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Eliminating Drift in Inventory and Order Based Production Control Systems

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Abstract

An Inventory and Order Based Production Control System lies at the heart of many commercial and bespoke ordering systems based on periodic review of stock and production targets. This simple and elegant control system works well, even when dealing with scenarios in which there are many competing value streams. However, such "interferences" inevitably cause some uncertainty in pipeline delivery times. We show via linear z-transform analysis that the consequences may include the possibility of inventory drift and instability. In this paper we establish the stability boundaries for such systems, and demonstrate an innovative method of eliminating inventory drift due to lead-time effect. This new principle is confirmed by simulation results.

Key Words

Order-up-to policy, lead-times, work-in-progress, stability, z-transforms, dynamics

1. Introduction

This paper is solely concerned with periodic review ordering systems i.e. the system states are reviewed and a decision is made on placing orders on the upstream supply pipelines at regular and equally spaced points in time. Such control systems have been advocated by, amongst others, Burbidge, 1984. The reasons include the avoidance of bullwhip as caused by multi-phased (EBQ) demands that in turn cause excessive stock volatility. Although we strongly support the use of periodic review systems, our starting point, as highlighted in the next section on the history of **I**nventory and **O**rder **B**ased **P**roduction **C**ontrol **S**ystems (IOBPCS), is via continuous control theory. For the purposes of this paper the continuous time approach to control may be regarded as the limiting case of periodic review.

If Jack Burbidge is rightly considered one of the "fathers" of modern supply chain principles, then equally so is Jay Forrester (Towill, 1997). It should be remembered that the latter's classic simulation model (Forrester, 1958) readily demonstrating demand amplification (now known, thanks to the important contribution of Lee et al, 1997 as bullwhip) is based on continuous review. This was also true of the even earlier contributions for example see Simon (1952) and Vassian (1955) who considered the periodic review case in continuous and discrete time respectively. Periodic review is also at the heart of the Lean Thinking Paradigm (Womack and Jones, 1996) with its abhorrence of the EBQ approach when seeking "batch of one" operation. This contrasts with the current EBQ approach to logistics in supply chains advocated by Lutz et al (2003).

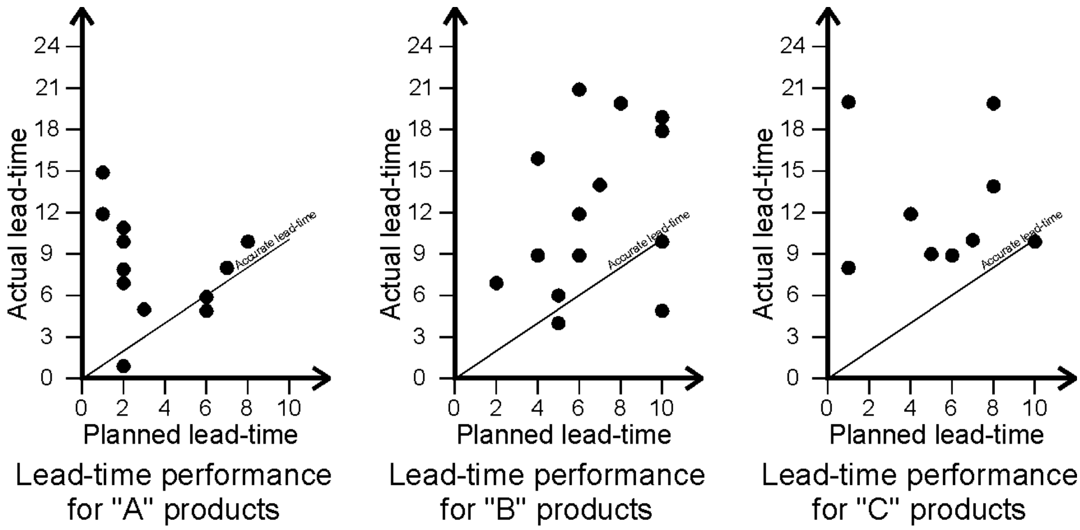


Figure 1. Summary of lead-time performance in an industrial setting (taken from Coleman 1988)

We are concerned herein with a production and inventory control system that incorporates pipeline (or Work In Progress, WIP) feedback. This type of ordering policy requires an estimate of the delivery lead-times before it can generate orders. Unfortunately it is readily shown these policies suffer from inventory drift if the lead-time estimate is not always equal to the current lead-time. Inventory drift is a term that we will use to describe the phenomena that, over time, inventory levels do not “lock on” to target levels when a step change in the consumption rate has occurred. As Fig. 1 clearly demonstrates, in a multi-product manufacturing facility with thousands of items in a current catalogue, this lead-time variation is often a problem.

One method of compensating for this situation is to use a Proportional plus Integral (PI) Controller in the inventory compensation loop. This classical solution has the customary drawback that in order to retain good dynamic response, the “recovery” component due to integral action is inevitably very “long-tailed”. So we eliminate drift but only very slowly, hence this approach is only suitable for products with an extended life cycle. A second solution is the continuous monitoring of actual lead-times. This continuous estimation of the lead-time is utilised via a non-linear feedback loop in the ordering policy. Provided the estimate is actually made available, and is noise and bias free, this solution has also been demonstrated to work. However to be effective the estimate has to be available in real-time and this requires a significant amount of management effort. This could also involve prediction, ahead of “interference” between value streams via simulation of the impact of scheduling decisions within the delivery process (Belk and Steels, 1998). Furthermore there is to date no theory supporting the stability and hence range of operation of this non-linear model.

For these reasons both the foregoing solutions proposed to date have sufficient implementation drawbacks to justify the search for further alternative scheme to eliminate inventory drift. There we propose an innovative solution based on the manipulation of the WIP loop that offers a faster response and is still user controllable. Early results show that the method eliminates inventory drift under the particular test conditions selected. An analytical solution to the linear stability of this new system is derived, and verified via simulation of systems in the region of the critical stability boundary. Our current expectation is that this updated system will offer improved performance in such a multi-product pipeline scenario.

2. Review of the IOBPCS Family of Decision Support Systems

The IOBPCS family of Decision Support Systems is summarised in Table I. IOBPCS (an acronym for **I**nventory and **O**rder **B**ased **P**roduction **C**ontrol **S**ystem) is the basic periodic review algorithm for issuing orders into a supply pipeline, in this case based on the current inventory deficit and incoming demand from our customer. At regular intervals of time the available system "states" are monitored and used to compute our next set of orders. According to Coyle (1977) such a system is frequently observed in action in many market sectors. This author studied the expected behaviour via industrial dynamics simulation models. Towill (1982) then recast the problem into a control engineering format with emphasis on predicting dynamic recovery, inventory drift, and noise bandwidth (leading importantly to variance estimations). Limited optimisation was thereby enabled within the constraints imposed by having only two adjustable parameters controlling a third order model. An important feature of this 1982 paper was relating the model to established hardware system "best practice" thus identifying good, workable, yet conservative designs capable of transfer into the production control arena.

IOBPCS system variant	**CONTROL "DRIVERS"**		
	Customer demand feedforward	**Inventory target**	**Pipeline target**
Order Based Production Control System (OBPCS)	Yes	N/A	N/A
Inventory Based Production Control System (IBPCS)	N/A	Fixed	N/A
Inventory and Order Based Production Control System (IOBPCS)	Yes	Fixed	N/A
Variable Inventory and Order Based Production Control System (VIOBPCS)	Yes	Variable	N/A
Pipeline, Inventory, and Order Based Production Control System (PIOBPCS)	Yes	Fixed	Fixed
Pipeline, Variable Inventory, and Order Based Production Control System (PVIOBPCS)	Yes	Variable	Fixed
Automatic Pipeline, Inventory and Order Based Production Control System (APIOBPCS)	Yes	Fixed	Variable
Automatic Pipeline, Variable Inventory and Order Based Production Control System (APVIOBPCS)	Yes	Variable	Variable

Table 1. Some common IOBPCS variants

Edghill and Towill (1989) extended the model, and hence the theoretical analysis, by allowing the target inventory to be a function of observed demand. This **V**ariable **I**nventory OBPCS is representative of that particular industrial practice where it is necessary to update the "inventory cover" over time. Usually the moving target inventory position is estimated from the forecast demand multiplied by a "cover factor". The latter is a function of pipeline lead-time often with an additional safety factor built in. A later paper by John et al (1994) demonstrated that the addition of a further feedback loop based on orders in our pipeline provided the "missing" third control variable. This **A**utomatic **P**ipeline IOBPCS model was subsequently optimised in terms of dynamic performance via the use of genetic algorithms, Disney et al, (2000).

However a further important conclusion emerged from John et al (1994) which showed that inventory drift would occur if the pipeline lead time *estimate* used as part of the control algorithm was different from the current *actual* lead time. This was recognised by the requirement (wherever possible) for updating lead-time estimates on-line thus enabling the **A**djustable APIOBPCS. Finally however, if we additionally include the Variable inventory target within the APIOBPCS ordering system, then we also encompass the Order-Up-To (OUT) Model developed initially within the OR fraternity and described recently by Looman et al (2002). The equivalence of the OUT and APVIOBPCS models was subsequently established by Dejonckheere et al, (2003). Thus Fig. 2 summarises the components of the IOBPCS family of models.

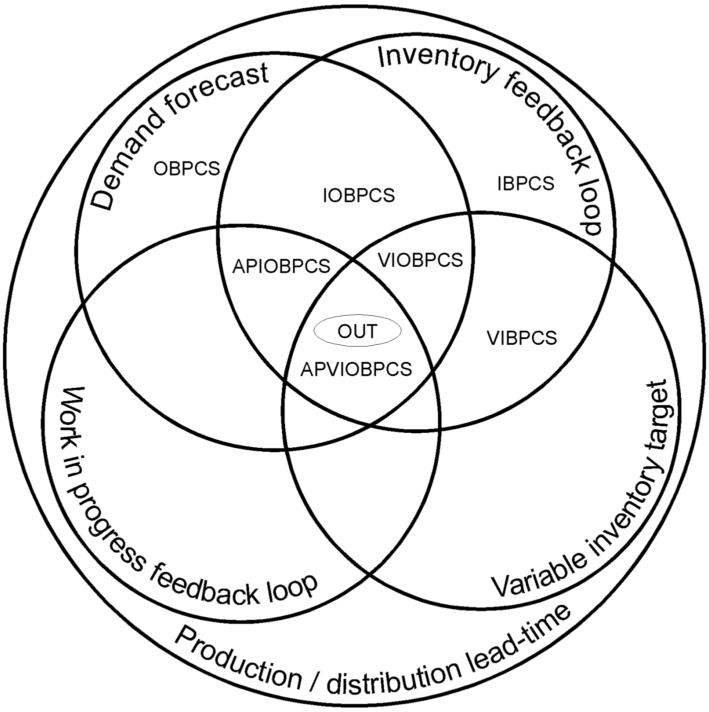


Figure 2. The IOBPCS family

3. IOBCPCS within the Scheme of Periodic Review Systems

Clearly what has emerged pragmatically via the study of the IOBPCS family of ordering polices is an architecture in which the inventory, pipeline, and sales forecasting contributions are clearly identifiable. So in general terms the Order Rate applied to our delivery pipeline may be written:

$$\begin{aligned} \text{ORATE} = &\ \text{function (inventory deficit)} \\ &+ \text{function (pipeline deficit)} \\ &+ \text{function (sales forecast)} \end{aligned} \tag{1}$$

where the sales forecast term manifestly includes present demand.

Typically, but not obligatorily, the inventory error controller and the pipeline error controller will be proportional devices, and the sales forecast will be an exponentially smoothed estimate based on actual demand from the customer. Note that Equation 1 exactly replicates the statement by Bonney (1990) who argued that all orders placed should be a function of sales forecast, inventory deficit, and goods in the pipeline (i.e. orders previously placed but not yet delivered). A later paper by Bonney et al (1994) highlighted the errors likely to arise in systems control caused by over/under recording of stock and over/under recording of issues. Hopefully bar coding and other relatively recent electronic monitoring will reduce their presence. However, as a precaution both noise and distortion effects have been included in recent simulation models with the intention of ensuring robust design, Cheema (1994).

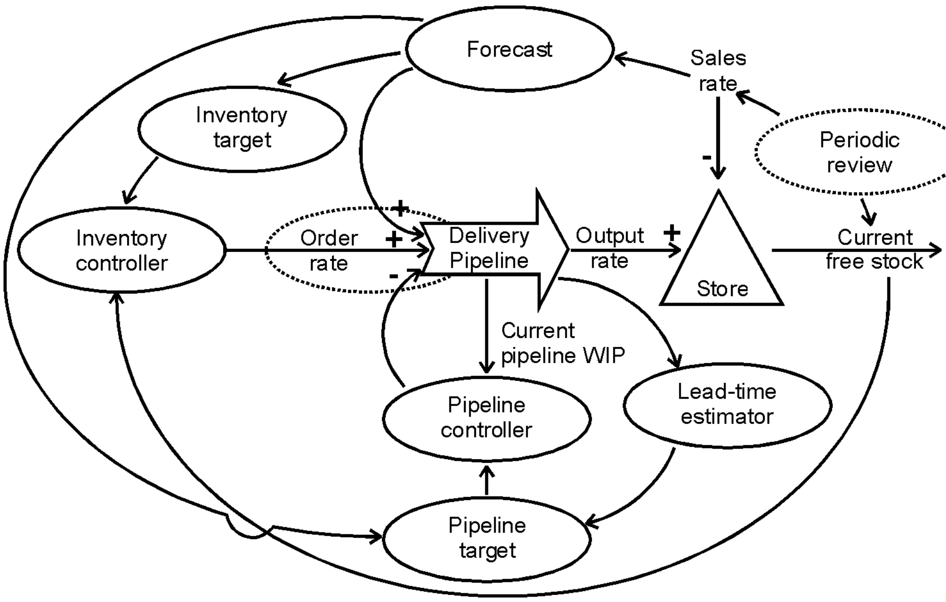


Figure 3. Architecture of the IOBPCS family of periodic review decision support systems

The IOBPCS family architecture of Fig. 3 has evolved over the last three decades. Originally based on an analogue (continuous time) approach, all the variants may be expressed in discrete time (Disney, 2001). This enables pure time delays to be handled exactly, an important consideration, when for example, stability boundaries are to be determined analytically (Disney and Towill, 2002). Also it is not just the early industrial dynamics models of Coyle (1977) that are embraced by this family. The IOBPCS family also encompasses many of the early applications of control theory to production and inventory control. Typical of these are Simon (1952, continuous time), and Vassian (1955, discrete time).

The Deziel and Eilon (1967) paper which combined a control approach embedded within an OR context is clearly an IOBPCS variant. In our view the Deziel and Eilon contribution to knowledge was greatly undervalued. For example, their intuitive approach to set the inventory loop gain equal to the pipeline loop gain. This not only made mathematical analysis tractable using the techniques available back in 1966, but also resulted in a "docile" and very stable control system. It is only the advent of modern software which has enabled us to rapidly cross-check their designs and put them into the context of other competitive systems (Disney and Towill, 2003).

4. IOBPCS in the Real World

As the IOBPCS family has evolved, it has become obvious that we were mimicking much production control practice additional to the situations described by Coyle (1977) with links back to Roberts (1981) and thence to Jay Forrester (1958). Equation 1 seems to have been “discovered” and “re-discovered” many times by management consultants who provide appropriate software to realise the algorithm. For example, process flow analysis back-up by times series modelling clearly identified an IOBPCS structure located within an automotive supply chain (Edghill et al, 1988). This was at the aggregate level an application advocated by Axsäter (1985). But writing software using a generic Equation 1 is not merely a necessary condition for good system design. The functions inherent in Equation 1 may take a standard form, but their settings within an operating scenario are context specific. There is little evidence that these settings are properly tuned, with a substantial gap in the knowledge exploited by user and system designer. This is at variance with the recommendations made by Feltner and Weiner (1985) and endorsed by Solberg (1992) that “users” should intellectually own such software.

In sharp contrast an IOBPCS variant has been designed and implemented “in-house” successfully controlling the delivery of 6000 health care aid products from the “active” catalogue (Cheema et al, 1997). This product range was divided up on a Pareto Curve basis (Koch, 1997) with availability targets varying according to the product ranking (A, B, C etc). Each product therefore had its own control law that was operated automatically and in parallel with the requisite control laws for all other items. This application shows that IOBPCS works at the individual product level (rather than just at the aggregate level as advocated in Axsäter (1985)). It is also an exemplifier of the management “rule of thumb” of “simplifying the operating scenario first” so that simple models are justifiable. This in turn is in line with the empirical evidence produced by New (1998). The concept was to use the ordering system to drive the factory MRP software that was responsible for solving detailed scheduling problems.

Because of the inevitable and considerable “interference” between product routings on the shop floor, the actual lead-time for each product varied from target. Hence these lead-times can be considered to vary both between products and within products. One proposal for compensating for lead-time variation was further investigated by Cheema (1994), having been implemented in an ad-hoc manner within the health care products company. This required the on-line monitoring and automatic updating of pipeline targets via an exponentially smoothed non-linear feedback loop. Company Annual Reports substantiated that customer service levels and stock levels were considerably improved when this so-called “To Make” ordering system was incorporated as the MRP driver but the costs of maintaining accurate records of actual lead-times must have been notable.

5. Inventory drift in APIOBPCS

Even with the “user as owner” scenario existing in this particular application some problems still occurred from time to time which analysis may soon cure (Evans et al, 1997). This in turn emphasises the need for published guidelines that match recommendations to specific operating scenarios. In other words there is no point in writing or purchasing control software if this is not supported by adequate supporting studies. Fortunately some guidelines are sufficiently useful and written in a userfriendly format (Sterman, 1989, Bonney et al, 1994, John et al, 1994 and Mason-Jones et al, 1997).

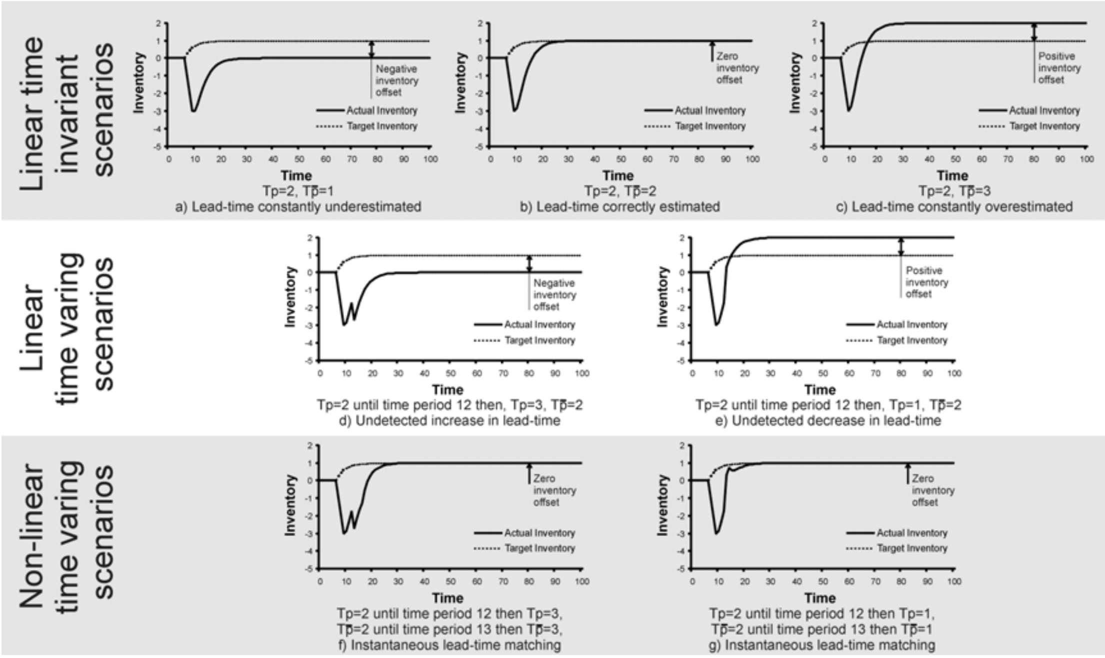


Figure 4. Inventory drift after a step change in demand due to lead-time variation
(*a*=1, *Ta*=3, *Ti*=*Tw*=2)

We have already mentioned the need to cope with lead-time variation as a core function of the production control system. Initial results obtained by John et al (1994) on the importance of matching pipeline targets to current lead-times was confirmed by Cheema (1994). Fig. 4 highlights our understanding via some simulation results with lead-time issues. These lead-time effects mimic a situation where either there is an unavoidable material shortage or queue for usage of a particular manufacturing facility, both events that typically occurred in the health care products company. Does the inventory revert to its target position (i.e. zero error) under such conditions? Fig. 4 shows that this goal is only achieved if the estimated lead-time $\left(T\overline{p}\right)$ is eventually equal to the actual lead-time (Tp) otherwise there is a positive or negative offset. Note that Figs. 4(f) and 4(g), with zero off-set, correspond to the health care products control system with the exponential smoothing constant in the lead-time feedback loop set equal to unity.

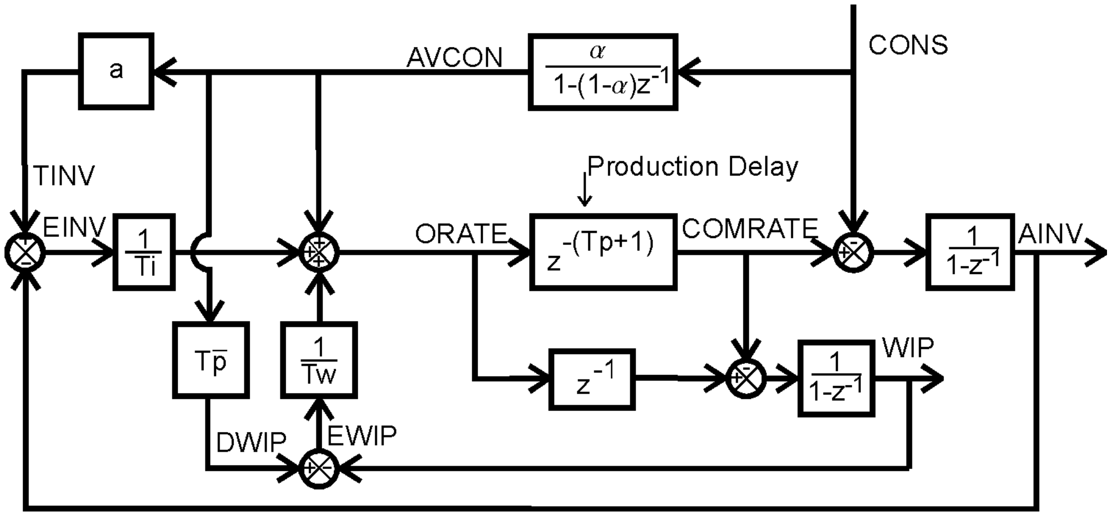


Figure 5. Block Diagram of APVIOBPCS in discrete time

The APVIOBPCS model used for this demonstration of inventory drift as caused by pipeline lead-time variation is shown in Fig. 4. It follows directly from the IOBPCS architecture previously met in Fig. 3 and actually corresponds to the conventional OUT policy (Loomans et al, 2002 and Dejonckheere et al, 2003). The variables available for dynamic control are the feedback gains ($\frac{1}{Ti}$ and $\frac{1}{Tw}$), the inventory cover a, and the exponential smoother ($\alpha = \frac{1}{1+Ta}$) used in sales forecasting feed forward channel. This model is derived in discrete time, hence the use of the z-transform. Note that to enable consistency of results the model adheres strictly to the "order-of-events" sequence initially defined by Vassian (1955).

Proof of the existence of the inventory offset can be obtained via the normal z-transform analysis as now follows. The first step is to find the inventory transfer function. This is easily achieved using standard block diagram techniques such as those highlighted in Nise (1995) and is given by;

$$\frac{AINV}{CONS} = \frac{\left(z\left(\begin{array}{l}-TaTi - T\overline{p}Ti - aTw - TiTw + Tiz + TaTiz + T\overline{p}Tiz + aTwz + TiTwz - \\ Ti(1+Tw(-1+z))z^{Tp}(Ta(-1+z)+z)\end{array}\right)\right)}{((-1+z)(Ta(-1+z)+z)(Tw+Ti(-1+(1+Tw(-1+z))z^{Tp})))} \tag{2}$$

To which we may apply the final value theorem (Equation 2), where $I(z) = \frac{1}{1-z^{-1}}$, the unit step input and $F(z)$ is the inventory transfer function (Equation 2).

$$\lim_{t\to\infty}\{f(t)i(t)\} = \lim_{z\to 1}\{(1-z^{-1})F(z)I(z)\} \tag{3}$$

Manipulation then yields the following final value of inventory for a step change in demand:

$$FV_{AINV} = a + \frac{Ti(\overline{Tp} - Tp)}{Tw} \tag{4}$$

As the required target inventory level is a, we note from Equation 4 that if the estimate of the lead-time is wrong then an error is produced, unless;

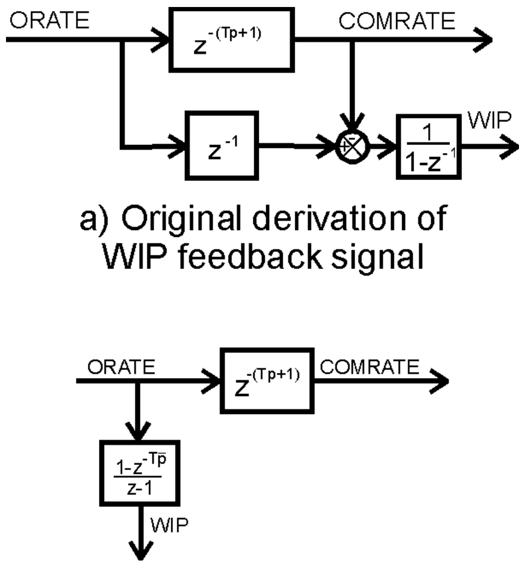
- The error between our perception of the lead-time and the actual lead-time is zero.
- *Tw* is set to infinity (i.e. there is no WIP feedback. Hence we have the original Inventory and Order Based Production Control System IOBPCS) model, Towill, (1982). This then makes the dynamic response more difficult to shape to match a desired behaviour.

Assuming that we wish to maintain a finite *Tw* (and indeed if we use the Deziel and Eilon 1967 rule of $Tw = Ti$ then we have no further choice in the matter) we must either accurately and continuously update our lead-time estimates, or, alternatively, find a different solution to the problem. We note that lead-time estimates may be complicated if there are complex/interacting product channels (Burbidge, 1990). Also accurate estimates are sometimes difficult even within our own company: if the pipeline crosses a number of process boundaries this problem multiplies. Designing for simplified material flow is one possible answer (Childerhouse and Towill, 2003) but is outside the scope of this paper. A further possibility is the "best matching" of the manufacturer/material supplier levels of flexibility (Garravelli, 2003). Also there now exists improved scheduling software, as reviewed by Knolmeyer et al (2002).

6. Our proposed solution to the inventory drift problem

Note that the integrator in the established APVIOPBCS policy model WIP feedback loop sums the difference between ORATE and COMRATE (as shown in Fig. 5). But the WIP level can also be estimated as the sum of the previous Tp + ORATE signals as shown below in Equation 5. It can be appreciated that the reason why the Final Value of the inventory levels (of APVIOPBCS) experience an offset is because the desired WIP level is based on the *"perception"* of the production lead-time $(\overline{Tp})$ and the actual WIP is based on the *"actual"* production lead-time. Therefore as already shown by the Final Value Theorem in Equation 4 if our perception of the production lead-time is wrong then there is a difference between the desired WIP and actual WIP in the steady state.

$$\frac{WIP_{APVIOBPCS}}{ORATE_{APVIOBPCS}} = \frac{z^{-1} - z^{-Tp-1}}{1 - z^{-1}} = \frac{1 - z^{-Tp}}{z - 1} = z^{-1} + z^{-2} + \ldots + z^{-Tp} = \sum_{n=1}^{Tp} z^{-n} \tag{5}$$



a) Original derivation of WIP feedback signal

b) Proposed replacement WIP estimator

Figure 6. Block diagram of the new WIP estimator

So we propose a new system aimed at avoiding this effect by replacing the "actual" WIP signal with a WIP signal that would have been generated if our previous $\overline{Tp}$ (rather than Tp) ORATE signals, as shown in Equation 6. Of course if $\overline{Tp} = Tp$ then the two systems are equivalent. However, if $\overline{Tp} <> Tp$ then the systems will produce a different dynamic response. Hence the stability will be affected, but provided the system is still stable, the steady state inventory offset is eliminated. Thus there is a need to establish stability boundaries for such a system as will be done later in the paper. We call this new model EPVIOBPCS (**E**stimated **P**ipeline **V**ariable **I**nventory and **O**rder **B**ased **P**roduction **C**ontrol **S**ystem) for clarity. This model is different from the adaptive lead-time APIOBPCS modification considered by Evans et al (1997) because we do not attempt to obtain real-time accurate estimates of the lead-time and exploit them via a non-linear feedback loop. Instead the new model requires that the WIP element within our block diagram is replaced by an equivalent WIP estimator as shown in Fig. 6.

$$\frac{WIP_{EPVIOPBCS}}{ORATE_{EPVIOBPCS}} = z^{-1} + z^{-2} + \ldots + z^{-\overline{Tp}} = \sum_{k=1}^{\overline{Tp}} z^{-k} = \frac{1-z^{-\overline{Tp}}}{z-1} \tag{6}$$

7. Time Domain Responses

The new transfer function that relates inventory to consumption (CONS) or demand is as follows:

$$\frac{AINV}{CONS} = \frac{z\begin{pmatrix}(\overline{Tp}Ti + (a+Ti)Tw)(-1+z)z^{Tp} - \\ Tiz^{Tp}(Ta(-1+z)+z)(-1+(1+Tw(-1+z))z^{\overline{Tp}})\end{pmatrix}}{(-1+z)(Ta(-1+z)+z)(Twz^{\overline{Tp}} + Tiz^{Tp}(-1+(1+Tw(-1+z))z^{\overline{Tp}}))} \tag{7}$$

Replacing *F(z)* with Equation 7 and *I(z)* with the Heaviside Step function, in the Final Value Theorem of Equation 3, shows via Equation 8 that there is zero steady state error in inventory level. This is because the system is now constrained so that the actual inventory and target inventory are brought into alignment. Therefore, it is shown that the EPVIOBPCS can cope with constant errors in the estimation of production lead-time.

$$FV_{AINV} = \lim_{z \to 1} \left\{ \frac{z\left(\begin{array}{l} \left(T\bar{p}Ti + (a+Ti)Tw\right)(-1+z)z^{Tp} - \\ Tiz^{Tp}(Ta(-1+z)+z)(-1+(1+Tw(-1+z))z^{T\bar{p}}) \end{array} \right)}{(-1+z)(Ta(-1+z)+z)(Twz^{T\bar{p}} + Tiz^{Tp}(-1+(1+Tw(-1+z))z^{T\bar{p}}))} \right\} = a \qquad (8)$$

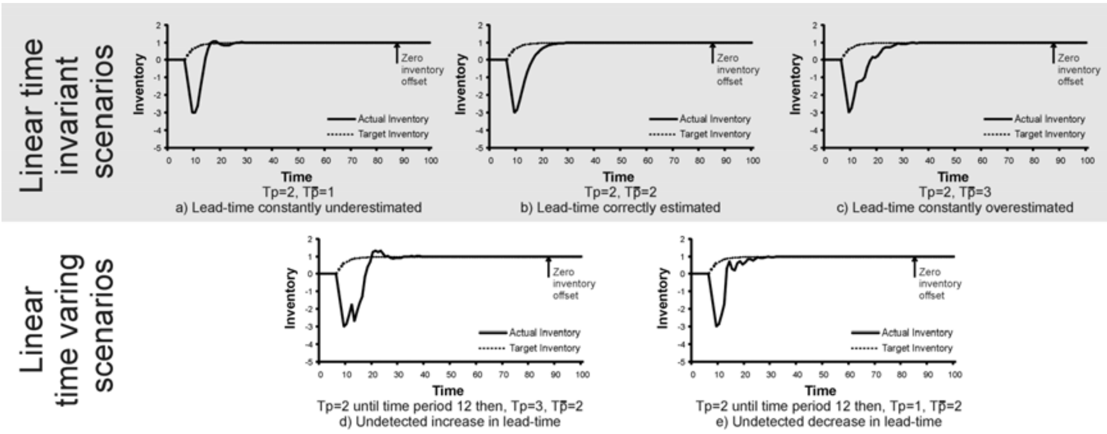


Figure 7. Recovery of inventory deficit with the new WIP calculation method (*a*=1, *Ta*=3, *Ti*=*Tw*=2)

We have developed a spreadsheet model to confirm the elimination of inventory offset by the new method of calculating WIP. Fig. 7 shows some sample time domain inventory domain responses for cases of both over-estimation and under-estimation of pipeline lead-times. The first row shows the linear time invariant case to confirm the implementations of Equation 8. The next row considered the linear time varying case. From these responses it is clear that under these particular conditions the modified model copes with inventory drift due to lead-time variation. To date we have not found a scenario where this method does not work but our knowledge of time varying control theory is limited here. Of course there also remains the problem of system stability, which will be addressed in the next section.

8. Establishing the stability boundaries for EPVIOBPCS

It is essential that any production and inventory control system is fundamentally stable. A stable system will respond to any finite input and return to its initial conditions, either with damped exponential decay or with damped sinusoidal and exponential decay. An unstable system, on the other hand will respond to a finite input with oscillations of ever increasing magnitude (or explode exponentially without bounds). A critically stable system will result in oscillations about the initial conditions of a constant magnitude i.e. limit cycling. We note that the two feedback loops (feed-forward loops have no impact on stability) can be tuned so as to create an unstable, or a critically stable, or a stable time domain response. Thus we need to identify the conditions for stability in order to ensure the new system produces a desirable response to real inputs over the expected operating range of the system.

Given Tp=	**And** $\overline{Tp}$ =	**Then either** Ti>	**Or** Tw>
1	1	1	$Tw > \frac{2Ti}{1+2Ti}$
1	2	1	$Tw > \frac{-Ti+\sqrt{Ti}\sqrt{-8+9Ti}}{2(-1+Ti)}$
1	3	1	$Tw > \frac{1+2\sqrt{-1+Ti}\sqrt{Ti}-Ti}{-1+Ti}$
2	1	1.61803	$Tw > -\frac{Ti(1+Ti)}{-1-Ti+Ti^2}$
2	2	1.61803	$Tw > \frac{-Ti(3+Ti)-Ti\sqrt{1-2Ti+9Ti^2}}{2(-1-Ti+Ti^2)}$
2	3	1.61803	$Tw > \frac{-\sqrt{-1+Ti}\sqrt{Ti}-Ti-2\sqrt{-1+Ti}Ti^{3/2}-Ti^2}{-1-Ti+Ti^2}$
3	1	2.24698	$Tw > \frac{-Ti(1+Ti^2)-(-1+Ti)Ti\sqrt{5+6Ti+Ti^2}}{2(1-Ti-2Ti^2+Ti^3)}$
3	2	2.24698	$Tw > \frac{-Ti^2(3+Ti)-(-1+Ti)Ti\sqrt{8+16Ti+9Ti^2}}{2(1-Ti-2Ti^2+Ti^3)}$
3	3	2.24698	$Tw > \frac{-Ti(-3+4Ti+2Ti^2)-Ti(-1+2Ti)\sqrt{1+4Ti^2}}{2(1-Ti-2Ti^2+Ti^3)}$

Table 2. Stability conditions for the EPVIOBPCS model

In fact it is possible to determine analytically the criteria for stability of discrete time using the procedure highlighted in Disney and Towill (2002). This is based on transforming the characteristic equation into the ω-plane and then using the well-established Routh-Hurwitz stability criterion. Application hereto yields the criteria of Table 2 that we have also plotted in Figure 8. This can be used to tune the feedback loops to avoid instable responses. There is no limit to the length of the lead-times that may be considered by this approach, but we have limited the table to lead-times of up to three periods. We also note that the D-E (after Deziel and Eilon (1967)) line always results in a stable system and has other important desirable properties, as also reported in Disney and Towill (2002).

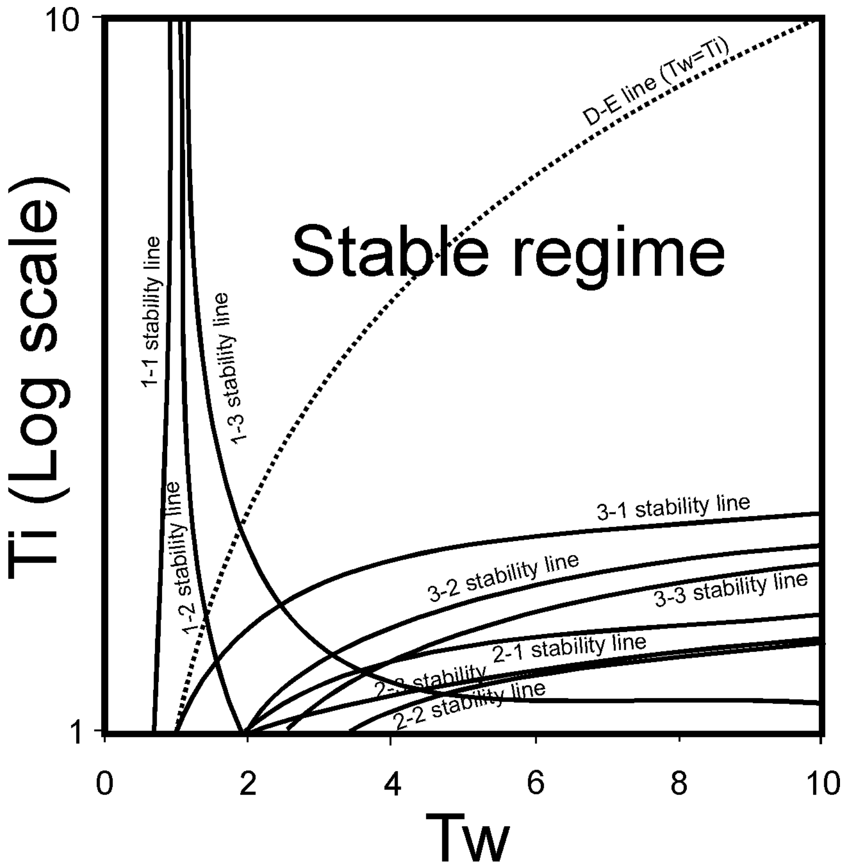
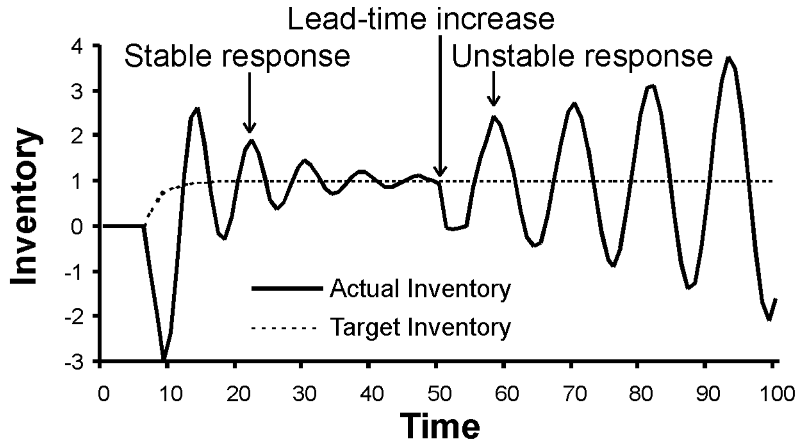


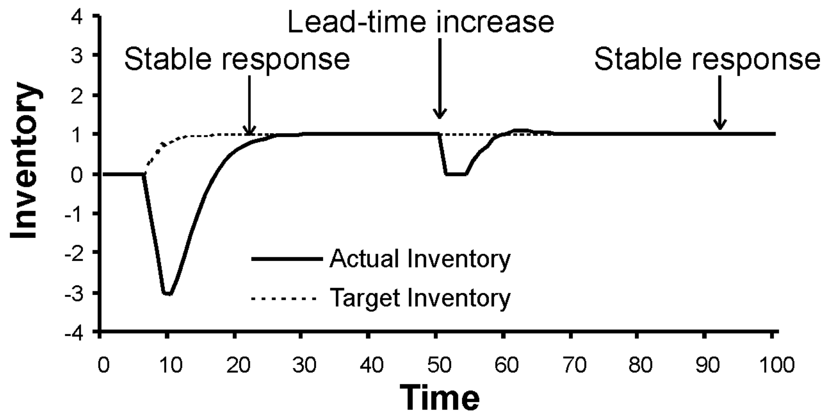
Figure 8. Stability conditions for EPVIOBPCS on the *Ti*-*Tw* plane (Key: Labels for the stability lines are the format of Tp-$T\overline{p}$)

We note from Table 2 and Figure 8 that our new system is slightly less stable than the APVIOBPCS model with automatic updating of the current lead-time. Hence extra care must be taken when setting system parameters. As an example suppose that for the purpose of illustration a system is set up with the following parameters; $Tp = T\overline{p} = 2, Ta = 2, Ti = 1.2, Tw = 3.3$. We note that this system is very lively but under nominal conditions it is stable (but not recommendable for any application we have met). Now suppose that the lead-time increases by one time period (say in time period 50 so that that *Tp*=3), but we do not automatically update our system settings. We thus predict via Fig. 7 that the system will become unstable due to the lead-time shift. This is indeed confirmed in simulation plot in Figure 8.

Thus it is important that system designers think carefully about parameter settings and avoid unstable regions both for nominal lead-times and for lead-times that could actually be the case sometime in the future before the system is "re-tuned". The case where $Tw = Ti = 3.3$ is particularly helpful in this situation as it is stable for all values of *Tp* (when $T\overline{p}$ =*Tp*) and the equivalent response is also shown in Fig. 9. Not only is this system extremely well behaved both under nominal conditions and additionally following the change in pipeline lead-time, it confirms the superiority of the design setting advocated on an intuitive basis by Deziel and Eilon (1967), even for our new WIP controlled system.



a) Confirmation of onset of instability
for "lively" design parameters (Ti=1.2, Tw=3.3)



b) Confirmation of a very stable response
with D-E design parameters (Ti=Tw =3.3)

Figure 9. Comparison of response of the EPIOBPCS design during an undetected pipeline lead-time change

9. Conclusions

It is well known that pipeline feedback has a beneficial effect on shaping the dynamic response of the IOBPCS class of periodic review production control systems (John et al, 1994). But unless the pipeline target uses the current value of delivery lead-time there is the potential for inventory drift to occur. This in turn will result in either excess stock build-up or deterioration of customer service level. Such a situation is apparently worsened if the lead-times vary during normal plant operation (Cheema, 1994). Our proposed solution to this problem is to use a WIP estimator based on the expected value of lead-time and to incorporate this within the pipeline control loop. Early simulation results show that the inventory offset is indeed eliminated. Furthermore the penalty of incurring the "long-tailed" response characteristic of the Proportional plus Integral control solution proposed by Cheema et al (1997) is avoided. Our solution also avoids dealing with noisy, stale, or biased information that may affect the alternative (Cheema et al, 1997) approach utilising non-linear lead-time feedback.

What we can say from this initial investigation is that if the control parameters remain "fixed" at their nominal values then the real-world case where lead-times increase will erode stability margins. This may cause a system designed to operate near the boundary to become unstable. Hence we have used a z-transform model, transformed it into the ω-plane and applied the Routh-Hurwitz method to determine these stability boundaries for a range of delivery lead-times. We have illustrated the procedure for the case of a volatile but initially stable system, which is de-stabilised by the lead-time change. However we have also demonstrated how such behaviour can be easily avoided by selecting a conservative design with parameters set to give a well damped dynamic response which is little affected by lead-time changes.

The IOBPCS range of models is easily exploitable via spreadsheet based decision support systems. These are currently being exploited in new applications (for example Disney et al, 2001). The long-term aim is to provide comprehensive analyses that enable the "best match" by selecting the most appropriate model for a given scenario, together with recommendations for good parameter settings. A virtue of the IOBPCS family is that by selecting extreme values we may mimic the wide spectrum of delivery scenarios ranging between the two extreme cases from "Level Scheduling" to "Pass-On-Orders". Hence when dealing with many parallel pipelines it is easy to exploit the same IOBPCS structure but tune the parameters according to Pareto Curve product classification. This enables availability targets to be met without excessive stockholdings as judged according to whether products are "A", "B", or "C" rated.

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