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## A Pragmatic Approach to the Design of Bullwhip Controllers

Li Zhou, Stephen Disney, and Denis R. Towill

### Abstract

It is well known that forecasting mechanisms can greatly increase “bullwhip” - demand variance amplification of orders as processed by both human and algorithmic decision makers. This paper is concerned with the application of the well established APIOBPCS Decision Support System (a variant of the Order-Up-To Rule) in such circumstances. It has two feedback controls (based on the inventory and the orders-in-pipeline respectively) with gains set equal according to the Deziel-Eilon Rule. There is one feed-forward control based on exponential forecasting, although this is not a restriction on the application of this system.

We consider the pragmatic role of APIOBPCS in the situation where the echelon decision maker may be handling a wide range of SKU's in a non-altruistic environment where upmarket information may either be withheld or simply unavailable. Under such circumstances it has been established via site-based studies that the decision makers output (the orders) reflect a wide range of strategies (or maybe ignorance). Three strategies may be regarded as “appropriate”, i.e. Pass-orders-Along; Demand Smoothing; and Level Scheduling depending, on context. APIOBPCS can be adapted to each of these *modus operandi*. In the first case with the added capability of smoothing the “sharp edges” with a modicum of inventory variation, and in the last case with the advantage of built-in trend detection.

“Players” in non-altruistic supply chains must be able to cope with added uncertainties due to lead-time variations. We show that APIOBPCS may be well matched to such situations and is hence “copable” as well as “capable”. The paper includes recommended parameter settings according to desired decision-making policies.

### 1. Introduction

1.1. The extent to which “demand amplification” disturbs supply chain activities is partly due to external factors, and partly due to decision making on behalf of the

various “players” in the system. External factors include interactions with both customers and suppliers (Larsen et al. 1999). The extent of high fidelity information flows is a key factor in interface management at the “front end” and the “back end” of the chain. Also important are the minimisation of customer schedule changes and the supply of materials, in which all must be present and correct (Corbett et al. 1999). Comprehensive summaries of bullwhip causes are given in Lee et al. (1997) with an updated and extended list in Geary et al. (2006). Quality of forecasting is seen as an important factor in its control, as identified by Lambrecht and Dejonckheere (1999) and Chen et al. (2000). Subsequently in a paper by Dejonckheere et al. (2002) an analytical assessment of the impact of forecasting was established via transfer function analysis. The present contribution continues that approach.

Supply Chain Process	HVLM Product Group	LVHM Product Group
	$\sigma/\mu$	$\sigma/\mu$
Consumer Purchases	0.1	0.07
Shop Orders	0.26	0.22
Wholesale Orders	0.75	0.67
Factory Internal Orders	0.54	1.6
Factory Supplies to Market	0.9	22

**Table 1. Demand Amplification in a European Confectionary Supply Chain**  
(Source: Holmström, 1997)

1.2. Early practical proof of the intensity of the bullwhip effect was demonstrated by Holmström, (1997). He studied two value streams, a High Volume Low Margin (HVLM) product group and a Low Volume High Margin Product Group. Table 1 shows his estimates expressed as (standard deviation/average) at each echelon. It is obvious that in most instances there is substantial amplification at each stage. The exception is at the factory for scheduling HVLM where no doubt this decision has been made based on past experience, and “market savvy”. The practical effect of the data shown in Table 1 is that for HVLM the range of fluctuation about the average) for factory supplies is of the order of 9 to 1 greater than Point-of-Sale deviations for the same product group. On a similar basis the Low Volume, High Margin (LVHM) product range experiences an increase of 28.6 to 1. These comparisons, crude though they are, give a rough indication of what actually happens in a typical supply chain.

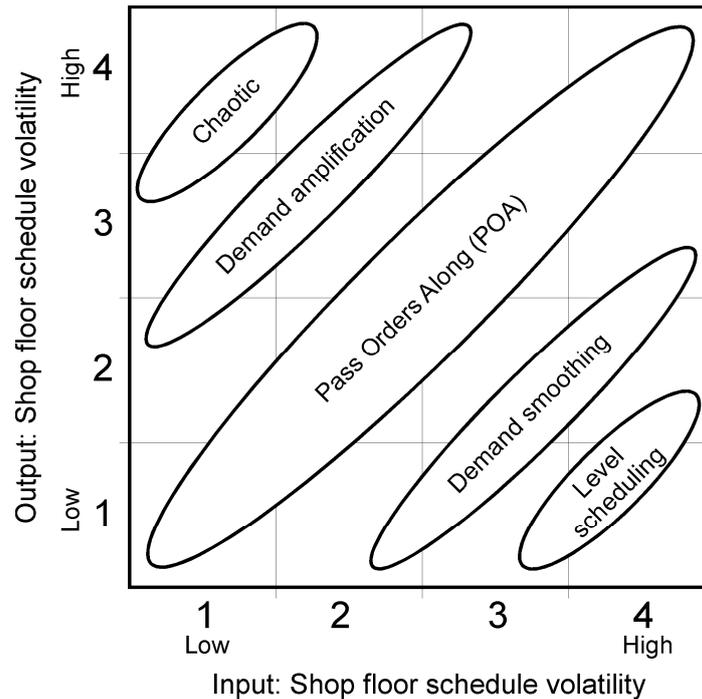
1.3. Bullwhip is potentially a very expensive occurrence (Metters, 1997). When “players” i.e. the various echelons work together altruistically then significant improvements in dynamic performance can result (Disney et al., 2008). This is much easier when operating as an “analytic corporation” (Davenport, 2006). Examples include Sport Obermeyer, (Fisher and Raman, 1996); Dell, (Kapuchinski et al. 2004) and Phillips Electronics, (de Kok et al. 2005). Many guidelines are available to decision makers in such supply chains, including Bertrand (1986), Wikner et al. (1991), Berry et al. (1995), Bonney et al. (1994), Chen et al. (2000), Edwards et al. (2001), Cachon and Lariviere, (2005), Chatfield et al (2004) and Hoberg et al. (2007). Note that bullwhip is the modern phrase coined by Lee et al. (1997) to describe a phenomenon labelled “demand amplification” by Jay Forrester, (1958). It is also well known in practice since at least as far back as 1919 when observed and damped down in Procter and Gamble supply chains (Schmenner, 2001).

1.4. In contrast to the foregoing problem solving for the analytic corporation, this paper is concerned with applications at the other end of the spectrum. Here the scenario may be poorly defined, customers may be short term i.e. “electronic auction” acquired (Busalacchi, 2001), or in the situation where many companies may need to intelligently trade-off capacity between “regular” and “occasional” customers (Potter et al. 2009) despite uncertainties clouding the issue. By “capable” we mean that the Decision Support System operating via readily remembered rules-of-thumb, will give acceptable rather than optimal performance. In other words, we seek a parameter space for designing “adequate” response at the expense of not selecting settings which we “absolutely best”, a theme first explored by Graham and Lathrop, (1953). The chosen DSS has a feed-forward path incorporating exponential forecasting and feedback controls utilising inventory and WIP levels. The paper shows that simple rules-of-thumb related to expected delivery lead times can also provide a “copable” bullwhip controller reacting well to uncertain operational requirements.

## **2. Real-World Bullwhip Scenario**

2.1. It has been evident since Jay Forrester’s, (1958) seminal research that bullwhip may exist at any echelon in a supply chain. Furthermore it is a multiplicative phenomenon. Hence as Holmström, (1997) demonstrated in Table 1 the effect is

greatly magnified when moving upstream from the marketplace. Later, in seeking to establish the comparative decision-making tactics of production schedulers, Childerhouse et al. (2008), defined the five categories of implied bullwhip generation shown in Figure 1. This was based on a site based assessment of the relationship between “our” customer demand and consequential volatility induced on the shop floor.



**Figure 1. Categories of Production Scheduler Strategies**

(Source: Childerhouse et al. 2008)

2.2. Pass Orders Along (POA) is an important logistics decision making benchmark in which, as the name implies, that perceived demand is relatively unfiltered when it hits the delivery process. Level Scheduling is another extreme favoured by the Toyota Production System (Womack et al. 1990) and means ignoring current demand and ensuring little volatility hits the shop floor. “Chaos” on the shop floor in the behavioural sense defined by Burger and Starbird, (2005) is self-explanatory. The intermediate options shown in Figure 1 are Demand Smoothing which reduces volatility somewhat, and (some) Demand Amplification, which may, depending on other system factors, have relative advantages in better customer service levels across a set of product groups (Potter et al. 2009).

2.3. What actually happens in practice? Table 2 shows the results of a site-based world-wide survey of 59 supply chains. The numerical volatility estimates thereby derived show that:

- All the posited categories of decision-making have been observed in industrial settings.
- The largest single grouping of 42% approximates to Pass Orders Along (POA)
- About 10% of the sample exhibit the extremes of behaviour (i.e. either Level Scheduling (LS) or Chaotic Response (CR)).
- 19% of the schedulers demonstrate Demand Amplification (DA).
- 29% practice Demand Smoothing (DS).

<b>Dynamics of Scheduler Strategy</b>	<b>Automotive Sector (n=22)</b>	<b>Non-Automotive Sector (n=37)</b>	<b>Total Sample (n=59)</b>
Level Scheduling	1 (5%)	1 (3%)	2 (3%)
Demand Smoothing	5 (22%)	12 (32%)	17 (29%)
Pass-on-Orders	10 (45%)	15 (40%)	25 (43%)
Demand Amplification	3 (14%)	8 (22%)	11 (19%)
Chaotic Response	3 (14%)	1 (13%)	4 (6%)

**Table 2. Classification of Observed Real-World Production Scheduler Strategies**  
(Source: Authors based on data from Childerhouse et al. 2009)

2.4. Hence it may be concluded that either by informed guesswork or by utilising a Decision Support System (DSS) that a significant percentage of Production Schedulers are in a major way influencing dynamic performance. Companies already operating as an “Analytical Corporation” (Davenport 2006) will already be using quite sophisticated software aids. However, not all bullwhip is “bad” in the sense that one product may sensibly be balanced against the needs of another within given capacity constraints (Potter et al. 2009). The need is to ensure “useful bullwhip” is thus constrained and does not cause unwanted disturbances to the delivery process. Such “interference” reduces both efficiency and effectiveness. It should therefore be designed out of the system (Burbidge, 1989).

### 3. An Industrial Perspective

3.1. It is a false assumption to suppose that all businesses are [yet] operating in the altruistic supply chain (Disney et al. 2008) where collaboration is the norm. Indeed the business may well be suffering from customers switching supplier allegiance according to the results of electronic auctions (Busalacchi, 1999) and have to keenly compete even in the absence of long term relationships with all or any of its suppliers. For example supermarkets are notorious for their fickle vendor relationships (Blythman, 2004). Nevertheless logistical and scheduling problems have to be solved over a time-scale somewhat larger than an individual product lead-time. How do such companies cope? The survey of 42 Australian businesses as studied by Buxey, (2001) in Table 3 is indicative of current practice. In particular there is some broad equivalence between the apparent modus operandi of this survey and the site-based investigation of Table 2.

3.2. It is very clear that many of these Australian companies face considerable uncertainty in product demand in all three critical dimensions, defined by Christopher and Towill, (2000). These are volume (*what is the average demand?*) volatility (*how much does the demand vary?*), and variety (*which particular versions are popular?*). These are complex issues for which scheduling answers are required in real-time. The need in such circumstances for a robust but simple DSS is best understood by reference to the seminal study of MIT Beer Game results by John Sterman, (1989). He analysed the performance of some 2000 “players” controlling the dynamic behaviour of a beer delivery value stream. Although it is frequently argued that the game is an over-simplification of the real-world, it nevertheless provides tremendous insight into human decision making in logistics. The most striking feature output from Sterman’s (1989) analysis was just how poorly some players performed; even some of the highly experienced executives were frequently caught out. But when faced with evidence from the actual model used in the game, many refused to believe that the bullwhip and alternating “boom and bust” situations resulted from their actions alone.

<b>PRODUCTION STRATEGY</b>	<b>TYPICAL OPERATIONAL CHARACTERISTICS</b>
<b>Level Scheduling Strategy (4 Companies)</b>	<ul style="list-style-type: none"> <li>• Limited variety of stable products with modest seasonality</li> <li>• Demand forecasting is fairly reliable</li> <li>• Little scope for volume flexibility</li> <li>• Marketing specifies the model mix each month</li> <li>• Marketing responsible for selling the annual production output</li> </ul>
<b>Chase Strategy (19 Companies)</b>	<ul style="list-style-type: none"> <li>• Predetermined strategy shapes production schedule</li> <li>• Predetermined strategy plans acquisition of resources</li> <li>• No attempt to balance marginal costs</li> <li>• Strategy means the business suffers minimum financial exposure</li> <li>• Retain enough flexibility to react quickly to seasonal demands</li> </ul>
<b>Modified Strategy (12 Companies)</b>	<ul style="list-style-type: none"> <li>• Practical amendment of chase strategy to track maximum demand</li> <li>• Planners engage in a limited form of stockpiling</li> <li>• Large orders have to be placed on long lead times</li> <li>• Big jobs may be slotted into the MPS earlier than needed</li> <li>• Work transferred from peak periods into slack ones.</li> </ul>
<b>Demand Management Strategy (4 Companies)</b>	<ul style="list-style-type: none"> <li>• This strategy appears the ideal way to tackle the seasonal problem</li> <li>• Strives to develop complementary product range</li> <li>• Two semi-independent production schedules result in a level workload</li> <li>• At facilities change-over adequate stock must be in place</li> </ul>
<b>Others (3 Companies)</b>	<ul style="list-style-type: none"> <li>• Dominated by labour and learning considerations</li> <li>• Hire and fire policy wasteful; need for stable core workforce</li> <li>• During the high season extensive overtime is scheduled</li> <li>• Only unskilled positions filled by temporary recruits</li> <li>• Limited idle time tolerated off season</li> </ul>

**Table 3. Typical Operational Characteristics of Production Strategy Options Evolving from Study of 42 Australian Manufacturers**  
(Based on the description by Buxey, 2001)

3.3. Hence as is frequently the case the “system” and not just the “players” wrongly got the blame. But just how bad were the players? To obtain a benchmark Serman, (1989) curve fitted and hence modelled the dynamic behaviour of each player using a simple but frequently used DSS formant. He also incorporated a comprehensive performance criterion which weighted a combination of order rate fluctuations and

stock level deviations. The baseline “anchor” decision regime he then adopted was the simple “Pass-Along-Orders” (PAO) benchmark. His statistical analysis of results are as follows:

- 75% of the players performed worse (some much worse) than simply using Pass-on-Orders as the DSS.

however

- 10% of the players significantly out-performed the performance achieved via adoption of PAO.

3.4. Obviously slightly different proportions of the “players” performance would result from variation of the weighting function. Nevertheless the logistical messages are clear:

- *It is very easy to get extremely poor performance from an inadequately tested DSS (whether algorithmic or not).*
- *Some decision rules (carefully matched to the operating scenario) will give much superior performance than POA.*
- *It is quite possible that acceptable (copable) performance will be output from using a relatively simple DSS.*

It is the aim of this paper to assist in obtaining workable solutions in these circumstances by adopting a pragmatic approach to parameter setting for such a Decision Support System. Typically this enables the scheduler to automate the decisions thereby output on (say) 95% of products (Lewis, 1997). Active involvement is required only occasionally, leaving the remnant of the “tricky” 5% of customer requirements to be very actively managed.

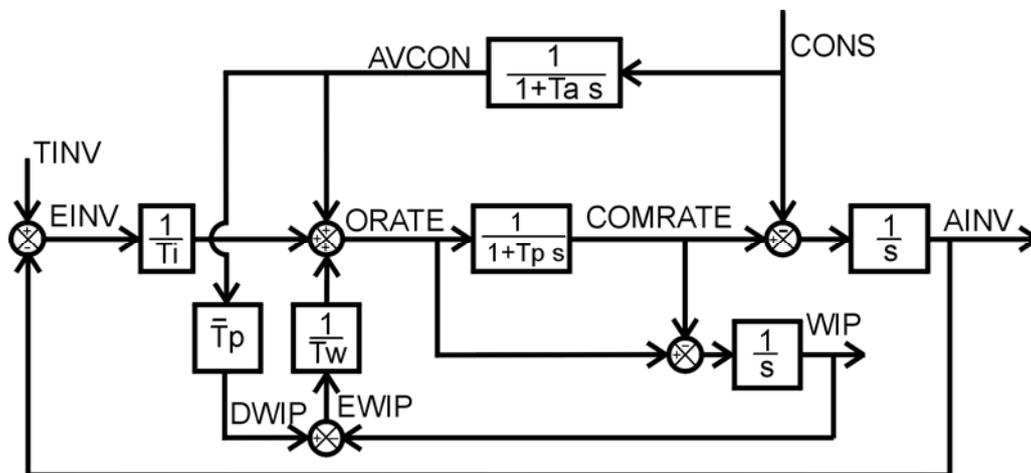
#### **4. The APIOBPCS Decision Support System (DSS)**

4.1. The particular DSS advocated and exploited in this paper is a special case of the Order-up-To Algorithm known as the Automatic Pipeline Inventory and Order-based Production System (APIOBPCS). It is designed to actively control delivery in the situation where the actual lead-time is  $T_p$ . This rule readily expressed in words as:

“Let the replenishment orders be equal to the sum of an exponentially smoothed demand (averaged over  $T_a$  time periods), plus a fraction

( $1/T_i$ ) of the inventory difference between target stock and actual stock, plus a fraction of ( $1/T_w$ ) of the difference between target ‘orders placed but not yet received’ and actual ‘orders placed but not yet received’ (John et al. 1994).

4.2. John et al. (1994) also highlight the importance of utilising the ‘best’ lead-time estimate,  $T_{\bar{p}}$ , currently available in setting the target ‘orders placed but not yet received’ (i.e. expected WIP level) if inventory drift is to be reduced. Figure 2 shows the corresponding block diagram in Laplace Operator ( $s$ ) format. There is one feed-forward control path (AVCON), and two feedback controls ( $EWIP/T_w$ ) and ( $EINV/T_i$ ). They compensate for errors in WIP and inventory respectively. Herein we are concerned with selecting  $T_a$ ,  $T_w$ , and  $T_i$  to provide good bullwhip and inventory controls simultaneously.  $T_a$  (the forecasting time constant) is related to  $\alpha$  (the conventional smoothing constant) via the simple formulae  $\alpha = (1 + \frac{\tau a}{\Delta t})^{-1}$  where  $\Delta t$  is the sampling period (Towill, 1977). A complete description of APIOBPCS variants is given in Disney and Towill, (2005) including system architecture and possible additional feed-forward and feedback controls such as the lead-time adaptive design proposed by Evans et al. (1997).



**Figure 2. Block Diagram of APIOBPCS Control System**  
(Source: John et al. 1994)

4.3. The APIOBPCS model encapsulates the general principles for replenishment rules as advocated by Popplewell and Bonney (1987). In particular it gives due prominence to the importance of including pipeline (WIP) feedback in replenishment decision-making, a factor further emphasised by Bonney (1990). Of course the APIOBPCS

principle is not a new concept. It is empirically well-established in industry (Coyle, 1977), and as already seen has the additional advantage of reasonably describing curve-fitted performance data from 2000 Beer Game ‘plays’ as elegantly modelled by Sterman (1989). Furthermore the particular variant known as the ‘to-make model’ has successfully if rather pragmatically controlled 6000 multi-product pipelines in the UK orthopaedic components industry. This was achieved via exploitation of empirically derived parameter settings (Cheema et al. 1989). It readily satisfies the requirement for provision of a simple yet DSS (Monniot et al. 1987)

4.4. Desired customer service levels can be ensured with the APIOBPCS model by the provision of a target stock level. This can be set arbitrarily and does not affect the system stability or variance ratios between the demand variance and the order (or inventory variance) in a linear system (an assumption we make herein). Thus in theory, with a high enough target stock level, any desired availability or fill-rate (or any other measure) can be achieved. Indeed, if desired by exploiting the classic ‘newsboy’ principle, the target stock level can be set to the critical fraction which ensures that the optimum economic stock-out probability is achieved (Disney et al. 2006a). Although for some delivery systems the target inventory level might theoretically be set to zero to try and achieve a “stockless” system, in practice it is much more realistic to aim for “Minimum Reasonable Inventory” (MRI), Grunwald and Fortuin, (1992).

## **5. Exploring the Step Response**

5.1. In studying system dynamic behaviour, it is usual to start with exploring the response to simple deterministic inputs. This is true for both simulation studies (Forrester, 1958) and analytic approaches (Truxal, 1955). The former firstly exploited analogue computation based on expressing the problem in terms of the principles of integration (Johnson, 1956), and latterly, (now exclusively) via digital computation, such as the Bullwhip Explorer (Lambrecht and Dejonckheere and, 1999). The analysis route utilised either classical methods (Piaggio, 1954), or more likely the Laplace Transform approach (Truxal, 1955). After falling out of favour with the advent of inexpensive and powerful simulation methods such as iThink©, there has been a recent resurgence of interest now that the associated algebra can be readily solved via computer software such as Matlab© and Mathematica©. This enables problems to be tackled via such analysis techniques which could not possibly have

been contemplated a few decades ago. Dejonckheere et al. (2002), and Disney et al. (2008) are good examples of this approach.

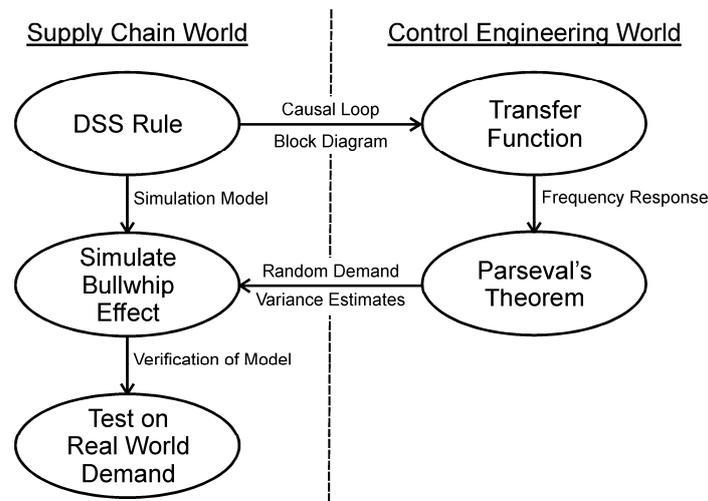
5.2. This brings us naturally to the big question of gaining insight into the usefulness of such methods in those branches of industry and commerce which for good operating scenario reasons are not part of an “analytical corporation” (Davenport, 2006). But what is known about each delivery process (whether internal, supplier or outbound) where a busy production scheduler is handling decisions usually involving hundreds if not thousands of SKU’s? Certainly a nominal, or target value of hoped for lead time, possibly with associated uncertainty (confidence) levels and maybe some idea whether all the goods would arrive together, or in a particular sequence. So it is reasonable to explore the behaviour of the APIOBPCS model of Figure 2 by expressing feed-forward and feedback controls as functions of this assumed lead time.

Design	Parameters	Comments	
DE-APIOBPCS	A <sub>1</sub>	$T_i=T_w=4$ $T_a=4$	The “Benchmark” conservative design with all parameters set equal to the expected delivery lead time.
	A <sub>2</sub>	$T_i=T_w=2$ $T_a=4$	Speeding up both inventory and OPL feedback loops.
	A <sub>3</sub>	$T_i=T_w=4$ $T_a=3$	Speeding up the Exponential Smoothing in the feed-forward path.
	A <sub>4</sub>	$T_i=T_w=4$ $T_a=20$	Slowing down the exponential smoothing in the feed-forward path.
	A <sub>5</sub>	$T_i=T_w=20$ $T_a=20$	Slackening inventory and OPL feedback loops plus slowing down feed-forward path smoothing.
APIOBPCS	B <sub>1</sub>	$T_i=4$ $T_w=20$ $T_a=4$	Loosening the effect of the OPL feedback loop.
	B <sub>2</sub>	$T_i=20$ $T_w=4$ $T_a=20$	Loosening the effect of the inventory feedback loop and slowing down the exponential smoothing in the feed-forward path.
	B <sub>3</sub>	$T_i=1$ $T_w=28$ $T_a=1$	Speeding up the inventory loop and smoothing in feed-forward path plus slackening OPL feedback loop.
	B <sub>4</sub>	$T_i=20$ $T_w=4$ $T_a=4$	Loosening the effect of the inventory path loop but speeding up the smoothing in the feed-forward path.
	B <sub>5</sub>	$T_i=4$ $T_w=1$ $T_a=4$	Strengthening the effect of the OPL feedback loop.

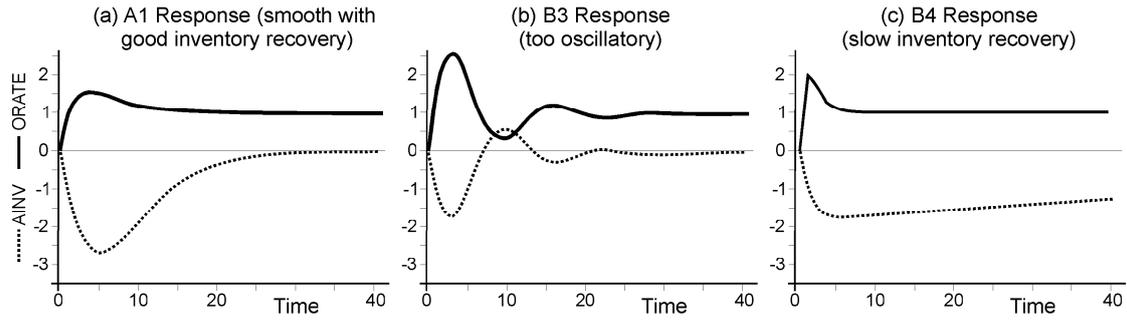
NB:  $T_p=T_p=4$  in all cases

**Table 4. Sample Parameter Settings**  
(Source: Authors)

5.3. Such an approach is reminiscent of that used in practical control engineering refined for this logistics application as shown in Figure 3. Furthermore via using “standard forms” of transfer functions written in terms of the Laplace Operators full advantage may be taken of evidence available from “good practice” in hardware system design. The result is arguably a somewhat conservative configuration i.e. safe but not particularly adventurous (Towill, 1982). Table 4 shows 10 sample designs for exploration via analysis and simulation. The starting point has been the various combinations of parameter settings first studied by Mason-Jones et al. (1997) when assessing bullwhip in extended supply chains. The “A” designs are all constrained to have ( $T_i=T_w$ ), which we refer to as Deziel-Eilon designs, in respect to the originator of this simple, but brilliantly effective simplification (Deziel and Eilon, 1967). The “B” designs are not so constrained (a potential fault, as demonstrated by John et al. 1994). Table 4 also contains brief explanatory notes concerning the choice of feed-forward and feedback control settings.



**Figure 3. Summary of the Control Engineering Methodology**  
(Source: Authors)



**Figure 4. Sample Step Responses**  
(Source: Authors)

5.4. All continuous transfer functions may be written as a ratio in polynomials of “ $s$ ”. Thus a general expression is,

$$F(s) = \frac{b_0 + b_1s + b_2s^2 + \dots + b_ms^m}{a_0 + a_1s + a_2s^2 + \dots + a_ns^n} \quad (1)$$

which is of order “ $n$ ”, thus requiring solution of a differential equation of similar order when the response is obtained via analytical methods. The two transfer functions of interest herein, (assuming at this stage that  $Tp$  is an exponential lag) follow from the paper by John et al. (1994),

$$O(s) = \frac{ORATE}{CONS} = \left( \frac{1 + sTp}{1 + sTa} \right) \frac{Tw + s(TpTi + (Ta + Ti)Tw)}{Tw + sTi(Tp + Tw + sTpTw)} \quad (2)$$

for determining delivery process ordering orate patterns given the demand rate CONS.

$$I(s) = \frac{AINV}{CONS} = Ti \left( \frac{Tp - sTaTw - (1 + sTa)Tp(1 + sTw)}{(1 + sTa)(Tw + sTi(Tp + Tw + sTpTw))} \right) \quad (3)$$

and hence similarly evaluating inventory levels (AINV) for the appropriate demand rate.

5.5. Solution of the step response either via analysis or simulation is now straightforward. ORATE and AINV plots for a small sample are shown in Figure 4, and indicate that suitable adjustment of control parameters yields a wide range of behaviour (assuming that in practice the model still holds under such varying requirements). Some patterns are obviously acceptable, others are not. A detailed comparison of the

various models is left until bullwhip has been studied, hence Section 7 will bring these topics together in a precise tabular format. Note that for the “shock” lens for viewing bullwhip (Towill et al. 2007), all 10 designs exhibit this phenomenon since their step responses overshoot (which is the requisite criterion).

## 6. Estimating Order Rate and Inventory Variances

6.1. As we have indicated previously the bullwhip phenomenon may be viewed through a number of user orientated lens (Towill et al. 2007). For example, the step responses shown in Figure 3 are related to the “shock” lens. In this section we shall evaluate ORVAR (as the ratio of ORATE variance over the CONS variance for random demand  $\sim$  strictly speaking “white noise”) via the “filter” lens. The required relationship for linear systems may be expressed as the general integral equation:

$$\frac{VAR(ORATE)}{VAR(CONS)} = \frac{1}{2\pi j} \int_{-jw}^{jw} F(-s)F(s)ds \quad (4)$$

which was first solved by James et al. (1947) based on research conducted by the MIT Radar Laboratory during WWII. They published the solution to equation 4 in tabular form up to  $n=10$ . The algebra involved is, however, somewhat arduous beyond  $n=3$ , hence the use of appropriate software is highly recommended. Equation (4) has since been widely exploited in many fields, including communications (Lee, 1960), control systems (Newton et al. 1957), and weapons guidance (Garnell and East 1977). It thus has a well established pedigree. When viewing logistics decisions via the variance lens (Towill et al. 2007) originating from the Operations Research Community, the condition for bullwhip to exist is ORVAR greater than unity.

6.2. Application of these published solutions to equation (4) with  $O(s)$  and  $I(s)$  transfer functions substituted in turn yields the formulae shown in Table 5. Both the Deziel-Eilon and the general case ( $T_i \neq T_w$ ) have been solved. We note in passing that as expected these are all much simpler than the corresponding solution previously obtained for the case when  $T_p$  is a pure time delay, as instanced in Disney and Towill, (2003). The hope is that what is lost in accuracy will be gained in additional insight. As we shall see later in the paper, our conservative design will cope with changes not only in

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**Table 4. Sample Parameter Settings**  
(Source: Authors)

lead times but also their distribution. An immediate observation following scrutiny of Table 5 is that  $T_p$  increases are reflected in both greater bullwhip and inventory variance, thus providing yet further proof and re-emphasis of a previously known result.

6.3. In Table 6 the bullwhip and inventory variances have been normalised by dividing by  $T_p$  to yield expressions in terms of  $\lambda_i$ ,  $\lambda_w$  (the feedback controllers) and  $\lambda_a$  (the feed-forward controller). The Deziel-Eilon formulae in the table make particularly interesting reading. For this special case it is obvious that the solutions are symmetrical in  $\lambda_i = \lambda_w$  and  $\lambda_a$ . This result has been confirmed by simulation of dynamic responses. The surprisingly apparent interchange-ability of  $\lambda_i$  and  $\lambda_a$  implies that for a linear system a similar effect (at least on bullwhip and inventory

Parameter settings	Performance Index	Formula (via Parseval's Theorem)
$T_i = T_w$ # $\bar{T}_p = T_p$	Bullwhip variance	$\frac{Ta^2 + (Tp + Tw)^2 + Ta(2Tp + 3Tw)}{2TaTw(Ta + Tw)}$
	Inventory variance	$\frac{\left( Tp^2Tw^4 + TaTw(3Tp^2Tw^2 + 3TpTw^3) + \right.}{2Tw(Tp + Tw)(Ta^2Tw + TpTw^2 + TaTw(Tp + Tw))} \left. Ta^2Tw(TpTw^2 + Tw(Tp + Tw)^2) \right)$
$T_i \neq T_w$ $\bar{T}_p = T_p$	Bullwhip variance	$\left( \frac{TaTiTw \left( (TiTp + (Ta + Ti + Tp)Tw)^2 - \right)}{2TpTw(TaTw + Ti(Tp + Tw))} + \frac{TaTi^2Tw^2(TpTw + Ta(Tp + Tw)) + Tp(TaTw + Ti(Tp + Tw))^3}{2TaTi^2Tw(Tp + Tw)(Ta^2Tw + TiTpTw + TaTi(Tp + Tw))} \right)$
	Inventory variance	$\frac{\left( Ta^2Tw(TpTw^2 + Ti(Tp + Tw)^2) + \right.}{2Tw(Tp + Tw)(Ta^2Tw + TiTpTw + TaTi(Tp + Tw))} \left. TiTp^2Tw^3 + TaTi(3Tp^2Tw^2 + 3TpTw^3) \right)$

# Deziel-Eilon rule,  $T_i = T_w$

**Table 5. Formulae for Bullwhip and Inventory Response to Random Signal Demands**

Parameter settings	Performance Index	Normalised formula
$T_i = T_w$ $\bar{T}_p = T_p$	Bullwhip variance	$\frac{1 + 2\lambda a + \lambda a^2 + 2\lambda w + 3\lambda a\lambda w + \lambda w^2}{2Tp(\lambda a^2\lambda w + \lambda a\lambda w^2)}$
	Inventory variance	$Tp \frac{\lambda w^2 + 3\lambda a\lambda w(1 + \lambda w) + \lambda a^2(1 + \lambda w(3 + \lambda w))}{2(1 + \lambda a)(1 + \lambda w)(\lambda a + \lambda w)}$
$T_w \neq T_p$ $\bar{T}_p = T_p$	Bullwhip variance	$\frac{\left( \lambda a^3\lambda w^3 + \lambda i^3(1 + \lambda w)^2(1 + \lambda w + \lambda a\lambda w) + \right.}{2Tp\lambda a\lambda i^2\lambda w(1 + \lambda w)(\lambda a\lambda i + (\lambda i + \lambda a(\lambda a + \lambda i))\lambda w)} \left. \lambda a\lambda i\lambda w^2(\lambda w + \lambda a(3 + (3 + \lambda a)\lambda w)) + \lambda a\lambda i^2\lambda w(3 + \lambda w(6 + 4\lambda w + 3\lambda a(1 + \lambda w))) \right)$
	Inventory variance	$Tp \frac{\lambda i\lambda w^2 + 3\lambda a\lambda i\lambda w(1 + \lambda w) + \lambda a^2\left( \lambda w^2 + \lambda i(1 + \lambda w)^2 \right)}{2(1 + \lambda w)(\lambda a\lambda i + (\lambda i + \lambda a(\lambda a + \lambda i))\lambda w)}$

Where:  $\lambda a = \frac{Ta}{Tp}$ ,  $\lambda w = \frac{Tw}{Tp}$ ,  $\lambda i = \frac{Ti}{Tp}$ .

**Table 6. Normalised Form of Inventory and Bullwhip Variances**  
(Source: Author)

variance) is obtained by changing either feed-forward or feedback controls (in Deziel-Eilon mode only). This relationship may well break down in the presence of non-linear behaviour such as strict capacity constraints etc. Note that the normalised format has the benefit that it does enable such insights to be obtained by inspection. This fact will be further exploited later in the paper.

## 7. Capability and Copability Review

7.1. Table 7 is a comprehensive performance review of the selected 10 system designs  $A_1 \rightarrow A_5$  and  $B_1 \rightarrow B_5$ . It covers the requirements of viewing decision making implications through both the “variance lens” and the “shock lens” as defined by Towill et al. (2007). The former viewpoint is catered for by estimating order rate variance (ORVAR) and inventory variance (INVAR) in response to a random signal drawn from a normal distribution. The latter is represented by the percentage overshoot (PV) following a step input in demand and the time at which this occurs (PT). However, because the shape of the dynamic response varies quite considerably across the 10 designs an additional measure has been included. This is the 95% settling time (ST) defined as the time taken before the response is finally within  $\pm 5\%$  of the final value. Table 7 shows the wide variation in ST as a ratio of PT (from less than 2 to 1 up to greater than 10 to 1). Hence the change in the shape of the response cannot be ignored. Note that as stated earlier PT is greater than unity (implying demand amplification) even when bullwhip (based on variance ratio) is much less than one. This is exactly the dilemma which the “bullwhip lens” concept of Towill et al. (2007) was created to study and resolve.

7.2. If LS is required (a characteristic of many Japanese industries according to research by Mollick, 2004), then Design A5 is a reasonable choice since the range in ORATE will be reduced by about 75%. Similarly Design A4 reduces the range by over 50%, thus being a prime candidate for DS. Finally Design A1 reduces range by about 20% for relatively small fluctuations in inventory, so is useful for POA. Hence on the basis of the nominal value of exponential lead time Table 7 indicates that APIOBPCS covers the industrial range of requirements for this kind of DSS (Childerhouse et al. 2009). Hence APIOBPCS is proving to be *capable* of satisfying the needs of such

decision makers perhaps with some additional fine tuning readily undertaken via Tables 5 and 6.

Design No.	$T_p = \bar{T}_p$	$T_a$	$T_i$	$T_w$	Bullwhip	Inventory variance	Peak Value	Peak Time	Settling Time (95%)
A1	4	4	4	4	0.625	3.0	1.45	5.78	20.99
A2	4	4	2	2	1.125	2.3	1.56	3.57	16.29
A3	4	1	4	4	2.125	1.8	1.67	2.49	14.03
A4	4	20	4	4	0.225	4.3	1.2	11.4	46.05
A5	4	20	20	20	0.073	8.3	1.18	31.8	89.38
B1	4	4	4	20	0.608	3.6	1.63	6.53	25.2
B2	4	4	20	4	0.315	3.5	1.12	8.45	43.2
B3	4	1	1	20	4.612	2.4	2.6	2.73	24.7
B4	4	1	20	1	5.979	1.1	1.97	1.09	5.72
B5	4	4	4	1	0.925	2.4	1.2	3.2	30.1
A1	8	4	4	4	1.063	3.7	1.79	5.25	22.9
A2	8	4	2	2	2.125	2.8	2.04	3.50	18.2
A3	8	1	4	4	4.325	2.2	2.29	2.3	16.1
A4	8	20	4	4	0.288	5.9	1.32	12.1	52.4
A5	8	20	20	20	0.085	10.7	1.25	34.5	92.3
B1	8	4	4	20	0.828	5.3	1.99	8.00	37.5
B2	8	4	20	4	0.616	4.0	1.37	5.75	42.7
B3	8	1	1	20	6.516	3.7	3.32	3.75	43.2
B4	8	1	20	1	19.592	1.2	3.43	1.0	6.73
B5	8	4	4	1	2.448	2.6	1.75	2.5	29.5
A1	12	4	4	4	1.625	4.0	2.15	5	24.3
A2	12	4	2	2	3.458	3.0	2.53	3.25	19.4
A3	12	1	4	4	7.325	2.4	2.91	2.25	17.6
A4	12	20	4	4	0.358	6.9	1.45	11.5	58.2
A5	12	20	20	20	0.097	12.5	1.32	32.5	96.1
B1	12	4	4	20	1.020	6.5	2.26	8.63	45.1
B2	12	4	20	4	1.049	4.3	1.71	5.00	36.1
B3	12	1	1	20	8.032	4.6	3.85	4.25	63.0
B4	12	1	20	1	41.212	1.2	2.9	1.0	7.25
B5	12	4	4	1	4.781	2.7	2.36	2.25	25.3

**Table 7. Summary of Performance When Controlling Different Exponential Lags**  
(Source: Authors)

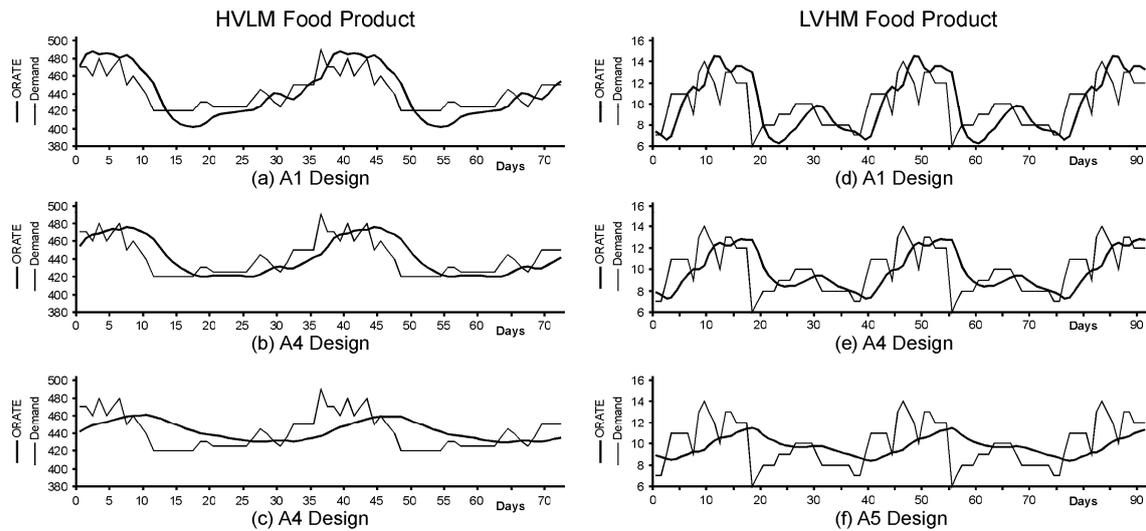
7.3. To illustrate the “copability” problem Table 7 shows the results obtained when the lead time is an exponential lag of 4; 8; and 12 days respectively. However these metrics are for the 10 designs with parameters still set assuming the lead time is 4 days. In other words the DSS has been designed in all instances to match this

delivery time but the actual value has deteriorated up to a factor of three. Hence we now have a guideline as to how each design would cope under such circumstances (obviously some perform very well, some poorly). These simulations thus realistically represent what might happen if the DSS is left to operate at these settings. (They may be periodically changed, but it is very unlikely the parameters are constantly adjusted). Table 7 is thus a realistic guideline as to what happens if lead time is “mis-guessed” when setting up the DSS. It can be seen by inspection that the Designs A1 through A5 are surprisingly robust to changes in lead time, as will be investigated further in Section 9. This we attribute to the effective use of feedback controls. As Horowitz (1959) has argued, their real purpose is to ensure coping with uncertainty rather than just adequately shaping the response under ideal conditions.

## **8. Application of APIOBPCS to Real-World Data**

8.1. From what we have said earlier, it should be possible to use the foregoing guidelines in any real-world environment expecting a “good” workable system, rather than homing in on optimal performance. So these designs have been tested via simulation with output as shown in Figure 5. in both cases the delivery delay has been arbitrarily assumed to be an exponential lag of 4 time periods. The real-world demand data comes from Plunkett, (2004) but has been replicated twice end-on to provide a longer time series. As with Holmström, (1997) there are two sets available: a HVLM food product and a LVHM food product. In this instance the ranges are less than 8% of average and nearly 100% of average respectively. In other words HVLM is a regular, stable seller. In contrast LVHM is somewhat inconsistent in demand.

8.2. The three designs simulated ( $A_1$  to enable PAO); ( $A_4$  to enable Demand Smoothing); and ( $A_5$  to enable Level Scheduling). All of them do the desired job reasonably well. PAO gives a little smoothing; the degree of Demand Smoothing is manifest from the display; and Level Scheduling achieves this and also picks out trend changes. If the decision maker is not satisfied with the trade-off between ORATE and AINV fluctuations then further fine tuning of  $\lambda_i/\lambda_w$  can be performed. (An increase in these parameters will generally further dampen ORATE at the expense of increased AINV and vice versa).



**Figure 5. Application to Real-World Data**  
(Source: Plunkett, 2004)

8.3. The foregoing Plunkett, (2004) data has provided us with two interesting test cases. However the variations in the Demand signal mask significant trend changes for both product groups. For example, with HVLM there is a significant drop in average orders (from 470 cases/day to 420 cases/day at day 11). This is followed by a slow recovery eventually reaching 450/day by day 34. For the LVHM product group the average demand is 9 cases/day until day 17, followed by 7 cases/day until day 24, then 12 cases/day thereafter. This is a further illustration of business competing within a non-altruistic supply chain where such trends are left for the vendor to detect (and cope with). There is thus an advantage in using APIOBPCS to automatically detect these changes rather than to use “pure” LS (ignore all demand patterns) where inventory would either build up excessively or alternatively lead to a stock-out scenario. Of course the scheduler still has to take action, but has a graphical display to serve as a wake-up call, rather than it be left to a complaint from an irate customer or harassed inventory manager.

## 9. The Ultimate Test For Copability

9.1. In further assessing the “copability” of APIOBPCS we decided to perform one final set of experiments. Although it may be argued that Table 7 gives the clue that as a structure it is robust in the presence of “ignorance” of the exact value of exponential lead time, this is not the critical case. As anticipated from control theory, this is dealing

with a pure time delay (Disney et al. 2006b). It is, of course, possible to make rational approximations (such as those due to Padé) but these all have round-off errors which eventually become significant. Hence our decision to evaluate the five designs A1-A5 in the scenario where the lead time is twice that expected, and moreover it is a pure delay. The controllers will all have been designed on the assumption of an exponential delay of  $Tp=4$ .

9.2. For these experiments the delivery process in Figure 6 now operates and in discrete time and the production delay is now a pure time delays of 8 periods, rather than operating in continuous time with an exponential delay as assumed in Figure 4 and the transfer function analysis we presented earlier. Preliminary tests showed that when viewed through the “shock” lens the differences between the step responses was little changed from those listed in Table 7 for  $Tp=8$ . In other words APIOBPCS coped well with the double uncertainty of lead time value formulation. To illustrate this point further, the responses to a random input were also examined in detail. These are shown in Figure 6 (remember that the pipeline transfer function is now includes an eight period pure time delay, but the controllers are set for a continuous time exponential lag of four periods. For comparison the CONS, ORATE, and EINV waveforms are all shown superimposed. It is now abundantly clear that even with such a high level of lead time ignorance that:

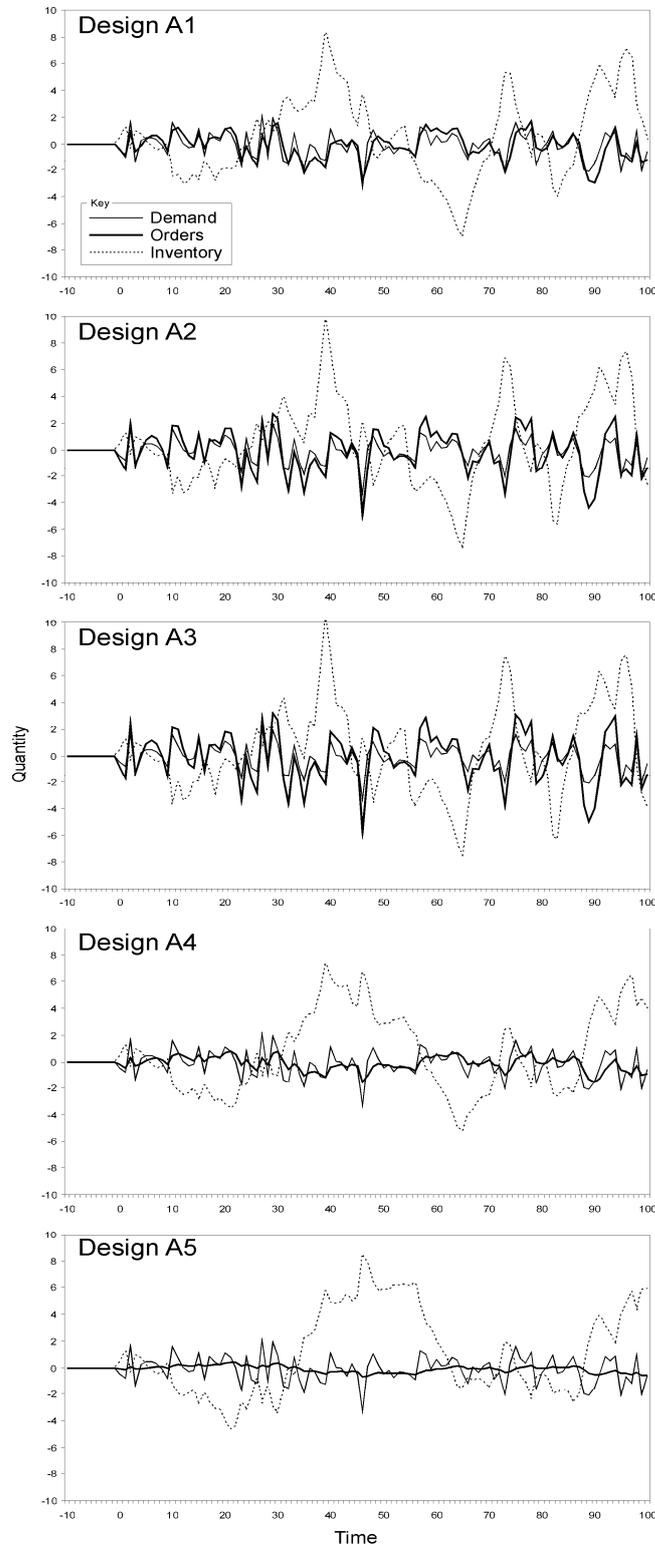
- Design A1  $\approx$  PAO
- Design A4  $\approx$  DS
- Design A5  $\approx$  LS

even more obviously than when applied to the Plunkett Data in Figure 5.

## 10. Discussion

10.1 An interesting comparison may be made with the real-world bullwhip estimates for a European confectionary supply chain (Holmström, 1997). These have previously been analysed for the two contrasting value streams shown in Table 1. The Holmström, (1997) estimates shown are expressed as  $\frac{\sigma}{\mu}$  at each stage in the delivery process. These results are even more startling if the normal bell-shape distribution should apply at any of these echelons i.e. with range of  $6\sigma$ . However

this is somewhat unlikely if the very spiky EBQ decision making is applied within any echelon, as implied by Burbidge, (1989).



**Figure 6. Random Responses in Discrete Time:  
Deziel-Eilon Designs (Pure Time Delay = 8 Days)  
(Source: Authors)**

10.2. Of course we do not know what DSS was used at each stage of the European Confectionary supply chain. But assuming that the original CONS is random, it is possible to make an informed estimate from exploiting the APIOBPCS concepts. For HVLM products we may recommend an A3 design (highly responsive for the retailer), A2 design (responsive for both wholesaler and factory deliveries), and A1 design (PAO for the factory schedule). This should give a well behaved pipeline with the range of factory deliveries about 150% of POS. Even if the lead time at all levels was doubled, for the same bullwhip settings (and an improbable level of ignorance for HVLM), the range would increase to 450% POS. This is very dramatic, but still well below the reported European confectionary supply chain of 900% by Holmström, (1997).

10.3. Similar reasoning may posit for LVHM a more volatile design at each echelon. If this were set at A3, then the LVHM range would be about 450% POS. Again this is well below the Holmström (1997) published data of 2860% (28.6:1). So provided the pipeline is operating on the “seamless flow” principle (Towill and Childerhouse, 2006) it is reasonable to suppose that bullwhip induced via the value stream decision making process can be suitably constrained. Better still, of course, to be able to operate the supply chain in altruistic mode (Disney et al. 2008). Nevertheless, as the survey by Buxey (2001) and highlighted by Busalacchi, (1999) in “lightweight procurement” mode many businesses do not have the good fortune to have such an opportunity, or to be part of an extended enterprise via the Davenport (2006) “Analytic Corporation” concept. The resultant gap is where a conservative approach to DSS selection is advised, and APIOBPCS fits the bill on the basis of evidence currently available.

10.4. The range of applicability of this simple but robust system is potentially very wide. Much of the information required by the user is in Table 7. This can be supplemented where necessary via the simple formulae of Table 6. We believe two principles are key. Firstly, as shown by John et al, (1994), the OPL feedback control requires the presence of a continuously updated estimate of current lead time to provide the necessary target value. Secondly we strongly recommend adherence to the Deziel-Eilon rule of equalising the inventory and OPL controllers (Deziel and

Eilon, 1967; and Disney and Towill, 2002). These two steps build trend-detection and robustness capabilities into the DSS. The remaining rule-of-thumb guidelines:

- For PAO, make  $\lambda a$  small and  $\lambda i / \lambda w$  about unity.
- For Demand Smoothing make  $\lambda a = \lambda i = \lambda w$  and all of them between unity and two.
- For [intelligent] Level Scheduling make  $\lambda a = \lambda i = \lambda w$  at least five.

In any given situation these designs will not be optimal, but they will be workable and reliable. Between them Tables 6 and 7 give adequate guidance for any “fine-tuning” considered necessary.

## 11. Conclusions

11.1 Bullwhip has been observed at many levels and for many years from “boom-and-bust” economic cycles (Sterman, 2000) to individual Proctor and Gamble product streams (Schmenner, 2001). Furthermore, it was known for its deleterious effects almost a century before its transformation from the less flamboyant “demand Amplification” of Jay Forrester, (1958). It is an expensive phenomenon (Metters, 1997), as determined from recognisable on-costs alone. The seminal work of Marshall Fisher, (1997) has shown that this is only half the story. Obsolescence costs and lost sales due to stock-outs can erode any profits made during the upswing. Hence the advent of innovative models which exploit “intelligent” use of expert forecasts to partition products into those that are supplied via regular replenishment, single batch, or initial batch plus agile top-up (Fisher and Raman, 1996). The proposed “Analytical Corporation” (Davenport, 2006) builds on such ideas.

11.2. But this leaves many businesses unsupported. They are neither part of an extended enterprise backed by such analytic capability, nor associated with an altruistic supply chain with information and philosophy sharing. The current paper has shown that such organisations can readily exploit the APIOBPCS as a DSS via simple rules-of-thumb which do not require exactly matching to individual products. The outcome is a *capability* (of matching goals when operating in the expected scenario) and surprisingly *capable* when surrounded by uncertainty. Earlier site-based research by Childerhouse et al. (2009) has categorised preferred decision-making strategies as Pass Orders Along (with limited smoothing); Demand

Smoothing (by say at least 50%); and Level Scheduling (but with trend detection capability). APIOBPCS can be suitably adjusted to perform any of these functions via feed-forward (forecasting) and feedback (inventory and orders-in-pipeline) controls.

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