Hosoda, T. and Disney, S.M., (2006), "The governing dynamics of supply chains: The impact of altruistic behavior", Automatica, Vol. 42, No. 4, pp1301–1309. DOI: 10.1016/j.automatica.2006.03.013.

The governing dynamics of supply chains: The impact of altruistic behaviour ⋆

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Abstract

This paper analyses an infinite horizon two-echelon supply chain inventory problem and shows that a sequence of the optimum ordering policies does not yield globally optimal solutions for the overall supply chain. First-order autoregressive demand pattern is assumed and each participant adopts the order-up-to (OUT) policy with a minimum mean square error forecasting scheme to generate replenishment orders. To control the dynamics of the supply chain, a proportional controller is incorporated into the OUT policy, which we call a generalised OUT policy. A two-echelon supply chain with this generalised OUT policy achieves over 10% inventory related cost reduction. To enjoy this cost saving, the attitude of first echelon player to cost increases is an essential factor. This attitude also reduces the bullwhip effect. An important insight revealed herein is that a significant amount of benefit comes from the player doing what is the best for the overall supply chain, rather than what is the best for local cost minimisation.

Key words: Multi-echelon inventory, order-up-to policy, collaboration, inventory costs, optimisation, base-stock policy

1 Introduction

Arrow *et al.* (1951) introduced the (s, S) ordering policy; Karlin (1960) studies the order-up-to (OUT) policy, that is, the $s = S$ case of the (s, S) policy. Karlin shows that if the purchase cost is linear and set-up costs do not exist, the optimal policy in each period can be characterised by a single critical number. Assuming an ARIMA (Box *et al.*, 1994) demand process, minimum mean square error (MMSE) forecasting, linear inventory holding and stock-out costs and zero lead-time, Johnson and Thompson (1975) show that the OUT policy is optimal. The OUT policy is widely employed in the real business world. Indeed, at least 2 of the 4 largest UK grocery retailers use this policy to replenish stores and DC's (e.g. Potter *et al.*, 2004). Focusing on the OUT policy, we examine a collaboration scheme that minimises the total supply chain costs for a two-echelon case.

For a single echelon of a supply chain, Vassian (1955) introduced an ordering policy with a Work In Progress (WIP) feedback loop and showed that this ordering policy minimises the variance of the net inventory levels. In addition, Vassian showed that the minimised variance of the net inventory level is identical to the variance of the error in the forecast of demand over the lead-time. In this paper, we call Vassian's ordering policy *the traditional OUT policy*. Note that several researchers adopt an alternative expression for the OUT policy that exploits a time varying OUT target (e.g. Lee *et al.*, 2000; Alwan *et al.*, 2003; Zhang, 2004), however, the dynamics given by these two expositions is identical (Hosoda, 2005; Hosoda and Disney, 2006).

From Vassian's seminal contribution, it is obvious that in a single echelon of a supply chain, the traditional OUT policy is an optimal policy for minimising the variance (or standard deviation) of inventory levels over time. In a multi-echelon supply chain scenario, however, it might be reasonable to assume that a sequence of the traditional OUT policies may not be optimal anymore as there is no guarantee that a succession of local minimisations will result in a global optimum.

The traditional order-up-to policy does not provide much freedom to manipulate the dynamics of the ordering process. By incorporating a proportional controller into the traditional OUT policy, however, a much richer policy is created where we have more flexibility to shape the ordering process. Using control theory techniques,

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several researchers have successfully manipulated the variances of the net inventory level with the addition of proportional controllers (e.g. Dejonckheere *et al.*, 2003; Disney and Towill, 2003; Disney *et al.*, 2004) in a policy's feedback loops. A comprehensive review on ordering policies with proportional controllers can be seen in Disney and Towill (2005). This research is motivated by a question; by incorporating a proportional controller into the traditional OUT policy and tuning the value of it properly, can the performance of the traditional OUT policy supply chain be improved upon? Our research purpose is to identify whether a particular form of collaboration (redistributing inventory costs) can achieve a better overall performance, and to quantify the benefit of this collaboration.

Assuming the market demand process follows the first-order autoregressive (AR(1)) process, Hosoda and Disney (2006) analyse a three echelon supply chain with a traditional OUT policy and MMSE forecasting. They present a formula for the variances of net inventory levels at each echelon level and conclude that there is no benefit of the information sharing in terms of lowering these variances. This paper is a sequel to Hosoda and Disney (2006). From here, we refer it as HD and will use HD's model as a benchmark for performance comparisons. [1]

2 Literature review

Many types of collaboration between participants in the supply chain have been studied from the point of reducing uncertainties in a supply chain. However, counter intuitively, not all results strongly support the benefit of collaboration.

Graves (1999) studies a two-echelon supply chain with the OUT policy and a non-stationary demand process and finds that sharing demand information brings no benefit to the upstream player. Kim and Ryan (2003) analyse the value of demand information sharing using a model with an unknown demand process and the exponential smoothing forecasting mechanism. They conclude that sharing demand data can significantly reduce up-stream costs in the supply chain. However, the benefit is limited when a large amount of historical order data is available. Assuming a known demand process and an MMSE forecast, Raghunathan (2001) reports similar results in that the set of order history data contains all the necessary information to reduce up-stream costs. Gavirneni *et al.* (1999) find that the benefit of information sharing increases as capacity increases since higher capacity provides the supplier with some flexibility in production planning. Assuming that the manufacturer can receive market demand information from the retailer even during time periods in which the retailer does not order, Simchi-Levi and Zhao (2003) report that there is a benefit of information sharing if the production capacity is very large and that the benefit partially depends upon the timing of information sharing. In their model, i.i.d. demand is assumed. Aviv and Federgruen (1998) conclude that the benefit from sharing demand information only is limited and that the Vendor Managed Inventory (VMI) program (where information on inventory levels is also shared) has much more potential and can reduce costs on average by 4.7%.

Bourland *et al.* (1996) study the impact of the frequency of market demand information sharing on the inventories in a two-echelon supply chain with normally distributed demand. They show that in a certain setting, as a result of more frequent demand information sharing, the expected inventories at the second echelon can be lowered by 26%. However, at the same time, those at the first echelon have increased by 4.2%. Using a two-echelon supply chain model, Aviv (2001) studies the benefit of collaborative forecasting and finds that the reduced level of uncertainty in the forecasting improves the cost performance of the supply chain. As the traditional OUT policy ensures that the variance of net inventory levels and the variance of forecast error over the lead-time are identical, HD indicates that to minimise the variance of net inventory levels, the MMSE forecast is an essential ingredient. They also show that each player does not necessarily need to share any information to improve its performance, since all the necessary information required to increase performance is already contained in the ordering process.

From the literature review, a useful general insight might be drawn. If market demand information sharing is already transmitted to the supply chain frequently, the benefit coming only from the reduced uncertainties by a collaboration is at best minor.

3 The objective function and model assumptions

We consider an infinite horizon two-echelon inventory problem. Assuming that the inventory related costs in the supply chain are directly proportional to standard deviation of the net inventory levels (e.g. Zipkin, 1995) at each echelon, we employ an objective function that is the sum of these standard deviations. The objective function can be expressed as

$$J = \sqrt{\mathrm{Var}(NS_1)} + \sqrt{\mathrm{Var}(NS_2)},$$

where $\mathrm{Var}(NS_n)$ is the variance of net inventory levels at echelon n. It should be noted that there is no fixed ordering cost in our model, as is commonly assumed (e.g. Johnson and Thompson, 1975; Aviv and

[1] Due to the space limitation, we have used Hosoda (2005) as the reference to all proofs in this paper. However, appendices proving the assertions claimed in this paper are available upon request.

Federgruen, 1998; Gavirneni *et al.*, 1999; Simchi-Levi and Zhao, 2003). The aim of this paper, therefore, is to analyse a form of supply chain collaboration between players that reduces the value of J.

A periodic review system is assumed and all of the results here are consistent whichever review period is adopted (day, week, month, *etc.*). The sequence of events at each echelon is as follows; at the beginning of a period, the replenishment orders placed earlier are received, the demand is fulfilled, the inventory levels and the WIP (the sum of the all orders that are already placed but not yet received) are reviewed and an ordering decision is made at the end of the period. Excess inventory is returned without penalty (i.e., the order quantity can be negative). This assumption may not be critical as shown in Lee *et al.* (1997), especially when the mean demand level is greater than 4 standard deviations of the demand variance. Excess demand is back-ordered (i.e., the net stock level can be negative at the end of time period) until the necessary stock becomes available. The replenishment lead-time at echelon n, L_n, is a constant positive integer. This lead-time includes replenishment, order processing delays, and a sequence of events delay. Capacity is unrestricted in our model. If the capacity limit is added to the model, the quantified benefit herein will be decreased to a certain level since replenishment decisions at each echelon will be affected by this limit (e.g. Gavirneni *et al.*, 1999; Simchi-Levi and Zhao, 2003; Aviv and Federgruen, 1998). Clearly decisions affected in this way cannot yield the minimum inventory related costs. Similar discussions can be seen in Cachon and Fisher (2000).

For the demand pattern, we assume an AR(1) pattern, as HD does. This assumption is common when autocorrelation exists among the demand process (e.g. Kahn, 1987; Lee *et al.*, 2000; Alwan *et al.*, 2003; Kim and Ryan, 2003; Zhang, 2004). The AR(1) process can be expressed as,

$$D_t = d + \rho(D_{t-1} - d) + \varepsilon_t, \tag{1}$$

where D_t is the demand at time period t, d is the mean demand, ρ is the autoregressive coefficient, $|\rho| < 1$, and ε_t is an i.i.d. white noise process with a mean of zero and a variance of σ_ε^2. $\mathrm{Var}(D)$, the variance of the AR(1) process, is given by $\sigma_\varepsilon^2/(1-\rho^2)$. For the sake of simplicity, we may set $d = 0$ without loss of generality.

4 Scenario 1: A supply chain that exploits the traditional OUT policy

In this scenario we assume that the supply chain consists of a sequence of traditional OUT policies. Each player employs the traditional OUT policy and the MMSE forecasting scheme. This policy minimises the variance of net inventory levels for a given demand and order process at each echelon. This scenario has been studied before in HD and here we will now summarise their results.

4.1 Traditional Order-up-to policy

The traditional OUT policy can be described as follows (Vassian, 1955)

$$O_t = \hat{D}_t^L - (WIP_t + NS_t),$$

where O_t is the order quantity placed at time period t, $\hat{D}_t^L$ is the conditional estimate of the total demand over the lead-time, $L(= 1, 2, 3, \ldots)$ made at time period t, WIP_t is the total orders that are already placed but not yet received, and NS_t is the net inventory level at the end of period t. The WIP at time t, WIP_t, can be expressed as;

$$WIP_t = \begin{cases} 0 & \text{if } L = 1, \\ \sum_{i=1}^{L-1} O_{t-i} & \text{otherwise.} \end{cases}$$

Let us use $\mathrm{Var}(\overline{NS}_n)$ to denote the variance of the net inventory levels at the echelon n, in the supply chain where the traditional OUT policy is exploited. HD provides the following expressions for $\mathrm{Var}(\overline{NS}_1)$ and $\mathrm{Var}(\overline{NS}_2)$;

$$\mathrm{Var}(\overline{NS}_1) = \frac{\left(L_1(1-\rho^2) + \rho(1-\rho^{L_1})(\rho^{L_1+1} - \rho - 2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2,$$

$$\mathrm{Var}(\overline{NS}_2) = \frac{\begin{pmatrix} L_2\left(1-\rho^2\right) + \\ \rho^{L_1+1}\left(1-\rho^{L_2}\right)\left(\rho^{L_1+1} + \rho^{L_1+L_2+1} - 2\rho - 2\right) \end{pmatrix}}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2.$$

Therefore the objective function for Scenario 1, J_{S1}, becomes

$$\begin{aligned} J_{S1} &= \sqrt{\mathrm{Var}(NS_1)} + \sqrt{\mathrm{Var}(NS_2)} \\ &= \sqrt{\mathrm{Var}(\overline{NS}_1)} + \sqrt{\mathrm{Var}(\overline{NS}_2)} \\ &= \sqrt{\frac{\left(L_1(1-\rho^2) + \rho(1-\rho^{L_1})(\rho^{L_1+1} - \rho - 2)\right)}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2} + \\ &\quad \sqrt{\frac{\begin{pmatrix} L_2\left(1-\rho^2\right) + \rho^{L_1+1}\left(1-\rho^{L_2}\right) \\ \left(\rho^{L_1+1} + \rho^{L_1+L_2+1} - 2\rho - 2\right) \end{pmatrix}}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2}. \end{aligned}$$

4.2 The ordering process, MMSE forecasts and the value of information sharing

In a traditional OUT policy, the accuracy of the forecast directly affects the variance of the net inventory level.

HD shows that O_{t+1} can be described as an ARMA(1, 1) process

$$O_{t+1} = \rho O_t + (1 + \rho\Lambda_{L_1})\varepsilon_{t+1} - \rho\Lambda_{L_1}\varepsilon_t, \tag{2}$$

where $\Lambda_{L_1} = (1-\rho^{L_1})/(1-\rho)$ and its variance is given as

$$\mathrm{Var}(O) = \frac{\begin{pmatrix} \left(1-\rho^{L_1+1}\right)^2 + \rho^2\left(1-\rho^{L_1}\right)^2 - \\ 2\rho^2\left(1-\rho^{L_1+1}\right)\left(1-\rho^{L_1}\right) \end{pmatrix}}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2.$$

Therefore, we may conclude that if historical data of O_t is available and it is analysed via the ARMA(1, 1) model (Box *et al.*, 1994), we can obtain estimated values of ρ and ε_t. Thus, from Eq. 2, we find that the unknown error term in the ordering process is $(1+\rho\Lambda_{L_1})\varepsilon_{t+1}$ for the case of no-information sharing. HD shows that O_{t+1} can also be expressed as

$$O_{t+1} = D_{t+1} + \rho\Lambda_{L_1}(D_{t+1} - D_t).$$

After substituting Eq. 1 into the equation above, some algebraic simplification yields

$$O_{t+1} = \rho^{L_1+1}D_t + (1+\rho\Lambda_{L_1})\varepsilon_{t+1}.$$

From this we can see that even if the up-to-date demand information D_t and the value of ρ are shared, the value of the unknown error term, $(1+\rho\Lambda_{L_1})\varepsilon_{t+1}$, is the same as in the case of no-information sharing. Therefore, in terms of the forecast accuracy, there is no difference between the case of information sharing and the case of no-information sharing. Since the MMSE forecast accuracy leads to the lower variance of net inventory levels, it is obvious that information sharing alone does not contribute to a lower standard deviation of net stock levels.

5 Scenario 2: A supply chain that exploits the generalised OUT policy

Scenario 2 assumes that the first echelon player employs the OUT policy with a proportional controller, F, added into the inventory position feedback loop. We call this ordering policy *the generalised OUT policy.* The second echelon player, however, still uses the traditional OUT policy with an updated MMSE forecasting scheme to minimise the variance of its own net inventory levels. At the end of this section, we will discuss the adequacy of our choice of employing the traditional OUT policy at the second echelon. In Scenario 2, a collaboration scheme is assumed. This means that the supply chain players are concerned only with the overall supply chain cost. The first echelon player manages its ordering process, by tuning the value of F, the proportional controller, to allow the second echelon player to reduce the cost at the second echelon. As a result of tuning the value of F, the cost at the first echelon may increase. If the cost reduction at the second echelon is large enough, however, the overall supply chain cost may decrease.

5.1 *The generalised Order-Up-To policy*

To realise our generalised OUT policy, let us begin by reviewing the traditional OUT policy.

$$\begin{aligned} O_t &= \hat{D}_t^{L_1} - (WIP_t + NS_t) \\ &= \acute{D}_t^{L_1} + \hat{D}_t^{L_1-1} - (WIP_t + NS_t) \\ &= \acute{D}_t^{L_1} + \left(\hat{D}_t^{L_1-1} - (WIP_t + NS_t)\right) \\ &= \acute{D}_t^{L_1} + \left(DIP_t - (WIP_t + NS_t)\right), \end{aligned} \tag{3}$$

where $\acute{D}_t^{L_1}$ is the conditional estimate of the demand in time period $t+L_1$ made at time period t. Therefore, $\hat{D}_t^{L_1} = \acute{D}_t^{L_1} + \hat{D}_t^{L_1-1}$. DIP_t is a Desired Inventory Position at time period t, that can be described as $\hat{D}_t^{L_1-1}$. Note that $DIP_t = 0$, if $L_1 = 1$; $DIP_t = \hat{D}_t^{L_1-1}$, if $L_1 > 1$. Incorporating a proportional controller, F, into Eq. 3 surrenders a generalised OUT policy

$$O_t = \acute{D}_t^{L_1} + F\left(DIP_t - (WIP_t + NS_t)\right), \tag{4}$$

where $0 < F < 2$ as shown in Hosoda (2005). Obviously, if $F = 1$, the generalised OUT policy is identical to the traditional OUT policy. From here, we will use the expression $\mathrm{Var}(\widetilde{NS}_n)$ for the variance of the net inventory of the generalised OUT policy at the echelon n, and J_{S2} for the objective function for Scenario 2. F^* represents the optimum value of F minimising J_{S2}, and the minimised value of J_{S2} is denoted as J^*_{S2}.

$\mathrm{Var}(\widetilde{NS}_1)$ can be written as

$$\begin{aligned} \mathrm{Var}(\widetilde{NS}_1) &= \frac{\begin{pmatrix} L_1(1-\rho^2)+ \\ \rho(1-\rho^{L1})(\rho^{L_1+1}-\rho-2) \end{pmatrix}}{(1-\rho)^2(1-\rho^2)}\sigma_\varepsilon^2 + \\ &\quad \frac{\left(\rho^{L_1}-1\right)^2(F-1)^2}{(\rho-1)^2(2-F)F}\sigma_\varepsilon^2 \\ &= \mathrm{Var}(\overline{NS}_1) + \frac{\Omega^2\Psi^2}{(\rho-1)^2(1-\Psi^2)}\sigma_\varepsilon^2, \end{aligned} \tag{5}$$

where $\Psi = 1 - F$ and $\Omega = \rho^{L_1} - 1$. Hosoda (2005) provides details. Clearly, the second term of the RHS of Eq. 5 is always equal to or greater than zero. This means that with the generalised OUT policy the first echelon inventory investments are never less than with the traditional OUT policy. From this, it is easy to see that;

- If F^* is not equal to 1, then $\sqrt{\mathrm{Var}(\widetilde{NS}_1)} > \sqrt{\mathrm{Var}(\overline{NS}_1)}$. In this paper, we characterise this situation as *altruistic* behaviour or an *altruistic* contribution because the first echelon accepts increased local costs to enable the total supply chain costs to be reduced.
- If $F^* = 1$, the supply chains for Scenario 1 and Scenario 2 are identical. In that case, we can conclude that incorporating proportional controller into the traditional OUT policy brings no benefit in terms of reducing our objective function, the sum of the standard deviations of the net inventory levels.

5.2 The ordering process, MMSE forecasts and the value of information sharing

To minimise the objective function, the second echelon or supplier who employs the traditional OUT policy, must complete an MMSE forecast of the orders placed by the first echelon player. The ordering process at time period $t+1$ placed by the first echelon player can be written as

$$\begin{aligned} O_{t+1} &= \acute{D}_{t+1}^{L_1} + F\big(DIP_{t+1} - (WIP_{t+1} + NS_{t+1})\big) \\ &= (1-F)O_t + \rho^{L_1}(\rho + F - 1)D_t + \\ &\quad \big(\rho^{L_1} + F\Lambda_{L_1}\big)\,\varepsilon_{t+1}, \end{aligned} \tag{6}$$

and its variance is

$$\mathrm{Var}(\widetilde{O}) = \frac{\begin{pmatrix} 2F\rho^{L_1}(\rho+1)(\rho+F-1)- \\ 2\rho^{2L_1}(\rho+F-1)^2- \\ F(\rho+1)\left(1+(F-1)\rho\right) \end{pmatrix}}{(F-2)\,(\rho-1)^2\,(\rho+1)\left(1+(F-1)\rho\right)}\sigma_\varepsilon^2.$$

Details are shown in Hosoda (2005). Eq. 6 tells us that to complete the MMSE forecast, the upper-stream player needs up-to-date information of D_t, and knowledge of the values of ρ, F and L_1, that we assume to be constants herein. It should be noted that Hosoda (2005) shows that Eq. 6 can be rewritten as an ARMA(1, ∞) process and interestingly, the value of the autoregressive coefficient is $1-F$, not ρ. Theoretically, by keeping an infinite number of historical orders, the second echelon player can do an MMSE forecast against the ARMA(1, ∞) demand process without the sharing of any up-to-date information. Therefore, the benefit of the up-to-date market information sharing decreases as the availability of the historical order data increases. [2]

[2] The block diagrams for the case of demand information sharing and the case of no demand information sharing are presented in Hosoda (2005). The transfer functions and dynamics of these two cases are equivalent. Thus we may conclude with our assumptions that there is no value of information sharing in our infinite horizon two-echelon supply chain.

5.3 The objective function

$\mathrm{Var}(\widetilde{NS}_2)$ can be obtained from the forecast error over the lead-time, as shown in Hosoda (2005).

$$\begin{aligned} &\mathrm{Var}(\widetilde{NS}_2) = \\ &\quad \frac{\sigma_\varepsilon^2}{(\rho-1)^2}\Bigg(L_2 + \frac{\left(\Psi^{2L_2}-1\right)\Omega^2\Psi^2}{\Psi^2-1} + \\ &\quad \frac{\rho^{L_1+1}\left(\rho^{L_2}-1\right)\left(\rho^{L_1+L_2+1}+\rho^{L_1+1}-2\rho-2\right)}{\rho^2-1} + \\ &\quad 2\Omega\Psi\left(\frac{1-\Psi^{L_2}}{1-\Psi} - \frac{\rho^{L_1+1}\left((\rho\Psi)^{L_2}-1\right)}{\rho\Psi-1}\right)\Bigg) \\ &= \mathrm{Var}(\overline{NS}_2) + \\ &\quad \frac{\sigma_\varepsilon^2}{(\rho-1)^2}\Bigg(\frac{\left(\Psi^{2L_2}-1\right)\Omega^2\Psi^2}{\Psi^2-1} + \\ &\quad 2\Omega\Psi\left(\frac{1-\Psi^{L_2}}{1-\Psi} - \frac{\rho^{L_1+1}\left((\rho\Psi)^{L_2}-1\right)}{\rho\Psi-1}\right)\Bigg). \end{aligned} \tag{7}$$

Thus, using Eq. 5 and Eq. 7, the objective function for the Scenario 2, J_{S2}, may be written as

$$\begin{aligned} J_{S2} &= \sqrt{\mathrm{Var}(NS_1)} + \sqrt{\mathrm{Var}(NS_2)} \\ &= \sqrt{\mathrm{Var}(\widetilde{NS}_1)} + \sqrt{\mathrm{Var}(\widetilde{NS}_2)} \\ &= \sqrt{\mathrm{Var}(\overline{NS}_1) + \frac{\Omega^2\Psi^2}{(\rho-1)^2\left(1-\Psi^2\right)}\sigma_\varepsilon^2} + \\ &\quad \sqrt{\begin{aligned} &\mathrm{Var}(\overline{NS}_2) + \\ &\frac{\sigma_\varepsilon^2}{(\rho-1)^2}\left(\frac{\left(\Psi^{2L_2}-1\right)\Omega^2\Psi^2}{\Psi^2-1} + \right. \\ &\left. 2\,\Omega\Psi\left(\frac{1-\Psi^{L_2}}{1-\Psi} - \frac{\rho^{L_1+1}\left((\rho\Psi)^{L_2}-1\right)}{\rho\Psi-1}\right)\right) \end{aligned}}. \end{aligned} \tag{8}$$

From Eq. 8, we have the following proposition. It is possible to prove that F^* never has unit value. The proof is provided in Hosoda (2005). Further analytical results are difficult to present due to the rather unwieldy expressions for the roots of Eq. 8. Thus we are now forced to resort to numerical investigations. However, first let us conclude this section with an investigation of the consequences of our selection of the traditional OUT policy at the second echelon.

5.4 On the second echelon ordering policy

Here, we have incorporated another proportional controller, F_2, into the second echelon. Thus, the model we use here is a two-echelon supply chain model where both two players employ the generalised OUT policy. Following Bellman's *Principle of Optimality* (Bellman, 1957, pp. 83), we will now show that even if we assume both two echelon players use the generalised OUT policy, the results from that model reduce to the results from our model previously considered. In other words, we are now going to prove the optimal value of F_2 is unity. Under the condition that the values of ρ, σ_ε^2, and lead-times are known, the variance of the net inventory levels at each echelon can be seen as a function of proportional controllers. Here, we introduce a new term, $f_{NS_2}(F_2|F)$ which represents the variance of the net inventory level at the second echelon with a proportional controller F_2, subject to F. If $F_2 = 1$, $f_{NS_2}(F_2|F)$ becomes identical [3] to $\mathrm{Var}(\widetilde{NS}_2)$. Now, a new objective function for Scenario 2, J_{S2,F_2} can be expressed as

$$J_{S2,F_2} = \sqrt{\mathrm{Var}(\widetilde{NS}_1)} + \sqrt{f_{NS_2}(F_2|F)}.$$

Remember that $\mathrm{Var}(\widetilde{NS}_1)$ is a function of F, as shown by Eq. 5. Therefore, J^*_{S2,F_2}, the minimised value of J_{S2,F_2}, can be written as

$$\begin{aligned} J^*_{S2,F_2} &= \min_{F,F_2}\left[\sqrt{\mathrm{Var}(\widetilde{NS}_1)} + \sqrt{f_{NS_2}(F_2|F)}\right] \\ &= \min_{F}\left[\sqrt{\mathrm{Var}(\widetilde{NS}_1)} + \min_{F_2}\left[\sqrt{f_{NS_2}(F_2|F)}\right]\right]. \end{aligned}$$

From Vassian's finding that the traditional OUT policy minimises the variance of the net inventory levels, we can deduce that setting $F_2 = 1$ yields the minimum value of $f_{NS_2}(F_2|F)$ at any given value of F. Now, we will have the final expression.

$$\begin{aligned} J^*_{S2,F_2} &= \min_{F}\left[\sqrt{\mathrm{Var}(\widetilde{NS}_1)} + \sqrt{f_{NS_2}(F_2 = 1|F)}\right] \\ &= \min_{F}\left[\sqrt{\mathrm{Var}(\widetilde{NS}_1)} + \sqrt{\mathrm{Var}(\widetilde{NS}_2)}\ \right] \\ &= J^*_{S2}. \end{aligned}$$

By expanding this result, we can have more general conclusion: In supply chains of greater than two echelons, when the downstream echelons use the generalised OUT policy, then the upper echelon should always use the traditional OUT policy to minimise the overall objective function, the sum of the standard deviations of the net inventory levels at each echelon.

6 Numerical investigations

In this section, we will enumerate the two objective functions, J_{S1} and J^*_{S2}, for different lead-time settings of $L_1 = L_2 = 2$ and $L_1 = 1, L_2 = 3$. In all cases, we have set $\sigma_\varepsilon^2 = 1$. Using Eq. 8, we have plotted the values of J_{S2} in Fig. 1 and Fig. 2 with the restriction that $0 < F < 2$, when $\rho = 0.7, 0.0$ and -0.7 for both lead-time settings. From these figures, we can see that J_{S2} has a unique minimum value for the given values of ρ, L_1 and L_2. Using the cylindrical algebraic decomposition algorithm (i.e. Caviness and Johnson, 1998), the value of F^*, which minimises J_{S2} under the model settings, has been obtained. Tables 1–2 show the results of two scenarios with the lead-time setting $L_1 = 2, L_2 = 2$ and Tables 3–4 highlight the case of $L_1 = 1, L_2 = 3$. In Table 3, the value of $\sqrt{\mathrm{Var}(\widetilde{NS}_1)}$ is independent from the value of ρ. This is because in the traditional OUT policy with unit lead-time, if an AR (or ARMA) demand process is assumed, the variance of the forecast error over the unit lead-time is identical to the variance of the error term in the demand process, σ_ε^2 for all ρ. From Tables 1−4, we can conclude that

- $J^*_{S2} < J_{S1}$ for all values of ρ. This means that our generalised OUT policy supply chain always outperforms the traditional OUT policy supply chain.
- F^* never has unit value, as expected.

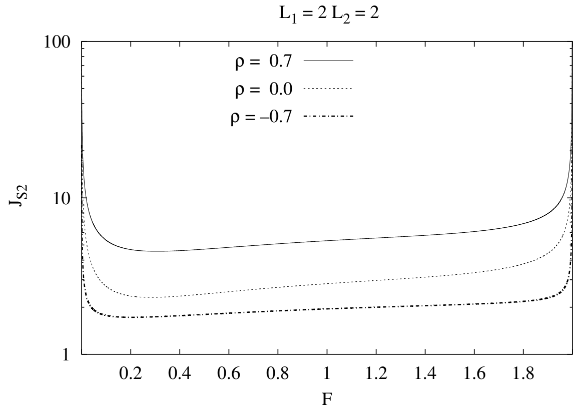

Fig. 1. The value of J_{S2} when $L_1 = 2, L_2 = 2$

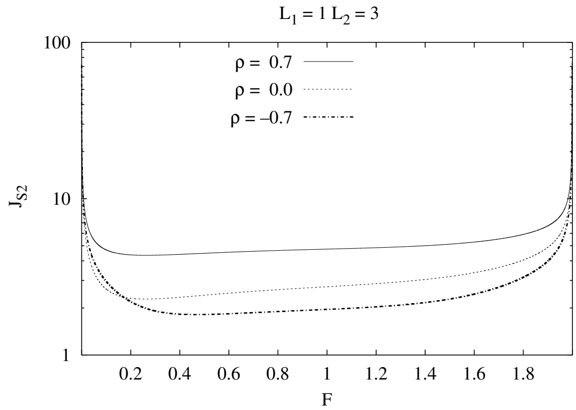

Fig. 2. The value of J_{S2} when $L_1 = 1, L_2 = 3$

[3] In Hosoda (2005), after showing that the net inventory level follows ARMA(1, $L_n - 1$) process, the exact expressions of $f_{NS_2}(F_2|F)$ and $f_{NS_2}(F_2 = 1|F) = \mathrm{Var}(\widetilde{NS}_2)$ are shown.

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Table 1
Calculated values of J_{S1}: $L_1 = 2, L_2 = 2$

Scenario 1			
ρ	$\sqrt{\mathrm{Var}(\widetilde{NS}_1)}$	$\sqrt{\mathrm{Var}(\widetilde{NS}_2)}$	J_{S1}
−0.9	1.005	0.928	1.933
−0.8	1.020	0.902	1.922
−0.7	1.044	0.908	1.952
−0.6	1.077	0.935	2.012
−0.5	1.118	0.976	2.094
−0.4	1.166	1.031	2.197
−0.3	1.221	1.098	2.319
−0.2	1.281	1.182	2.463
−0.1	1.345	1.286	2.632
0.0	1.414	1.414	2.828
0.1	1.487	1.570	3.057
0.2	1.562	1.759	3.321
0.3	1.640	1.985	3.625
0.4	1.720	2.252	3.972
0.5	1.803	2.565	4.368
0.6	1.887	2.929	4.815
0.7	1.972	3.348	5.321
0.8	2.059	3.830	5.889
0.9	2.147	4.378	6.526

- Value of F^* is affected by the value of ρ and lead-time settings.
- $J^*_{S2} < J_{S1}$ is achieved by altruistic behaviour of the first echelon player. That is, by accepting $\sqrt{\mathrm{Var}(\widetilde{NS}_1)} > \sqrt{\mathrm{Var}(\overline{NS}_1)}$.

Now, let us employ the equation $\Delta J = (J_{S1} - J^*_{S2})/J_{S1}$ as a measure of the benefit of altruistic behaviour and information sharing in the supply chain. The calculated values of ΔJ are shown in Fig. 3. The maximum value of ΔJ, 18.3%, is achieved at $\rho = 0.0$ when the lead-time setting is $L_1 = 2, L_2 = 2$. The average values of the ΔJ are 15.0% and 11.6% for each lead-time settings. If we introduce an assumption that the value of autocorrelation coefficient ρ is positive, these average values become 15.8% and 11.1%, respectively.[4] Table 5 shows variance ratios of order rates; a measure of the bullwhip effect. The variance ratio for Scenario 1 is obtained by $\mathrm{Var}(O)/\mathrm{Var}(D)$ and that for Scenario 2 is $\mathrm{Var}(\widetilde{O})/\mathrm{Var}(D)$. If the variance ratio is greater than unit value, we conclude the bullwhip has occurred. Under both lead-time settings,

[4] The assumption of positive values of autocorrelation coefficient is more realistic assumption from the point of the real business world. See Lee *et al.* (1997), Lee *et al.* (2000) and Disney *et al.* (2003).

Table 2
Calculated values of J^*_{S2}: $L_1 = 2, L_2 = 2$

Scenario 2				
ρ	F^*	$\sqrt{\mathrm{Var}(\widetilde{NS}_1)}$	$\sqrt{\mathrm{Var}(\widetilde{NS}_2)}$	J^*_{S2}
−0.9	0.14180	1.019	0.831	1.850
−0.8	0.18193	1.059	0.704	1.762
−0.7	0.20502	1.116	0.609	1.724
−0.6	0.22104	1.186	0.540	1.726
−0.5	0.23410	1.267	0.495	1.762
−0.4	0.24597	1.354	0.470	1.824
−0.3	0.25708	1.447	0.463	1.910
−0.2	0.26707	1.544	0.476	2.019
−0.1	0.27553	1.645	0.507	2.152
0.0	0.28237	1.750	0.561	2.311
0.1	0.28787	1.859	0.642	2.501
0.2	0.29238	1.971	0.756	2.727
0.3	0.29618	2.085	0.907	2.993
0.4	0.29945	2.202	1.102	3.304
0.5	0.30230	2.320	1.344	3.664
0.6	0.30483	2.440	1.639	4.079
0.7	0.30711	2.561	1.993	4.554
0.8	0.30921	2.561	1.993	4.554
0.9	0.31117	2.805	2.899	5.704

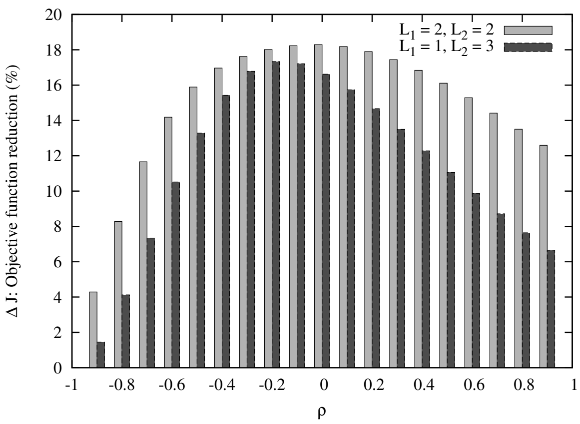

Fig. 3. ΔJ: Objective function reduction (%)

Scenario 1 always generates bullwhip when ρ is positive. On the other hand, in Scenario 2, for almost any values of ρ, the bullwhip is mitigated and the variance ratio is less than unit value. This means that the variance of orders placed by the first echelon is less than that of market demand. Furthermore, the variance ratio of Scenario 2 is always lower than that of Scenario 1 when $\rho \geq -0.5$. If we again exploit the assumption of positive value of ρ, we may conclude that the properly managed altruistic

Table 3
Calculated values of J_{S1}: $L_1 = 1, L_2 = 3$

Scenario 1			
ρ	$\sqrt{\mathrm{Var}(\overline{NS}_1)}$	$\sqrt{\mathrm{Var}(\overline{NS}_2)}$	J_{S1}
−0.9	1.000	0.933	1.933
−0.8	1.000	0.924	1.924
−0.7	1.000	0.956	1.956
−0.6	1.000	1.017	2.017
−0.5	1.000	1.097	2.097
−0.4	1.000	1.192	2.192
−0.3	1.000	1.302	2.302
−0.2	1.000	1.428	2.428
−0.1	1.000	1.570	2.570
0.0	1.000	1.732	2.732
0.1	1.000	1.917	2.917
0.2	1.000	2.130	3.130
0.3	1.000	2.373	3.373
0.4	1.000	2.652	3.652
0.5	1.000	2.971	3.971
0.6	1.000	3.337	4.337
0.7	1.000	3.755	4.755
0.8	1.000	4.232	5.232
0.9	1.000	4.773	5.773

Table 4
Calculated values of J^*_{S2}: $L_1 = 1, L_2 = 3$

Scenario 2				
ρ	F^*	$\sqrt{\mathrm{Var}(\widetilde{NS}_1)}$	$\sqrt{\mathrm{Var}(\widetilde{NS}_2)}$	J^*_{S2}
−0.9	0.65464	1.066	0.840	1.905
−0.8	0.54418	1.124	0.721	1.844
−0.7	0.46560	1.183	0.629	1.812
−0.6	0.40500	1.244	0.560	1.805
−0.5	0.35655	1.306	0.512	1.818
−0.4	0.31807	1.367	0.487	1.854
−0.3	0.28993	1.420	0.496	1.916
−0.2	0.27300	1.456	0.550	2.007
−0.1	0.26509	1.475	0.653	2.127
0.0	0.26207	1.482	0.796	2.278
0.1	0.26105	1.484	0.974	2.458
0.2	0.26074	1.485	1.186	2.671
0.3	0.26067	1.485	1.432	2.917
0.4	0.26073	1.485	1.718	3.203
0.5	0.26091	1.485	2.048	3.532
0.6	0.26126	1.484	2.426	3.910
0.7	0.26181	1.482	2.859	4.341
0.8	0.26257	1.481	3.351	4.832
0.9	0.26356	1.478	3.910	5.389

behaviour at the first echelon reduces not only the overall inventory related costs in a supply chain but also the bullwhip related costs at the second echelon. This is the case despite the fact that no bullwhip cost information was included in the objective function. In Fig. 4, an example of the discrete simulation results is shown in order to demonstrate the impact of the altruistic behaviour at the first echelon on the supply chain dynamics. [5]

7 Conclusions

We have investigated an infinite horizon two-echelon inventory problem assuming the OUT policy and AR(1) demand pattern. By incorporating a single controller, F, into the traditional OUT policy at the first echelon, we may obtain significant savings in the total supply chain cost, as measured by the sum of the standard deviations of the net stock levels at each echelon. To enjoy this high benefit, we show that altruistic behaviour by the first echelon is essential. In addition, this behaviour can mitigate the bullwhip effect. Since the benefit at the second echelon is large enough to compensate for the loss at the first echelon, a central planner should provide an incentive to the first echelon player for his altruistic contribution. An important insight from our results is that a significant amount of benefit comes from each player in the supply chain *doing what is the best for itself and the supply chain, rather than doing what is the best for its own selfish interests.*

In terms of the shared information, in addition to the values of ρ and F, we used up-to-date demand information D_t to enable the upper-stream player to complete an MMSE forecast. This leads to not only the minimum inventory related cost at the second echelon, but also to the minimisation of inventory costs in the overall supply chain. We also show that instead of up-to-date demand information sharing, the historical order data also enables the second echelon player to achieve the same performance in the supply chain. The benefit, therefore, does not depend on reducing the uncertainties in a supply chain. Instead, *it has been achieved by governing the dynamics of the supply chain through the process of minimising the objective function.* This point is the most significant difference from other relevant research schemes that usually exploit the well-known principle that less uncertainty leads to a more efficient supply chain.

[5] Simplified versions of our supply chain simulation models can be seen at http://www.bullwhip.co.uk/bwExplorer.htm.

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Table 5
Measured bullwhip in the first echelon player's order

	$L_1 = 2, L_2 = 2$		$L_1 = 1, L_2 = 3$	
ρ	Scenario1	Scenario2	Scenario1	Scenario2
−0.9	0.689	0.659	0.658	0.732
−0.8	0.516	0.421	0.424	0.545
−0.7	0.436	0.265	0.286	0.403
−0.6	0.416	0.170	0.232	0.293
−0.5	0.438	0.119	0.250	0.210
−0.4	0.489	0.098	0.328	0.151
−0.3	0.569	0.098	0.454	0.114
−0.2	0.677	0.111	0.616	0.100
−0.1	0.820	0.133	0.802	0.112
0.0	1.000	0.164	1.000	0.151
0.1	1.220	0.208	1.198	0.214
0.2	1.476	0.270	1.384	0.301
0.3	1.759	0.355	1.546	0.409
0.4	2.048	0.472	1.672	0.535
0.5	2.313	0.624	1.750	0.673
0.6	2.505	0.810	1.768	0.817
0.7	2.564	1.014	1.714	0.954
0.8	2.405	1.196	1.576	1.063
0.9	1.927	1.261	1.342	1.106

The success of the altruistic behaviour in a real business is completely dependent on the redistribution of the cost savings, since the benefit will be generated at the second echelon only. Thus, without the promised redistribution of benefit, the first echelon player has no incentive for his altruistic contribution. A VMI type of scheme, where the first echelon player lets the second echelon player own and manage all inventory in a supply chain, might be a suitable scheme in which to exploit this altruistic behaviour. Under a VMI scheme, the second echelon player is solely responsible for the overall inventory related costs so that the redistribution of costs between players is not necessary.

Finally, let us highlight the limitations of our research. We have recognised that the results shown herein are conditional upon our model settings; a known demand process, no capacity limitations, and no incentive conflicts between the two players. We have assumed that the sum of the standard deviations of the net inventory levels represents the costs of the total supply chain. However, adding order standard deviations (to capture, for example, bullwhip costs) into the objective function might be more reasonable. If the unit cost of the inventory at the first echelon is different from that at the second echelon, an alternative objective function (that includes weights for the standard deviations) might be a better choice.

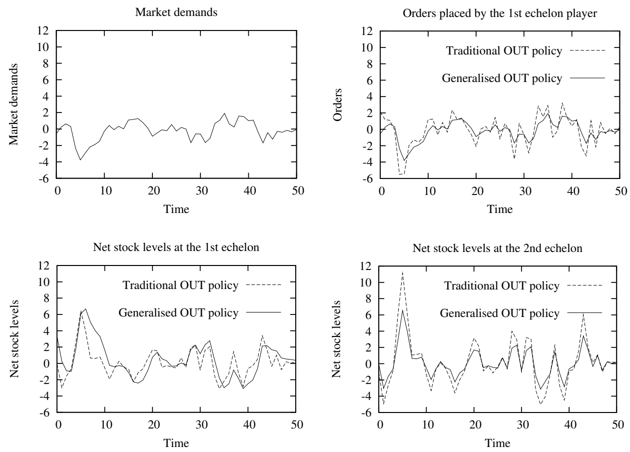

Fig. 4. An example of model behaviour: $\rho = 0.7$, and $L_1 = 2, L_2 = 2$

Acknowledgements

Authors are thankful for two anonymous referees and editors for all their valuable comments.

References

Alwan, L. C., J. J. Liu and D. Yao (2003). Stochastic characterization of upstream demand process in a supply chain. *IIR Transactions* **35**, 207–219.

Arrow, K. J., T. Harris and J. Marschak (1951). Optimal inventory policy. *Econometrica* **19**(3), 250–272.

Aviv, Y. (2001). The effect of collaborative forecasting on supply chain performance. *Management Science* **47**(10), 1326–1343.

Aviv, Y. and A. Federgruen (1998). The operational benefits of information sharing and vendor managed inventory (VMI) programs. Working paper, Washington University, St. Louis, MO.

Bellman, R. (1957). *Dynamic Programming.* Princeton University Press. Princeton, NJ.

Bourland, K. E., S. G. Powell and D. F. Pyke (1996). Exploiting timely demand information to reduce inventories. *European Journal of Operational Research* **9**, 239–253.

Box, G. E. P., G. M. Jenkins and G. C. Reinsel (1994). *Time Series Analysis: Forecasting and Control.* 3rd ed. Prentice Hall. Englewood Cliffs, NJ.

Cachon, G. P. and M. Fisher (2000). Supply chain inventory management and the value of shared information. *Management Science* **46**(8), 1032–1048.

Caviness, B. F. and Johnson, J. R., Eds. (1998). *Quantifier Elimination and Cylindrical Algebraic Decomposition: Texts and monographs in symbolic computation.* Springer–Verlag.

Dejonckheere, J., S. M. Disney, M. R. Lambrecht and D. R. Towill (2003). Measuring and avoiding the bullwhip effect: A control theoretic approach. *European Journal of Operational Research* **147**, 567–590.

Disney, S. M. and D. R. Towill (2003). Vendor-managed inventory and bullwhip reduction in a two-level supply chain. *International Journal of Operations & Production Management* **23**(6), 625–651.

Disney, S. M. and D. R. Towill (2005). Eliminating drift in inventory and order based production control systems. *International Journal of Production Economics* **93-94**, 331–344.

Disney, S. M., D. R. Towill and W. Van de Velde (2004). Variance amplification and the golden ratio in production and inventory control. *International Journal of Production Economics* **90**, 295–309.

Disney, S. M., I. Farasyn, M. Lambrecht, D. R. Towill and W. Van de Velde (2003). Creating win-win scenarios from the bullwhip problem ∼ By design not accident. EUROMA POMS Conference. Como Lake, Italy.

Gavirneni, S., R. Kapuscinski and S. Tayur (1999). Value of information in capacitated supply chain. *Management Science* **45**(1), 16–24.

Graves, S. C. (1999). A single-item inventory model for a nonstationary demand process. *Manufacturing & Service Operations Management* **1**, 50–61.

Hosoda, T. (2005). The principles governing the dynamics of supply chains. PhD thesis. Cardiff University. Wales.

Hosoda, T. and S. M. Disney (2006). On variance amplification in a three-echelon supply chain with minimum mean square error forecasting. *OMEGA, The International Journal of Management Science* **34**(4), 344–358.

Johnson, G. D. and H. E. Thompson (1975). Optimality of myopic inventory policies for certain dependent demand processes. *Management Science* **21**(11), 1303–1307.

Kahn, J. A. (1987). Inventories and the volatility of production. *American Economic Review* **77**, 667–679.

Karlin, S. (1960). Dynamic inventory policy with varying stochastic demands. *Management Science* **6**(3), 231–258.

Kim, H. and J. K. Ryan (2003). The cost impact of using simple forecasting techniques in a supply chain. *Naval Research Logistics* **50**, 388–411.

Lee, H. L., K. C. So and C. S. Tang (2000). The value of information sharing in a two-level supply chain. *Management Science* **46**(5), 626–643.

Lee, H. L., V. Padmanabhan and S. Whang (1997). Information distortion in a supply chain: The bullwhip effect. *Management Science* **43**(4), 546–558.

Potter, A., M. M. Naim and S. M. Disney (2004). Utilising a production system archetype to attenuate bullwhip in a grocery supply chain. The 13th International Working Seminar on Production Economics. Igls, Austria.

Raghunathan, S. (2001). Information sharing in a supply chain: A note on its value when demand is nonstationary. *Management Science* **47**(4), 605–610.

Simchi-Levi, D. and Y. Zhao (2003). The value of information sharing in a two-stage supply chain with production capacity constraints. *Naval Research Logistics* **50**(8), 888–916.

Vassian, H. J. (1955). Application of discrete variable servo theory to inventory control. *Journal of the Operations Research Society of America* **3**(3), 272–282.

Zhang, X. (2004). The impact of forecasting methods on the bullwhip effect. *International Journal of Production Economics* **88**, 15–27.

Zipkin, P. H. (1995). Performance analysis of a multi-item production-inventory system under alternative policies. *Management Science* **41**(4), 690–703.