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Altruistic Behaviour in a Two-Echelon Supply Chain with Unmatched Proportional Feedback Controllers

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Abstract

We study a two echelon supply chain with first order autoregressive demand and unit replenishment lead-times. Each echelon of the supply chain uses conditional expectation to generate Minimum Mean Squared Error (MMSE) forecasts. Both echelons use these forecasts inside the "Order-Up-To" policy to generate replenishment orders. We investigate three different scenarios: The first is when each echelon aims to minimize their own local inventory holding and backlog costs. The second scenario is concerned with an altruistic retailer who is willing and able to sacrifice some of his own performance for the benefit of the total supply chain. The retailer does this by smoothing the demand placed on the manufacturer by using a matched proportional controller in the inventory and WIP feedback loops. The third scenario is concerned with an altruistic retailer with two, unmatched, controllers. The matched controller case out-performs the traditional case by 14.1%; the unmatched controller case outperforms the matched controller case by 4.9%.

Keywords: Supply chains, multi-echelon inventory, order-up-to policy, collaboration, co-ordination, altruistic behavior

1 INTRODUCTION

There has been a growing interest in supply chains ever since Christopher (1998) argued that it is not individual companies but rather supply chains that compete against each other. Assuming the unit of competition is at the supply chain level creates a number of interesting challenges as actions in one part of the supply chain will have consequences in other parts of supply chain. This could be in a different company, a different industry, or even in another part of the world. Thus there is a need to understand how decisions taken by one member of the supply chain can affect other members of the supply chain. Collaboration in and co-ordination of, supply chains is therefore an important area of study.

It is often appropriate to consider a supply chain to be a series of decisions. In these decisions, demand is observed, forecasts are made, and orders are placed on supplier or production facilities so as to replenish inventory positions. These decisions also have to account for the lead-time between placing the order and receipt of that order into stock. Thus supply chains are concerned with the upstream flow of information and the downstream flow of materials.

It is common to assume a simple linear chain, with one retailer and one manufacturer, for example. This is appropriate in many practical situations, as in divergent supply chains (many retailers, one manufacturer), the many retailer demands can simply be aggregated into one. However, in the general case there can be many retailers, many manufacturers and a complex and time varying trading relationship between the members. This type of supply chain is very hard to understand and analyse. Thus we focus herein on a simple dyadic supply chain (of one retailer and one manufacturer).

Decision making processes often introduce a dynamic effect in supply chains. This dynamic effect is known as the bullwhip effect after the seminal contribution of Lee, Padmanabhan and Whang (1997), who used the term to highlight that the variance of the demand was amplified as demand signal was processed by the replenishment algorithm. This bullwhip effect is quite common in industry, as the order-up-to policy with unit feedback controllers and simple forecasting techniques (such as moving average and exponential smoothing) is guaranteed to amplify demand (Dejonckheere et al. (2003). The economic consequences of the bullwhip are not insignificant. Metters (1997) suggests that factory gate profit could be increased by 30% if bullwhip can be removed from a supply chain. The typical industrial response to combat the bullwhip effect has been to reduce lead-times, improve forecasts and to use information sharing. Chen, Drezner, Ryan and Simchi-Levi (2000) have elegantly quantified the benefits of these strategies in a multi-echelon supply chain scenario. Another, equally important, strategy is to reduce the use of discounting and promotions, via an everyday low pricing strategy, Fisher et al. (1997).

Luo (2007) considers a coordination scheme in a two-echelon supply chain consisting of a manufacturer and a retailer. It is assumed that a manufacturer can modify the retailers order pattern by offering incentives in order to lower total costs. The manufacturer asks the retailer to change its order quantity to reduce his own set up, ordering and inventory holding costs. To entice the retailer to behave in a manner that is advantageous to the manufacturer, a credit period incentive is offered by the manufacturer. It is shown that the benefit of the manufacturer is always greater than the loss to the retailer. Thus this cooperation scheme can bring the benefits to overall supply chain.

In another two-echelon supply chain scenario Gavirneni (2006) assumes that the supplier can alter the pattern of orders placed by the retailer by altering his prices. As the result of this incentive, the retailer's ordering pattern is not locally optimum anymore and thus the retailer's cost increases. However, the benefit to the manufacturer is sufficient enough to compensate for the increase at the retailer. With the aid of information sharing, the overall supply chain performance can be improved by 5%, on average.

Other incentives to encourage the retailer to incur cost increases include quantity flexibility (Tsay, 1999), quantity discounts (Weng, 1995), and revenue sharing (Giannoccaro and Pontrandolfo, 2004). Cachon and Lariviere (2005) discuss revenue sharing contracts and their relationship to other types of contracts in supply chains with risk-neutral agents. Gan, Sethi and Yan (2004) investigate decision making by risk adverse agents in a supply chain. Cachon (2003) provides a review of supply chain coordination mechanisms and emphasises coordination actions and transfer payments that ensures each firm's objective becomes aligned with the supply chain's objective.

In this paper we suggest a method to reduce supply chain inventory costs by coordinating, or aligning, the parameters of replenishment rules. We consider the case of a two echelon supply chain consisting of a retailer and a manufacturer, with one product in an environment where only inventory related (holding and backlog) costs exist. For the sake of simplicity these costs are identical at both the retailer and the manufacturer.

The role of aligning the replenishment rules and their parameters in a multi-echelon supply chains has been analysed before. Using a serially linked two-echelon supply chain with an AR(1) market demand, Hosoda and Disney (2006b) investigated the impact of altruistic behaviour on the overall supply chain inventory holding and backlog costs. To realize altruistic behaviour at the retailer, they introduced matched proportional controllers into the inventory and Work-In-Progess (WIP) feedback loops of the traditional Order-Up-To policy, Vassian (1955). These matched proportional controllers enabled the order placed by the retailer to be manipulated to reduce the total supply chain inventory cost. It is suggested that altruistic behaviour by the retailer mitigates the bullwhip effect, and this lower bullwhip is the source of the benefit at the manufacturer. Also, the cost benefit at the manufacturer is large enough to compensate the loss at the retailer. It is shown that on average more than 10% cost reduction can be achieved in the overall supply chain.

Hosoda (2005) extends the model shown in Hosoda and Disney (2006b) to a three-echelon case. Using numerical experiments, Hosoda showed that in a three-echelon case, not only the first echelon (retailer), but also the second echelon player, has to be altruistic to minimise the overall supply chain costs. It is shown that the average cost benefit in this setting is around 25%.

Disney et al. (2008) assumed that the market demand follows a stable i.i.d. white noise process and investigates the benefit of altruistic behaviour in a serially linked two-echelon supply chain. Two matched proportional controllers are incorporated into the replenishment (ordering) policy and this modified ordering policy was exploited by both echelons of the supply chain (the retailer and the manufacturer). Here the objective function included both inventory and bullwhip costs at both echelons of the supply chain.

2 CONTRIBUTION OF THIS PAPER

In this paper we will focus on the retailer's ordering policy that can yield a globally minimal supply chain inventory holding and backlog cost irrespective of incentives given by the manufacturer. We will use a classical OUT policy with two *unmatched* (that is, independent) proportional controllers in the inventory and WIP feedback loops. It is the analysis of the unmatched controllers that makes this paper unique in the literature.

Assuming first order autoregressive (AR(1)) demand we study a serially linked two-echelon supply chain that exploits a generalized order-up-to OUT policy with unmatched feedback controllers at the first echelon (the retailer) and a traditional OUT policy at the second echelon (the manufacturer). We also assume that Minimum Mean Square Error (MMSE) forecasting is used and unit lead-times are present at each echelon.

The benefit from the retailer's altruistic behavior enabled by the generalized OUT policy with unmatched feedback controllers will be investigated. Each player acts to minimize global inventory costs. To quantify its benefit, this strategy will be compared with other two strategies; 1) a traditional strategy where each player minimizes local inventory costs, Hosoda and Disney (2006a), and 2) an altruistic strategy achieved by the generalized OUT policy with matched feedback controllers, Hosoda and Disney (2006b).

As an indicator of the supply chain performance, we will employ a metric that consists of the sum of the stationary standard deviation of the net inventory levels at each echelon. This is a valid approach when safety stock have been optimized via the newsvendor principle as inventory costs are then linearly related to the standard deviation to the inventory levels, Disney et al. (2006c). We also quantify the bullwhip effect in the supply chain, although we do not use this information in the objective function.

We reveal the exact analytical expressions of the performance indicators. We highlight that the generalized OUT policy with unmatched controllers enable us to manipulate the dynamics of a supply chain with higher degree of freedom than the generalized OUT policy with matched controllers. Furthermore, we also discuss what kind of information should be shared between these two players to achieve the benefits from the altruistic strategy we highlight herein.

We proceed by first defining the supply chain model in section 3. Section 4 highlights the objective function. Sections 5, 6 and 7 analyse three supply chain co-ordination schemes analytically. Section 8 compares them numerically. Section 9 concludes.

3 DIFFERENCE EQUATION REPRESENTATION OF THE TWO-ECHELON SUPPLY CHAIN

We assume the demand faced by the retailer is a mean centered autoregressive stochastic process of the first order, AR(1). Thus,

$$d_{1,t} = \mu_d + \rho(d_{1,t-1} - \mu_d) + \varepsilon_t, \tag{1}$$

and the demand faced by the manufacturer in the second echelon is the retailers order,

$$d_{2,t} = o_{1,t} \; . \tag{2}$$

In Eq (1), ρ is the autoregressive constant ($-1<\rho<1$), $d_{1,t}$ is the demand at the first echelon (the retailer) at time t and μ_d is the average demand. We assume $\mu_d >> 4\sigma_d$ so that the possibility of negative demand is negligible. ε_t is a stochastic white noise process. In Eq (2), $d_{2,t}$ is the demand at the second echelon (the manufacturer) at time t and $o_{1,t}$ is the orders placed by the first echelon (the retailer) at time t. It is useful to consider the mean centered version of AR(1) demand process as then the mean has no influence on the variance ratios and there is no initial transient response.

We will also assume that there is a unit replenishment lead-time at each echelon. Additionally, there is a one period, order of events delay. Thus at both echelons (where the first part of the subscript is used to indicate the echelon in question; x=1 for the retailer and x=2 for the manufacturer), the following inventory balance equation holds,

$$ns_{x,t} = ns_{x,t-1} - d_{x,t} + o_{x,t-2} \ , \tag{3}$$

where *ns* is the net stock (inventory on hand). In each ordering policy we will also need two forecasts of demand. One of these forecasts is the conditional expectation of the demand in the next period and this is used to generate a desired WIP (or pipeline, orders placed but not yet received) target. The other forecast is the conditional expectation of demand in the period after the replenishment order arrives, that is, the forecast of demand in the next, next period. For the retailer these forecasts are

$$dwip_{1,t} = E[d_{1,t+1}] = \rho d_{1,t} \tag{4}$$

$$\hat{d}_{1,t} = E[d_{1,t+2}] = \rho^2 d_{1,t} \tag{5}$$

However, these two forecasts are considerably more complex for the manufacturer. They are

$$dwip_{2,t} = E[o_{1,t+1}] = \rho^3 d_{1,t} + \frac{\rho^2 d_{1,t} - o_{1,t}}{Tw} + \frac{tns - ns_{1,t} - o_{1,t-1} + \rho d_{1,t}}{Ti} \tag{6}$$

$$\hat{d}_{2,t} = E[o_{1,t+2}] = \left((\rho^3 + \rho^4) d_{1,t} + \frac{1}{Tw}\left(\rho^2 d_{1,t} - o_{1,t} - \frac{\rho^2 d_{1,t} - o_{1,t}}{Tw} - \frac{tns - ns_{1,t} - o_{1,t-1} + \rho d_{1,t}}{Ti} \right)\right) \tag{7}$$

We may use these forecasts in the following difference equation to generate orders (which holds at both echelons),

$$o_{x,t} = \hat{d}_{x,t} + \frac{1}{Ti}\left(tns - ns_{x,t}\right) + \frac{1}{Tw}\left(dwip_{x,t} - o_{x,t-1}\right) \tag{8}$$

These last few difference equations (Eqs 6-8) contain some new notation. The first is *tns*, the target net stock, a time invariant target safety stock that is used to ensure a desired fill-rate or availability of stock is achieved. The other two new terms are *Ti* and *Tw*. These are linear feedback gains in the net stock and WIP feedback loops respectively. Feedback gains are a very simple and very well known technique from the field of control theory for manipulating the response of a dynamic system. When $Tw=Ti=1$ we say then the supply chain consists of two serially linked, "traditional" OUT policies; when $Tw=Ti$, we say there are "matched controllers; when $Tw \neq Ti$ we say there are "unmatched controllers".

4 THE OBJECTIVE FUNCTION

We will consider minimising the following objective function

$$J = \sqrt{Var[NS_1]} + \sqrt{Var[NS_2]} \,. \tag{9}$$

This is an appropriate objective function when there are inventory holding and backlog costs that are linear in the inventory position in cases when *tns* has been set to the critical fractile to minimize the costs via the newsboy principle Disney et al. (2006c). The fact that we have simply added the two standard deviations together also implies that the retailer's inventory holding and backlog costs are as important as the manufacturer's inventory holding and backlog costs.

Hosoda and Disney (2006b) shows that setting *Ti*=*Tw*=1 at the manufacturer (the second echelon) yields a minimum value of *J* in a given scenario. Therefore, in our two-echelon model, only the first echelon (the retailer) exploits the feedback controllers (*Ti* and *Tw*) to manipulate the dynamics of the supply chain. The manufacturer simply uses a traditional order-up-to policy with MMSE forecasting to minimise *J*. This is a natural consequence of our objective function, (Eq 9). If the objective function contains bullwhip related costs then this does not hold and the manufacturer should incorporate feedback controller(s) into his replenishment rule. This is outside the scope of this paper. However, we will quantify order variance at both echelons in our model for completeness.

In the rest of the paper we will compare three scenarios:

- The traditional, local optimisation. This scenario considers the case when both the retailer and the manufacturer are solely concerned with minimising their own, local inventory holding and backlog costs. We will study this scenario in section 5.
- The altruistic retailer, global optimisation with matched controllers. This scenario considers the case when the retailer is able and willing to alter his replenishment rule (by tuning *Ti*) in order to minimise the total supply chain costs. We assume in this case that the retailer uses a generalised OUT policy with matched controllers, *Tw*=*Ti*. We will study this scenario in section 6.
- The altruistic retailer, global optimisation with unmatched controllers. We will study this scenario in section 7 and is essentially the same as the previous strategy but with independent, unmatched controllers in the retailer's replenishment rule, that is $Tw \neq Ti$.

5 ANALYSIS OF THE TRADITIONAL OUT POLICY SCENARIO; THE LOCAL OPTIMISATION

Here the retailer uses *Ti*=*Tw*=1 and thus the supply chain consists of two serially linked OUT policies with MMSE forecasting. As *Ti*=*Tw*=1 there are no stability issues in the supply chain and the variance of the two net stock positions turns out to be

$$Var[NS_1] = \left(2 + 2\rho + \rho^2\right)\sigma_\varepsilon^2 \tag{10}$$

$$Var[NS_2] = (2 + \rho(4 + \rho(6 + \rho(6 + \rho(4 + \rho(2 + \rho))))))\sigma_\varepsilon^2 = \left(2 + 4\rho + 6\rho^2 + 6\rho^3 + 4\rho^4 + 2\rho^5 + \rho^6\right)\sigma_\varepsilon^2 \tag{11}$$

These expressions for the variances may, in general, be obtained by a variety of ways, from stochastic analysis Hosoda and Disney (2006a) via the frequency domain Dejonckheere et al. (2004), control theory Disney and Towill (2003), or state space methods Gaalman and Disney (2006). However we will not provide further details here due to space requirements. Figure 1 illustrates the inventory costs (via the standard deviations used in the objective function, Eq (9)) as a function of the autoregressive parameter, ρ.

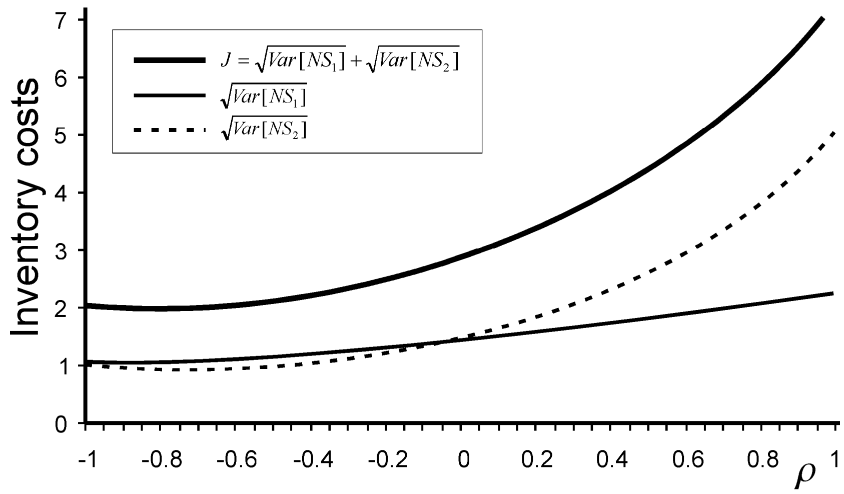

Figure 1: The inventory variances in the traditional supply chain

The variance expressions for the demand and the two order rates are;

$$Var[D_1] = \sum_{t=0}^{\infty} \left(\rho^t\right)^2 = \frac{1}{1-\rho^2}\sigma_\varepsilon^2 \tag{12}$$

$$Var[O_1] = \frac{2\rho\left(\rho^3+\rho^4-1-\rho\right)-1}{\rho^2-1}\sigma_\varepsilon^2 \tag{13}$$

$$Var[O_2] = \frac{2\rho(\rho-1)(1+\rho)\left(1+\rho^2\right)\left(1+\rho+\rho^2+\rho^3+\rho^4\right)-1}{\rho^2-1} \tag{14}$$

which have been plotted in Figure 2. Note from Figure 2 that when $\rho > 0$ then a bullwhip effect exists as $Var[O_1] > Var[D_1]$ and $Var[O_2] > Var[D_1]$.

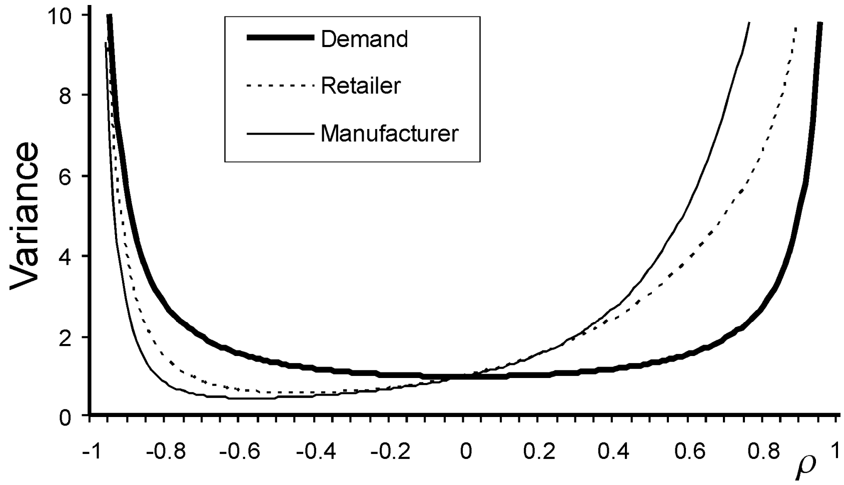

Figure 2: Order variances in the traditional supply chain

6 ANALYSIS OF THE GENERALISED OUT POLICY SCENARIO WITH MATCHED CONTROLLERS

By setting $Tw=Ti$, at the retailer we have matched feedback controllers. This yields a new set of variance ratio formulas. These are:

$$Var[NS_1] = \frac{Ti\left(2 + Ti(1+p)^2\right) - 1}{2Ti - 1}\sigma_\varepsilon^2 \qquad (15)$$

$$Var[NS_2] = \frac{\begin{pmatrix} (1+\rho)^2 - 4Ti(1+\rho)^2 + 2Ti^3\rho^2(1+\rho)(3+2\rho) \\ -Ti^2(1+\rho)^2\left(2\rho^2 - 5\right) + Ti^4\rho^4\left(2+\rho(2+\rho)\right) \end{pmatrix}}{Ti^4}\sigma_\varepsilon^2 \qquad (16)$$

From Eq (15) we can see that the valid range of *Ti* to ensure stability is $0.5 < Ti < \infty$. We may use these variance ratios in the objective function (Eq 9) and determine the value of *Ti* that minimises the objective function, *Ti**. Analytically this appears to be very difficult to achieve. However, using numerical techniques is considerably less complex and results in the following graphical relationship, see Figure 3.

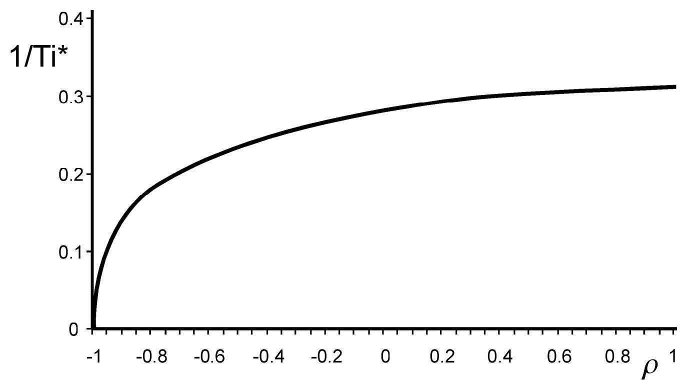

Figure 3. The optimal *Ti* when matched controllers exist with an altruistic retailer

Using this optimal *Ti* inside the objective function results in Figure 4 which describes the minimised inventory costs in our supply chain.

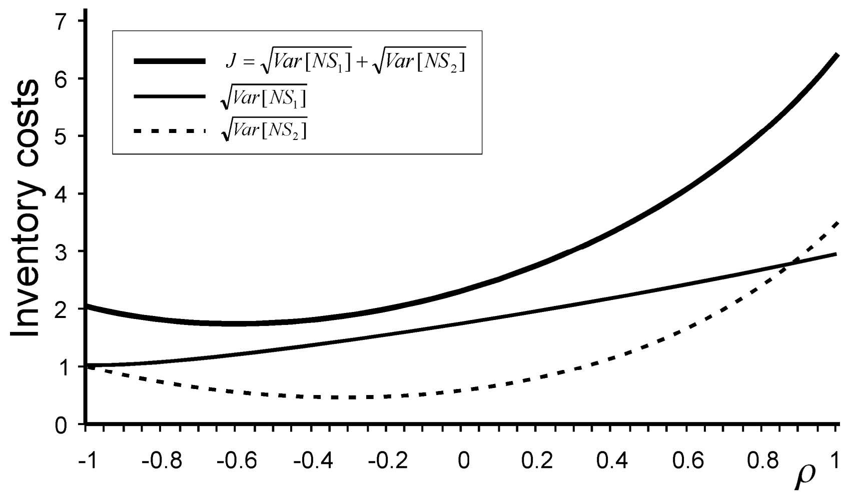

Figure 4. The objective function with matched controllers at *Ti*=*Ti with an altruistic retailer**

The general expressions for the order variances are given by Eqs (17) and (18).

$$Var[O_1]=\frac{\left(Ti+\rho+Ti\rho+2Ti\rho^2+2(Ti-1)\rho^3+2(Ti-2)Ti\rho^4-2Ti^2\rho^5\right)}{(2Ti-1)(Ti(\rho-1)-\rho)\left(\rho^2-1\right)}\sigma_\varepsilon^2 \tag{17}$$

$$Var[O_2]=\frac{\begin{pmatrix}2Ti(\rho-1)(1+\rho)^3(7\rho-1)-2(\rho-1)\rho(1+\rho)^3+\\ 2Ti^2(\rho-1)(1+\rho)^3\left(6-20\rho+\rho^3\right)+2Ti^7\rho^4\left(1+\rho-2\rho^3-2\rho^4+2\rho^6\right)-\\ 2Ti^3(\rho-1)(1+\rho)^2(14+\rho(\rho(\rho(5+\rho(5+\rho))-30)-15))+\\ 2Ti^4(\rho-1)(1+\rho)^2(15+\rho(\rho(\rho(9+\rho(9+5\rho))-26)-6))+\\ Ti^6\left(1+\rho\left(1-2(\rho-1)\rho\left(5+\rho\left(10+\rho\left(4+\rho\left(5\rho^3+3\rho^4-5-7\rho\right)\right)\right)\right)\right)\right)+\\ Ti^5\left(12+\rho\left(13+2\rho\left(\rho\left(\rho\left(26+\rho\left(22+\rho\left(\rho^3+\rho^4-5-15\rho-8\rho^2\right)\right)\right)-15\right)-20\right)\right)\right)\end{pmatrix}}{\left(Ti^5(2Ti-1)(Ti(\rho-1)-\rho)(\rho-1)(1+\rho)\right)}\sigma_\varepsilon^2 \tag{18}$$

When *Ti* has been set to *Ti** then the order variances can be plotted as shown in Figure 5. Comparison of Figures 2 and 5 shows that the altruistic contribution of the retailer results in a smoothing of the retailers order variance. Thus, if the retailer incurs some bullwhip related costs in his retail, warehousing or transportation activities, then he may in fact, be even more willing to use the proportional feedback controller to minimize costs at the supplier than this stylized analysis suggests.

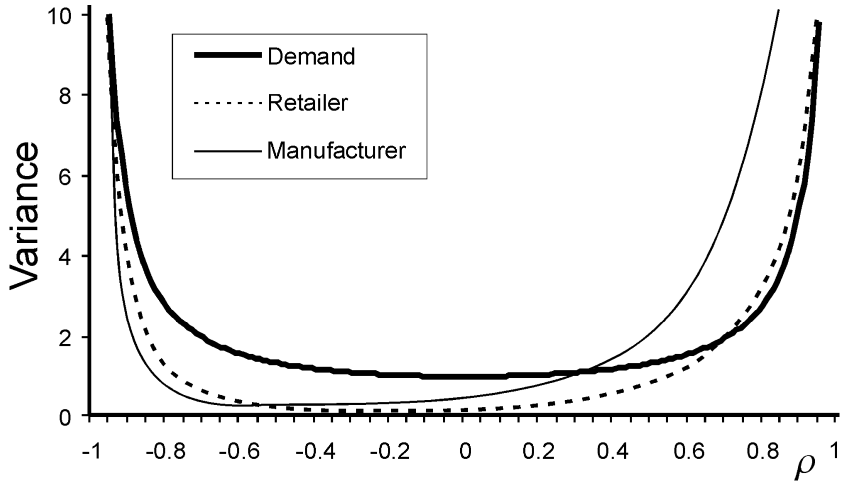

Figure 5. The demand and order variances with matched controllers at *Ti*=*Ti with an altruistic retailer**

7 ANALYSIS OF THE GENERALISED OUT POLICY SCENARIO WITH UN-MATCHED CONTROLLERS

As there are two unmatched controllers then there is a need to conduct a stability analysis at the retailer. There is no need to consider such issues at the manufacturer as here $Ti=Tw=1$ which results in a stable system. Thus the analysis for the generalised OUT policy with unmatched controllers consists of a two stage approach.

7.1. Stability analysis

Stability can be readily investigated via transfer functions and we will exploit Jury's Inners approach to conduct the analysis, Jury (1974). This transfer function of the retailers order rate is

$$\frac{O_1(z)}{\varepsilon(z)}=\frac{z^2\left(Ti(z-1)\rho+Tw\left(z+Ti(z-1)\rho^2\right)\right)}{(Tw+Ti(z-1)(1+Twz))(z-\rho)}. \tag{19}$$

It is known that stability only depends upon feedback loops and thus we may ignore the feed-forward autoregressive term. Setting $\rho=0$ and simplifying results in

$$\frac{O_1(z)}{\varepsilon(z)}=\frac{Twz^2}{Tw+Ti(z-1)(1+Twz)} \tag{20}$$

Jury's stability test requires us to expand out the denominator and collect together powers of z.

$$A(z)=Tw-Ti+z(Ti(1-Tw))+z^2TiTw \tag{21}$$

The first part of Jury's stability test is to ensure that $A(1)>0$ where $A(1)=A(z)|_{z\to 1}$. Thus it follows that $Tw>0$.

The second stage of Jury's test is that $(-1)^n A(-1) > 0$. This is true if and only if,

$$Ti < \frac{Tw}{2(1-Tw)}. \tag{22}$$

The third and final stage of Jury's stability test is that certain matrices of the co-efficient of the denominator of the systems transfer function are positive innerwise. Because our transfer function is only of second order, this criteria easily reduces to the fact that

$$Ti > \frac{Tw}{1+Tw} \tag{23}$$

and

$$Ti > \frac{Tw}{1-Tw}. \tag{24}$$

Numerical investigation reveals that (24) is non critical as it is entirely encompassed by (22). It is interesting to note that the stability bound in first step of Jury's test results in *Tw*>0, but (23) shows us that *Tw* can in fact be negative. Careful investigation shows that the unstable region of *Tw*, becomes stable when *Tw*<-1. For confirmation, a more direct stability test is given in Disney (2008) and results in *Tw*>0, Eq 22 and Eq 23. The redundant condition produced by Jury's Inners Test is not generated. Figure 6 illustrates the stability region.

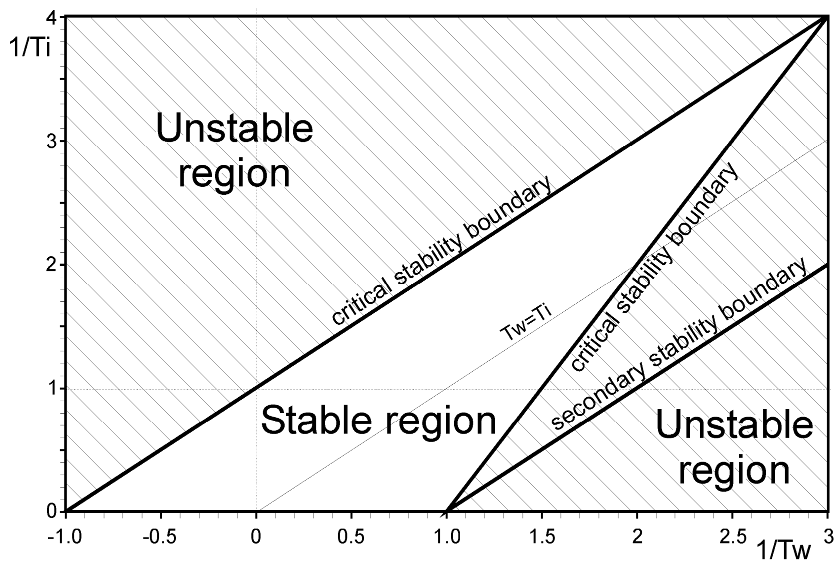

Figure 6: The stability boundary for the generalized OUT policy with unit lead-times

7.2. Variance analysis

The variance of the retailer's net stock is given by,

$$Var[NS_1] = \frac{Ti^2\left(Tw + Ti(Tw-1)(1+Tw+Tw\rho)^2 + Tw^2\left(Tw+2\rho+Tw\rho^2\right)\right)}{Tw\left(2Ti(Tw-1)+Tw\right)\left(Ti+(Ti-1)Tw\right)}\sigma_\varepsilon^2 \tag{25}$$

The variance of the manufacturer's net stock is given by,

$$Var[NS_2] = \left(\rho^2 + \rho^3 + \frac{\rho}{Tw} - \frac{\rho}{Tw^2} - \frac{1}{TiTw} + \frac{2+\rho}{Ti}\right)^2 \sigma_\varepsilon^2 + \left(\rho^2 + \frac{\rho}{Tw} + \frac{1}{Ti}\right)^2 \sigma_\varepsilon^2 \tag{26}$$

Using these variance ratios in the objective function we may find the optimal values of the unmatched feedback controllers. Again, analytically this is very difficult, but numerical techniques do exist and they result in values for *Ti* and *Tw* as shown in the Figure 7. Figure 7 contains some very remarkable features. For $\rho < -0.25734$ both *Ti** and *Tw** are positive. However, as there are two local optimums in the solution space near $\rho \approx -0.25734$, the optimal *Ti** and *Tw** is discontinuous in ρ. Interestingly, the optimal *Tw** is negative for $\rho > -0.25734$.

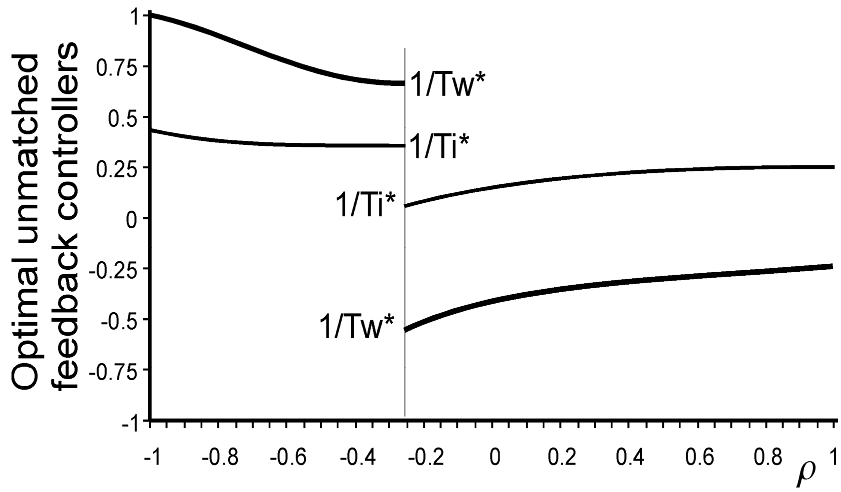

Figure 7. Tuning the unmatched feedback controllers to minimise supply chain inventory costs

Using these values in the objective function we may illustrate the inventory costs as shown in Figure 8. Here we can see the impact of the discontinuous *Ti** and *Tw**.

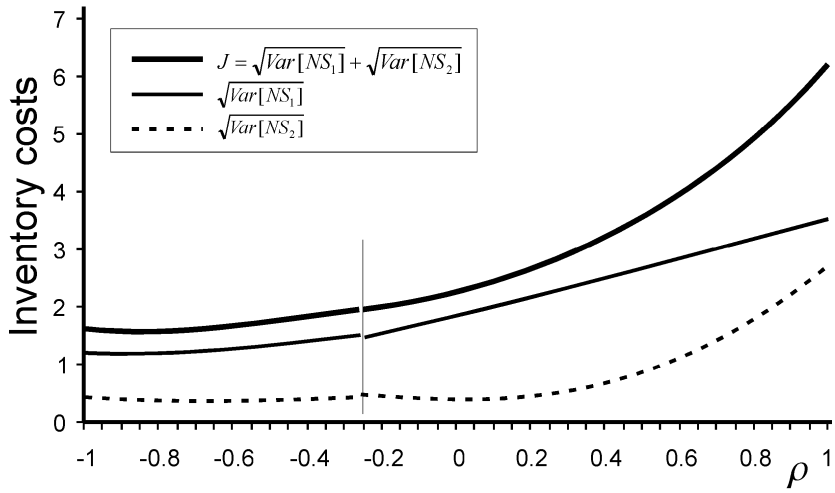

Figure 8. The objective function with unmatched controllers with an altruistic retailer

Returning now to the impact of altruistic retailer with unmatched controllers on the variances of the order rates we have;

$$Var[O_1] = \frac{4Tw\left(\begin{array}{l} Tw^3\rho^2 + 2Ti^3(Tw-\rho)(\rho-1)\rho^2(1+Tw\rho)^2 + TiTw^2\left(\rho(2\rho(\rho-2)-1)+Tw(1+\rho(\rho-1))(2\rho^2-1)\right)+ \\ Ti^2Tw\left(\begin{array}{l}\rho+\rho^2(3+2\rho(\rho-2))+Tw(\rho-1)(2\rho^2-1)(1+\rho(2\rho-1))+ \\ Tw^2(1+\rho)(2\rho^2(\rho-1)(1+\rho(\rho-1))-1)\end{array}\right)\end{array}\right)}{\left(\begin{array}{l}(2Ti(Tw-1)+Tw)(Ti+(Ti-1)Tw)\left(\sqrt{Ti}(Tw(\rho-2)-\rho)-\rho\sqrt{Ti(1+Tw)^2-4Tw^2}\right) \\ \left(\sqrt{Ti}(Tw(\rho-2)-\rho)+\rho\sqrt{Ti(1+Tw)^2-4Tw^2}\right)(\rho^2-1)\end{array}\right)}\sigma_\varepsilon^2 \tag{27}$$

$$Var[O_2] = \frac{-\left(16\left(\begin{array}{l}\alpha\beta + 2Ti^6(\rho-1)\rho^2(1+Tw\rho)^2(\chi)+2Ti^2Tw^4(\rho^2-1)(\phi+\omega)+ \\ Ti^3Tw^3\tau + Ti^4Tw^2(\varphi+\upsilon+\eta)+Ti^5Tw(\xi+\lambda)\end{array}\right)\right)}{\left(\begin{array}{l}Ti^3Tw(Ti+(-1+Ti)Tw)(2Tw^2+Ti(-1+Tw^2)-\psi)(2Tw^2+Ti(-1+Tw^2)+\psi) \\ \left(\sqrt{Ti}(Tw(-2+\rho)-\rho)-\zeta\right)\left(\sqrt{Ti}(Tw(-2+\rho)-\rho)+\zeta\right)(-1+\rho^2)\end{array}\right)}\sigma_\varepsilon^2 \tag{28}$$

where we have used the following substitutions as the formula is rather complex.

$$\psi = \sqrt{Ti}(Tw-1)\sqrt{Ti(1+Tw)^2-4Tw^2}\ ,\ \zeta = \rho\sqrt{Ti(1+Tw)^2-4Tw^2} \tag{29}$$

$$\alpha = 2Tw^7\rho^2(1-\rho^2)(Tw(2+\rho)-1)-2TiTw^5(1+\rho^2) \tag{30}$$

$$\beta = \left(\rho^2 + Tw\rho(\rho-1-4\rho^2)+Tw^3(\rho-2)(\rho+\rho^2-1)(1+\rho+\rho^2)+Tw^2(2\rho-1)(1+\rho(\rho(2+\rho)-1))\right) \tag{31}$$

$$\chi = \left(\begin{array}{l} 2Tw^2(1+2\rho)(-1+\rho^2)^2 - 2Tw^4(-1+\rho)(1+\rho)(-1+\rho^2(2+3\rho)) + \\ Tw^6(-1+2\rho(-1-\rho+\rho^3+\rho^4)) + Tw^5(2+\rho(3+2\rho^2(1+\rho)(-2+\rho^2))) - \\ 2Tw(-1+\rho)(1+\rho)^2(-1+2\rho) - \\ 2Tw^3(-1+\rho)\rho(1+\rho)(2+\rho)(-1+(-1+\rho)\rho) + 2\rho(-1+\rho^2) \end{array} \right) \tag{32}$$

$$\phi = \rho(2\rho(2-\rho)-1) + Tw^4(1+\rho)(1+\rho(1+\rho)(1+2\rho+\rho^3+\rho^4)) \tag{33}$$

$$\varphi = \left(\begin{array}{l} 2(\rho-1)\rho(1+\rho)(\rho(8+\rho(4\rho-11))-2) + \\ 2Tw(\rho-1)(1+\rho)(\rho(2+\rho)(7+\rho(10\rho-13))-2) - \\ 4Tw^2(3+\rho(\rho^2(12+\rho(1+\rho)(2\rho-3)(2+\rho^2))-9)) + \\ Tw^5(\rho(2\rho(30+\rho(8+\rho(15\rho^3+11\rho^4+\rho^5-3\rho^6-4\rho^7-28-19\rho)))-3)-16) \end{array} \right) \tag{34}$$

$$\xi = \left(\begin{array}{l} 2Tw^2(\rho-1)\rho(1+\rho)(8+\rho(\rho(2-\rho+5\rho^3)-10)) + \\ 2Tw^4\rho(4+\rho(\rho(\rho(9+\rho(1-2(\rho-1)\rho)^2)-9)-5)) - \\ 2Tw(\rho-1)\rho(1+\rho)(3\rho-4)(3\rho-1) + 2\rho^2(4+\rho(\rho-7+7\rho^2-5\rho^3)) - \\ 2Tw^3(\rho^4-4\rho+2\rho^5+7\rho^6-3\rho^7-8\rho^8+5\rho^9) \end{array} \right) \tag{35}$$

$$\lambda = \left(\begin{array}{l} Tw^6(\rho-1)(\rho(9+2\rho^2(\rho(1+\rho)(3+\rho(7-2\rho-3\rho^3+\rho^4))-7))-1) - \\ Tw^7(1+\rho)(2\rho^2(\rho(2+\rho(4+\rho(4+\rho(-3+\rho(-4+\rho(-1+\rho+2\rho^2))))))-5)-1) + \\ Tw^5\rho(\rho(5+2\rho(9+\rho(\rho(3+\rho(11+\rho(\rho(2\rho^2+5\rho^3-9-2\rho)-5)))-9)))-17) \end{array} \right) \tag{36}$$

$$\tau = \left(\begin{array}{l} 2Tw^2(\rho-1)(1+\rho)(\rho(11+\rho(13+\rho(\rho-16-3\rho^2+\rho^3)))-8) - \\ 2(\rho-1)\rho(1+\rho)((\rho-7)(\rho-1)\rho-3) - 2Tw(\rho-1)(1+\rho)(\rho(13+\rho(\rho(11\rho-5)-8))-3) + \\ Tw^5(\rho(2+\rho(33+2\rho(6+\rho(2\rho^3+\rho^4+\rho^5+\rho^6+\rho^7-12-11\rho))))-14) - \\ 2Tw^4(\rho-1)(1+\rho)(15+\rho(\rho(\rho(\rho(4+\rho(3+\rho)(4+3\rho))-14)-22)-5)) - \\ 2Tw^3(\rho-1)(1+\rho)(\rho(7+\rho(17+\rho(\rho(1+\rho((\rho-12)\rho-4))-2)))-11) \end{array} \right) \tag{37}$$

$$\upsilon = \left(\begin{array}{l} 2Tw^4\rho(2+\rho(\rho(\rho(19+\rho^2(\rho(13\rho-1)-16))-1)-16)) + \\ 2Tw^3(\rho-1)(1+\rho)(\rho(8+\rho(\rho(\rho(5+\rho(\rho(4\rho-15)-1))-2)-8))-6) \end{array} \right) \tag{38}$$

$$\omega = \left(\begin{array}{l} Tw^2\rho(7+\rho^2(21+\rho(6+\rho))) + Tw(\rho(1+\rho(3\rho(\rho-4)-4))-1) + \\ Tw^3(3-\rho(6+\rho(1+\rho)(11+\rho(7+\rho(3\rho-4))))) \end{array} \right) \tag{39}$$

$$\eta = Tw^6(13+\rho(3+\rho(2\rho(\rho(16+\rho(13+\rho^2(\rho(\rho^3-7-2\rho)-8)))-5)-33))) \tag{40}$$

These expressions are plotted in Figure 9.

Disney, S.M. and Hosoda, T., (2009), "The benefit of altruistic behaviour achieved by the order-up-to policy with unmatched proportional feedback controllers in a two-echelon supply chain", *International Journal of Intelligent Systems, Technologies and Applications*, Vol. 6, No. 3/4, pp269–286. ISSN 1740-8865. DOI: 10.1504/IJISTA.2009.024257.

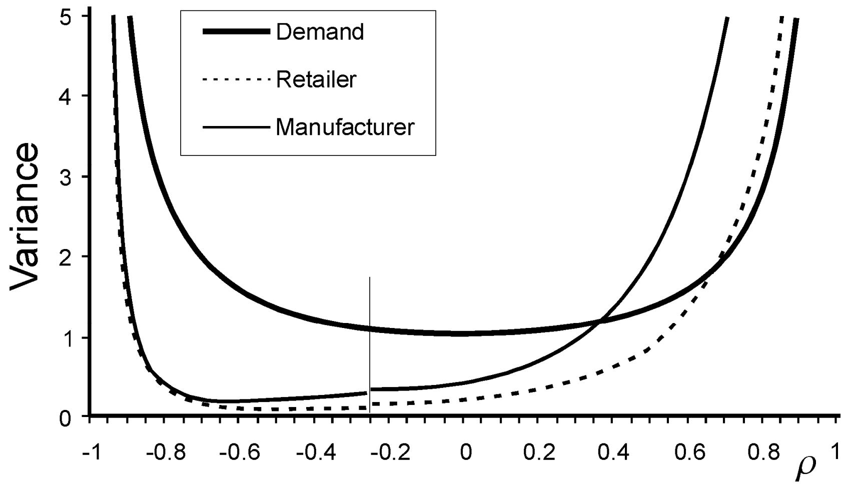

Figure 9. The order variances with unmatched controllers (of *Ti*=*Ti and *Tw*=*Tw**) at with an altruistic retailer**

8 NUMERICAL INVESTIGATIONS

In order to highlight the benefit of the unmatched controllers with altruistic retailer we will now enumerate the inventory costs and order variances for a range of values in the autoregressive demand parameter. This is shown in Table 1.

Table 1. Enumeration of the three supply chain scenarios

	Traditional Supply Chain, Ti=Tw=1			Altruistic Retailer with Matched Controllers					Altruistic Retailer with Unmatched Controllers					
ρ	Retailers Inventory Cost	Man. Inventory Cost	Total Inventory Costs	Tw^*=Ti^*	Retailers Inventory Cost	Man. Inventory Cost	Total Inventory Costs	% benefit above traditional supply chain	Ti^*	Tw^*	Retailers Inventory Cost	Man. Inventory Cost	Total Inventory Costs	% benefit above matched supply chain
-1	1	1	2	99999	1	1	2	0	2.2916	1	1.210608	0.43637	1.64698	17.650818
-0.9	1.004987	0.927826	1.93281	7.0522	1.015004	0.836012	1.85101	4.23202052	2.5032	1.0538	1.212717	0.36862	1.58134	14.568868
-0.8	1.019803	0.901767	1.92157	5.4967	1.058742	0.703698	1.76244	8.28125428	2.2592	1.12469	1.222485	0.34969	1.57219	10.795020
-0.7	1.044030	0.907694	1.95172	4.8776	1.115600	0.608615	1.72421	11.6568244	2.7322	1.21156	1.245786	0.35458	1.60038	7.1824312
-0.6	1.077033	0.934631	2.01166	4.5240	1.186125	0.540233	1.72635	14.1825429	2.7945	1.30595	1.283655	0.36894	1.6526	4.2723559
-0.5	1.118034	0.976281	2.09431	4.2717	1.266789	0.494717	1.76150	15.8910170	2.8298	1.39529	1.334536	0.38756	1.7221	2.2372178
-0.4	1.166190	1.030543	2.19673	4.0655	1.354407	0.469610	1.82401	16.9668312	2.8380	1.46523	1.396672	0.40978	1.80646	0.9628588
-0.3	1.220655	1.098303	2.31895	3.8898	1.446924	0.463397	1.91032	17.6215702	2.8195	1.50119	1.468354	0.43780	1.90616	0.2178938
-0.2	1.280624	1.182294	2.46291	3.7443	1.543644	0.475552	2.01919	18.0161371	11.318	-1.9466	1.532943	0.45499	1.98794	1.5480031
-0.1	1.345362	1.286227	2.63158	3.6294	1.644620	0.507137	2.15175	18.2335213	8.0763	-2.16987	1.68466	0.41019	2.09486	2.6443885
0	1.414213	1.414213	2.82842	3.5414	1.749812	0.561211	2.31102	18.2929485	6.6096	-2.39687	1.8387	0.39577	2.23447	3.3125049
0.1	1.486606	1.570484	3.05709	3.4738	1.869447	0.631878	2.50132	18.1795635	5.7847	-2.6212	1.994565	0.41342	2.40799	3.7314174
0.2	1.562049	1.759289	3.32134	3.4201	1.970832	0.756007	2.72684	17.8994008	5.2690	-2.83001	2.153179	0.46502	2.6182	3.9840359
0.3	1.640121	1.984940	3.62506	3.3762	2.085404	0.907326	2.99273	17.4433165	4.9208	-3.01765	2.314919	0.55435	2.86928	4.1251672
0.4	1.720465	2.251883	3.97234	3.3394	2.202038	1.101507	3.30354	16.8364639	4.6693	-3.18779	2.479179	0.6869	3.16611	4.1602403
0.5	1.802775	2.564786	4.36756	3.3080	2.320380	1.343665	3.66404	16.1077385	4.4777	-3.34643	2.645139	0.86864	3.51379	4.1009575
0.6	1.886796	2.928579	4.81537	3.2805	2.440127	1.639021	4.07914	15.2890968	4.3265	-3.49769	2.812214	1.10518	3.9174	3.9651666
0.7	1.972308	3.348462	5.32077	3.2561	2.561007	1.992942	4.55395	14.4118412	4.2041	-3.64383	2.980042	1.40206	4.38211	3.7734610
0.8	2.059126	3.829869	5.88899	3.2340	2.682787	2.410940	5.09372	13.5043066	4.1033	-3.78616	3.148372	1.76476	4.91314	3.5453700
0.9	2.147091	4.378447	6.52553	3.2137	2.805271	2.898642	5.70391	12.5908988	4.0187	-3.92556	3.317006	2.19882	5.51583	3.2973641
1	2.236067	5	7.23606	3.1947	2.928303	3.461763	6.39006	11.6914462	3.9468	-4.06275	3.485779	2.70988	6.19566	3.0422908
							Average >	14.1585114					Average >	4.9103729

We can see that if the retailer is able to alter his replenishment rule to incorporate matched feedback controllers then total supply chain inventory costs may be reduced by as much as 14% when compared to a traditional supply chain. However, if the retailer is will to go even further and use appropriately tuned unmatched controllers, then a further 4.9% reduction in total supply chain inventory costs may be gained.

9 CONCLUSIONS

The unmatched controller generalised OUT policy dominates the matched controller case with an altruistic retailer who is concerned with minimising the global supply chain inventory costs. The benefit appears to be approximately 5% reduction in the inventory holding and backlog costs. Closer inspection reveals that the altruistic contribution of the retailer, in the unmatched case, is even higher than in the matched case. However, the rewards are even higher when compared to the traditional supply chain where members are only concerned with their local inventory holding and backlog costs as the unmatched controller case is 18.5% better, on the average.

In order to gain this advantage the first echelon needs to be able to understand the manufacturer's cost structure, the demand signal and the lead-times in the supply chain and then alter the structure of his replenishment rules. This is, indeed, a very complex task and we imagine that it will take considerable industrial engineering efforts to achieve. Even if this could technically be done then the manufacturer has to understand and use market place information and be willing to share some of the economic benefit with his customer. Otherwise, the retailer will have no incentive to make the altruistic contribution and smooth his replenishment orders. Of course, we have also assumed a linear system exists, and thus all unmet demand has been backordered and the statistical properties of the demand signal are time invariant.

As a final point, the analysis therein is very complex and rather ugly. Recent work by Gaalman and Disney (2007) suggests that much more elegant results can be found by exploiting the "full-state-feedback" controller, a technique advocated by modern control theory. This will be explored in future research.

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