# Investigating the effects of fiscal policy in the UK under the zero lower bound on nominal interest rates

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# Abstract

This thesis investigates the effects of the UK post-crisis fiscal policy while the nominal interest rate hits the zero bound. With a limited ability to use conventional monetary policy to support the economy, fiscal policy gets more attention as the alternative to bring the economy back to normal. To evaluate the effectiveness and implications of the fiscal policy during the financial crisis, this thesis introduces a New Keynesian dynamic stochastic general equilibrium (DSGE) model with an explicit government sector and monetary policy governed by a Taylor rule with interest rate constrained by the zero lower bound. The simple New Keynesian framework without the presence of the financial frictions serves as the baseline model. The model is then extended to include financial frictions since the financial crisis of 2008 revealed the imperfection of financial markets. The model is estimated on filtered data for the period from 1989Q2 to 2017Q2 using the Bayesian estimation to find that fiscal policy is more effective under the zero lower bound. The extended version is then estimated and tested against the same period's unfiltered data using the Indirect Inference method. The estimated model shows that financial frictions work to amplify the effects of fiscal policy. Facing with the liquidity trap, fiscal stimulus could help the economy to escape the zero lower bound and improve social welfare. However, the UK government did not utilise enough of these fiscal policy and thus there was almost no recovery done by the fiscal policies. The fiscal policy had the scope to help the economy recover.

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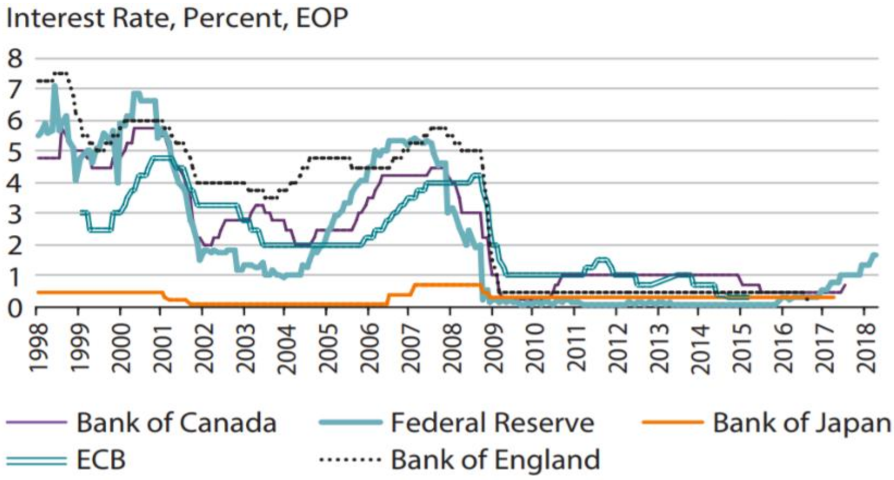
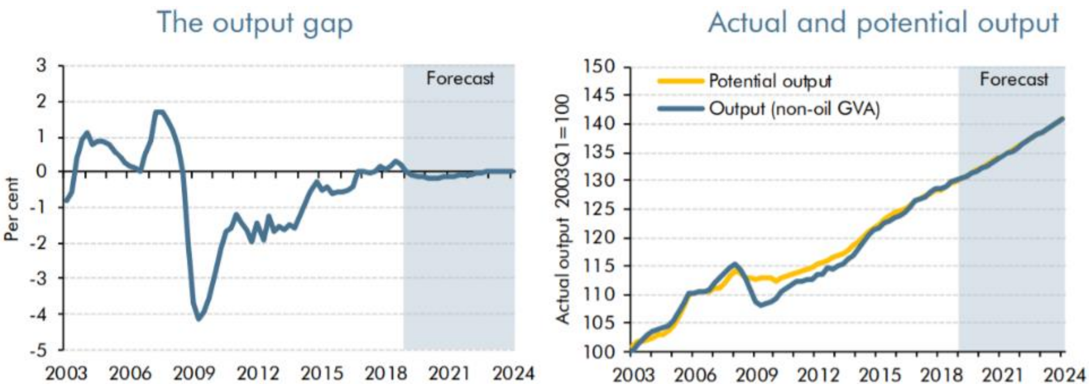
Figure 1.1 Central bank policy rates




Source: Haver Analytics, the Federal Reserve, and the International Monetary Fund.

Reacting to the global financial crisis, the central banks around the industrialized countries slashed their policy rates to historical low to stimulate the aggregate demand (see Figure 1.1). The short-term nominal rates are effectively 0% after the crisis, and these central banks cannot stimulate the economy by manipulating their bank rates any more. In parallel, these economies experience depressed aggregate activity and low inflation (see Figure 1.2 and Figure 1.3).

Figure 1.2 GDP of the U.K.[1]




Source: Office for Budget Responsibility, Economic and fiscal outlook (March 2019)

[1] Output gap estimates on a quarterly basis, based on the latest National Accounts data and expressed as actual output less potential output as a percentage of potential output (non-oil basis).

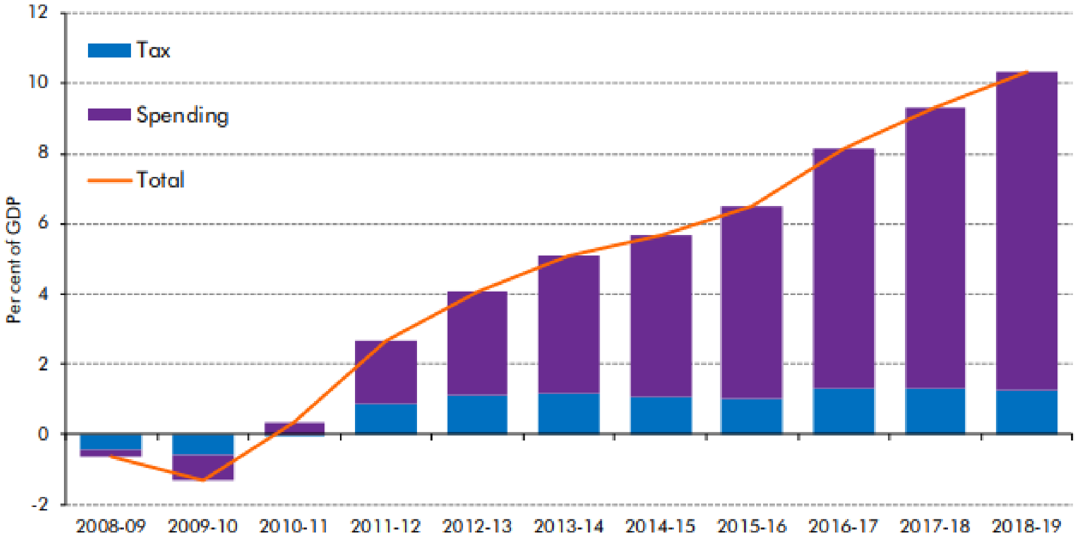
Figure 1.3 Inflation (CPI) of the U.K.


Source: Raw data CPI annual % change (D7G7) at Office of National Statistics

As the traditional policy tool is not available, central banks resorted to unconventional monetary policies, such as Quantitative Easing. However, heavy financial regulations imposed on banks prevented them from extending credit, especially to 'risky' smaller businesses. As remarked by Blinder et al. 2017, many financial institutions with huge financial stresses kept their funds instead of lending them out. Consequently, the effects of Quantitative easing are controversial and limited. With limited abilities of monetary policy, fiscal policy gets more attention as the alternative to bring the economy back to normal. There is an increasing number of studies discussing the role of fiscal policy in assisting the economic recovery and getting out of the liquidity trap. For the US, recent studies suggest that fiscal policy can be more effective under the zero lower bound. Christiano et al (2011), Eggertsson (2011) and Woodford (2011) show that government spending multiplier can be larger at zero nominal interest rate and government spending can improve social welfare. However, the supply-side fiscal stimulation appears less efficient, remarking by Eggertsson (2011) that a lower labour tax could lower the employment. Therefore, the effectiveness of the fiscal policy may depend on which side of the economy is affected.

In U.K., the government immediately responded to the financial crisis by implementing fiscal strategies. At the first two years just after the crisis, a package of the fiscal stimulus was installed by carrying out more government spending and cutting the tax rates (see Figure 1.4).

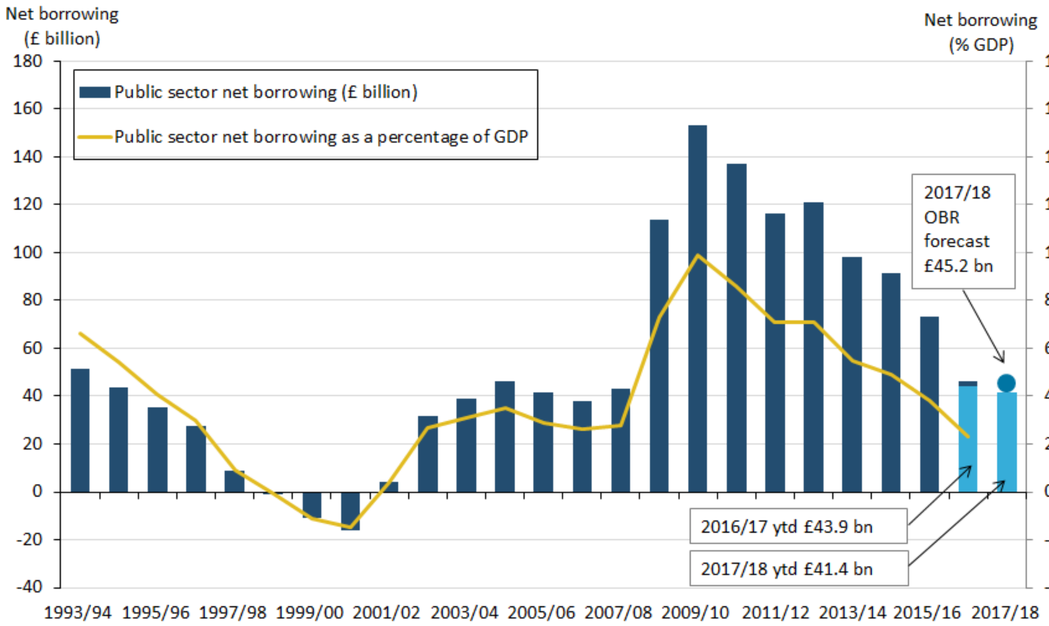
Figure 1.4 The U.K. fiscal policy measures since Budget 2008: tax and spending




Source: Institute for Fiscal Studies (IFS).

These stimulus measures were consistent with Eggertsson (2011) and Woodford (2011) who argue that the fiscal stimulus can expand output and generate inflation when zero lower bound is binding, and this is the optimal policy to stimulate the economy in a liquidity trap. While later on 2010, the U.K. government announced that the most urgent target was to reduce the burden of the deficit due to an increasing structural deficit and a struggling structural position faced by the country (see Figure 1.5). Confronting with this problem, the government decided to alter the fiscal strategy from undertaking stimulus measures to a sizable fiscal austerity in order to achieve a balanced government budget overall.

Figure 1.5 Public sector net borrowing (excluding public sector banks), U.K.[2]




Source: office for National Statistics

This thesis aims to investigate the effectiveness of the U.K. fiscal policy in the period after the financial crisis and asks what are the most effective fiscal policies in helping the economic recovery and how much of it should be applied? To answer these questions, we use New Keynesian model with an explicit government sector. Firstly, the model assumes perfect financial markets and is estimated using the Bayesian method. The multiplier analysis implies that increasing the government expenditures and cutting the consumption tax are expansionary regardless of the existence of the constraint on the nominal interest rate, and particularly, these measures become more effective at zero interest rate. While cutting the labour income tax and capital income tax are expansionary in a positive-interest-rate regime but contractionary in the liquidity trap.

Secondly, financial crisis shows that financial markets are not perfect, and as a

[2] (1) Financial year 2017/2018 represents the financial year ending 2018 (April 2017 to March 2018).

(2) Office for Budget Responsibility (OBR) full financial year forecast of £45.2 billion for public sector net borrowing excluding public sector banks (March 2018 Economic and Fiscal Outlook).

(3) Ytd equals year-to-date (April to February).

result, this feature is incorporated into an extended version of the macroeconomic model to allow for interactions richer transmission mechanism and fiscal policies. I estimate this version model with the financial imperfection assumption. It suggests that fiscal measures are more effective with the presence of financial frictions, more specifically in the liquidity trap. Although a Bayesian estimation is popular and used up to this point, it does not provide a statistical test of fitness of the model to data. Thus, I re-estimate and test the model using Indirect inference method. Unlike earlier sections, the data here is unfiltered to preserve all information. The resulted model therefore can be confidently used for policy analysis. The policy analysis finds that facing with the liquidity trap, fiscal policy can help the economy to get out of the zero lower bound and improve social welfare.

The thesis is organised as follows. The literature review is outlined in Chapter 2. Chapter 3 discusses the methodologies of estimation methods, Bayesian estimation and Indirect Inference, with the comparison. Chapter 4 introduces the baseline model built on the New Keynesian small open economy DSGE model with the zero lower bound. Chapter 5 extends the baseline model with financial frictions to incorporate the financial channels for specifying a significant source of aggregate shocks, and to explain the role of financial frictions in affecting the fiscal policy under different interest rate regimes. Chapter 6 evaluates and re-estimates the model by applying the Indirect Inference method to obtain more reliable results and improve the capacity of the model in fitting the data. Finally, Chapter 7 concludes the thesis.

# Chapter 2 Literature Review

The research question is to evaluate the post-crisis effectiveness and implications of the U.K. fiscal recovery. Following this motivation, this chapter surveys the literature on fiscal policy in terms of three aspects: (1) studying the effects of the fiscal policy including the government expenditure rules and tax rules in the economy where the nominal interest rate is constrained by the zero lower bound; (2) considering the fiscal effects in an open economy perspective; (3) discussing the fiscal policy in models with the presence of the financial friction.

**The literature survey of fiscal policy in a liquidity trap**

The zero lower bound on nominal interest rates creates a challenging situation for policies. In the standard Keynesian frameworks, the ineffective monetary policy in a liquidity trap entails the fiscal policy to play a more crucial role in stimulating and recovering the economy. The effectiveness of the expansionary fiscal policy in a liquidity trap is supposed to be fairly large. However, it is criticized for its lack of dynamics, expectations, and micro-foundations in the classic macroeconomic IS-LM model which cannot deliver a convincing analysis of policy.

Most of recent literature on effectiveness of fiscal policy utilizes the New Keynesian framework. Eggertsson (2008) studies the US recovery from the Great Depression regard to the policy actions by using a DSGE model with sticky prices and rational expectations. He finds that the expansionary effect of increasing government spending is great in the liquidity trap. The reason is when economic recovery is driven by an expectation shift from "contractionary" (expecting an economic and deflation in the future) to "expansionary" (expecting an economic expansion and inflation in the future), the expectation of higher future inflation in the liquidity trap helps the expansionary fiscal policy to lower the real interest rates, thus the aggregate demand is stimulated, and the expectation of higher future income level encourages the individual demand by increasing the permanent income.

Christiano, Eichenbaum and Rebelo (2011) evaluate the size of government spending multiplier in a more well-defined estimated DSGE model. With an effective Taylor rule, the nominal interest rate increases reacting to an expansionary fiscal policy shock that drives up the output and inflation. However, when the nominal interest rate is constrained by the zero lower bound, increasing the government spending substantially becomes a socially optimal policy which promotes the output and expected inflation. If the nominal interest rate stays at zero, higher expected inflation will push downward pressure on the real interest rate and raise the private consumption, thus, lead to another increase in output, and expected inflation and an additional reduction in the real interest rate. Hence, the multiplier effects of rising government spending are fairly large and offset the deflationary spiral accompanied the zero interest rate state. Additionally, Woodford (2011) discusses a large government spending multiplier in a liquidity trap by proposing the properties of the New Keynesian DSGE model and accounting for the intertemporal optimization and expectations in determining the aggregate economic activity. It suggests that the setting of sticky price or wage allow for a larger value of multiplier compared to the value in a neoclassical model, although this value mainly relies on the performance of the monetary policy. When the nominal interest rate is stuck at zero, the value of the multiplier can be greater than one, and, under this circumstance, if the expansion of the government spending can partly fill the output gap, the social welfare will be improved.

Bouakez, Guillard, and Roulleau-Pasdeloup (2017) classify the government spending into public consumption and investment, which implies that the government spending is not completely wasteful and does affect the marginal productivity in the private sector directly. They analyse the efficiency of increasing the government investment under the zero lower bound and compare this with the results in an unconstrained interest rate regime. It is illustrated that raising the government investment can expand the aggregate demand at zero interest rate, and also improve the natural output level even when the economy has escaped from the liquidity trap and promote the aggregate demand further in the short term, as long as the duration of accumulating the stock of public capital is long enough, which give rise to the conclusion that the fiscal stimulus plan should completely target on the measures of public investment.

Recently, Bilbiie, Monacelli and Perotti (2019) investigate the relationship between the social welfare and the value of the government spending multiplier under zero lower bound by building a medium-scale DSGE model and calibrating it to feature the dynamics of the main macroeconomic variables during Great Recession in the US. It is concluded that increasing government spending is notable welfare-improving at the zero lower bound, particularly executing the stimulus package in advance.

Regards to the effectiveness of the tax measures in a liquidity trap, there are several pieces of research studying the model with tax policy at the zero interest rate. Eggertsson and Woodford (2004) study the optimal tax policy in response to the liquidity trap to find which policy can diminish the distortions. The results show that adjusting the tax rates in the short term is the optimal strategy responding to the binding interest rate, but when the changes in the taxes only affect the supply side of the economy, they are required to be increased during the liquidity trap and committed to cut below their long-term level later. Besides, the optimal policy is also history-dependent. When the fiscal policy reacts to the shock with an appropriate response, the returns obtained from diverging from a strict inflation target are modest.

Eggertsson (2011) considers a standard New Keynesian DSGE model to evaluate the effects of cutting taxes and raising government spending under the zero lower bound when the economy suffers deflation and an output collapse. This paper argues that cutting the labour income tax is contractionary in the liquidity trap, since the decline in the tax rate derives downward pressure on inflation by decreasing the firms' marginal costs, hence raising the real interest rate. Due to the inability of the monetary authority to cut the bank rate further, the increase in the real interest rate generates a contractionary effect. Moreover, cutting the capital income tax is also contractionary at zero interest rate. Capital tax cuts encourage the households to save more rather than consuming, and then output declines. Since the problem of staying in the zero lower bound state is the lack of aggregate demand instead of the production capacity of the economy, less capital tax exacerbates the problem of inadequate aggregate demand when the exact opposite is required. While this problem can be solved by increasing government spending, which effectively expands the aggregate demand. Therefore, this paper summarises that cutting the labour and capital taxes is not the optimal policy in a liquidity trap because they are contractionary, but government spending stimulus is one of the optimal policy, though it should be accompanied with the sales tax cuts and investment tax credits.

Correia et al. (2013) consider the zero lower bound in the formation of a classic New Keynesian model and observe that a scheme of unconventional fiscal policy, involving an increasing path for consumption tax and a decreasing path for labour tax with a temporary cut in capital income tax, could be the optimal policy in the liquidity trap. As demonstrated, the unconventional fiscal policy can be implemented to duplicate the effects of a negative interest rate and avoids the problems of being stuck in the liquidity trap completely.

Boneva, Braun and Waki (2016) solve an analytically tractable New Keynesian DSGE model with the zero lower bound that imitates the output collapse and deflation in the Great Recession of the US. They detect that the effectiveness of the fiscal policy is sensitive to the persistence degrees of the zero lower bound. If the duration remaining at zero interest rate is sufficiently long, the cuts in labour tax derive expansionary effect and government spending multiplier becomes less than one, which are important as they increase the possibility of using the supply-side fiscal policy to stimulate the economy in the liquidity trap.

**The literature survey of fiscal policy in an open economy perspective**

Lately, several studies analyse the fiscal policy based on the perspective of an open economy. To detect how the features of the open economy reconstruct the traditional interpretation of the fiscal policy, the following papers are notably discussed.

When an economy is no longer a closed one, the imbalance in the current account will affect the transmission of policies substantially, particularly, the fiscal policy in a country with an unfavourable trade balance. Kim and Roubini (2008) empirically analyse the impacts of the fiscal rules caused by the government budget shocks on the real exchange rate and national current account with a floating exchange rate. Contrary to the literature, the results generated by the US data indicates that a budget deficit shocks, or an expansionary fiscal shock, derives the depreciation in the exchange rate and promotes the current account. More savings and less private investment give rise to the expansion in the current account. As a consequence, there exists a 'twin divergence' between fiscal deficit and current account deficit, implying that the current account worsens when government account improves. However, this conclusion is criticized by Monacelli and Perotti (2010). In this paper, they extend the study of Kim and Roubini (2008) to take account of the real exchange rate and trade balance and generally analyse the reactions of the real exchange rate, trade balance, and their joint movements with private consumption and output to the government spending shocks. This paper finds that an increase in the government spending (fiscal balance worsens) leads to the trade balance deficit and real exchange rate depreciation, which provides the evidence for the conventional 'twin deficit' assumption but is contrary to the findings of Kim and Roubini (2008). This is mainly because of a negative wealth effect impacting on the performance of private consumption.

Ilzetzki, Mendoza and Végh (2013) identify the effects of the government spending shocks based on the features of the open economy, the regime of the exchange rate and the openness degree of the trade. This paper finds that the government spending multiplier is fairly great in the countries with the fixed exchange rates while is zero when the exchange rates are floating. Government spending multiplier is also closely related to the openness degree since the values of fiscal multipliers in the open economy are significantly less than those in the closed economy, even the values become negative in certain economies with high-degree of openness.

Additionally, recent developments in the fields of open economy aroused the renewed interests in studying fiscal policy taking zero lower bound into account. Several attempts have been made by Jeanne (2009), Cook and Devereux (2013), Fujiwara and Ueda (2013) and Erceg and Lindé (2013).

Jeanne (2009) introduces a two-country DSGE model with nominal stickiness where the economies could be hit by the demand shocks. With a negative demand shock hitting to one economy, not only the interest rate is declined in the economy shocked by the disturbance but also in the other economy, which implies that the global liquidity trap with unemployment and zero nominal interest rates may be caused by an international spillover effect when one country hits by a demand shock and falls into the liquidity trap. In that case, the fiscal policy can play a role in helping the economies return to the full employment level. An increase in government spending can expand the aggregate demand when private consumption is not a perfect substitute for government consumption. Thus, the fiscal stimulus can recover the economy by reaching full employment during the period of a global liquidity trap. But this paper argues that the fiscal stimulus may not be the optimal policy to achieve the first-best level of social welfare, since the allocation of the consumption between the private and government sectors may be distorted, which means increasing the government spending may crow out the private consumption and lead to overconsuming the public goods.

Cook and Devereux (2013) study the optimal policy strategies to manage the global liquidity trap by using a two-country model. The crucial characteristic of this model is that the relative prices react perversely. The declines in the demand of the home country lead to an appreciation in terms of trade, intensifying the disturbance. In the liquidity trap, the home country cannot respond to this shock. Then the optimal strategy may require the foreign country to increase the interest rate at a positive rate even its natural level is negative. The optimal policy involves close cooperation between the monetary and fiscal policies, which strictly relies on the degree of trade integration. In the highly-open-trade markets, the optimal policy is to implement the equal fiscal stimulus in both the home and the foreign counties when they are in the liquidity trap simultaneously. However, if the trade is inadequate, the optimal strategy is to implement an impressive fiscal stimulus in the home country, accompanied by a positive interest rate and modest fiscal stimulus in the foreign country. This result suggests that the monetary policy and fiscal policy should be mutually supportive to manage the global liquidity trap.

Different from the above literature, Fujiwara and Ueda (2013) assume that both two economies are trapped in the liquidity trap in a medium-sized two-country DSGE model and investigate the fiscal multiplier and fiscal spillover when both these two economies stay at zero interest rate regime. In this paper, the contrary results are generated compared with the traditional economic theories. In the economy where government spending occurs, the value of the fiscal multiplier is greater than one, the terms of trade and currency depreciate. The fiscal spillover has a negative effect when the intertemporal substitution elasticity of the consumption is below one, or a positive effect when the elasticity exceeds one. One of the significant factors in explaining the effects of fiscal policy in terms of the view of an open economy is the incomplete stabilization of marginal costs caused by the zero lower bound. Though the model includes the incomplete markets or endogens stock of capital, the results stay the same. The assumption of local-currency pricing derives the effects of the fiscal spillover becoming positive regardless of the size of the substitution elasticity.

Besides discussing government spending, Erceg and Lindé (2013) include the tax policy in their model. They introduce a two-country DSGE model to evaluate the effects of tax-based fiscal consolidation compared to the spending-based fiscal consolidation in a union of currency. There are three important findings. (1) with a limited ability for the monetary authority to manipulate the nominal interest rate, the negative effects of the spending-based fiscal consolidation are supposed to be larger than the effects of the tax-based consolidation, although the tax-based consolidation tends to be more expensive in the long-term. (2) when the nominal interest rate is constrained, implementing a large size of spending-based consolidation possibly generate counterproductive effects to the economy, implying that the losses of the output increase at the margin. (3) the mixed policy which incorporates a temporary and sharp increase in taxes and gradual and slow cuts in government spending is an appropriate strategy in minimizing the output losses caused by the fiscal consolidation.

**The literature survey of fiscal policy with the presence of financial frictions**

In the traditional New Keynesian model, it is assumed that the nominal interest rate set by the monetary authority is uniquely decided by the cost of funds, which implies a perfect financial market. However, the recent crisis in 2008 reveals the shortcomings of this straightforward assumption, and it is more promising to present the financial frictions in the macroeconomic model in the wake of the financial crisis. Based on these facts, several studies contribute to the fields of investigating the effects of fiscal policy with the presence of financial frictions.

To evaluate the effectiveness of the fiscal policy, Fernández-Villaverde (2010) introduces a DSGE model with financial frictions, calibrates this model to observe the performance of the US economy, and analyses the dynamics of the output to various fiscal shocks. With the presence of financial frictions, increasing government spending can be more efficient compared to the cuts in taxes in stimulating the GDP in an extremely short term. This phenomenon is caused by the dynamics of the real wealth generated by the 'Fisher effect' and the endogenous finance premium derived by the fiscal shocks.

Another notable contribution to the literature is that of Eggertsson and Krugman (2012). In this paper, they introduce a simplifying New Keynesian model to describe the slumps induced by excess debt held by the households causing a shrink in the aggregate demand. This paper finds that when the deleveraging shock pushes the economy into an unusual and chaotic world where saving is harmful, increasing productivity will lower the output level and flexible wages raise the unemployment rate. In that case, the expansionary fiscal policy could be efficient, since the effects of the shock are intrinsically temporary, which requires a short-term fiscal response to stimulate the economy. Suggesting by the paper, the temporary increase in government spending not only does not crowd out the private consumption but also increases the expenditures of liquidity-constrained debtors.

Corsetti et al. (2013) consider the financial frictions by introducing a sovereign risk channel to investigate the stabilisation policy and macroeconomic dynamics. The mechanism of this channel is that increasing the risk pushes an upward pressure on the costs of private borrowing. When the ability of the monetary authority to offset the widened credit spreads is limited, the sovereign risk turns to play a crucial role in determining the macroeconomic dynamics. It suggests that the relationship between the sovereign risk channel and fiscal multipliers is very close. Precisely, the impact of the government spending on the output depends on: (1) the sensitivity of the risk premium to adjustments in government indebtedness; (2) the length of the expected duration when zero lower bound constrains the policy rate; (3) the responsiveness of tax policy to economic activity. Besides, this paper finds that the pro-cyclical fiscal strains are generally more favourable to the economy since the decreases in the budget deficits enhance the ability of private financing. In extreme examples where the fiscal tightening is extremely harsh and the expected duration of being constrained by the zero lower bound is persistent, the fiscal retrenchment in a downward path may even have an expansionary effect to the economy. Besides, the cases of the self-defeating fiscal consolidation with an average level of fiscal tightening and extremely long duration of zero lower bound are found out being detrimental to output and increasing the deficits.

Carrillo and Poilly (2013) find that the imperfections in the credit market considerably raise the effects of increasing the government spending by studying a New Keynesian model with financial frictions à la Bernanke, Gerler and Gilchrist (1999) (BGG). As in BGG, the source of the financial accelerator which propagates the shocks to the economy is the existing of the asymmetric information between borrowers and lenders, implying that if there is an idiosyncratic disturbance to the return of the firm, it can only be observed by the entrepreneurs, the borrowers, but not by the bank, the lenders. Hence, it is risky for the bank to lend to the entrepreneurs as the result of idiosyncratic shocks to the return of the potential investment. To monitor the realized return of the entrepreneurs, the bank has to pay an extra fixed monitoring cost, which is called a "costly state verification" problem assumed by Townsend (1979) and Gale and Hellwig (1985). To mitigate the problems between borrowers and lenders due to the asymmetric information and agency costs, the entrepreneurs need to sign a standard debt contract with the bank to get the loans. According to the assumption of the BGG, the setup of the optimal contract indicates a negative relationship between the entrepreneurs' wealth and the agency costs. Particularly, an entrepreneur with a strong balance sheet position revealing a high level of net worth and relatively low leverage relies on the external finance comparatively less, thus the risk of bankruptcy and the premium on the external finance for this borrower is small, and vice versa. In the model setting of BGG, the cost of funding to a firm is determined endogenously with the cycle by a 'financial accelerator' effect. The mechanism of this effect illustrates a mutual effect between the cost of funding and the net worth of the entrepreneurs. Therefore, the mechanism of the presence of financial frictions affecting the fiscal effectiveness can be described as that when there is an increase in aggregate demand due to an expansionary fiscal policy, both the demand for capital stock and its price rise, which improves the value of the entrepreneurs' net worth. The growth in the net worth drives down the leverage of the firm, which induces a decrease in the costs of funding and stimulates the investment. Hence, there is a feedback effect caused by an increase in capital demand, which improves the net worth further. Finally, the multiplier effect, or so-called 'accelerator' effect, on investment demand completely proceeds. The model in this paper not only incorporates the mechanism of the financial accelerator as in BGG but also includes a debt-deflation effect that causes the cost of funding fluctuations and is absent in BGG. The basic assumption of this effect is that the debt contracts are signed in nominal terms, which transfers the deflation effect as introduced by Fisher (1933). Consequently, any unpredicted oscillation in the price level of the economy could change the actual value of the debt burden of the entrepreneurs, since any risk from macroeconomic uncertainty is avoided by the lenders. More specifically, as entrepreneurs, if an increase in the government spending pushes up the inflation, their balance sheets will be ameliorated because the contracts are written regard to the nominal interest rate, which means a better financial position enhances the firm's capacity to invest, and vice versa. Indeed, this is the mechanism of the debt deflation effect, and it is the central financial element to shape the dynamics of the model economy and one of the essential derivation of nominal rigidities in the model.

By relying on previous literature discussed above, this thesis contributes to building a New Keynesian DSGE small open economy model of the U.K. economy with the zero lower bound to evaluate the effectiveness of the fiscal policy responses to the financial crisis of 2008 in the U.K. This thesis also attempts to fill the gap in existent literature regard to the effects of the mutual interaction between a liquidity trap and the financial frictions on the effectiveness of the fiscal policy and to explain the role of financial frictions in affecting the effectiveness of the fiscal policy under different interest rate regimes.

# Chapter 3 Methodology of estimation and comparison

The models are estimated by Bayesian estimation and then estimated and evaluated by the Indirect Inference method. This chapter explains each method in details. Besides, the method of solving the DSGE model with occasionally binding constraint is also presented.

## 3.1 Bayesian estimation

### 3.1.1 Introduction of Bayesian approach

Traditional statistical estimation methods typically suppose that there is no link between variables. Hence, the null hypothesis generally assumes no relationship between the variables and no prior information about them. Nevertheless, it is very common that the academics do know the variables possibly based on the previous investigations or empirical evidence. Accordingly, the Bayesian technique takes the background information into account to proceed with the estimation, which is distinctly different from the traditional method referred to a frequentist framework that relies on repeated experiments over several times. In other words, the Bayesian statistic differs from the traditional statistic, such as the maximum likelihood, in holding different idea about the unknown variables.

To clarify the statement above, an example is presented. Considering a regression equation $y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \varepsilon$, where $y$ denotes the dependent variable; $x_1$ and $x_2$ denote the independent variables; $\varepsilon$ is the residual; $\alpha$, $\beta_1$ and $\beta_2$ are the parameters given unknown before the estimation. The traditional methods assume that there is only one fixed solution for each parameter and no information about this value before the estimation. While Bayesian method provides a probability distribution containing the true value for each parameter, which means each parameter is given a distribution to specify the uncertainty. The distribution defined before observing the data is so-called the prior distribution. Next, by observing the data, the evidence is described in the form of the likelihood function of the data. The prior uncertainty is then updated by the data likelihood to derive a posterior distribution for each parameter which contains less uncertainty. Hence, these three ingredients, considering the prior distribution, the likelihood function and the posterior distribution, constitute the Bayes' theorem.

In the latest years, the Bayesian estimation has been widely applied in several macroeconomic studies and becomes a substantial method to estimate the macroeconomic models (Smets and Wouters, 2003 and 2007; An and Schorfheide, 2007; Adolfson et al, 2007; Del Negro and Schorfheide, 2008; Fujiwara, Hirose, and Shintani, 2011). As Guerrón-Quintana and Nason (2013) explained, the reason why the Bayesian approach turns to be widely used is that it can offer the academics the opportunity to evaluate the macroeconomic models where the frequentist econometrics is found difficult to implement, particularly in the DSGE model. Even though the DSGE model can be evaluated by adopting conventional optimization approaches, the Bayesian technique is still popular in estimation. On one hand, the Bayesian method provides a growing array of tools which can be employed by the investigators to evaluate the DSGE model. On the other hand, the Bayesian technique is also popular in estimating a medium or large scale DSGE model with its powerful computing capability by applying Markov chain Monte Carlo (MCMC) simulators. But for frequentist econometricians, these DSGE models have the problems of identification which cannot be solved by themselves. The frequentist econometricians also criticize the DSGE model for misspecifying the true model, since the DSGE model is usually built as an abstraction of the actual economy. But

Bayesian estimators eschew the existence of such a true model and argues that there is no DSGE model exactly fits the true model.

### 3.1.2 Bayesian estimation.

Nowadays, Bayesian inference is commonly used to estimate a macroeconomic model. One of the most essential features of this method is to estimate the model cooperating with the prior information. The detailed estimation procedure of Bayesian theorem will be presented in the following section.

Assume there are two random variables $A$ and $B$. The joint probability of $A$ and $B$ occurring simultaneously is defined:

$$p(A,B) = p(A|B)p(B), \tag{3.1}$$

where $p(A|B)$ is the conditional probability of $A$ happening relying on $B$ having happened and $p(B)$ refers to the marginal probability of $B$. In contrast, the joint probability can be defined as follows as well:

$$p(A,B) = p(B|A)p(A). \tag{3.2}$$

By equating the RHS of equation (3.1) and (3.2), $p(B|A)$ can be obtained as:

$$p(B|A) = \frac{p(A|B)p(B)}{p(A)}, \tag{3.3}$$

which is the crucial rule for Bayes' theorem. While it becomes more complicated when this rule is practised to an economic model. Usually, when considering the data in an economic model, there are more various parameters and observations. Assume $y$ denotes a vector or matrix of the data and $\theta$ is a vector or matrix of parameters used to describe the data. Then in an economic model, the question for a Bayesian estimation alters to detect the values for the parameters $\theta$ based on the data $y$. By applying the above Bayesian rule (equation (3.3)) and substituting $A$ and $B$ by y and θ respectively. The result is obtained as:

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}. \qquad (3.4)$$

The conditional probability of $\theta$ above refers to a fundamental question in estimating the model what the information of the parameters $\theta$ can be found by giving the data $y$. In equation (3.4), the denominator $p(y)$ can be ignored because the only interest is the coefficients $\theta$ in regression. By neglecting the marginal probability of $y$, the rule can be rewritten as:

$$p(\theta|y) \propto \; p(y|\theta)p(\theta), \qquad (3.5)$$

which presents a familiar relationship in implicating the Bayesian method to estimate an economic model. The above relationship indicates that the posterior is proportional equals to the product of the likelihood and prior as $p(\theta|y)$ denotes the posterior density, $p(y|\theta)$ refers to the likelihood function and $p(\theta)$ is the prior density respectively. It is also named as the kernel of posterior in Koop (2003), which means that the knowledge of the parameters can be obtained by giving the prior information of the parameters and updating it by the observed data.

In the Bayes' view, the prior density, $p(\theta)$, is a non-data distribution which does not involve any information for the data. Normally, the prior distribution is obtained from the literature as the prior beliefs on the coefficients before the regression. At this stage, Bayesian estimation is argued as a controversial method. The likelihood function, $p(y|\theta)$, describes the process of generating the data, which is the distribution of the data conditional on the parameters in the model. Based on the information by observing the data, the prior beliefs can be updated to form the posterior density. The posterior density, $p(\theta|y)$, describes all the updated prior information after observing the data, which takes both data and non-data information into account.

Having generally introduced the fundamental concepts of Bayesian econometrics, it is the time to discuss the estimation process. Firstly, the model in this chapter is solved by an occasionally binding approach. Then providing the prior distribution and obtaining the likelihood function by applying an inverse filter which will be explained in details in the next part of this section. Next, the posterior distribution can be calculated out by maximising the kernel of the posterior subject to every parameter. Finally, the moments of the parameters are found by applying a Random Walk Metropolis-Hastings (RWMH) algorithm.

The RWMH is a general algorithm that finds out the Markov chain with stationary distribution relating to the posterior density. This algorithm is first built up by Metropolis et al. (1953) and then summarized by Hastings (1970). In this chapter, the draws of the posterior distribution are generated by the RWMH algorithm following the steps in An and Schorfheide (2007) as:

1. Use a numerical optimization routine to maximize the log posterior kernel $\ln p(y|\theta) + \ln p(\theta)$. Denote the posterior mode by $\tilde{\theta}$.
2. Let $\tilde{\Sigma}$ be the inverse of the Hessian computed at the posterior mode $\tilde{\theta}$.
3. Draw $\theta^{(0)}$ from $\mathcal{N}\left(\tilde{\theta}, c_0^2\tilde{\Sigma}\right)$ or directly specify a starting value.
4. For $s = 1, \ldots, n_{sim}$, draw $\vartheta$ from the proposal distribution $\mathcal{N}\left(\theta^{(s-1)}, c^2\tilde{\Sigma}\right)$. The jump from $\theta^{(s-1)}$ is accepted $\left(\theta^{(s)} = \vartheta\right)$ with probability $min\left\{1, r(\theta^{(s-1)}, \vartheta|Y)\right\}$ and rejected $\left(\theta^{(s)} = \theta^{(s-1)}\right)$ otherwise. Here
$$r\left(\theta^{(s-1)}, \vartheta \mid Y\right) = \frac{p(Y|\vartheta)p(\vartheta)}{p(Y|\theta^{(s-1)})p(\theta^{(s-1)})} = \frac{Posterior(\vartheta)}{Posterior(\theta^{(s-1)})}.$$
5. Approximate the posterior expected value of a function $h(\theta)$ by $\frac{1}{n_{sim}}\sum_{s=1}^{n_{sim}} h(\theta^{(s)})$. (An and Schorfheide (2007), p.20)

### 3.1.3 Accounting for the occasionally binding constraint.

The Bayesian estimation method applied in this thesis follows the methodology in Guerrieri and Iacoviello (2017) that estimate the model concerning the occasionally binding constraint of zero lower bound on the nominal interest rate. By using the occasionally binding approach based on the previous study, the model can be solved nonlinearly. In that case, assuming that the model can be solved in two regimes, one regime with a binding zero lower bound and the other one without. In each regime, the first-order approximation is around the same point, which is linked by the solution approach of occasionally binding. The details of the solution method refer to Section 3.4.

Based on the result in Section 3.4, the solution form of the model can be described as:

$$X_t = P(X_{t-1}, \epsilon_t)X_{t-1} + D(X_{t-1}, \epsilon_t) + Q(X_{t-1}, \epsilon_t)\epsilon_t. \qquad (3.6)$$

All the variables except the innovations processes are collected by the vector $X_t$, while the shocks are arranged in vector $\epsilon_t$. The coefficients in reduced-form are located in matrix $P$, $Q$ and vector $D$. These matrices and vectors are functions of the lagged state vector and of the current innovations. However, while the current innovations can trigger a change in the reduced-form coefficients, $X_t$ is still locally linear in $\epsilon_t$.

To express the solution in terms of the observed variables, each term needs to multiply a matrix $H_t$ which is the matrix collects the observed series since the vector of observed variables $Y_t$ is equal to $H_t X_t$. For matrix $H_t$, it collects coefficients, since the interest rate is dropped from the observed vector under zero lower bound. In this situation, the monetary shock is assumed to be zero as well, unless the national rate which is implied by the model is a positive number otherwise the observed rate is zero. Under that circumstance, the national rate is selected as observed and the monetary shock is reinstated.

As the coefficients in reduced-form in equation (3.6) rely on the current innovations $\epsilon_t$ endogenously which cannot apply Kalman filter to retrieve the estimates of the innovation in $\epsilon_t$, alternatively the innovations $\epsilon_t$ are solved by given $X_{t-1}$ and the current realization of $Y_t$ recursively following Fair and Taylor (1983). The structure of the non-linear functions as following:

$$Y_t = H_t P(X_{t-1}, \epsilon_t) X_{t-1} + H_t D(X_{t-1}, \epsilon_t) + H_t Q(X_{t-1}, \epsilon_t)\epsilon_t. \qquad (3.7)$$

In principle, there is the possibility that $\epsilon_t$ has multiple solutions to the extent that equation (3.7) is highly nonlinear. However, in theory, the likelihood function can be constructed without depending on a one-to-one mapping between $Y_t$ and $\epsilon_t$ by using the approach introduced in this chapter. When building up the likelihood function, a general correspondence between $Y_t$ and $\epsilon_t$ is allowed to invoke the standard results.

The vector $\mathrm{X_t}$ encompasses the unobserved components, hence an initialization is required by the scheme of filtering. Assume that the steady-state of the model coincides with the initial value of $\mathrm{X_0}$ and the filter is trained by using the first 22 observations. Besides, the distribution of innovations $\epsilon_t$ is assumed to be a multivariate Normal distribution with the covariance matrix $\Sigma$. According to that assumption, the transformation of the likelihood function $f$ for the observed data $Y^T$ can be expressed in the following logarithmic transformation which is implied by the change of variable argument:

$$log\big(f(Y^T\ )\big) = -\frac{T}{2} log(det(\Sigma)) - \frac{1}{2}\sum_{t=1}^{T} \epsilon_t'(\Sigma^{-1})\epsilon_t + \sum_{t=1}^{T} log\left(\left|det\frac{\partial\epsilon_t}{\partial Y_t}\right|\right). \qquad (3.8)$$

To obtain the inverse transformation above from the innovations to the observation variables, the Jacobian matrix $\frac{\partial \epsilon_t}{\partial Y_t}$ is needed. It can be only solved implicitly by $\left(H_t Q(X_{t-1}, \epsilon_t)\right)\epsilon_t - (Y_t - H_t P(X_{t-1}, \epsilon_t) X_{t-1} - H_t D_t) = 0$. Due to the nonzero term of determinant $H_t Q(X_{t-1}, \epsilon_t)$, the implicit differentiation can proceed. Therefore, it is a locally invertible implicit transformation and the Jacobin matrix of the inverse filter is written as:

$$\frac{\partial \epsilon_t}{\partial Y_t} = \left(H_t Q(X_{t-1}, \epsilon_t)\right)^{-1}. \qquad (3.9)$$

The local linearity in the innovation $\epsilon_t$ of the solution of the model (namely, $\frac{\partial P(X_{t-1},\epsilon_t)}{\partial \epsilon_t} = \frac{\partial D(X_{t-1},\epsilon_t)}{\partial \epsilon_t} = \frac{\partial Q(X_{t-1},\epsilon_t)}{\partial \epsilon_t} = 0$, where these derivatives are specified) depends on the derivation of this Jacobin matrix above. Obtaining this result and observing that $\left|det\left(H_t Q(X_{t-1}, \epsilon_t)\right)^{-1}\right| = 1/|det H_t Q(X_{t-1}, \epsilon_t)|$, equation (3.8) can be rewritten as:

$$log\left(f(Y^T\ )\right) = -\frac{T}{2} log(det(\ \Sigma)) - \frac{1}{2}\sum_{t=1}^{T} \epsilon_t'(\Sigma^{-1})\epsilon_t - \sum_{t=1}^{T} log(|det H_t Q(X_{t-1}, \epsilon_t)|). \qquad (3.10)$$

Under this circumstance, there is no further requirement on calculations, and the Jacobin matrix of the inverse filter can be solved out based on the solution of the model. Hence, the evaluation of the likelihood function in a matter of seconds is allowed by this characteristic of the solution, which significantly reduces the time compared to the general method in Fair and Taylor (1980) and makes the estimation possible.

## 3.2 Indirect Inference

### 3.2.1 Introduction of Indirect Inference

The pioneer in applying the Indirect Inference method to the DSGE model is Smith (1993), which is further improved as a general technology by Gourieroux, Montfort and Renault (1993). These outstanding studies along with the additional well-known researches, such as Gregory and Smith (1991), Gourieroux and Montfort (1995) and Canova (1998) comprise the core of the literature of the indirect inference estimation that is viewed as a general concept of a simulated method of moments. Recently the academics including Le et al. (2011 and 2016), Liu and Minford (2014), and Meenagh et al. (2019) are inspired by the literature and extend the method of Indirect Inference to evaluate and estimate a previously estimated or calibrated DSGE model. It is noted by Meenagh et al. (2019) that the Indirect Inference is a preferably suitable approach to test a DSGE model which is already estimated by certain techniques, for example, the Bayesian method. As known, the DSGE model could not 'truly' represent the real economy since it is a simplified form of the reality, thus, a DSGE model should be tested on whether it is 'pseudo-true' rather than 'literally true', and the test should provide a powerful technique to identify the ability of a model in revealing the essential dynamics of the real economy concerned by the researchers. This utility of a test not only is acted according to the simplifications in the DSGE model but also follows the theory of the Friedman test. It is remarked by Friedman (1994) that the objective of a test should not be set on answering whether an economic model is the 'literal truth' but whether the model could behave 'as if it is true'.

The fundamental design of processing the Indirect Inference method is to introduce an auxiliary model to provide the representations from both the data and the theory performance, and further to generate a function of criterion by comparing these two descriptions. Different from the conventional method to evaluate the model, such as 'matching the moments' by which various moments are derived to evaluate the fitness of the calibrated model to the data, the Indirect Inference method is a formal statistic test which is obtained to generate a unique statistic to summarize the model. It is argued by Le et al (2011) that 'matching the moments' is an informal method whose procedure is free of distribution and supplying lock of formal statistical inference to judge the closeness of the various moments, e.g. variances, co-variances, cross-correlation and auto-correlation, which causes that the model is tested based on an unknown critical region. Furthermore, the statement of DeJong and Dave (2011) supports this view by declaring that experimental results could be ambiguous analysed as if the statistical formality is neglected, and it is undoubted that the judgements of whether the observed data is matched by the theory can be subjective.

The statement about the formal formation of the Indirect Inference can be explained in details based on the methodology designed for Indirect Inference evaluation in Canova (2007). When applying the Indirect Inference method to evaluate a macroeconomic model, the simulated data can be derived based on the given parameters of the structural model and the error distributions. By choosing the parameters, the estimation results of the auxiliary model from simulated data are close to those generated from the actual data. Suppose that there is a set of observed data $y_t$ with $n$ dimension whose probability density function is $f(y_t|\beta)$in which $\beta$ is the vector of parameters in the structural model. Additionally, assume that an auxiliary model can be defined correctly with a tractable probability density function $p(y_t|\theta)$ in which $\theta$ is the vector of parameters in the auxiliary model. This auxiliary model could be simply specified as a data-analysing model that does not explain the mechanism in observing data, with which Durlauf and Blume (2008) present a similar view as "The auxiliary model serves as a window through which to view both the actual, observed data and the simulated data generated by the economic model: it selects aspects of the data upon which to focus the analysis".

By maximizing the likelihood function $p(y_t|\theta)$, the estimates of the parameters in the auxiliary model using the actual data can be obtained as follows:

$$\hat{\theta} = arg\,\max_{\theta} \sum_{t=1}^{T} log\, p(y_t|\theta), \tag{3.11}$$

where certain properties of the actual data can be captured by $\hat{\theta}$ which is generally consistent with the estimator $\beta$.

Besides that, the auxiliary model also needs to be estimated using the simulated data. By simulating the structural model, the replicates of the simulated data can be produced by drawing $s$ times from the data independently, which is denoted as $x_t(\beta)$, and a certain value $\beta_0$ is assumed to allow $\{x_t(\beta_0)\}_{s=1}^{S}$ following the same distribution with $\{y_t\}_{t=1}^{T}$. Based on these pseudo observations $\{x_t(\theta)\}_{s=1}^{S}$ by setting some initial values for the variables and parameters, the parameter estimates of the auxiliary model using the simulated data can be obtained via the above methodology written as following:

$$\tilde{\theta}(\beta) = arg\,\max_{\theta} \sum_{s=1}^{S} \sum_{t=1}^{T} log[p(x_t(\beta)|\theta)]. \tag{3.12}$$

To make sure $\tilde{\theta}(\beta)$ is the most approximated value to $\hat{\theta}$, the Indirect Inference applies the method of simulated quasi-maximum likelihood (SQML) which serves to select the values of parameters to match the representation of the actual and simulated data in the following way:

$$b(\beta) = arg\max_{\beta} \sum_{s=1}^{S} \sum_{t=1}^{T} log\left[p(y_t | \tilde{\theta}(\beta))\right], \tag{3.13}$$

where a particular value of $\theta$ given by $b(\beta)$ is produced to maximize the likelihood function based on the finding function of $\beta$. Suppose that such a particular parameter $\theta$ truly exists, then the actual data and simulated data are expected to satisfy a condition as:

$$\theta = plim\hat{\theta} = plim\tilde{\theta}(\beta). \tag{3.14}$$

Even though the above condition holds, it does not mean that this structural model fits the actual data well, but it is a much weaker requirement for the structural parameters which can make the 'shallow' parameter estimated from the actual data and simulated data identical. In other words, the auxiliary model is required to be parameterized in a rich enough set to mimic the main features of the actual data. The details about how to choose an auxiliary model will be discussed in the subsequent section of 'the choice of the auxiliary model'.

### 3.2.2 Indirect Inference test

It is important to understand the procedure and the essence of the Indirect Inference test before turning to argue the methodology of the estimation since Indirect Inference is a simulation-based approach for parameters estimation. The fundamental method of the test is to adopt an auxiliary model which is generally in a form of a Vector Auto Regression with the Exogenous variable (VARX) model to compare the performance based on the actual data and simulated data generated from the structural model and to develop a criterion function. To measure the distance between the actual and simulated data, the test employs the Wald statistic as a suitable metric. The Wald statistic can be computed as follows:

$$W = \left(\beta^{\alpha} - \overline{\beta^{s}(\hat{\theta}_0)}\right)'\Omega^{-1}\left(\beta^{\alpha} - \overline{\beta^{s}(\hat{\theta}_0)}\right), \tag{3.15}$$

where $\beta^{\alpha}$ is the VAR coefficients in the auxiliary model generated from the actual data; $\overline{\beta^{s}(\hat{\theta}_0)} = E\left(\beta^{i}(\hat{\theta}_0)\right) = \frac{1}{s}\sum_{i=1}^{s}\beta^{i}(\hat{\theta}_0)$ is the sample average of the estimated coefficients obtained from $s$ sets of simulated data from the structural model whose parameters are calibrated or estimated as $\hat{\theta}_0$ as given; the variance-covariance matrix of the distribution of $\beta^{i}$ is denoted as $\Omega = cov\left(\beta^{i}(\hat{\theta}_0) - \overline{\beta^{s}(\hat{\theta}_0)}\right) = \frac{1}{s}\sum_{i=1}^{s}\left(\beta^{i}(\hat{\theta}_0) - \overline{\beta^{s}(\hat{\theta}_0)}\right)\left(\beta^{i}(\hat{\theta}_0) - \overline{\beta^{s}(\hat{\theta}_0)}\right)'$.

To execute the Indirect Inference test, there are three main steps, which is presented in Minford et al. (2009) originally, and then developed by Le et al. (2011) introducing Monte Carlo experiments and also by Le et al. (2016) using non-stationary data. The full-scale explanation of the test is referred to those original papers. The following is a brief description of the procedure of applying the Indirect Inference test to the DSGE model.

**Step 1: Calculate shock processes**

The residuals along with the innovations of the DSGE model conditional on the observed data and the set of structural parameters need to be computed at first. Within the model system, if there is no future expectation in an equation, the errors can be directly calculated from the data and the parameters; instead, the terms of the rational expectations could be computed by the methods of robust instrumental variables introduced by McCallum (1976) and Wickens (1982), where the instruments are the lagged endogenous data and thus the instrumental variables regression is the auxiliary VAR model. Then the autoregressive coefficients (shock persistence) and the innovation are estimated by treating the errors as the autoregressive processes and using the OLS method to clarify the autoregressive behaviours.

**Step 2: Derive the simulated data by bootstrapping**

After obtaining the innovations from the previous step, the simulated data can be generated by bootstrapping these innovations. The innovations need to be bootstrapped according to the time vector, which can preserve any simultaneity between them and then put them back to the original shock processes, thus the size and the distribution of the new sample are identical to those of the original shocks. The shock processes are then drawn by an overlapping manner and used for generating the results of the model. At the starting period, the model is solved by drawing a vector of shocks and given its initial lagged values using Dynare (Juilliard, 2001), and then this solution turns to be the vector of lagged variables for next period. At the period 2, the model is solved by drawing the second vector of shocks with a replacement for this period, and this solution becomes the lagged variable vector for period 3, and so forth until there is a serious of full-size bootstrapped simulations. Finally, this process needs to be repeated $S$ times to generate $S$ sets of bootstrapped simulation prepared for the test, where $S = 1000$. Generally speaking, the procedure of obtaining the bootstrapped simulations includes drawing the bootstrapped innovations by time vector with replacement uniformly and solving the model by using these pseudo-random innovations to obtain $S$ simulated datasets, each of which generates a set of estimated parameters of the auxiliary model as $\beta^i(\hat{\theta}_0)$.

**Step 3: Calculate the Wald statistic**

For the model evaluation, the Wald statistic can be calculated by equation (3.15) which is used as the test statistics based on the distribution of the difference between $\beta^{\alpha}$ and $\overline{\beta^{s}(\hat{\theta}_{0})}$. In principle, the Wald statistic quantifies the distance between the descriptions of the macroeconomic model performance and the actual data behaviour. Under the null hypothesis $\beta^{\alpha} = \overline{\beta^{s}(\hat{\theta}_{0})}$, the non-rejection region implies that there is no significant difference between the performance of the structural model and the dynamic behaviour of the actual data. While the rejection region suggests that there may be a problem in the model specification.

To compare the test statistic based on the critical value and obtain the conclusion precisely, the confidence level is set to be 95%, which means that the Wald statistic from the actual data should be less than 95th percentile of that from the simulated data, where the corresponding p-value is computed as $\frac{(100-the\ Wald\ percentile)}{100}$. Alternatively, the same result can be yielded more simply by introducing a transformed Mahalanobis distance as following:

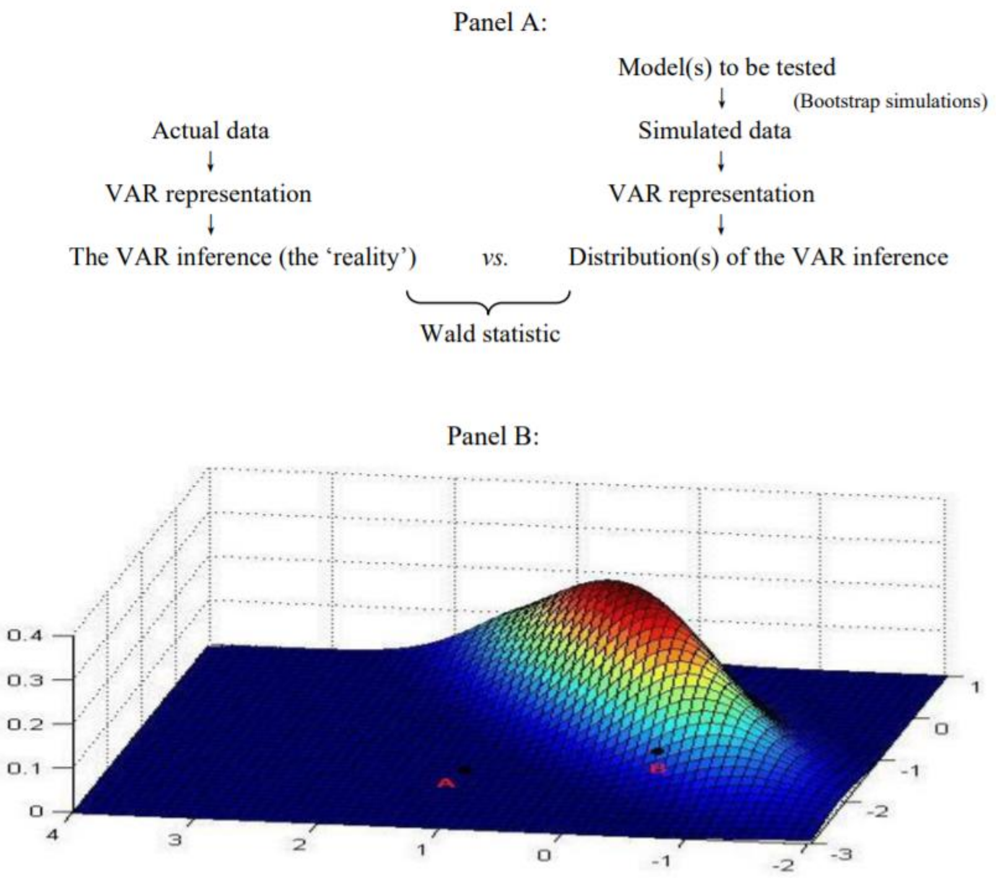
$$MD = 1.645\left(\frac{\sqrt{2w^{\alpha}}-\sqrt{2k-1}}{\sqrt{2w^{0.95}}-\sqrt{2k-1}}\right), \tag{3.16}$$

where $w^{\alpha}$ is the Wald statistic from the actual data; $w^{0.95}$ is the Wald statistic for the 95th percentile of the simulated data; $k$ is the length of $\beta^{\alpha}$. For a transformed Mahalanobis distance, if the statistic is less than 1.645 the null hypothesis is not rejected, and vice versa. This normalized way is established by Le et al. (2012), where the Mahalanobis Distance is organized based on the same joint distribution and normalized as a t-statistic so that the resulting t-statistic is 1.645 at the 95% point the distribution, and thus anything beyond would lead to the rejection of the model.

The steps of methodology above describe the procedure of testing a DSGE model given a set of particular values of parameters. Following the findings in Minford and

Ou (2010), this procedure can also be presented graphically as Figure 3.1 shown below. In this figure, Panel A represents the fundamental processes of the Indirect Inference test discussed in previous content. Panel B presents a mountain-shaped graph, which demonstrates that how ‘reality’ is compared to the predictions of the model based on the Wald test by considering two various sets of parameters reflecting different properties chosen from the data. In this panel, the mountain-shaped graph represents the corresponding joint distribution obtained from the simulated data, and either of the points is supposed to be estimates generated from the actual data with chosen properties. If point A turns to be the estimates based on the actual data, it implies that the structural model fails to explain the performance of the real world appropriately and do not pass the test, since the model prediction is seriously far away from the reality suggestions. Instead, if the result of the estimates lies at point B which is on the mountain, it implies that the joint distribution of the chosen properties reflected by the structural model captures the features of the performance in the real economic world. These distances are evaluated by the Wald statistic introduced above.

Figure 3.1 The Principle of Testing using Indirect Inference



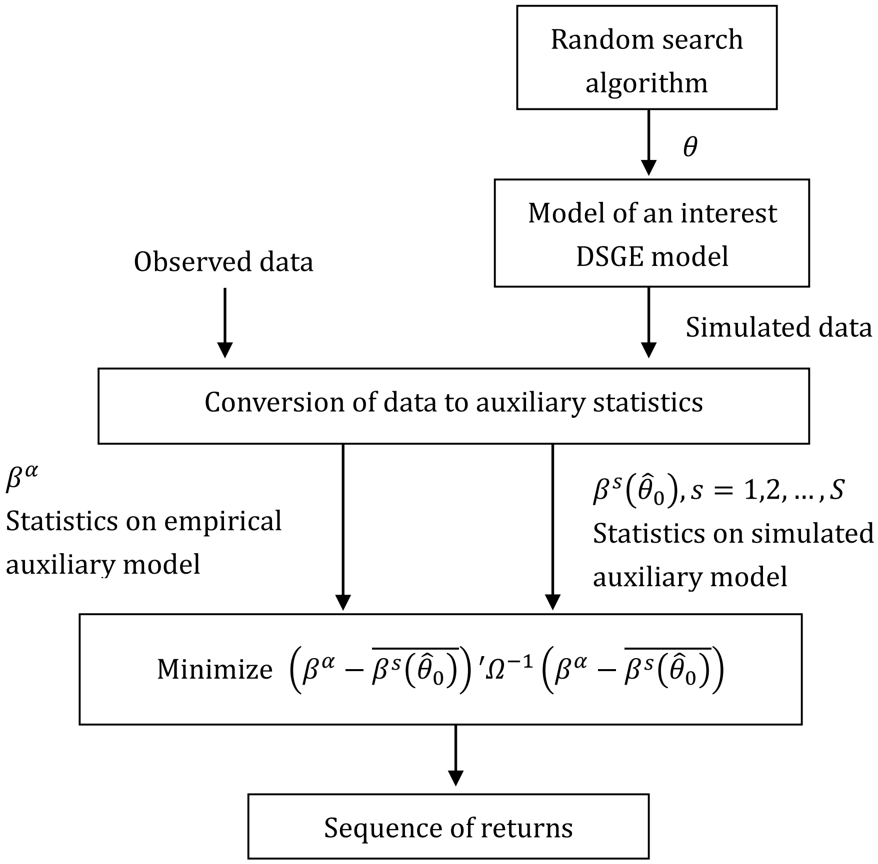

Source: Minford and Ou (2010)

Having discussed the procedure of testing a structural model and the principles of processing the test, it can be detected that the Indirect Inference test is powerful, in other words, there is a relatively high probability of being rejected if the values of the structural parameters are inaccurate. Hence, it is necessary to search for a set of ideal coefficients which can mimic the dynamics of the actual data accurately. By following this logic, the reason of applying the Indirect Inference method to estimate the model becomes sensible, since the core idea of this method is to search for the optimal set of values of the parameters which minimizes the Wald statistics or makes the performance of the model fits the behaviour of the actual data well. The next section of this thesis will introduce the Indirect Inference estimation in details.

### 3.2.3 Indirect Inference estimation

The Indirect Inference estimation is the most preferable approach to estimate a structural DSGE model by minimizing the distance between these two sets of estimates of the auxiliary model based on the actual data and simulations. Different from the usual algorithm based on Simulated Annealing introduced by Le et al. (2012), this thesis adopts the pure random search that samples repeatedly from a feasible region based on a uniform sampling distribution, which is firstly defined by Brooks (1958). Compared with other algorithms, the main advantage of using the random search is that it takes place over a wider range within the structural parameter state and to prevent the searching from being trapped in the local minima. That is to say, the results are going to describe almost all the possibilities based on a given feasible region with an increasing number of searching times. For the random search algorithm, when the initial value is set for the vector of parameters, the Wald statistics at this point can be calculated out through the above steps 1-3. Then the algorithm moves randomly to search within the parameter state for a new set of values which is used for solving the Wald statistic. After repeating this process, there is a sequence of results for all the outcomes including the minimum Wald statistics obtained from an optimal set of estimates. In this thesis, the stopping rule for the algorithm is intuitively to set the number of searching which should be theoretically as large as possible but is set to 200,000 for a practicable operating. The procedure for applying the Indirect Inference method is revealed in Figure 3.2.

Figure 3.2 The steps for estimating DSGE model by Indirect Inference estimation method




Overall, the most essential intention of introducing the Indirect Inference method to evaluate and estimate the macroeconomic model is that the model can be tested unconditionally against the actual data and be re-estimated to search for a certain set of estimates to ensure it is the optimal one to fit.

### 3.2.4 The choice of the auxiliary model

As known, the solution of a log-linearized DSGE model can be generally represented as a restricted vector autoregressive and moving average (VARMA) model which can also be represented by a reduced form of a VAR model approximately. The details about applying a VAR to rewrite a DSGE model is discussed to a greater extent by De Jong and Dave (2007), Del Negro et al. (2007), Del Negro and Schorfheide (2008), and Canova (2007). Thus, by following the above literature, a

VAR model can be chosen as an auxiliary model naturally for the evaluation of a DSGE model, and the observed data can be described as an unrestricted VAR model. The advantage of applying an auxiliary model is that it can fit the data alone easily. When a DSGE model is identified by a restricted VAR model, the simulated data can reflect these structural restrictions in the model which is also consistent with the VAR model, whereas the auxiliary model can be then unrestrictedly estimated from both the actual data and simulated data.

Remarked by the Meenagh et al. (2019), a VARX model serves as an auxiliary model when the exogenous variables or the shocks in the model follow non-stationary processes. The non-stationary exogenous variables will transmit non-stationary processes to the residuals in one or more structural model equations. Since the shocks are generated from the actual data, when these processes are treated as the observed variables, the number of the co-integrating vectors would be less than that of the endogenous variables. Hence, the VARX model where the non-stationary residuals present as the observed variables is allowed to represent the solution of the estimation model, and an unrestricted formation of this VARX model is performed as the auxiliary model.

Based on Le et al. (2016), the VARX model is an approximation form of the reduced DSGE model and can be defined as a co-integrated VARX model. Suppose that the structural DSGE model is simplified in a log-linearized form as the following function shown:

$$A(L)y_t = B(L)E_t y_{t+1} + C(L)x_t + D(L)e_t. \qquad (3.17)$$

Assume that the exogenous variable $x_t$ is driven by:

$$\Delta x_t = \alpha(L)x_{t-1} + d + c(L)\epsilon_t, \qquad (3.18)$$

where $y_t$ is a vector of the endogenous variables with the size $p \times 1$; $x_t$ is a vector of the exogenous variables with the size $q \times 1$; $E_t y_{t+1}$ is a vector of future expected endogenous variables with the size $r \times 1$. $x_t$ is assumed to be non-stationary, which means $y_t$ linearly depended on $x_t$ is also non-stationary. Besides, $e_t$and $\epsilon_t$ are both the vectors of i.i.d error processes with zero mean and covariance matrix $\Sigma$. The lag operator is denoted as $L$, as $y_{t-s} = L^s y_t$, and $A(L)$, $(B(L)$ etc.) is the matrix polynomial functions in the lag operator of order $h$ whose roots of determinantal polynomial lie outside the complex unit circle.

The basic solution of $y_t$ is then denoted as:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t, \qquad (3.19)$$

where $f$ is a vector of constant and polynomial functions in the lag operator. Due to the non-stationary features of both $y_t$ and $x_t$, The model solution above can be rewritten with the $p$ co-integrated relationship as:

$$y_t = [I - G(1)]^{-1}[H(1)x_t + f] = \Pi x_t + g. \qquad (3.20)$$

The matrix $\Pi$ with size $p \times p$ has the rank $0 \leq r < p$ in which $r$ is denoted as the number of linearly independent co-integrated vectors. The difference between the LHS and RHS, as $\eta_t = y_t - (\Pi x_t + g)$, stands for the error correction term. Since in the short run, $y_t$ seems like a function of the deviation from an equilibrium, in the long run, the model solution can be written as:

$$\bar{y}_t = \Pi \bar{x}_t + g, \qquad (3.21)$$

$$\bar{x}_t = [1 - a(1)]^{-1}[dt + c(1)\xi_t], \qquad (3.22)$$

$$\xi_t = \sum_{s=0}^{t-1} \epsilon_{t-s}, \qquad (3.23)$$

where $\bar{y}_t$ and $\bar{x}_t$ are the solutions of $y_t$ and $x_t$ in the long run, respectively, and $g$ is a vector of constant. Notably, the exogenous variable can be decomposed into two components, as $\bar{x}_t = \bar{x}_t^d \bar{x}_t^s$, where $\bar{x}_t^d = [1 - a(1)]^{-1} dt$ is denoted as a deterministic trend and $\bar{x}_t^s = [1 - a(1)]^{-1} c(1) \xi_t$ is a stochastic trend. In this case, a co-integrated vector error correction model (VECM) can be obtained by following the methodology in Meenagh et al. (2019) as:

$$\Delta y_t = P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + M(L)e_t + N(L)\epsilon_t - [I - G(1)](y_{t-1} - \Pi x_{t-1})$$

$$= P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t - [I - G(1)](y_{t-1} - \Pi x_{t-1}), \quad (3.24)$$

$$\omega_t = M(L)e_t + N(L)\epsilon_t, \quad (3.25)$$

where $\omega_t$is a mixed moving average process. Meanwhile, the above VECM can be approximately represented in a form of VARX model as:

$$\Delta y_t = -K(y_{t-1} - \Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \varsigma_t, \quad (3.26)$$

where $\varsigma_t$ is denoted as a i.i.d process with mean zero. Based on the solutions of $\bar{x}_t = \bar{x}_{t-1} + [1 - a(1)]^{-1}[d + \epsilon_t]$ and $\bar{y}_t = \Pi \bar{x}_t + g$, the VARX model can also be described as:

$$\Delta y_t = K(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1}) + R(L)\Delta y_{t-1}$$

$$+ S(L)\Delta x_t + h + \varsigma_t, \quad (3.27)$$

where the deterministic and time trends are embodied in $\bar{x}_t$.

According to the properties of the auxiliary model, either equation (3.26) or (3.27) can be chosen as the auxiliary model. Additionally, the equation (3.27) can be rewritten as:

$$y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + t + q_t, \qquad (3.28)$$

where $q_t$ is the vector of errors which comprises the lagged difference regressions and the deterministic time trend in $\bar{x}_t$ that disturbs both the endogenous and exogenous variables.

By following Le et al. (2016), the equation (3.27) serves as the auxiliary model. This equation illustrates the distinctions between the temporary deviation of $x_t$ from the trend and the disturbance from the trend component, which is beneficial to estimate the parameters in the VARX model by adopting a classical OLS method that is a relatively simple but effective approach. It is also verified by Meenagh et al. (2019) that this procedure can be highly precise by applying the Monte Carlo experiments.

As mentioned, the auxiliary model serves as a 'bridge' by which both the descriptions of the actual data and the simulated data can be represented. There is no need for an auxiliary model to be specified correctly based on the view of Gourieroux et al. (1993) remarking that a correct inference can rely on an 'incorrectly' specified auxiliary model. If the auxiliary model is specified too precisely, there will be no difference between the Indirect Inference and the Maximum Likelihood, which makes it possible to consider a 'Directed Wald' test rather than a 'Full Wald' test on the auxiliary model. It is found by Le et al. (2011) that the Wald statistic can be divided into two types as the 'Full Wald' whose criterion is calculated from the full joint distribution of the coefficients of the VARX model with the full covariance matrix, implying that all the endogenous variables would be taken into account in the auxiliary model, and the 'Directed Wald' which is generated based on the performance of the model in terms of one or more aspects instead of involving all variables in the model. The core idea of the Directed Wald statistic also supports the point of view that 'too many good models were rejected' due to the problems of misspecification in a model discussed in the previous section. The Directed Wald statistic avoids that dilemma by selecting a group of endogenous variables rather than all of them concerning the essential features or interests for evaluating the model.

## 3.3 Why indirect inference?

Among the DSGE literature, the most popular and widely-used estimation approach is the Bayesian method. The core idea of this method is to launch a compromise between the powerful use of the prior information about the structural parameters and the classic likelihood estimation dealing with the fitness of the structural model to the dynamics in the real economic world.

It is necessary to take account of this combination since a crucial feature of the DSGE model is that it is a simplification of the reality and a structural model possibly or completely being misspecified. The misspecified models are more probably rejected by the classical likelihood methods that assess all of the characteristics and parameter values of the model. For instance, a model may behave well in explaining the long-run properties of the economy while being poor in short-run, since there is a lack of information to specify. As a consequence, a model with acceptable long-run properties could be rejected by the classical likelihood methods which fails to involve sufficient lags in the dynamics. Therefore, the likelihood methods are criticized for that "those tests were rejecting too many good models" as remarked by Tom Sargent interviewed by Evans and Honkapohja (2005). There is a counter-criticism on the conventional macroeconomic models and likelihood methods due to their powerful data-mining feature. Under this circumstance, “calibration was intended as a vehicle for quantitative analysis” argued by Tom Sargent at the interview. Thus, proposing the prior information in the calibration and the uncertainty of the prior can help the classical estimation to make the model still be “good” in a sense of prior knowledge although it may be rejected by a formal statistic inference.

Typically, the Bayesian estimation is a method evaluating the posterior distribution of the parameter as a weighted average of the prior distribution and the likelihood functions. The precision of these two elements can be mirrored by the weights in the estimating process, in a way that the more powerful the prior beliefs are or the less informative the data of the observables are, the more the results of the posterior are determined by the prior information, and vice versa. Commonly, the posterior distribution for the parameters is similar to their prior distributions. There are two possible reasons for explaining this phenomenon. Firstly, the information contained in the data of the observations are dominated by that contained in the prior beliefs, which leads to the result that posterior is not much different from an informative prior. Secondly, there is a small likelihood as the model could not fit the data well by employing the Bayesian inference, suggesting either that the model specification is poor, or that the data of observations are relatively uninformative. The identification problems are mostly caused by the lack of informative data.

Therefore, there are two significant weaknesses when estimating the model though the Bayesian method. First, the Bayesian estimation requires the assumption of the prior being well-defined. While many researchers criticized the prior knowledge by disputing that it is too subjective, and a strong prior could bias the final estimation results. Additionally, an informative prior which dominates the posterior could, thus, sabotage the inspection on the model specification. Second, the specification of the model affects the results of the posterior distribution strongly, specifically the misspecification bias can lead the inference and prediction to an inaccurate conclusion.

In addition to the Bayesian method, the maximum likelihood (ML) and generalized method of moments (GMM) are also the conventional estimators favoured by the macroeconomists. However, the parameters of the model are identified by either the ML or the GMM depending on the same sample and theoretical information for the first moments, which is clarified by Hall (1996) who explains the reason why there could be an identification problem when practising these two estimation methods. More details about the similar identification problem existing in applying both ML and GMM are discussed by Fernández-Villaverde, et al. (2009). The assumption of a true model could bind the identification problem. But it is still a question that whether any parameters can be identified when the model is misspecified, such as that the pure ML estimation suffers the problem in identifying the parameters in a misspecified DSGE model. Therefore, the identification and misspecification problems of the DSGE models seems still to be left unsolved by applying the estimators discussed above.

Respond to these problems, the Indirect Inference is applied. It is a methodology explored well in the classical likelihood literature but being less concerned in the Bayesian paradigm. The Indirect Inference is firstly practised on DSGE model by Smith (1993). It is noted by Smith (1993) and Gourieroux, Montfort, and Renault (1993) that the Indirect Inference technique provides the tests on model specification and estimator, and its asymptotic properties are standard even though the fitness of the structural model to the data, or the true likelihood, is unknown. The core concept of the Indirect Inference methodology is to yield a statistical criterion in minimizing the distance between the likelihoods of an auxiliary model generated by the actual data and simulated samples.

The Indirect Inference method applied here is followed the methodology established in Le et al. (2011) originally, which is similar to what has been discussed in the previous section that employing a Wald test as the statistical criterion when evaluating the distances between the vector of relevant coefficients generated from the actual and simulated data to assess the performance of the model in fitting the data and explaining the economy. If the model performs well, the simulated data produced from the model should be similar to the actual data, and thus, the estimates generated from these two sets of data should not be distinctly different. If the model fails to pass the Wald test or the behaviour of the simulated data is significantly different from that of actual data, the performance of the model could be improved by applying the Indirect Inference estimation which is used to search for a proper set of coefficients. The Indirect Inference estimation is based on the principle of the Indirect Inference test in a manner that the procedure of testing is repeating until the global minima of the Wald statistic are detected. Typically, the design of the Indirect Inference is trying to search for a group of coefficients which is the most appropriate one to meet the criterion of the Wald test.

Generally, it can be summarized that the DSGE model could be evaluated in two different categories of approaches. One of the categories applies the likelihood function to find out how precisely the model fits the data for a set of variables. While the other one tries to detect how well the model explains the behaviour of a group of variables over the real business cycle, where the Indirect Inference method belongs to. Additionally, the power, or in other words the probability of rejecting a false model, of the Indirect Inference test is much greater than the likelihood ratio test, and the features of the variables in which the researchers interested can be focused on, but it cannot be done by the likelihood ratio test. These conclusions have been proved by Le et al. (2012) using Monte Carlo experiments to do the comparison between these two methods.

Additionally, econometricians often specify a model relating parameters and exogenous variables to some observable variables. In several cases, the macroeconomic models are too complicated to apply appropriate expressions for the probability distributions of the endogenous variables. These expressions may even not exist. Under this circumstance, the estimation procedures, such as ML and Bayesian methods, may not be applied, but the Indirect Inference method can deal with it. Therefore, the Indirect Inference is a powerful and statistically robust technique to deeply estimate in a complex macroeconomic model and can provide more convincing results for the thesis to conduct the policy analysis.

## 3.4 Solution method

Base on Guerrieri and Iacoviello (2015), the DSGE model with occasionally binding constraint of zero lower bound can be solved by using the first-order perturbation theory and implicating it in a piecewise linear perturbation method to manage occasionally binding constraint.

There are two essential reasons for following the approach in Guerrieri and Iacoviello (2015) to solve the occasionally binding model. First, general perturbation approaches can only solve for a local approximation, while the occasionally binding constraint cannot be captured. In the occasionally binding approach, the constraint is concerned as distinguishing the model for two different regimes: one refers to a normal regime where the constraint unbinds, the other is the alternative regime with a binding constraint. The local approximation of the model around the same point under each regime is linked by applying the piecewise linear perturbation method. Particularly, the solution solved by this algorithm is not just a linear solution, but rather, it can be highly nonlinear. The high nonlinearity is produced by the interaction between the duration of staying at one of the two regimes and the state vector of each regime. In other words, the expected duration depends on the dynamics in each regime, and in the opposite, the state vector for each regime is depended by how long it is expected to stay at this regime. Second, there is a package of numerical routines, OccBin, provided by Guerrieri and Iacoviello (2015) to outline the algorithm of the piecewise linear solution with Dynare, which makes the process of this algorithm more convenient and intuitive.

To clarify the process of this algorithm clearly, assume there is a model with an occasionally binding constraint, namely a zero lower bound. In other words, this model has two regimes, one with a binding interest rate regime, the other one with a slack constraint. For each regime, the model is linearized around the steady-state, even though the point may be different. The regime applying the linearization point is referred to as a 'normal case' (C1), and the other is an 'alternative case' (C2). And whether the binding constraint is being with a normal case or an alternative case is immaterial.

For applying the approach of the algorithm of the occasionally binding constraint solution, it is important to satisfy two central requirements. First, the Blanchard and Kahn (1980) condition should be held in a normal case to ensure the existence of a solution of a linearized model under rational expectations. Second, when the model is moved from a normal case to an alternative case by a structural shock, the model should return to the normal case within a finite period under a draconian condition that there are no future unexpected shocks.

In general, when the constraint is slack, the linearized structure of the model under a normal case is suggested as:

$$\mathcal{A}E_tX_{t+1} + \mathcal{B}X_t + \mathcal{C}X_{t-1} + \mathcal{E}\epsilon_t = 0, \qquad (C1)$$

where $X$ denotes a vector of endogenous variables in the model with size $n$; $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ are matrices of coefficients in the linearized equations for the vector $X$ with size $n \times n$; $\epsilon_t$ is a vector for all structural shocks with size $m$; and $\mathcal{E}$ is the matrix of the parameters of the structural shocks with size $n \times m$.

Similarly, when the constraint is binding, the linearized structure can be written as:

$$\mathcal{A}^*E_tX_{t+1} + \mathcal{B}^*X_t + \mathcal{C}^*X_{t-1} + \mathcal{D}^* + \mathcal{E}^*\epsilon_t = 0, \qquad (C2)$$

where $\mathcal{A}^*$, $\mathcal{B}^*$ and $\mathcal{C}^*$ are matrices of coefficients with size $n \times n$; $\mathcal{E}^*$ is the matix of parameters with size $n \times m$; and $\mathcal{D}^*$ is a vector of constant arising from the fact that the linearized system is achieved around the steady-state applied by the normal case (C1) with size $n$.

Having outlined the characteristics of the two regimes, now the solution of the model can be characterized by adopting the definition in Guerrieri and Iacoviello (2015):

> Definition 1. A solution for a model with an occasionally binding constraint is a function $f: X_{t-1} \times \epsilon_t \rightarrow X_t$ such that the conditions under the system (C1) or the system (C2) hold, depending on the evaluation of the occasionally binding constraint (Guerrieri and Iacoviello (2015), p.24).

By giving the equilibrium conditions in C1 and C2 and cooperating with the occasionally binding constraint, the algorithm of the piecewise linear solution $f$ can be defined. Additionally, this algorithm applies a guess-and-verify method by positioning an initial guess and then verifying it to decide whether it is necessary to update the initial guess. The details are described as follows:

1. Let $T$ be the data when the current guess implies that the model will return to the regime (C1). Then for any $t \geq T$, using the standard perturbation methods, one can characterize the linear approximation to the decision rule for $X_t$,given $X_{t-1}$, as
$$X_t = PX_{t-1} + Q\epsilon_t, \tag{C1DR}$$
where $P$ and $Q$ are $n \times n$ and $n \times m$ matrices of reduced-form parameters, respectively. Then, using the notation of the equation $X_t = P_t X_{t-1} + R_t$, for any $t \geq T$, $P_t = P, R_t = 0$.
2. Using $X_T = PX_{T-1}$ and equation (C2), coupled with the assumption that agents expect no shocks beyond the first period, the solution in period $T-1$ will satisfy the following matrix equation:
$$\mathcal{A}^* PX_{T-1} + \mathcal{B}^* X_{T-1} + \mathcal{C}^* X_{T-2} + \mathcal{D}^* = 0. \tag{1}$$
Solve the equation above for $X_{T-1}$ to obtain the decision rule for $X_{T-1}$, given $X_{T-2}$:
$$X_{T-1} = -(\mathcal{A}^* P + \mathcal{B}^*)^{-1}(\mathcal{C}^* X_{T-2} + \mathcal{D}^*). \tag{2}$$
Accordingly, $P_{T-1} = -(\mathcal{A}^* P + \mathcal{B}^*)^{-1}\mathcal{C}^*$ and $R_{T-1} = -(\mathcal{A}^* P + \mathcal{B}^*)^{-1}\mathcal{D}^*$.
3. Using $X_T = P_{T-1}X_{T-2} + R_{T-1}$ and either (C1) or (C2), as implied by the current guess of regimes, solve for $X_{T-2}$ given $X_{T-3}$.
4. Iterate back in this fashion until $X_0$ is reached, applying either (C1) or (C2) at each iteration, as implied by the current guess of regimes.
5. Depending on whether regime (C1) or (C2) is guessed to apply in period 1, $Q_1 = -(\mathcal{A}P_2 + \mathcal{B})^{-1}\mathcal{E}$, or $Q_1 = -(\mathcal{A}^* P_2 + \mathcal{B}^*)^{-1}\mathcal{E}^*$. Trivially, in the special case in which (C1) is guessed to apply in all periods, one can see that $Q_1 = Q$, consistent with equation (C1DR).
6. Using the guess for the solution obtained in step 1 to 5, compare paths for $X$ to verify the current guess of regimes. If the guess is verified, stop. Otherwise, update the guess for when regimes (C1) and (C2) apply and return to step 1. (Guerrieri and Iacoviello (2015), p.24)

It is feasible to find an initial guess by implicating the standard perturbation method on the normal case (C1) given the initial values for $X_0$ and $\epsilon_1$. In principle, the initial guess should be updated due to a switch between the two regimes leading to a related change in the dynamics of the endogenous variables.

# Chapter 4 Fiscal policy in a model under zero lower bound

## 4.1 Introduction

This chapter aims to build a small open economy DSGE model for the U.K. economy with zero lower bound and endogenous fiscal instruments to analyse and evaluate the effectiveness and implementation of the fiscal policy in the liquidity trap. All fiscal instruments are specified endogenously rather than assuming to be exogenous stochastic processes. The fiscal rules are defined with two essential roles: (1) responding to the degree of economic activities as automatic stabilizers; (2) responding to the state of real government debt level to keep the dynamics of real debt under control and avoid a high debt to GDP ratio. This setup of fiscal policy follows Leeper et al. (2010). The main fiscal rules are distortionary taxes (consumption, labour income and capital income taxes) and government expenditure measures (government spending, investment and lump-sum transfers).

The structure of this chapter is organised as follows. The next section outlines the baseline New Keynesian DSGE model of the U.K. economy. Section 3 presents the essential information of estimation including the data description, calibration, prior distribution and posterior estimates. Section 4 and Section 5 discuss the impulse response functions and values of fiscal multipliers implied by the shocks, respectively. Section 6 provides suggestions about the fiscal policy implementation based on the results of fiscal multipliers and actual fiscal responses of the U.K. economy. Section 7 concludes.

## 4.2 The model economy

The model is built by extending the New Keynesian small open economy DSGE model of Kollmann (2002) and Adolfon et al. (2007) with zero lower bound and endogenous fiscal rules following Leeper at el (2010).

Households maximise their utilities in terms of consumption and leisure. Households also supply the capital services to the rental market and determine their level of investment based on the specified adjustment costs. In this structure of the small open economy model, the households deposit by holding domestic and foreign bonds. The amount of bond holdings is balanced by the UIP condition which pins down the changes in the expected exchange rate.

In the sector of firms, the final goods producers by combining domestic and imported intermediate goods to produce the final goods in a perfectly competitive market. The domestic, imported and exported goods are produced by the differential intermediate firms who are monopolistic competition and set their prices in the Calvo-style sticky prices with variant indexations. The amount of labour and capital employed in production activity is decided according to the technology of the domestic producers of the intermediate goods.

Besides, the government plays the role of monetary and fiscal authorities by setting the monetary policy and fiscal policy.

### 4.2.1 Representative Household

A continuum of households obtains their utilities by leisure and consumption. The households maximise their intertemporal utilities by choosing the current consuming level, working hours and amount of domestic bonds and foreign bonds to hold. At the meantime, they provide the capital services and determine their investment level.

The representative household's preference is described by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \qquad (4.1)$$

where $C_t$ denotes the household's consumption level, and $L_t$ denotes the working hour; $\beta$ $(\beta \in (0,1))$ is the discount factor, and $E_0$ is an expectation operator depending on the information in period 0. The organization of the household's utility function $U(.)$ is consistent with Kollmann (2002) which depicts a simple version of small open economy and illustrates the features of $U_C > 0$ and $U_L < 0$. Additionally, I include an exogenous stochastic process $\mu_t^{pre}$ in utility function to capture the fluctuations in labour supply.

$$U(C_t, L_t) = ln(C_t) - \mu_t^{pre} L_t \ . \qquad (4.2)$$

The preference shock follows an autoregressive process with $i.i.d.$ error $\sigma_t^{pre}$ which is normally distributed with zero mean and standard deviation $\epsilon_t^{pre}$

$$\hat{\mu}_t^{pre} = \rho^{pre} \hat{\mu}_{t-1}^{pre} + \sigma_t^{pre}.$$

The private physical capital is accumulated according to the law of motion which is identical to each household:

$$K_t^p = (1-\delta) K_{t-1}^p + \mu_t^{ip} F(I_t^p, I_{t-1}^p), \qquad (4.3)$$

where $\delta$ is the depreciation rate of private physical capital.

The private investment is turned into private capital based on the function $F(I_t^p, I_{t-1}^p)$ following Christiano, Eichenbaum and Evans (2005) with an adjustment cost on investment and defined as:

$$F\left(I_t^p, I_{t-1}^p\right) = (1 - S\left(\frac{I_t^p}{I_{t-1}^p}\right) )I_t^p, \tag{4.4}$$

where $(1) = 0$ , $S'(1) = 0$ and $S'' > 0$ , implying that there is no cost when the capital stock remains a constant level. Additionally, equation (4.4) implies that:

$$F_1\left(I_t^p, I_{t-1}^p\right) \equiv \frac{\partial F(I_t^p, I_{t-1}^p)}{\partial I_t^p} = 1 - S\left(\frac{I_t^p}{I_{t-1}^p}\right) - S'\left(\frac{I_t^p}{I_{t-1}^p}\right)\frac{I_t^p}{I_{t-1}^p} \ ,$$

$$F_2\left(I_t^p, I_{t-1}^p\right) \equiv \frac{\partial F(I_t^p, I_{t-1}^p)}{\partial I_{t-1}^p} = S'\left(\frac{I_t^p}{I_{t-1}^p}\right)\left(\frac{I_t^p}{I_{t-1}^p}\right)^2 .$$

Hence, the following conditions hold in steady-state:

$$F_1(I^p, I^p) = -S'(1) + \left(1 - S(1)\right) = 1,$$

$$F_2(I^p, I^p) = 0.$$

In equation (4.3), $\mu_t^{ip}$ is the investment-specific technology shock following the AR(1) exogenous process:

$$\hat{\mu}_t^{ip} = \rho^{ip}\hat{\mu}_{t-1}^{ip} + \sigma_t^{ip},$$

where $\sigma_t^{ip}$ is $i.i.d.$ error with mean zero and standard deviation $\epsilon_t^{ip}$.

In the private capital accumulation equation (4.3), the adjustment cost on the investment performs as a nominal rigidity in this model. Without such constraint on the investment, there will be an instantaneous adjustment on building and installing capital stock used in production. However, in reality, capital stock is characteristic of rigidity for some degree, which means that a gradual adjustment is taken on building capital rather than immediate action. The quicker adjustment on installing the capital is, the higher the cost is required.

In this model economy, the period $t$ budget constraint in nominal terms for a representative household is given by:

$$(1+\tau_t^c)P_tC_t + P_tI_t^p + B_t + S_tB_t^f = (1-\tau_t^w)W_tL_t + R_{t-1}B_{t-1} + S_t\Phi_{t-1}R_{t-1}^fB_{t-1}^f + \left[(1-\tau_t^k)R_t^k + \delta\tau_t^k\right]K_{t-1}^p + TR_t + Prof_t. \qquad (4.5)$$

The left-hand side of the above budget constraint function demonstrates the household's total spending, while the total income is shown on the right-hand side. Households can spend their total income on consumption ($C_t$), investment ($I_t^p$) with price $P_t$, and buying one-period nominal domestic government bonds ($B_t$) and foreign government bonds ($B_t^f$). Besides, $\tau_t^c$ denotes the rate of the consumption tax which displays a wedge between the price paid by the consumer and the price set by the producer.

For the package of the total income, households obtain their wages ($W_t$) and returns on capital ($R_t^k$) by supplying labour ($L_t$) and providing physical capital ($K_{t-1}^p$) to the firms, respectively. Then they receive total after-tax labour income $(1-\tau_t^w)W_tL_t$ where $\tau_t^w$ denotes the rate of the labour income tax and after-tax capital income $\left[(1-\tau_t^k)R_t^k + \delta\tau_t^k\right]K_{t-1}^p$ where $\tau_t^k$ denotes the rate of the capital income tax. The capital income tax is only taxed on the returns except for the depreciation value of the capital. Furthermore, households receive lump-sum transfers ($TR_t$) from the government, firm's profit ($Prof_t$) as the owner of the firms, and the interest rate on holding domestic and foreign bonds ($B_{t-1}$ and $B^f_{t-1}$) which are matured in period $t$ with the interest rates $R_{t-1}$ and $R^f_{t-1}$, respectively; $S_t$ is the nominal exchange rate, a unit of foreign currency in terms of home currency, which implies that exchange rate depreciates when $S_t$ increases and exchange rate appreciates when $S_t$ decreases. Meanwhile, the interest rate on foreign bonds is adjusted by a bond premium $\Phi_{t-1}$.

Based on Adolfson et al. (2007) and Schmitt and Uribe (2003), the bond premium is determined by the position of the net foreign asset in the domestic country and defined concerning the share of the domestic output as:

$$A^f_t \equiv \frac{S_t B^f_t}{P_t Y_t}. \tag{4.6}$$

It is assumed that the bond premium function $\Phi(A^f_t, \chi_t)$ is a strictly decreasing function of net foreign asset position ($A^f_t$) and satisfied $\Phi(0,0) = 1$, where $\chi_t$ denotes the bond premium shock following the process as shown as:

$$\hat{\chi}_t = \rho^{\chi} \hat{\chi}_{t-1} + \sigma^{\chi}_t,$$

where $\sigma^{\chi}_t$ is $i.i.d.$ error with mean zero and standard deviation $\epsilon^{\chi}_t$. Therefore, the features of an imperfect international financial market captured by the functions above imply that the premium on the nominal interest rate of the foreign asset is charged on the domestic households when the whole domestic country is a net borrower ($B^f_t < 0$), while the households would obtain a less pay-off when the domestic country becomes a net lender in the global economy ($B^f_t > 0$).

Moreover, Schmitt and Uribe (2001) stated that there is also a technical reason for introducing a bond premium function. In a small open economy, the foreign interest rate is assumed to be a completely exogenous variable. Under this condition, the whole economy presents nonstationary dynamics when responding to stationary shocks. Consequently, the non-linearized equilibrium system may not be approximated validly by the solution of the log-linear equilibrium model due to the existence of non-stationarity. To identify the dynamics of equilibrium appropriately by the log-linear equilibrium model, the premium on interest rate is assumed to be a variable term. Hence, a well-defined steady-state is ensured by involving the bond premium to close the small open economy model.

The Lagrangian problem for households is presented as follows:

$$\max_{\{C_t,\ L_t,\ B_t,\ B_t^f,\ K_t^p,\ I_t^p\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\widetilde{L_t}\right] \ ,$$

$$\widetilde{L_t} = \left\{ \begin{array}{c} ln(C_t) - \mu_t^{pre} L_t \\ +\lambda_t \left[ \begin{array}{c} (1-\tau_t^w) W_t L_t + \mathrm{R_{t-1}} B_{t-1} + S_t \Phi_{t-1} R_{t-1}^f B_{t-1}^f + \left[(1-\tau_t^k) R_t^k + \delta \tau_t^k\right] K_{t-1}^p \\ +TR_t + Prof_t \\ -\left((1+\tau_t^c) P_t C_t + P_t I_t^p + B_t + S_t B_t^f\right) \end{array} \right] \\ +\lambda_t^k \left[(1-\delta) K_{t-1}^p + \mu_t^{ip} F\left(I_t^p, I_{t-1}^p\right) - K_t^p\right] \end{array} \right\}.$$

Households generate their strategies to maximise their expected utilities, where $\lambda_t$ is the Lagrangian multiplier for budget constraints and $\lambda_t^k$ is the Lagrangian multiplier for capital accumulation. While Tobin's Q is defined as, $q_t = \frac{\lambda_t^k}{\lambda_t}$, the ratio of these two Lagrangian multipliers. Notice that the lower-case letters represent the real terms for variables. The first-order conditions are:

$$w.r.t.\ \ C_t: \qquad \lambda_t = \frac{1}{C_t} \cdot \frac{1}{1+\tau_t^c}\ , \tag{4.7}$$

$$w.r.t.\ \ L_t: \quad \lambda_t = \frac{\mu_t^e}{(1-\tau_t^w)w_t}\ , \tag{4.8}$$

$$w.r.t.\ \ b_t: \quad \lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}}\ , \tag{4.9}$$

$$w.r.t.\ \ b_t^f: \quad \lambda_t S_t = \beta E_t \lambda_{t+1} \frac{R_t^f S_{t+1} \Phi_t}{\pi_{t+1}}\ , \tag{4.10}$$

$$w.r.t.\ \ k_t^p: \quad q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \begin{matrix} \left[(1-\tau_{t+1}^k) r_{t+1}^k + \delta \tau_{t+1}^k\right] \\ +q_{t+1}(1-\delta) \end{matrix} \right\}\ , \tag{4.11}$$

$$w.r.t.\ \ i_t^p:\ \lambda_t = q_t \lambda_t \mu_t^{Ip} F_1(i_t^p, i_{t-1}^p) + \beta E_t\left[q_{t+1}\lambda_{t+1}\mu_{t+1}^{Ip} F_2(i_{t+1}^p, i_t^p)\right], \tag{4.12}$$

where $\pi_t$ denotes the inflation in the home country as $\pi_t = \frac{P_t}{P_{t-1}}$.

By combining the first-order conditions for domestic and foreign bonds (equation (4.9) and (4.10), respectively), it yields:

$$R_t = R_t^f \frac{E_t S_{t+1}}{S_t} \varphi_t.$$

After log-linearizing the above function, the modified UIP condition can be obtained as follows:

$$\hat{R}_t - \hat{R}_t^f = E_t \hat{S}_{t+1} - \hat{S}_t - \lambda_\varphi \hat{A}_t^f + \hat{\chi}_t,$$

where $\hat{R}_t = \frac{R_t - \bar{R}_t}{\bar{R}_t}$; $\hat{R}_t^f = \frac{R_t^f - \bar{R}_t^f}{\bar{R}_t^f}$; $\hat{S}_t = \frac{S_t - \bar{S}_t}{\bar{S}_t}$ ( a bar denotes the steady-state value of the variable). Due to the imperfect integration within the global financial market, the net foreign assets position of the home country is involved in UIP condition, which causes a departure to a standard UIP condition.

### 4.2.2 Technologies and Firms

There are two categories of firms engaged in the home country: the final goods firms and the intermediate goods firms. There are three different types of intermediate firms: the domestic intermediate firms, the exported intermediate firms and imported intermediate firms.

The final goods firm combines a continuum of domestic and imported intermediate goods to produce a single final good which is sold in the domestic market and is also a non-tradable good. The trade between the home country and the rest of the world is only allowed in some intermediate firms. The imported intermediate firms buy homogenous goods in the world market and transform them into differentiated goods to sell to the final good producers in the home country. The exported intermediate goods firms, they work in a similar pattern as imported firms and transform domestic goods into differentiated exported good by brand naming and sell them to the world market. While the domestic intermediate goods firms produce non-traded goods for the home market only.

The market for final goods is perfectly competitive, whereas the intermediate goods market is a monopolistic competitive. The intermediate firms who produce differentiated goods set their prices based on the setting of the Calvo sticky price and allowing for the price indexations. Due to the nominal rigidities in imported and exported prices, there is an incomplete exchange rate pass-through in both these prices, which is in line with Adolfson et al. (2007) and is proved by Campa and Goldberg (2001) by criticizing the full exchange rate pass-through and the law of one price adopting the data of Euro area.

#### 4.2.2.1 Final goods firms

The final good is produced depending on the aggregate technology by a CES production function indexed with domestic intermediate goods and imported intermediate goods:

$$Z_t = \left[(1-\alpha^m)^{\frac{1}{\theta}}(Y_t^d)^{\frac{\theta-1}{\theta}} + (\alpha^m)^{\frac{1}{\theta}}(Y_t^m)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \qquad (4.13)$$

where $Z_t$ denotes the aggregate output for final goods; $Y_t^d$ and $Y_t^m$ are quantities of domestic and imported intermediate goods used for producing final goods respectively; $\theta$ is the substitution between domestic and imported goods, $\theta > 0$; $\alpha^m$ is the share of imported goods in final good production which is generated by the steady-state level of openness degree of the home country to the world market.

By solving the maximization problem of equation (4.13) subject to the budget constraint, $P_t Z_t = P_t^d Y_t^d + P_t^m Y_t^m$, the demands for domestic and imported intermediate goods are obtained as follows:

$$Y_t^d = (1-\alpha^m)\left(\frac{P_t^d}{P_t}\right)^{-\theta} Z_t, \qquad (4.14)$$

and

$$Y_t^m = \alpha^m \left(\frac{P_t^m}{P_t}\right)^{-\theta} Z_t, \qquad (4.15)$$

where $P_t^d$ and $P_t^m$ are the prices for domestic and imported goods in the market of the home country; $P_t$ denotes the consumer price index (CPI) of the home country. The CPI is given by:

$$P_t = \left[(1-\alpha^m)\left(P_t^d\right)^{1-\theta} + \alpha^m (P_t^m)^{1-\theta}\right]^{\frac{1}{1-\theta}}. \qquad (4.16)$$

Because of the perfect competition in the final good market, the CPI is the marginal cost of the final good which is the minimal cost for purchasing one unit of the final good as well.

Additionally, the demand function for exported goods is assumed to resemble the domestic and imported demand functions (equation (4.14) and (4.15)) described as follows:

$$Y_{i,t}^x = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{-v} Y_t^x,\ Y_t^x = \left(\frac{P_t^x}{P_t^f}\right)^{-\eta} Z_t^f. \qquad (4.17)$$

#### 4.2.2.2 Domestic intermediate goods firms

The intermediate good $i$ is produced in the home country following a Cobb-Douglas production function. In this case, the intermediate firms face constant return to scale in terms of two private factors and increasing return to scale regarding all public and private factors because of the positive externality of public capital. The firms adopt public infrastructure combined with private capital stock and labour for proceeding the production. The production function is:

$$Y_{i,t} = A_t (K_{i,t}^p)^{\alpha_1} {L_{i,t}}^{(1-\alpha_1)} (K_t^g)^{\alpha_2}, \qquad (4.18)$$

where $K_t^g$ denotes the capital stock of government required at time $t$ which is equal in all $i$ firms; $\alpha_1$ and $\alpha_2$ are the elasticities of production level related to the private and public capital stock, respectively; $\alpha_1$ also implies the participation level for private capital in the productive activity, then it can be derived that $(1-\alpha_1)$ is the participation level of labour factor; $A_t$ denotes the productivity shock in all $i$ firms identically which follows the exogenous process as:

$$\hat{A}_t = \rho^A \hat{A}_{t-1} + \sigma_t^A,$$

where $\sigma_t^A$ is $i.i.d.$ error with mean zero and standard deviation $\epsilon_t^A$. Similar to the setup in Baxter and King (1993) and Leeper et al. (2010), an increasing return to scale regarding productive public capital is proposed. A higher stock of public capital not only could increase the output level by its right but also crowds in private capital and labour by promoting the marginal productivity of other inputs. The strength of this mechanism is determined by the elasticity of output regarding the public capital.

The problem of the intermediate firm $i$ at time $t$ is to minimise its expense constrained by the productive process, then the problem can be set as follows:

$$\min_{\left\{K_{i,t}^p,\ L_{i,t}\right\}_{t=0}^{\infty}} W_t\ L_{i,t} + R_t^k K_{i,t}^p + \nu_t P_{i,t}\left[Y_{i,t} - A_t (K_{i,t}^p)^{\alpha_1} {L_{i,t}}^{(1-\alpha_1)} (K_t^g)^{\alpha_2}\right], \quad (4.19)$$

where $R_t^k$ is the nominal rental rate for private capital, $W_t$ is the nominal wage, $\nu_t$ is the Lagrangian multiplier.

The first-order conditions for $K_{i,t}^p$ and $L_{i,t}$ are yielded as follows:

$$w.r.t. L_{i,t}: \quad W_t = (1-\alpha_1)\ \nu_t P_{i,t} A_t (K_{i,t}^p)^{\alpha_1} {L_{i,t}}^{(-\alpha_1)}, \quad (4.20)$$

$$w.r.t. K_{i,t}^p: \quad R_t^k = \alpha_1 \nu_t P_{i,t} A_t {K_{i,t}^p}^{(\alpha_1-1)} {L_{i,t}}^{(1-\alpha_1)}. \quad (4.21)$$

Then the real rental rate of capital can be generated by combing the equation (4.20) and (4.21) as follows:

$$\frac{r_t^k}{w_t} = \frac{\alpha_1}{(1-\alpha_1)} \frac{L_{i,t}}{K_{i,t}^p}. \tag{4.22}$$

The nominal marginal cost, which is the cost for supplying each additional domestic intermediate good, can be explained by the Lagrangian multiplier, $\nu_t P_{i,t}$, in equation (4.19). Accordingly, $\nu_t$ is obtained as the real marginal cost ($mc_t \equiv \nu_t$ ) for the domestic intermediate firms as follows by adopting the first-order conditions of equation (4.20) and (4.22):

$$mc_t = \frac{1}{A_t (K_t^g)^{\alpha_2}} \left(\frac{r_t^k}{\alpha_1}\right)^{\alpha_1} \left(\frac{w_t}{1-\alpha_1}\right)^{1-\alpha_1}. \tag{4.23}$$

These intermediate goods serve as both non-traded goods and traded goods sold to the domestic and foreign market, respectively.

Each domestic intermediate firm $i$ produces the domestic intermediate good $Y_i^d$ sold to the final firm. There is a continuum of the domestic firms and each of them is a monopoly producer. The monopolistic competition of the market is introduced by adopting the framework of Dixit and Stiglitz (1977):

$$Y_t^d = \left[\int_0^1 \left(Y_{i,t}^d\right)^{\upsilon-1} di\right]^{\frac{\upsilon}{\upsilon-1}}, \upsilon > 1, \tag{4.24}$$

where $\nu$ denotes the parameter determining the steady-state of mark-up for domestic firms.

By solving the profit maximization problem for domestic producers as $max\, Y_t^d P_t^d - \int_0^1 Y_{i,t}^d P_{i,t}^d di$ and taking the price of final goods $P_t^d$ and input price $P_{i,t}^d$ as given, the demand function for domestic intermediate goods is obtained:

$$Y_{i,t}^{d} = \left(\frac{P_{i,t}^{d}}{P_{t}^{d}}\right)^{-v} Y_{t}^{d}. \tag{4.25}$$

Integrating equation (4.25) by adopting equation (4.24), the price of final goods $P_t^d$ is a CES aggregate of the intermediate goods price $P_{i,t}^d$:

$$P_t^d = \left[\int_0^1 \left(P_{i,t}^d\right)^{1-v} di\right]^{\frac{1}{1-v}}. \tag{4.26}$$

The domestic intermediate producers set their prices by implementing the mechanism indicated in Calvo (1983), which is similar to the setting in Smets and Wouters (2003). It is assumed that the producers can re-optimize their prices with a random probability of $(1-\xi_d)$ to obtain new prices denoted as $P_t^{d,new}$ which is identical for all firms. With probability $\xi_d$, the producers do not update their prices and set the prices indexed to the previous inflation as $P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d} P_t^d$, where $\pi_t^d = \frac{P_t^d}{P_{t-1}^d}$ is the domestic inflation; $\kappa_d$ is the price indexation parameter. If at period $t$ the producers do not update their prices during $s$ periods ahead, the price will be $(\pi_t^d \pi_{t+1}^d \dots \pi_{t+s-1}^d)^{\kappa_d} P_t^{d,new}$ in period $t+s$. Thus, the profit maximum problem of the intermediate firm $i$ when optimizing the prices is written as:

$$\max_{\left\{P_t^{d,new}\right\}_{t=0}^{\infty}} E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \lambda_{t+s} \left(\Pi_{t+s}^d P_t^{d,new} - MC_{t+s}\right) Y_{i,t+s}^d, \tag{4.27}$$

where the stochastic discount factor $(\beta\xi_d)^s \lambda_{t+s}$ is adopted to ensure the firm's profit conditions on the life utility with the discount factor $\beta$ and marginal utility of households' income $\lambda_{t+s}$ at period $t+s$ which is an exogenous variable in solving the intermediate firms' problem; $\Pi_{t+s}^d$ denotes the scheme of the price indexation as $\Pi_{t+s}^d = \prod_{k=1}^{s} \left(\pi_{t+k-1}^d\right)^{\kappa_d}$.

The first-order condition of domestic firms' problem can be solved by using equation (4.25):

$$E_t \sum_{s=0}^{\infty} (\beta \xi_d)^s \lambda_{t+s} \left[ \left( \frac{P_{t+s-1}^d}{P_{t-1}^d} \right)^{\kappa_d} \frac{P_t^d}{P_{t+s}^d} \right]^{-v} Y_{t+s}^d \times \left[ \left( \frac{P_{t+s-1}^d}{P_{t-1}^d} \right)^{\kappa_d} P_t^{d,new} - \frac{v}{v-1} MC_{t+s} \right] = 0. \qquad (4.28)$$

For the continuum of domestic intermediate firms, they make their price decision based on the price indexation scheme and first-order condition of the optimal reset price above. Then the evolution of the aggregate price index $P_t^d$ for domestic firms is obtained as:

$$P_t^d = \left[ \left( \int_0^{\xi_d} \left( P_{t-1}^d \left( \pi_{t-1}^d \right)^{\kappa_d} \right)^{1-v} + \int_{\xi_d}^1 \left( P_t^{d,new} \right)^{1-v} \right) \right]^{\frac{1}{1-v}}$$

$$= \left[ \xi_d \left( P_{t-1}^d \left( \pi_{t-1}^d \right)^{\kappa_d} \right)^{1-v} + (1-\xi_d) \left( P_t^{d,new} \right)^{1-v} \right]^{\frac{1}{1-v}}. \qquad (4.29)$$

The aggregate Phillips curve in the domestic sector can be obtained by combining the equation (4.28) and (4.29), then, and log-linearizing as:

$$\hat{\pi}_t^d = \frac{\beta}{1+\beta\kappa_d} E_t \hat{\pi}_{t+1}^d + \frac{\kappa_d}{1+\beta\kappa_d} \hat{\pi}_{t-1}^d + \frac{(1-\beta\xi_d)(1-\xi_d)}{(1+\beta\kappa_d)\xi_d} \widehat{mc}_t. \qquad (4.30)$$

Noted that when $\kappa_d = 1$ the Phillips curve above reduces to the form as in Altig et al. (2003); when $\kappa_d = 0$ the Phillips cure is in purely forward-looking style.

#### 4.2.2.3 Imported intermediate goods firms

There is a continuum of imported firms who buy the homogenous goods from the world market and sell these undifferentiated goods to the final goods firms who provide final goods to the domestic market by combining domestic and imported intermediate goods.

The imported firms take the price $P_t^f$ for the homogenous good from the world market and set their prices at a local currency price in Calvo stickiness which allows each firm re-optimizes the price with a random probability of $(1-\xi_m)$ and sets the new price as $P_t^{m,new}$. With the probability $\xi_m$, the firms do not alter their prices and the prices are indexed to the previous inflation as $P_{t+1}^m = (\pi_t^m)^{\kappa_m}\ P_t^m$, where $\pi_t^m = \frac{P_t^m}{P_{t-1}^m}$ is the imported inflation and $\kappa_m$ is the price indexation parameter for imported firms. If at period $t$ the imported firms do not re-optimize their prices during $s$ periods ahead, the price will be $(\pi_t^m \pi_{t+1}^m \dots \pi_{t+s-1}^m)^{\kappa_m} P_t^{m,new}$ in period $t+s$. Then the optimization problem for each imported firm $i$ is:

$$\max_{\{P_t^{m,new}\}_{t=0}^{\infty}} \ E_t \sum_{s=0}^{\infty} (\beta\xi_d)^s \lambda_{t+s} \left(\Pi_{t+s}^m P_t^{m,new} - S_{t+s}\ P_{t+s}^f\right) Y_{i,t+s}^m, \qquad (4.31)$$

where $\Pi_{t+s}^m$ denotes the scheme of the imported price indexation as $\Pi_{t+s}^m = \prod_{k=1}^{s} (\pi_{t+k-1}^m)^{\kappa_m}$.

The imported good is produced by adopting a continuum of $i$ imported intermediate goods supplied by each firm following a CES function as:

$$Y_t^m = \left[\int_0^1 \left(Y_{i,t}^m\right)^{v-1} di\right]^{\frac{v}{v-1}}, v > 1. \qquad (4.32)$$

Accordingly, the demand function for each imported intermediate good $i$ is obtained:

$$Y_{i,t}^m = \left(\frac{P_{i,t}^m}{P_t^m}\right)^{-v} Y_t^m. \qquad (4.33)$$

Thus, the optimization problem for each imported firm $i$ is solved for the demand for imported goods denoted by equation (4.33) and the first-order condition is obtained as:

$$E_t \sum_{s=0}^{\infty} (\beta \xi_m)^s \, \lambda_{t+s} \left[ \left( \frac{P_{t+s-1}^m}{P_{t-1}^m} \right)^{\kappa_m} \frac{P_t^m}{P_{t+s}^m} \right]^{-v} Y_{t+s}^m \times$$
$$\left[ \left( \frac{P_{t+s-1}^m}{P_{t-1}^m} \right)^{\kappa_m} P_t^{m,new} - \frac{v}{v-1} S_{t+s} \; P_{t+s}^f \right] = 0. \qquad (4.34)$$

Then the aggregate imported price is solved as:

$$P_t^m = \left[ \int_0^1 \left( P_{i,t}^m \right)^{1-v} di \right]^{\frac{1}{1-v}}$$

$$= [\xi_m (P_{t-1}^m (\pi_{t-1}^m)^{\kappa_m})^{1-v} + (1-\xi_m)(P_t^{m,new})^{1-v}]^{\frac{1}{1-v}}. \qquad (4.35)$$

The log-linearized form for the imported pricing function can be expressed as:

$$\hat{\pi}_t^m = \frac{\beta}{1+\beta\kappa_m} E_t \hat{\pi}_{t+1}^m + \frac{\kappa_m}{1+\beta\kappa_m} \hat{\pi}_{t-1}^m + \frac{(1-\beta\xi_m)(1-\xi_m)}{(1+\beta\kappa_m)\xi_m} \widehat{mc}_t^m, \qquad (4.36)$$

where $\widehat{mc}_t^m = \hat{P}_t^f + \hat{S}_t - \hat{P}_t^m$.

#### 4.2.2.4 Exported intermediate goods firms

The exported firms sell the continuum of differentiated intermediate goods to the world market by purchasing the domestic goods and differentiating these goods through name branding. Hence, the marginal cost of the exported firms is the domestic good price $P_t^d$. The demand for each firm $i$ is defined as:

$$Y_{i,t}^x = \left( \frac{P_{i,t}^x}{P_t^x} \right)^{-v} Y_t^x, \qquad (4.37)$$

where the exported price $P_t^x$ is assumed to be expressed in the local currency of the world market.

To take an incomplete exchange rate pass-through into account, the exported prices are assumed in Calvo stickiness and invoiced in the foreign currency. With a random probability $(1-\xi_x)$, the firms receive the Calvo price-reset signal and derive the new prices, $P_t^{x,new}$. While the firms maintain the prices indexed to the previous inflation with probability $\xi_x$ as $P_{t+1}^x = (\pi_t^x)^{\kappa_x} P_t^x$, where $\pi_t^x = \frac{P_t^x}{P_{t-1}^x}$ represents the inflation in the exported sector and $\kappa_x$ is the price indexation parameter. Thus, each exported intermediate producer $i$ maximize the profits in terms of local currency by solving the problem as:

$$\max_{\{P_t^{x,new}\}_{t=0}^{\infty}} E_t \sum_{s=0}^{\infty} (\beta\xi_x)^s \lambda_{t+s} \left( \Pi_{t+s}^x P_t^{x,new} - \frac{P_{t+s}^d}{S_{t+s}} \right) Y_{i,t+s}^x, \tag{4.38}$$

where $\Pi_{t+s}^x = \Pi_{k=1}^s (\pi_{t+k-1}^x)^{\kappa_x}$ is the scheme of the exported price indexation.

Hence, the first-order condition of the optimization problem is solved subject to the demand function as equation (4.37) that is expressed as:

$$E_t \sum_{s=0}^{\infty} (\beta\xi_x)^s \lambda_{t+s} \left[ \left( \frac{P_{t+s-1}^x}{P_{t-1}^x} \right)^{\kappa_x} \frac{P_t^x}{P_{t+s}^x} \right]^{-v} Y_{t+s}^x \times \left[ \left( \frac{P_{t+s-1}^x}{P_{t-1}^x} \right)^{\kappa_x} P_t^{x,new} - \frac{v}{v-1} \frac{P_{t+s}^d}{S_{t+s}} \right] = 0. \tag{4.39}$$

The average exported price in period $t$ is then obtained as follows:

$$P_t^x = [\xi_x (P_{t-1}^x (\pi_{t-1}^x)^{\kappa_x})^{1-v} + (1-\xi_x)(P_t^{x,new})^{1-v}]^{\frac{1}{1-v}}. \tag{4.40}$$

The log-linearized form could be generated as the aggregate Phillips curve for exported producers as follows:

$$\hat{\pi}_t^x = \frac{\beta}{1+\beta\kappa_x} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\beta\kappa_x} \hat{\pi}_{t-1}^x + \frac{(1-\beta\xi_x)(1-\xi_x)}{(1+\beta\kappa_x)\xi_x} \widehat{mc}_t^x, \qquad (4.41)$$

where $mc_t = \frac{P_t^d}{S_t P_t^x}$.

### 4.2.3 Relative prices

Various relative prices are proposed in this model. There are four different types of stationary relative prices in domestic, imported and exported sectors:

$$\psi_t^d = \frac{P_t^d}{P_t} = \frac{\pi_t^d}{\pi_t} \psi_{t-1}^d, \qquad (4.42)$$

$$\psi_t^m = \frac{P_t^m}{P_t} = \frac{\pi_t^m}{\pi_t} \psi_{t-1}^m, \qquad (4.43)$$

$$\psi_t^x = \frac{P_t^x}{P_t^f} = \frac{\pi_t^x}{\pi_t} \psi_{t-1}^x, \qquad (4.44)$$

$$\psi_t^f = \frac{P_t^f}{P_t} = \frac{\pi_t^f}{\pi_t} \psi_{t-1}^f, \qquad (4.45)$$

where $\pi_t^f = \frac{P_t^f}{P_{t-1}^f}$ denotes the inflation of the foreign country. These relative prices are the prices operated when the various firms in each sector manage their production faced different prices.

### 4.2.4 Government

#### 4.2.4.1 Monetary policy

The performance of the monetary policy in this model is approximated by an instrument rule that follows Taylor (1993), instead of optimizing a loss function to describe the behaviour of the monetary authority. Based on the recent literature on monetary policy regimes as Smets and Wouters (2003), the baseline monetary rule is set to stipulate that the adjustment in the short-run nominal interest rate conforms to the deviations of inflation and output gap, and also incorporate with the interest rate smoothing. Although the aim of the instrument rules is not to optimize the behaviour, the Taylor style monetary policy is shown to be operated well based on the empirical studies. The findings in Onatski and Williams (2004) reveals that in the model of Smets and Wouters (2003) the instrument rules perform better than the optimal rules in terms of the welfare-based loss.

Hence, the monetary policy is approximated by the following instrument rule in a log-linearized form as:

$$\hat{R}_t^{tr} = \gamma^r \hat{R}_{t-1}^{tr} + (1-\gamma^r)\left(\gamma^\pi \hat{\pi}_t^d + \gamma^y \hat{Y}_t\right) + \hat{\mu}_t^{tr}, \qquad (4.46)$$

where $\pi_t^d$ denotes the domestic PPI inflation which is the growth factor of the price index of domestic intermediate goods; $Y_t$ is the real GDP; $\mu_t^{tr}$ is the shock to the nominal interest rate implemented by the monetary authority following an autoregressive process:

$$\hat{\mu}_t^{tr} = \rho^{tr} \hat{\mu}_{t-1}^{tr} + \sigma_t^{tr}.$$

It is assumed that the monetary policy is approximated only when $R_t \geq 0$ is satisfied which serves as a zero lower bound for the rule. In other words, if the Taylor rule generates a negative interest rate, the monetary policy is then inactive and constrained at zero by the zero bound. Therefore, the monetary policy is set based on a modified Taylor rule subject to the zero lower bound:

$$\hat{R}_t = max\left[0, \hat{R}_t^{tr}\right]. \qquad (4.47)$$

As remarked by Christiano et al. (2011) and Basu and Bundick (2017), introducing a zero lower bound on the nominal interest rate generates a significant non-linearity additionally. When any shock deriving the aggregate price level and output in the same direction, it could be magnified by an inactive policy since the interest rate could not be adjusted by the monetary authority.

#### 4.2.4.2 Fiscal policy

The government budget constraint satisfies that the aggregate public expenditure of spending, investment, transfers and the interests on previous debt is equal to the gross revenue on distortionary taxes and insurance of public bond which is written as:

$$G_t + I_t^g + \frac{R_{t-1}b_{t-1}}{\pi_t} + TR_t = \tau_t^c C_t + \tau_t^k \left(R_t^k - \delta\right) K_{t-1}^p + \tau_t^w w_t L_t + b_t, \tag{4.48}$$

where the government capital is accumulated according to the law of motion as:

$$K_t^g = \left(1 - \delta_g\right) K_{t-1}^g + I_t^g, \tag{4.49}$$

where the government capital depreciates following the depreciation rate $\delta_g$. The fiscal policy is approximated by the instrument rules which are introduced based on the fiscal set-up in Leeper et al (2010). The fiscal rules have two characteristics. First, the rules are modelled to involve a contemporaneous response to the deviation of the GDP from its steady-state value. Accordingly, the public expenditure rules respond to the output deviation countercyclically, while the tax rules behave procyclically, which means that these fiscal variables are featured as the automatic stabilizers in the model economy. Second, to protect the government debt to GDP ratio growing to an unsustainably high level, all the fiscal rules are allowed for a component of the state of government debt in their responses for keeping the movements of the real debt under control. Thus, the fiscal rules associated to the government debt and cyclical position of the economy are written in the log-linearized form as follows:

$$\hat{G}_t = -\gamma^{g,b}\hat{b}_{t-1} - \gamma^{g,y}\hat{Y}_t + \hat{\mu}_t^g, \qquad \hat{\mu}_t^g = \rho^g\hat{\mu}_{t-1}^g + \sigma_t^g, \qquad (4.50)$$

$$\hat{I}_t^g = -\gamma^{i,b}\hat{b}_{t-1} - \gamma^{i,y}\hat{Y}_t + \hat{\mu}_t^{ig}, \qquad \hat{\mu}_t^{ig} = \rho^{ig}\hat{\mu}_{t-1}^{ig} + \sigma^{ig}, \qquad (4.51)$$

$$\widehat{TR}_t = -\gamma^{t,b}\hat{b}_{t-1} - \gamma^{t,y}\hat{Y}_t + \hat{\mu}_t^{tra}, \qquad \hat{\mu}_t^{tra} = \rho^t\hat{\mu}_{t-1}^{tra} + \sigma_t^{tra}, \qquad (4.52)$$

$$\hat{\tau}_t^c = \gamma^{c,b}\hat{b}_{t-1} + \gamma^{c,y}\hat{Y}_t + \hat{\mu}_t^c, \qquad \hat{\mu}_t^c = \rho^c\hat{\mu}_{t-1}^c + \sigma_t^c, \qquad (4.53)$$

$$\hat{\tau}_t^k = \gamma^{k,b}\hat{b}_{t-1} + \gamma^{k,y}\hat{Y}_t + \hat{\mu}_t^k, \qquad \hat{\mu}_t^k = \rho^k\hat{\mu}_{t-1}^k + \sigma_t^k, \qquad (4.54)$$

$$\hat{\tau}_t^w = \gamma^{w,b}\hat{b}_{t-1} + \gamma^{w,y}\hat{Y}_t + \hat{\mu}_t^w, \qquad \hat{\mu}_t^w = \rho^w\hat{\mu}_{t-1}^w + \sigma_t^w, \qquad (4.55)$$

where $\gamma^{s,j}$ $(\gamma^{s,j} \geq 0, s = g, i, t, c, k, w;\ \ j = b, y)$ describes the responses of the fiscal rules to the GDP deviation and dynamics of the government debt; the related fiscal shocks $\hat{\mu}_t^i$ $(i = g, ig, tra, c, k, w)$ are considered to allow for persistent exogenous changes in policies with $i.i.d.$ errors $\sigma^i$ $(i = g, ig, tra, c, k, w)$ distributed $N(0, \epsilon^i)$.

### 4.2.5 Market-clearing conditions

The market-clearing conditions should be held in equilibrium, meaning that the demand and supply in each market need to be identical, which is consistent with the Walras' Law in general equilibrium theory. Hence, the market-clearing conditions for the final goods market, the labour market and the capital market should satisfy:

$$Z_t = C_t + I_t^p + I_t^g + G_t, \quad (4.56)$$

$$L_t = \int_0^1 L_{i,t} di, \quad (4.57)$$

$$K_t^p = \int_0^1 K_{i,t}^p di, \quad (4.58)$$

where equation (4.56) represents the resource constraint in equilibrium reflecting that the aggregate supply of final goods meets the aggregate demand of consumption and investment goods from both the private and public sectors; the left-hand side of equation (4.57) and (4.58) is the total supply of the market, while the right-hand side denotes the aggregate demand of the intermediate goods producers for production.

In the intermediate goods market, the domestic producers supply the non-traded and internationally traded intermediate goods to the domestic and foreign markets, respectively. Each of them performs as the price setter in the monopolistic competition markets. Thus, the aggregate supply of the differentiated goods should satisfy the domestic and foreign demands in equilibrium as follows:

$$Y_t = \int_0^1 Y_{i,t}^d di + \int_0^1 Y_{i,t}^x di. \quad (4.59)$$

Accordingly, the nominal term of the GDP is derived as:

$$Y_t^{nom} = \int_0^1 P_{i,t}^d Y_{i,t}^d di + \int_0^1 S_t P_{i,t}^x Y_{i,t}^x di. \quad (4.60)$$

As for the domestic bond market, it is assumed that the domestic bond is not held by foreign households in a small open economy. Thus, in equilibrium, the stock of the domestic bond evolves in terms of the government budget constraint, which implies that the government issues the debt over time to finance its public deficit. The equilibrium outstanding debt is zero under the assumption that the government closes the budget by collecting taxes in every period. Hence, the market-clearing condition in the domestic bond market yields:

$$B_t = \int_0^1 B_{h,t} dh = 0. \tag{4.61}$$

Since there is a fully elastic supply of foreign bond which meets the demand accumulated by the domestic agents, the market-clearing condition in the foreign bond market requires:

$$B_t^f = \int_0^1 B_{h,t}^f dh. \tag{4.62}$$

### 4.2.6 Evolution of net foreign assets

The evolution of the aggregate net foreign assets requires as follows:

$$P_t^x Y_t^x - P_t^f Y_t^m = B_t^f - \Phi_{t-1} R_{t-1}^f B_{t-1}^f, \tag{4.63}$$

where $R_{t-1}^f \Phi_{t-1}$ denotes the gross risk-adjusted nominal interest rate; the left-hand side represents the trade balance of the home country. Remind that the position of net foreign assets is defined regarding the share of the domestic output, which is reported as:

$$A_t^f \equiv \frac{S_t B_t^f}{P_t Y_t}. \tag{4.64}$$

Rearrange the equation (4.63) by dividing both sides $P_t Y_t$, and substituting $A_t^f$ into, it yields:

$$S_t \psi_t^x \psi_t^f \frac{Y_t^x}{Y_t} - S_t \psi_t^f \frac{Y_t^m}{Y_t} = A_t^f - \Phi_{t-1} R_{t-1}^f \frac{A_{t-1}^f}{\pi_t} \frac{Y_{t-1}}{Y_t} \frac{S_t}{S_{t-1}}. \qquad (4.65)$$

### 4.2.7 Foreign economy

It is assumed that the foreign economy is exogenously given in the small open economy, which means the foreign inflation, interest rate and consumption are exogenous variables. The foreign economy is modelled as a vector autoregressive model based on Adolfson et al. (2007) and Justiniano and Preston (2005). Hence, the processes are outlined in the following form:

$$F_t = A * F_{t-1} + \sigma_t^F, \qquad (4.66)$$

where $F = \left[\hat{\pi}_t^f, \hat{R}_t^f, \hat{Z}_t^f\right]$ is the vector of foreign variables; $A$ is the matrix of the persistence parameters estimated in the following section.

## 4.3 Estimation

### 4.3.1 Data description

To estimate the model, the data set obtains the observations for 14 series: GDP, consumption, private investment, inflation, nominal interest rate, total hours worked, export, import, government spending, government investment, transfers, and effective rates on distortionary taxes (consumption, labour income and capital income taxes). All the series span the period from 1989Q2 to 2017Q2 on U.K. data. All the data for real variables are converted to per capita basis by being weighted by a working-age population index. Except specified, all variables are adjusted seasonally and expressed in constant prices. The majority of the data are obtained directly from the ONS and BoE as displayed in Table 4.1.

Table 4.1 Data description

| Variable | Notation | Source | Description |
|---|---|---|---|
| Output | $Y$ | ONS:AMBI | Gross domestic product; SA, CP[3] |
| Consumption | $C$ | ONS:ABJR | Household final consumption expenditure; SA, CP |
| Private investment | $I^p$ | ONS:NPQS | Total gross fixed capital formation minus government investment; SA, CP |
| Price level | $\pi$ | ONS:CGBV | Percentage change in GDP deflator, Quarterly |
| Nominal interest rate | $R$ | BoE:IUQAAJNB | 3-months Treasury Bill; quarterly average rate of discount |
| Working hours | $L$ | ONS:MGRZ, YBUS | Average weekly hours worked[4] |
| Export | $Y^x$ | ONS:IKBK | Balance of payments: Exports: Total trade; SA, CP |
| Import | $Y^m$ | ONS:IKBL | Balance of payments: Imports: Total trade; SA, CP |
| Government spending | $G$ | ONS:NMRP | Total final consumption expenditure by general government; SA, CP |
| Government investment | $I^g$ | ONS:RPZG | Government gross fixed capital formation; SA, CP |
| Total labour force | $pop$ | ONS:BCJD,DYDC | Working population[5] |

The rest of the time series comprising the government transfers and distortionary tax rates are computed following Jones (2002), Leeper et al. (2010) and Bhattarai and Trzeciakiewicz (2017). Before generating the data for effective rates on the labour income tax and the capital income tax, the average tax rate on income ($\tau^i$) needs to be computed since the ONS does not distinguish the data between the labour income and capital income taxes.

The average income tax rate is defined as:

$$\tau^i = \frac{[QWMQ] + [NVCO]}{[QWLW] + [QWME] + [QWLS] + [QWLT]},$$

where [QWMQ] is the income taxes paid by households (HH) and non-profit institutions serving households (NPISH); [NVCO] is the other current taxes paid by

[3] SA stands for "seasonally adjusted"; CP stands for "current price".

[4] $L = \frac{YBUS}{MGRZ} * MGRZindex$ where the $MGRZindex$ is calculated by normalizing the number of MGRZ so that the value in 2010Q1 is one.

[5] Total labour force is the sum of total claimant count and workforce jobs, and used to scale the data as per capita.

HH and NPISH; [QWLW] is the wages and salaries of HH and NPISH; [QWME] is property income of HH and NPISH; [QWLS] is the gross operating surplus of HH and NPISH; [QWLT] is the gross mixed income of HH and NPISH.

Then, the effective rate on labour income tax yields:

$$\tau^{w} = \frac{([QWLW] + 0.5 * [QWLT]) * \tau^{i} + [QWLX]}{[QWLW] + [QWLX]},$$

where [QWLX] is employers' social contributions of HH and NPISH.

The effective rate on capital income tax is:

$$\tau^{k} = \left\{ \begin{matrix} ([QWME] + [QWLS] + 0.5 * [QWLT]) * \tau^{i} + \\ [NMZJ] + [NVCM] - [QWMQ] - [NVCO] + [NSSO] + [NMYD] \end{matrix} \right\} * \frac{1}{[ABNF]},$$

where [NMZJ] is current taxes on income received by the general government; [NVCM] is other current taxes received by the general government; [NSSO] is capital taxes of HH and NPISH; [NMYD] is other taxes on production and [ABNF] is the gross operating surplus of the total economy.

The effective rate on consumption tax is:

$$\tau^{c} = \frac{[NVCC]}{C - [NVCC]},$$

where [NVCC] is total taxes on products and $C$ is the household final consumption expenditure.

Lastly, the amount of public transfers is calculated as:

$$TR = [NNAD] + [NNAN] + [NMRL] + [NNBC],$$

where [NNAD] is social benefits other than transfers in kind; [NNAN] is total other current transfers; [NMRL] is the total paid on subsidies; [NNBC] is total capital transfers.

Assume that the variables are defined as:

$$X = ln\left(\frac{x}{pop}\right) * 100,$$

where $x$ denotes output, consumption, private investment, working hours, export, import, government spending, government investment and transfers. According to Leeper (2010), all variables need to be detrended with the linear trend. Before estimating the model, the data are detrended by adopting the one-sided HP filter to eliminate the low-frequency components following Guerrieri and Iacoviello (2017). There are two benefits for dealing the data with the one-sided HP filter. First one is that it derives plausible estimates of the trend and cycle for the data. Second, the correlation of current observations with subsequent ones would not affect the one-sided HP filter, which is remarked by Stock and Watson (1999).

### 4.3.2 Calibration and estimation

Table 4.2 reports values of calibrated coefficients. Most of these parameters are relative to the value of steady-state in the model, hence the calibration is applied to match the sample mean of the steady-state.

The discount factor $\beta$ is set to 0.995 in line with Guerrieri and Iacoviello (2017) for an annual equilibrium real interest rate of 2 per cent. Consist with Harrison and Oomen (2010), the depreciation rate for private capital is set to 0.025, which indicates that the private capital depreciates 10 per cent annually, and the private capital share in production technology is fixed at 0.3, hence the share of labour in production is 70 per cent. The openness degree $\alpha^m$ is set to 0.3 implied the steady-state import to GDP ratio is 30 per cent and the price elasticities of aggregate imports $\theta$ and exports $\eta$ are fixed at 0.6 for both, which follows Kollmann (2002). The steady-state of marginal cost markup parameter $\frac{v}{v-1}$ for the intermediate producers in domestic, imported and exported sectors is fixed at 1.2 which is in line with Blanchard and Gali (2010). The bond premium parameter $\lambda_\varphi$ is set to 0.01 based on the findings of Lindé et al. (2009).

To match the data for capital stock and investment in the government sector, the public capital depreciation rate and government investment to GDP ratio are fixed to 0.015 and 0.02 respectively, which gives the government capital stock to GDP ratio at 0.32 and is consistent with Bhattarai and Trzeciakiewicz (2017). The share of public capital in production function is set to 0.015 based on the estimation range in the literature of Leeper et al. (2010) and Stähler and Thomas (2012). To match the sample mean of the U.K. data from 1989:Q2 to 2017:Q2, the average VAT (consumption tax rate $\tau^c$), labour income tax $\tau^w$ and capital tax rates $\tau^k$ are 18 per cent, 29 per cent and 28 per cent, respectively. The average government spending during the sample period is pinned down at 0.16 and the endogenous lump-sum transfer is calibrated to 0.28.

Table 4.2 Calibrated parameters and steady-state ratios

| Parameters | Definition | Value |
|---|---|---|
| $\beta$ | Discount factor | 0.995 |
| $\delta$ | Private capital depreciation rate | 0.025 |
| $\delta_g$ | Public capital depreciation rate | 0.015 |
| $\alpha^m$ | The steady state imports/GDP ratio | 0.3 |
| $\alpha_1$ | Share of private capital in production function | 0.3 |
| $\alpha_2$ | Share of public capital in production function | 0.015 |
| $\theta$ | Price elasticity of aggregate imports | 0.6 |
| $\eta$ | Price elasticity of aggregate exports | 0.6 |
| $\lambda_\varphi$ | Bond premium parameter | 0.01 |
| $\tau^c$ | Consumption tax rate | 0.18 |
| $\tau^w$ | Labour tax rate | 0.29 |
| $\tau^k$ | Capital tax rate | 0.27 |
| $TR/GDP$ | Government transfer to GDP ratio | 0.28 |
| $b/GDP$ | Government debt to GDP ratio | 0.6 |
| $G/GDP$ | Government spending to GDP ratio | 0.16 |
| $I^g/GDP$ | Government investment to GDP ratio | 0.02 |

Table 4.3 reports the prior distribution for the estimated coefficients. These are based on studies of Smets and Wouters (2003 and 2007), Adolfson et al. (2005) for an open economy perspective and Bhattarai and Trzeciakiewicz (2017) for the fiscal policy analysis on data of U.K.

Table 4.3 Prior distribution

| Parameters | Prior distribution | | |
|---|---|---|---|
| | type | mean | std. dev./df |
| Investment adj. cost $\phi$ | normal | 4.00 | 1.50 |
| Calvo domestic prices $\lambda^d$ | beta | 0.75 | 0.10 |
| Calvo export prices $\lambda^x$ | beta | 0.75 | 0.10 |
| Calvo import prices $\lambda^m$ | beta | 0.75 | 0.10 |
| Indexation domestic prices $\kappa^d$ | beta | 0.50 | 0.15 |
| Indexation export prices $\kappa^x$ | beta | 0.50 | 0.15 |
| Indexation import prices $\kappa^m$ | beta | 0.50 | 0.15 |
| Con. tax resp. to debt $\gamma^{c,b}$ | normal | 0.20 | 0.10 |
| Con. tax resp. to GDP $\gamma^{c,y}$ | normal | 0.50 | 0.50 |
| Cap. tax resp. to debt $\gamma^{k,b}$ | normal | 0.20 | 0.10 |
| Cap. tax resp. to GDP $\gamma^{k,y}$ | normal | 0.50 | 0.50 |
| Lab. Tax resp. to debt $\gamma^{w,b}$ | normal | 0.20 | 0.10 |

| | | | |
|---|---|---|---|
| Lab. Tax resp. to GDP $\gamma^{w,y}$ | normal | 0.50 | 0.50 |
| Trans. Resp. to debt $\gamma^{tr,b}$ | normal | 0.20 | 0.10 |
| Trans. Resp. to GDP $\gamma^{tr,y}$ | normal | 0.50 | 0.50 |
| Gov. spend. Resp. to debt $\gamma^{g,b}$ | normal | 0.20 | 0.10 |
| Gov. spend. Resp. to GDP $\gamma^{g,y}$ | normal | 0.50 | 0.50 |
| Gov. inv. Resp. to debt $\gamma^{ig,b}$ | normal | 0.20 | 0.10 |
| Gov. inv. Resp. to GDP $\gamma^{ig,y}$ | normal | 0.50 | 0.50 |
| AR(1) cons. tax $\rho^{c}$ | beta | 0.75 | 0.10 |
| AR(1) cap. tax $\rho^{k}$ | beta | 0.75 | 0.10 |
| AR(1) lab. tax $\rho^{w}$ | beta | 0.75 | 0.10 |
| AR(1) transfers $\rho^{tra}$ | beta | 0.75 | 0.10 |
| AR(1) gov. spend. $\rho^{g}$ | beta | 0.75 | 0.10 |
| AR(1) gov. inv. $\rho^{ig}$ | beta | 0.75 | 0.10 |
| Interest rate smoothing $\gamma^{r}$ | beta | 0.40 | 0.10 |
| Inflation response $\gamma^{\pi}$ | normal | 1.50 | 0.10 |
| Output response $\gamma^{y}$ | normal | 0.13 | 0.05 |
| AR(1) tfp. $\rho^{A}$ | beta | 0.75 | 0.10 |
| AR(1) preference $\rho^{pre}$ | beta | 0.75 | 0.10 |
| AR(1) private inv. $\rho^{ip}$ | beta | 0.75 | 0.10 |
| AR(1) monetary policy $\rho^{tr}$ | beta | 0.50 | 0.10 |
| AR(1) bond premium $\rho^{\chi}$ | beta | 0.75 | 0.10 |
| AR(1) foreign interest rate $\rho^{Rf}$ | beta | 0.75 | 0.10 |
| AR(1) foreign inflation $\rho^{\pi f}$ | beta | 0.75 | 0.10 |
| AR(1) foreign consumption $\rho^{Zf}$ | beta | 0.75 | 0.10 |
| s.d. tfp. shock $\sigma^{A}$ | inv. gamma | 0.10 | 2 |
| s.d. preference shock $\sigma^{pre}$ | inv. gamma | 0.10 | 2 |
| s.d. private inv. shock $\sigma^{ip}$ | inv. gamma | 0.10 | 2 |
| s.d. monetary policy shock $\sigma^{tr}$ | inv. gamma | 0.10 | 2 |
| s.d. bond premium shock $\sigma^{\chi}$ | inv. gamma | 0.10 | 2 |
| s.d. foreign interest rate shock $\sigma^{Rf}$ | inv. gamma | 0.10 | 2 |
| s.d. foreign inflation shock $\sigma^{\pi f}$ | inv. gamma | 0.10 | 2 |
| s.d. foreign consumption shock $\sigma^{Zf}$ | inv. gamma | 0.10 | 2 |
| s.d. con. tax shock $\sigma^{c}$ | inv. gamma | 0.10 | 2 |
| s.d. cap. tax shock $\sigma^{k}$ | inv. gamma | 0.10 | 2 |
| s.d. lab. tax shock $\sigma^{w}$ | inv. gamma | 0.10 | 2 |
| s.d. transfers shock $\sigma^{tra}$ | inv. gamma | 0.10 | 2 |
| s.d. gov. spend. shock $\sigma^{g}$ | inv. gamma | 0.10 | 2 |
| s.d. gov. inv. shock $\sigma^{ig}$ | inv. gamma | 0.10 | 2 |

There are three types of prior distributions for all estimated parameters, i.e. beta, normal and inverse gamma distributions. For all parameters drop in an interval between 0 and 1, they are assumed to follow the beta distributions which include parameters of price stickiness $\lambda$, price indexations $\kappa$, interest rate smoothing $\gamma^r$ and shock persistence $\rho$. The prior mean of price stickiness is set to be 0.75 in all sectors, hence, the domestic, imported and exported producers update their prices every 3 quarters accordingly. The prior mean of indexation to previous inflation is set to be 0.5 for domestic, imported and exported prices. While the standard deviation of the prior distribution of price indexation is assumed to be larger than that of Calvo price stickiness since the uncertainty for inflation persistence in prices is higher. As for the coefficients of shock processes, the most of the prior mean is set to be 0.75, except for the persistence of the monetary policy shock that is set with a lower value of 0.5 assumed to be serially uncorrelated.

For the parameters assumed to be positive are set to follow the inverse gamma distributions. The standard deviation of shocks $\sigma$ follows the inverse gamma distribution with a standard mode of 0.10 and a degree of freedom of 2, which demonstrates the fact that quite a few prior information can be collected for those parameters.

For the rest parameters whose prior distribution is assumed to follow the normal distribution, they are parameters in investment adjustment cost, $\phi$, and policy responses in monetary and fiscal rules, $\gamma$. For fiscal policy rules, the parameters indicating the response of fiscal policy to GDP fluctuation is assumed with mean 0.5 and standard deviation 0.5, whereas the distribution means and standard deviation for the parameters of debt innovation responses are 0.2 and 0.1, respectively. For Taylor rule, the prior mean of the parameters of responses to inflation and output are set to 1.5 and 0.13, respectively, which is in line with the literature.

The posterior of all estimated parameters is generated by two steps. Firstly, the mode of the posterior distribution is calculated by standard patterns of numerical optimization. The maximum likelihood is found by solving the model and then adopting the inverse filter. Secondly, the posterior densities of these estimated parameters are obtained by using a standard random walk Metropolis-Hastings algorithm with a 50,000-draw chain. The estimated results of posterior distribution statistics are presented in Table 4.4.

Table 4.4 Posterior distributions.

| Parameter | Posterior distribution | | | |
|---|---|---|---|---|
| | mode | mean | 5% | 95% |
| Investment adj. cost $\phi$ | 4.7898 | 4.8502 | 4.7599 | 5.0166 |
| Calvo domestic prices $\lambda^d$ | 0.8296 | 0.8311 | 0.8248 | 0.8355 |
| Calvo export prices $\lambda^x$ | 0.8017 | 0.8135 | 0.8013 | 0.8241 |
| Calvo import prices $\lambda^m$ | 0.5947 | 0.6015 | 0.5926 | 0.6119 |
| Indexation domestic prices $\kappa^d$ | 0.5716 | 0.5679 | 0.5621 | 0.5796 |
| Indexation export prices $\kappa^x$ | 0.5674 | 0.5662 | 0.5607 | 0.5754 |
| Indexation import prices $\kappa^m$ | 0.6240 | 0.6129 | 0.6018 | 0.6276 |
| Con. tax resp. to debt $\gamma^{c,b}$ | 0.0372 | 0.0323 | 0.0231 | 0.0415 |
| Con. tax resp. to GDP $\gamma^{c,y}$ | 0.3114 | 0.3152 | 0.2989 | 0.3368 |
| Cap. tax resp. to debt $\gamma^{k,b}$ | 0.0835 | 0.0717 | 0.0603 | 0.0893 |
| Cap. tax resp. to GDP $\gamma^{k,y}$ | 0.9918 | 0.9861 | 0.9671 | 0.9978 |
| Lab. Tax resp. to debt $\gamma^{w,b}$ | 0.1502 | 0.1456 | 0.1288 | 0.1658 |
| Lab. Tax resp. to GDP $\gamma^{w,y}$ | 0.7973 | 0.7987 | 0.7797 | 0.8190 |
| Trans. Resp. to debt $\gamma^{tr,b}$ | 0.0972 | 0.1049 | 0.0940 | 0.1172 |
| Trans. Resp. to GDP $\gamma^{tr,y}$ | 0.3935 | 0.3946 | 0.3817 | 0.4118 |
| Gov. spend. Resp. to debt $\gamma^{g,b}$ | 0.1348 | 0.1489 | 0.1348 | 0.1663 |
| Gov. spend. Resp. to GDP $\gamma^{g,y}$ | 0.0616 | 0.0337 | -0.0187 | 0.0680 |
| Gov. inv. Resp. to debt $\gamma^{ig,b}$ | 0.2124 | 0.2268 | 0.2084 | 0.2465 |
| Gov. inv. Resp. to GDP $\gamma^{ig,y}$ | 0.9544 | 0.9669 | 0.9589 | 0.9731 |
| AR(1) cons. tax $\rho^c$ | 0.8078 | 0.8096 | 0.8055 | 0.8131 |
| AR(1) cap. tax $\rho^k$ | 0.8061 | 0.8066 | 0.7998 | 0.8164 |
| AR(1) lab. tax $\rho^w$ | 0.7982 | 0.7997 | 0.7970 | 0.8027 |
| AR(1) transfers $\rho^{tra}$ | 0.7990 | 0.7954 | 0.7922 | 0.8000 |
| AR(1) gov. spend. $\rho^g$ | 0.9344 | 0.9349 | 0.9296 | 0.9394 |
| AR(1) gov. inv. $\rho^{ig}$ | 0.2069 | 0.1984 | 0.1900 | 0.2086 |
| Interest rate smoothing $\gamma^r$ | 0.4282 | 0.4222 | 0.4172 | 0.4295 |
| Inflation response $\gamma^\pi$ | 1.5696 | 1.5656 | 1.5553 | 1.5768 |
| Output response $\gamma^y$ | 0.1028 | 0.1024 | 0.1004 | 0.1043 |

| | | | | |
|---|---|---|---|---|
| AR(1) tfp. $\rho^{A}$ | 0.7621 | 0.7605 | 0.7502 | 0.7709 |
| AR(1) preference $\rho^{pre}$ | 0.7893 | 0.7897 | 0.7815 | 0.7978 |
| AR(1) private inv. $\rho^{ip}$ | 0.7716 | 0.7695 | 0.7602 | 0.7745 |
| AR(1) monetary policy $\rho^{tr}$ | 0.3670 | 0.3732 | 0.3630 | 0.3883 |
| AR(1) bond premium $\rho^{\chi}$ | 0.7267 | 0.7457 | 0.7244 | 0.7569 |
| AR(1) foreign interest rate $\rho^{Rf}$ | 0.8127 | 0.8169 | 0.8104 | 0.8219 |
| AR(1) foreign inflation $\rho^{\pi f}$ | 0.7914 | 0.7916 | 0.7879 | 0.7949 |
| AR(1) foreign consumption $\rho^{Zf}$ | 0.7637 | 0.7609 | 0.7553 | 0.7666 |
| s.d. tfp. shock $\sigma^{A}$ | 0.0528 | 0.0501 | 0.0477 | 0.0528 |
| s.d. preference shock $\sigma^{pre}$ | 0.0664 | 0.0639 | 0.0604 | 0.0667 |
| s.d. private inv. shock $\sigma^{ip}$ | 0.2516 | 0.2503 | 0.2439 | 0.2540 |
| s.d. monetary policy shock $\sigma^{tr}$ | 0.0157 | 0.0171 | 0.0136 | 0.0213 |
| s.d. bond premium shock $\sigma^{\chi}$ | 0.5089 | 0.5049 | 0.4923 | 0.5168 |
| s.d. foreign interest rate shock $\sigma^{Rbf}$ | 0.2971 | 0.2973 | 0.2939 | 0.3000 |
| s.d. foreign inflation shock $\sigma^{\pi f}$ | 0.0263 | 0.0281 | 0.0263 | 0.0307 |
| s.d. foreign consumption shock $\sigma^{Zf}$ | 0.0547 | 0.0527 | 0.0507 | 0.0547 |
| s.d. con. tax shock $\sigma^{c}$ | 0.0229 | 0.0225 | 0.0211 | 0.0235 |
| s.d. cap. tax shock $\sigma^{k}$ | 0.0180 | 0.0171 | 0.0154 | 0.0185 |
| s.d. lab. tax shock $\sigma^{w}$ | 0.0239 | 0.0230 | 0.0216 | 0.0247 |
| s.d. transfers shock $\sigma^{tra}$ | 0.0186 | 0.0177 | 0.0157 | 0.0194 |
| s.d. gov. spend. shock $\sigma^{g}$ | 0.0256 | 0.0240 | 0.0226 | 0.0261 |
| s.d. gov. inv. shock $\sigma^{ig}$ | 0.0495 | 0.0499 | 0.0488 | 0.0510 |
| Marginal likelihood | 2589 | | | |

First result concerned the parameter $\phi$, where $\frac{1}{\phi}$ serves as the elasticity of investment subject to the current shadow price of installed capital. According to the estimation results, the investment elasticity is 0.21. Table 4.5 provides literature investment elasticity which ranges from 0.17 (indicating the largest estimate of investment adjustment cost) in Smets and Wouters (2003) to 0.56 (the smallest estimate) in Levin et al (2005). The estimated result for elasticity is within this range, which implies that each one per cent increase in capital price leads to a 21 per cent rise in investment goods.

Table 4.5 Literature for investment elasticity.

| Aggregate (quarterly) data | | |
|---|---|---|
| | Estimation methodology | Elasticity |
| Levin et al. (2005) | Bayesian DSGE | 0.56 |
| Levin et al. (2005) | Bayesian DSGE | 0.56 |
| Altig et al. (2011) | Impulse response matching | 0.45 |
| Christiano et al. (2005) | Impulse response matching | 0.40 |
| Justiniano and Primiceri (2008) | Bayesian DSGE | 0.30 |
| Smets and Wouters (2003) | Bayesian DSGE | 0.17 |

Secondly, the model displays a considerable degree of price stickiness. The estimated price stickiness for domestic price setter $\lambda_d$ is around 0.83, which implies that the average duration for domestic producers to alter their prices is around 6 quarters (average duration= $(1-\lambda_d)^{-1}$ ). This result is broadly comparable with Galí et al. (2001) which detects a relatively lower fraction of domestic producers who does not reset their prices by estimating the data of the Euro area. In this paper, the interval for price stickiness parameter is estimated between 0.77 and 0.87. While Smets and Wouters (2003) indicate a value of 0.91. The lower degree of nominal rigidity implies that the Phillips curve of the domestic producer is rather sharp, more specifically, the inflation becomes quite sensitive to the dynamics in aggregate marginal cost. Once there is a positive marginal cost shock, the inflation boosts up promptly to the maximum degree and then return to its equilibrium. The degree of the impact of a cost shock to the inflation and the speed of reverting to the equilibrium is managed by the degree of price rigidity. More exactly, the less degree of price stickiness is, the more producers who are willing to re-optimal their prices in a given period, the greater response of inflation is and the quicker the inflation converges to its equilibrium.

For the price stickiness in import sector $\lambda^m$, the parameter is estimated for a lower value of 0.59, which implies a relatively high degree of pass-through of the exchange rate to the imported prices, already for a short period. Meanwhile, the parameter for exported price stickiness, $\lambda^x$, is estimated as 0.80 which is substantially lower than $\lambda_d$, suggesting a 5-quarter stickiness in the exported sector.

Regarding the indexation parameters in price setting ($\kappa^d$, $\kappa^x$ and $\kappa^m$), the results indicate a slightly higher inflation persistence in the imported price for 0.62 but around 0.57 for the both domestic and exported price which are in the relative moderate degree of persistence.

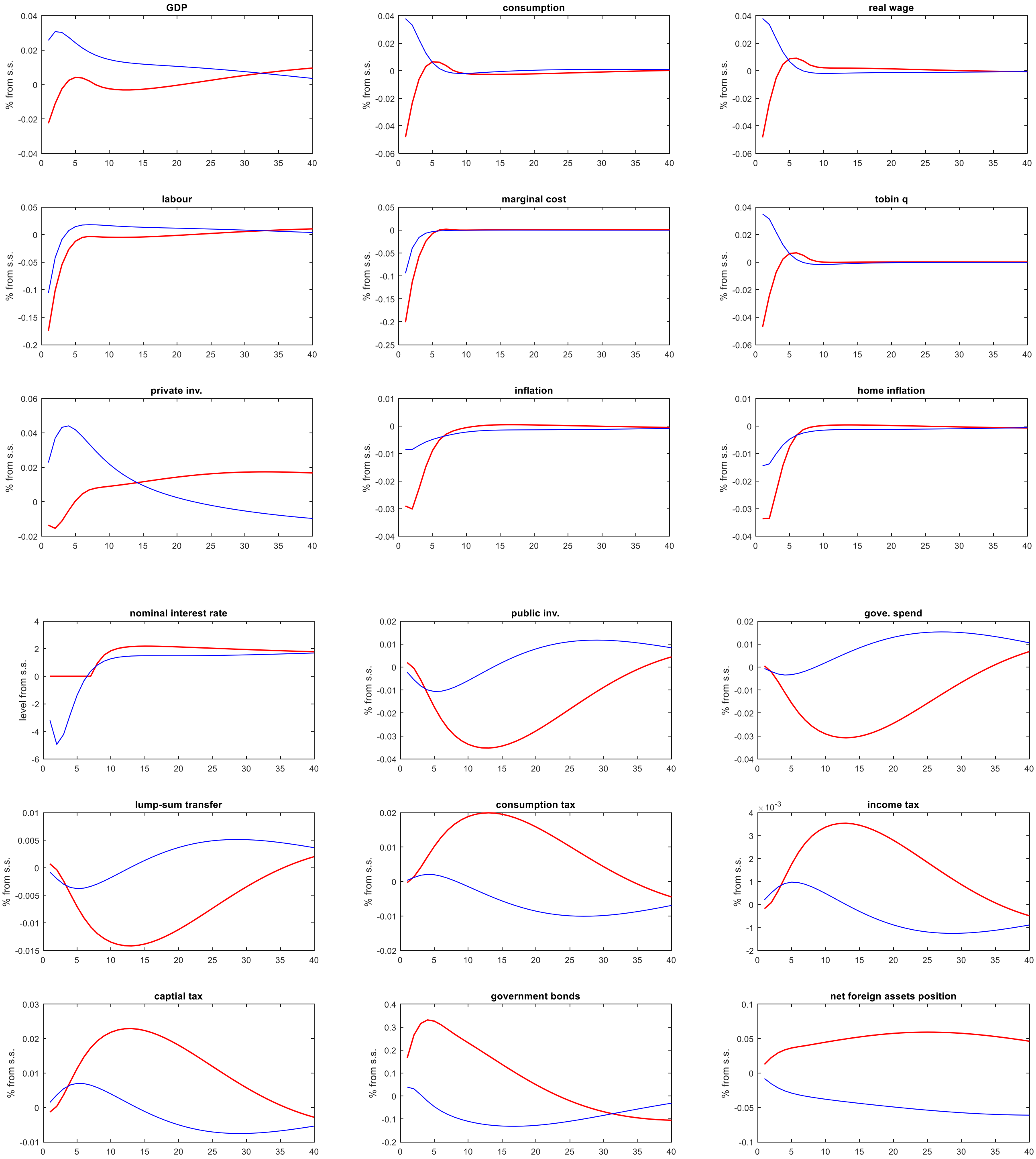
Thirdly, turning to the parameters in the government sector, the coefficients of monetary policy are broadly consistent with the estimated results in Smets and Wouters (2003), except the parameter of interest rate smoothing that is comparatively low. The estimated response of inflation is above one and similar to the value in Taylor (1993) and Clarida, Gali and Gertler (1999) which find an aggressive inflation target monetary policy. Besides, the interest rate rule does not respond strongly to output with a parameter of 0.10, which can be supported by works of literature. Based on the results of parameters of fiscal policy, the public debt is mainly controlled by government spending, government investment and labour income tax, and fluctuations in aggregate output is generally managed by government investment and capital tax during sample year, which is in line with Bhattarai and Trzeciakiewicz (2017) analysing the fiscal policy in an open economy by using the data of U.K. Additionally, the 90 per cent confidence interval of the parameter of government spending to GDP includes zero, implying that government spending was not applied to control the GDP fluctuations systematically.

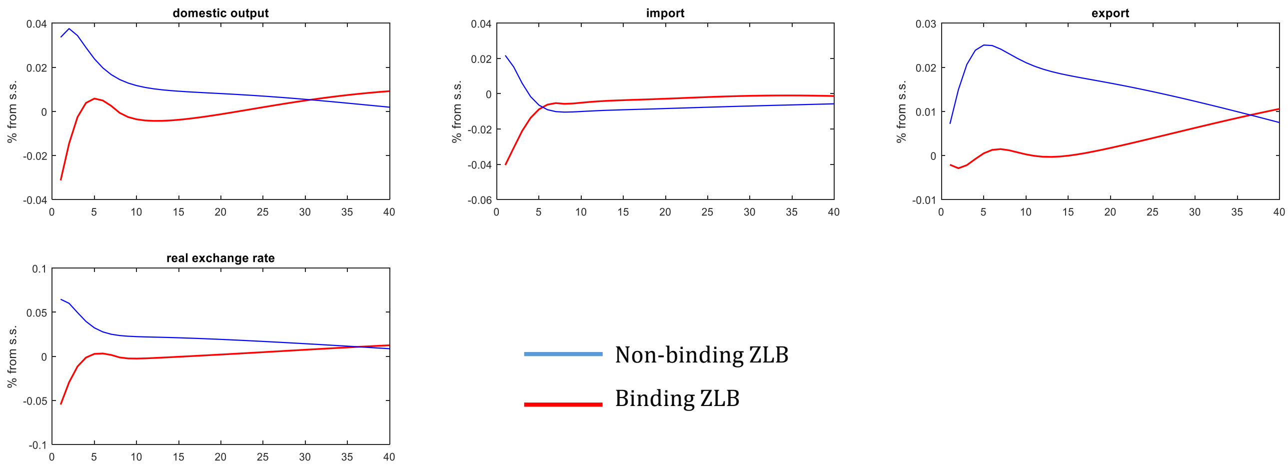
Lastly, for the structural shocks, they are divided into two groups, non-fiscal shocks including productivity shock, investment shock and preference shock, and fiscal policy shocks. The persistence coefficients of non-fiscal shocks are essentially lower than that estimated by Smets and Wouters (2003). This phenomenon is in line with Adolfson et al. (2007) and explains that the open economy feature of this model supplies an extra source of internal propagation. The open economy setup extends the model to induce foreign interest rate shock, foreign inflation shock and foreign consumption shock, which account for a considerable amount of the most persistent shocks. The persistence coefficients of fiscal policy shocks are similar to the estimated results in Bhattarai and Trzeciakiewicz (2017). Government spending shock is the most persistent shock with the $AR(1)$ estimated parameter of 0.93. Government investment shock is the least persistence shock whose $AR(1)$ parameter is approximately 0.21.

## 4.4 Impulse response functions

Figure 4.1 shows the impulse response functions to 10% productivity shock under the scenario with and without the zero lower bound. The estimation implies that the nominal interest rate is constrained at zero for 7 periods due to this structural shock, which means that the economy would escape from a liquidity trap after almost two years.

Figure 4.1 Impulse responses to a productivity shock*






Footnote:

* In each graph, the period by quarters is denoted by the horizontal axis, and the percentage or level deviation from the equilibrium is referred by the vertical axis.

Without zero lower bound constraint, in the labour market, progress in technology promotes an increase in real wage through marginal productivity, since in a fully flexible wage-setting model the real wage and marginal productivity are identical. Meanwhile, the working hours were lowered by increasing productivity and supply of labour, since the income effect dominates the substitution effect. In the goods market, the aggregate demand including private consumption and investment is stimulated by the rise in real income and the real wage. While the degree of the increase in consumption induced by an unanticipated growth in real income is larger than the increase in income itself since the household expects a higher income in the future. Then the domestic country borrows from the rest of the world to finance its current spending.

Due to the expansion in the supply of domestic goods market, the relative price of domestic goods to foreign price drops to restore the external balance. Domestic currency depreciates and exports improve. Although the currency depreciation, the demand for foreign goods is still improved, which can be explained by the international co-movements of the output from Kollmann (2001): when there is an expansion in domestic aggregate demand which is caused by the decline in the domestic interest rate, part of the expansion transfers to foreign goods. Hence, the demand for foreign goods increase. Here, the distortionary taxes are introduced, which would distort the behaviour of household on consuming and investing. Although the aggregate demand expands, part of the growth of private consumption and investment is offset by the distortionary tax but direct to the demand for foreign goods.

In the government sector, since there is sufficient demand in the market, as an automatic stabilizer, government cuts their expenditure for public consumption and public investment, while rises the rate of distortionary tax.

When zero lower bound is binding, the whole economy falls into a recession with dropping in real GDP, private consumption, investment after the unexpected shock. The insufficient demand in the domestic market also reveals a climbing in private saving due to a positive real interest rate under the binding zero lower bound. During the duration of that liquidity trap lasting about two years, there are less consumption, investment and savings. Eggertsson (2011) argued that at zero interest rates, the output is demand-determined and the fundamental problem is lack of demand. When the nominal interest rate is constrained long enough, it could generate the collapse in output and induce the recession.

In the international goods market, the demand falls in the liquidity trap due to an appreciation of the domestic currency. This decline in exported demand can also be explained by Bodenstein, Erceg and Guerrieri (2017) who stated that the decrease in demand for exported goods leads to a decline in the marginal cost of production and inflation which is not followed by a lower interest rate. The binding zero lower bound on nominal interest rate raises the real interest rate sharply in a short period, which limits the expansion in aggregate demand compared to the situation when the nominal interest rate can be cut immediately. The aggregate demand can even decline if the initial recession is more pronounced. With the decline in net export and this gap not filling by the aggregate demand, the output decreases by almost the same as in domestic country and abroad. As to the demand for imported goods, it also falls. Since the imported goods are used to produce the domestic final goods, the obvious reduction in domestic aggregate demand dampens the demand for imported goods.

There are some noticeable features in the behaviour of government when the monetary policy is restricted by the zero lower bound. At the first period during the recession, the government attempts to install a fiscal stimulus package to encourage the economy by cutting the tax rates (i.e. consumption, labour income and capital taxes) and expanding the government expenditures (i.e. government spending, investment and lump-sum transfer). However, these measures are quickly inversed at next period due to the climbing amount of government debt. To balance the government budget and keep the real debt under control, the government needs to reverse its measures from the fiscal expansion to fiscal austerity by raising taxes and cutting expenditures.

In general, the results presented are consistent with the facts of fiscal responses in the U.K. after the 2008 financial crisis. At the first two years just after the crisis, a package of the fiscal stimulus was implemented by carrying out more government spending and cutting in the tax rates. While later on 2010, due to an increasing structural deficit and a struggling structural position faced by the country, the U.K. government had to announce that the most urgent target of fiscal policy was to reduce the burden of the deficit. Confronting with an upward revision of expected structural deficits, the government decided to change the strategy of fiscal measures from undertaking a stimulus package to projecting the size of fiscal consolidation to achieve a balanced government budget overall.

## 4.5 Fiscal multipliers

As known, the fiscal multipliers are conventional tools applied to quantitatively study the short-term effects of discretionary fiscal policy on GDP, consumption and investment. In this chapter, the fiscal multipliers are calculated following Leeper (2010) that quantitatively summarizes the impacts of fiscal shocks on output, consumption and private investment frequently. The multipliers are calculated in present value by using the method in Mountford and Uhlig (2009). Since the present value of the fiscal multiplier illustrates the full economic dynamics caused by the disturbances of fiscal shocks rather than a one-period effect, it is more preferable than the impact multipliers. Besides, the macroeconomic impact is also properly discounted in the future. The present value of the fiscal multiplier over a k-period horizon is written as:

$$Present\ \ Value\ \ Multiplier\ \ (k) = \frac{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta\tilde{Y}_{t+j}}{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta\tilde{X}_{t+j}},$$

where $\tilde{Y} =$ {GDP, consumption, private investment} and $\tilde{X} =$ {government spending, government investment, transfers, consumption tax, labour income tax, capital tax}. It is noticeable that the multiplier is the initial impact multiplier when $k = 0$ which is widely discussed in previous literature as in Blanchard and Perotti (2002) and Forni et al. (2009).

The results for the fiscal multipliers are presented in Table 4.6 for a normal case and a case with a binding zero lower bound, respectively.

Table 4.6 Fiscal multipliers[6]

| | without ZLB[7] | | | ZLB[8] | | |
|---|---|---|---|---|---|---|
| | Y | C | I | Y | C | I |
| Gov. spend. | | | | | | |
| Impact | 0.44 | -0.19 | -0.08 | 1.26 | 0.63 | 0.08 |
| 4 quarters | 0.25 | -0.16 | -0.19 | 1.21 | 0.37 | 0.12 |
| 12 quarters | -0.20 | -0.16 | -0.40 | 1.05 | 0.06 | 0.03 |
| 20 quarters | -0.67 | -0.20 | -0.58 | 0.86 | -0.06 | -0.10 |
| Gov. inv. | | | | | | |
| Impact | 0.64 | -0.05 | -0.01 | 0.67 | 0.02 | 0.00 |
| 4 quarters | 0.58 | -0.06 | -0.04 | 0.59 | -0.01 | -0.02 |
| 12 quarters | 0.51 | -0.05 | -0.13 | 0.45 | -0.06 | -0.13 |
| 20 quarters | 0.48 | -0.02 | -0.22 | 0.39 | -0.06 | -0.24 |
| Trans. | | | | | | |
| Impact | 0.00 | 0.01 | 0.00 | 0.32 | 0.39 | 0.06 |
| 4 quarters | -0.07 | -0.02 | 0.01 | 0.34 | 0.33 | 0.13 |
| 12 quarters | -0.27 | -0.08 | 0.05 | 0.35 | 0.20 | 0.20 |
| 20 quarters | -0.38 | -0.14 | 0.14 | 0.43 | 0.17 | 0.20 |
| Cons. tax | | | | | | |
| Impact | -0.30 | -0.48 | 0.02 | -0.47 | -0.69 | -0.01 |
| 4 quarters | -0.24 | -0.47 | 0.05 | -0.46 | -0.66 | -0.01 |
| 12 quarters | -0.07 | -0.49 | 0.13 | -0.41 | -0.59 | 0.03 |
| 20 quarters | 0.09 | -0.49 | 0.19 | -0.34 | -0.57 | 0.10 |
| Cap. tax | | | | | | |
| Impact | -0.01 | 0.00 | -0.01 | 0.02 | 0.03 | 0.00 |
| 4 quarters | -0.03 | 0.01 | -0.03 | 0.03 | 0.03 | -0.01 |
| 12 quarters | -0.08 | 0.02 | -0.07 | 0.04 | 0.04 | -0.03 |
| 20 quarters | -0.13 | 0.02 | -0.09 | 0.04 | 0.04 | -0.04 |
| Lab. tax | | | | | | |
| Impact | -0.36 | -0.33 | -0.10 | 0.25 | 0.34 | 0.02 |
| 4 quarters | -0.63 | -0.36 | -0.27 | 0.56 | 0.32 | 0.02 |
| 12 quarters | -1.21 | -0.28 | -0.59 | 0.83 | 0.20 | -0.23 |
| 20 quarters | -1.69 | -0.25 | -0.73 | 0.67 | 0.15 | -0.49 |

When the economy is not restricted by the zero lower bound on the nominal interest rate, increasing the government expenditures and cutting tax rates are

[6] A positive value of the multiplier means an increase in fiscal rules increases the output, consumption and private investment, while a negative multiplier means an increase in the fiscal rules has the negative impacts in contrast.

[7] ZLB refers to the zero lower bound.

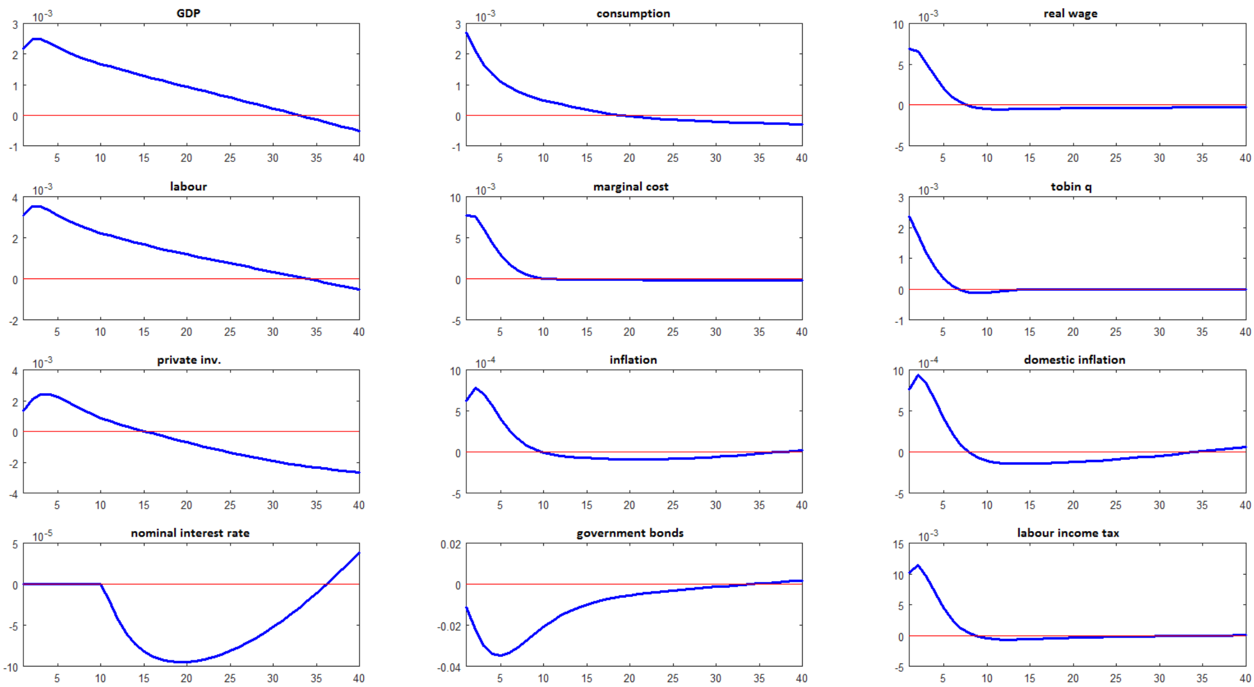
[8] The results are obtained by holding a zero nominal interest rate by ten quarters.

expansionary, which is consistent with the previous discussion. While there are three noticeable differences when the economy stays in a liquidity trap with zero interest rate.

### (1) The effects of increasing the labour income tax rate

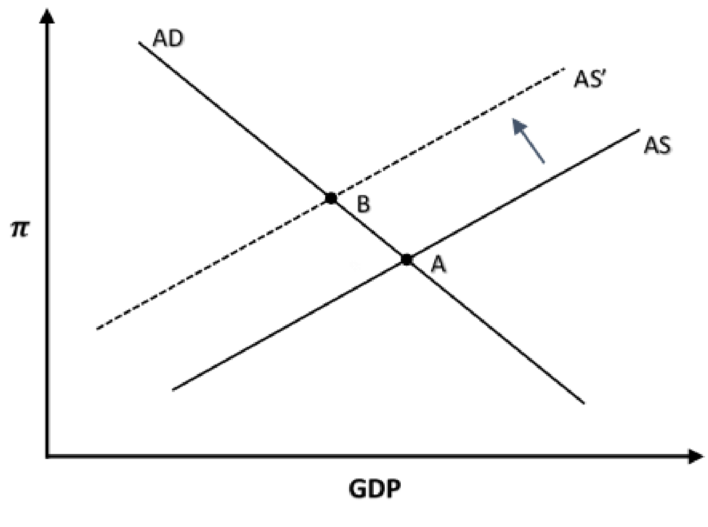
However, the positive multiplier of the labour income tax reported in Table 4.6 implies that cutting the labour income tax is contractionary in a liquidity trap, which is consistent with the findings in Eggertson (2011) and Mckay and Reis (2016). In other words, an increase in the labour income tax rate has an expansionary effect at zero interest rate. The impulse response functions to this shock is presented in Figure 4.2.

Figure 4.2 Impulse response functions to labour income tax shock with a zero lower bound



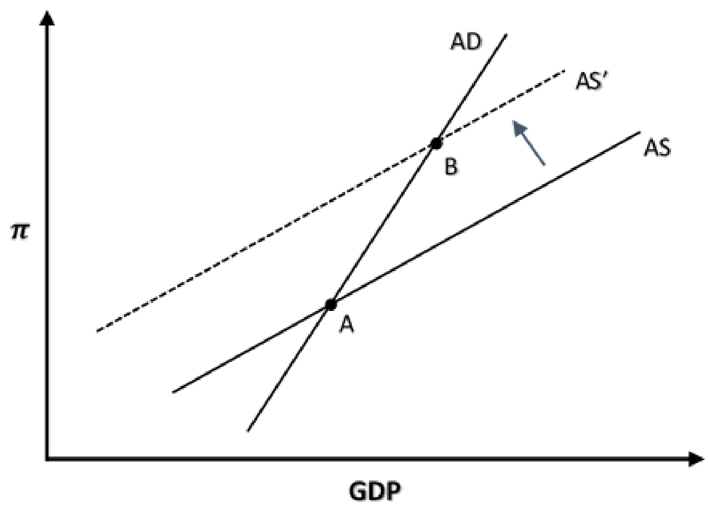
I would start with this paragraph first. Raising labour income tax rate is contractionary under normal circumstance without the zero lower bound. The mechanism of this effect can be clarified precisely respecting to aggregate demand and aggregate supply of the economy. In Figure 4.3 where the aggregate demand (AD) and aggregate supply (AS) curves are displayed, an increase in labour income tax rate shifts up the AS curve, which generates an inflationary effect. The reason is that the higher labour tax rate causes the households to work less since they earn less for each working hour, which raises the real wage. Hence, the firms produce the same amount of goods but cost more due to a higher wage, which induces the firms to charge a higher price for the goods, thus the higher inflation is generated. Response to the pressure of inflation, the central bank increases the nominal interest rate more than the increasing degree of the inflation to offset this inflationary effect, leading to a reduction in output with higher inflation, which explains the reason for generating a downward-sloping AD curve. Therefore, the dynamic of the equilibrium (moving from point A to point B) illustrates that rising the labour tax rate at a positive nominal interest rate provokes the contraction of the economy.

Figure 4.3 The contractionary effect without the zero lower bound



This positive labour tax shock increases the real wage and labour supply since the substitution effect dominates the income effect. With a higher wage, the households give up their leisure to work more due to a greater reward. Hence, the marginal cost for the production also increases, which causes the firms to supply fewer goods to the market and sets a higher price. The movement of the AS curve at zero interest rate shown in Figure 4.4 is the same as that in Figure 4.2. However, rising the labour tax rate impacts the economy oppositely with a binding zero lower bound, which is mainly caused by a different AD curve. The slope of the AD curve is changed from downward sloping to upward sloping shown in Figure 4.4. This graph is similar to that in Eggertsson (2011) which explains that the upward sloping AD curve results from a positive relationship between inflation and output in the liquidity trap. For a given zero nominal interest rate, any increase in inflation reduces the real interest rate, leading to relatively cheaper spending and thus increasing demand. In contrast, the deflationary effect generates a positive real interest rate which depresses the aggregate demand. Therefore, the new equilibrium (point B) in Figure 4.4 explains that the inflationary effect generated by increasing the labour income tax rate could derive an expansionary effect under the zero lower bound.

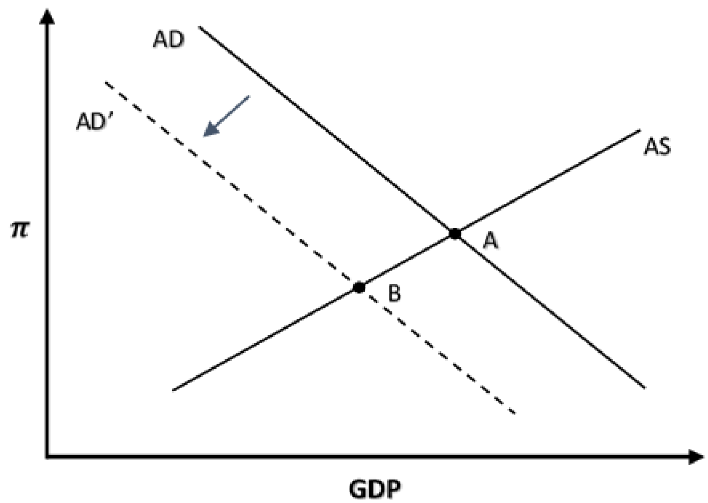
Figure 4.4 The expansionary effect with the zero lower bound



### (2) The effects of increasing the capital tax rate

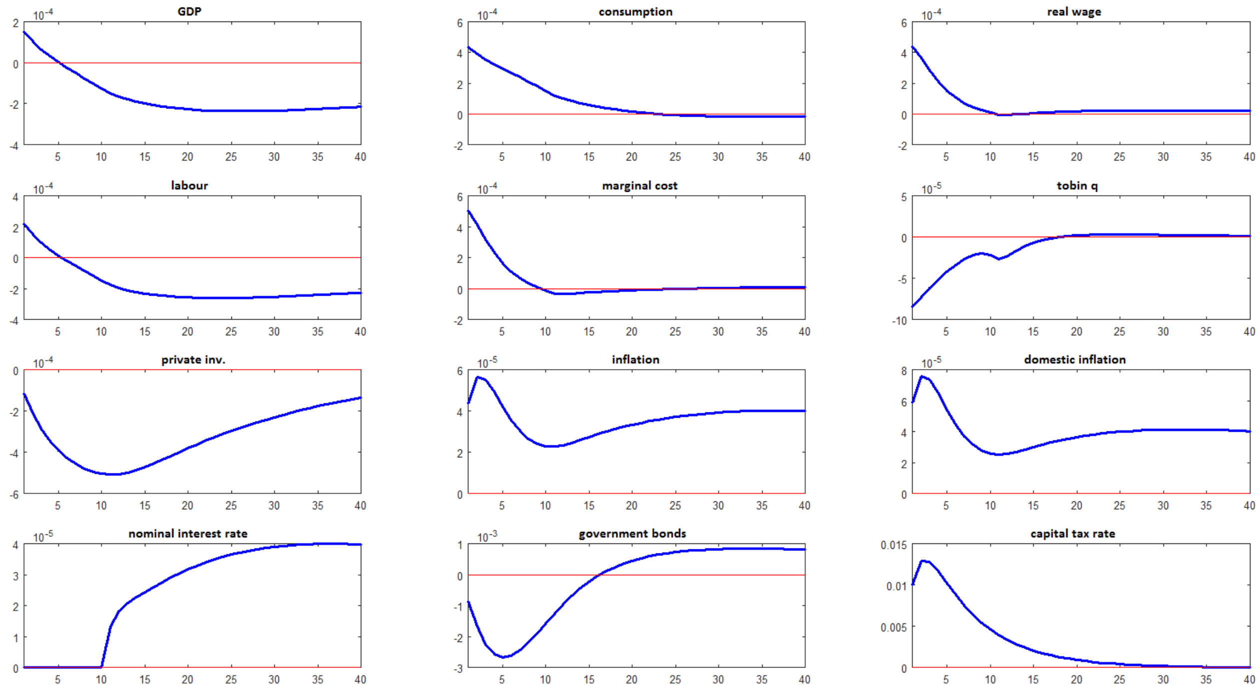
Likewise, the increase in capital tax rate has an expansionary effect at zero interest rate, which is also proved by Eggertsson (2011) and Christiano, Eichenbaum and Rebelo (2011). In the model of this chapter, the capital stock is specified as holding by the households, hence the capital tax is paid by the households. Without the zero lower bound, it does not make a significant difference between the situations of that capital tax is paid by the household or the firm. While it matters much on who holds the capital stock when considering the zero lower bound. When the households hold the capital stock as in this chapter, the changes in the capital tax rate affect the aggregate demand only and the impact on the aggregate supply can be ignored.

Figure 4.5 The contractionary effect without the zero lower bound



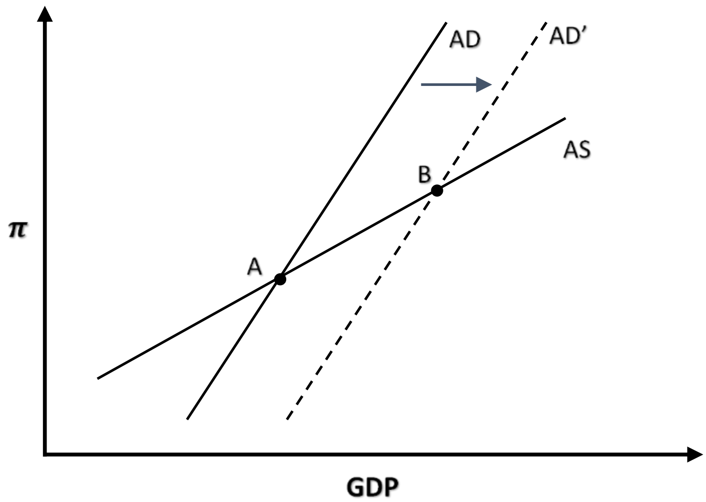

At a positive nominal interest rate, the returns to the capital are reduced by raising the capital tax rate, which dampens the investment and results in shifting the AD curve to the left as in Figure 4.5. The lack of aggregate demand generates a deflationary effect to the economy, thus the original equilibrium (point A) moves to the intersection (point B) of the dashed AD curve and the AS curve at lower output and inflation. Therefore, increasing the capital tax rate has a contractionary effect under a normal circumstance.

Figure 4.6 Impulse-response for the capital tax shock with a zero lower bound



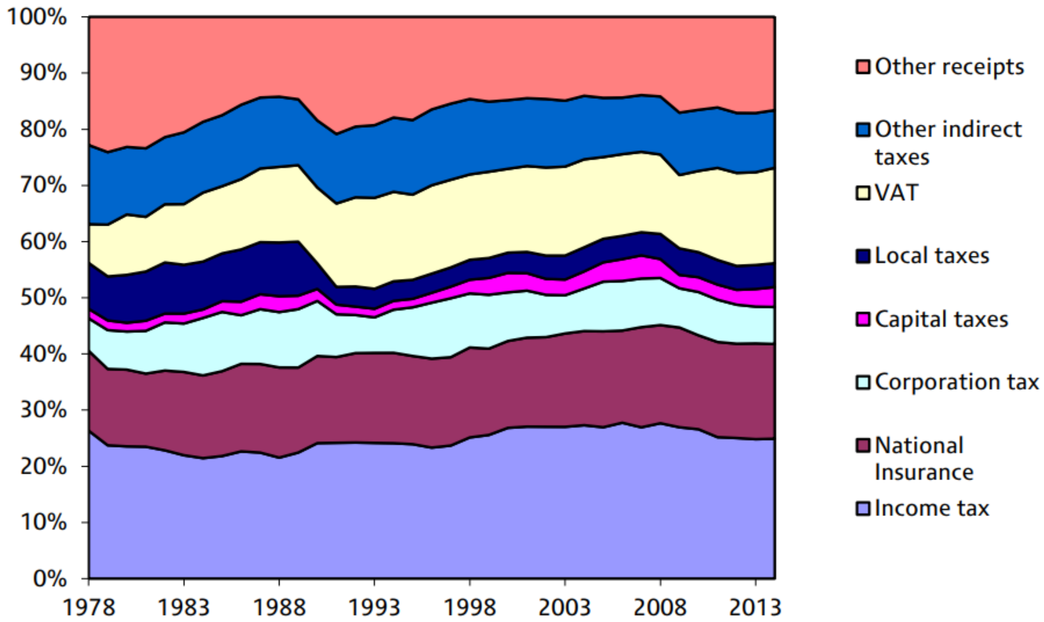

Even though the production capacity is weakened by a rise in capital tax rate at a positive interest rate, in a liquidity trap, the problem of the economy is not the insufficient capacity of production but the inadequate aggregate demand, which is an essential point of view remarked by Eggertsson (2011). It is obvious in Figure 4.6 that a positive capital tax shock reduces the investment and saving but encourages the consumption otherwise, which creates more aggregate demand of the economy. In Figure 4.7, the AD curve shifts to the right due to the expansion in the aggregate demand. Since the AD curve is an upward sloping curve in the liquidity trap, the new equilibrium is formed at point B at higher inflation and more output. Therefore, the increase in the capital tax produces an expansionary effect under the zero lower bound.

Figure 4.7 The expansionary effect with the zero lower bound




Although increasing the rates of both labour income tax and capital tax is efficient, but the measure of the labour income tax seems to be more powerful. Figure 4.8 shows that the total government revenue of the U.K. government is always dominated by the resource of the labour income tax, while the role of the capital tax is limited. According to the results in Table 4.6, the multiplier for the labour income tax is much higher in absolute value than that of the capital tax (-0.01 and -0.36 under a normal circumstance, 0.02 and 0.25 under the zero lower bound). This conclusion is also in consistence with the literature of the optimal tax theory pioneered by Chamley (1986) and Judd (1985) suggesting that the optimal tax strategy is to claim a zero capital tax in the long term, and Mirrlees (1971) arguing that designing a labour income tax can optimize the social welfare. In the theory of the optimal capital tax, the capital income is conceptualized as a future consumption, the tax on the capital income represents a differentiated consumption tax on current and future consumption. Therefore, a capital tax measure leads to the distortion of households' saving and consumption behaviours, since the excessively taxed consumption in the future is substituted with the present consumption by the households. Consequently, a zero capital tax rate might be optimal, postulated by Atkinson and Stiglitz (1976) based on the Chamley and Judd theorem. The optimal labour income tax theory is developed by Conesa and Krueger (2006) where a DSGE model with a utilitarian social welfare function is introduced, and they compute the optimal income tax that plays a valuable role in affecting the households' behaviours of consumption, saving and labour supply.

Figure 4.8 The composition of government receipts, 1978-79 to 2014-15[9]




Source: HM Treasury (see http://www.ifs.org.uk/tools_and_resources/fiscal_facts).

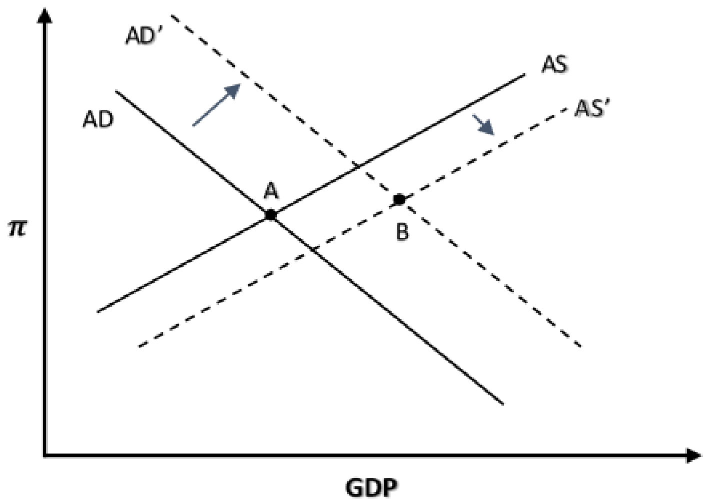
### (3) The effects of increasing government spending

The values of multipliers in Table 4.6 indicate that the stimulus effect of increasing government expenditure including public spending, investment and transfer, and

[9] Years are fiscal years, so 2008 means 2008–09. 'National Insurance' excludes NI surcharge when it existed, and 'VAT' is net of refunds paid to other parts of central and local government; these are both included in 'other receipts'. 'Other indirect taxes' are excise duties and customs duties. 'Corporation tax' includes petroleum revenue tax, the supplementary charge and the 1997–98 windfall tax. 'Capital taxes' are capital gains tax, inheritance tax (and its predecessors) and stamp duties. 'Local taxes' are council tax, the community charge, domestic rates and business rates before 1990; from 1990, business rates are included in 'other receipts'.

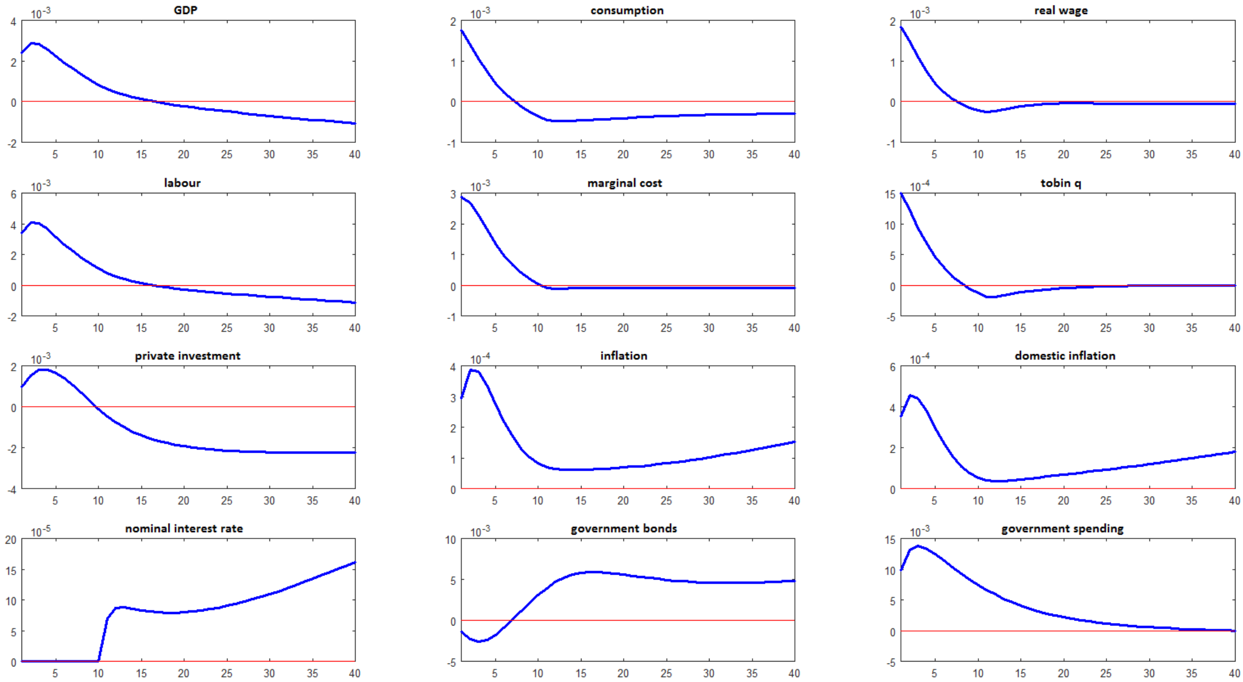
cutting the consumption tax rate is more effective in a liquidity trap. It can be explained as that higher inflation is induced by all these four fiscal measures by stimulating the economy in the demand side. Due to a constant nominal interest rate in the liquidity trap, the real interest rate declines with higher inflation. Hence, private demand is expanded by this inflationary effect. Particularly, the effect of increasing government spending is extremely intensified by the zero lower bound, which will be discussed in the following content.

Figure 4.9 The expansionary effect without the zero lower bound



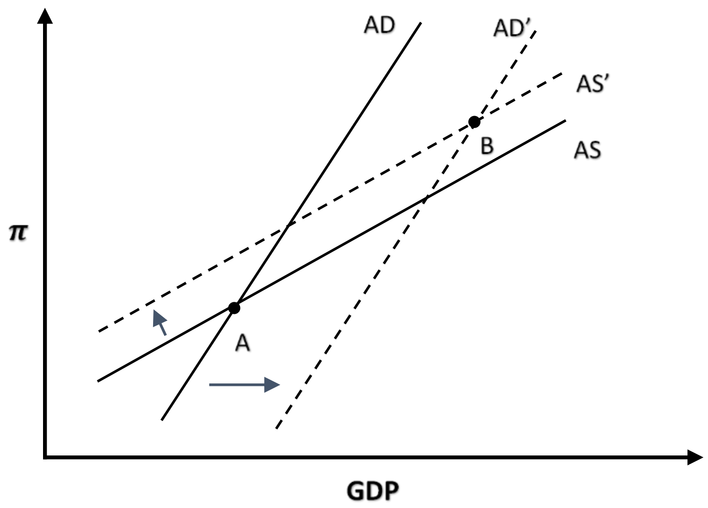

When the nominal interest rate is slack, the rise in the government spending increases the aggregate demand since it is an increase in autonomous spending, thus the AD curve in Figure 4.9 is shifted to the right. Meanwhile, the AS curve is also shifted by an increase in government spending. In this case, private consumption is crowded out by government spending since part of the resources is taken away by the government from the private sector. To compensate for the loss in consumption, the households chose to work more, which induces a decline in the real wage and thus shifts the AS curve down. However, this deflationary effect is not strong enough to counteract the simulative effect of rising the government spending, hence the equilibrium is relocated to the point B as in Figure 4.9 at more output and slightly higher inflation.

Figure 4.10 Impulse-response for the government spending shock with a zero lower bound




Compared to the expansionary effect of increasing government spending under a normal circumstance, the effect under a zero lower bound is strikingly larger. The intuition behind a larger government spending multiplier can be detected from Figure 4.10. Furthermore, Figure 4.11 indicates that a positive government spending shock increases the output and shifts the upward-sloping AD curve in a liquidity trap to the right. Likewise, the real wage and marginal cost is increased, which shifts up the AS curve and generate an inflationary effect. Since the nominal interest rate is bound at zero, the real interest rate is driven down by higher inflation, causing a stimulus in private consumption and thus a further increase in aggregate demand, marginal cost and inflation. Thus, the multiplier effect of increasing government spending causes an extremely expansion in output in the liquidity trap.

Figure 4.11 The expansionary effect with the zero lower bound




The conclusion of stimulating the economy by raising the government spending is more effective at zero interest rate is also proved in a cross-country example presented in Table 4.7 by comparing the fiscal multipliers in Boubaker, Nguyen and Paltalidis (2018) who use the data of France, Germany, Italy and Spain and in this chapter using U.K. data.

Table 4.7 The cross-country comparison of fiscal multipliers

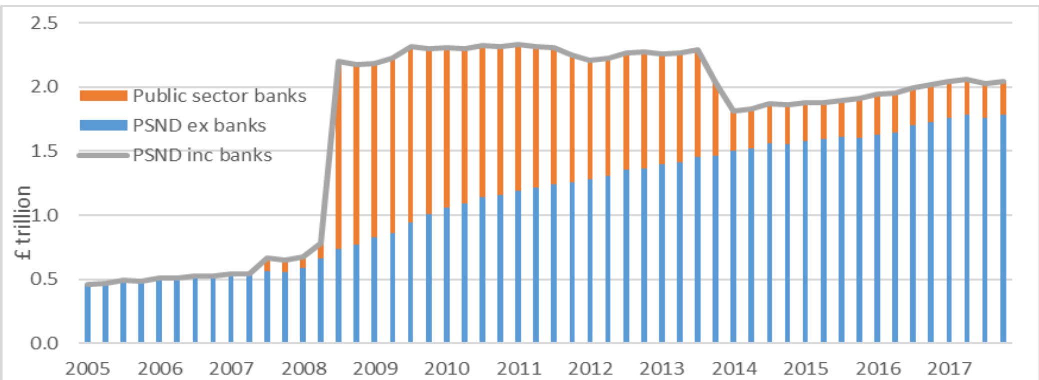
| | Without ZLB | ZLB |
|---|---|---|
| *U.K.* | | |
| Government spending | 0.44 | 1.26 |
| Consumption tax | 0.30 | 0.47 |
| *France* | | |
| Government spending | 1.09 | 1.84 |
| Consumption tax | 0.48 | 0.62 |
| *Germany* | | |
| Government spending | 1.17 | 1.83 |
| Consumption tax | 0.64 | 0.85 |
| *Italy* | | |
| Government spending | 1.14 | 1.97 |
| Consumption tax | 0.27 | 0.45 |
| *Spain* | | |
| Government spending | 1.12 | 1.99 |
| Consumption tax | 0.20 | 0.37 |

This table reveals the effects of increasing government spending and cutting consumption tax with and without the zero lower bound. It is noticeable that the impact of increasing government spending is stronger than cutting the tax in each country regardless of the liquidity trap, and this impact becomes much stronger especially at zero interest rate. In a binding ZLB regime, the government spending multiplier ranges from 1.26 (U.K.) to 1.99 (Spain) compared to the consumption tax multiplier varies from 0.37 (Spain) to 0.85 (Germany). Generally, the results in Boubaker, Nguyen and Paltalidis (2018) support the findings in this chapter that the expansionary effect of increasing government spending is greater than cutting the tax, notably in the liquidity trap.

## 4.6 Policy application

The fiscal multipliers analysis suggests that increasing the government expenditures and cutting the consumption tax, coupled with the rising labour income and capital taxes are effective in helping the economy recover from the crisis. However, relating to the actual fiscal responses of the U.K. government to the global financial crisis in 2008, there are several differences between the model implications and actual fiscal reacts.

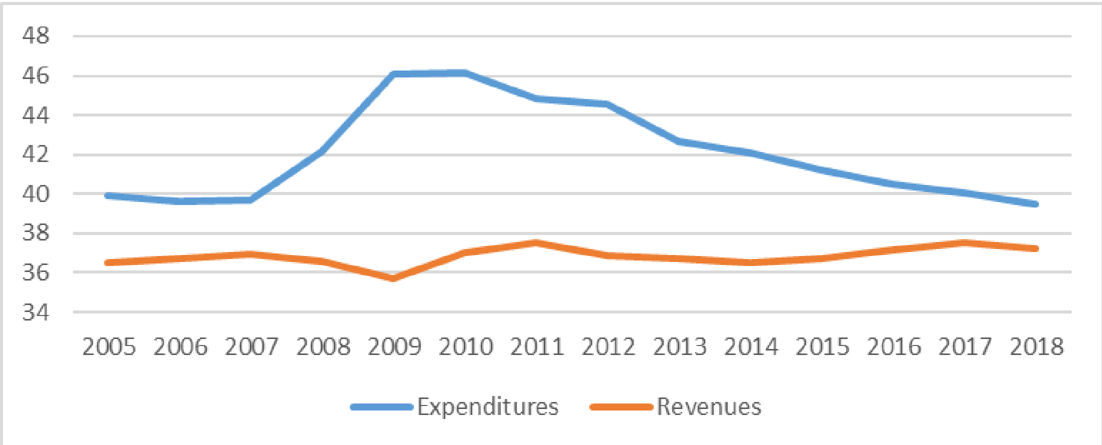
Figure 4.12 Public sector net debt




Source: Fiscal sustainability report (July 2018) by Office for Budget Responsibility (see https://obr.uk/fsr/fiscal-sustainability-report-july-2018/)

Initially, the U.K. government announced an expansionary fiscal policy with bringing forward a £3 billion capital spending and cutting the main rate of VAT from 17.5% to 15% as a fiscal stimulus package in the 2008 budget report, which aimed at regaining the economy growth by a conventional fiscal scheme. However, this stimulus package was reversed quickly by the fiscal consolidation announced in 2010 due to severely large structural budget deficits faced by the U.K. (see Figure 4.12). Before and at the start of the crisis, the government implemented a sustainable public investment rule attempting to keep the ratio of net debt to GDP at a prudent and stable level of under 40 per cent over the economic cycle. Nevertheless, the financial crisis and consequent recession raised the public net debt above the stable level dramatically. Hence, instead of promoting economic growth, the most urgent issue faced by the government was to reduce the deficit. In 2010, a programme of fiscal austerity with cutting government expenditures and increasing the taxes for multiple fiscal years was installed. The transformation for the fiscal policy from a loosen policy to tighten one is shown in Figure 4.13.

Figure 4.13 Government expenditures and revenues




Source: Figures up to 2018 are calculated using the Office for National Statistics series for total managed expenditure (KX5Q) and revenues (JW2O) divided by GDP (BKTL).

There are discrepancies between the model implied fiscal multipliers analysis and actual fiscal responses as one suggests an increasing path and the other requires a decreasing path of the government spending. Apart from the difference in the government expenditures, the U.K. government increased the standard rate of value-added tax (VAT) as one of the most significant measures of fiscal consolidation during the recession, which is also in contrast to the theoretical conclusion of an efficient fiscal strategy of cutting the consumption tax in the previous section. Table 4.8 presents the historical path of the VAT rate. The multiplier analysis implies an expansionary effect of cutting the consumption tax, and this effect will be more effective in a liquidity trap. While cutting the VAT is not supported by the practical policy during the crisis.

Table 4.8 Historical VAT rate

| From | To | Rate |
|---|---|---|
| 19 March 1991 | 30 November 2008 | 17.5% |
| 01 December 2008 | 31 December 2009 | 15% |
| 01 January 2010 | 03 January 2011 | 17.5% |
| 04 January 2011 | Present | 20% |

Sources: HM Revenue & Customs (see http://www.hmrc.gov.uk/vat/vat-introduction.htm).

The main reason for the U.K. government raising the VAT rate instead of cutting it could be that the burden of the deficit is a more urgent problem compared with economic expansion and GDP growth recovery. The U.K. government aimed for a better structural deficits position and a balanced budget overall during the crisis. Thus, although the multiplier analysis proves that cutting the consumption tax could stimulate the private consumption by an immediate decline in the purchasing price, and the prices will be expected to increase at a relatively higher level as soon as the recession is over, but raising VAT is implemented for the need of fiscal consolidation, although it leads to weak economic growth.

Within the scheme of fiscal austerity, the labour income tax and capital tax were increased by the U.K. government after the financial crisis of 2008 (see Table 4.9 and Table 4.10). In general, for the labour income tax, the government sets a new

50% top rate on the person whose income is above £150,000 and reduces the tax-free personal allowance; for the capital tax, a 28% top rate is set for an individual and a higher 28% tax rate is set for the trusts and personal representatives. Even though the purpose of raising labour income tax and capital tax is to meet the requirements of the fiscal consolidation, but these strategies are also approved as the expansionary fiscal policies under zero lower bound theoretically. Intuitively, the increases in the rates of both labour income tax and capital tax can generate an inflationary effect in a liquidity trap which can induce the expansion in the economy.

Table 4.9 Rate of income tax: 2008-09 to 2016-17.

| | 2008-09 | | 2009-10 | | 2010-11 | |
|---|---|---|---|---|---|---|
| | Bands of taxable income[10] £ | Rate of tax % | Bands of taxable income[10] £ | Rate of tax % | Bands of taxable income[10] £ | Rate of tax % |
| Basic rate[11] | 1-34,800 | 20 | 1-37,400 | 20 | 1-37,400 | 20 |
| Higher rate | Over 34,800 | 40 | Over 37,400 | 40 | Over 37,400 | 40 |
| Additional Rate | Not Applicable | | Not Applicable | | Over 150,000 | 50 |
| | 2011-12 | | 2012-13 | | 2013-14 | |
| | Bands of taxable income £ | Rate of tax % | Bands of taxable income £ | Rate of tax % | Bands of taxable income £ | Rate of tax % |
| Basic rate | 1-35,000 | 20 | 1-34,370 | 20 | 1-32,010 | 20 |
| Higher rate | Over 35,000 | 40 | Over 34,370 | 40 | Over 32,010 | 40 |
| Additional Rate | Over 150,000 | 50 | Over 150,000 | 50 | Over 150,000 | 45 |
| | 2014-15 | | 2015-16 | | 2016-17 | |
| | Bands of taxable income £ | Rate of tax % | Bands of taxable income £ | Rate of tax % | Bands of taxable income £ | Rate of tax % |
| Basic rate | 1-31,865 | 20 | 1-31,785 | 20 | 1-32,000 | 20 |
| Higher rate | Over 31,865 | 40 | Over 31,785 | 40 | Over 32,000 | 40 |
| Additional Rate | Over 150,000 | 45 | Over 150,000 | 45 | Over 150,000 | 45 |

Source: HM Revenue & Customs (see https://www.gov.uk › Income Tax).

10 Taxable income is defined as gross income for income tax purposes less any allowances and reliefs available at the taxpayer's marginal rate.

11 From 2008-09 the starting rate is abolished for all non-savings income (e.g. employment, self-employed trading profits, pensions and property income), which is the first slice of income to be charged to income tax. The starting rate and the starting rate limit for savings is shown in the table below. Where taxable non-savings income does not fully occupy the starting rate limit the remainder of the starting rate limit is available for savings income.

Table 4.10 Capital gains tax rates

| Year | Rates | | |
|---|---|---|---|
| | Individuals | Discretionary and accumulation trusts | Other trusts and personal representatives |
| 2008-09 | 18%[12] | 18% | 18% |
| 2009-10 | 18%[12] | 18% | 18% |
| 2010-11 | 18%/28%[12] [13] [14] | 18%/28%[13] | 18%/28%[13] |
| 2011-12 | 18%/28%[12] [14] | 28% | 28% |
| 2012-13 | 18%/28%[12] [14] | 28% | 28% |
| 2013-14 | 18%/28%[12] [14] | 28% | 28% |
| 2014-15 | 18%/28%[12] [14] | 28% | 28% |

Source: HM Revenue & Customs (see http://www.hmrc.gov.uk/rates/cgt.htm).

In conclusion, increasing the labour income tax and capital tax can satisfy both the requirements of fiscal consolidation and expansionary effects in the liquidity trap, but the decision making on government expenditures and consumption tax depends on the aim of the fiscal policy. In terms of the fiscal consolidation, it requires the cuts in government expenditures and tax-raising measures to reduce the public borrowing from a high status to a planned overall fiscal surplus. While these measures generate deflationary effects in a liquidity trap and are unfavourable for economic growth, which is not suggested to implement on the purpose of recovering the economy during the crisis based on the findings in this chapter.

## 4.7 Concluding remarks

This chapter studies the effectiveness and implications of the fiscal policy in a

[12] For certain types of business asset, 'Entrepreneurs' relief reduces this rate to 10% subject to a 'lifetime' limit of gains set at £1m for disposals between 6 April 2008 and 5 April 2010; £2m between 6 April 2010 and 22 June 2010; £5m between 23 June 2010 and 30 May 2011; and £10m on or after 1 April 2011.

[13] Applies only to gains realized after 23 June 2010. Gains before this date were charged at the 2009-10 rates

[14] The 18% rate applies to gains below the basic rate income tax band limit, and the 28% rate to gains above.

liquidity trap based on the data of the U.K. economy from 1898Q2 to 2017Q2. The results generated on the estimated parameter illustrates that rising the government expenditures and cutting the consumption tax are expansionary regardless of the existence of the zero lower bound. In particular, these expansionary effects are more effective in the liquidity trap. While cutting the labour income tax and capital income tax are expansionary in a positive-interest-rate regime but contractionary at zero interest rate. These conclusions of the U.K. fiscal policy implementation are consistent with the findings of Eggertsson (2011) that evaluates the effect of the recovery bill, the largest fiscal expansion in U.S. economic history since New Deal, in the recession after the financial crisis of 2008 to stimulate the U.S. economy.

Moreover, this chapter also discusses the actual fiscal responses of U.K. government of fiscal consolidation during the financial crisis, and it suggests that increasing the labour income tax and capital income tax is favourable for both the fiscal consolidation and economic stimulus in the liquidity trap. However, there is a dilemma for cutting government expenditures and increasing the consumption tax between the economic stimulus and consolidation. Even though these fiscal measures impede economic growth from a theoretical perspective, but they are implemented for reducing the burden of the deficit and achieving a balanced government budget overall to bring the public finances to a sustainable path, while still supporting the recovery. Indeed, converting an unsustainable budget deficit to a sustainable deficit would build social confidence. The improvement in the state of public deficit though effects of restoring the confidence and expectations could stimulate the private investment and consumption and then support the economic growth and recovery.

Besides, there is also a confidence problem in the financial sector due to the credit crunch as the aftermath of the financial crisis in the U.K., leading to lower economic growth. However, the DSGE model in this chapter is built based on the conventional assumption of the standard New Keynesian model that there is perfect competition between the interest rate set by the monetary authority and the cost of credit for firms in the financial market. Since the financial crisis of 2008 has witnessed the failure of this theory, and the effectiveness of the fiscal policy is sensitive to the imperfection of the credit market, argued by Fernández-Villaverde (2010) and Eggertsson and Krugman (2012), the model needs to be modified by incorporating the financial sector to bring insight into the fiscal study for the U.K. economy during the crisis. In the next chapter, this thesis will introduce the presence of the financial frictions to evaluate the effectiveness of the fiscal policy under the impact of the financial frictions in a liquidity trap.

# Chapter 5 Fiscal policy in a model with financial frictions under zero lower bound

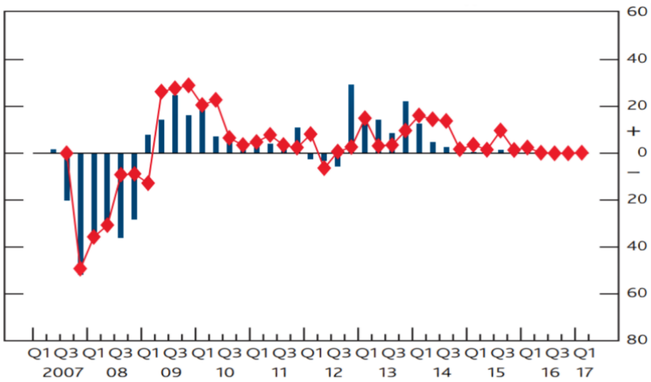
## 5.1 Introduction

In the conventional New Keynesian model, it is assumed that the financial assets, specifically the bonds and credit, are perfect substitutes. However, the global financial crisis of 2008 revealed the fragility of this assumption. In the aftermath of the financial crisis, many developed countries experienced a recession. A combination of real shocks caused the economic recession; and the financial shocks and financial mechanisms topped this resulting a financial crisis. which is remarked by Le et al. (2016 and 2018).

In the U.K., the recovery from the crisis was obstructed by the credit crunch where banks limited their lending causing a rapid decline in the availability of credit (see Figure 5.1).

Figure 5.1 Cooperate credit availability[15]

Net Percentage balances[16]



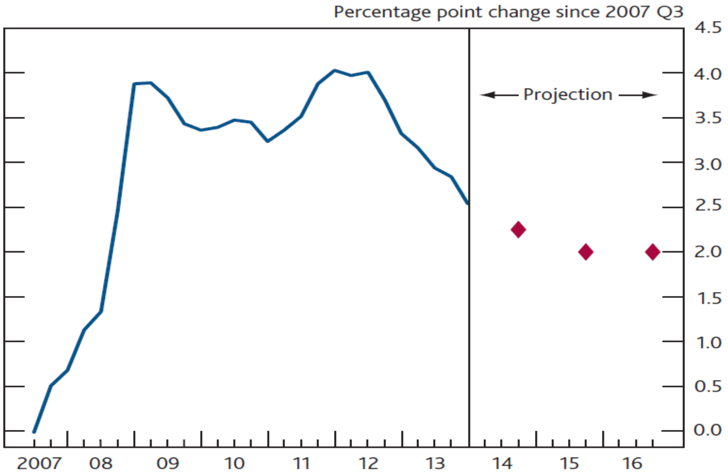

Source: Credit Conditions Survey 2016Q4, Bank of England (see https://www.bankofengland.co.uk/credit-conditions-survey/2016/2016-q4.htm).

[15] Net percentage balances are calculated by weighting together the responses of those lenders who answered the question. The blue bars show the responses over the previous three months. The red diamonds show the expectations over the next three months. Expectations balances have been moved forward one quarter so that they can be compared with the actual outturns in the following quarter.

[16] A positive balance indicates that more corporate credit is available.

Despite, the economy saw a wider credit spread for households and firms (see Figure 5.2). The credit conditions became tighter and then exacerbated the depression by restricting the economic activities of consumption and investment.

Figure 5.2 Credit spreads since the start of the financial crisis[17]




Sources: Bank of England, BDRC Continental SME Finance Monitor, Bloomberg, BofA Merrill Lynch Global Research, used with permission, British Household Panel Survey, Department for Business, Innovation and Skills and Bank calculations.

In the previous chapter, it was found that the fiscal policies are more effective under the zero lower bound; increasing the government expenditures, cutting the consumption tax and raising the labour and capital tax can generate inflationary effect at zero interest rate. But these conclusions are yielded under the perfect financial market assumption. As Fernández-Villaverde (2010) and Eggertsson and Krugman (2012) argued, the effectiveness of the fiscal policy is indeed susceptible to the imperfection of the credit market. Hence, it is necessary to include this financial imperfection in this chapter and investigate whether the effects of fiscal policies are going to be amplified by the credit frictions during the recession. Besides, this chapter will use the Bayesian method to analyse the question of interest.

[17] The diamonds show the profile of the summary credit spread at one, two and three years ahead that was published in the May 2014 Inflation Report. The blue line shows the back data.

To explain the role of financial frictions, this chapter extends the baseline model in Chapter 4 by incorporating the imperfection financial market based on the financial accelerator mechanism of Bernanke, Gertler and Gilchrist (1999) (BGG). According to the BGG, the entrepreneurs have to borrow externally to finance capital purchases used in the production. They are considered to be risky and thus they have to pay an external finance premium on their borrowing. The idea is whenever shocks cause the asset price to rise, they lower the external finance premium, leading to a higher investment and aggregate demand. This in turn triggers more increases in the asset prices and further economic boom. Also the model incorporates a debt-deflation effect, so that any unpredicted change in the price level of the economy could change the actual value of the debt burden of the entrepreneurs, since any risk from macroeconomic uncertainty is avoided by the lenders, and then influence the investment decision by the firms.

This chapter investigates how financial imperfection can affect the expansionary and inflationary effects of fiscal policy. Moreover, this chapter studies the impact of the interaction between a liquidity trap and the financial frictions on the effectiveness of the fiscal policy. To achieve this aim, the present values of the fiscal multipliers for government expenditures and distortionary taxes policies are calculated: the cases considering financial frictions and not under the environment with and without the zero lower bound on nominal interest rate (Carrillo and Poilly, 2013). The differences between the values of multipliers can reveal the role of financial frictions. It finds the existence of the financial frictions magnifies the effect of the fiscal policies irrespective of the interest rate regimes. More importantly, the presence of the financial frictions contributes to the fiscal effectiveness more when the nominal interest rate is constrained at zero, i.e. the sizes of the fiscal multipliers are larger in the liquidity trap. The former one could be explained by that the final effect of the financial frictions on the fiscal policies is magnified propagating through two channels together, the financial accelerator and the Fisher deflation channels; the latter is caused by the iteration between these two effects.

The remaining sections of this chapter are organized as follows: Section 2 presents the model set; the data description, calibration and the estimation results are discussed in Section 3; Section 4 explains the impulse response functions; Section 5 analyses the results of the fiscal multipliers; Section 6 concludes.

## 5.2 The model economy

The model is based on the baseline model in Chapter 4 and incorporates financial frictions a la BGG (1999). There are three reasons for why the financial elements are significant in modelling. Firstly, the recession in the U.K. is exacerbated by the credit crunch in the banking system. To demonstrate the impact of the presence of the financial frictions, it is necessary to embody the financial sector and financial shocks to capture the disturbance and dynamics of the economy. Secondly, the finance premium disturbance can generate the situation of the liquidity trap in a model with zero lower bound. Amano and Shukayev (2012) state that the probability of the nominal interest rate being lowered to its zero bound can be increased by assuming that there is a risk premium shock hitting the economy. Additionally, the wedge between the funding cost and the nominal interest rate is naturally entailed by a credit premium shock, which implies the existence of arbitrage between the capital and bounds. Thirdly, with an endogenous credit spread, the roles of the consumers and the entrepreneurs are automatically separated, which helps to study the individual behaviours of consumption and investment more intuitively.

Moreover, the baseline model is also modified by replacing the simple version of utility function with a general form of it to get a better fit to the economy and provide more convincing suggestions. There are five sectors generally: households, capital producers, entrepreneurs (intermediate firms), final goods firms and government. This section explains the differences between the households and production sectors with those in the previous chapter, while the details of the remaining sectors refer to Section 4.2.

### 5.2.1 Representative household

In line with the baseline model, the households save in domestic and foreign bonds, which balances into a UIP condition specifying the changes in the exchange rate. They also gain their utilities by leisure and consumption. Each household chooses consumption level, working hour and amount of holdings in domestic bonds and foreign bonds to maximize the intertemporal utility. However, this model modifies the simple version of utility function in Chapter 4 by including a habit formation that affects the preferences of the individual consumption. Habit formation is introduced in the model with the aim of explaining the main dynamics observed in empirical evidence and addressing the adjustment principle. The existence of the consumption habits can reveal a mild response of the consumption to the fluctuations in the level of output over time, meaning that the consumption response to a shock in terms of the income level is described as a bell-shaped curve. Hence, the preference of the representative household is denoted as:

$$E_0\beta^t[ln(C_t - hC_{t-1}) - \mu_t^{pre}\frac{L_t^{1+\psi}}{1+\psi}], \tag{5.1}$$

where $C_t$ and $L_t$ are each household's consumption level and working hour, respectively. The habit persistence introduced in the preference is interpreted as $hC_{t-1}$, which illustrates friction in the pattern of the consumption. The habit formation is characterized in a popular functional form that adopts a quasi-difference between the current and previous level of the consumption, where $h > 0$ is the coefficient of the persistence. Hence, the utility of the representative household is not determined by the current consumption level, but by the quasi-difference in consumption. The preference shock is defined as:

$$\hat{\mu}_t^{pre} = \rho^{pre}\hat{\mu}_{t-1}^{pre} + \sigma_t^{pre}.$$

Since the households don't own the capital stock in this model, they only allocate their total income on consumption and saving in the period-$t$ matured domestic bonds, $B_{t-1}$, and foreign bonds, $B_{t-1}^f$, which is paid with an interest rate $R_t$ and a risk-adjusted interest rate $\Phi_{t-1}R_{t-1}^f$, respectively. Accordingly, the budget constraint in every period faced by all households is described as:

$$(1 + \tau_t^c)P_tC_t + B_t + S_tB_t^f =$$

$$(1 - \tau_t^w)W_tL_t + R_{t-1}B_{t-1} + S_t\Phi_{t-1}R_{t-1}^fB_{t-1}^f + TR_t, \qquad (5.2)$$

where $\tau_t^i, i = c, w$, are the distortionary taxes on consumption and labour income; $TR_t$ refers to the lump-sum transfers obtained from the government.

Hence, the optimization problem for the households can be written as:

$$\max_{\{C_t, L_t, B_t, B_t^f\}_{t=0}^{\infty}} E_0\sum_{t=0}^{\infty}\beta^t\{[ln(C_t - hC_{t-1}) - \mu_t^{pre}\frac{L_t^{1+\psi}}{1+\psi}]$$

$$+ \lambda_t[\ (1 - \tau_t^w)W_tL_t + R_{t-1}B_{t-1} + S_t\Phi_{t-1}R_{t-1}^fB_{t-1}^f + TR_t$$

$$- (1 + \tau_t^c)P_tC_t - B_t - S_tB_t^f]\}.$$

After rearranging the functions in real terms, the solutions for the first-order conditions are denoted as:

$$w.r.t.C_t: \quad \frac{1}{C_t-hC_{t-1}} - \beta h E_t\left(\frac{1}{C_{t+1}-hC_t}\right) - \lambda_t(1+\tau_t^c) = 0, \tag{5.3}$$

$$w.r.t.L_t: \quad \frac{\mu_t^{pre}L_t^{\psi}}{(1-\tau_t^w)w_t} = \lambda_t, \tag{5.4}$$

$$w.r.t.b_t: \quad \beta E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}\right) = 1, \tag{5.5}$$

$$w.r.t.b_t^f: \quad \beta E_t\left(\frac{\lambda_{t+1}S_{t+1}\Phi_t R_t^f}{\lambda_t S_t \pi_{t+1}}\right) = 1, \tag{5.6}$$

where the lower-case letters indicate the real terms of variables; $\pi_t$ denotes the inflation in the home country as $\pi_t = \frac{P_t}{P_{t-1}}$.

### 5.2.2 Capital producers

The stock of physical capital is produced by capital producers in a perfectly competitive market. The production of the capital producer is proceeded by combining the existing capital and investment goods and includes adjustment costs to increase the capital price in terms of the sold amount of the capital. Then the installed capital stock is rented to the entrepreneurs to produce the domestic and the exported intermediate goods.

Suppose there is a sufficient amount of indifference capital producers who are perfectly competitive and take given prices in the market. When period $t$ ends, they purchase the existing stock of capital $K_t^p$ and combine this with investment goods $I_t$ to produce newly installed capital stock $K_{t+1}^p$ which is then purchased by the entrepreneurs. Consistent with the setting of capital accumulation in the baseline model, the capital producers accumulate the physical capital depending on the law of motion as follows:

$$K_{t+1}^{p} = (1-\delta)K_t^p + \mu_t^{ip} F\left(I_t^p, I_{t-1}^p\right), \qquad (5.7)$$

where the investment adjustment costs are defined as the function $F\left(I_t^p, I_{t-1}^p\right) = \left(1 - S\left(\frac{I_t^p}{I_{t-1}^p}\right)\right) I_t^p$ following the formation in Christiano, Eichenbaum and Evans (2005).

The problem of the capital producers is to maximize their profits subjected to the chosen investment level and capital stock, which is a dynamic process due to the presence of the adjustment costs. It needs to be solved as:

$$\max_{\{I_{t+s}, K_{t+s}\}_{t=0}^{\infty}} E_t\left[\sum_{s=0}^{\infty} \beta^s \lambda_{t+s} \pi_{t+s}^{cp}\right],$$

$$\pi_{t+s}^{cp} = Q_{t+s}\left[(1-\delta)K_{t+s}^p + \mu_t^{ip} I_{t+s}^p \left(1 - S\left(\frac{I_{t+s}^p}{I_{t+s-1}^p}\right)\right)\right] - Q_{t+s}(1-\delta)K_{t+s}^p - I_{t+s}^p \; .$$

The capital producers' profits, $\pi_t^{cp}$, are composed of that the income of selling the new capital $K_{t+1}^p$ at the real price $Q_t$ minus the payment of buying the previously installed capital $K_t^p$ and investment goods $I_t^p$. Since the marginal rate of the transformation from the depreciated previous capital to newly installed capital is utility, the real prices for both the new and old capital are identical.

Hence, the first-order condition for the capital producers' problem is indicated as:

$$1 = Q_t \mu_t^{ip}\left[1 - S\left(\frac{I_t^p}{I_{t-1}^p}\right) - S'\left(\frac{I_t^p}{I_{t-1}^p}\right)\frac{I_t^p}{I_{t-1}^p}\right] +$$

$$\beta E_t\left[Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}^{ip} S'\left(\frac{I_t^p}{I_{t-1}^p}\right)\left(\frac{I_t^p}{I_{t-1}^p}\right)^2\right], \qquad (5.8)$$

which is the function of the investment demand. In this function, the real price of the capital is assigned to the investment adjustment costs and its marginal cost. In other words, the impact on the investment from a certain shock can be mitigated and then influences the capital price.

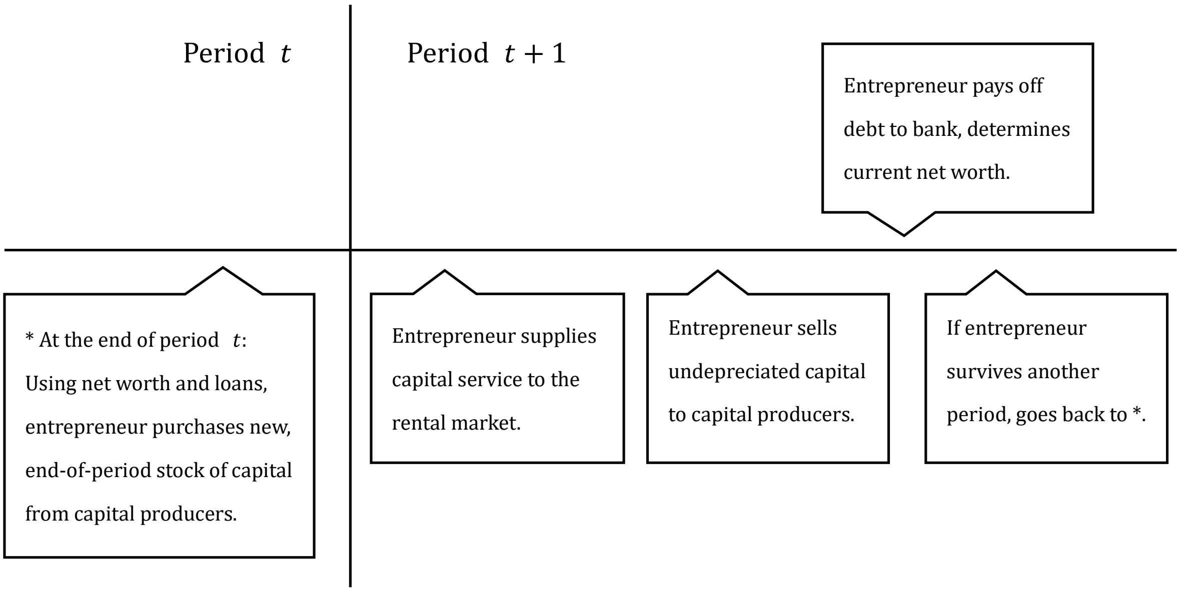
### 5.2.3 Intermediate goods firms

#### 5.2.3.1 Entrepreneurs

Due to the presence of the financial frictions in this model, the setting of the intermediate goods firms is different from the setup in the baseline model. The intermediate goods firms are acted by the entrepreneurs who produce the intermediate goods by hiring labour and buying new capital in terms of the production function, and whilst their expenses are covered by the net worth combining with the funds borrowed from the financial intermediates, which is consistent with the structure in Christiano, Motto and Rostagno (2014) following the BGG mechanism closely. More specifically, the value of acquiring the new capital is always less than the net worth due to the risk-neutral entrepreneurs, and there is a fixed surviving rate for the firms to the next period.

The details of the entrepreneurs' activities are discussed in the following content. At the end of period $t$, the entrepreneurs summarize their financial statements by obtaining the net worth $NW_{t+1}$ which is viewed as their internal funds. Then combining it with the external funds borrowed from a bank to purchase the new capital $K_{t+1}^{p}$ installed at period $t$ at the real price $Q_t$ for the time- $t+1$ production. At next period $t+1$ the entrepreneurs gain the return of the marginal production in supplying the capital services to the rental market and also receive the income in selling the existing undepreciated capital $(1-\delta)K_{t+1}^{p}$ at price $Q_{t+1}$. Additionally, the debt to the bank which is borrowed for the capital acquisition for time-$t+1$ production needs to be paid off by the entrepreneurs at the amount of $Q_t K^p_{t+1} - NW_{t+1}$. Meanwhile, the banks need to pay the opportunity cost for the funds gained from the depositors as $R_t$ which is the risk-free interest rate in the economy. The overall process for the entrepreneurs' activity is illustrated in Figure 5.3.

Figure 5.3 The entrepreneurs' one-period life




For entrepreneurs, the optimal capital demand is determined by the expected marginal return on capital (or the expected marginal costs of the external funds) at period $t+1$. Thus, the optimal demand for capital in equilibrium is followed as:

$$E_t f_{t+1} = E_t \left[ \frac{(1-\tau^k_{t+1}) R^k_{t+1} + (1-\delta) Q_{t+1}}{Q_t} + \tau^k_{t+1} \delta \right], \qquad (5.9)$$

where the expected marginal return on capital equivalents to the after-tax rental rate of the capital that depends on the marginal productivity of it at period $t+1$, $(1-\tau^k_{t+1}) R^k_{t+1}$, adds the value of the undepreciated capital which is sold to the capital producer at the end of the period $t+1$ at price $(1-\delta) Q_{t+1}$, against the cost of capital acquisition for the time-$t+1$ production, $Q_t$, plus the tax rebates on the depreciated capital, $\tau_{t+1}^{k}\delta$. As presented, equation (5.9) indicates the correlation between the financial state of the entrepreneurs and the external financing cost, which consequently determines the capital demand.

Based on the assumptions in the BGG, the presence of the agency problem in the financial market causes the costs for the external funds are higher than that of the internal finance. However, the problem of the asymmetric information between the borrowers and lenders could be solved by introducing the costly state verification, remarking by Townsend (1979). When the observation of the ex-post return is free of cost to the entrepreneurs subjected to a random result, the financial intermediates could afford a monitoring cost to observe the realized return of the entrepreneurs. Having observed the outcome of the ex-post return, the entrepreneurs choose whether to pay off their debt to banks or default. If the debt is fully repaid, the realized return does not need to be verified, while if the entrepreneur defaults, the external leadings need to be audited by the banks that the outcome of the projects less the monitoring cost need to be recovered.

Accordingly, a financial contract is solved by BGG to maximize the entrepreneurial payoff subjecting to the required rate of return earned by the financial intermediates. It is demonstrated that the discounted return to capital or the external finance premium is implied by the financial contract depending on the firms' leverage ratio, the features of the distribution of the realized returns and the entrepreneurs' expected life span. Hence, the elasticity of the external finance premium regard to the leverage is determined by these basic values.

The marginal cost of external finance is equivalent to a gross of the external finance premium and riskless interest rate. The optimal condition should be satisfied by the capital as follows:

$$E_t f_{t+1} = E_t \left[ EP_{t+1}(\bullet) \frac{R_t}{\pi_{t+1}} \mu_t^{ep} \right], \qquad (5.10)$$

where the expected real interest rate is denoted as $E_t(R_t/\pi_{t+1})$, and it is implied by the equation that the entrepreneurs could obtain the loans from the financial intermediaries only if the risk of default can be offset, then a finance premium would be charged over the opportunity costs of the funds by the lenders. Also, the nominal contracts can clarify the reality of the debt deflation effect in Fisher (1933). Furthermore, $\mu_t^{ep}$ refers to a financial premium shock, which could distort the return on capital observed by the entrepreneurs, following the assumption in Carrillo and Poilly (2013). The finance premium shock drives a wedge between the rate of return on capital and the risk-free interest rate. When there is a positive shock, the price of the capital and investment demand is declined by the widened credit spread and vice versa.

The external finance premium is defined as:

$$EP_{t+1} = \left( \frac{Q_t K_{t+1}^p}{NW_{t+1}} \right)^{\omega}, \qquad (5.11)$$

where $\frac{Q_t K_{t+1}}{NW_{t+1}}$ is the leverage ratio of the entrepreneurs; $\omega$ denotes the elasticity of the premium with respect to leverage ratio implying that when the leverage declines, the entrepreneurs depend on the collateralized loans to a larger extent to finance their projects, which means that a higher entrepreneurs' stake in the project could reduce the riskiness of the loans and the cost of external borrowing.

As the capital is purchased by the capital producers and the debt contract is signed with the financial intermediaries, the net worth of the entrepreneurs evolves based on the law of motion as follows:

$$NW_{t+1} = \kappa\big[f_t Q_{t-1} K_t^p - E_{t-1} f_t \big(Q_{t-1} K_t^p - NW_t\big)\big]\mu_t^{nw} = \kappa V_t, \qquad (5.12)$$

where $\kappa$ is the surviving rate to the next period faced by the entrepreneurs, which implies that the fraction of the entrepreneurs, $(1-\kappa)$, died from the market, the remaining value of them would be transferred to the newly entered entrepreneurs used as their net worth to purchase the capital stock. Accordingly, Christensen and Dib (2008) remark that the assumption of the surviving rate ensures that it is never sufficient for the entrepreneurs to entirely finance the purchase of the newly installed capital relying on their net worth alone, thus they obtain the funds by borrowing from the financial intermediates to satisfy their excess investment demand over the net worth. Besides, there is a fixed scale of the entrepreneurial sector with an adequate number of newly entered members to replace the departed one. Furthermore, $f_t$ is the ex-post return on capital owned at period $t$, and $E_{t-1}f_t$ denotes the ex-post cost of external funding; $Q_{t-1}K_t^p - NW_t$ refers to the aggregate amount of debt needed to be paid off at period $t$; $\mu_t^{nw}$ is the net worth ( the firm equity) shock which represents in an autoregressive process:

$$\hat{\mu}_t^{nw} = \rho^{nw}\hat{\mu}_{t-1}^{nw} + \sigma_t^{nw}.$$

Hence, the earning from the operation at period $t$ turns to be the entrepreneurial net worth for next period $t+1$. The propagation of the two channels of the financial frictions emerges from equation (5.10) and (5.12). The financial accelerator mechanism as in BGG is implied by that a reduction in the capital price deteriorates the entrepreneurial balance sheet as the external finance premium is increased by a decline in the net worth, which raises the cost of external funding. Then the capital demand is reduced due to a higher cost of borrowing, and the investment and the output are further decreased. The mechanism of the debt deflation channel is that the repayment on the loan agreed in real terms depends on the ex-post real interest rate when the debt contract is signed in terms of the nominal terms. Hence, an unpredicted rise in inflation decreases the real cost of debt and, in turn, increases the entrepreneurs' net worth.

The entrepreneurs who are bankrupt at period $t$ consume their remaining resources in the amount of:

$$C_t^e = (1 - \kappa)V_t. \tag{5.13}$$

Thus, the aggregate amount of the net worth from the departed entrepreneurs should be eliminated from this market.

Generally, the frictions in the financial market can magnify the impacts of a certain type of disturbances on the output through the net worth evolution of the entrepreneurs. The current entrepreneurial net worth is the sum of the previous net worth, the capital rental and the current value of the capital stock minus the obligations of the previous loans. Consequently, when the rate of return or current price of the capital is disturbed by a certain shock, the investment demand of the firms could be impacted by the changes in the net worth. Besides, when the aggregate price level is reduced (or raised) by a shock, the increase (or decrease) in the real value of the debt contracts could also lead to the fluctuations in the net worth, which implies the transmission of debt deflation effect. Therefore, when a shock impacts the rate of return on capital or the aggregate price level, the final effect of this shock on the output level is magnified by the presence of the financial frictions.

#### 5.2.3.2 Production firms

The entrepreneurs also serve as the production firms to produce intermediate goods in each sector. The domestic intermediate goods firms produce non-traded goods for the home market by adopting the capital stock and hiring labour. The exported intermediate goods firms transform the domestic goods into differentiated exported goods by brand naming and sell them to the world market. While the imported intermediated goods firms purchase the homogenous goods in the world market and convert them into differentiated goods sold to the final goods producers in the home country.

As the domestic intermediate goods firms, the output of the domestic firm $i$ is yielded by following a Cobb-Douglas production function that in a similar set with Leeper et al. (2010) as follows:

$$Y_{i,t} = A_t \left(K_{i,t}^{p}\right)^{\alpha_1} L_{i,t}^{1-\alpha_1} \left(K_t^{g}\right)^{\alpha_2}, \tag{5.14}$$

where $K_t^g$ is the stock of the government capital; $A_t$ is the exogenous technology shock which is identical across the sector of the domestic producers and follows the AR(1) process as:

$$\hat{A}_t = \rho^A \hat{A}_{t-1} + \sigma_t^A.$$

Assume $R_t^k$ and $W_t$ denotes the rate of return on capital and wage rate. By solving the cost minimization problem of the firm $i$, it yields that

$$\frac{R_t^k}{W_t} = \frac{\alpha_1}{1-\alpha_1} \frac{L_{i,t}}{K_{i,t}^{p}}. \tag{5.15}$$

The marginal cost is then generated as follows:

$$MC_t = \frac{1}{A_t \left(K_t^{g}\right)^{\alpha_2}} \left(\frac{W_t}{1-\alpha_1}\right)^{1-\alpha_1} \left(\frac{R_t^k}{\alpha_1}\right)^{\alpha_1}. \tag{5.16}$$

The price setting of the intermediate firms in the domestic, imported and exported sectors follows the mechanism introduced in Calvo (1983) which is in line with the set-ups presented in the baseline model. The details of the process of solving the firm's profit maximization problem for the re-optimal prices in each sector refer to the previous chapter.

The evolution of the aggregate price index $P_t^d$ for the domestic firms is produced as:

$$P_t^d = \left[\xi_d\left(P_{t-1}^d\left(\pi_{t-1}^d\right)^{\kappa_d}\right)^{1-v} + (1-\xi_d)\left(P_t^{d,new}\right)^{1-v}\right]^{\frac{1}{1-v}}, \tag{5.17}$$

where $1-\xi^d$ denotes the probability of receiving the Calvo price-reoptimized signal; $\pi_{t-1}^d = \frac{P_t^d}{P_{t-1}^d}$ is the domestic inflation; $\kappa_d$ is the price indexation parameter as the prices are set to $P_{t+1}^d = \left(\pi_t^d\right)^{\kappa_d} P_t^d$ when the firms don't receive the Calvo signal.

Analogously, the relationship described in equation (5.17) also holds for the aggregate price index $P_t^x$ that is set for selling the differentiated exported intermediate goods abroad as:

$$P_t^x = [\xi_x(P_{t-1}^x(\pi_{t-1}^x)^{\kappa_x})^{1-v} + (1-\xi_x)(P_t^{x,new})^{1-v}]^{\frac{1}{1-v}}. \tag{5.18}$$

Lastly, the solution of the decision problem of setting the aggregate price index $P_t^m$ for the imported goods is described as follows:

$$P_t^m = [\xi_m(P_{t-1}^m(\pi_{t-1}^m)^{\kappa_m})^{1-v} + (1-\xi_m)(P_t^{m,new})^{1-v}]^{\frac{1}{1-v}}. \tag{5.19}$$

## 5.3 Estimation

### 5.3.1 Data description

The data set is built by providing two extra observations of external finance premium and net worth in addition to the data of the baseline model to feature prominently the financial frictions presented in this chapter. The two additional observations are quarterly U.K. time-series data during the period 1989Q2 to 2017Q2 obtained from DataStream displayed in Table 5.1.

Table 5.1 Data description

| Variable | Notation | Source | Description |
|---|---|---|---|
| External finance premium | $EP$ | Reuters, DataStream | Difference between primary banking lending rate and bank official rate |
| Entrepreneur net worth | $NW$ | Reuters, DataStream | FTSE all share index, divided by the GDP deflator |

In line with the baseline model, the data are detrended by using the one-sided HP filter to eliminate the low-frequency components from the time series processes (Guerrieri and Iacoviello, 2017).

### 5.3.2 Calibration

Most calibrated parameters have the same values as in the baseline model of Chapter 4. Due to the modifications in households and production sectors, this chapter requires two additional calibrated parameters.

Table 5.2 Calibrated parameters

| Parameters | Definition | Value |
|---|---|---|
| $\psi$ | Labour supply elasticity | 1 |
| $\kappa$ | Survival rate of entrepreneurs | 0.99 |

The labour supply elasticity, $\psi$ ,is calibrated to 1 based on Christiano, Eichenbaum and Evans (2005). The entrepreneurs' survival rate, $\kappa$ ,is fixed at 0.99, i.e. an average surviving duration of entrepreneurs lasts over six years, following Le et al. (2012). This value is also obtained based on Christiano et al. (2010) in which the value is fairly close to 0.9728 in BGG (1999).

### 5.3.3 Prior distributions

The extra prior distributions of the estimated parameters are summarized in Table 5.3, and the remaining distributions are referred to Chapter 4. The prior distributions are broadly related to Smets and Wouters (2003, 2007), Adolfson et al. (2007) and Bhattarai and Trzeciakiewicz (2017), which is discussed in the baseline model. While regarding financial frictions, the setup is suggested by BGG (1999).

For the parameters whose values are restricted between 0 and 1, their prior distributions are assumed to be a standardized beta distribution. Accordingly, this type of distribution is applied to the habit information parameter $h$, the Calvo stickiness parameters $\xi$, the indexation parameters $\kappa$, interest rate smoothing $\gamma^r$ and shock persistence $\rho$.

The parameters, considering the standard deviations of the structural shock $\sigma$, are estimated by following an inverse gamma distribution, since they are restrained to be greater than zero.

Lastly, the prior distributions for the rest parameters, for instance, the adjustment cost parameter $\phi$, elasticity of financial premium $\omega$ and the policy responses $\gamma$ are set following nominal distributions.

Table 5.3 Prior distribution

| Parameters | Prior distribution | | |
|---|---|---|---|
| | type | mean | std. dev./df |
| Habit persistence $h$ | beta | 0.70 | 0.10 |
| Elasticity of financial premium $\omega$ | normal | 0.05 | 0.02 |

### 5.3.4 Posterior distributions

The estimation results are reported in Table 5.4. The first column of the posterior distribution exhibits the mode values of the estimated parameters generated by maximizing the posterior distribution of the model. The values of mean, the 5 and 95 per cent percentiles of the distribution are presented in the remaining three columns which are derived by adopting a Metropolis-Hastings algorithm rely on a 50,000-draw Markov chain.

Table 5.4 Posterior distributions

| Parameter | Posterior distribution | | | |
|---|---|---|---|---|
| | mode | mean | 5% | 95% |
| Investment adj. cost $\phi$ | 4.5136 | 4.5878 | 4.4838 | 4.7271 |
| Habit formation $h$ | 0.7590 | 0.7607 | 0.7553 | 0.7659 |
| Elasticity of premium w.r.t. leverage $\chi$ | 0.0406 | 0.0404 | 0.0373 | 0.0429 |
| Calvo domestic prices $\lambda^d$ | 0.7772 | 0.7775 | 0.7734 | 0.7840 |
| Calvo export prices $\lambda^x$ | 0.7657 | 0.7658 | 0.7617 | 0.7717 |
| Calvo import prices $\lambda^m$ | 0.7753 | 0.7776 | 0.7737 | 0.7828 |
| Indexation domestic prices $\kappa^d$ | 0.5791 | 0.5849 | 0.5780 | 0.5903 |
| Indexation export prices $\kappa^x$ | 0.5674 | 0.5662 | 0.5607 | 0.5754 |
| Indexation import prices $\kappa^m$ | 0.5701 | 0.5774 | 0.5697 | 0.5832 |
| Con. tax resp. to debt $\gamma^{c,b}$ | 0.1975 | 0.1866 | 0.1751 | 0.1988 |
| Con. tax resp. to GDP $\gamma^{c,y}$ | 0.4889 | 0.4854 | 0.4752 | 0.4979 |
| Cap. tax resp. to debt $\gamma^{k,b}$ | 0.2337 | 0.2349 | 0.2179 | 0.2486 |
| Cap. tax resp. to GDP $\gamma^{k,y}$ | 0.6998 | 0.7131 | 0.6955 | 0.7325 |
| Lab. Tax resp. to debt $\gamma^{w,b}$ | 0.2724 | 0.2767 | 0.2667 | 0.2895 |
| Lab. Tax resp. to GDP $\gamma^{w,y}$ | 0.3257 | 0.3328 | 0.3238 | 0.3440 |
| Trans. Resp. to debt $\gamma^{tr,b}$ | 0.2081 | 0.2183 | 0.2045 | 0.2369 |
| Trans. Resp. to GDP $\gamma^{tr,y}$ | 0.3135 | 0.3132 | 0.3006 | 0.3249 |
| Gov. spend. Resp. to debt $\gamma^{g,b}$ | 0.2781 | 0.2565 | 0.2296 | 0.2842 |
| Gov. spend. Resp. to GDP $\gamma^{g,y}$ | 0.5102 | 0.5260 | 0.5006 | 0.5542 |

| | | | | |
|---|---|---|---|---|
| Gov. inv. Resp. to debt $\gamma^{ig,b}$ | 0.2534 | 0.2939 | 0.2500 | 0.3312 |
| Gov. inv. Resp. to GDP $\gamma^{ig,y}$ | 0.9521 | 0.9521 | 0.9482 | 0.9561 |
| AR(1) cons. tax $\rho^{c}$ | 0.8026 | 0.8032 | 0.7964 | 0.8104 |
| AR(1) cap. tax $\rho^{k}$ | 0.8502 | 0.8489 | 0.8433 | 0.8544 |
| AR(1) lab. tax $\rho^{w}$ | 0.8050 | 0.8052 | 0.8006 | 0.8086 |
| AR(1) transfers $\rho^{tra}$ | 0.6747 | 0.6735 | 0.6499 | 0.6860 |
| AR(1) gov. spend. $\rho^{g}$ | 0.9088 | 0.9088 | 0.9044 | 0.9144 |
| AR(1) gov. inv. $\rho^{ig}$ | 0.4710 | 0.4700 | 0.4608 | 0.4846 |
| Interest rate smoothing $\gamma^{r}$ | 0.4479 | 0.4465 | 0.4332 | 0.4528 |
| Inflation response $\gamma^{\pi}$ | 1.6676 | 1.6730 | 1.6616 | 1.6932 |
| Output response $\gamma^{y}$ | 0.0982 | 0.0995 | 0.0968 | 0.1034 |
| AR(1) tfp. $\rho^{A}$ | 0.8065 | 0.8065 | 0.8025 | 0.8111 |
| AR(1) preference $\rho^{pre}$ | 0.8607 | 0.8571 | 0.8508 | 0.8614 |
| AR(1) private inv. $\rho^{ip}$ | 0.8641 | 0.8662 | 0.8613 | 0.8709 |
| AR(1) monetary policy $\rho^{tr}$ | 0.4250 | 0.4192 | 0.3985 | 0.4380 |
| AR(1) bond premium $\rho^{\varepsilon}$ | 0.7816 | 0.7792 | 0.7747 | 0.7841 |
| AR(1) foreign interest rate $\rho^{Rf}$ | 0.8471 | 0.8489 | 0.8419 | 0.8602 |
| AR(1) foreign inflation $\rho^{\pi f}$ | 0.8515 | 0.8517 | 0.8494 | 0.8543 |
| AR(1) foreign consumption $\rho^{Zf}$ | 0.8503 | 0.8484 | 0.8404 | 0.8564 |
| AR(1) external finance premium $\rho^{ep}$ | 0.8751 | 0.8789 | 0.8742 | 0.8846 |
| AR(1) net worth $\rho^{nw}$ | 0.8838 | 0.8860 | 0.8755 | 0.8920 |
| s.d. tfp. shock $\sigma^{A}$ | 0.5833 | 0.5453 | 0.4810 | 0.5837 |
| s.d. preference shock $\sigma^{pre}$ | 0.0250 | 0.0258 | 0.0247 | 0.0268 |
| s.d. private inv. shock $\sigma^{ip}$ | 0.1391 | 0.1408 | 0.1227 | 0.1683 |
| s.d. monetary policy shock $\sigma^{tr}$ | 0.0113 | 0.0118 | 0.0097 | 0.0133 |
| s.d. bond premium shock $\sigma^{\varepsilon}$ | 0.0114 | 0.0122 | 0.0110 | 0.0144 |
| s.d. foreign interest rate shock $\sigma^{Rf}$ | 0.0217 | 0.0215 | 0.0183 | 0.0239 |
| s.d. foreign inflation shock $\sigma^{\pi f}$ | 0.0144 | 0.0147 | 0.0130 | 0.0172 |
| s.d. foreign consumption shock $\sigma^{Zf}$ | 0.0200 | 0.0197 | 0.0185 | 0.0221 |
| s.d. external finance prem. shock $\sigma^{ep}$ | 0.0143 | 0.0140 | 0.0129 | 0.0158 |
| s.d. net worth shock $\sigma^{nw}$ | 0.0169 | 0.0169 | 0.0156 | 0.0184 |
| s.d. con. tax shock $\sigma^{c}$ | 0.0461 | 0.0454 | 0.0402 | 0.0502 |
| s.d. cap. tax shock $\sigma^{k}$ | 0.0108 | 0.0112 | 0.0104 | 0.0123 |
| s.d. lab. tax shock $\sigma^{w}$ | 0.0085 | 0.0084 | 0.0076 | 0.0096 |
| s.d. transfers shock $\sigma^{tra}$ | 0.0384 | 0.0403 | 0.0381 | 0.0426 |
| s.d. gov. spend. shock $\sigma^{g}$ | 0.1315 | 0.1306 | 0.1276 | 0.1332 |
| s.d. gov. inv. shock $\sigma^{ig}$ | 0.0634 | 0.0564 | 0.0465 | 0.0665 |
| Marginal likelihood | 4803.1953 | | | |

First, the estimate of the habit information, $h$ ,is estimated at 0.76, which implies a moderate degree of habit formation for consumers and is in line with the value of

0.71 estimated by Smets and Wouters (2007) and 0.69 by Adolfson et al. (2007). Second, the estimated elasticity of the financial premium with respect to the leverage $\omega$ is evaluated at 0.04, which is slightly lower than a standard calibrated value of 0.05 in BGG (1999), Bernanke and Gertler (2000), but it is in consistence with Christensen and Dib (2008) which report an estimate value of 0.042 for a sticky-price DSGE model. Third, the results of the price stickiness parameters ($\xi^d, \xi^x$ and $\xi^m$) in domestic, exported and imported sectors are around 0.77 which are not statistically different from the prior means and suggest that an average duration for producers to re-optimize their price is about 4.3 quarters. Likewise, the estimated indexation parameters in price setting ($\kappa^d, \kappa^x$ and $\kappa^m$) are about 0.57 which are broadly comparable with the prior setting in literature, suggesting that the inflation persistences in the domestic, exported and imported sectors are moderate.

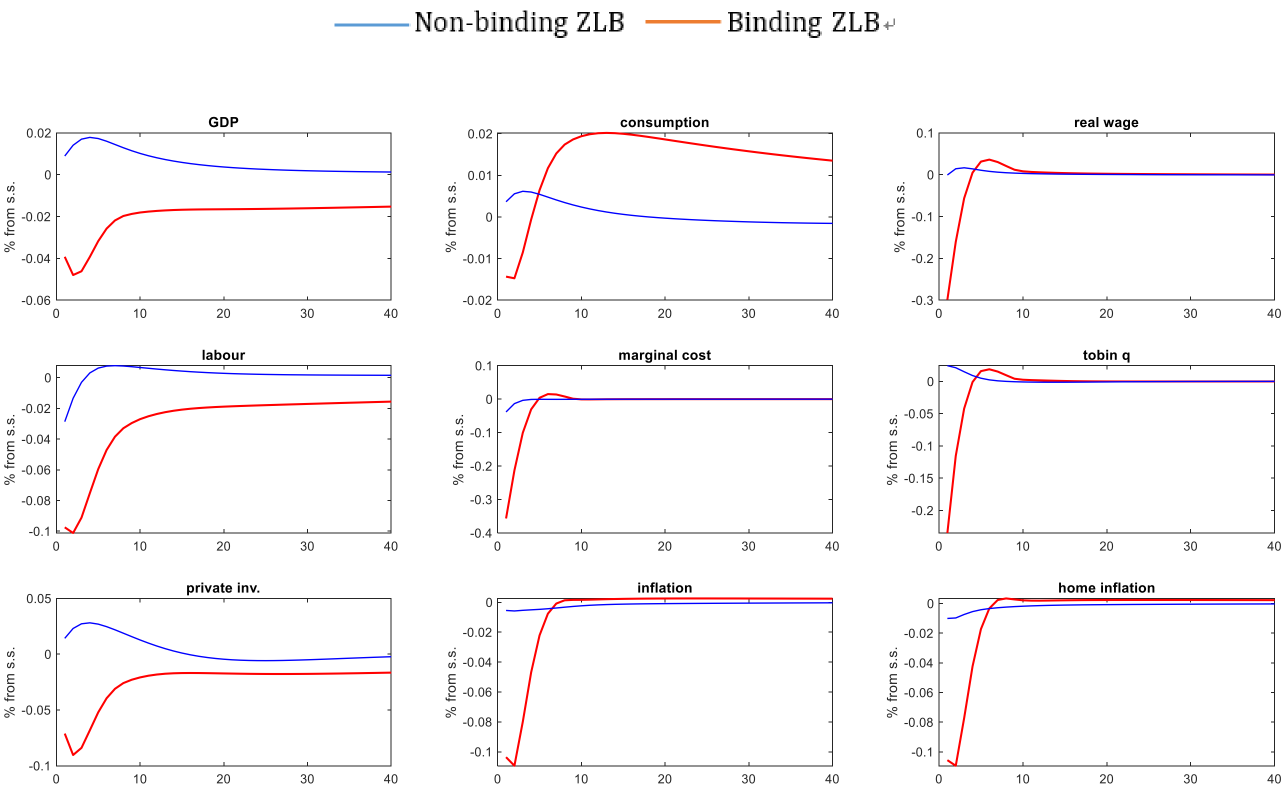
Fourth, turning to the specification of the government performance, the monetary policy coefficients of inflation response ($\gamma^\pi = 1.67$) and output response ($\gamma^y = 0.10$) are broadly close to the estimates in Bhattarai and Trzeciakiewicz (2017). Regard to the estimates of the fiscal policy, the fluctuations in the government debt level are generally governed by the fiscal rules of the government spending, government investment and labour income tax, while the capital income tax and government investment contribute more in managing the output variations, which is in consistence with the conclusions in the previous chapter. Additionally, the responses to the GDP deviations are relatively stronger than the responses to the government debt for all the fiscal rules.

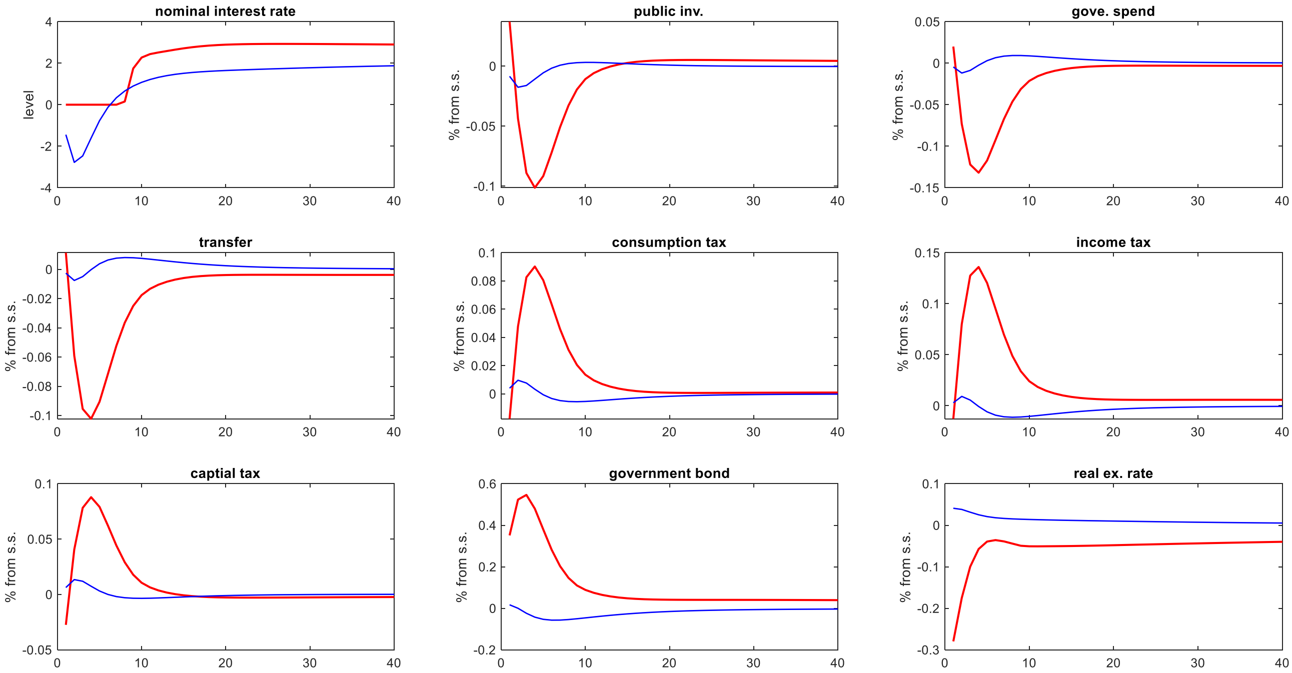
Fifth, based on the estimation results of the structural shock, it is observed that none of the autoregressive coefficients for these shocks is estimated greater than 0.90, which implies that there isn't any extremely persistent process within the structural shocks.

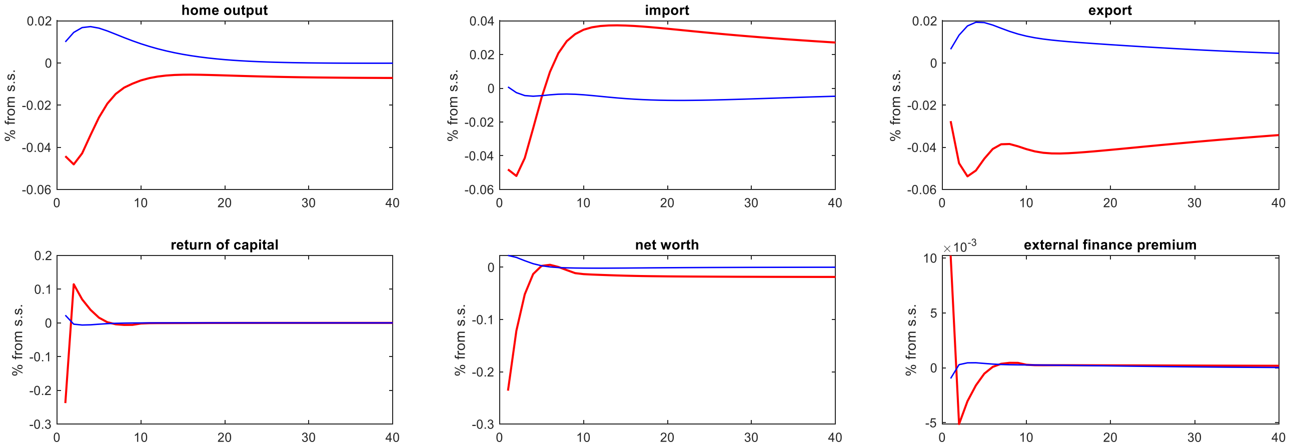
## 5.4 Impulse response functions

To illustrate how the presence of the zero lower bound affects the model with the financial frictions, this section discusses the impulse response functions for fundamental elements of this model economy to a temporary positive 10% productivity shock under two interest rate regimes.

Figure 5.4 Impulse responses to a productivity shock (with financial frictions)






Without the zero lower bound, output, consumption and private investment rise with a positive technology shock. While the government, as an automatic stabilizer, cuts the expenditures on public consumption, public investment and transfers, and raises the distortionary tax rates for consumption, labour income and capital income due to the sufficient demand of the economy. Exports rise due to the depreciation for the domestic currency, and imports decline.

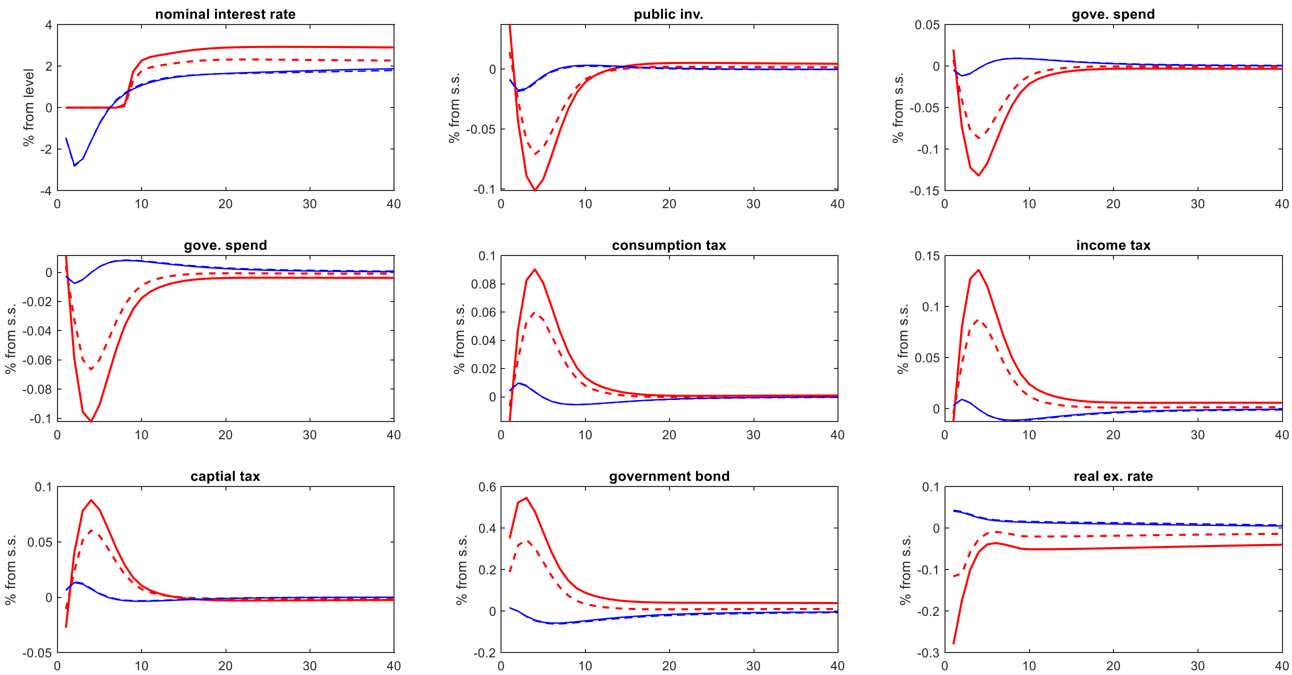
The demand for the private capital stock expands and, its price (Tobin's Q) is derived up improving the entrepreneurs' balance sheet. Even though the net worth is lowered by the deflation, this negative effect on the net worth is over-compensated by the positive effect generated by the relative higher return on capital. Since the response of the monetary authority to the fluctuations of the inflation is estimated more aggressive ( $\gamma^{\pi} = 1.67$ ) than the response to the changes in the output ($\gamma^{y} = 0.10$), the decrease in the inflation is less pronounced. Thus, with the higher net worth, the external finance premium decreases causing investment and aggregate demand to increase, which stimulates further expansionary effect through the financial accelerator mechanism.

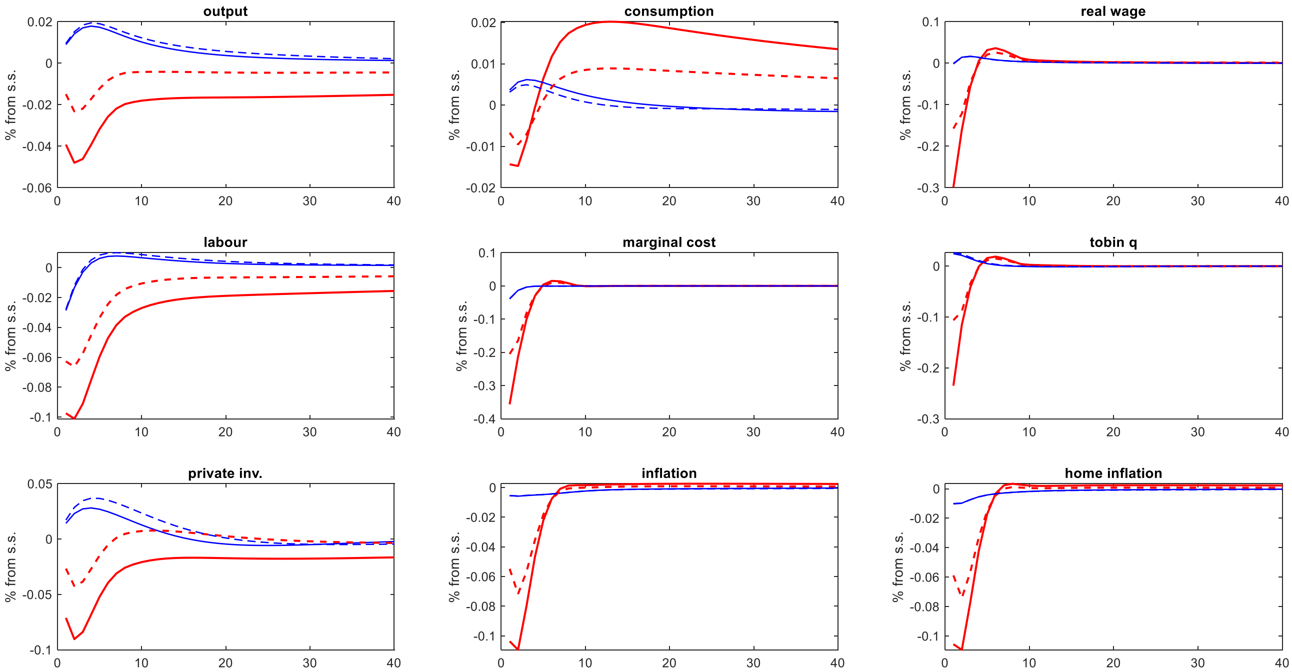
However, with the zero lower bound, the whole economy falls into recession accompanying the slumps in the real GDP, consumption and investment following a productivity shock. In the liquidity trap, the monetary authority cannot adjust the nominal interest rate to offset the deflationary effect by further reducing the rate. Hence, the severer the deflation is in the liquidity trap, the more the real interest rate increases, compared to the situation in the non-binding ZLB regime. As for entrepreneurs, the real costs for repaying the existing debt turns to be more expensive, which induces a negative impact through the debt deflation channel on their balance sheet. Meanwhile, the lack of capital demand leads to the decline in capital price, which lowers the firms' collateral value and then derives up their leverage through the financial accelerator channel. Therefore, the collapse in the net worth is amplified by the adverse effects from both the debt deflation effect and financial accelerator effect. The external finance premium is pushed up sharply, which deteriorates the investment further.

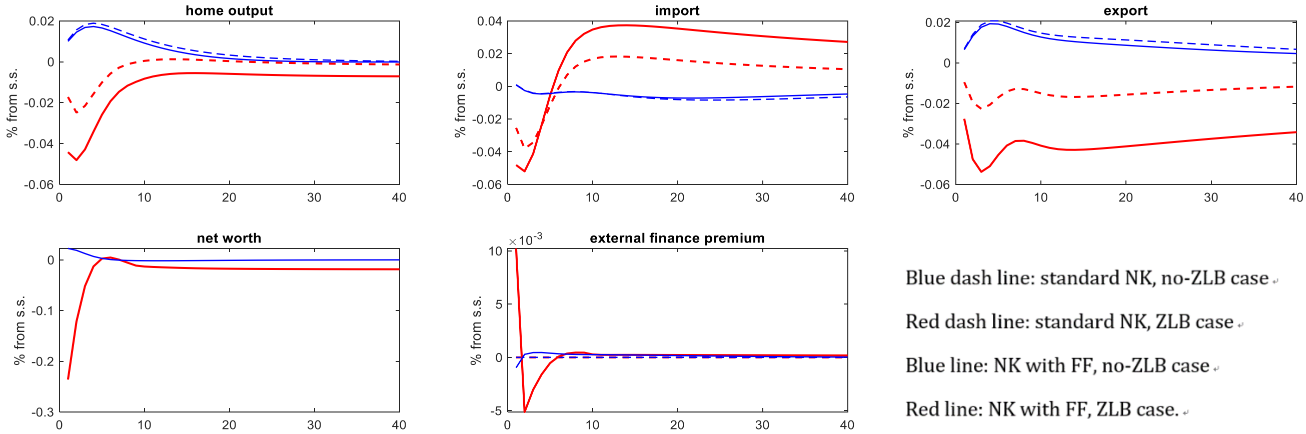
Regarding the public sector, the fiscal stimulus package is installed with the increasing path of government expenditures and the decreasing path of distortionary taxes to improve the economy in the recession. However, these stimulating measures are quickly inverted by the government to manage the continued increases in the public deficit. The government reverses the measures from the fiscal expansion to fiscal austerity by raising the distortionary tax rates and cutting down the public expenditures to balance the government budgets and keep the growth of the real debt under control.

In the following section, it concentrates on explaining the significant role of financial frictions by plotting the impulse responses of the key macroeconomic variables to a temporary technology shock in two DSGE models under two different interest rate regimes (see Figure 5.5): (1) the NK model with financial frictions under binding ZLB regime and non-binding ZLB regime; (2) the standard NK model without financial frictions under binding ZLB regime and non-binding ZLB regime.

Figure 5.5 Impulse responses to a productivity shock (with and without financial frictions)






There are two effects generated by the presence of financial frictions, the financial accelerator effect and debt-deflation effect. However, the economic outcome depends on which effect is dominant, and this also depends on whether the economy is under the binding ZLB regime.

**(1) Comparing the impulse responses of the models with and without the financial frictions in a non-binding ZLB regime.**

Following a temporary positive technology shock, the impact of the financial frictions on the output is limited under non-binding ZLB regime. For the components of the aggregate demand, there is a marginal amplified effect on the consumption but a dampening effect on the raise of the investment, since the debt deflation effect weakens the financial accelerator effect. In the non-binding ZLB regime, the monetary authority cuts the nominal interest rate to protect the decline in inflation from becoming more pronounced, hence, the negative effect generated by the debt-deflation mechanism on the net worth and then on the investment demand of the firm is offset by the expansionary monetary policy to a certain extent. Nevertheless, the growth of the investment is smaller than its response to the shock in an NK-noFF model. Thus, the presence of the financial frictions dampens the rise of the output and investment through the debt deflation channel when the model is hit by a positive technology shock. This conclusion is consistent with Iacoviello (2005) and Christensen and Dib (2008) who state that the setting of the nominal debt contracts dampens the effects of the supply shock. Therefore, the increase in the investment is less significant in the model considering the financial frictions, since the positive effect produced by the financial accelerator mechanism is weakened by the negative effect from the debt deflation channel, implying that the investment could not be fully stimulus by the expansion in the demand of the capital stock in a model with the financial frictions existing.

**(2) Comparing the impulse responses of the models with and without the financial frictions in a binding ZLB regime.**

When the nominal interest rate is constrained by the zero lower bound, the whole economy falls into a recession with collapses in real GDP, private consumption and investment after the unexpected shock hitting the model. While the decline in the investment is amplified to a large degree by the financial accelerator in a binding ZLB regime, which implies that the reduction in the output is severer than that in the model with the perfect credit market in the liquidity trap. This phenomenon can be explained by that the performance of the investment in the model with the financial frictions is impacted by two mechanisms jointly. As the return of capital drops, the demand for capital decreases, which puts downward pressure in both the price of capital and investment demand. The former reduces the value of firms' collateral that discourages the investment further, which illustrates that the financial accelerator channel generates a negative effect on the investment. Meanwhile, this negative effect is reinforced through the debt deflation channel. When the nominal interest rate stays at zero for a few periods, and the central banks could not react to the deflation by cutting the interest rate further, the real interest rate increases immediately. Thus, the real cost of funds becomes more expensive, which leads to a deterioration of the firms' leverage ratio. Then the increasing external finance premium reduces the entrepreneurial net worth in turn, which pushes down the investment demand further. Therefore, with the negative effects yielded from both the financial accelerator channel and the debt deflation channel, there is a significant collapse in the investment in the model with financial frictions, and the decline becomes severer under the binding ZLB regime.

## 5.5 Fiscal multipliers

The effects of the fiscal policy in a model with financial frictions under the non-binding ZLB regime and binding ZLB regime are summarized by the fiscal multipliers. The present-value multipliers in terms of GDP, private consumption and investment are calculated following the algorithm of Mountford and Uhlig (2009). The present value of the fiscal multiplier over a $k$-period horizon is described as follows:

$$Present\ \ Value\ \ Multiplier\ \ (k) = \frac{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta\tilde{Y}_{t+j}}{\sum_{j=0}^{k}\left(\prod_{i=0}^{j} R_{t+i}^{-1}\right)\Delta\tilde{X}_{t+j}},$$

where $\tilde{Y} =$ {GDP, consumption, private investment} and $\tilde{X} =$ {government spending, government investment, transfers, consumption tax, labour income tax, capital tax} are the level changes in essential macroeconomic variables and fiscal measures relative to their steady states.

The computed cumulative fiscal multipliers are presented in Table 5.5 for the non-binding ZLB and binding ZLB regime, respectively. By comparing these two groups of multipliers, it is illustrated that the fiscal policy plays a different role in various interest rate regimes.

Table 5.5 Fiscal multipliers[18]

| | without ZLB | | | ZLB[19] | | |
|---|---|---|---|---|---|---|
| | Y | C | I | Y | C | I |
| *Gov. spend.* | | | | | | |
| Impact | 0.535 | -0.117 | -0.052 | 1.839 | 0.190 | 0.469 |
| 4 quarters | 0.378 | -0.253 | -0.111 | 1.756 | 0.031 | 0.495 |
| 12 quarters | 0.195 | -0.489 | -0.175 | 1.587 | -0.478 | 0.369 |
| 20 quarters | 0.144 | -0.639 | -0.161 | 1.699 | -0.833 | 0.356 |
| *Gov. inv.* | | | | | | |
| Impact | 0.624 | -0.047 | -0.016 | 1.104 | 0.069 | 0.176 |
| 4 quarters | 0.579 | -0.170 | -0.031 | 1.324 | 0.039 | 0.455 |
| 12 quarters | 0.906 | -0.550 | 0.097 | 1.552 | -0.659 | 0.948 |
| 20 quarters | 1.409 | -0.871 | 0.315 | 1.774 | -1.454 | 1.309 |
| *Trans.* | | | | | | |
| Impact | 0.035 | -0.002 | -0.015 | 0.350 | 0.092 | 0.144 |
| 4 quarters | 0.004 | -0.010 | -0.053 | 0.538 | 0.112 | 0.247 |
| 12 quarters | -0.206 | 0.020 | -0.186 | 0.752 | -0.053 | 0.309 |

[18] A positive value of the multiplier means an increase in fiscal rules increases the output, consumption and private investment, while a negative multiplier means an increase in the fiscal rules has the negative impacts in contrast.

[19] The results are obtained by holding a zero nominal interest rate by ten quarters.

| | | | | | | |
|---|---|---|---|---|---|---|
| 20 quarters | -0.702 | 0.130 | -0.264 | 0.991 | -0.225 | 0.338 |
| *Cons. tax* | | | | | | |
| Impact | -0.004 | -0.016 | 0.004 | -0.155 | -0.054 | -0.058 |
| 4 quarters | 0.007 | -0.033 | 0.012 | -0.215 | -0.067 | -0.080 |
| 12 quarters | 0.034 | -0.051 | 0.029 | -0.270 | -0.024 | -0.084 |
| 20 quarters | 0.049 | -0.051 | 0.032 | -0.345 | 0.033 | -0.094 |
| *Cap. tax* | | | | | | |
| Impact | -0.003 | 0.002 | -0.003 | -0.025 | -0.003 | -0.012 |
| 4 quarters | -0.004 | 0.007 | -0.007 | -0.037 | 0.001 | -0.021 |
| 12 quarters | -0.010 | 0.016 | -0.017 | -0.052 | 0.018 | -0.031 |
| 20 quarters | -0.020 | 0.023 | -0.024 | -0.068 | 0.032 | -0.039 |
| *Lab. tax* | | | | | | |
| Impact | -0.029 | -0.011 | -0.010 | 0.068 | 0.014 | 0.029 |
| 4 quarters | -0.053 | -0.026 | -0.024 | 0.191 | 0.011 | 0.073 |
| 12 quarters | -0.062 | -0.051 | -0.029 | 0.485 | -0.143 | 0.150 |
| 20 quarters | -0.048 | -0.072 | -0.010 | 0.798 | -0.349 | 0.250 |

Under the non-binding ZLB regime, increasing the government expenditures including government spending, government investment and transfers, and cutting the distortionary tax rates are expansionary, which is consistent with the conclusion in the previous chapter. Under the binding ZLB regime, the expansionary effects of rising the government expenditures and cutting the consumption tax rate turn to be more aggressive; in contrast, increasing the labour income tax becomes expansionary instead of cutting the rate, which is also the main finding in the previous chapter when the nominal interest rate is at zero. Besides, there is a noticeable distinction between the multiplier of the capital income tax in this and previous chapters. In Chapter 4, rising the capital income tax has a similar effect with increasing the labour income tax as both of them are expansionary under the binding ZLB regime but contractionary under the non-binding ZLB regime. However, in this chapter with the presence of the financial frictions, when the nominal interest rate is bound at zero, the growth in the capital income tax derives a contractionary effect even to a greater degree compared to the effect under the non-binding interest rate. It can be explained by that the capital stock is assumed being owned by the firms rather than the households in the model setting of this chapter, thus, the capital income tax is paid by the firms. In other words, for a given level of expected return on capital $R_{t+1}^{k}$, the rise in the capital income tax rate leads to the decline in the demand for investment goods of the firms. In equilibrium, this effect induces the reduction in private investment, output and inflation. If other things equal, the effect of raising the capital income tax can be potentially magnified by a binding interest rate. Additionally, the increase in the capital income tax rate born by the firms can seem as isomorphic as a more expensive cost of finance in the model with the financial frictions.

The exercise reported in Table 5.6 implies that with the presence of the financial frictions, the effects of the fiscal rules are magnified regardless of the interest rate regimes. Besides, the fiscal measures are more effective reinforced by the financial frictions when the nominal interest rate stays at zero.

Table 5.6 Fiscal multipliers (with and without financial frictions)

| Non-binding ZLB regime | | | | | | |
|---|---|---|---|---|---|---|
| Fiscal multipliers | $G$ | $I^g$ | $TR$ | $\tau^c$ | $\tau^k$ | $\tau^w$ |
| Model with f.f. | 0.535 | 0.624 | 0.035 | -0.004 | -0.003 | -0.029 |
| Model without f.f. | 0.498 | 0.615 | 0.033 | -0.003 | -0.001 | -0.026 |
| Binding ZLB regime | | | | | | |
| Fiscal multipliers | $G$ | $I^g$ | $TR$ | $\tau^c$ | $\tau^k$ | $\tau^w$ |
| Model with f.f. | 1.839 | 1.104 | 0.350 | -0.155 | -0.025 | 0.068 |
| Model without f.f. | 1.411 | 0.950 | 0.154 | -0.082 | -0.014 | 0.015 |

As for the former fact, except for cutting the labour income tax which is deflationary at zero interest rate but inflationary under the non-binding rate, rising the government expenditures and reducing the consumption and capital income taxes are inflationary irrespective of the interest rate regimes. As stated by Eggertsson and Krugman (2012) and Fernández-Villaverde (2010), inflation growth improves the firms' balance sheet when the debt contracts are signed in nominal terms. With a better financial position, the firms' net worth increases and then leading to the enhancement in the investment ability. This phenomenon is eventually worked by the debt deflation effect when the credit frictions present in the finance market.

The latter implies that the size of the fiscal multipliers is affected by the mutual effect between the financial frictions and zero lower bound. Typically, the contribution of the financial frictions is relatively greater to the size of fiscal multipliers. While the final effect of the financial accelerator mechanism is potentially affected by the response of the monetary authority to the fiscal disturbances. If the monetary authority raises the nominal interest rate to the inflationary effect generated by the shocks, the multiplier effect will be weakened by a more expensive credit, implying that the demand of investment is discouraged slightly by the debt deflation effect. On the contrary, if the economy falls into the liquidity trap for a few periods and the monetary authority could not react the inflationary effect by adjusting the nominal interest rate, the macroeconomic transmission of the fiscal shocks to the investment will be fully amplified. A few periods of cheaper credit encourage the firms to invest. The investment demand is stimulated both by the effects of the financial accelerator channel and the debt deflation channel. In conclusion, the final multiplier effect of the financial accelerator mechanism is partially offset by the debt deflation effect under the non-binding ZLB regime but magnified under the binding ZLB regime.

## 5.6 Concluding remarks

This chapter studies the role of financial frictions in affecting the effectiveness of the fiscal policy under different interest rate regimes and the interaction between the financial frictions and the liquidity trap on the macroeconomic transmission of the shocks.

The following findings are revealed by the experiments. The implications of fiscal rules are more effective when there are credit frictions in the financial market since the inflationary effect improves the firms' balance sheets, which strengthens their capability of investment. Moreover, the fiscal effectiveness is more noticeable under the binding ZLB regime, since the restricted monetary policy allows for the decline in the real interest rate to accommodate the fiscal measures, which in turn expands the private expenditures. Additionally, the mutual effects of the two channels of propagating the financial frictions are also discussed. Even though the debt-deflation channel plays a significant role in magnifying the fiscal measures, the financial accelerator channel is the main resource of generating the more effective fiscal measures in the model with an imperfect credit market.

However, this conclusion based on the Bayesian estimates may not be highly reliable, since this estimation method is criticized for its subjectivism due to its assumption of prior knowledge. Hence, it is worthy to further study the empirical performance of the macroeconomic model and improve the capacity of the model in fitting the data. This would be the essential work for the following Chapter 6 to further evaluate and re-estimate the model.

# Chapter 6 Evaluate and estimate the DSGE model by Indirect Inference method

## 6.1 Introduction

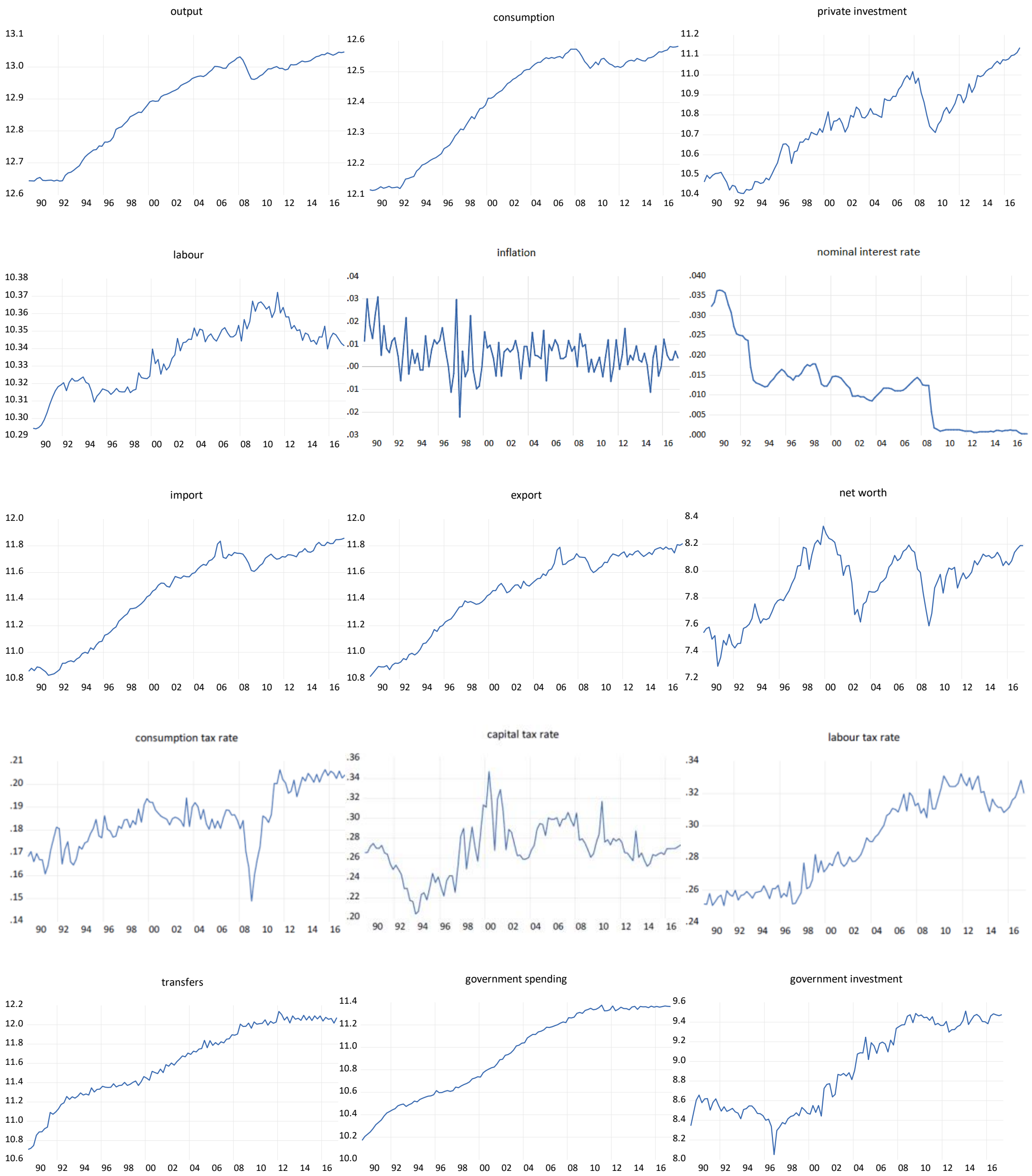
The model with financial friction and zero lower bound was estimated in Chapter 5 using the Bayesian estimation method, but this method doesn't test the fitness of the model against the data. In this chapter, the same model is re-estimated and tested using Indirect Inference method. The rationale for this alternative method was described in the Methodology chapter. Another extension in this chapter is to consider non-filtered data to avoid losing information due to arbitrary data filters. With the statistically robust Indirect Inference estimation results, this chapter could proceed to a further study of the policy implications and recommendations.

The structure of this chapter is as follows. Section 2 presents the data summary. Section 3 reports the estimation results and robust check. Section 4 discusses the policy implications. Lastly, Section 5 concludes.

## 6.2 Data summary

This chapter uses the same data set for the U.K. from 1989Q2 to 2017Q2, but instead of filtering it, unfiltered data is used (see Figure 6.1).

Figure 6.1 Actual data series




Nelson and Plosser (1982) argue that the majority of the macroeconomic time series data are non-stationary, since a great amount of time series contains unit roots and is found to be co-integrated. Regarding the existent researches, there are two types of non-stationary data: (1) the trend stationary process in which the mean trend is deterministic, implying that as long as the trend is eliminated the residual is stationary; (2) difference stationary process in which the mean trend is stochastic, and the stationary process can be derived by differencing the series.

Remarking by the Wickens (1982), there is a noteworthy implication by using the non-stationary data for modelling. It allows the researchers to identify the persistence of the shocks. With the existence of a stationary or trend stationary data, the memories of the endogenous variables only last a short time and the impacts of the shocks on the economy is temporary. After hitting by the shock, the variables will converge to their steady trends. Conversely, the memories of the endogenous variables are persistent and the impacts of the shocks are everlasting with the existence of a unit root process. The variables will not return to their previous path after a random disturbance. When the shock is permanent, levels of the variables shift way eternally, since the endogenous variables involving the identical balanced growth path (BGP) are transmitted by levels of the shock. The former case can be defined as the business cycle effect and the latter one is the effect of the long-run growth path.

The majority of the macro research use stationary filtered data for their analysis. One of the idea of this approaches is to study the short-term fluctuations around the trend and investigate when the disturbances might be stabilized or intensified by the macroeconomic policies. Within macroeconomic models, the stochastic dynamics of the variable is expressed in the form of the temporary deviation from its steady-state. Typically, researchers derive the non-stationary data by using linear detrending for deterministic trends or first differencing. However, the former is criticized for generating the data process improperly when including the stochastic trends, while the later may inappropriately amplify the high-frequency noise component in data. If the dynamics are eliminated or amplified within the range of certain frequency, the impact of a permanent shock on the stationary detrended data may be potentially non-negligible.

It is also popular to detrend the time series by using HP filter or Band Pass (BP) filter, which decompose the non-stationary data into the trend and cyclical components, so does the previous chapters in this thesis applying the one-side HP filter before the Bayesian estimation. However, the applications of the HP filter or the BP filter cause great controversy in macroeconomic research. Firstly, it may spuriously derive inexistence cycles in the detrended data, which changes the dynamic structure of the time series. Argued by Harvey and Jaeger (1993), the stochastic dynamics of a detrended data may systematically different from its non-stationary counterpart along the dimension of the original interest in the empirical research. Secondly, it can produce marked distortions in the main stylised facts of the model. As explained by Meenagh et al. (2019), the filter may transform the forward-looking properties of the model, which becomes seriously flawed in estimating a DSGE model in which both the expectations structure and the impulse responses are of importance to the interest usually. Furthermore, when applying the HP filter or the BP filter to the non-stationary data, it could not certainly recognize the difference between the trend-stationary and difference-stationary specifications (Cogley and Nason 1995, Murray 2003).

Accordingly, there are two substantial reasons for applying the unfiltered non-stationary data in this chapter. First, the existence detrending techniques seem inappropriate and imprecise in processing the non-stationary data. Second, certain interactions between the variables of interests and the dynamic properties of the model may be distorted by the stationary data, which is not easy to uncover, which is also proved by Andrle (2008) who argues that the true business cycle dynamics may not be captured by models using the detrended data. Hence, the features of the non-stationarity and the stochastic trend remain in the data applied in this chapter.

The method of handling the non-stationary data follows Meenagh et al. (2019). When applying the Indirect Inference technique to evaluate the model in terms of the non-stationary data, the solution of the model described as a VECM or VARX model, after log-linearization. Once there are certain unobservable non-stationary variables, for example, the technology shock, the number of the endogenous variable will be greater than that of the co-integrated vectors, reflecting that the non-stationary residual will present in one or more of the long-term structural equations. As all the parameters are known from the estimates and calibration, the residuals can be constructed from the data. When these residuals are treated as observable variables, the number of co-integrated relations can be equal with that of endogenous variables. In other words, by assuming the endogenous variables are co-integrated with certain non-stationary exogenous variables, the non-stationary data can be handled by applying the mechanism of Indirect Inference.

## 6.3 Estimation results and robust check

This section presents the process of estimation and robust check for the results. Section 6.3.1 reports the results of the Indirect Inference test of the model with Bayesian estimates found in Chapter 5 and the model estimated by Indirect Inference itself. Section 6.3.2 presents the estimation results of the Indirect Inference method. Section 6.3.3 illustrates the properties of this model. The power of Indirect Inference test is examined by designing the Monte Carlo experiments in Section 6.3.4. Section 6.3.5 analyses the performance of the main macroeconomic variables by accessing the impulse response functions to several structural shocks. Section 6.3.6 reports the variance and historical decomposition.

### 6.3.1 Indirect Inference test results

Recall the section 3.2.4, the choice of the auxiliary model in this thesis is a $VARX(1)$ which is the approximation form of the reduced DSGE model. The $VARX(1)$ performs as the unrestricted auxiliary model throughout the process of the Indirect

Inference test and estimation in this chapter, and also describes the essential features of certain key macroeconomic variables in this DSGE model.

The main interests of the model are the aggregate output, $y_t$, the nominal interest rate, $r_t$, and inflation, $\pi_t$. The reasons for choosing the key variables are fairly straightforward. Firstly, the output is one of the most important macroeconomic factors in explaining economic behaviours. Secondly, since this model incorporates the zero lower bound on the nominal interest rate, and compares the dynamics of the economy under different interest regimes, the behaviour of the nominal interest rate should be considered. Thirdly, the core idea of this thesis is to evaluate the effectiveness and implementations of the fiscal policy which mainly depends on the state of inflation of the economy, hence, inflation should be chosen.

Therefore, a $VARX(1)$ in terms of three key endogenous variables involved in the Direct Wald statistics is defined as:

$$\begin{bmatrix} y_t \\ r_t \\ \pi_t \end{bmatrix} = B \begin{bmatrix} y_{t-1} \\ r_{t-1} \\ \pi_{t-1} \end{bmatrix} + C \begin{bmatrix} A_{t-1} \\ T \\ const \end{bmatrix} + \begin{bmatrix} e_{yt} \\ e_{rt} \\ e_{pt} \end{bmatrix}, \tag{6.2}$$

where $B = \begin{bmatrix} b_{yy} & b_{yr} & b_{yp} \\ b_{ry} & b_{rr} & b_{rp} \\ b_{py} & b_{pr} & b_{pp} \end{bmatrix}$.

The VARX (1) presented above includes the lagged endogenous variables, the lagged productivity trend, $A_{t-1}$, which is the non-stationary shock in the model, time trend term, $T$, which denotes the deterministic trend in the observed data and simulations, and the constant term, $const$. The matrix $C$ is viewed as the driving factors of non-stationarity which explains the effect of the exogenous variables. The vector of the coefficient $\beta^s$ in equation (6.1) is required to contain all estimates in $B$ matrix and the variances of the fitted residuals generated from each set of simulations. The same group of coefficients constitute the vector $\beta^{\alpha}$ derived by the observed data:

$$\beta = [b_{yy}, b_{yr}, b_{yp}, b_{ry}, b_{rr}, b_{rp}, b_{py}, b_{pr}, b_{pp}, var(e_{yt}), var(e_{rt}) var(e_{pt})]'. \quad (6.3)$$

Accordingly, the Indirect Inference test is expected to check whether the model-implied VARX can derive the identical VARX based on observed data. More precisely, the test is to check whether the DSGE model can imitate the behaviour of the output, nominal interest rate and inflation jointly.

Following the mechanism of the Indirect Inference test discussed in Chapter 3, the test is applied regarding the Bayesian estimates, reported in Chapter 5, and also the Indirect Inference estimates. The test results are presented in Table 6.1. Note that this thesis applies the empirical estimate of its small sample distribution gained by bootstrap illustrated in Section 3.2, rather than using the asymptotic distribution of the Wald statistic, which implies that the estimated Wald statistic does not follow the Chi-square distribution as usual. Hence, the transformed Mahalanobis Distance can serve as a reference assessment. Additionally, the transformed Mahalanobis Distance is normalized as a statistic following t-distribution so that any p-value that is less than 5% would lead to the rejection of the model, of which the detailed explanation is discussed in Section 3.2.

Table 6.1 Wald test results

| Endogenous | Method of estimation | Wald statistic | P-value |
|---|---|---|---|
| $y_t, r_t, \pi_t$ | Bayesian estimation | 328.81 | 0.0000 |
| | Indirect Inference estimation | 0.0577 | 0.0735 |

Not surprisingly, the test results with the Bayesian estimates indicate a severe rejection on the null hypothesis, namely the Wald percentile of the joint distribution of $y_t$, $r_t$ and $\pi_t$ is 328.81 normalizing into the Mahalanobis

Distance statistic of 15.51, which implies that there is a significant difference between the performance described by the structural model and the dynamic behaviour of the actual data. This may cause by either the Bayesian estimated parameters are inappropriate or the structural DSGE model fails to explain the data behaviours. To exclude the latter reason, the structural parameters are re-estimated by Indirect Inference method to minimise the Wald statistic. With the most satisfactory parameters evaluated by the Indirect Inference method, if the structural model passes the test, it can be concluded that the improper values for certain parameters generated by the Bayesian estimation lead to the failure of the test rather than the rejection in the structural model itself. Fortunately, the test results are greatly improved by applying the Indirect Inference estimates. It shows that the p-value of the statistics is significantly greater than 5%, which means that the observed data lies in a 95% confidence interval implied by the sampling distribution of $y_t$, $r_t$ and $\pi_t$. In other words, the distributions of the three main macroeconomic variables in the auxiliary $VARX(1)$ model jointly pass the test. When applying the Indirect Inference estimates, the fitness of the overall performance of the structural model to the behaviour of the actual data is fairly acceptable.

### 6.3.2 Estimation results

As discussed in Methodology chapter, the core idea of the Indirect Inference estimation is to find out the most satisfactory parameters of the structural model that "the observed data and the simulated data look statistically the same from the vantage point of the chosen window" (Durlauf and Blume, 2008), or rather, minimizing the distance between these two sets of estimates of the auxiliary model based on the actual data and simulations with a given criterion. Commonly, the scores, impulse response function, or actual coefficients are chosen as the criterion. This section presents the values of the actual coefficients which act as the data descriptor. To calculate the minimum value of the Wald statistic, a powerfully searching algorithm, the random search, is introduced, which takes place over a wider range within the structural parameter state and to prevent the searching from being trapped in the local minima. That is to say, the results are going to describe almost all the possibilities based on a given feasible region with an increasing number of searching times. The estimation results are shown in Table 6.2.

Table 6.2 Estimates of the structural model with financial frictions

| Parameters | Calibration | 95% Confidence Interval | | Estimation |
|---|---|---|---|---|
| | | Upper | Lower | |
| Investment adj. cost $\phi$ | 4 | 6.7841 | 4.4596 | 6.5727 |
| Habit persistence $h$ | 0.7 | 0.4755 | 0.3435 | 0.3986 |
| Elasticity of financial premium $\omega$ | 0.04 | 0.0655 | 0.0523 | 0.0626 |
| Calvo domestic prices $\lambda^d$ | 0.75 | 0.3649 | 0.2134 | 0.3263 |
| Calvo export prices $\lambda^x$ | 0.75 | 0.9657 | 0.8334 | 0.9755 |
| Calvo import prices $\lambda^m$ | 0.75 | 0.4537 | 0.3014 | 0.3203 |
| Indexation domestic prices $\kappa^d$ | 0.5 | 0.4638 | 0.3113 | 0.3193 |
| Indexation export prices $\kappa^x$ | 0.5 | 0.3841 | 0.2325 | 0.2790 |
| Indexation import prices $\kappa^m$ | 0.5 | 0.2557 | 0.1041 | 0.1398 |
| Con. tax resp. to debt $\gamma^{c,b}$ | 0.2 | 0.5326 | 0.4662 | 0.5120 |
| Con. tax resp. to GDP $\gamma^{c,y}$ | 0.5 | 0.9183 | 0.7830 | 0.8575 |
| Cap. tax resp. to debt $\gamma^{k,b}$ | 0.2 | 0.5247 | 0.4591 | 0.4796 |
| Cap. tax resp. to GDP $\gamma^{k,y}$ | 0.5 | 0.5957 | 0.4794 | 0.5614 |
| Lab. Tax resp. to debt $\gamma^{w,b}$ | 0.2 | 0.3038 | 0.1980 | 0.2743 |
| Lab. Tax resp. to GDP $\gamma^{w,y}$ | 0.5 | 0.7318 | 0.6958 | 0.7110 |
| Trans. Resp. to debt $\gamma^{tr,b}$ | 0.2 | 0.1673 | 0.0922 | 0.0838 |
| Trans. Resp. to GDP $\gamma^{tr,y}$ | 0.5 | 0.1892 | 0.0932 | 0.1267 |
| Gov. spend. Resp. to debt $\gamma^{g,b}$ | 0.2 | 0.2288 | 0.1023 | 0.0764 |
| Gov. spend. Resp. to GDP $\gamma^{g,y}$ | 0.5 | 0.6021 | 0.5258 | 0.5751 |
| Gov. inv. Resp. to debt $\gamma^{ig,b}$ | 0.2 | 0.3245 | 0.2694 | 0.2866 |
| Gov. inv. Resp. to GDP $\gamma^{ig,y}$ | 0.5 | 0.8662 | 0.7009 | 0.8530 |
| Interest rate smoothing $\gamma^r$ | 0.4 | 0.5717 | 0.2393 | 0.3314 |
| Inflation response $\gamma^\pi$ | 1.61 | 1.4128 | 1.0771 | 1.1260 |
| Output response $\gamma^y$ | 0.06 | 0.0721 | 0.0592 | 0.0799 |

To explain it more clearly, the results can be divided into groups of non-fiscal and fiscal parameters. Firstly, comparing the estimates of non-fiscal parameters with the calibration. For the parameter of investment adjustment cost $\phi$, it is estimated to be 6.57 which is higher than the starting value. In other words, the investment elasticity here is about 0.15 that is close to the elasticity of 0.17 estimated by Smets and Wouters (2003), implying that each one per cent increase in capital price leads to a 15 per cent rise in investment goods. While the estimate of the habit persistence $h$ is decreased from 0.7 to 0.4, which implies that the impact of the accumulated stock of previous private consumption on the current level of utility is estimated to be much smaller. Regarding the financial parameter, the elasticity of the external financial premium $\omega$ is slightly increased to 0.06 which is similar to the standard calibrated value of 0.05 in BGG (1999). In the production sector, the Calvo parameters are much less than the calibration, except for the export price $\lambda^x$. That is to say, the average duration for domestic and imported producers to reset their prices is shorter than that for exported producers. The degrees of nominal rigidities in domestic and imported sectors are lower than that in the exported sector, or specifically, the domestic and imported inflation is more sensitive to the dynamics of the marginal cost. The estimates of the indexation parameters ($\kappa^d$, $\kappa^x$ and $\kappa^m$) are decreased in all the sectors. This indicates that the degree of inflation persistence is lower in all the prices. Generally, the nominal interest rate is estimated to be less sensitive to the changes in inflation, but more responsive to the fluctuations in output, and less auto-correlated. The sensitivity of the interest rate to inflation deviations decreases from 1.61 to 1.13, while to output fluctuations increases from 0.06 to 0.08. Besides, the parameter of interest rate smoothing reduces to 0.33.

As for the estimated fiscal parameters, it finds that the fiscal instruments respond to the fluctuations in aggregate output stronger than the changes in government debt level. The government investment and consumption tax have the greatest responses to the GDP fluctuations. While the capital income tax plays an important role in controlling government debt.

### 6.3.3 Shock processes

Most of the structural shocks in the model of this chapter are autocorrelated. These serially correlated residuals in the DSGE structural model are defined as the exogenous disturbances to the model. In this model, there exist 16 shocks including 6 fiscal policy shocks and 10 non-fiscal shocks. Due to the unobservability of these shocks, they are obtained from the estimated parameters and structural errors generated from the non-stationary data.

The stationarity of each shock is evaluated by conducting statistical tests: The Augmented Dickey-Fuller (ADF) test. The test results and the values of the autoregressive coefficients are presented in Table 6.3. For the ADF test, it assumes the null hypothesis of a unit root presenting in the shock process and the alternative hypothesis of a stationary process.

Table 6.3 Stationarity of Shocks and Estimated $\boldsymbol{AR(1)}$ Parameters

| Shocks | AR(1) | ADF p-value | Process |
|---|---|---|---|
| Productivity[20] | 0.3543 | 0.9446 | Non-stationary |
| Preference | 0.2280 | 0.0000 | Stationary |
| Private investment | 0.5949 | 0.0000 | Stationary |
| Monetary policy | 0.6022 | 0.0000 | Stationary |
| Bond premium | 0.3411 | 0.0000 | Stationary |
| External premium | 0.3409 | 0.0005 | Stationary |
| Net worth | 0.7418 | 0.0000 | Stationary |
| Foreign interest rate | 0.2921 | 0.0006 | Stationary |
| Foreign inflation | 0.8874 | 0.0000 | Stationary |
| Foreign consumption | 0.7112 | 0.0181 | Stationary |
| Con. tax | 0.8699 | 0.0251 | Stationary |
| Cap. tax | 0.1120 | 0.0063 | Stationary |
| Lab. tax | 0.8609 | 0.0299 | Stationary |
| Transfers | 0.8238 | 0.0139 | Stationary |
| Gov. spend. | 0.7882 | 0.0972 | Stationary |
| Gov. Inv. | 0.2524 | 0.0063 | Stationary |

The test results indicate that the hypothesis of stationarity of productivity shock is rejected by ADP test, which implies that the specification of the model where the productivity shock is expressed as a deterministic trend-stationary process is excluded, and it is more promising to treat the productivity shock as the integration of order one, $I(1)$, containing a stochastic trend. Hence, the production function with the $I(1)$ productivity shock need to be re-specified as $Y_{i,t} = A_t(K_{i,t}^{p})^{\alpha_1}L_{i,t}{}^{(1-\alpha_1)}(K_t^{g})^{\alpha_2}$, where $A_t$ denotes the total factor productivity shock follows the ARIMA(1,1,0) process as $\hat{A}_t = \hat{A}_{t-1} + \rho^A(\hat{A}_{t-1} - \hat{A}_{t-2}) + \sigma^A$.

Moreover, all the shocks but productivity shock are strongly rejected by the null hypothesis of a unit root at 10% significant level by the ADF test, except the productivity shock. Hence, these shocks are defined as the stationary exogenous processes in this chapter. Even though there are few empirical studies provide solid evidence that the fitness can be improved by applying different specification of the

[20] Productivity shock follows a ARIMA(1,1,0) process.

shocks, but this chapter assumes that the exogenous processes, other than the productivity shock, reveal AR(1) dynamics and the productivity shock follows an ARIMA(1,1,0) process. Therefore, the AR(1) parameters for stationary shocks are evaluated according to the formation of $\hat{\mu}_t^i = \rho^i \hat{\mu}_{t-1}^i + \sigma_t^i$, while the AR(1) parameter for the productivity shock is estimated from $\Delta\hat{A}_t = \rho^A \Delta\hat{A}_{t-1} + \sigma^A$. Suggested by the estimated AR(1) persistent values, the persistence of these AR(1) processes are tremendously different among these shocks.

After log-linearizing the model, it is assumed that the solution of the model is expressed by a VARX model. When the unobservable variables are non-stationary, for example, a technology shock, then the number of co-integrated variables will be less than that of the endogenous variables. In other words, one or more of the long-term structural equations will exist a non-stationary residual. When the estimates of all the coefficients are generated out, the residuals could be constructed from the data. When these residuals are treated as observable variables, then the number of co-integrated relations could be identical with that of endogenous variables. This allows the solution of the estimated model to be written as a VARX model where the non-stationary residuals appear as observable variables, and an unrestricted form of this VARX model is used as the auxiliary model. To apply this mechanism, the auxiliary model has to include these non-stationary residuals generated from the DSGE model. If we did this, then the auxiliary model will contain key variables required for co-integration; hence, there would be co-integration and the VARX model would be stationary after allowing for error correction. It follows the idea that the auxiliary model is partly conditioned by the DSGE model which is treated as the null hypothesis. That is to say, the VARX model is constructed under the null hypothesis. All that the constraint of the null hypothesis does is to guarantee that the VARX model achieves co-integration based on the null. This residual assumption ensures that the DSGE model achieves co-integration.

# Figure 6.2 Residual calculated from the log-linearized model using estimated parameter

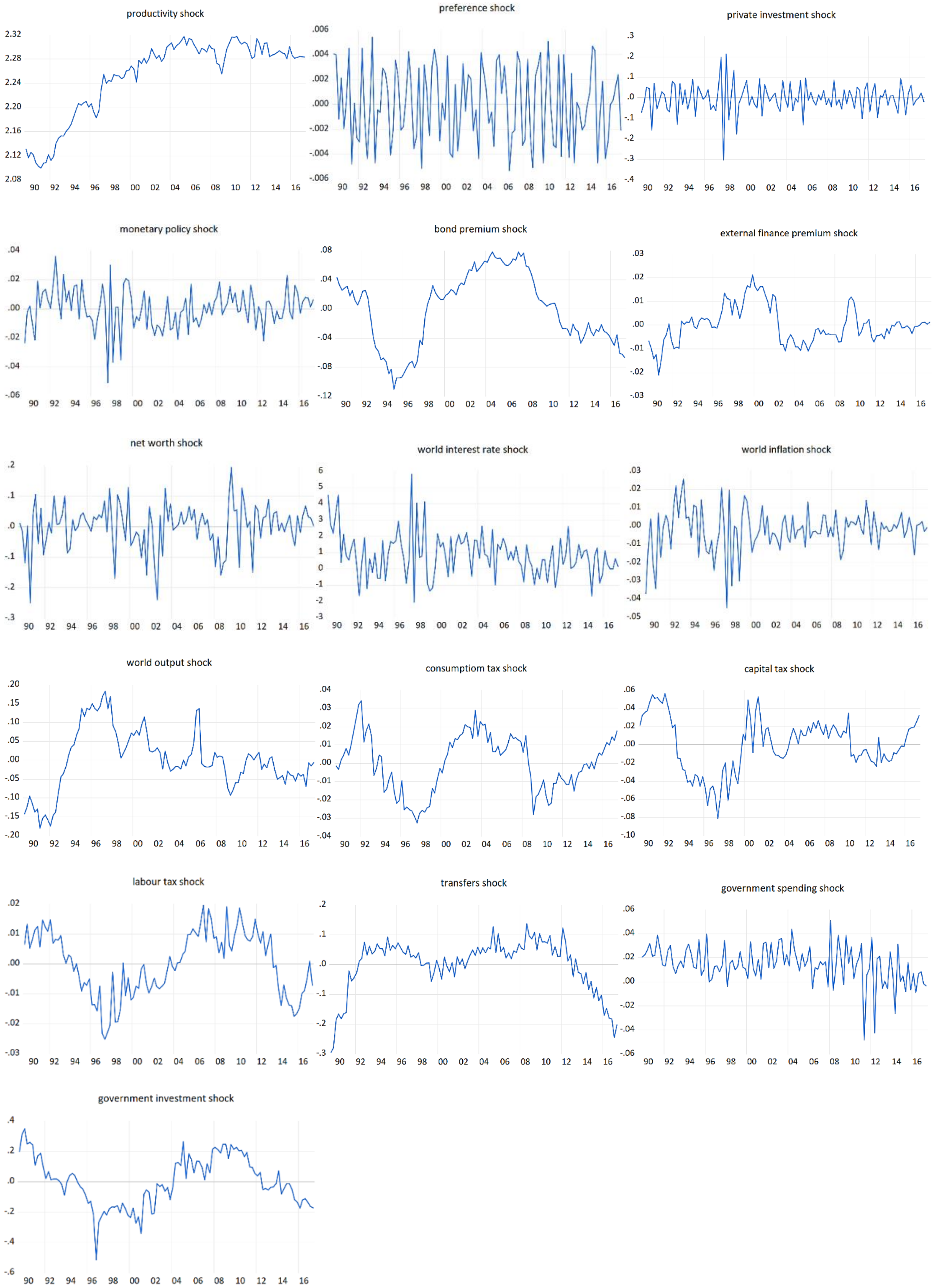
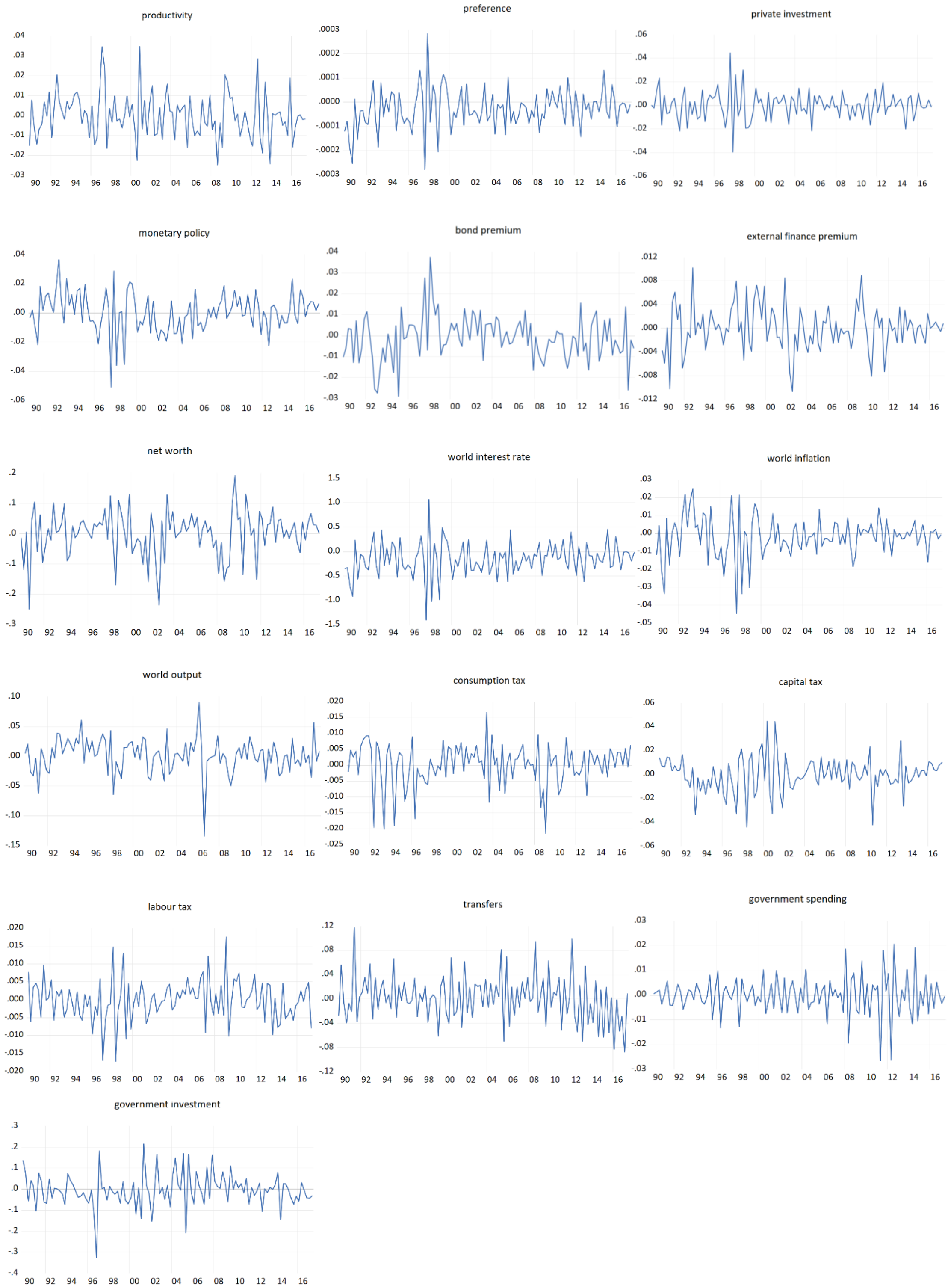


Figure 6.3 Innovations of structural shocks



### 6.3.4 Power of the Indirect Inference test

In the previous section, the Indirect Inference test is applied to evaluate the fitness of an estimated model to the data. However, this test may be questioned of whether it is a reliable test, or how powerful the test is. Typically, the power of a test denotes the probability of rejecting a null hypothesis when the alternative hypothesis is true. The more powerful the test is, the less likely it makes a Type II error. In Le et al. (2016), the power of the Indirect Inference test is assessed by using the Monte Carlo experiments on some representative macroeconomic models, such as Christiano et al. (2005) and Smets and Wouters (2003 and 2007), and compared with the power of a Likelihood Ratio test. It suggests that although both of these two tests reject the false null hypothesis successfully, the Indirect Inference test is significantly more powerful than the Likelihood Ratio test when the number of observed data is finite or adopting the non-stationary data.

This section will concentrate on examining the power of the Indirect Inference test by supposing a variety of models in different degree of falseness. The falseness is derived by considering an increasing degree of numerical misspecification in the parameters. Since the distribution of the Wald statistic is unknown, it is fairly difficult to generate the power of the Indirect Inference test through the straight algebra. In such a case, the examination relies on the Monte Carlo simulation to find out the distribution of Wald. The process of the experiment for investigating the power of the test in terms of a True model and various False models is described in details as follows:

*Step 1* Generate the residuals and innovations of the structural estimated model, which is treated as the True model, based on the actual data and Indirect Inference estimates. Using the Monte Carlo procedure to obtain 1000 sets of simulations. Then these samples including the variances, skewness and kurtosis of the shocks and innovations serve as the artificial data and also the True data.

*Step 2* Falsify the parameters of the True model by alternately increasing or decreasing the value of each coefficient by identical $+/-\ x\%$ to construct a False model that can be considered as a misspecified version of the True model. Similarly, simulating this false model for 1000 times to obtain the False data.

*Step 3* Calculating the Wald statistic for each set of False data following the procedure of the Indirect Inference test introduced in Section 3.2.2. Hence, the empirical distribution of these 1000 number of Wald statistics based on equation (6.4) can be derived to solve out the 95 percentile.

$$W_i^{False} = \left(\beta_i^{False} - \overline{\beta^{False}}\right)' \Omega_{False}^{-1} \left(\beta_i^{False} - \overline{\beta^{False}}\right). \qquad (6.4)$$

Then obtaining the Wald statistics by using True data based on the False model as follows:

$$W_i^{True} = \left(\beta_i^{True} - \overline{\beta^{False}}\right)' \Omega_{False}^{-1} \left(\beta_i^{True} - \overline{\beta^{False}}\right). \qquad (6.5)$$

*Step 4* Find out how many out of the 1000 simulations based on the True data would reject the False model on this calculated distribution at 95% confidence level. Consequently, the rejection rate for a given percentage level $+/-\ x\%$ of misspecification represents the power of the test.

The results of this Monte Carlo experiment are reported in Table 6.4 where the rejection rates are set at 95% confidence level. Furthermore, the True model is falsified at variety of misspecification level of 1%, 3%, 5%, 7%, 10%, 15% and 20%.

Table 6.4. Power of the Indirect Inference test

| Percent Misspecified | True | 1 | 3 | 5 | 7 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|---|---|
| Rejection rate | 5.0 | 18.4 | 38.6 | 47.1 | 76.4 | 98.5 | 100 | 100 |

There are two findings obtained from the table above. Firstly, the rejection rate increases with the degree of falseness increasing. When the degree of falseness is sufficiently high, the False model is rejected at 100%. Secondly, the Indirect Inference test is pretty sensitive to the degree of falseness. As long as the True model is misspecified at or above 10%, the Indirect Inference Wald statistic rejects completely at approximately 100% of the time. In other words, the coefficients could not be estimated further than 10% from the true values otherwise the model could be doubtless rejected by the data. The findings imply that the indirect Inference test is a highly powerful test which can provide some security to the policymaker to choose the most appropriate model in explaining the sample data behaviour.

### 6.3.5 Impulse response functions

To take a further analysis of the estimated model dynamics, this section presents the impulse response functions to a couple of shocks.

### 6.3.5.1 Productivity shock

Figure 6.4 Impulse responses to a productivity shock

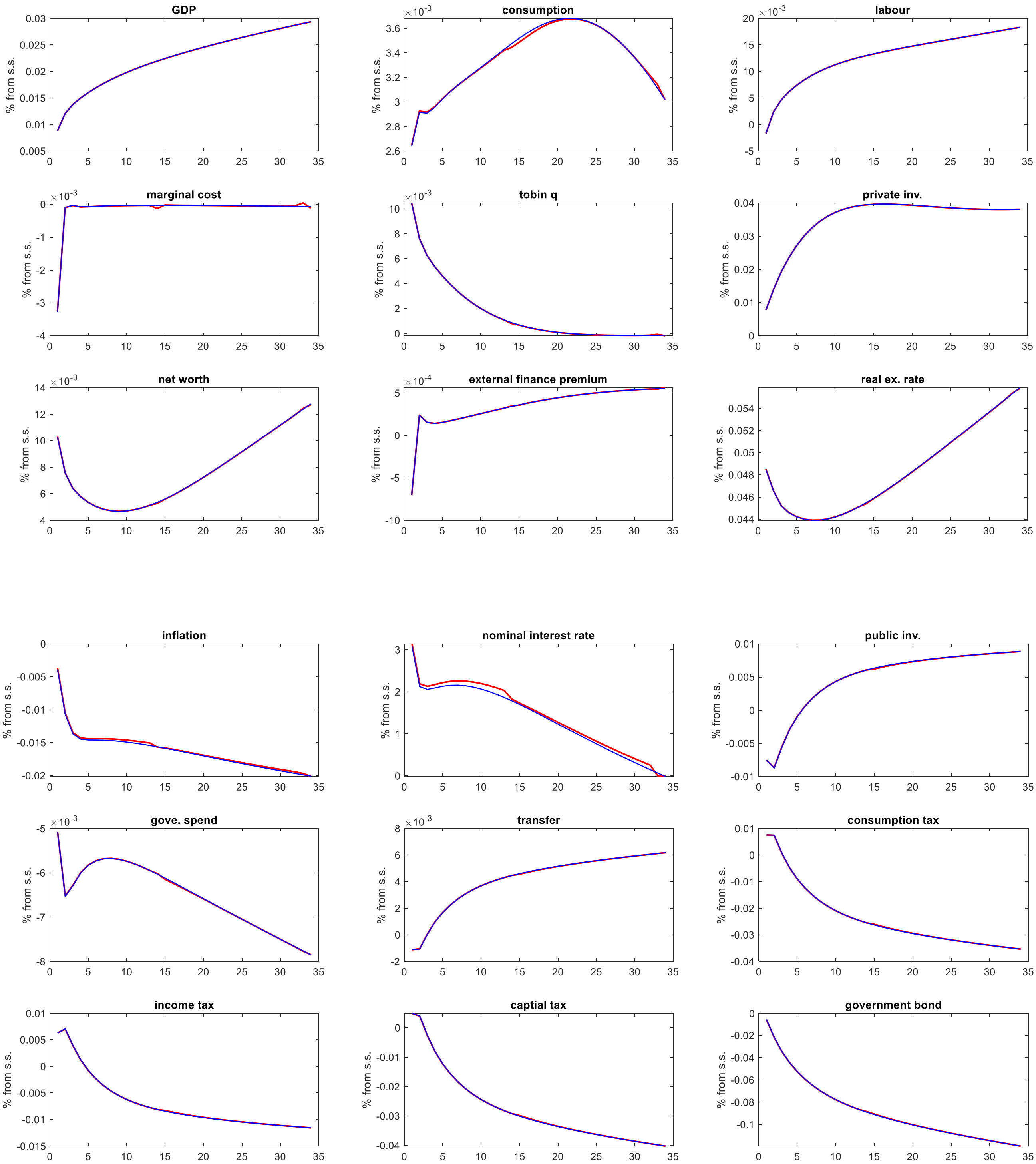
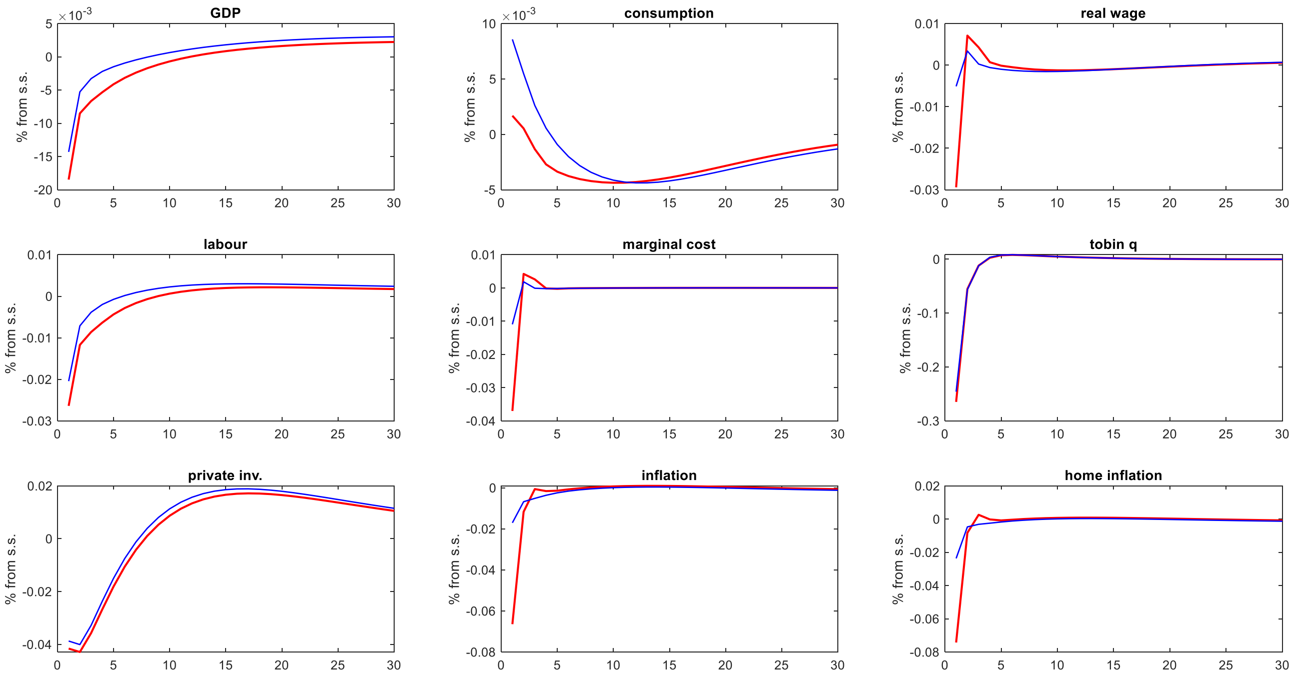


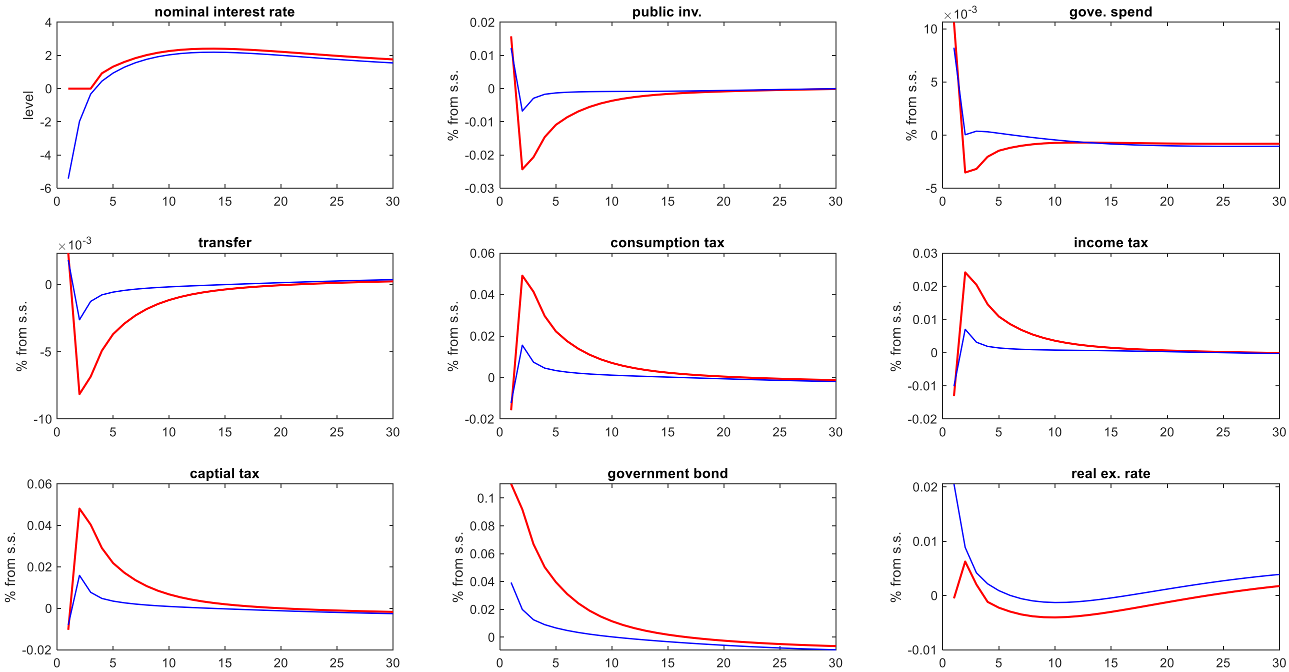
Figure 6.4 presents the dynamics of the key macroeconomic variables to a 1% productivity shock. It is obvious that the nominal interest rate decreases throughout the entire periods and is constrained by the zero lower bound. After a highly persistent productivity shock following ARIMA(1,1,0) process, the key macroeconomic variables including the GDP, private consumption and investment react positively permanently. While the inflation declines dramatically due to a persistently raising in goods supply market where the firms produce with a superior technology and decreasing marginal cost. Meanwhile, the monetary authority reacts to this negative effect by cutting the nominal interest rate to maintain an appropriate real interest rate. In the labour market, increasing productivity pushes up the real wage. Higher wage consequently encourages the households to work more, since the substitution effect dominates the income effect. With a higher wage, the households give up their leisure to work more due to a greater reward. In the goods market, the expanding demand for capital stock pushes up its price (Tobin's Q), which also builds up the entrepreneurial net worth. Accompanying this positive effect, the external finance premium drops, which in turn generates more demand for investment. In the public sector, the government expenditures respond to the deviations of GDP and government debt countercyclically, while the distortionary taxes respond to them procyclically. Since the output boosts instantly after hitting by the productivity shock, the responses to the output dominate the responses to the government debt, the fiscal policy serves as the automatic stabilizer by reducing the government expenditures and raising the distortionary taxes. However, with the continued building up in government debt, the problem of the debt burden becomes serious. Thus, the government reverses some tight fiscal policies with stimulus fiscal measures including increasing the government investment and transfers and cutting taxes to balance the government budget. In the international goods market, since the domestic firms become more productive, these domestic tradable goods are sold to foreign markets by lowing their prices to restore the external balance. Then the real exchange rate depreciates which in turn promote the competitiveness of the domestic tradable goods but reduces the import demand.

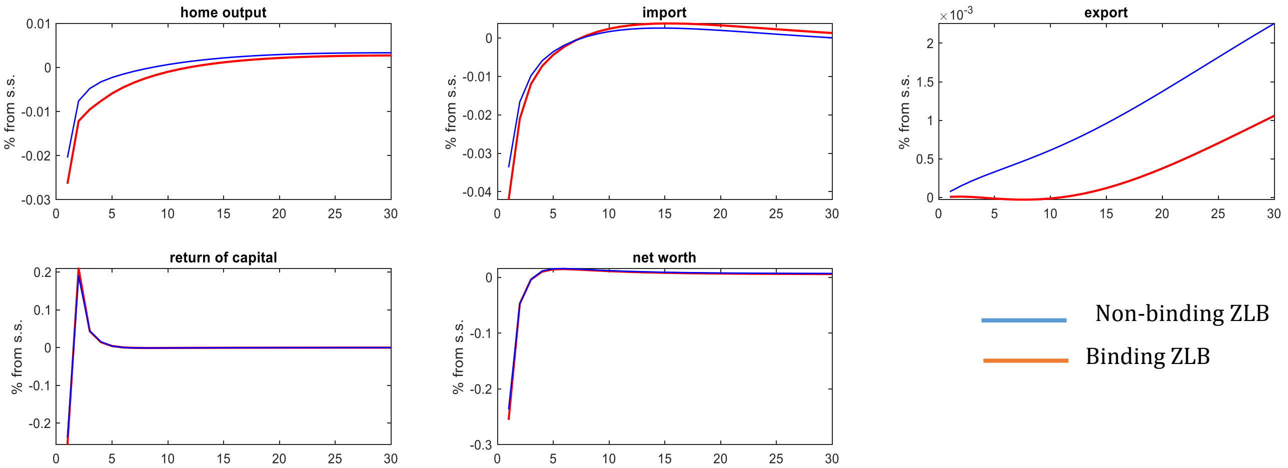
### 6.3.5.2 External finance premium shock

With a temporary 1% finance premium shock, Figure 6.5 presents the impulse response functions for essential macroeconomic variables under two different interest rate regimes in which the blue line shows the responses in a non-binding ZLB regime while the red line refers the responses in a binding ZLB regime. As shown, a positive finance premium shock pushes the economy into the liquidity trap for 3 periods.

Figure 6.5 Impulse responses to a finance premium shock






Without the zero lower bound, the credit spread widens in the aftermath of a positive finance premium shock, which discourages the private investment and causes the output and inflation to plummet. But the consumption is stimulated since the flow of resources into private investment is reduced. After the cost of funding is driven up by a positive premium shock, Tobin' Q increases immediately and then net worth decreases. The reduction in the net worth exacerbates the firms' balance sheet effect, which means that the entrepreneurs' stake in financing capital expenditures is cut down. Thus, the entrepreneurs are forced to borrow at a higher finance premium over the Bank Rate. The investment drops further by the financial accelerator effect. Additionally, the falling inflation leads the investment to decrease by the debt deflation effect. In general, with the presence of financial friction, the downward trend in the investment is accelerated after an initial decline. While in this non-binding ZLB regime, the deflationary effect is mitigated by the cuts in the nominal interest rate to a certain degree.

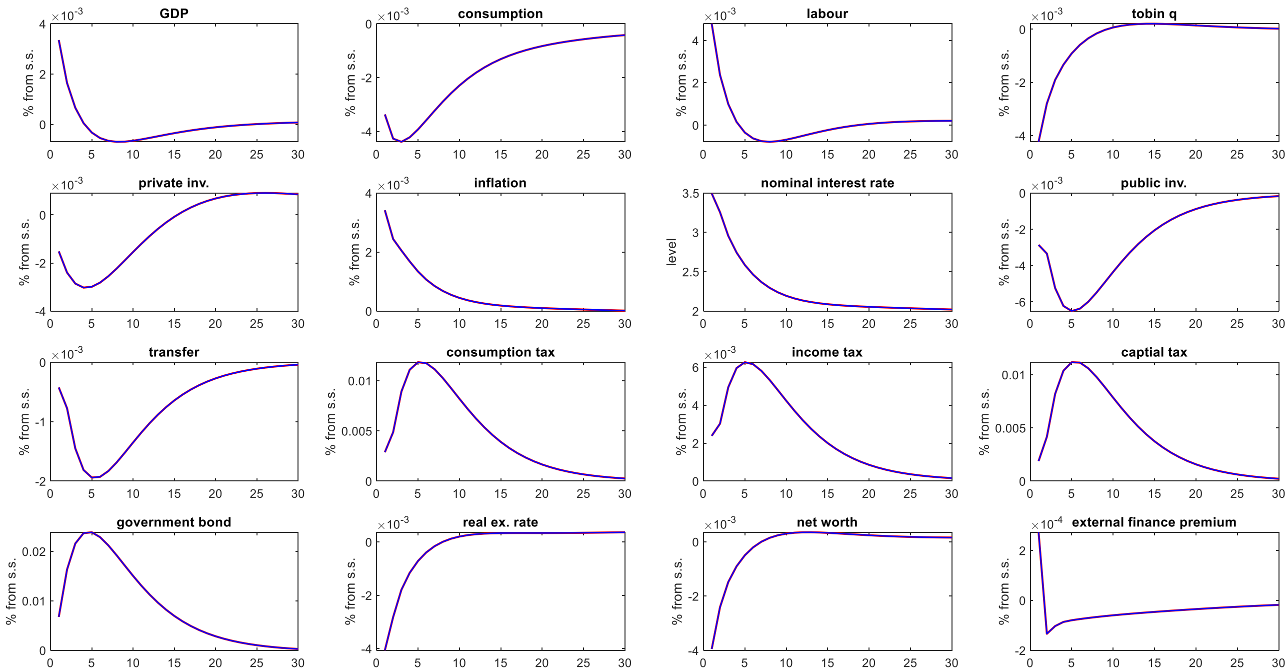
As for the public sector, since the fiscal rules are governed by the fluctuations in the output and government debt level, namely government expenditures including government spending, investment and transfers response counter-cyclically while taxes response pro-cyclically. Since the effect of the declining output dominates, the government implement a fiscal stimulus package with the increases in government expenditures and cuts in all distortionary taxes. Moreover, the depreciation in the real exchange rate causes the export turning out to be more competitive, and thus, the export increases, while the import reduces.

When the interest rate is constrained by the zero lower bound, the recession is amplified, since the monetary authority can no longer manipulate the nominal interest rate to manage the negative effects of an adverse shock. With the sever decreasing in the GDP, the economy is pushed into a recessionary-deflationary path and is lack of demand in the liquidity trap. Due to the worsening deflation in ZLB regime, the investment demand decreases slightly more than that in the non-binding ZLB regime, which is indeed the debt deflation effect at work. Furthermore, with a positive real interest rate, the saving increases sharply. A looser fiscal policy is installed to support the economic growth and recovery in the recession that also builds up government debt.

#### 6.3.5.3 Fiscal shocks

The following section analyses the impulse responses to four fiscal shocks, government spending consumption tax, labour income tax and capital tax. The policy experiment is considered as a temporary fiscal stimulus with a 1% increase in government spending and a 1% cut in each taxes.

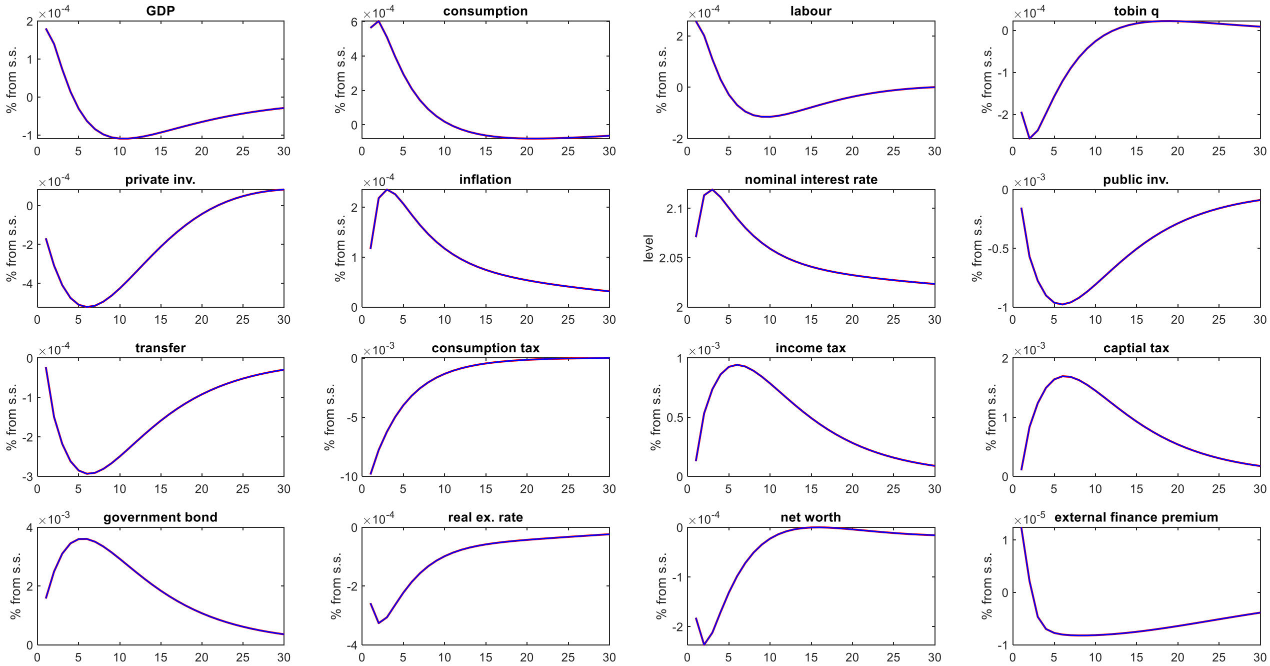
Figure 6.6 Impulse responses to a positive government spending shock




In Figure 6.6, it shows the IRFs to a temporary 1% rising of government spending shock. With a rise in government spending, the aggregate demand expands, which pushes up the general price level. The monetary authority reacts to this stimulus by raising the nominal interest rate, then the demand for investment decreases since the credit becomes more expensive which derives a downward pressure on the entrepreneurial net worth. In such a case, the final effect on the capital price is negative, thus the financial accelerator channel generates a negative effect on the investment. An increase in the basic rate implies a lower price for government bond and higher demand for it. The households manage their budget by reducing their demand for consumption goods, which leads to a decline in the public revenue from consumption tax. Besides, the increase in the interest rate also leads to a fall in the present discounted value of consumers' future incomes, which induces them to work harder to offset this loss. Government deals with thes negative results of the public coffers (higher spending and lower revenue) by issuing new debt to increasing the public funds. To cover the cost of a higher level of public debt, the government budget is adjusted by reducing the expenditures (government investment and transfers) and raising the distortionary taxes (consumption, labour income and capital income taxes). Additionally, the real exchange rate appreciates in response to an increase in the domestic demand due to an unanticipated increase in government spending.

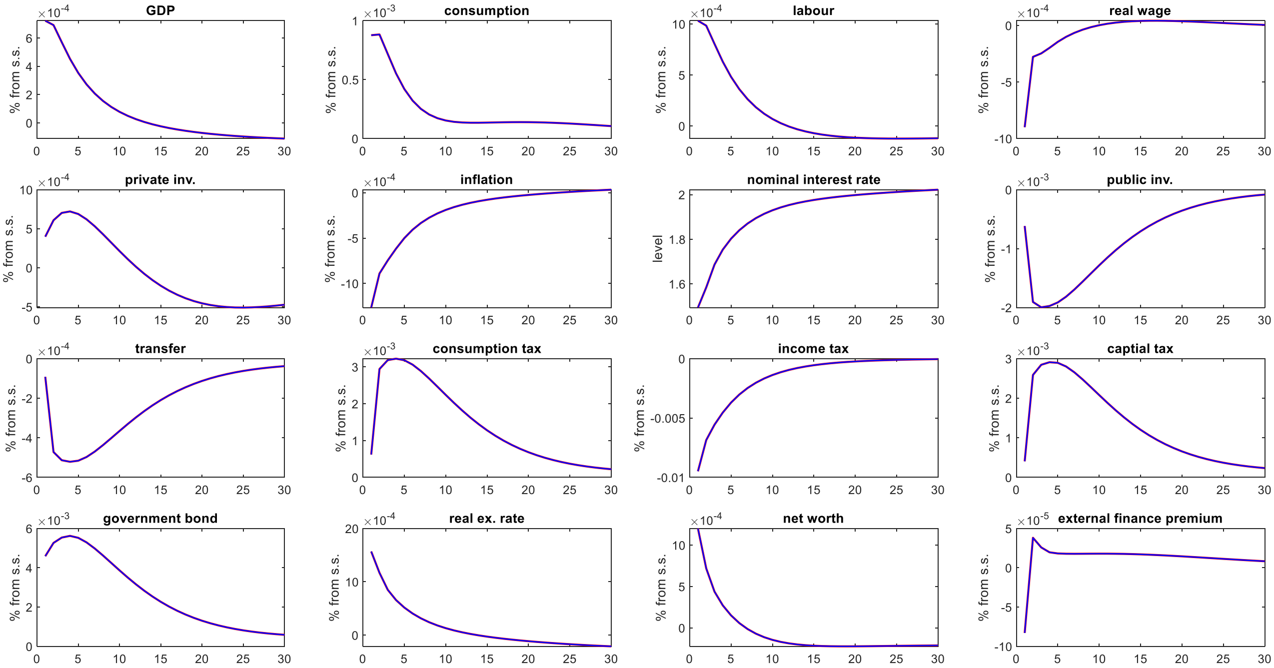
In general, the government spending shock, in this case, generates the crowding-out effects on both private consumption and investment. This finding is consistent with Cogan et al. (2010) who estimates the Smets-Wouters model and detects that although both consumption and investment decline due to the government spending shock, this negative effect can be offset by the increase in the government spending and, thus, the final multiplier effect on GDP is positive.

Figure 6.7 Impulse responses to a negative consumption tax shock




The results for a 1% decrease in the consumption tax is shown in Figure 6.7. The relaxation of the consumption tax reduces the cost of private consumption directly. On the supply side of the economy, this shock narrows the gap between the real wage and marginal rate of substitution between consumption and leisure, hence the households are willing to work more. On the demand side, the cheaper after-tax consumption and more labour supply increase incomes and private consumption. The increases in the aggregate demand increase inflation, and so does the nominal and real interest rates. Through the financial accelerator channel, there is a negative effect on the firm's investment demand with higher external finance premium and lower entrepreneurial net worth. Similar to the government spending shock, the fall in the public coffer (less public revenue owing to a tax cut) is compensated by cutting public expenditures and raising revenues to increase the government debt.

Figure 6.8 Impulse responses to a negative labour income tax shock




As in Figure 6.8, a cut in the labour income tax increases the after-tax payment for each hour of labour supply, and thus, household raises labour supply since the substitution effect dominates the income effect, but the real wage before tax drops. With the higher disposable income, the households increase their consumption. The inflation falls as a result of the downward pressure on the pre-tax real wage. The monetary authority responses to this deflationary effect by cutting the nominal interest rate. Since this negative effect is not strong enough, the nominal interest rate is not constrained by the zero lower bound. A drop in the real interest rate leads to a lower cost of funding and stimulates the private investment. Also, the government offsets the decline in public revenue by reducing expenditures and raising other taxes. Moreover, a decline in the real interest rate leads to the depreciation of the domestic currency.

Figure 6.9 Impulse responses to a negative capital income tax shock

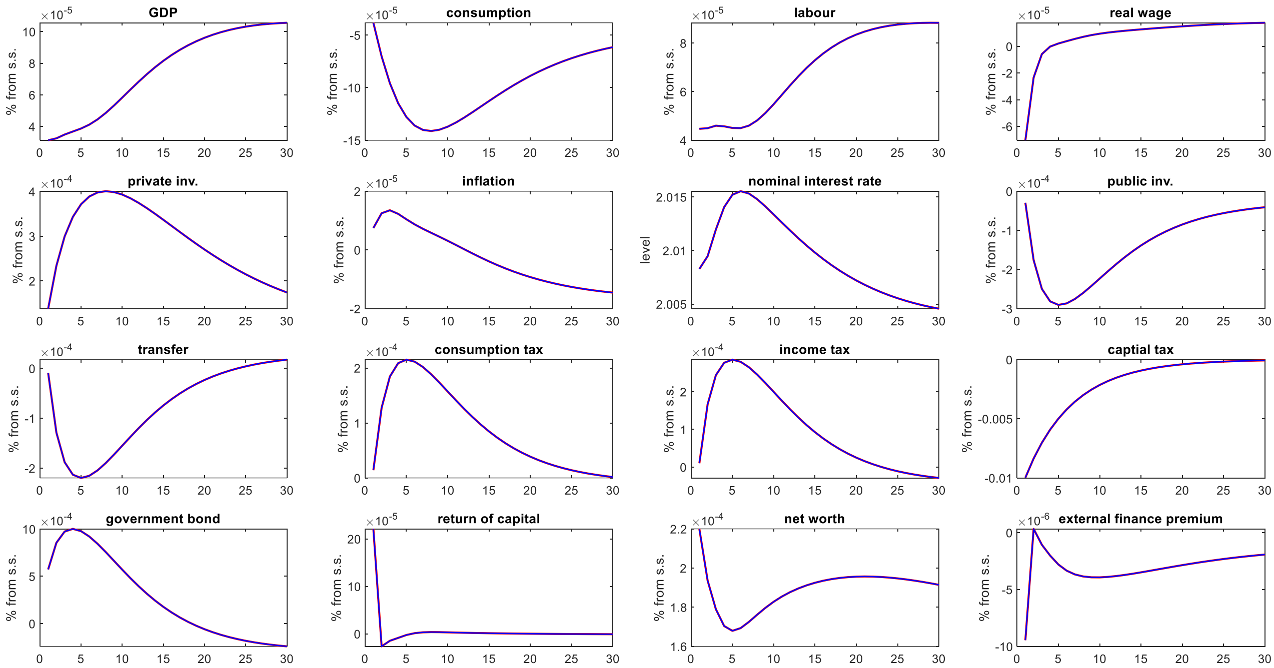



Figure 6.9 reports the model dynamics after a cut in the capital income tax. The instant effect of this shock is the reallocation of production inputs from labour to capital, which discourages the labour demand. The lower demand for labour pushes down the real wage. While a decrease in the real wage induces the households to work more since the income effect dominates. However, real income still reduces, which results in a slight decline in private consumption. Lower capital income tax rate raises the rate of return on capital, leading to the increases in both investment demand and the price of the capital stock. The higher price improves the firm's balance sheet, which generates a further financial accelerator effect on the investment. The expansion in the aggregate demand puts an upward pressure in inflation. Consequently, the nominal interest rate increases.

### 6.3.6 Variance and historical decomposition

#### 6.3.6.1 Variance decomposition of the episodes

This section investigates the main driving forces contributing to the movements of the main macroeconomic variables during the episode of 2008-2017 by studying the variance decomposition (seeTable 6.5).

Table 6.5. Variance decomposition for 2008Q4 to 2017Q2

| | Variables | | | | |
|---|---|---|---|---|---|
| Shocks | Output | Interest rate | Inflation | Consumption | Ex. rate |
| Productivity | 31.16 | 15.09 | 35.34 | 30.23 | 20.36 |
| Preference | 24.63 | 14.48 | 27.97 | 38.53 | 18.98 |
| Private investment | 7.25 | 11.49 | 5.22 | 5.39 | 3.74 |
| Monetary policy | 5.47 | 26.21 | 18.06 | 7.05 | 5.04 |
| Bond premium | 0.31 | 4.51 | 0.54 | 0.36 | 6.39 |
| External premium | 16.16 | 16.13 | 3.34 | 4.24 | 12.11 |
| Net worth | 8.25 | 5.07 | 1.35 | 1.58 | 4.37 |
| Foreign interest rate | 1.93 | 2.41 | 1.25 | 1.57 | 5.25 |
| Foreign inflation | 2.03 | 0.94 | 2.24 | 0.88 | 6.58 |
| Foreign consumption | 0.61 | 0.28 | 0.25 | 3.05 | 14.27 |
| Con. tax | 0.09 | 0.52 | 0.34 | 0.57 | 0.23 |
| Cap. tax | 0.02 | 0.27 | 0.28 | 0.28 | 0.08 |
| Lab. tax | 0.05 | 0.19 | 0.35 | 0.23 | 0.11 |
| Transfers | 0.14 | 0.22 | 0.47 | 0.72 | 0.34 |
| Gov. spend. | 1.30 | 1.46 | 2.24 | 3.35 | 1.63 |
| Gov. Inv. | 0.61 | 0.72 | 0.76 | 1.97 | 0.52 |
| Total | 100 | 100 | 100 | 100 | 100 |

The productivity shock and preference shock explain the most variations in output. The financial shocks (external premium and net worth shocks) contribute almost a quarter of the movement, showing that financial shocks are important sources of fluctuations too, which is in line with De Graeve (2008), Christiano, Motto and Rostagno (2010) stating that the financial risk shock dominants the output variance. The monetary policy shock during this period has little influence on real variables' movement, but it's role is relatively important to explain the inflation variation. The fiscal policies contribute almost nothing to the business cycle, i.e. its role is minimum.

### 6.3.6.2 Historical decomposition

More specifically to identical what shocks have a particular effect in certain periods, this section presents the historical decomposition for the period from 2006Q4 to 2017Q2.

Figure 6.10. Historical decomposition of output

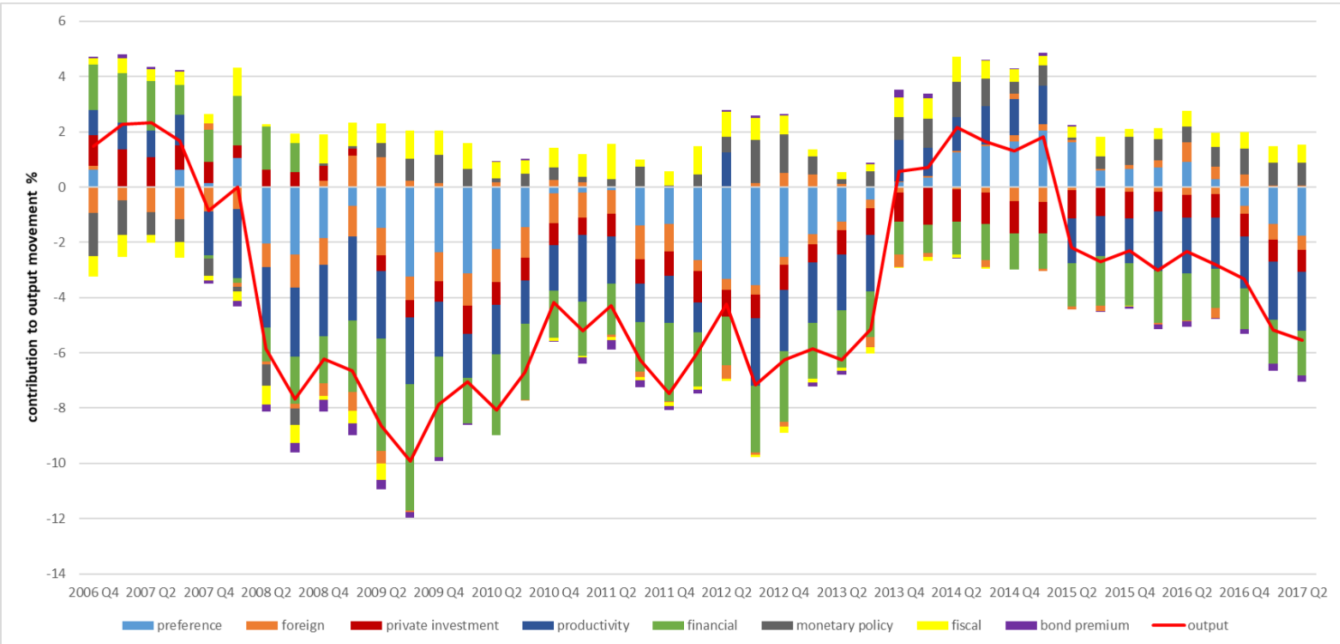
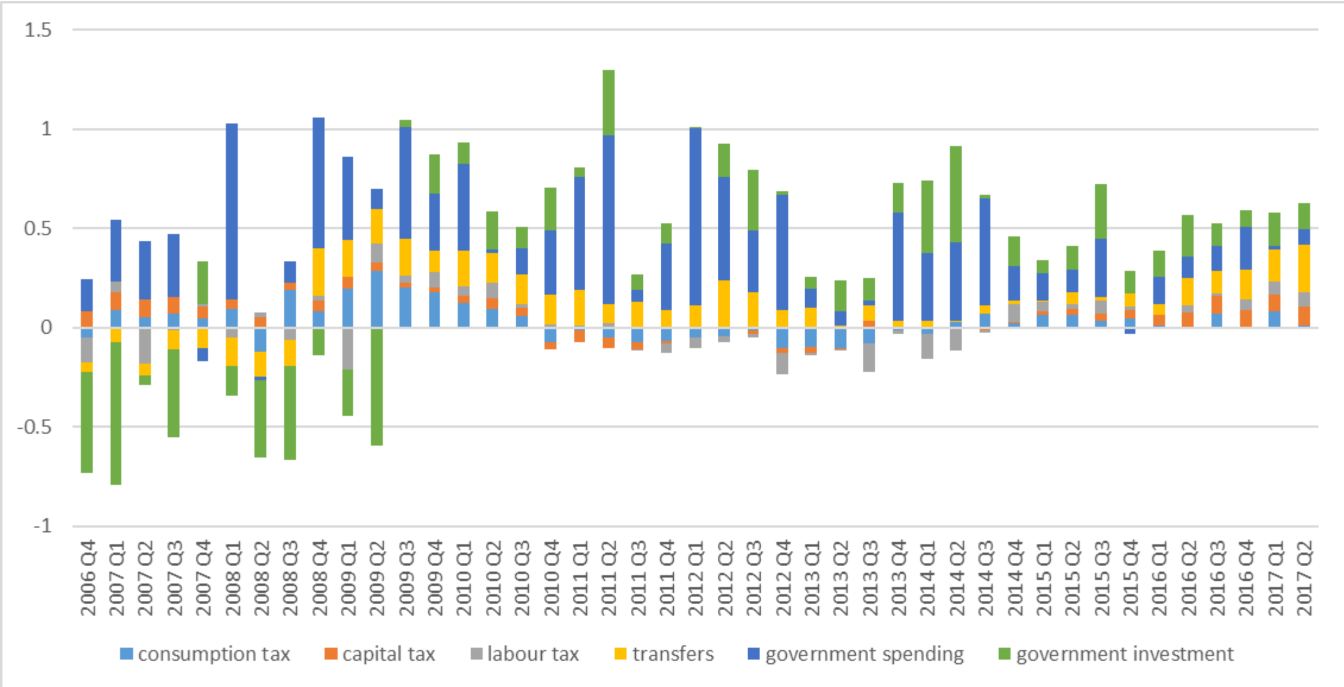



Figure 6.10 depicts the historical decomposition of the structural shocks to the output. During the financial crisis of 2008, the productivity shock dominates the sharply dampening in the output, and then it contributes positively to the economic recovery period of 2013 to the start of 2015. Labour supply shock is the main factor in driving the recession and recovery. Besides, financial shocks also make a significant contribution to the reduction in output, especially the finance premium shock is one of the main sources of driving the output collapse subsequent to the

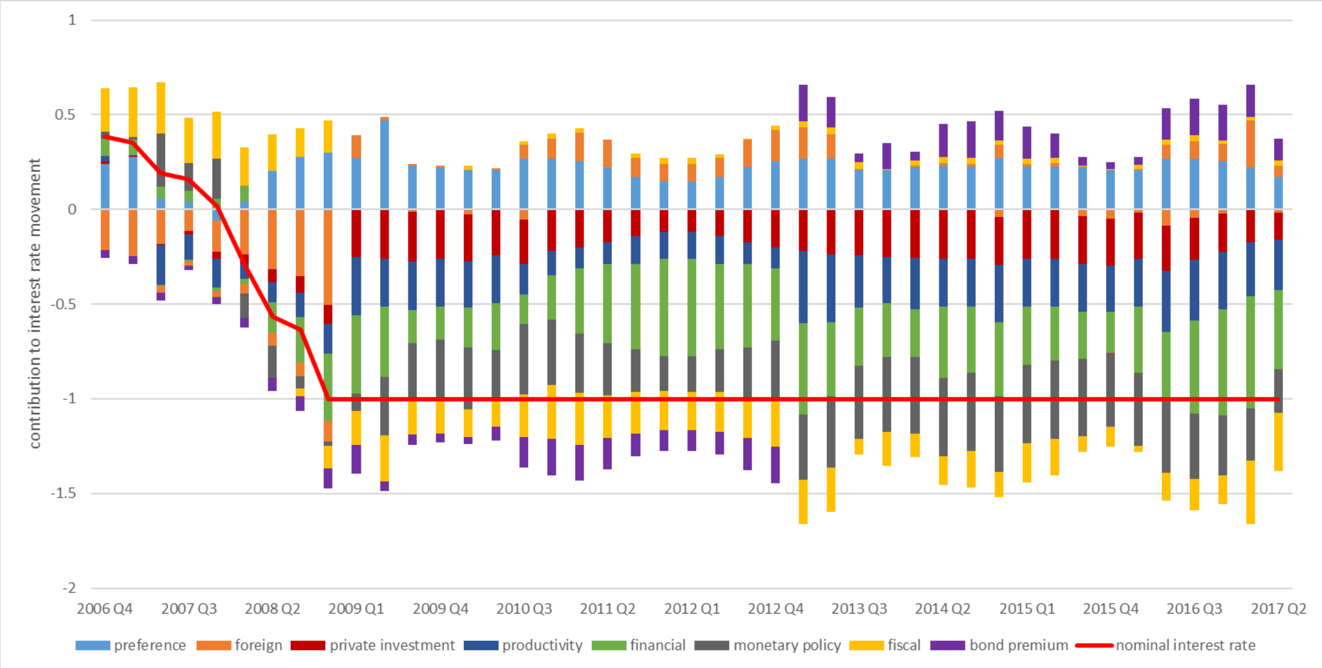
financial crisis and continuously depresses the output growth onwards. Private investment shock generates a negative impact on output during the sample period, which may be derived from the financial accelerator mechanism pushing the private investment drop further and the aggregate demand shrink.

Figure 6.11 Fiscal contributions to the output fluctuation




On the contrary, the monetary policy shock partly offsets the downward pressure in output since the monetary authority cuts the nominal interest rate regularly. Meanwhile, the fiscal policy contributes positively but small to improving the economy (see Figure 6.10). The government expenditure shocks attempt to promote the output over the sample period, and government spending shock is the most effective fiscal shock during the financial crisis (see Figure 6.11). The distortionary tax shocks positively impact the output from 2008 to 2010 due to a fiscal stimulus package conducting by the U.K. government. While these impacts turn to be slightly negative after 2010, which may be caused by the fiscal austerity with increasing the tax rates. By controlling the government debt, the fiscal consolidation supports the economic recovery and thus the tax revenue expands, then the impacts of tax shocks return positive from 2013 onwards to the end of periods.

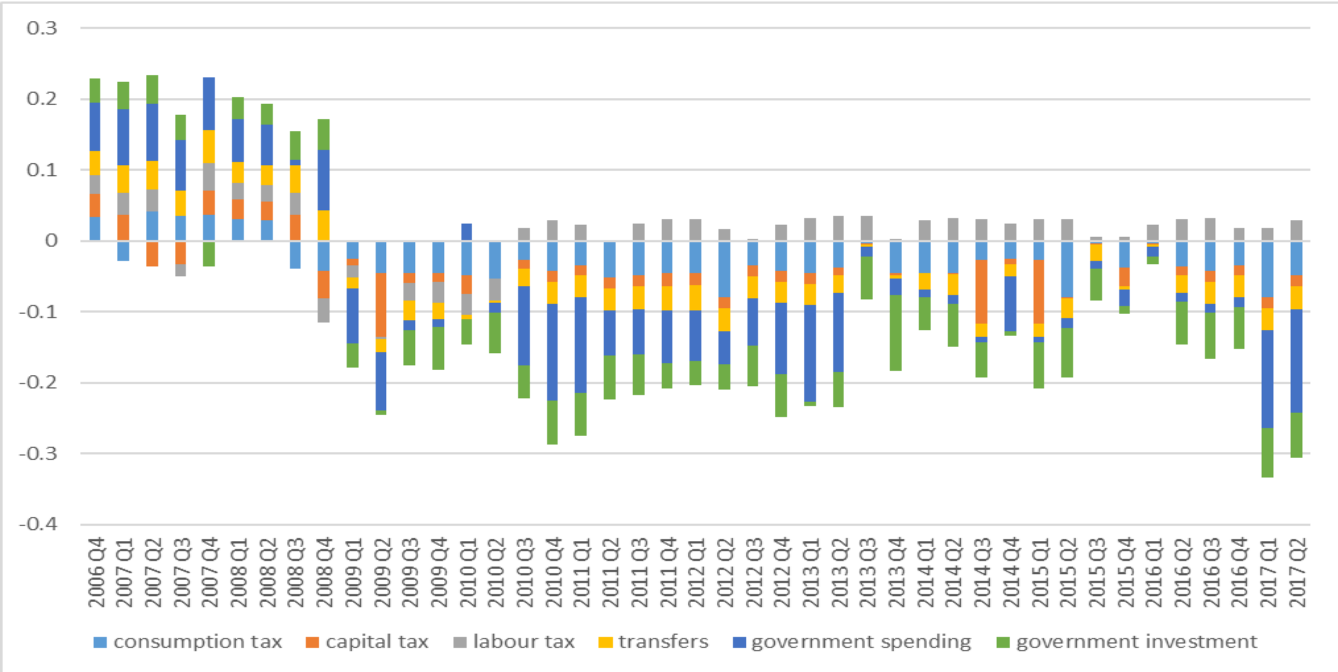
Figure 6.12 Historical decomposition of nominal interest rate*




*When the deviation of the nominal interest rate falls below to its steady-state ($\bar{R} = \frac{\bar{\pi}}{\beta}$), the interest rate will be constrained by the zero lower bound and not drop further.

Figure 6.12 presents how the estimated model with zero lower bound suggests the structural shocks drive fluctuations of the nominal interest rate during the financial disruptions in the U.K. Notably, the nominal interest rate drops dramatically in the aftermath of the financial crisis, and then it is constrained by the zero lower bound from 2008 onwards with the combination of the impacts of the various disturbances. During the crisis, the monetary policy shock is obviously and not surprisingly the major negative impact on the interest rate, since the U.K. government continuously conducts the tightening monetary policy over the horizon. Besides, the productivity shock and private investment shock also contribute to the interest rate movements negatively in the recession. Under these combined effects, the finance premium rises and then the financial shocks yield further downward pressure on the nominal interest rate. With the joint influence of these shocks, the model economy falls into the liquidity trap and the nominal interest rate is constrained at zero for several periods. While these negative impacts are partially offset by a labour supply shock. Specifically, the fiscal shocks contribute negatively to the interest rate variance except for the labour income tax shocks in the crisis. Particularly, within these fiscal shocks, the government spending shock contributes most to the fluctuations, which implies that government spending is the most effective fiscal policy in influencing the economy (see Figure 6.13).

Figure 6.13 Fiscal contributions to the nominal interest rate fluctuation




According to the results of the historical decomposition, the fiscal policy did not contribute much to the variation of the macroeconomic variables, of which the reason could be explained as in Sawyer (2012) and Riley and Chote (2014) that these policies were implemented for stabilizing the public confidence to the government debt, rather than getting the economy out of the liquidity trap (this job was left for unconventional monetary policy), as they did not help to push upward pressure on the nominal interest rate. While the reason for requesting the policies to help the economy escape from the liquidity trap is probably because when getting out of the zero lower bound, there will be more room for the monetary policy to control the economy by manipulating the policy rate in response to unanticipated shocks. From the lesson of the Keynesian literature, the fiscal policy is fairly effective in coping with the zero interest rate. Hence, there comes the problem of how to use fiscal measures to confront the liquidity trap and what are their effects on the rest of the economy. The details of these questions will be discussed in the next section.

## 6.4 Policy implications

Based on the historical composition analysis, the fiscal measures seem not playing significant roles in encouraging economic recovery. Consequently, it leads to a further question: what else fiscal policy could have done better when the economy is in the recession and facing a binding ZLB regime?

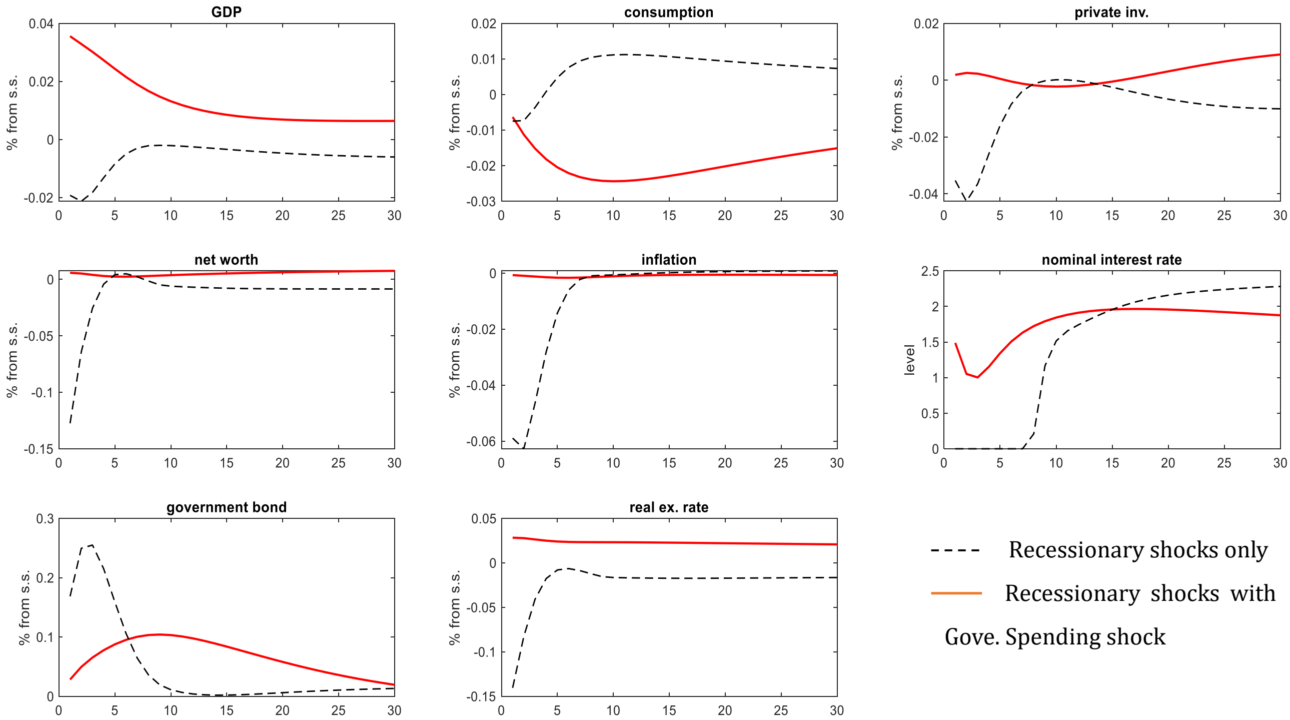
In the liquidity trap, the conventional monetary policy does not work. Although the unconventional monetary tools, such as the quantitative easing, may work (Le et al., 2018), it is not proposed in the model structure of this thesis. Therefore, the only policy tool is fiscal policy. If the liquidity trap is concerned, it is important to question how much the fiscal policy should do to help the economy escape from it. This section analyses this question in two steps. Firstly, assume the recessionary shocks send the nominal interest rate against the zero bound and evaluate to what extent the government should boost the fiscal measures to push the economy out of the liquidity trap. Then find which fiscal measure is the most effective tool. Secondly, calculate the welfare effect of these fiscal tools and further discuss the most useful policy in improving the economy.

### 6.4.1 The impact of fiscal tools on getting out of the liquidity trap

Mertens and Ravn (2014) argue that the recession and constrained interest rate can be a result of large economic shocks to the macroeconomic fundamentals. Eggertsson (2003), Eggertson (2011), Cook and Devereux (2011), Coenen et al. (2012) and Bouakez et al. (2017) discuss the quantitative effects of the fiscal policy in this liquidity trap. Following these studies, this section introduces the negative preference shock (one of the significant demand shock) and negative monetary shock serving as the recessionary shocks to bring the economy into the recession and the binding ZLB regime. The standard deviations of these two shocks are set as 1.5851% and 1.3559%, respectively, implying by the model testing and estimation. With these negative recessionary shocks, the model implies that the nominal interest rate is constrained for 7 periods. In the preliminary examination stage, each fiscal policy is assumed to be boosted to 10% attempting to disengage the nominal interest rate from the zero bound. In all below impulse response functions analysis, the black dotted line refers to the case with the recessionary shocks only; the red solid line refers to the case with both the recessionary shocks and fiscal shock.

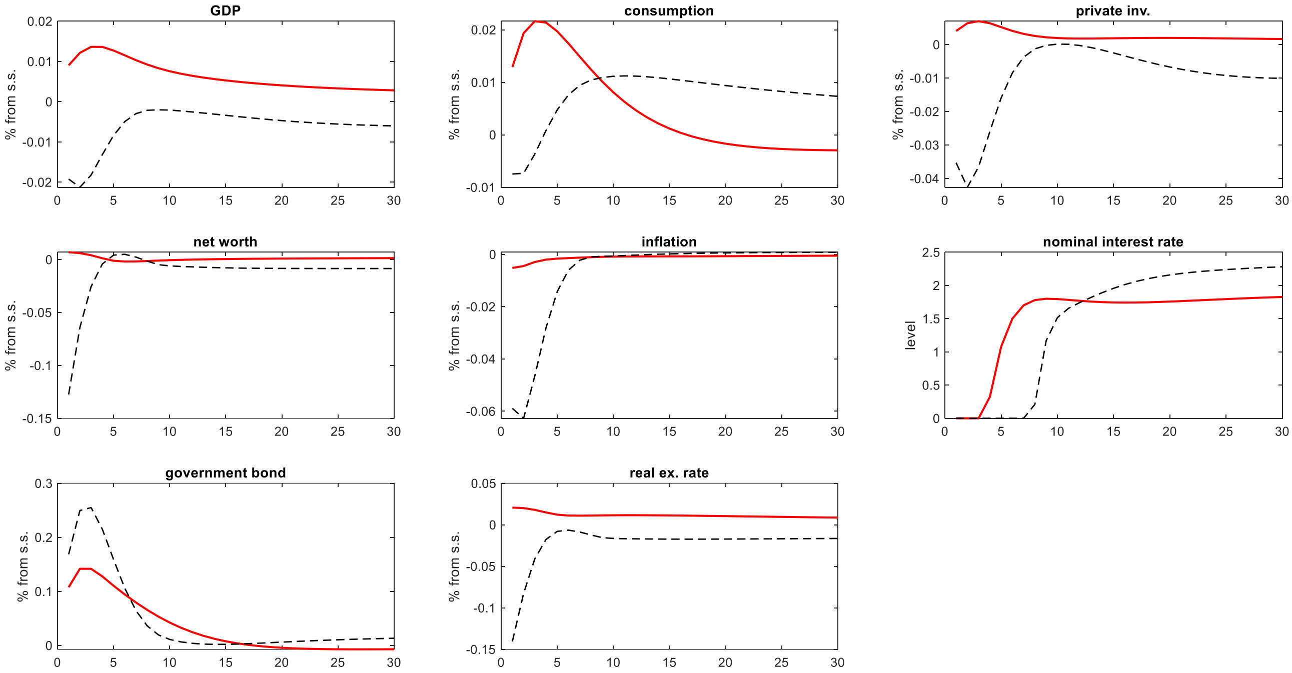
With 10% government spending stimulus, the aggregate demand expands and generates a great inflationary effect. The higher inflation creates more room for the monetary authority to set a positive nominal interest rate. Meanwhile, inflation increase also improves the entrepreneurial net worth propagated though the debt-deflation channel, leading in turn to the enhancement in private investment. However, the households rearrange their budget composition for consuming less since the crowding-out effects reduce private consumption by raising public spending. The government bond decreases owing to a lower real interest rate when the economy escapes from the liquidity trap. Moreover, the real exchange rate appreciates in response to an increase in domestic demand.

Figure 6.14 Impulse response to a government spending shock



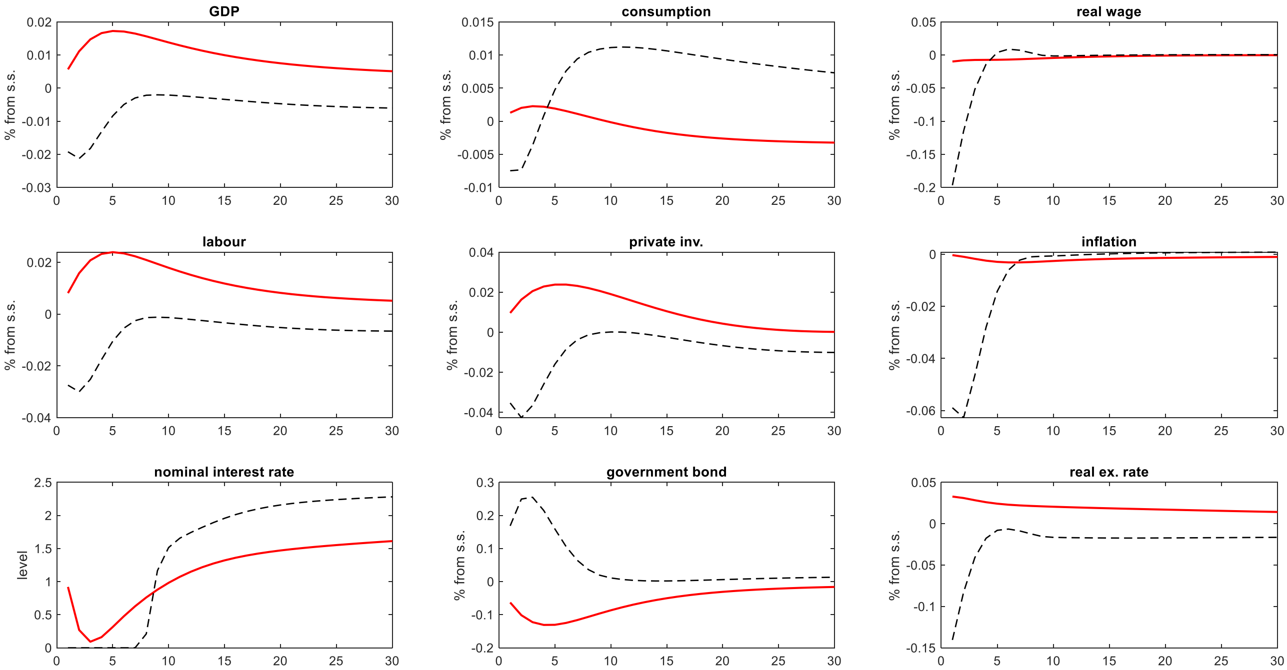

If consumption tax is cut by 10%, (see Figure 6.15), private consumption increases. The households are encouraged to consume more since the after-tax price for the same quantity of the goods is cheaper. Consequently, the aggregate demand increases, which puts upward pressure in inflation. In this binding ZLB regime, the inflationary effect yields a positive effect on the nominal interest rate and shortens the duration of the liquidity trap from 7 periods to 3 periods. Also, private investment rises due to the debt-deflation effect. Similar to the positive government spending shock, the demand for government debt decreases and domestic currency appreciates.

Figure 6.15 Impulse response to a consumption tax shock



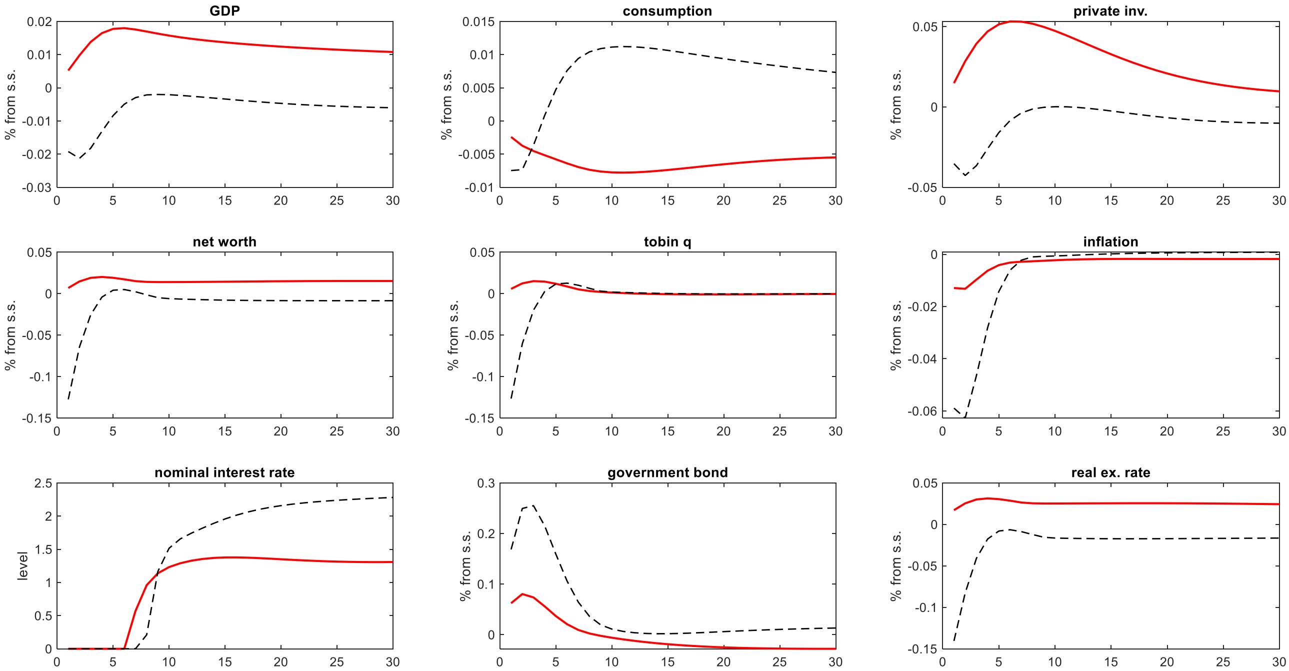

In Figure 6.16, an increase in labour income tax raises the real wage instantly and labour supply increases. In this case, the substitution effect dominates the income effect. With higher wages, households give up their leisure to work more due to greater rewards. Thus, for the firms, producing the same amount of goods costs them more in hiring labour, which requires them to set a higher price to compensate their losses leading to an increase in inflation. The monetary authority raises the nominal interest rate. As discussed in the previous chapter, increasing the labour income tax is expansionary in the binding ZLB regime, hence the reduction of the output caused by the recessionary shocks is ameliorated by a higher labour income tax. Besides, there is also a moderate improvement in private investment due to this inflationary effect. The households enhance their consumption due to the increase in disposable income. While government debt declines rapidly since the real interest rate drops sharply.

Figure 6.16 Impulse response to a labour income tax shock



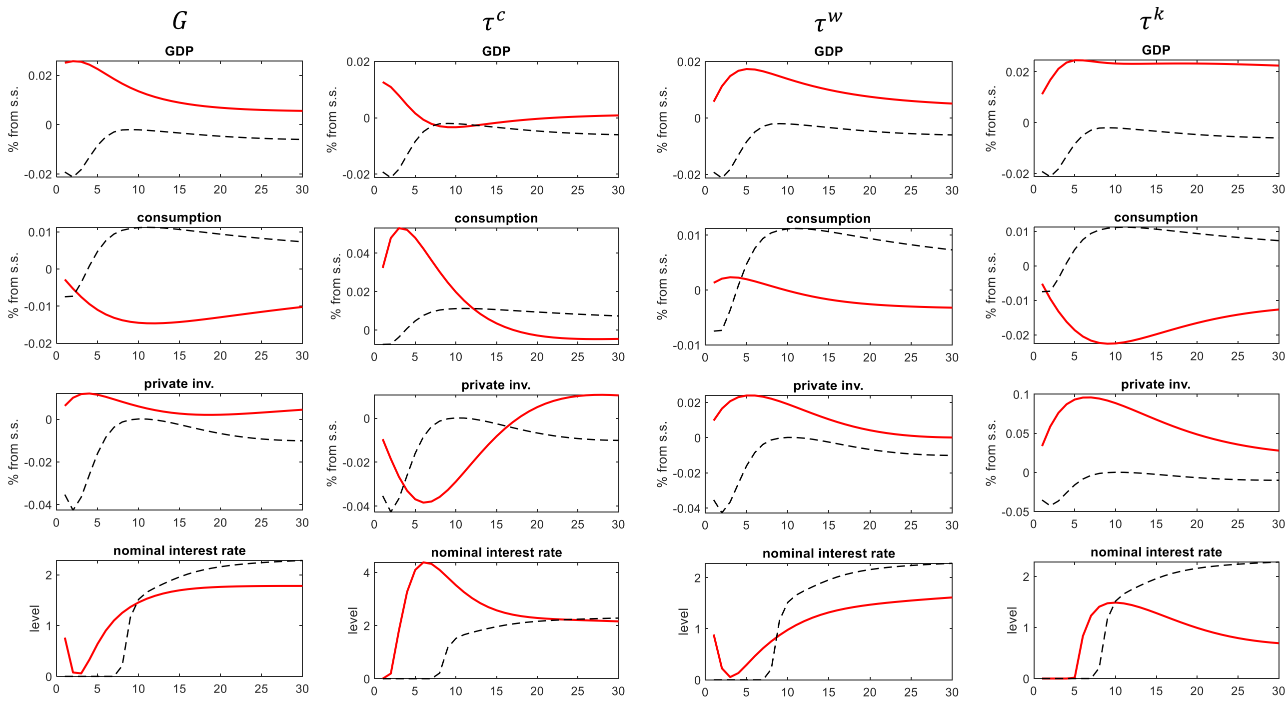

If capital income tax is cut by 10%, the private investment and capital stock increase. Consequently, asset price (Tobin's Q) rises, which pushes up the entrepreneurial net worth, stimulates the private investment and also improves the aggregate demand. While private consumption tends not to be promoted mush since more resources flowed in the investment goods market. Furthermore, it is found that the duration of the liquidity trap appears not to be shortened much by cutting the capital income tax. It only reduces from 7 periods to 6 periods. The reason could be that the inflationary effect generated in this case seems not great enough to create sufficient room to get the nominal interest rate above zero shortly.

Figure 6.17 Impulse response to a capital income tax shock




Having checked the effects of different fiscal measures, now consider helping the economy escape the liquidity trap. By assuming a 10% boost across all fiscal measures, now we can determine more carefully about what minimum percentages boosts needed for each fiscal measure to get the economy out of the binding ZLB regime (see Figure 6.18).

Figure 6.18 The limit of the fiscal policy*




* In the order from left to right, each column presents the impulse responses to the recessionary shocks with a positive government spending shock, a negative consumption tax shock, a positive labour income tax shock and a negative capital income tax shock.

By obtaining the simulations, the first three columns indicate that increasing government spending to 6.2%, cutting the consumption tax to 28.3% and increasing the labour income tax to 9.8% are just satisfied the condition that the nominal interest rate is pushed to the level of just above zero. While the fourth column shows that even the degree of the capital tax shock is set to be 30%, this aggressive negative shock still cannot derive a positive nominal interest rate at the initial period. The results of the distortionary taxes are consistent with Forni et al. (2009) which introduce an empirical analysis of the European economy and find a significant effect of labour income and consumption taxes on consumption and output while the smaller effect of the capital income tax in the short-run. Also, Mertens and Ravn (2013) remark that cuts in the capital tax do not affect or even lower the consumption, and there is no strong evidence that the changes in the capital income tax affect the short-term nominal interest rate significantly.

In conclusion, the package including increasing the government spending, rising labour income tax and cutting consumption and capital taxes could generate inflationary effects in the liquidity trap, which is in line with the results of fiscal multipliers in Chapter 5, and all of these fiscal measures put upward pressures on the nominal interest rate and shorten the duration of the liquidity trap. However, by comparing the impacts of various magnitudes of the fiscal measures, increasing government spending is the most effective fiscal measure in pushing the nominal interest rate above zero and rescuing the economy from the liquidity trap instantly. But cutting the labour income tax is also effective in raising the nominal interest rate and has a distinct advantage in stimulating the private consumption, investment and controlling the government debt. Hence, the remaining part of this section will further evaluate the welfare effects of these two fiscal measures to find out the most effective fiscal policy in getting the economy out of the binding ZLB regime and, at the same time, recovering the economy during the recession.

### 6.4.2 The analysis of welfare effect

This section uses an ad hoc loss function to evaluate fiscal policies. The objective of the government is to minimise the following quadratic loss function:

$$Loss = 0.5 * Var(Y) + 0.5 * Var(\pi).$$

The variances are obtained by applying 6.2% and 9.8% deviation of government spending and labour income tax shocks in a binding ZLB regime, respectively, which are generated above and find how much variance is for output and inflation.

Table 6.6 The results for welfare effect

| Fiscal policy | $Var(Y)$ | $Var(\pi)$ | $Loss$ |
|---|---|---|---|
| $G$ | 0.0133 | 0.0098 | 0.0116 |
| $\tau^w$ | 0.0092 | 0.0101 | 0.0096 |

The results indicate that the welfare loss of increasing the government spending is higher than that of cutting the labour income tax, which implies that the relaxation in the labour income tax is the most effective fiscal measure to stabilize inflation and the output jointly. Although government spending policy is slightly more anti-inflationary than labour income tax policy, labour income tax policy performances much better in controlling the volatility of the output. This conclusion is in line with Mattesini and Rossi (2012) who argue that labour income tax acts as a significant automatic stabilizer that the higher is the degree of the labour income tax, the lower is the volatility of the output and inflation. Besides, Auerbach and Feenberg (2000) also find the empirical evidence to support that the labour income tax could improve the output stabilization through its impact on labour supply and on aggregate demand.

After discussing the welfare effects of these two effective fiscal policies, we can also assess the measures by calculating the fiscal multipliers in the binding ZLB regime to supplement the analysis above (see Table 6.7).

Table 6.7 The results for fiscal multipliers

| Fiscal policy | Multipliers |
|---|---|
| $G$ | 1.0045 |
| $\tau^w$ | 0.0467 |

It shows that the value of impact government spending multiplier is greater than the value of labour income tax multiplier, which implies that government spending has more impact on output. This result is not distinctly different from what has been found in welfare analysis since the variance of output responding to the changes in government spending is also greater than that to the labour income tax. The difference just happens in the variance of the inflation, which suggests that the ability of government spending to stabilise inflation is slightly greater than the other fiscal measure.

There are two criteria, the welfare effect and fiscal multiplier, for choosing the policies out of these two fiscal measures. The choice is determined by what is concern by the policymaker. If the economic recovery and expansion is more concern, increasing government spending will be a better choice. While if the government is more attentive to minimise the welfare loss and improve the social welfare, raising the labour income tax is more effective than the other fiscal measure.

## 6.5 Concluding remarks

This chapter presents the methodology of the Indirect Inference method to test and re-estimate the structural DSGE model with the presence of financial frictions to explain the behaviour of the U.K. economy between 1989 and 2017 more precisely. Having tested this small open economy model, the result suggests that the Indirect Inference method is better than Bayesian technique in searching for an appropriate set of coefficients to mimic the behaviour of the model economy over the actual data by minimising the distance between the simulated-data-based model and actual-data-based model. Specifically, the Indirect Inference estimated model performances well in explaining the behaviour of the output, interest rate and inflation jointly over the sample period. Furthermore, the test result of the model with Indirect Inference estimates implies a comfortable acceptance of the model for this sample period at 5% significant level. Besides, the Monte Carlo experiments prove that the Indirect Inference test is a powerful test to rely on. Summarily, the model featuring an imperfect financial market can match the time-series properties of the U.K. data with the Indirect Inference estimates.

This chapter also discusses the effectiveness and implementation of the fiscal policy in terms of two aspects. Firstly, analysing the fiscal policy in terms of a conventional method by generating the impulse response functions to fiscal shocks. It suggests that that fiscal stimulus policy is effective in improving the economic growth in this model. Secondly, this chapter considers the cases of the recession and liquidity trap and evaluates the effects of the fiscal measures on coping with the zero interest rate and also on improving the economy. It is found that all of these fiscal measures are expansionary and effective in shortening the duration of the liquidity trap. More specifically, increasing government spending and labour income tax are the most useful fiscal policies in helping the economy escape from the liquidity trap. Then, by comparing the welfare effects of these two measures, it is concluded that increasing the labour income tax performances better than increasing the government speeding in stabilizing the output and inflation jointly regarding a welfare-based analysis.

# Chapter 7 Conclusion

The motivation behind this thesis is to investigate the effects of the UK post-crisis fiscal policy on the economies when the nominal interest rate hits the zero bound and whether the policy could have done more to help the economic recovery. To achieve this goal, this thesis introduces a New Keynesian DSGE model with government sector built-in. Due to the inability of the conventional monetary policy in the recession, Chapter 4 builds a small open economy DSGE model of the U.K. economy with zero lower bound to evaluate the effect of the fiscal policy in the liquidity trap. Particularly, the fiscal rules designed in this thesis are defined with two essential roles. On one hand, the fiscal rules serve as the automatic stabilizers to respond to the degree of economic activities. On the other hand, they are required to keep the government debt under control by responding to the fluctuation in the debt level. This thesis mainly focuses on various distortionary taxes and government expenditure measures.

The baseline model in Chapter 4 assumes that the financial markets are perfect, however, since data sample used also covers the period of the financial crisis, it is necessary to investigate whether the dynamics of the model and its implications would differ much by including the financial imperfections. Remarked by Fernández-Villaverde (2010) and Eggertsson and Krugman (2012), the effectiveness of the fiscal policy is fairly sensitive to the imperfection of the credit market. Thus, Chapter 5 extends the baseline model based on the financial accelerator mechanism of BGG and assumes that the existence of financial frictions impacts the magnitude and persistence of economic fluctuations by two channels, the financial accelerator and debt deflation channels.

Having done the previous work with the Bayesian estimation, I am aware that this method does not test the fitness of the model against the data and thus provides the model with no statistical power to facilitate the policy recommendations. To verify the model further, Chapter 6 aims to re-estimate and evaluate it using the Indirect Inference method. In this further work, to avoid losing data information due to arbitrary filter, the data is used in the unfiltered style. The method verified that the model estimated by the Bayesian estimation cannot mimic the dynamic properties of the main macroeconomic data. While, the behaviours of the output, interest rate and inflation over the sample period are well explained by the Indirect Inference estimated model. Besides, the experiments indicate that the Indirect Inference test is highly powerful and reliable.

Generally, the fiscal policy experiments in this thesis provide the following findings. Firstly, the expansionary fiscal policies including increasing government expenditures and cutting the consumption tax are more effective during the liquidity trap, since the monetary policy accommodates these fiscal expansions by allowing for a decline in the real interest rate which encourages the private spending and stimulates the economy. While cutting the labour income tax is expansionary in a normal case but contractionary in the binding ZLB regime, which is the same case for cutting the capital income tax in the model with perfect financial markets. The reason could be explained as that cutting these taxes at a zero interest rate may generate deflationary effects which are harmful to the economy in the liquidity trap. These results are in line with several standard New Keynesian analyses of fiscal policy, such as Eggertsson and Woodford (2004), Eggertsson (2011), Christiano, Eichenbaum and Rebelo (2011) and Boneva, Braun and Waki (2016). Secondly, when there is asymmetric information in the credit market, the fiscal policies can be very effective in the binding ZLB regime when they focus on strengthening the balance sheet of firms because, by doing so, they can stimulate the investment and then boost the economic activity. In addition to promoting the economy by financial accelerator channel, the impacts of fiscal measures can also be amplified by the debt deflation channel, since the inflationary effects derived by the fiscal policy could reduce the real cost of debt and in turn ameliorate entrepreneurial net worth. Thus, it is concluded that the presence of financial frictions could magnify the effects of fiscal policy, specifically in the binding ZLB regime, which is also the finding of Fernández-Villaverde (2010), Eggertsson and Krugman (2012) and Carrillo and Poilly (2013). Thirdly, after improving the fitness of the model to the data sample, the empirical results indicate that the fiscal policy did not contribute much to the U.K. recovery in the recession and gets the economy out of the binding ZLB regime. If the liquidity trap is concerned, the policy experiments suggest that increasing the government spending and labour income tax are the most useful policy measures when only the fiscal policy can be relayed on to restore the economy. Meanwhile, the labour income tax performances well in stabilizing the volatility of the output and inflation and serve as an important automatic stabilizer, which is also proved by Auerbach and Feenberg (2000) and Mattesini and Rossi (2012).

Summarily, the contributions of this thesis could be classified into two aspects. Theoretically, it introduces a New Keynesian DSGE model with government sector to describe the U.K. economy. It also considers the cases of the liquidity trap and the credit crunch to evaluate the effectiveness and implications of the fiscal policy in the recession. Technically, this thesis applies two approaches, the Bayesian and Indirect Inference methods, to estimate the model. The Bayesian estimated model could incorporate background knowledge and update the previous understanding with new information. While the Indirect Inference method provides a classical statistical inferential framework to improve the model fitness. Hence, although it is convenient and popular to apply Bayesian method nowadays, it is still more prudent to take a further study on the fiscal policy implications relying on the Indirect inference estimated model.

# Appendix 1 Log-linearized model list for Chapter 4

Euler equation

$$\left(\hat{C}_{t+1}-\hat{C}_t\right)+\frac{\tau^c}{1+\tau^c}(\hat{\tau}^c_{t+1}-\hat{\tau}^c_t)=\hat{R}_t-\hat{\pi}_{t+1}$$

Labour supply

$$\hat{C}_t+\frac{\tau^c}{1+\tau^c}\hat{\tau}^c_t+\hat{\mu}^e_t=\hat{w}_t-\frac{\tau^w}{1-\tau^w}\hat{\tau}^w_t$$

Uncovered interest rate parity

$$\hat{R}_t-\hat{R}^f_t=\hat{S}_{t+1}-\hat{S}_t-\lambda_\varphi\hat{A}^f_t+\hat{\chi}_t$$

Value of capital stock (Tobin's Q):

$$\hat{q}_t+\left(\hat{C}_{t+1}-\hat{C}_t\right)+\frac{\tau^c}{1+\tau^c}(\hat{\tau}^c_{t+1}-\hat{\tau}^c_t)$$

$$=\ \beta\left[r^k\frac{\tau^k}{\tau^k-1}\left(\hat{\tau}^k_{t+1}+\hat{r}^k_{t+1}\right)+\ \ \delta\tau^k\hat{\tau}^k_{t+1}+(1-\delta)\hat{q}_{t+1}\right]$$

Investment Euler equation

$$\hat{I}^p_t=\frac{\beta}{1+\beta}\hat{I}^p_{t+1}+\frac{1}{1+\beta}\hat{I}^p_{t-1}+\frac{1}{\phi+\phi\beta}(\hat{q}_t+\mu^{Ip}_t)$$

Capital accumulation equation

$$\hat{K}^p_t=(1-\delta)\hat{K}^p_{t-1}+\delta\hat{I}^p_t$$

Aggregate production equation

$$\hat{Y}_t=\hat{A}_t+\alpha_1\hat{K}^p_t+(1-\alpha_1)\hat{L}_t+\alpha_2\hat{K}^g_t$$

Labour demand (hours) equation

$$\hat{L}_t=\hat{r}^k_t+\hat{K}^p_t-\hat{w}_t$$

Consumer price index

$$\hat{\pi}_t=(1-\alpha^m)\hat{\pi}^d_t+\alpha^m\hat{\pi}^m_t$$

New Keynesian Philips Curve

$$\hat{\pi}^d_t=\frac{\beta}{1+\beta\kappa_d}E_t\hat{\pi}^d_{t+1}+\frac{\kappa_d}{1+\beta\kappa_d}\hat{\pi}^d_{t-1}+\frac{(1-\beta\xi_d)(1-\xi_d)}{(1+\beta\kappa_d)\xi_d}\widehat{mc}_t$$

$$\hat{\pi}^m_t=\frac{\beta}{1+\beta\kappa_m}E_t\hat{\pi}^m_{t+1}+\frac{\kappa_m}{1+\beta\kappa_m}\hat{\pi}^m_{t-1}+\frac{(1-\beta\xi_m)(1-\xi_m)}{(1+\beta\kappa_m)\xi_m}\widehat{mc}^m_t$$

$$\hat{\pi}_t^x = \frac{\beta}{1+\beta\kappa_x} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\beta\kappa_x}\hat{\pi}_{t-1}^x + \frac{(1-\beta\xi_x)(1-\xi_x)}{(1+\beta\kappa_x)\xi_x}\widehat{mc}_t^x$$

$$\widehat{mc}_t^x = \hat{\pi}_t^d - \hat{\pi}_t^x - \hat{S}_t + \hat{S}_{t+1} + \widehat{mc}_{t-1}^x$$

Real gross domestic product

$$\hat{Y}_t = \frac{Y^d}{Y}\hat{Y}_t^d + \frac{Y^x}{Y}\hat{Y}_t^x$$

Intermediate goods demand equation

$$\hat{Y}_t^d = -\theta\hat{\psi}_t^d + \hat{Z}_t$$

$$\hat{Y}_t^m = -\theta\hat{\psi}_t^m + \hat{Z}_t$$

$$\hat{Y}_t^x = -\eta\hat{\psi}_t^x + \hat{Z}_t^f$$

Relative prices

$$\hat{\psi}_t^d = \hat{\pi}_t^d - \hat{\pi}_t + \hat{\psi}_{t-1}^d$$

$$\hat{\psi}_t^m = \hat{\pi}_t^m - \hat{\pi}_t + \hat{\psi}_{t-1}^m$$

$$\hat{\psi}_t^x = \hat{\pi}_t^x - \hat{\pi}_t + \hat{\psi}_{t-1}^x$$

$$\hat{\psi}_t^f = \hat{\pi}_t^f - \hat{\pi}_t + \hat{\psi}_{t-1}^f$$

Monetary policy Taylor rule

$$\hat{R}_t^{tr} = \gamma^r \hat{R}_{t-1}^{tr} + (1-\gamma^r)\left(\gamma^\pi\hat{\pi}_t^d + \gamma^y\hat{Y}_t\right) + \hat{\mu}_t^{tr}$$

Zero lower bound

$$\hat{R}_t = max\left[0, \hat{R}_t^{tr}\right]$$

Government budget constraint

$$G\hat{G}_t + I^g\hat{I}_t^g + bR\left(\hat{b}_{t-1} - \hat{\pi}_t + \hat{R}_{t-1}\right) + TR\widehat{TR}_t$$

$$= b\hat{b}_t + \tau^c C\left(\hat{\tau}_t^c + \hat{C}_t\right) + \tau^w wL\left(\hat{\tau}_t^w + \hat{w}_t + \hat{L}_t\right)$$

$$+ \tau^k R_t^k K^p\left(\hat{\tau}_t^k + \hat{R}_t^k + \hat{K}_{t-1}^p\right) - \delta\tau^k K^p(\hat{\tau}_t^k + \hat{K}_{t-1}^p)$$

Public capital accumulation equation

$$\hat{K}_t^g = \left(1-\delta_g\right)\hat{K}_{t-1}^p + \delta\hat{I}_t^g$$

Fiscal rules

$$\hat{G}_t = -\gamma^{g,b}\hat{b}_{t-1} - \gamma^{g,y}\hat{Y}_t + \hat{\mu}_t^g$$

$$\hat{I}_t^g = -\gamma^{i,b}\hat{b}_{t-1} - \gamma^{i,y}\hat{Y}_t + \hat{\mu}_t^{ig}$$

$$\widehat{TR}_t = -\gamma^{t,b}\hat{b}_{t-1} - \gamma^{t,y}\hat{Y}_t + \hat{\mu}_t^{tra}$$

$$\hat{\tau}_t^c = \gamma^{c,b}\hat{b}_{t-1} + \gamma^{c,y}\hat{Y}_t + \hat{\mu}_t^c$$

$$\hat{\tau}_t^k = \gamma^{k,b}\hat{b}_{t-1} + \gamma^{k,y}\hat{Y}_t + \hat{\mu}_t^k$$

$$\hat{\tau}_t^w = \gamma^{w,b}\hat{b}_{t-1} + \gamma^{w,y}\hat{Y}_t + \hat{\mu}_t^w$$

Aggregate demand equation

$$\hat{Z}_t = \frac{C}{Z}\hat{C}_t + \frac{I^p}{Z}\hat{I}_t^p + \frac{I^g}{Z}\hat{I}_t^g + \frac{G}{Z}\hat{G}_t$$

Evolution of net foreign assets position

$$\frac{Y^x}{Y}\left(\hat{S} + \hat{\psi}_t^x + \hat{Y}_t^x - \hat{Y}_t\right) - \frac{Y^m}{Y}\left(\hat{S} + \hat{\psi}_t^f + \hat{Y}_t^m - \hat{Y}_t\right) = \hat{A}_t^f - R^f * \hat{A}_{t-1}^f$$

AR (1) shock processes

$$\hat{A}_t = \rho^A\hat{A}_{t-1} + \sigma_t^A$$

$$\hat{\mu}_t^{pre} = \rho^{pre}\hat{\mu}_{t-1}^{pre} + \sigma_t^{pre}$$

$$\hat{\mu}_t^{ip} = \rho^{ip}\hat{\mu}_{t-1}^{ip} + \sigma_t^{ip}$$

$$\hat{\mu}_t^{tr} = \rho^{tr}\hat{\mu}_{t-1}^{tr} + \sigma_t^{tr}$$

$$\hat{\chi}_t = \rho^{\chi}\hat{\chi}_{t-1} + \sigma_t^{\chi}$$

$$\hat{R}_t^f = \rho^{Rf}\hat{R}_{t-1}^f + \sigma^{Rf}$$

$$\hat{\pi}_t^f = \rho^{\pi f}\hat{\pi}_{t-1}^f + \sigma^{\pi f}$$

$$\hat{Z}_t^f = \rho^{Zf}\hat{Z}_{t-1}^f + \sigma^{Zf}$$

$$\hat{\mu}_t^g = \rho^g\hat{\mu}_{t-1}^g + \sigma_t^g$$

$$\hat{\mu}_t^{ig} = \rho^{ig}\hat{\mu}_{t-1}^{ig} + \sigma^{ig}$$

$$\hat{\mu}_t^{tra} = \rho^t\hat{\mu}_{t-1}^{tra} + \sigma_t^{tra}$$

$$\hat{\mu}_t^c = \rho^c\hat{\mu}_{t-1}^c + \sigma^c$$

$$\hat{\mu}_t^k = \rho^k\hat{\mu}_{t-1}^k + \sigma^k$$

$$\hat{\mu}_t^w = \rho^w\hat{\mu}_{t-1}^w + \sigma^w$$

## Appendix 2 Log-linearized model list for Chapter 5

Marginal utility of consumption

$$\hat{\lambda}_t = \frac{1}{(1-\beta h)(1-h)}\left[\beta h\left(\hat{C}_{t+1} - h\hat{C}_t\right) - \left(\hat{C}_t - h\hat{C}_{t-1}\right)\right] - \frac{\tau^c}{1+\tau^c}\hat{\tau}_t^c$$

Euler equation

$$\hat{\lambda}_{t+1} - \hat{\lambda}_t = \hat{\pi}_{t+1} - \hat{R}_t$$

Labour supply

$$\hat{\lambda}_t = \hat{\mu}_t^e + \psi\hat{L}_t - \widehat{w}_t + \frac{\tau^w}{1-\tau^w}\hat{\tau}_t^w$$

Uncovered interest rate parity

$$\hat{R}_t - \hat{R}_t^f = E_t\hat{S}_{t+1} - \hat{S}_t - \lambda_\varphi\hat{A}_t^f + \hat{\chi}_t$$

Capital accumulation equation

$$\widehat{K}_t^p = (1-\delta)\widehat{K}_{t-1}^p + \delta\hat{I}_t^p$$

Investment Euler equation

$$\hat{I}_t^p = \frac{\beta}{1+\beta}\hat{I}_{t+1}^p + \frac{1}{1+\beta}\hat{I}_{t-1}^p + \frac{1}{\phi+\phi\beta}\left(\hat{q}_t + \mu_t^{Ip}\right)$$

Aggregate production equation

$$\hat{Y}_t = \hat{A}_t + \alpha_1\widehat{K}_t^p + (1-\alpha_1)\hat{L}_t + \alpha_2\widehat{K}_t^g$$

Labour demand (hours) equation

$$\hat{L}_t = \hat{r}_t^k + \widehat{K}_t^p - \widehat{w}_t$$

Capital demand equation

$$\widehat{f}_t = \frac{(1-\tau^k)R^k\hat{R}_t^k - \tau^k(R^k-\delta)\hat{\tau}_t^k + (1-\delta)\hat{Q}_t - [(1-\tau^k)R^k + (1-\delta)]\hat{Q}_{t-1}}{(1-\tau^k)R^k + (1-\delta) + \delta\tau^k}$$

External finance premium equation

$$E_t\hat{f}_{t+1} - (\hat{R}_t - \hat{\pi}_{t+1}) = \omega\left(\hat{Q}_t + \widehat{K}_t^p - \widehat{NW}_t\right) + \hat{\mu}_t^{ep}$$

Evolution of entrepreneur's net worth

$$\frac{\widehat{NW}_t - \hat{\mu}_t^{nw}}{\kappa f} = \frac{K^p}{NW}\hat{f}_t - \omega\left(\frac{K^p}{NW} - 1\right)\left(\hat{Q}_t + \widehat{K}_t^p\right) - \left(\frac{K^p}{NW} - 1\right)\left(\hat{R}_{t-1} - \hat{\pi}\right) + \left[\omega\left(\frac{K^p}{NW} - 1\right) + 1\right]\widehat{NW}_{t-1}$$

Consumer price index

$$\hat{\pi}_t = (1-\alpha^m)\hat{\pi}_t^d + \alpha^m \hat{\pi}_t^m$$

New Keynesian Philips Curve

$$\hat{\pi}_t^d = \frac{\beta}{1+\beta\kappa_d} E_t \hat{\pi}_{t+1}^d + \frac{\kappa_d}{1+\beta\kappa_d} \hat{\pi}_{t-1}^d + \frac{(1-\beta\xi_d)(1-\xi_d)}{(1+\beta\kappa_d)\xi_d} \widehat{mc}_t$$

$$\hat{\pi}_t^m = \frac{\beta}{1+\beta\kappa_m} E_t \hat{\pi}_{t+1}^m + \frac{\kappa_m}{1+\beta\kappa_m} \hat{\pi}_{t-1}^m + \frac{(1-\beta\xi_m)(1-\xi_m)}{(1+\beta\kappa_m)\xi_m} \widehat{mc}_t^m$$

$$\hat{\pi}_t^x = \frac{\beta}{1+\beta\kappa_x} E_t \hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\beta\kappa_x} \hat{\pi}_{t-1}^x + \frac{(1-\beta\xi_x)(1-\xi_x)}{(1+\beta\kappa_x)\xi_x} \widehat{mc}_t^x$$

$$\widehat{mc}_t^x = \hat{\pi}_t^d - \hat{\pi}_t^x - \hat{S}_t + \hat{S}_{t+1} + \widehat{mc}_{t-1}^x$$

Real gross domestic product

$$\hat{Y}_t = \frac{Y^d}{Y}\hat{Y}_t^d + \frac{Y^x}{Y}\hat{Y}_t^x$$

Intermediate goods demand equation

$$\hat{Y}_t^d = -\theta\hat{\psi}_t^d + \hat{Z}_t$$

$$\hat{Y}_t^m = -\theta\hat{\psi}_t^m + \hat{Z}_t$$

$$\hat{Y}_t^x = -\eta\hat{\psi}_t^x + \hat{Z}_t^f$$

Relative prices

$$\hat{\psi}_t^d = \hat{\pi}_t^d - \hat{\pi}_t + \hat{\psi}_{t-1}^d$$

$$\hat{\psi}_t^m = \hat{\pi}_t^m - \hat{\pi}_t + \hat{\psi}_{t-1}^m$$

$$\hat{\psi}_t^x = \hat{\pi}_t^x - \hat{\pi}_t + \hat{\psi}_{t-1}^x$$

$$\hat{\psi}_t^f = \hat{\pi}_t^f - \hat{\pi}_t + \hat{\psi}_{t-1}^f$$

Monetary policy Taylor rule

$$\hat{R}_t^{tr} = \gamma^r \hat{R}_{t-1}^{tr} + (1-\gamma^r)\left(\gamma^\pi \hat{\pi}_t^d + \gamma^y \hat{Y}_t\right) + \hat{\mu}_t^{tr}$$

Zero lower bound

$$\hat{R}_t = max\left[0, \hat{R}_t^{tr}\right]$$

Government budget constraint

$$G\hat{G}_t + I^g\hat{I}_t^g + bR\left(\hat{b}_{t-1} - \hat{\pi}_t + \hat{R}_{t-1}\right) + TR\widehat{TR}_t$$

$$= b\hat{b}_t + \tau^c C\left(\hat{\tau}_t^c + \hat{C}_t\right) + \tau^w wL\left(\hat{\tau}_t^w + \hat{w}_t + \hat{L}_t\right)$$

$$+ \tau^k R_t^k K^p\left(\hat{\tau}_t^k + \hat{R}_t^k + \hat{K}_{t-1}^p\right) - \delta\tau^k K^p(\hat{\tau}_t^k + \hat{K}_{t-1}^p)$$

Public capital accumulation equation

$$\hat{K}_t^g = \left(1 - \delta_g\right)\hat{K}_{t-1}^p + \delta\hat{I}_t^g$$

Fiscal rules

$$\hat{G}_t = -\gamma^{g,b}\hat{b}_{t-1} - \gamma^{g,y}\hat{Y}_t + \hat{\mu}_t^g$$

$$\hat{I}_t^g = -\gamma^{i,b}\hat{b}_{t-1} - \gamma^{i,y}\hat{Y}_t + \hat{\mu}_t^{ig}$$

$$\widehat{TR}_t = -\gamma^{t,b}\hat{b}_{t-1} - \gamma^{t,y}\hat{Y}_t + \hat{\mu}_t^{tra}$$

$$\hat{\tau}_t^c = \gamma^{c,b}\hat{b}_{t-1} + \gamma^{c,y}\hat{Y}_t + \hat{\mu}_t^c$$

$$\hat{\tau}_t^k = \gamma^{k,b}\hat{b}_{t-1} + \gamma^{k,y}\hat{Y}_t + \hat{\mu}_t^k$$

$$\hat{\tau}_t^w = \gamma^{w,b}\hat{b}_{t-1} + \gamma^{w,y}\hat{Y}_t + \hat{\mu}_t^w$$

Aggregate demand equation

$$\hat{Z}_t = \frac{C}{Z}\hat{C}_t + \frac{I^p}{Z}\hat{I}_t^p + \frac{I^g}{Z}\hat{I}_t^g + \frac{G}{Z}\hat{G}_t$$

Evolution of net foreign assets position

$$\frac{Y^x}{Y}\left(\hat{S} + \hat{\psi}_t^x + \hat{Y}_t^x - \hat{Y}_t\right) - \frac{Y^m}{Y}\left(\hat{S} + \hat{\psi}_t^f + \hat{Y}_t^m - \hat{Y}_t\right) = \hat{A}_t^f - R^f * \hat{A}_{t-1}^f$$

AR (1) shock processes

$$\hat{A}_t = \rho^A\hat{A}_{t-1} + \sigma_t^A$$

$$\hat{\mu}_t^{pre} = \rho^{pre}\hat{\mu}_{t-1}^{pre} + \sigma_t^{pre}$$

$$\hat{\mu}_t^{ip} = \rho^{ip}\hat{\mu}_{t-1}^{ip} + \sigma_t^{ip}$$

$$\hat{\mu}_t^{tr} = \rho^{tr}\hat{\mu}_{t-1}^{tr} + \sigma_t^{tr}$$

$$\hat{\chi}_t = \rho^{\chi}\hat{\chi}_{t-1} + \sigma_t^{\chi}$$

$$\hat{R}_t^f = \rho^{Rf}\hat{R}_{t-1}^f + \sigma^{Rf}$$

$$\hat{\pi}_t^f = \rho^{\pi f}\hat{\pi}_{t-1}^f + \sigma^{\pi f}$$

$$\hat{Z}_t^f = \rho^{Zf}\hat{Z}_{t-1}^f + \sigma^{Zf}$$

$$\hat{\mu}_t^{ep} = \rho^{ep}\hat{\mu}_{t-1}^{ep} + \sigma_t^{ep}$$

$$\hat{\mu}_t^{nw} = \rho^{nw}\hat{\mu}_{t-1}^{nw} + \sigma_t^{nw}$$

$$\hat{\mu}_t^{g} = \rho^{g}\hat{\mu}_{t-1}^{g} + \sigma_t^{g}$$

$$\hat{\mu}_t^{ig} = \rho^{ig}\hat{\mu}_{t-1}^{ig} + \sigma^{ig}$$

$$\hat{\mu}_t^{tra} = \rho^{t}\hat{\mu}_{t-1}^{tra} + \sigma_t^{tra}$$

$$\hat{\mu}_t^{c} = \rho^{c}\hat{\mu}_{t-1}^{c} + \sigma^{c}$$

$$\hat{\mu}_t^{k} = \rho^{k}\hat{\mu}_{t-1}^{k} + \sigma^{k}$$

$$\hat{\mu}_t^{w} = \rho^{w}\hat{\mu}_{t-1}^{w} + \sigma^{w}$$

# Appendix 3 The methodology of variance and historical decomposition

Variance decomposition is a classical statistical approach to quantify the contribution of each disturbance to the variation of endogenous variables in a reduced form of the structural model. In other words, the variance of the forecast error for each variable is calculated as the proportion of the deviation generated by each structural shock at a specific time horizon, which can be written as follows:

$$\phi_{i,j}(h) = \frac{\omega_{i,j}(h)}{\Omega_i(h)},$$

where $\omega_{i,j}(h)$ refers to the forecast error variance of variable $i$ caused by shock $j$ at time horizon $h$; $\Omega_i(h)$ denotes the total effect. Particularly, $\omega_{i,j}(h)$ is equal to:

$$\omega_{i,j}(h) = \sum_{k=0}^{h} IMP_{i,j}(k)^2,$$

where $IMP_{i,j}(k)$ stands for the impulse response of variable $i$ to shock $j$ at period $k$.

To clarify the methodology of the historical decomposition, it is assumed that an estimated model is represented in a reduced form of the structural VAR model as follows:

$$y_t = A_0 y_t + A_1 y_{t-1} + \cdots + A_p y_{t-p} + v_t, \qquad (A.1)$$

where $A_i$ denotes the structural coefficients and $v_t$ stands for the structural shocks. Suppose the reduced form of the shocks is written as $u_t = (I - A_0)^{-1} v_t$ and $A_i^*$ represents the reduced form of coefficient matrices. Define $C(L) = A^*(L)^{-1}$, with $C_0 = I$, is the moving average matrix, thus the moving average representation of equation (A.1) is derived as:

$$y_t = A^*(L)^{-1} u_t = C(L) u_t = \sum_{s=0}^{\infty} C_s u_{t-s}, \qquad (A.2)$$

Equation (A.2) is organized regarding the reduced form of shocks. In terms of the structural shocks, it can be represented as:

$$y_t = \sum_{s=0}^{\infty} [C_s (I - A_0)^{-1}](I - A_0) u_{t-s} = \sum_{s=0}^{\infty} D_s \, e_{t-s}, \qquad (A.3)$$

where $e_t = (I - A_0) u_t$ and $D_s = C_s (I - A_0)^{-1}$. Therefore, for a specific period $t + j$, equation (A.3) can be described as:

$$y_{t+j} = \sum_{s=0}^{j-1} D_s e_{t+j-s} + \sum_{s=j}^{\infty} D_s e_{t+j-s}, \qquad (A.4)$$

which denotes the historical decomposition. The decomposition above is comprised of two terms. The second term on the right-hand side of equation (A.4) reveals the expectation of $y_{t+j}$ given the information at period $t$, which is defined as the 'base projection' of vector $y$. While the first term describes the gap between the actual data and the base projection owning to the structural innovations in the variables after period $t$. Thus, the actual data at period $t + j$ are the base projection adding the weighted structural innovations to the variables. In this section, the starting period $t$ is set as 2006Q4, and the sample period lasts 44 quarters until 2017Q2.

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