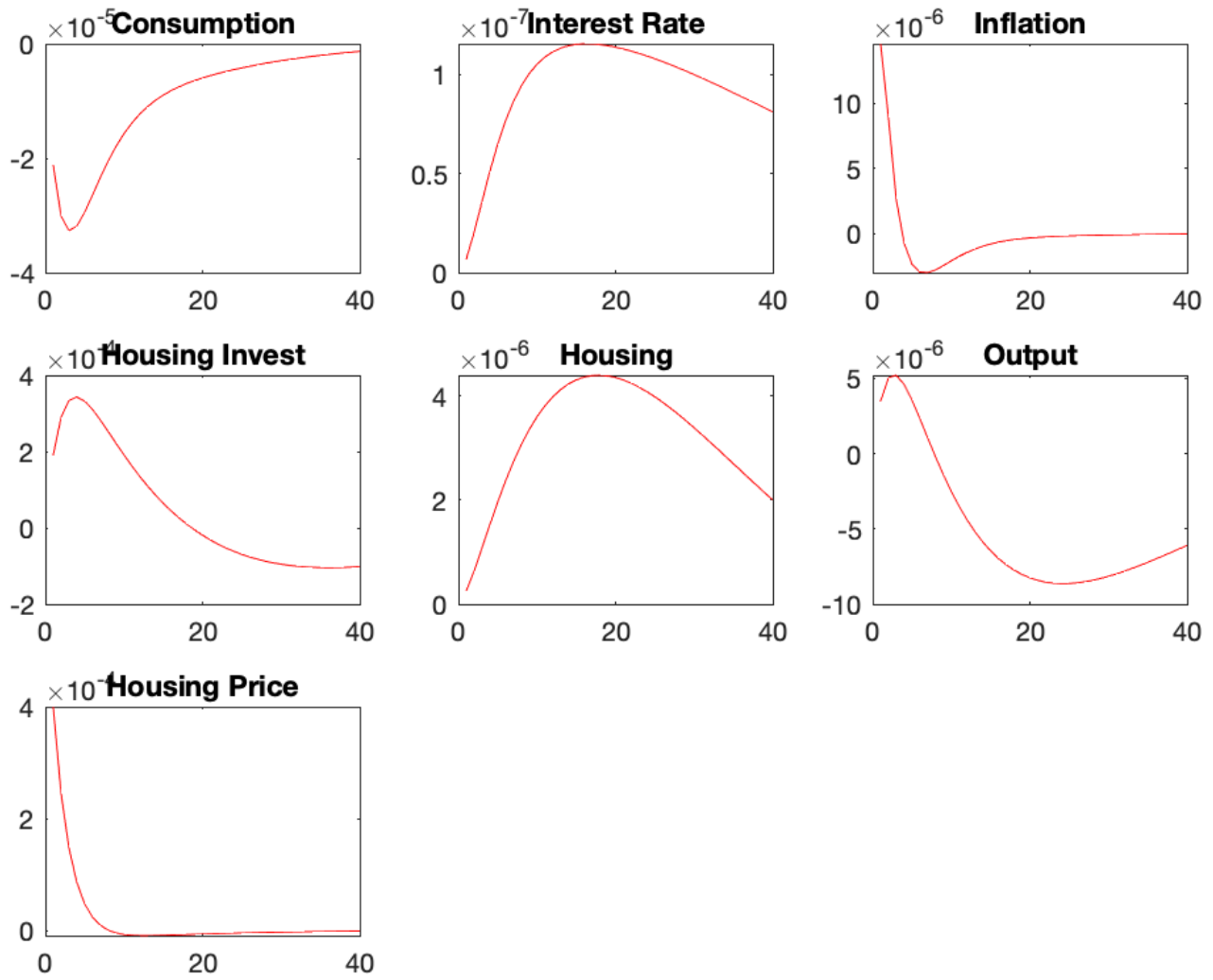


Figure 4.13: Estimated Response to 1% Non-Stationary Productivity Shock

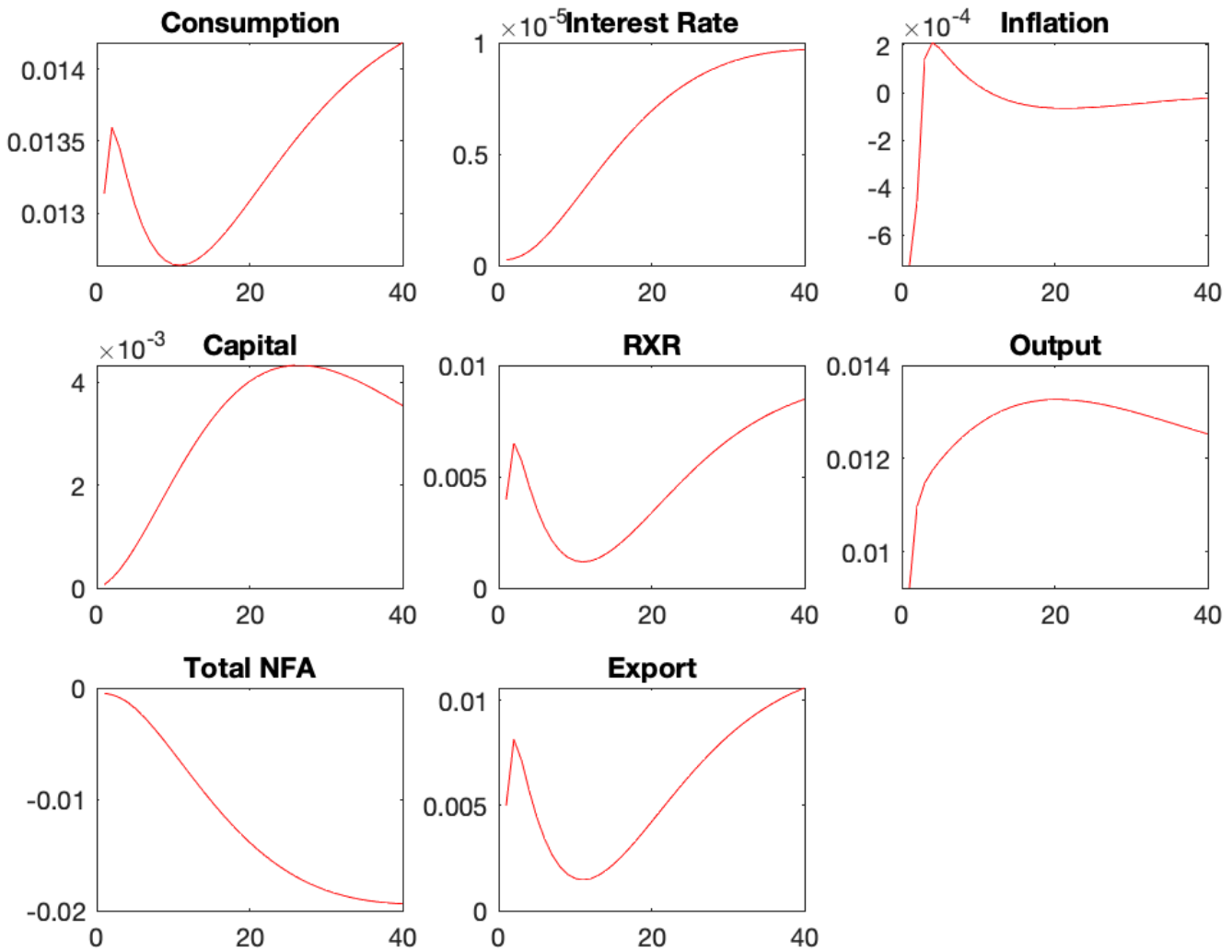


Figure 4.14: Estimated Response to 1% Government Spending Shock

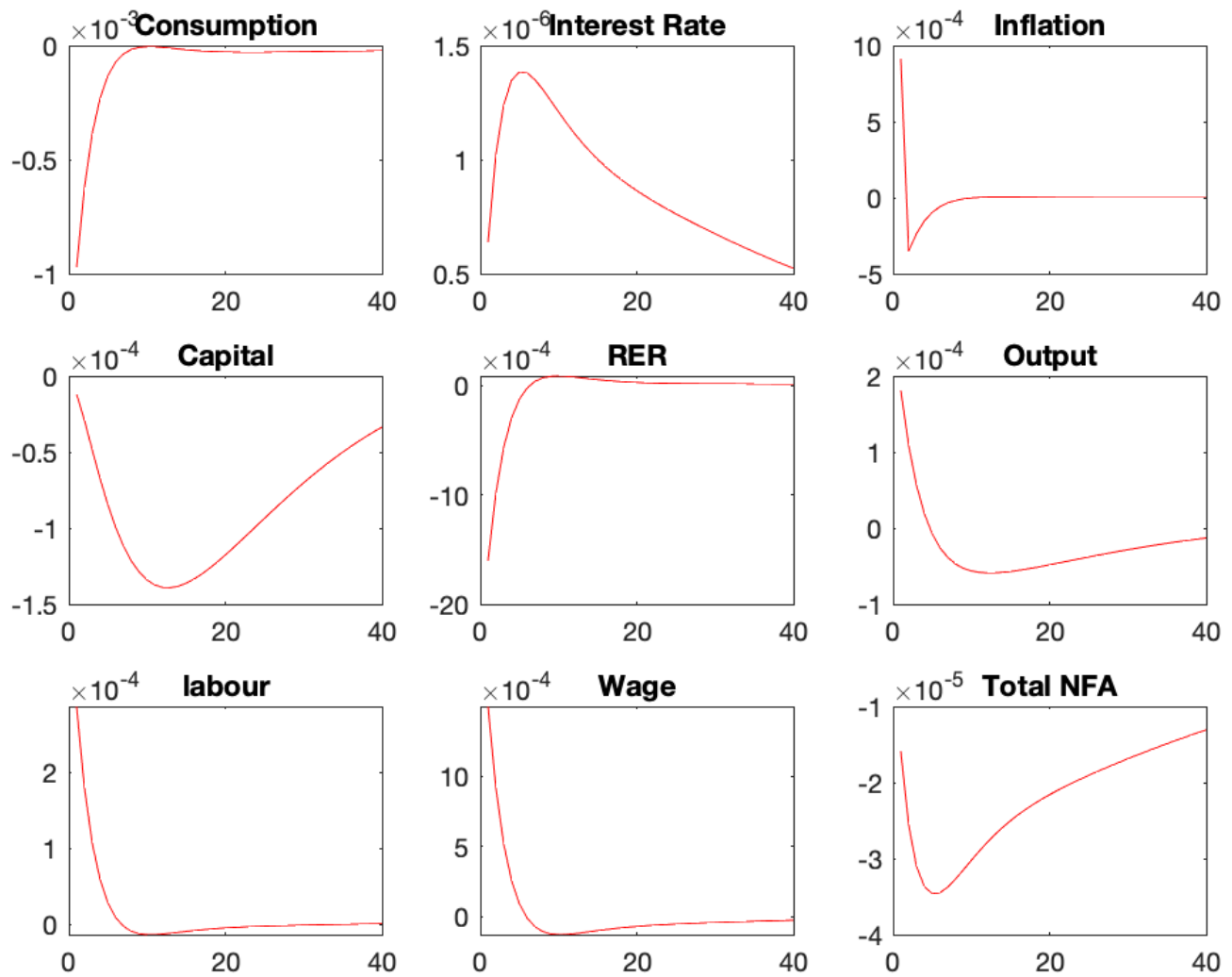




Figure 4.15: Estimated Response to 1% Foreign Consumption Shock

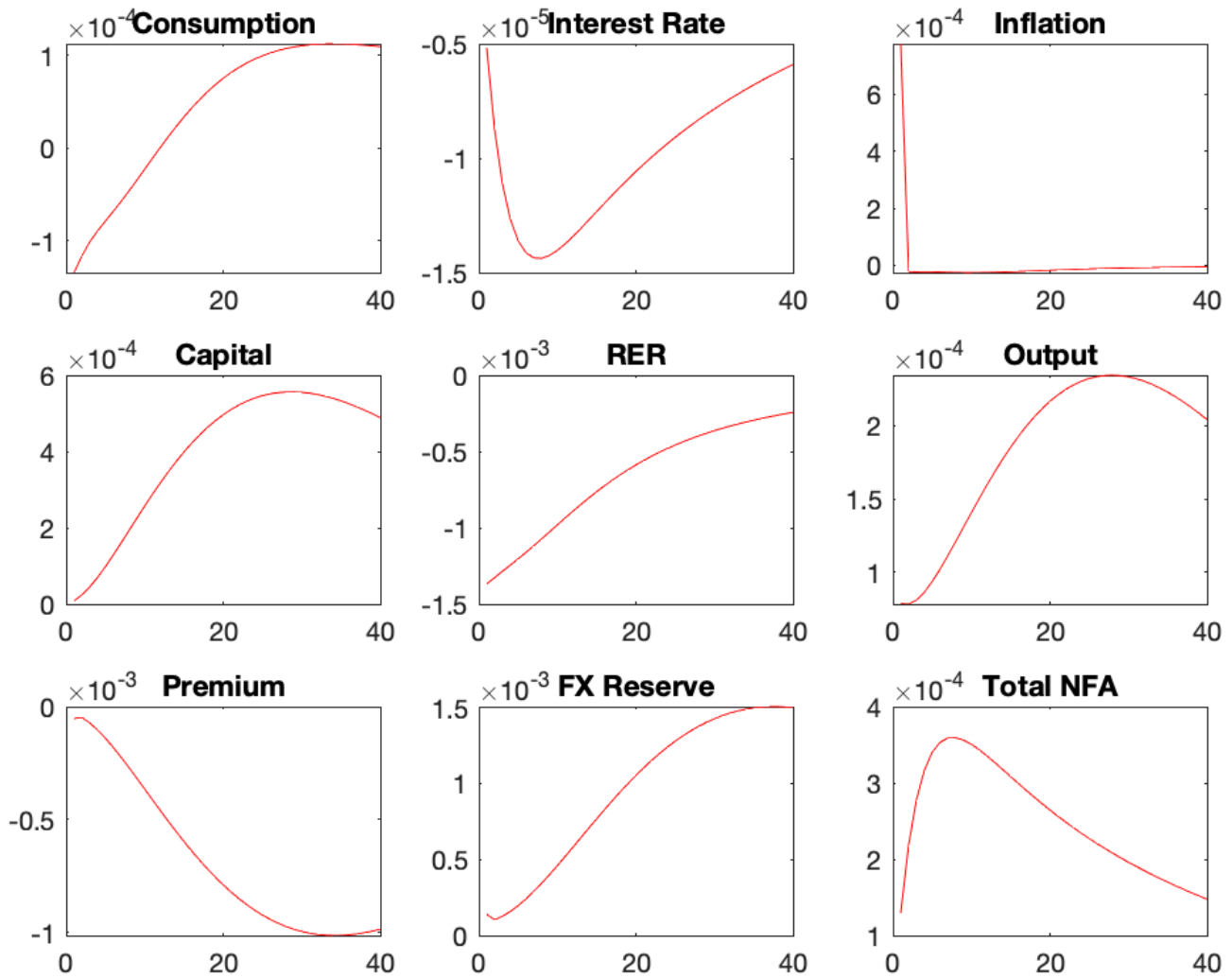
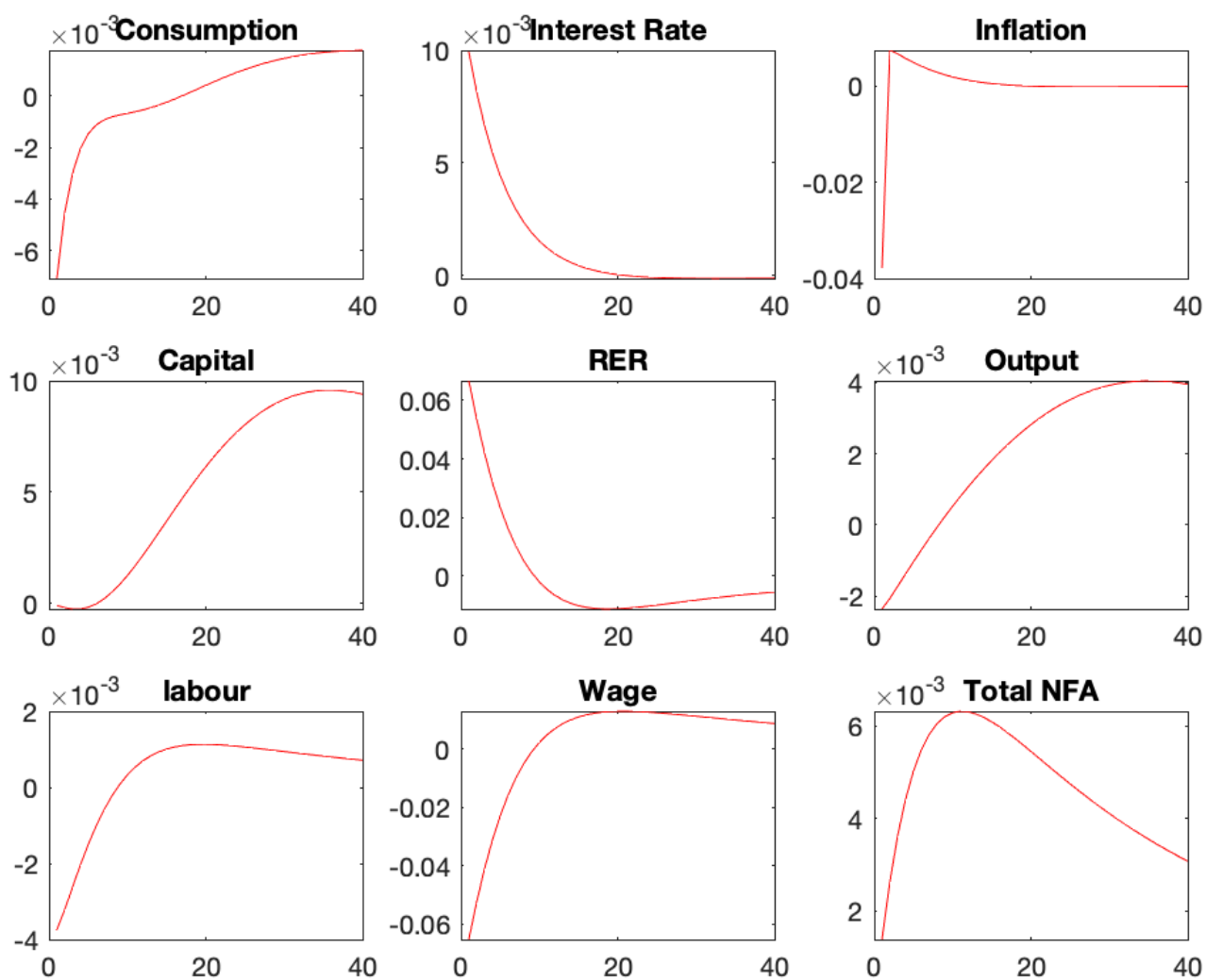


Figure 4.16: Estimated Response to 1% Foreign Interest Rate Shock



## 4.C Historical Decomposition

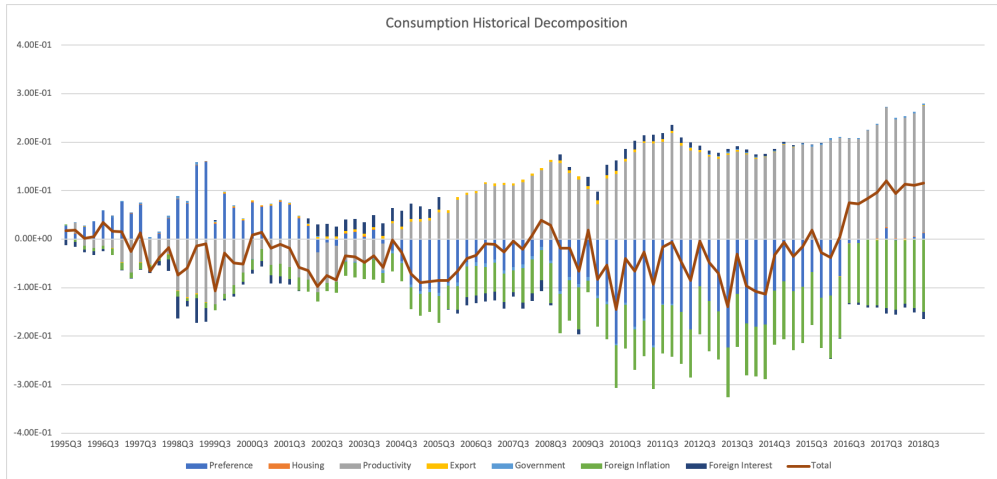


Figure 4.17: Historical Decomposition of Consumption

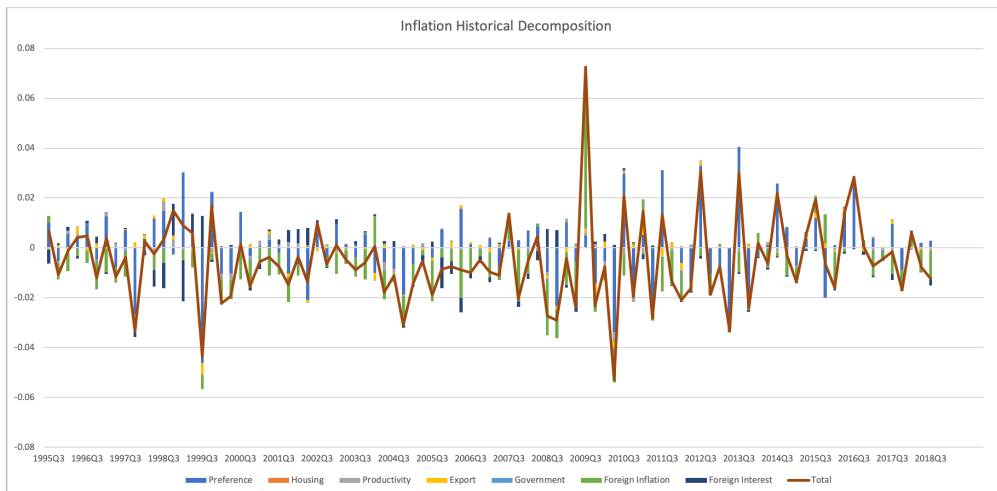


Figure 4.18: Historical Decomposition of Inflation

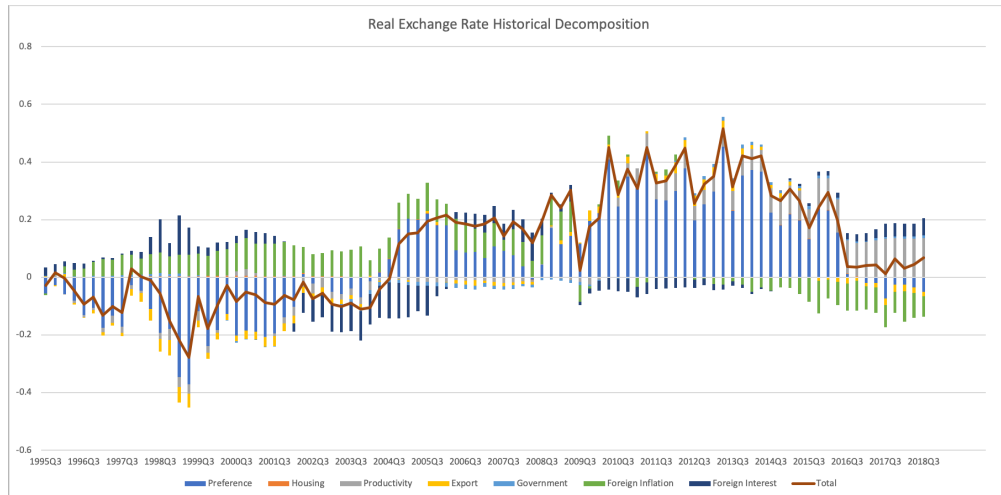


Figure 4.19: Historical Decomposition of Real Exchange Rate

## 4.D VARX(1) Parameter Distribution and Fit

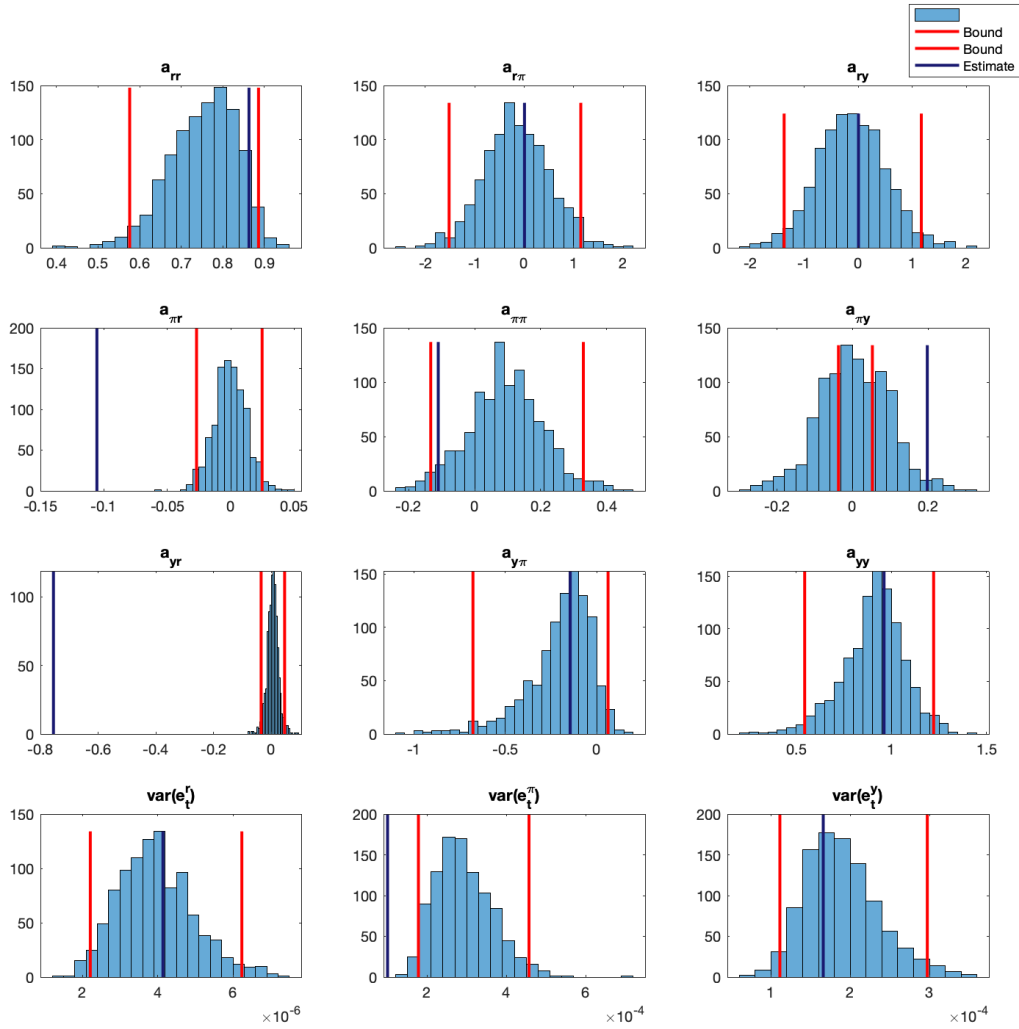


Figure 4.20: VARX(1) Parameter Distribution and Fit



## Chapter 5

# Model with Housing Collateral Constraint

### 5.1 Introduction

The benchmark model has provided an outline on how the monetary system works with the currency board framework. It passes the indirect inference test to explain the main economic activities, as well as fitting the behaviour in the financial crisis. This chapter would examine the model with housing collateral constraint. Like most Iacoviello (2005) type collateral model, the households sector is going to be split into patient and impatient group. Patient households behaves as the saver, providing funds to borrowing activity. While the impatient households borrow to cover the cost of consumption and housing purchase, the borrowing as an upper bond which cannot exceed a proportion of the housing value.

As in Iacoviello (2005), the nature of the collateral existing is from the fact that a large fraction of the loan and borrowing is secured by the value of real estate. In addition,

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we realise that housing market is significant to the business and economy fluctuation but the detail mechanism and transmission is not well understood. Together with the households heterogeneity by Kiyotaki and Moore (1995), the collateral constraint model is able to support the main argument: the rise in the housing price raises the capacity of the borrowing and the borrowers would tend to spend more and invest more. This finding is also supported by the VAR analysis that output and assets price move together by the assets price shock. Apart from the relation between housing price and output, the paper emphasises the policy issue that the economy has quite little gain in volatility if central bank responses to the housing price, even though the comovement of housing price and output is approved by the collateral constraint model.

Funke and Paets (2013) build the similar DSGE model with empirical application to Hong Kong. They find that loan-to-value ratio and housing preference shock contribute to the housing price movement the most. However, the borrower housing preference shock, which drives up the demand of housing, does not raise the corresponding borrower's consumption. The similar finding is that output is increased by the housing demand shock. In the loan-to-value ratio, Rabanal (2018) concludes the same result that a higher loan-to-value ratio has the same effect as by the housing demand shock, raising output. The higher loan-to-value ratio can raise the output because households can borrow more, spend more and investment more. Regarding to the policy issue, the tightening loan-to-value ratio is able to calm down the housing market by reducing the housing price. We assume that the cost of house production is the same as that of general production, so we do not alter our model of prices. Future work can work on the additional channel that the higher living costs of housing would transmit to the CPI through rental market, since over 30% of CPI is from the housing rent.

The housing collateral model has not been tested against the Hong Kong data with Indirect inference, those in the literature focus on policy implication and the comovement



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of housing price with other variables. In addition to the model without the collateral, which has been tested that the model without the collateral constraint can fit the Hong Kong data well, it comes with another point that: does the housing collateral model fit the Hong Kong data? Which can fit better, the model with the housing collateral or the model without this constraint? The structure of this chapter is that: First is to describe the model with collateral constraint and the calibration. Second is the testing and estimation result of the collateral model. Third is the empirical result and the comparison to the benchmark model.

## 5.2 Model Setups

The most difference from the benchmark model is the setup in households sector. This sector now contains patient households and impatient households, while the impatient households is treated as the borrower and patient households is treated as the saver. The difference between them is the discount factor that impatient households discount factor is smaller than the patient households discount factor, which also makes the collateral constraint in the impatient households sector bind in the steady state. Because the collateral constraint is in the households sector only, the rest settings of the collateral model would be kept the same as in the benchmark model in chapter three.

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## Patient Households

The representative patient households maximise the expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta_P^t [\gamma_t^c \log C_{P,t} + \gamma_t^h \log H_{P,t} - \frac{N_t^{1+\eta}}{1+\eta}] \quad (5.1)$$

Where patient households' utility is from current consumption  $C_{P,t}$ , housing  $H_{P,t}$  and disutility from working  $N_t$ . Here are inverse elasticity of labour  $\eta$ , consumption shock  $\gamma_t^c$ , housing demand shock  $\gamma_t^h$ . These two shocks follow AR(1) process with i.i.d normal distribution.

This maximisation problem is subject to households' budget constraint:

$$\begin{aligned} P_t C_{P,t} + P_t I_t^k + P_t^h [H_{P,t} - (1 - \delta^h) H_{P,t-1}] + D_t + B_t + S_t B_t^f = W_t N_t + R_t^k K_{t-1} + (1 + R_{t-1}) D_{t-1} \\ + (1 + R_{t-1}) B_{t-1} + (1 + R_{t-1}^f) S_t \phi_{t-1} B_{t-1}^f + T_t \end{aligned} \quad (5.2)$$

and capital accumulation function with investment adjustment cost:

$$K_t = (1 - \delta^k) K_{t-1} + [1 - S(\frac{I_t^k}{I_{t-1}^k})] I_t^k \quad (5.3)$$

$\kappa^k$  is a parameter measures the adjusting investment cost where is the cost is  $S(\frac{I_t^k}{I_{t-1}^k}) = \frac{\kappa^k}{2} (\frac{I_t^k}{I_{t-1}^k} - 1)^2$ , while  $S(1) = S'_k(1) = 0, S''_k(1) = \kappa^k$ .

For every period, households buy consumption goods, make investment decisions and purchase new housing with a relative housing price  $q_t^h = \frac{P_t^h}{P_t}$ , make deposit  $D_t$ , while purchasing domestic and foreign bonds. At the same time, households receive wage  $W_t$  from working, return from physical capital rent, return from deposit, domestic bonds and foreign bonds with their rates  $R_{t-1}, R_{t-1}^f$  respectively, where money is not interest-

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bearing.  $T_t$  is the lump-sum transfer. To ensure there is a well-defined steady state, this model follows Schmitt-Grohe and Uribe (2003), as well as Adolfson et.al (2007) that there is a risk premium which depends on the ratio of net foreign assets position.  $S$  is the nominal exchange rate and to be set at fixed for a currency board and fixed exchange rate regime.

$$\phi_t = \exp[-\phi_a(Z_t - \bar{Z})] \quad (5.4)$$

where  $\phi$  is the elasticity of country risk premium,  $Z_t$  is total foreign assets position contains the foreign bonds held in the public and those foreign reserve held in the monetary authority, which is  $Z_t = B_t^f + F_t$ .

By choosing  $C_{P,t}, H_{P,t}, I_t^k, K_t, N_t, D_t, B_t, B_t^f$ , FOCs of households are as following:

$$C_{P,t} : \lambda_t^P = \frac{\gamma_t^c}{P_t C_{P,t}} \quad (5.5)$$

$$I_t^k : q_t^k [1 - S(\frac{I_t^k}{I_{t-1}^k}) - S'(\frac{I_t^k}{I_{t-1}^k}) \frac{I_t^k}{I_{t-1}^k}] + \beta_P E_t [\frac{\lambda_{t+1}^P}{\lambda_t^P} q_{t+1}^k S'(\frac{I_{t+1}^k}{I_t^k}) (\frac{I_{t+1}^k}{I_t^k})^2] = 1 \quad (5.6)$$

$$K_t : q_t^k = \beta_c E_t \frac{\lambda_{t+1}^P}{\lambda_t^P} [(1 - \delta^k) q_{t+1}^k + R_{t+1}^k] \quad (5.7)$$

$$H_{P,t} : \frac{\gamma_t^h}{H_{P,t}} = \lambda_t^P P_t^h - \beta_P E_t \lambda_{t+1}^P P_{t+1}^h (1 - \delta^h) \quad (5.8)$$

$$N_t : N_t^\eta = \lambda_t^P W_t \quad (5.9)$$

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$$D_t : \lambda_t^P = \beta_c E_t \lambda_{t+1}^P (1 + R_t) \quad (5.10)$$

$$B_t : \lambda_t^P = \beta_P E_t \lambda_{t+1}^P (1 + R_t) \quad (5.11)$$

$$B_t^f : \lambda_t^P = \beta_P E_t \lambda_{t+1}^P (1 + R_t^f) \phi_t \frac{S_{t+1}}{S_t} \quad (5.12)$$

The Euler equation for consumption can be given by combining (5) and (10):

$$\frac{\gamma_t^c}{C_{P,t}} = \beta_c E_t \frac{\gamma_{t+1}^c}{C_{P,t+1}} \frac{(1 + R_t)}{\pi_{t+1}} \quad (5.13)$$

The optimal condition for housing is from (5) and (8):

$$\frac{\gamma_t^h}{H_{P,t}} = \frac{\gamma_t^c}{C_{P,t}} q_t^h - \beta_c E_t \frac{\gamma_{t+1}^c}{C_{P,t+1}} q_{t+1}^h (1 - \delta^h) \quad (5.14)$$

Given (5) and (9), the intratemporal condition yields. This condition equates that the marginal substitution between consumption and leisure is as a price, the nominal wage.

$$N_t^\eta C_{P,t} = \frac{W_t}{P_t} \gamma_t^c \quad (5.15)$$

The international no arbitrage condition can be taken from (11) and (12):

$$E_t \left( \frac{1 + R_t}{\pi_{t+1}} \right) = E_t \left( \frac{(1 + R_t^f) \phi_t}{\pi_{t+1}} \right) \frac{S_{t+1}}{S_t} \quad (5.16)$$

$\phi_t$  is the country risk premium discussed in equation (4) which depends on the net foreign

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assets position and a risk premium shock. On one hand, it is to explain the fact the lenders would require higher return with those countries in higher debt position. On the other hand, it is to avoid misspecification and singularity problem in closing the model.

The UIP in log-linearised:

$$\hat{r}_t = \hat{r}_t^f + \Delta S_{t+1} - \phi \hat{z}_t$$

As the Hong Kong has fixed exchange rate,  $\Delta S_{t+1} = 0$ , the UIP is:

$$\hat{r}_t = \hat{r}_t^f - \phi \hat{z}_t$$

### Impatient Households

The representative impatient household maximises the expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta_I^t [\gamma_t^c \log C_{I,t} + \gamma_t^h \log H_{I,t}] \quad (5.17)$$

Where patient households' utility is from current consumption  $C_{I,t}$ , housing  $H_{I,t}$ . Here are consumption shock  $\gamma_t^c$ , housing demand shock  $\gamma_t^h$ . These two shocks follow AR(1) process with i.i.d normal distribution.

This maximisation problem is subject to households' budget constraint:

$$P_t C_{I,t} + P_t^h [H_{I,t} - (1 - \delta^h) H_{I,t-1}] + (1 + R_{t-1}) L_{t-1} = L_t$$

equivalent to:

$$C_{I,t} + q_t^h [H_{I,t} - (1 - \delta^h) H_{I,t-1}] + \frac{(1 + R_{t-1}) l_{t-1}}{\pi_t} = l_t \quad (5.18)$$

and borrowing constraint:

$$L_t \leq mE_t \frac{P_{t+1}^h H_{I,t}}{1 + R_t} \quad (5.19)$$

equivalent to the real borrowing (or loan) constraint:

$$l_t \leq mE_t \frac{q_{t+1}^h \pi_{t+1} H_{I,t}}{1 + R_t}$$

by choosing  $C_{I,t}, H_{I,t}, L_t$ , the FOCs of the impatient households are:

$$C_{I,t} : \lambda_t^I = \frac{\gamma_t^c}{P_t C_{I,t}} \quad (5.20)$$

$$L_t : \beta_I E_t \lambda_{t+1}^I (1 + R_t) = \lambda_t^I - \lambda_t^{I'} \quad (5.21)$$

$$H_{I,t} : \frac{\gamma_t^h}{H_{I,t}} = \lambda_t^I P_t^h - \beta_I E_t \lambda_{t+1}^I P_{t+1}^h (1 - \delta^h) - \lambda_t^{I'} m \frac{E_t P_{t+1}^h}{1 + R_t} \quad (5.22)$$

Given equation (15) and (17), the housing condition (18) can be:

$$\frac{\gamma_t^h}{H_{I,t}} = \frac{q_t^h}{C_{I,t}} \gamma_t^c - \beta^I (1 - \delta^h) \frac{q_{t+1}^h}{C_{I,t+1}} \gamma_{t+1}^c - \left[ \frac{P_{t+1}}{C_t P_t} \gamma_t^c - \beta^I E_t \frac{1 + R_t}{C_{I,t+1}} \gamma_{t+1}^c \right] m E_t \frac{q_{t+1}^h}{1 + R_t} \quad (5.23)$$

### Production Entrepreneur Firms

$$P_t^d Y_t - W_t N_t - R_t^k K_{t-1} \quad (5.24)$$

Where  $P_t$  is the general price level,  $N_t$  is labour and  $K_{t-1}$  is capital. The corresponding nominal wage and rental rate are  $W_t$  and  $R_t^k$ .

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Subject to the following production technology:

$$Y_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \quad (5.25)$$

Here  $A_t$  is the technology process follows ARIMA(1,1,0) process, the log-linearised equation:

$$\hat{A}_t - \hat{A}_{t-1} = \rho_a(\hat{A}_{t-1} - \hat{A}_{t-2}) + \varepsilon_t^a \quad (5.26)$$

First order conditions of entrepreneur sector are: Marginal production of labour and labour demand:

$$\frac{W_t}{P_t^d} = (1 - \alpha) A_t K_{t-1}^\alpha N_t^{-\alpha} \quad (5.27)$$

Marginal production of capital and capital demand:

$$\frac{R_t^k}{P_t^d} = \alpha A_t K_{t-1}^{\alpha-1} N_t^{1-\alpha} \quad (5.28)$$

the modified credit premium equation in this thesis is:

$$E_t c y_{t+1} - (R_t - E_t \pi_{t+1}) = \chi(q_t^k + k_t - n w_t) - \mu m_t^d \quad (5.29)$$

The money demand is from the firm's balance sheet that firm holds money as collateral to its borrowing to finance the cost of capital, in the form as money to capital demand ratio together with the firm's net worth:

$$m_t^d = (1 + \xi) k_t - \xi n w_t \quad (5.30)$$

where  $\xi$  is the net worth to money ratio in steady state.

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Net worth evolution:

$$nw_t = \nu nw_{t-1} + \frac{K}{NW} (cy_t - E_{t-1}cy_t) + E_{t-1}cy_t \quad (5.31)$$

Entrepreneur consumption:

$$c_t^e = nw_t \quad (5.32)$$

### Housing Producer

$$\max E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^I [q_t^h (H_t - (1 - \delta^h)H_{t-1}) - I_t^h] \quad (5.33)$$

subject to the law of motion in housing:

$$H_t = (1 - \delta^h)H_{t-1} + [1 - \frac{\kappa^h}{2} (\frac{I_t^h}{I_{t-1}^h} - 1)^2] I_t^h \quad (5.34)$$

This dynamic profit maximisation problem can be solved with the real price of housing

$q_t^h = \frac{P_t^h}{P_t}$ :

$$q_t^h [1 - S(\frac{I_t^h}{I_{t-1}^h}) - S'(\frac{I_t^h}{I_{t-1}^h}) \frac{I_t^h}{I_{t-1}^h}] + \beta_P E_t [\frac{\lambda_{t+1}^I}{\lambda_t^I} S'(\frac{I_t^h}{I_{t-1}^h}) (\frac{I_t^h}{I_{t-1}^h})^2] = 1 \quad (5.35)$$

### Export, import, total net foreign assets

Domestic demand for foreign goods, which is hence the import demand:

$$IM_t = (1 - \omega) (\frac{SP_t^f}{P_t})^{-\theta} c_t \quad (5.36)$$

$P_t^d$  is the domestic goods price,  $P_t^f$  is the foreign price in domestic currency, leading to



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the consumer price index (CPI):

$$P_t = [\omega(P_t^d)^{1-\theta} + (1-\omega)(P_t^f)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (5.37)$$

Symmetrically, the export demand, or the foreign demand for domestic goods can be given as:

$$EX_t = (1-\omega^f)\left(\frac{P_t}{SP_t^f}\right)^{\theta^f} C_t^f \quad (5.38)$$

Total net foreign assets:

$$Z_t = (1 + R_{t-1}^f)Z_{t-1} + \frac{EX}{Q_t} - IM_t \quad (5.39)$$

$Q_t$  for real exchange rate.

$$Z_t = B_t^f + F_t \quad (5.40)$$

Foreign exchange intervention:

$$S_t F_t = M_t^s = M_t^d \quad (5.41)$$

## 5.3 Log-linearisation

### i. Patient Households

Consumption Euler Equation:

$$\hat{c}_t^p = E_t \hat{c}_{t+1}^p - (\hat{R}_t - E_t \pi_{t+1}) + \hat{\gamma}_t^c$$

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Capital Investment:

$$\hat{i}_t^k = \frac{1}{1 + \beta_p} \hat{i}_{t-1}^k + \frac{\beta_p}{1 + \beta_p} E_t \hat{i}_{t+1}^k + \frac{1}{\kappa^k (1 + \beta_p)} \hat{q}_t^k$$

Capital tobin's q:

$$\hat{q}_t^k = \beta_p (1 - \delta^k) E_t \hat{q}_{t+1}^k + [1 - \beta_p (1 - \delta^k)] E_t \hat{r}_{t+1}^k - (\hat{R}_t - E_t \pi_{t+1})$$

House Demand:

$$[1 - \beta_p (1 - \delta^h)] (\hat{\gamma}_t^h - \hat{h}_{p,t}) = \hat{q}_t^h - \hat{c}_t^p - \beta_p (1 - \delta^h) E_t (\hat{q}_{t+1}^h - \hat{c}_{t+1}^p)$$

Labour Supply

$$\eta m_{p,t} \hat{c}_t^p + \hat{\gamma}_t^c = \hat{w}_t - \hat{p}_t$$

Nominal UIP with Risk Premium:

$$\hat{r}_t = \hat{r}_t^f - \phi_a \hat{z}_t$$

Capital Accumulation:

$$\hat{k}_t = (1 - \delta^k) \hat{k}_{t-1} + \delta^k \hat{i}_t^k$$

## ii. Impatient Households

Housing Demand:

$$\begin{aligned} [1 - \beta_I (1 - \delta^h) - m + \beta_I m] (\hat{\gamma}_t^h - \hat{h}_{I,t}) = & \hat{q}_t^h + (1 - m) (\hat{\gamma}_t^c - \hat{c}_{I,t}) + (\beta_I \delta^h - m) E_t (\hat{q}_{t+1}^h - \hat{c}_{I,t+1}) \\ & + \beta_I \delta^h \hat{\gamma}_{t+1}^c - m (\hat{R}_t - E_t \pi_{t+1}) \end{aligned}$$

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Consumption:

$$\frac{\bar{C}_I}{\bar{Y}} c_{I,t} + \frac{\bar{q}_h \bar{H}_I}{\bar{Y}} [\delta^h \hat{q}_t^H + h_{I,t} - (1 - \delta^h) h_{I,t-1}] + \frac{\bar{L}(1 + \bar{r})}{\bar{Y}} (r_{t-1} + l_{t-1}) = \frac{\bar{L}}{\bar{Y}} \hat{l}_t$$

Borrowing:

$$\hat{l}_t = E_t q_{t+1}^h + h_{I,t} - (\hat{R}_t - E_t \pi_{t+1})$$

### iii. Entrepreneurs

Production Function:

$$\hat{Y}_t = \hat{A}_t + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t$$

Labour Demand (Marginal Product of Labour):

$$\hat{A}_t - \alpha \hat{n}_t + \alpha \hat{k}_{t-1} = \hat{w}_t - \hat{p}_t^d$$

Marginal Product of Capital:

$$\hat{A}_t + (1 - \alpha) \hat{n}_t + (\alpha - 1) \hat{k}_{t-1} = \hat{r}_t^k - \hat{p}_t^d$$

Credit premium:

$$E_t c y_{t+1} - (R_t - E_t \pi_{t+1}) = \chi (\hat{q}_t^k + \hat{k}_t - n \hat{w}_t) - \mu \hat{m}_t^d$$

Net worth evolution

$$n \hat{w}_t = \frac{\bar{K}}{\bar{N}\bar{W}} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t + \nu n \hat{w}_{t-1}$$

---

Money demand from entrepreneur:

$$\hat{m}_t^d = (1 + \xi)\hat{k}_t - \xi n\hat{w}_t$$

Entrepreneur consumption:

$$\hat{c}_t^e = n\hat{w}_t$$

#### iv. Housing Producer

Housing investment:

$$\hat{i}_t^h = \frac{1}{\kappa^h(1 + \beta_I)}\hat{q}_t^h + \frac{\beta_I}{1 + \beta_I}E_t\hat{i}_{t+1}^h + \frac{1}{1 + \beta_I}\hat{i}_{t-1}^h$$

Housing Accumulation:

$$\hat{h}_t = (1 - \delta^h)\hat{h}_{t-1} + \delta^h\hat{i}_t^h$$

#### v. Monetary Operation

Foreign Reserve intervention and Currency board balance sheet:

$$\hat{f}_t = \hat{m}_t^s$$

#### vi. Marketing Clearing and identity

Goods Market:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{c}_t + \frac{\bar{C}^e}{\bar{Y}}\hat{c}_t^e + \frac{\bar{I}^k}{\bar{Y}}\hat{i}_t^k + \frac{\bar{I}^h}{\bar{Y}}\hat{i}_t^h + \frac{\bar{G}}{\bar{Y}}\hat{g}_t + \frac{\bar{E}X}{\bar{Y}}\hat{e}x_t - \frac{\bar{I}\bar{M}}{\bar{Y}}\hat{i}m_t$$

Money Market:

$$\hat{m}_t^d = \hat{m}_t^s$$

---

Total consumption:

$$\hat{c}_t = \frac{\bar{C}_P}{\bar{C}} c_{p,t} + \frac{\bar{C}_I}{\bar{C}} c_{I,t}$$

Total housing:

$$\hat{h}_t = \frac{\bar{H}_P}{\bar{H}} h_{p,t} + \frac{\bar{H}_I}{\bar{H}} h_{I,t}$$

CPI and CPI inflation:

$$\hat{p}_t = \omega \hat{p}_t^d + (1 - \omega) \hat{p}_t^f$$

$$\pi_t = p_t - p_{t-1}$$

$$\pi_t^f = p_t^f - p_{t-1}^f$$

$$z_t = \frac{\bar{B}^f}{\bar{Z}} \hat{b}_t^f + \frac{\bar{F}}{\bar{Z}} \hat{f}_t$$

## vii. Trade

Balance of payment with foreign reserve:

$$\hat{z}_t = \bar{R} \hat{R}_{t-1}^f + (1 + \bar{R}) z_{t-1} + \frac{\bar{E}X}{\bar{Z}} (e\hat{x}_t - \hat{q}_t) - \frac{\bar{I}M}{\bar{Z}} (i\hat{m}_t)$$

$$i\hat{m}_t = -\theta \hat{q}_t + \hat{c}_t \text{ (Import Demand)}$$

$$e\hat{x}_t = \theta^f \hat{q}_t + \hat{c}_t^f \text{ (Export Demand)}$$

Real exchange rate:

$$\hat{q} = \hat{p}_t^f - \hat{p}_t^d$$

## viii. Structure Shocks Process

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Preference shock to consumption:

$$\gamma_t^c = \rho_c \gamma_{t-1}^c + \varepsilon_{c,t}$$

House demand shock:

$$\gamma_t^h = \rho_h \gamma_{t-1}^h + \varepsilon_{h,t}$$

Technology shock:

$$A_t - A_{t-1} = \rho_a (A_{t-1} - A_{t-2}) + \varepsilon_{A,t}$$

Hong Kong is a small open economy and can be treated as no effect to the rest of the world, world shock  $\varepsilon_{r^f,t}, \varepsilon_{\pi^f,t}$

$$R_t^f = \rho_{R^f} r_{t-1}^f + \varepsilon_{r^f,t}$$

$$\pi_t^f = \rho_{\pi^f} \pi_{t-1}^f + \varepsilon_{\pi^f,t}$$

Foreign consumption innovation and export demand shock  $\varepsilon_{c^f,t}$

$$c_t^f = \rho_{c^f} c_{t-1}^f + \varepsilon_{c^f,t}$$

Government spending shock  $\varepsilon_{g,t}$ :

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

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## 5.4 Calibration and Data

In order to simulate the model to see the behaviour of the economy, all parameters are calibrated according to the literatures like Smets and Wouters (2007), Bernanke et.al (1999), Funke and Paetz (2011) and Le et.al (2014) or the data. The parameter value is in summary of Table 1 and here gives some descriptions. The inverse elasticity of labour supply  $\eta$  is set to be 3. The patient households discount factor  $\beta_c$  is 0.9929, this is give the with the corresponding steady state interest 0.72%. The impatient households discount factor is set to be 0.98, where the lower value indicates that impatient households behave as the borrower and they are less patient.

Output elasticity of capital  $\alpha$  is standard to be 0.3 Capital depreciation rate  $\delta^k$  for 0.025, while the housing depreciation  $\delta^h$  is 0.01. In the bundle of consumption goods, the home bias  $\omega$  for 0.4 and elasticity between domestic goods and imported goods  $\theta$  for 1. Symmetrically, the foreign home bias  $\omega^f$  and foreign elasticity between foreign domestic goods and exported goods from home country  $\theta^f$  are 0.4 and 1 respectively. Capital adjustment cost parameters in physical capital and housing  $\kappa^k$  and  $\kappa^h$  are 6.

Most HK data are captured from the Datastream from 1994Q1 to 2018Q3, which is then taken from either the Census and Statistics Department of Hong Kong government or directly from the Hong Kong Monetary Authority, while some are from the World Bank and US Bureau of Economics Analysis, as well as the International Monetary Fund. All the data are in the constant price or those in current price in the original source will be transferred into constant price using the consumer price level. Detail of the data source is referred to the appendix in this chapter, but here would give a brief description. Since most macro model would use the quarterly data to conduct the analysis of the business cycle property, this frequency would naturally introduce the seasonal effect. To remove the seasonal effect, the original data needs to be seasonal adjusted. The most widely

used method is US Census Bureau's software package X12-ARIMA. According to Hong Kong households survey and HKMA, the share of patient households is set to be 66.6% and share of impatient households is to be 33.4%. The consumption and housing in these two types of households are calculated by these two ratios.

Table 5.1: Calibration Parameters in Collateral Model

<b>Parameter</b>	<b>Definition</b>	<b>Value</b>
$\beta_P$	Patient Households Discount Factor	0.9929
$\beta_I$	Impatient Households Discount Factor	0.98
$\eta$	inverse Elasticity of Labour Supply	3
$\alpha$	Capital Share	0.3
$\delta^k$	Capital Depreciation	0.025
$\delta^h$	Housing Depreciation	0.01
$\omega$	Domestic Home Bias	0.4
$\theta$	Elasticity between Domestic and Imported goods in Home	1
$\omega^f$	Foreign Home Bias	0.4
$\theta^f$	Elasticity between Domestic and Imported goods in Foreign	1
$\kappa^k$	Capital Investment Adjustment Cost	6
$\kappa^h$	Housing Investment Adjustment Cost	6
$\chi$	Feedback from Leverage to Finance Premium	0.05
$\mu$	Feedback from Money to Finance Premium	0.7
$\xi$	Response of Net Worth to Money ratio	0.2



Table 5.2: Steady State Ratio in Collateral Model

<b>Ratio</b>	<b>Definition</b>	<b>Value</b>
$\frac{\bar{C}^c}{\bar{Y}}$	Consumption to Output ratio	0.6367
$\frac{\bar{C}^e}{\bar{Y}}$	Entrep. Consumption to Output ratio	0.008
$\frac{\bar{I}^h}{\bar{Y}}$	Housing investment to Output ratio	0.1148
$\frac{\bar{I}^k}{\bar{Y}}$	Capital investment to Output ratio	0.1471
$\frac{\bar{G}}{\bar{Y}}$	Government Spending to Output ratio	0.1051
$\frac{\bar{EX}}{\bar{Y}}$	Total export to Output ratio	1.6803
$\frac{\bar{IM}}{\bar{Y}}$	Total import to Output ratio	1.6571
$\frac{\bar{F}}{\bar{Z}}$	Foreign Exchange Reserve to Total Foreign Assets ratio	0.1
$\frac{\bar{B}^f}{\bar{Z}}$	Foreign Bonds to Total Foreign Assets ratio	0.9
$\frac{\bar{K}}{\bar{NW}}$	Capital to Net Worth ratio	1.5
$\nu$	Survival Rate	0.9
$m$	Loan to Value Ratio	0.6
$\bar{r}$	Steady state interest rate	0.72%

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## 5.5 Indirect Inference Test and Estimation

Here I recall the Indirect Inference to do the estimation and testing. Indirect Inference Wald test is a simulation based testing, its key factors are those actual residual and innovations (shocks) obtained by model with actual data. Compare to the Bayesian, Indirect Inference do not assume or give a distribution for the parameters, giving the fact that Indirect Inference would rely much less on the priors. The process of Indirect Inference testing follows the steps in Le et.al (2012, 2016):

*Step 1. Calculate the errors of the model conditional on the observed data and calibrated parameter.*

Estimate the structural errors of the DSGE macroeconomic model, given the parameter and the observed data. The number of independent structural errors is taken to be less than or equal to the number of endogenous variables. The errors are not assumed to be normally distributed. When the equations contain no expectations the errors can simply be backed out of the equation and the data. When there are expectations which is quite often in the DSGE model, it is necessary to use the robust instrumental variables methods of McCallum (1976) and Wickens (1982), with the lagged endogenous data as instruments. Thus effectively the auxiliary model applied is VAR. An alternative method for expectations estimation is the exact method, here the model itself is to project the expectations and because these depend on the extracted residuals there is iteration between the two elements until convergence.

*Step 2. Bootstrapping the model with implied shock by many time to get a set of coefficients and simulated data.*

With those structure error get from the step 1, the simulated data can be bootstrapped. In the DSGE model, those structure shocks are assumed to be the autoregressive process. Most of those shock would be a stationary process like AR(1) with a high persistence, but

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in this thesis model, the technology shock is taken as a nonstationary process. To deal with this issue, the process of the technology is empirically set as first order difference stationary. To get N simulations, the randomly and independently drawing procedure is repeated for each sample by the projection method in Minford et.al (1983, 1986).

*Step 3. Get the Indirect Inference Wald Statistics*

To determine rejecting a model or not, we need estimated the auxiliary model, here the VAR, both by the actual data and simulated sample. By estimating the VAR, the estimates of actual data  $\alpha_T$  and estimates of simulation  $\alpha_S(\theta_S)$  can be obtained. Next is to calculate the asymptotic distribution of  $(\alpha_T - \overline{\alpha_S(\theta_S)})$ , where  $\overline{\alpha_S(\theta_S)}$  is the average of all repeated simulation samples, and the variance-covariance matrix  $\Omega$  which is estimated from bootstrapping  $\alpha_S(\theta_S)$ , the Wald Statistics is then calculated as:

$$WS = (\alpha_T - \overline{\alpha_S(\theta_S)})' \Omega (\theta_S - \overline{\theta_S})^{-1} (\alpha_T - \overline{\alpha_S(\theta_S)}) \quad (5.42)$$

In order to evaluate the fitness of model at a 95% confidence level, the Wald statistics from actual data is required to be less than the 95th percentile of the one from the simulated data. Wald statistics from simulated data follow  $\chi^2$  distribution with degree of freedom  $k - 1$  where  $k$  is the number of parameter. For convenience, the Wald can be transformed into t statistics as a transformed Mahalanobis distance by:

$$T = \left( \frac{\sqrt{2W^\alpha} - \sqrt{2k - 1}}{\sqrt{2W^{0.95}} - \sqrt{2k - 1}} \right) 1.65 \quad (5.43)$$

Where  $W^\alpha$  is the Wald statistic on the actual data and  $W^{0.95}$  is the Wald statistic on the 95th percentile of the simulated data. For a model to pass the test, T is required to be less than 1.65.

Table 5.3: Indirect Inference Estimates of the Structure Model with Housing Collateral

Parameter	Definition	Calibration	Estimation
$\alpha$	Capital Share in Production	0.3	0.2962
$\eta$	Inverse Elasticity of Labour Supply	3	5.4552
$\delta^k$	Capital Depreciation	0.025	0.0360
$\delta^h$	Housing Depreciation	0.01	0.0019
$\omega$	Domestic Home Bias	0.4	0.2873
$\theta$	Elasticity between Domestic and Imported goods in Home	1	1.0813
$\omega^f$	Foreign Home Bias	0.4	0.3442
$\theta^f$	Elasticity between Domestic and Imported goods in Foreign	1	1.2501
$\kappa^k$	Capital Investment Adjustment Cost	6	4.9879
$\kappa^h$	Housing Investment Adjustment Cost	6	2.2649
$\chi$	Feedback from Leverage to Finance Premium	0.05	0.0534
$\mu$	Feedback from Money to Finance Premium	0.7	1.0228
	Variable in the VARX(1)	Trans-W	p-value
Calibration	$Y, r, \pi$ (Output, Interest rate, Inflation)	3.8396	0.000
Estimation	$Y, r, \pi$ (Output, Interest rate, Inflation)	2.2017	0.024

Table 5.4: Comparison of Testing Result on Estimation between Base Model and Collateral Model

	Variable in the VARX(1)	Trans-W	p-value
Base	$Y, r, \pi$ (Output, Interest rate, Inflation)	1.0924	0.122
Collateral	$Y, r, \pi$ (Output, Interest rate, Inflation)	2.2017	0.024

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The results estimation of structure parameters from Indirect Inference are shown in table 5.3. In the households side, the only structure parameter is the inverse elasticity of labour supply  $\eta$ , the value has increased by 81.5% from 3 to 5.4552. This implements that households are less concern on smoothing the labour working so that when the wage moves, households would decide to change the supply of the labour more.

For the supply side and production, the capital share in the production  $\alpha$  is slightly less than 0.3. This is to say that in Hong Kong, capital is relative less applied to production. Given the fact that the value is still less than 0.5, which means the Hong Kong economy is still labour oriented. The housing depreciation  $\delta^h$  changes a lot as the calibrated value 0.01, 0.0019 by estimation, saying that housing actually depreciates much less than calibration. With respect to capital depreciation  $\delta^k$ , the value decreases from 0.025 to 0.0360, by 44% increasing. Compared to the calibration and the estimation in the base model, collateral model indicates that capital will depreciate more. Regarding to the capital and housing investment adjustment cost,  $\kappa^k$  and  $\kappa^h$ . Most other researchers, who study the economy other than Hong Kong, would find them most likely to be less than 1. Based on the calibration of them being 6 in the literatures of Hong Kong, the estimation with collateral finds them are 4.9879 for capital adjustment cost  $\kappa^k$  and 2.2649 for housing investment adjustment cost  $\kappa^h$ . Regarding to the large amount of difference, the calibration is not well specified in housing investment. Both capital investment and housing investment are more costly to be converted into capital and housing, while housing investment will cost less to be converted into housing relative to the capital investment. However, these two cost parameters by estimation are less than those in the base model. Because of the collateral constraint, both capital investment and housing investment can be less costly converted into capital and housing. Moreover, this change can be seen more on housing investment due to the setting that collateral is housing.

The rest two parts of parameters are the open economy and financial premium. The home

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goods bias of Hong Kong  $\omega$  moves from 0.4 to 0.2873, so that Hong Kong consumers actually prefer more on import goods. This is a tricky one because Hong Kong has extraordinary large amount of the export and import, and most of them are re-exported to other destinations. The corresponding home bias for rest of world  $\omega^f$  changes from 0.4 to 0.3442, saying that the rest of world in this model would prefer less on domestic goods. The elasticity between domestic goods and imported goods in home economy  $\theta$  rises to 1.0813 and the corresponding one in foreign economy  $\theta^f$  increases to 1.2501. So that 1% increase in the real exchange rate  $Q$  would make the imports drop by 1.0813% and the exports rise by 1.2501%. Feedback from leverage to finance premium  $\chi$  increases little from 0.05 to 0.0534 and feedback from money to finance premium increases by 14.6%. Compare to the calibration which is from the US data in Le et.al (2016), feedback to premium from the leverage and the money are relative more in Hong Kong in the model with collateral.

The testing result on collateral model is more than 1.65 in the Transformed-Wald test, which is to reject the collateral model. Recall the testing result from the base model, we can see that the base model is better than the collateral model in matching the behaviour of data. The Indirect Inference do not reject the base model, but reject the model with collateral.

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## 5.6 Residual and Shock Property

This section discusses the properties of these structure shock processes in the model. Those shock are calculated from the estimated coefficients by Indirect Inference estimation, and then fit into the non-stationary actual data. In order to test whether the shock is well specified in the form of AR(1) process or ARIMA (1,1,0) process for the productivity, the test of stationarity is taken for each shock by two method, the ADF test and KPSS test. Table 5.5 provides the result of unit root tests and their AR(1) coefficients. It worths noticing that sometime the test results from two types may have different interpretation, so it will rely on a personal judgement to decide the stationarity. Figure 5.1 provides graphs for the structure residuals from the estimated model, while Figure 5.2 shows those shocks from estimated model.

ADF test has the null hypothesis of unit root (non-stationary), while KPSS has the null hypothesis of stationary. The application of two types of test, not only one, is for the consideration of prudence. From the column two of ADF p-value, it is clear that housing demand shock, government spending shock, foreign consumption shock ( or can be treated as export demand shock) and foreign inflation shock are all rejected even at 1% significance level, while consumption preference shock and foreign interest rate shock are sure to be rejected at 5% which is the rate not that strict but more commonly applied. The productivity shock process is tested to be non-stationary under ADF with the p-value of 0.9699. In KPSS test, productivity is rejected against stationary clearly. In addition, the estimation of level productivity is 1.0004, which is another prove that productivity should be treated as non-stationary.

It can be seen all other processes look like a stationary process in Figure 5.1. Overall, for consumption preference, foreign Consumption (or export shock in this model), foreign inflation and foreign interest rate, the ADF test of unit root is rejected and the KPSS

of stationary is not rejected both at 5% significance level. So we can conclude these processes are all stationary. The housing demand can reject the unit root in ADF but it also rejects stationary in KPSS, while government spending cannot reject unit root in ADF but it cannot reject stationary as well. According to their AR(1) coefficients that they are both less than 1, I would say they are stationary.

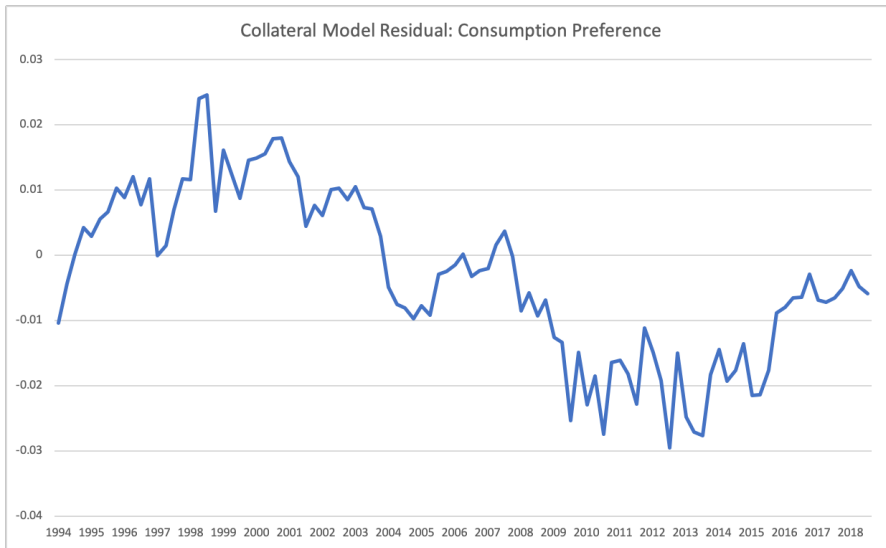
Table 5.5: Residual Stationarity Test and AR(1) Coefficients in Collateral Model

Shock	Stationarity Test		Conclusion	AR(1)
	ADF p-value	KPSS stats		
Consumption Preference	0.0328*	0.1474*	Trend Stationary	0.9127
Housing Demand	0.0015	0.2286***	Trend Stationary	0.8389
Productivity	0.9699***	1.1743***	Non-stationary	0.1653†
Government Spending	0.4189***	0.2955	Stationary	0.9852
Foreign Consumption	0.0055	0.1320	Stationary	0.8693
Foreign Inflation	0.003	0.4386*	Stationary	0.6868
Foreign Interest Rate	0.0106	0.0939	Stationary	0.8624

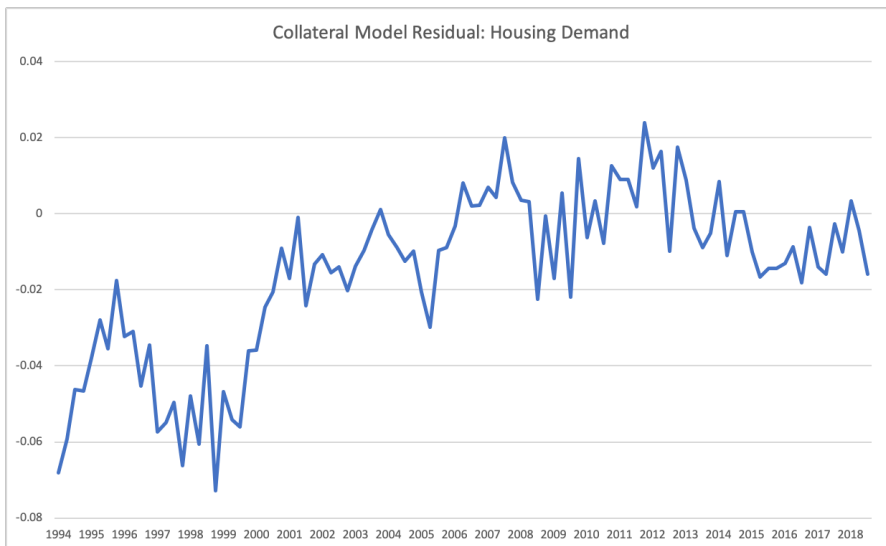
1. KPSS \*, \*\*\* indicates rejection of stationary at 10% and 1% respectively.
2. ADF p-value \*, \*\*\* indicates do not reject unit root at 1% and 10% respectively.
3. † The AR(1) coefficient of productivity is for the first order differenced one.



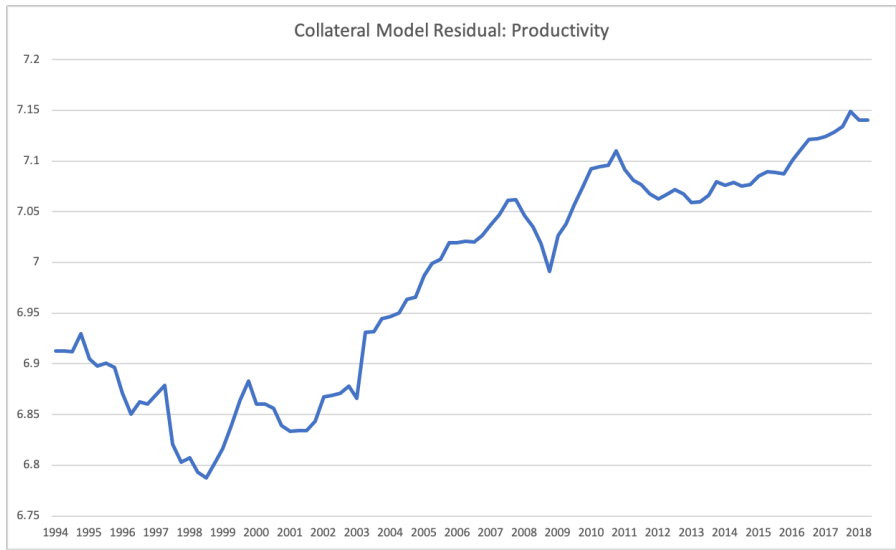
Figure 5.1: Residuals from the Collateral Model by Estimation



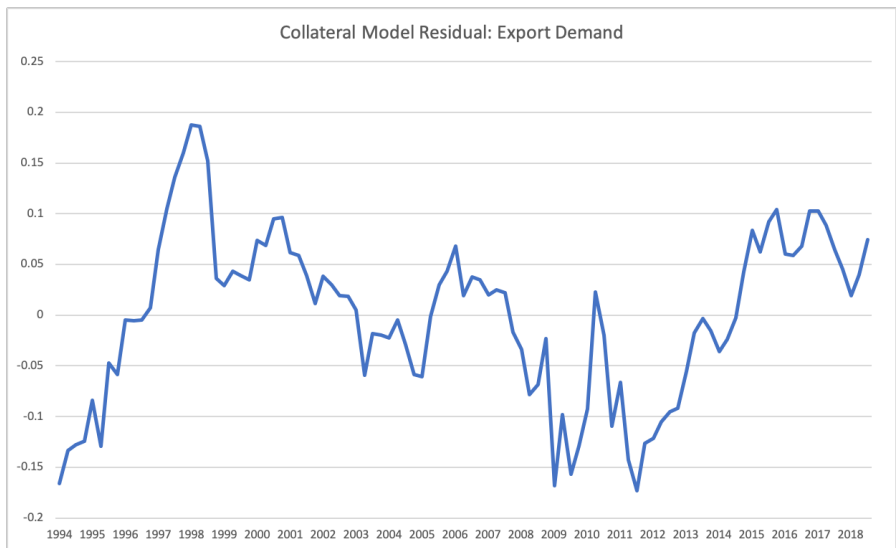
(a)



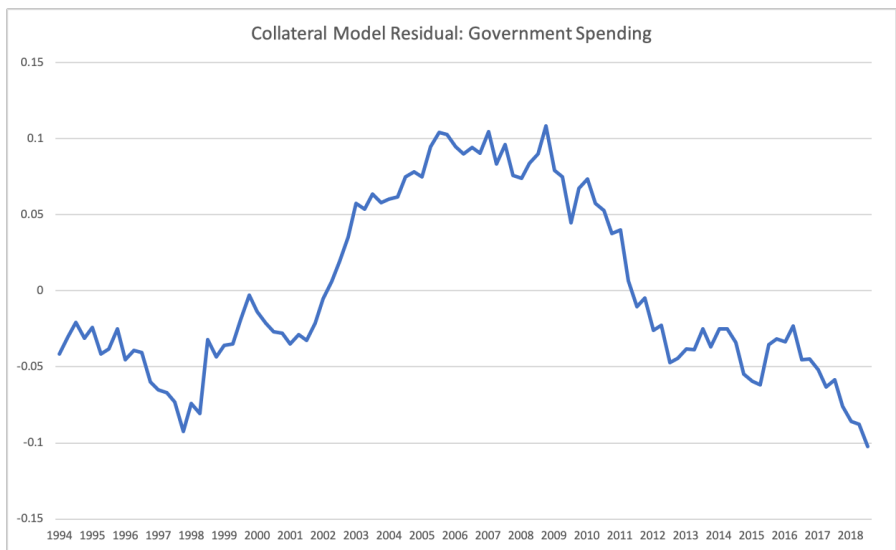
(b)



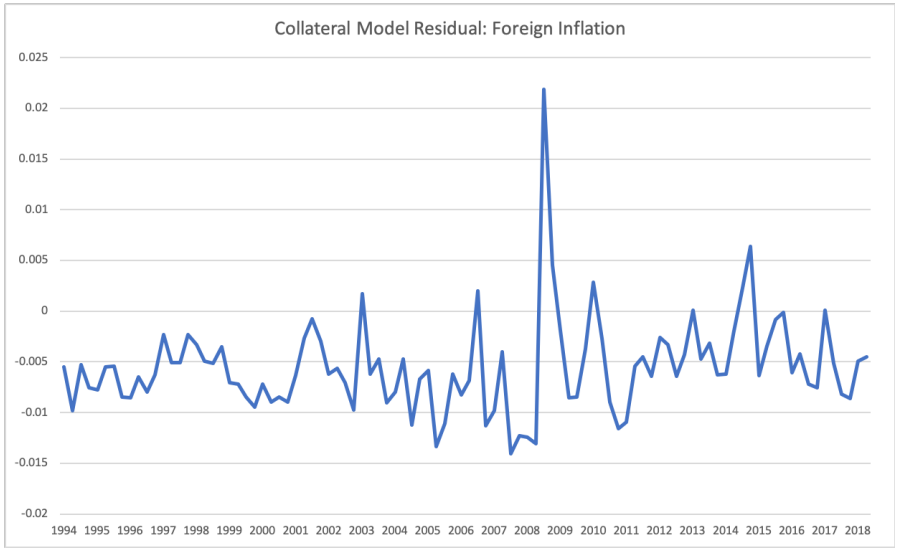
(c)



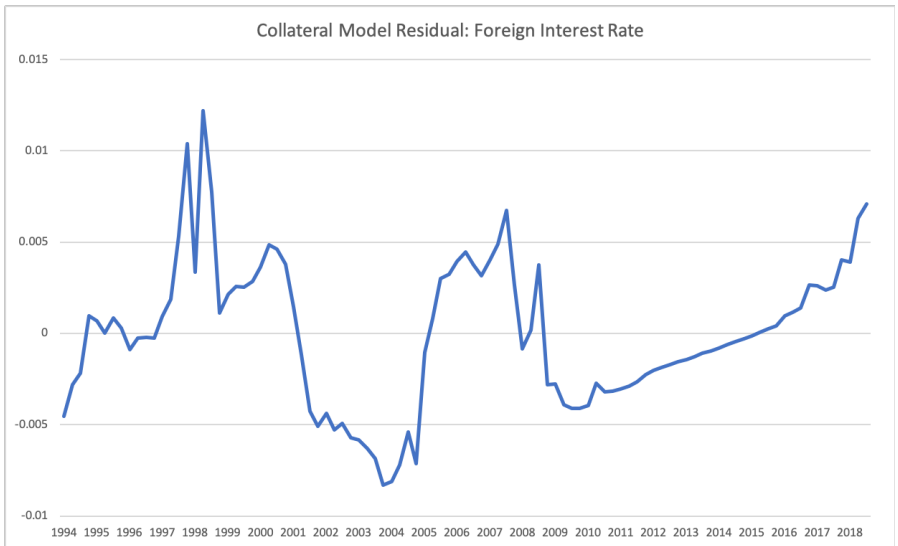
(d)



(e)

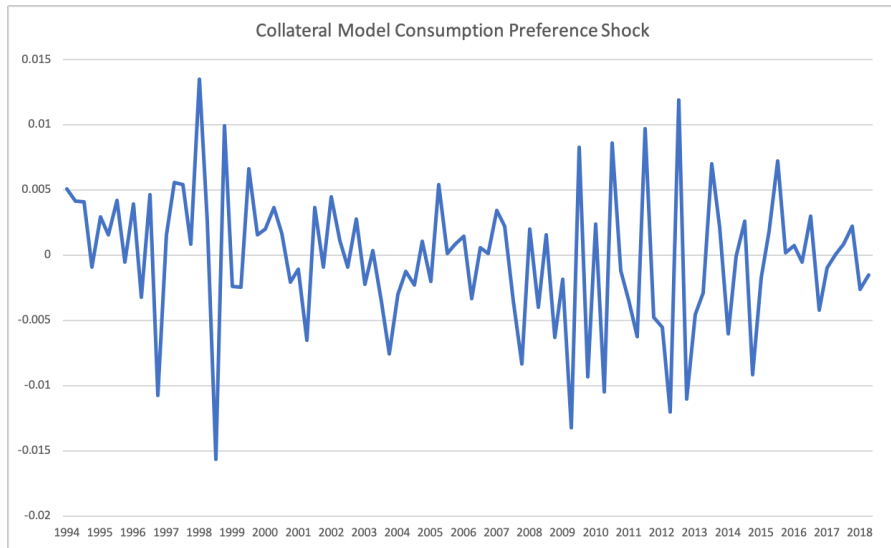


(f)

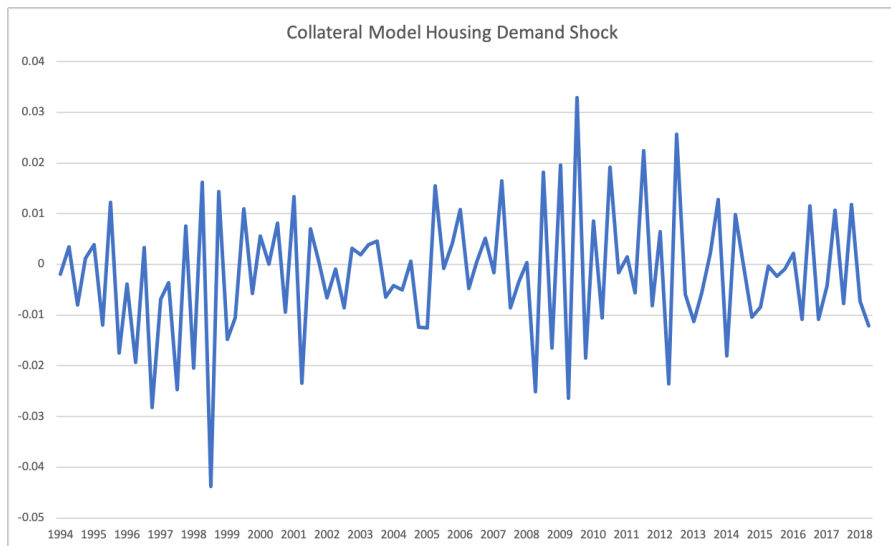


(g)

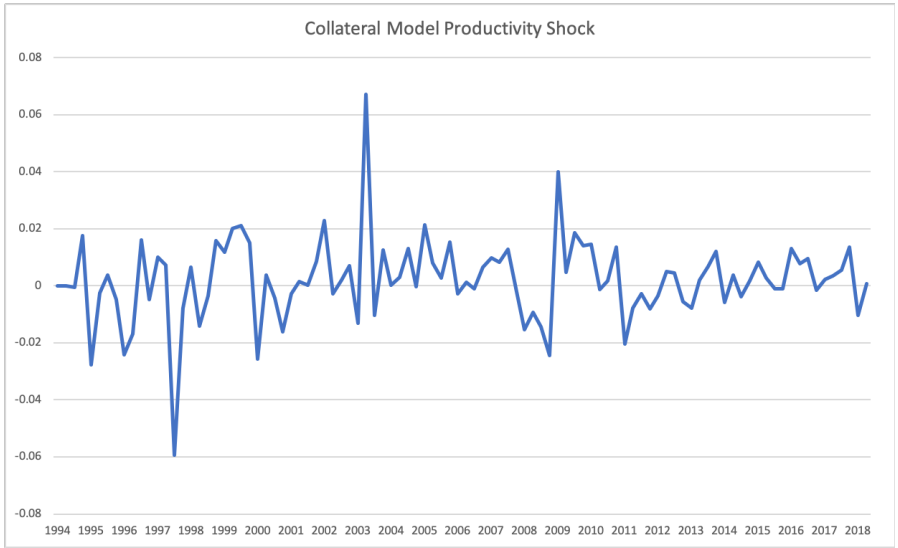
Figure 5.2: Shocks from the Collateral Model by Estimation



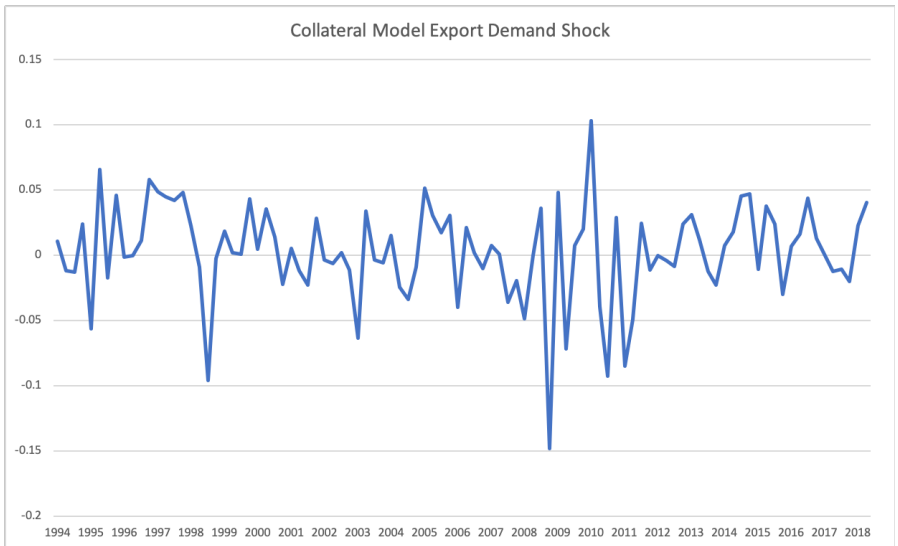
(a)



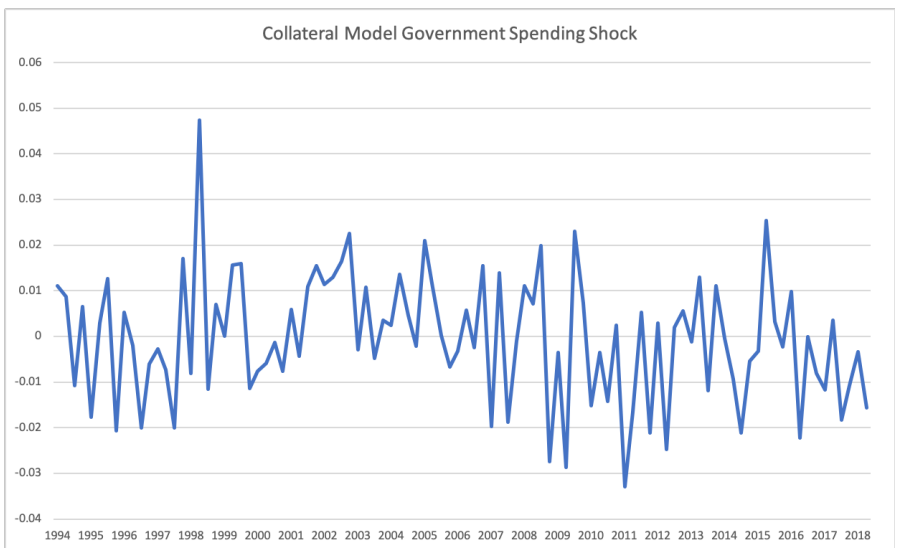
(b)



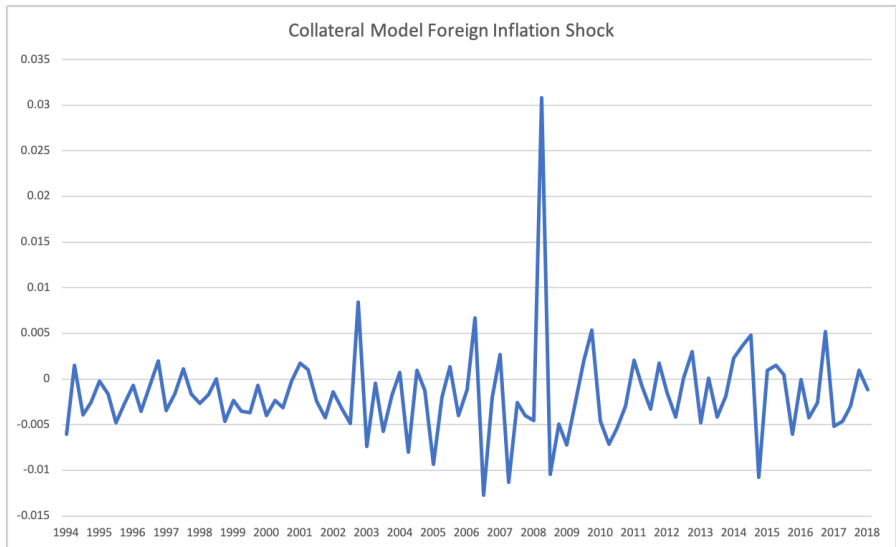
(c)



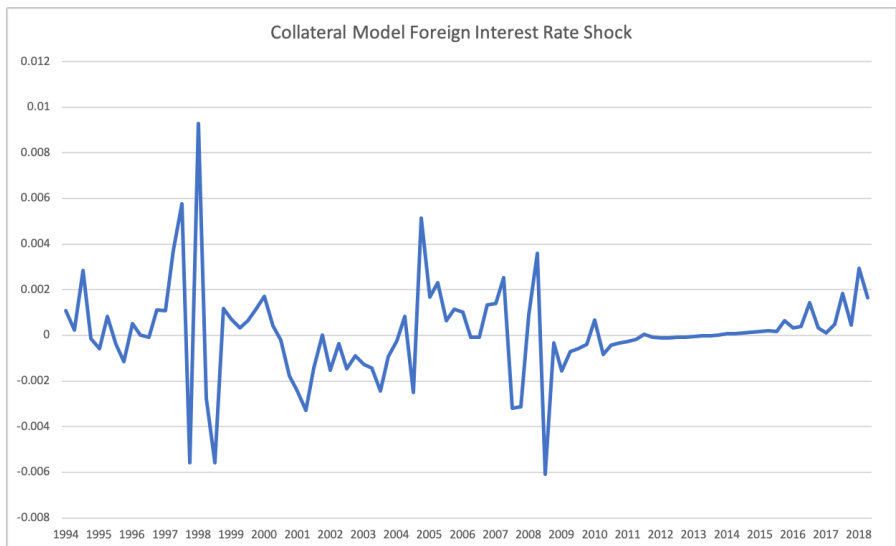
(d)



(e)



(f)



(g)

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## 5.7 Impulse Responses

This section displays analysis and compares the behaviour of base model and collateral model, by the impulse response functions. Because the collateral is rejected by the Indirect Inference, I then recall the base model and explain how the base model works.

*Consumer Preference Shock.*

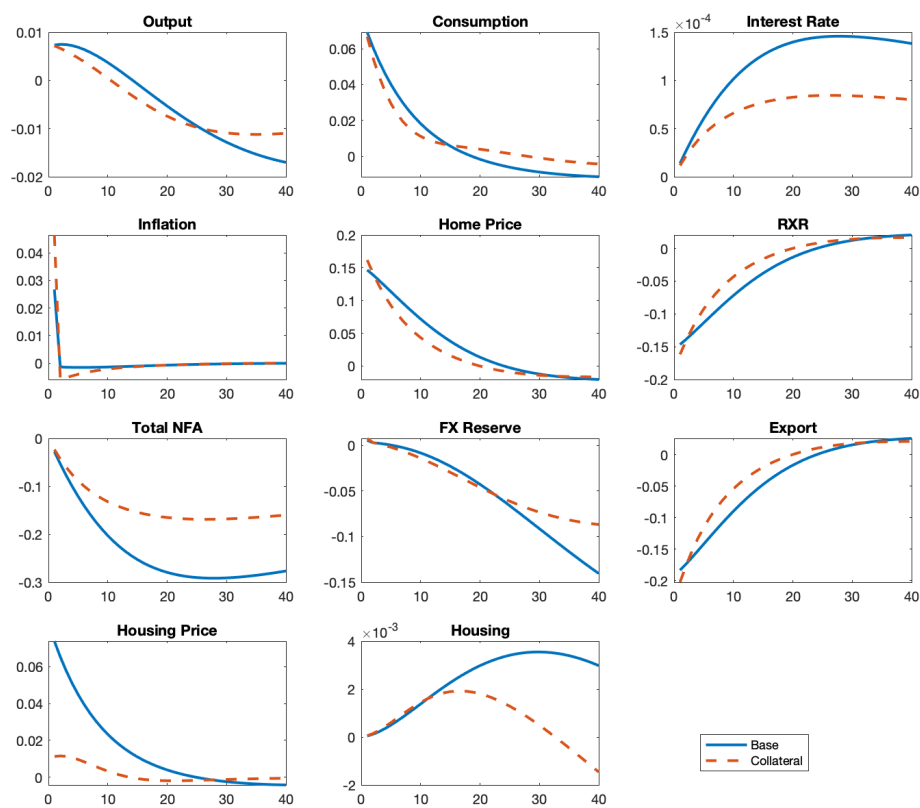


Figure 5.3: IRFs from Collateral and Base Model by Preference Shock

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As in Figure 5.3, consumer preference shock hits the consumption in households to have it increased in both models. The increase in the consumption is met by the more output produced by firms, then more money demand in the economy so the authority has to use the foreign exchange reserve to exchange for more supply of money under fixed exchange rate and currency board arrangement. The more supply of domestic goods is followed by an increase in the domestic price, inflation as well as real exchange rate. The increase of those prices clear the goods market, but have negative effect on the export, while the decrease in the export brings current account deficit and decline in the total net foreign assets.

The collateral model does not behaviour too much differently from the base model, but consumption returns to steady state quicker than in the base model. It is because that the higher interest rate hit the borrowing capacity by the collateral constraint, lowering the borrowing amount. In the collateral model, the initial consumption shock hits the Euler equation and increases the consumption. Later, the rising in the consumption raises the interest rate and the higher interest rate has negative impact on the borrowing capacity. The relative lower borrowing capacity further accelerates the returning process of the consumption. Without collateral constraint, such impact does not exist.

Thus, I recall the diagram below to explain the working of the base model with consumption preference shock. A positive consumption preference shock initially shifts the ISBB curve to the right by giving more demand for goods. The new temporary intersection of ISBB' and OS curve can have more output and lower real exchange rate. Therefore, we can see the intersection of the ISBB' and OS is to the right of the XM curve, which means there is a current account deficit, as shown in the Figure 5.3 that the Total NFA declines. The decline in the NFA shifts the ISBB' to its original place, ISBB. That is to say that the decline in NFA drives the output and real exchange rate back to its equilibrium,



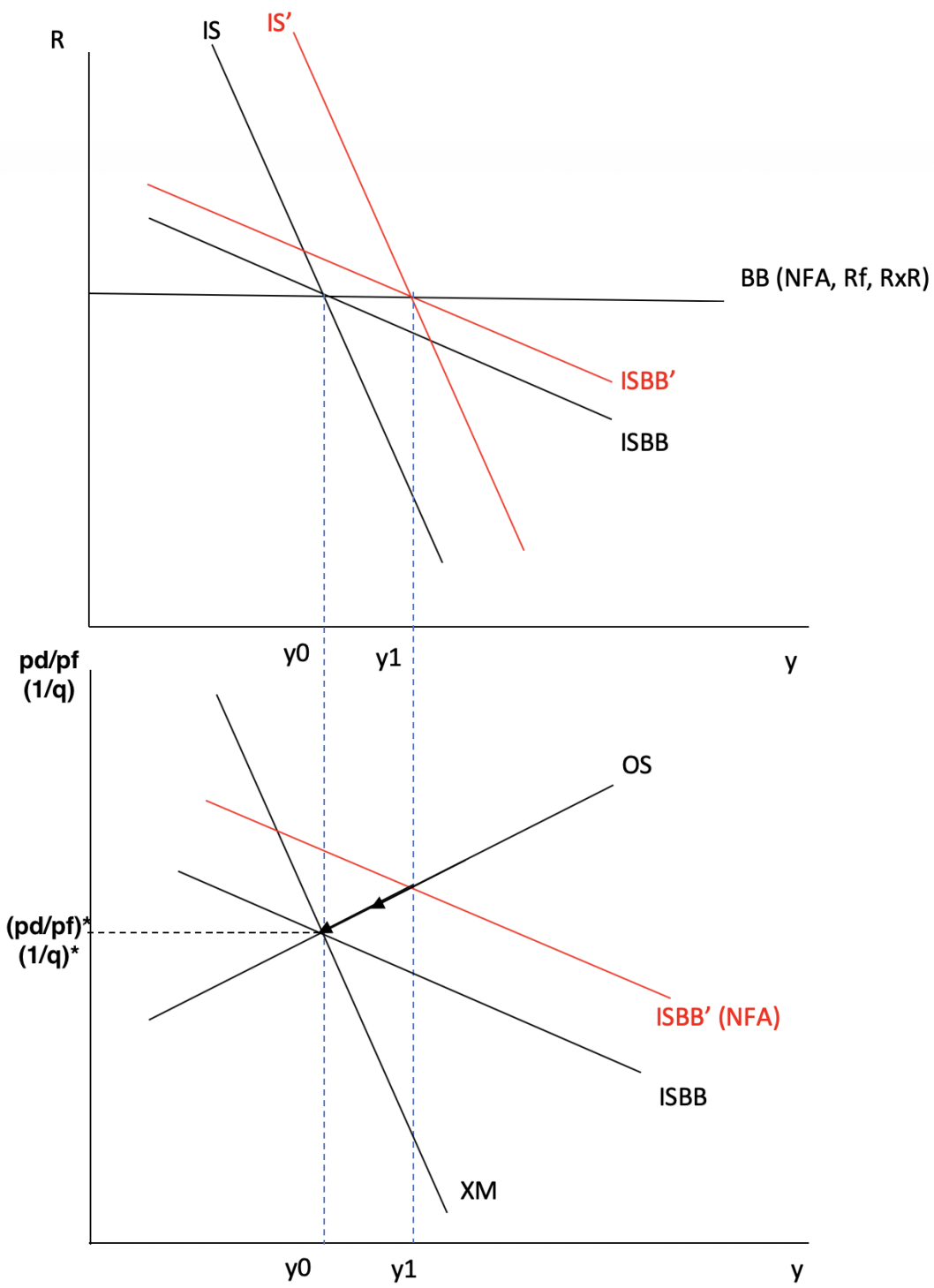


Diagram of the dynamics to Consumption Shock

*Export Shock.*

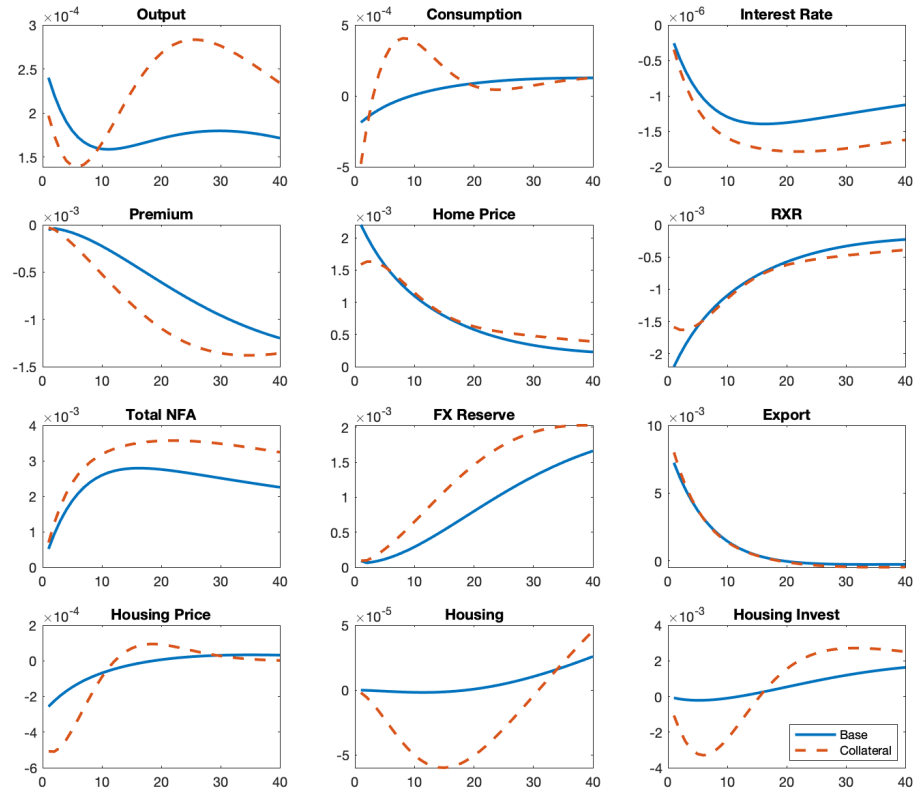


Figure 5.4: IRFs from Collateral and Base Model by Export Shock

As in Figure 5.4, an export demand shock drives up the foreign demand for the domestic goods, export increases. In the monetary condition, more money supply increases through the foreign exchange intervention. The monetary condition is loosen because the firm is willing to produce more as the domestic price is higher, as well as more demand from abroad. The rise in the output means there is more demand for capital, firm will need more funds to finance its cost on capital. Thus, the credit premium is decreased due to expansion in the credit market. The real exchange rate decreases because of the

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increase in the home price, while the total net foreign assets is accumulated more due to the current account surplus and rising in the export. Simultaneously, interest rate is decreased by more accumulated total net foreign assets through the risk premium UIP.

Compare to the base model, collateral has seen more drop in the consumption. It can be seen that export demand shock raises the export and output, but it crowds out the housing investment which is the input in producing the housing. The housing price drops more here and could have negative wealth effect on consumption via the collateral constraint. Because housing price decreases and even decreases more, the borrowing capacity drops because the housing value decreases. As impatient households can borrow less, their consumption decreases more. In the base model without the collateral constraint, this wealth effect is less significant than in collateral model.

The diagram below again helps to figure out how the export demand shock affects the economy. It drives up the demand for export, which directly shifts the XM to the right. Therefore it generates a current account surplus for the domestic economy as the new intersection of OS and ISBB is to the left of the new XM curve. Up to this point, output increases and real exchange rate decreases by the market clearing condition. A higher domestic price level would encourage firms to produce more to get more profits and meet the rising demand for goods to export. Then the net foreign assets increases from the current account surplus, which would shift BB curve down so domestic interest rate would decrease via the risk premium uncovered interest rate parity. Meanwhile, rising in the total net foreign assets shifts the ISBB curve to the right.

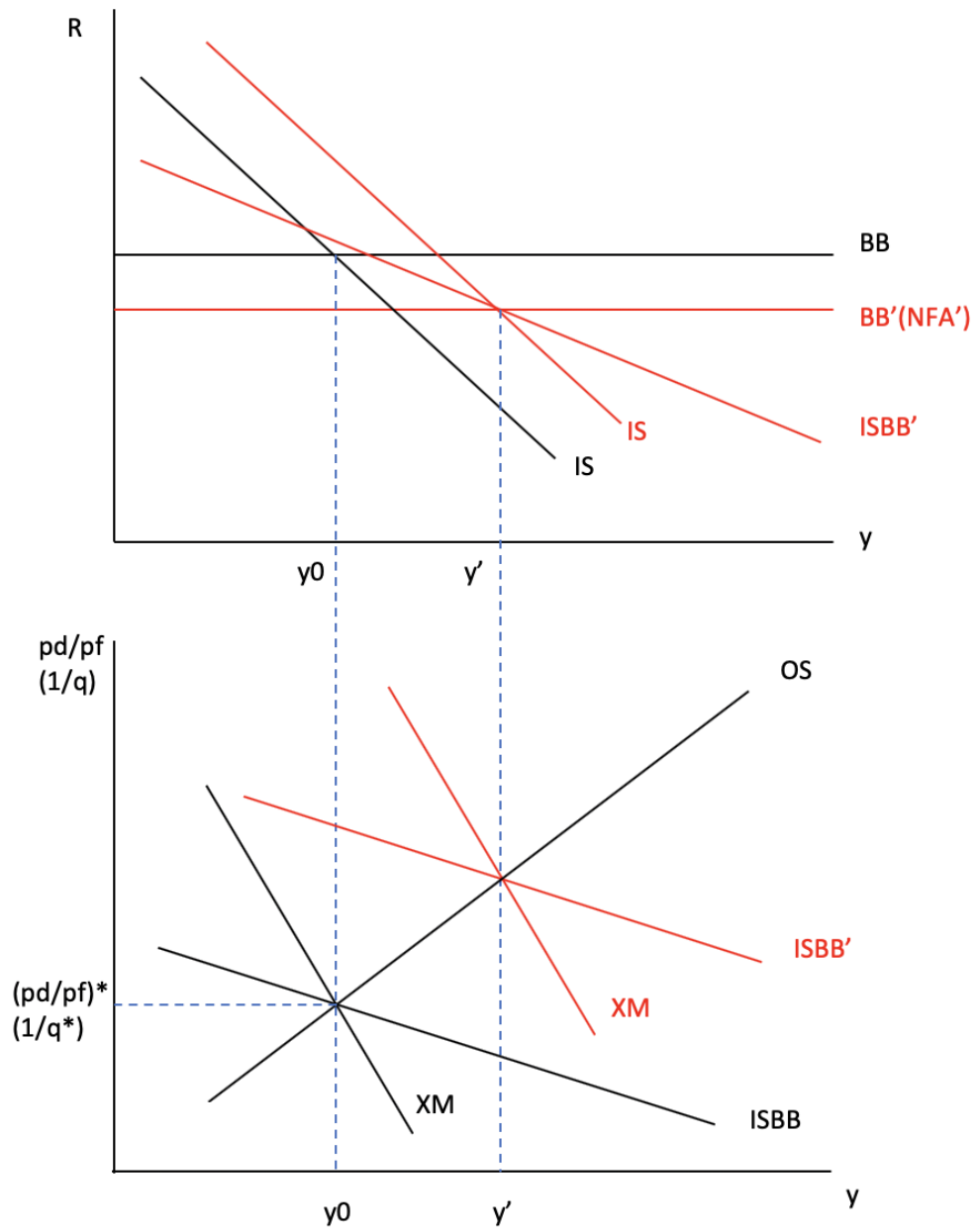


Diagram of the dynamics to Export Shock

*Foreign Interest Shock.*

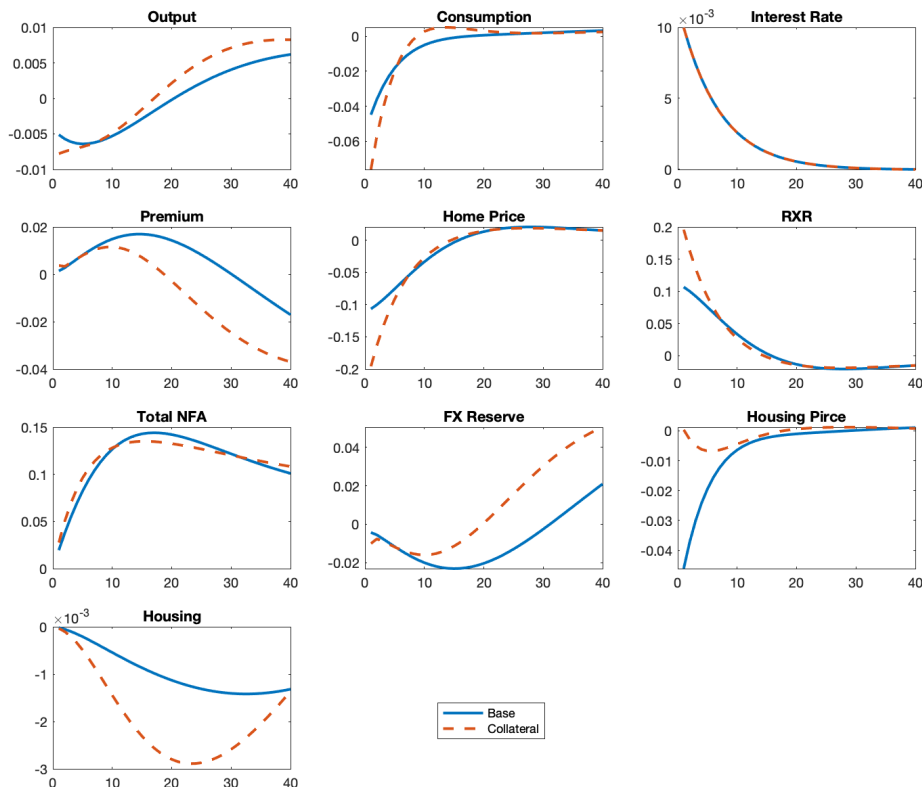


Figure 5.5: IRFs from Collateral and Base Model by Foreign Interest Shock

Figure 5.5 tells that a positive foreign interest shock affects the domestic economy through the domestic interest rate, taking the role similar to a tightening monetary policy. The domestic interest rate increases by 1% and moves back to steady state after 20 quarters. The increased funding cost means a contraction in the capital and investment. Output begins to decline by the reduction in capital and investment, with a following decline in the labour. Together with the increase in the real exchange rate, the domestic economy see a temporary current account surplus with more accumulated

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NFA. Finally, the rise in the NFA makes output and real exchange rate moves to their equilibrium when the domestic interest rate moves back to equilibrium again. Regarding to the monetary condition, the less demand for capital means there is less demand for money in the aggregate economy. The authority then must decrease the money supply through foreign exchange intervention, as it can be seen that foreign reserve decline with a tightened credit market.

Compared with the base model, collateral model generates similar dynamics but with different scale. In the base model, the positive foreign interest rate raises domestic interest rate through the uncovered interest rate parity. The higher interest rate decreases consumption by the Euler equation. However, the higher interest rate also lowers the borrowing capacity by the collateral constraint in the collateral model. The drop in the borrowing would have another negative effect on consumption, from the second transmission in the collateral constraint.

The behaviour and effect from positive foreign interest rate shock can be explained by the following diagram. It shifts up the BB curve and domestic interest decreases. The rising of the domestic interest rate leads to recession in the domestic economy that output, consumption and investment decrease. Then ISBB curve shifts to the left, intersecting with the OS curve to the left the XM curve. At this point, real exchange rate increases and home price decreases. This temporary current account surplus generates an increase in the total net foreign assets, shifting the ISBB curve to the right to its equilibrium.

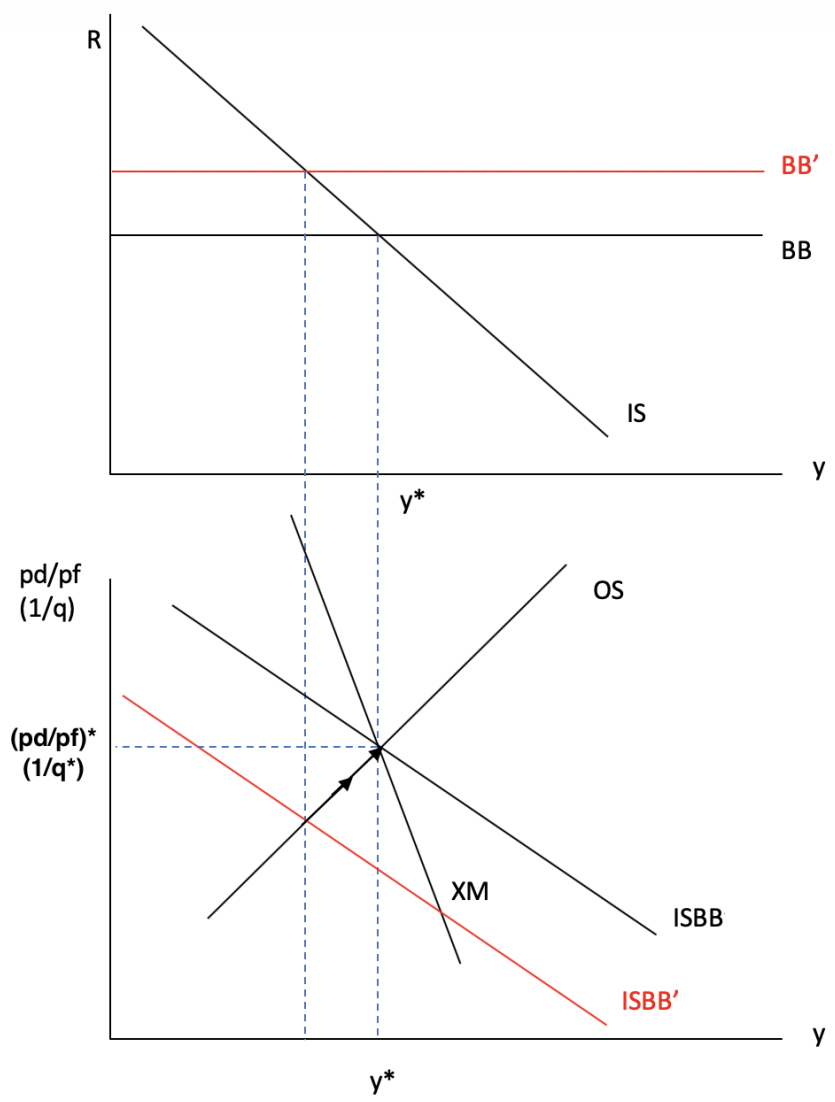


Diagram of the dynamics to Foreign Interest Shock

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## 5.8 Stability and Welfare Evaluation

Since the linked exchange rate and currency board were founded in 1983, the monetary system in Hong Kong has had many challenges, including several financial crises and speculative attacks. Although this mechanism has been seen as a success for Hong Kong, there are still some discussions and arguments on that if it could better to switch to another regime.

One alternative is to peg to Chinese Yuan, instead of US dollar. This argument comes from the fact that Hong Kong has a closer trade relation to mainland China, than US. One of the considerations in the initial setting is that US was the most important trading partner for Hong Kong, so it seems to be reasonable to peg to RMB. However, there are two technical problems. First, the Chinese Yuan is not fully convertible, and the Hong Kong monetary authority may not have enough access to Yuan liquid assets as a back-up foreign asset for the issue of Hong Kong dollars, which would lower investors' confidence and stability of the economy. Second, most foreign assets are held in the form of risk-free government bonds, and the supply of Chinese government bonds is limited. Third, US dollar is still the dominant currency in the world, most trading activities are completed in US dollar. It is not the time to peg to Chinese Yuan and US dollar would be the best choice if Hong Kong stays in the currency board arrangement. But perhaps the most powerful argument against adopting currency union with mainland China is the threat this would cause to Hong Kong as a separate economy outside Chinese economic control; this separation is the reason it attracts so much foreign investment and financial activity.

Another alternative is to abandon the currency board and have a floating exchange rate. Technically, it could be workable, but I suppose a floating exchange rate will have a lower stability and more uncertainties. Therefore, in this section, I first drop the foreign



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reserve intervention and currency board settings and add the floating exchange rate and simple Taylor rule, followed by the discussion of impulse responses to a Taylor rule shock. I then bootstrap the fixed exchange rate model and the alternative floating exchange rate model many times, by estimated parameters from fixed exchange rate model and actual shocks from data. For the Taylor rule error, because Hong Kong actually does not have Taylor rule, I cannot use Hong Kong data to get this error as it might not reflect the domestic feedback to interest rate. Instead, I use the error from US data as a proxy, where the standard error of the US Taylor rule is 0.0074. Lastly, I measure the stability by the output and inflation variances in the bootstrapping simulation, combined with welfare costs.

In the floating exchange regime, the Taylor rule is set as:

$$r_t = \rho_\pi \pi_t + \rho_y y_t + \gamma_t^r \quad (5.44)$$

Where  $r$  is interest rate,  $\pi$  is inflation and  $y$  is output;  $\rho_\pi$  is the feedback from inflation,  $\rho_y$  is the feedback from output;  $\gamma_t^r$  is the Taylor rule shock, following the AR(1) process:

$$\gamma_t^r = \rho^r \gamma_{t-1}^r + \epsilon_t^r \quad (5.45)$$

Figure 5.6 displays the impulse responses to a monetary shock in the alternative floating exchange rate model. A positive Taylor rule shock acts as a tightening monetary policy and it initially hit the interest rate, with an increase in the interest rate. A higher interest rate lowers the consumption in the Euler equation, while it also decreases investment because of a higher cost. This downward shift in the demand side then goes to the supply side, output and inflation decrease. The lower domestic price further results in a real depreciation and domestic goods are relatively more competitive, we can see export

increases with more accumulated net foreign assets.

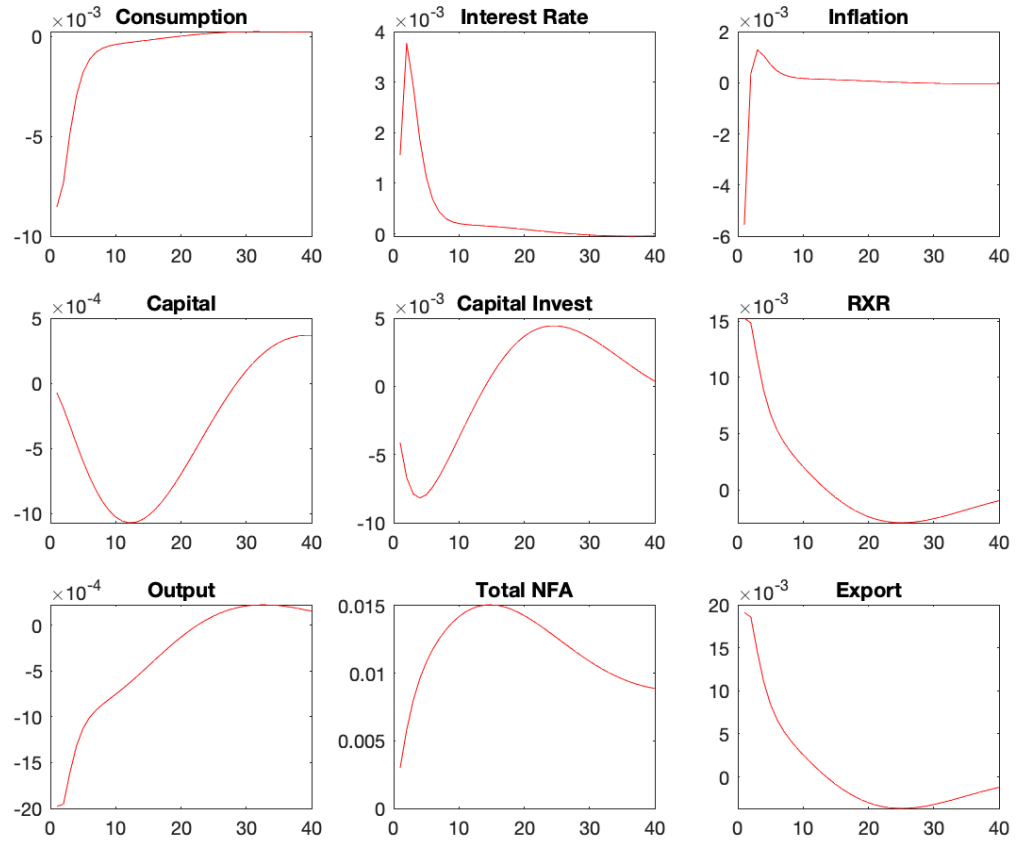


Figure 5.6: IRFs to Monetary Shock in Floating Exchange Rate

The floating exchange rate model behaves like those in the literature, I then ask the question which regime is better, fixed rate or floating rate? To answer this research question, I calculate the variance of output and inflation, together with the welfare cost measure which follows Galí and Monacelli (2005). In order to capture the variance, I bootstrap both models by their actual shocks from data 1000 times, get the variance of output and inflation in each simulation and then take the average.

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Table 5.6: Stability and Welfare Loss Under Two Exchange Rate Regime

	Fixed	Floating		
		$\rho_\pi : 1.5, \rho_y : 0.06$	$\rho_\pi : 1.7, \rho_y : 0.06$	$\rho_\pi : 1.5, \rho_y : 0.08$
Output Variance	0.0026	0.0069	0.0270	0.0038
Inflation Variance	0.0003	0.0006	0.0012	0.0005
Welfare Loss	0.0029	0.0075	0.0282	0.0043

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From Table 5.6, we can see that output and inflation are more stable under fixed exchange rate and currency board. Regarding to the floating exchange rate, all three experiments would generate more fluctuations and welfare loss. If we have more feedback from inflation, both inflation and output variances would increase. However, if we have more feedback from output, both variances decrease. Overall, by the actual data, Hong Kong would benefit from fixed exchange rate and currency board as they can have more stability and less welfare loss. Economically, it is not good for Hong Kong to switch to a floating exchange rate.

## 5.9 Conclusion

The benchmark model is extended with the housing collateral, to evaluate the collateral constraint and to see if the housing collateral model is better in fitting the data or not. By indirect inference testing, the model with collateral cannot fit the Hong Kong data while the model without housing collateral is superior. Comparing the impulse responses in these two models, we can see that the those variables in collateral model response more than in benchmark model. It is because collateral effect does not exist in the benchmark

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model and collateral model has one more channel by the collateral constraint. In addition to the monetary transmission by the full exchange intervention, the interest rate would further affect the economy through the collateral constraint. Apart from the substitution effect, the wealth effect from housing market is stronger than in the benchmark model. Although the collateral model shows some spillover from the housing market, which is suggested in past literatures, this chapter states that collateral constraint should be prudentially considered as it might be rejected by actual data in some empirical cases. Regarding to the policy, first I would suggest that housing plays no role in the economy, not only because the collateral model has been rejected, but also that housing demand shock contributes to nearly zero in the variance decomposition. Second, I would suggest Hong Kong to stay in the currency board and fixed exchange rate as they can provide more stability.

## 5.A Data Source in Collateral Model

Table 5.7: Data Source or Derivation in Collateral Model

Variable	Notation	Code or Source	Definition
$C^c$	Households Consumption	HKCNPFR	Private consumption expenditure, HKD, CP, SA
$C^P$	Patient Consumption	/	Derived from equation
$C^I$	Impatient Consumption	/	Derived from equation
$Y$	GDP output	HKGDP	GDP, HKD, CP, SA
$I^k$	Capital Investment	HKGFCF	Gross fixed capital information.exclude Housing investment, HKD, CP, SA
$I^h$	Housing Investment	HKCONIESC	Gross fixed capital information-Construction, HKD, CP, SA
$G$	Government Spending	HKCNGOV	Government consumption expenditure, HKD, CP, SA
$EX$	Export	HKEXNGS	Export, HKD, CP, SA
$IM$	Import	HKIMNGS	Import, HKD, CP, SA
$K$	Capital	/	Derived from equation
$H$	Housing	/	Derived from equation, scaled by the market size in 2017
$M^s$	Money Supply	HKXMON0	M0, HKD, CP, SA
$Z$	Total NFA	HKXNFA	Net foreign Assets owned by public and government, USD, CoP, NSA
$B^f$	Private NFA	/	Derived from Total NFA by excluding the government NFA, USD, CoP, SA
$CY$	Borrowing Rate	HKQ60P	Bank lending rate/4, Quarterly
$NW$	Net Worth	HNGKNGI	Hang Seng share index/GDP deflator, SA, Price index

Variable	Notation	Code or Source	Definition
$P^d$	Home Price	HKGDPIPDE	GDP deflator, Price index
$\pi^d$	Home Inflation	/	Quarterly percentage change in GDP deflator
$P$	CPI	HKCPI	Consumer Price Index, SA, Price index. 2017=100
$\pi$	CPI Inflation	/	Quarterly percentage change in CPI
$P^f$	Foreign Price	USQCP009F	US Consumer Price Index all item, SA, Price index, 2017=100
$\pi^f$	Foreign Inflation	/	Quarterly percentage change in US CPI
$r$	Domestic Interest Rate	HKMA	Three month HIBOR/4, Quarterly
$r^f$	Foreign Interest Rate	USGBILL3	Three month US Treasury bill rate/4, Quarterly
$C^f$	Foreign Consumption	US BEA	US GDP, CoP, SA
$H^P$	Patient Housing	/	Derived from equation
$H^I$	Impatient Housing	/	Derived from equation
<i>Premium</i>	Credit Premium	/	Difference between bank lending rate and HIBOR
$P^h$	Housing Price	HKBPPCN	Residential property price, Price index, SA
$q^k$	Capital Price	/	Derived from investment Euler equation
$R^k$	Capital Rent	/	Derived from equation
$N$	Labour Force	HKEMPTOTP	Total employment, SA
$W$	Wage	HKXWCMF.F	Manufacturer unit wage cost, SA. 2017=100
$Q$	Real Exchange Rate	HKBISRNR	Inverse of real effective exchange rate, SA. 2017=100

† CP=Constant Price, CoP=Current Price, SA=Seasonal Adjusted,

NSA=Not Seasonal Adjusted

‡ HKMA=Hong Kong Monetary Authority, US BEA=US Bureau of Economic Analysis

## 5.B Impulse Response from Estimated Collateral Model

Figure 5.7: Collateral Model Estimated Response to Government Spending Shock

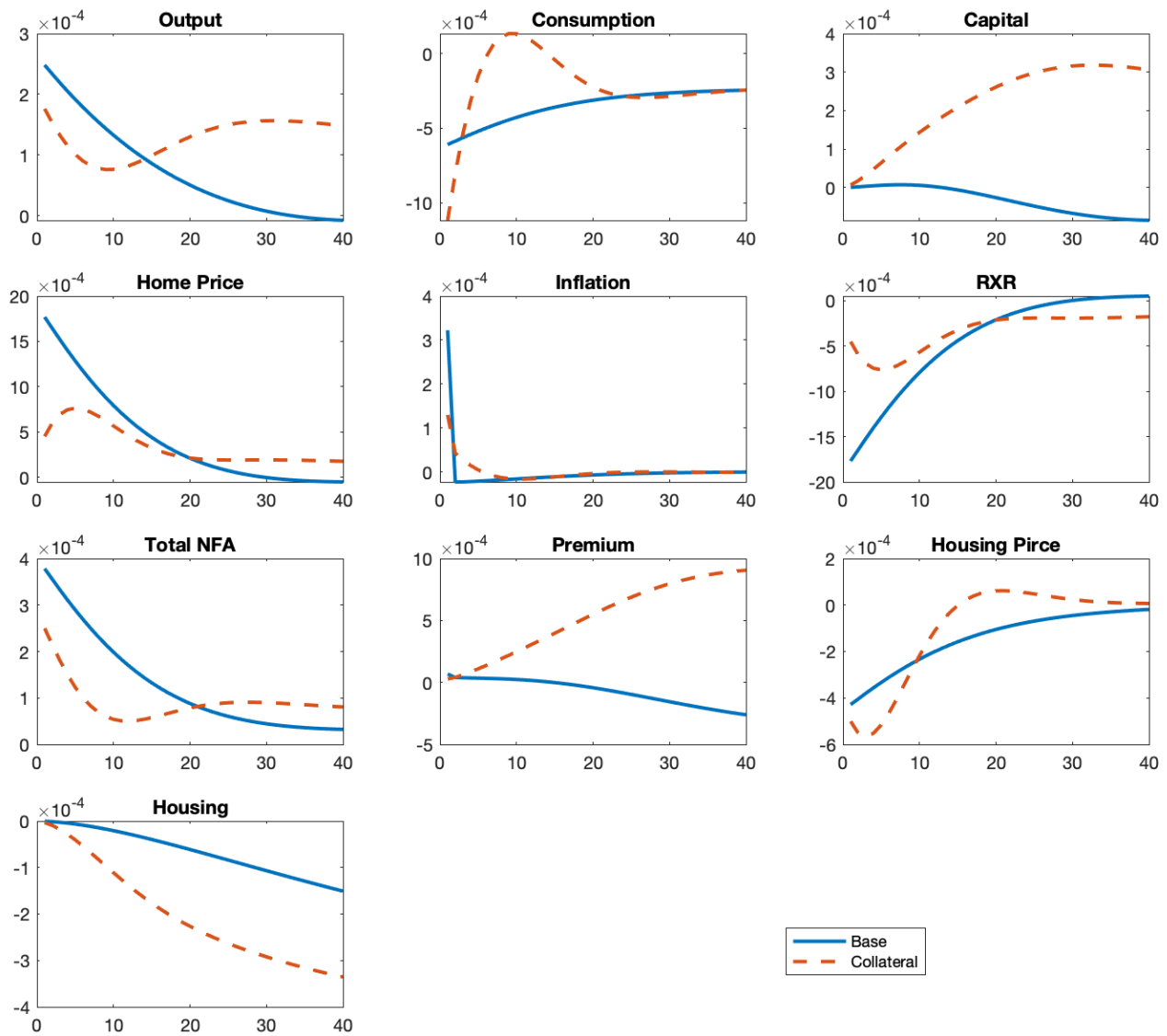


Figure 5.8: Collateral Model Estimated Response to Housing Demand Shock

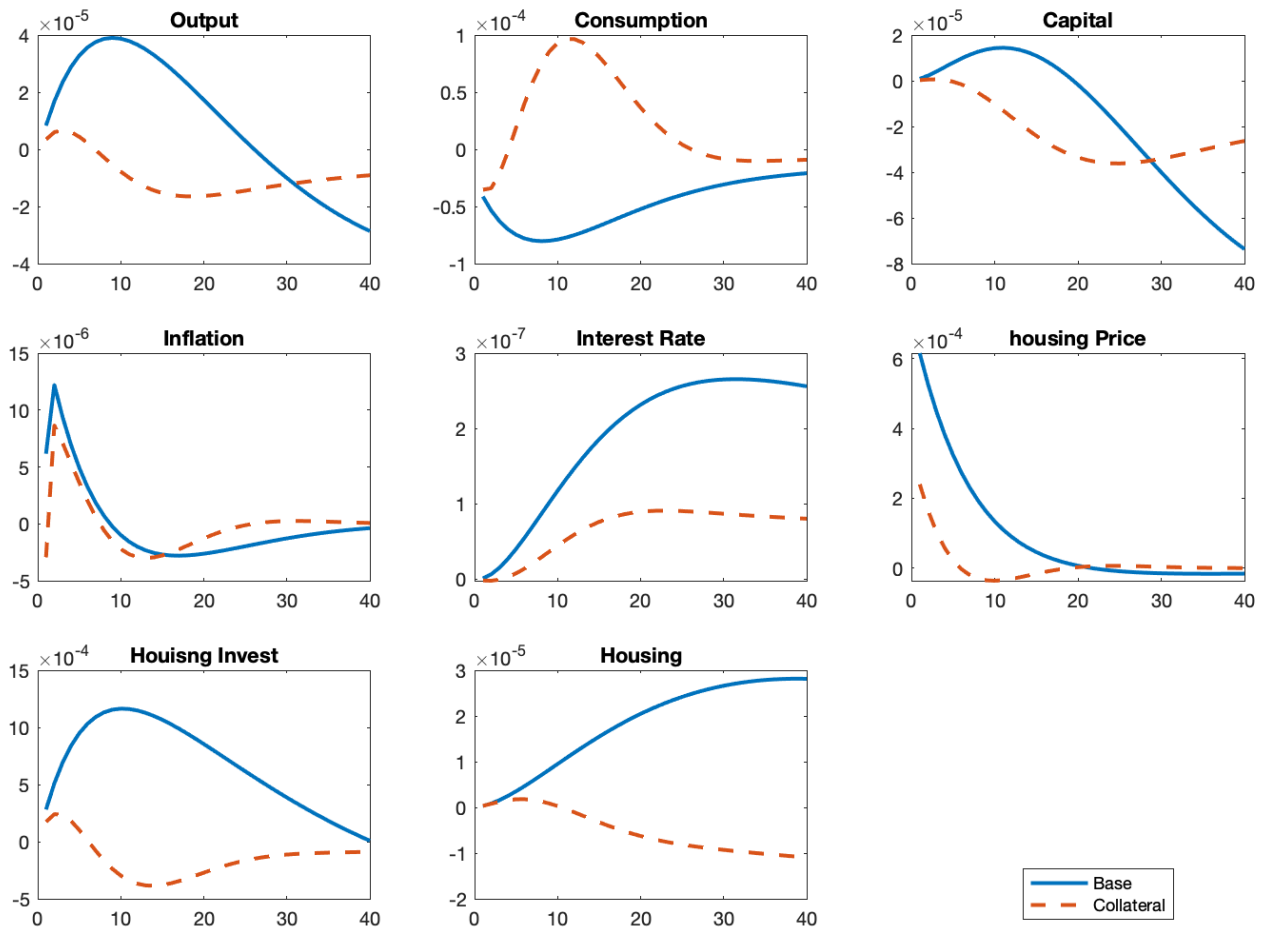
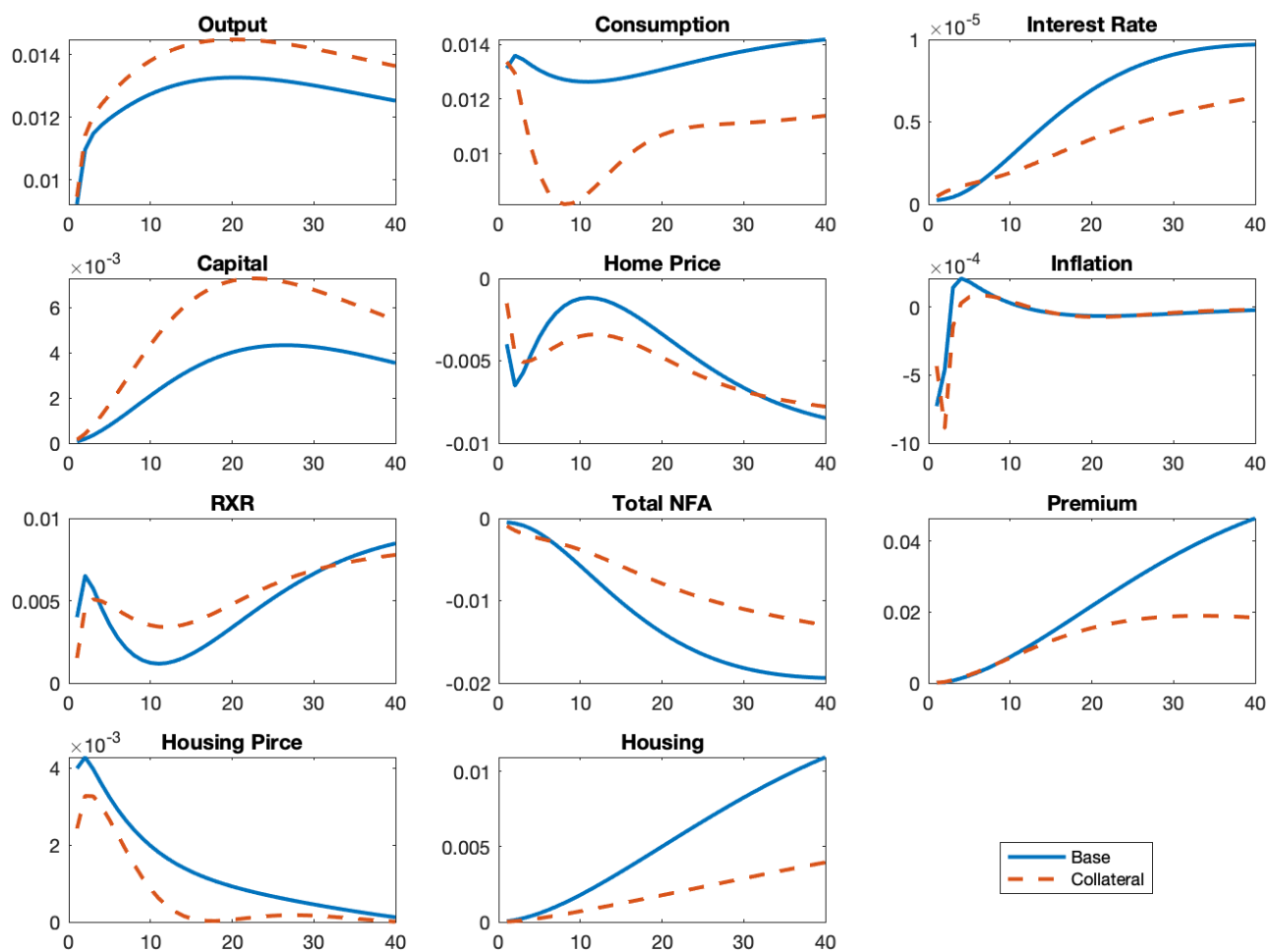




Figure 5.9: Collateral Model Estimated Response to Non-Stationary Productivity Shock





## Chapter 6

# Conclusion

A currency board implies no independence of monetary policy, with exchange rate fixity absolute. Hong Kong has been seen as a successful and typical case for currency board, with the experience in the handover to China, Asian Financial Crisis and Global Financial Crisis. The first aim of this thesis was to find out how the economy works under this currency board system. Most previous work either only features fixed exchange rate or a Taylor rule under a moving exchange rate. A further aim was to investigate whether housing collateral, given the high value of housing in Hong Kong, was an important channel for the transmission of shocks through the economy. Finally, we wanted to evaluate claims that Hong Kong would be better off under a regular floating exchange rate regime with an independent monetary policy

This thesis has studied the economy of Hong Kong through the lens of a small open economy DSGE model with a currency board exchange rate commitment. It assumes flexible prices and a banking system that provides credit to entrepreneurial household-firms, with both collateral and cost of verification. We estimated and evaluated the model by Indirect Inference over the sample period of 1994Q1-2018Q3; we found that

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it matches the data behaviour, as represented by a VAR. We also evaluated a second version of the model in which there is a housing collateral constraint on consumers as in Iacoviello and Neri (2010), and widely used in Hong Kong modelling. However, this version is rejected by the Hong Kong data. In addition, we found out that the housing market has no role in the economy as the housing demand shock accounts for nearly zero, even in the estimated collateral model. We examined the economy's volatility using bootstrapping of the model innovations, under both the estimated currency board model and a standard alternative regime with floating exchange rate and a Taylor rule; we found that Hong Kong welfare is higher in the currency board, as it substantially reduces output and inflation volatility.

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