Valuation and Appraisal Techniques for DTI Policies and Projects: A Report to the Department of Trade and Industry

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Valuation and Appraisal Techniques for DTI Policies and Projects: A Report to the Department of Trade and Industry

Executive Summary

DTI requested a report on appraisal techniques that were especially appropriate for the Department’s business, identifying real options and multi-criteria analysis as of particular interest. [NOTE: thought it would be useful to introduce economic appraisal within a wider process]

An economic appraisal’s key purpose is to inform policy decisions. Within a series of parameters such as ringfences and other rules, it identifies socially preferred policy options when considering interventions that have implications on how scarce resources are allocated across competing uses. At the core of a full economic appraisal are the valuations of the costs and benefits. The normal assumption should be that these valuations are based on the ‘willingness to pay’ and ‘willingness to accept’ of those individuals or agents directly and indirectly affected by a project. Methods of obtaining these valuations, when market prices are not available, are well established in the professional literature and included in Treasury guidance. Partly under the stimulus of government departments, the techniques for eliciting such valuations, and their understanding, have advanced considerably. In order to capture uncertain costs and benefits to society over time, full scale Net Present Value (NPV) calculations can be expensive, for example because eliciting individuals’ valuations may be difficult, and in this case there should be a preliminary evaluation in a pilot NPV. Only if the pilot NPV suggests that there is a good chance of the project being viable should a full scale NPV be undertaken.

There are cases where the ‘willingness to pay’ of individuals is not the appropriate measure of benefit because individuals are not informed about the likely impact on their welfare of a project. This might be because of the complexity of outputs, for example regarding policies intended to reduce the extent of global warming, or simply because the individuals are unfamiliar with the outputs of the project. If the problem is lack of familiarity then in some circumstances a pilot project may be used so that valuations can be obtained from individuals with experience of the service. If a pilot project is impractical then there may be no alternative to replacing individuals’ valuations with those of experts in an NPV calculation. Whenever expert valuations are employed it must be recognised that experts often have a vested interest in the outcome of the calculation.

There are some circumstances when even experts may struggle to place a plausible monetary value on the outputs. For example, it might be extremely difficult to value unlikely but extreme risks such as those involved in the choice of a site for the disposal of nuclear waste. In these cases expert opinion could be best summarised in multi-criteria analysis (MCA). MCA appraises options and project costs and benefits, not as measured by the recipients’ expressed values but by experts’, managers’ or stakeholders’ judgements about the outputs. Scores for different criteria are weighted and summed to reach decisions. One criterion should always be the NPV valuation for those inputs and outputs that can be quantified in meaningful monetary units. The MCA process encourages a disciplined approach to decision making, but cannot on its
own yield a simple positive or negative overall judgement because of the difficulty of aggregating scores on heterogeneous dimensions. A final decision will inevitably reflect a “political” judgement about the relative importance of the different criteria.

The private sector is more commonly now employing real option appraisal of projects. The DTI should consider following suit for projects whenever it is expected that more information about the values of outputs and inputs is expected to accrue over time. Real options analysis is a general approach that is applicable whenever there is an opportunity to make investment decisions at different dates and when later decisions will be better informed. There are many examples in the literature of how thinking that utilises real option concepts can improve decisions. For example the expected NPV of a project may vary with the date it is undertaken if more information is available with the passage of time. An initial investment decision may open up the possibility of further real investment opportunities at a later date. The variety of settings where real option analysis may be applied means that there is no single technique for valuing real options in the way that there is for more standard financial options. Every project will require specific thinking about what information is likely to arrive in the future and what decisions are possible at different dates.

The most robust approach to applying real option analysis is to set out clearly the discrete dates at which decisions may be taken and what information is expected to be available when. A useful framework is a decision tree that identifies the timing and nature of decisions, as well as what will be known at the point the decision is made. Real option analysis is not an alternative to the NPV approach, but is actually an intrinsic part of a thorough NPV evaluation. For example, real options analysis requires a comparison of the NPV of a project initiated today with the (discounted) expected NPV if it is delayed for a year until more information has accrued. The value of the option to delay is that the project will only proceed if in the light of more precise information it still appears profitable.

The real option approach is not a substitute for more conventional appraisal tools of risk analysis. One response to risk is to delay a decision until the information is clearer, but inevitably some risk will remain and must be factored in to the net present value. The conventional case for the public sector in general and DTI in particular, being concerned only with project expected values, and not with project risk as well, remains sound so long as the benefits of these projects are ultimately spread over the whole UK population. If the beneficiary group is reduced then there may be a case for considering risk as well.

A distinctive challenge for DTI arises from the need to appraise projects where the later stages are to be funded largely by the private sector and the earlier stages mainly by the public sector. Public sector discounting is typically at a lower rate than that of the private sector. Hence it may be that the private sector decides returns on later stages are too low, and the later stages will not go ahead even though they would be NPV positive at public sector discount rates. This fact must be recognised when the NPV calculation is undertaken. It is just one of many anomalies that are thrown up when the public sector discount rate is less than that of the private sector.

The public sector places more risk adjustment in the numerator of the net present value while the private sector tends to boost the discount rate for risk instead. There are good reasons to suppose the public sector approach – but not necessarily the
practice - is correct. Nonetheless with an agreed assessment of the risks of a project it should be possible for the two approaches to yield the same appraisal outcome. The alternative, of the public sector raising its discount rate, may encourage short-termism and would appear to be contrary to Treasury guidance.

Selection of multi-stage projects of the type appropriate to real options analysis may prove problematic in the face of ring-fenced departmental or divisional budgets. A core idea in the real options literature is flexibility in adapting investment decisions to information flows. If short-term budget constraints do not permit such substitution between time periods this should be taken into account in the appraisal. Consideration should be given to formulating budgets in such a way that divisions and departments are not given an incentive to choose an investment date for budgetary reasons that does not maximise value. The ‘ring fence’ difficulty arises in acute form when expenditure of competing projects would run over a number of years or phases, each of which is subject to a fixed budget.
Recommendations

1. The presumption should be that estimation of net present value (NPV) of social costs and benefits is the preferred technique for investment appraisal unless a strong argument to the contrary can be made in a particular case.

2. MCA ranking should be preferred to an NPV evaluation only if a large fraction of the project’s outputs cannot be reasonably and economically measured by market or quasi-market valuations, either by individuals directly affected or by experts.

3. Real option appraisal is appropriate where project-relevant information is expected to arrive in the future. An initial project may create an option, but not a commitment, for subsequent real investments, or the initial project might be delayed until more information has been acquired. Real option analysis reflects the fact that future decisions can be informed by more information than is available today and hence better decisions can be made. The value of any options created is one of the benefits of a project and as such should be included in the NPV calculation just like any other benefit. The option to delay is valued by comparing expected present values of a project decision today versus the expected value if the decision is taken in the light of more information in the future.

4. Real option analysis implies that the value of a project can vary with the date that it is initiated. If short term divisional budget constraints discourage the free choice of project start date then consideration should be given to formulating divisional budgets in such a way that departments are not given an incentive to initiate an activity for budgetary reasons at a time that is in conflict with the valuing maximising date.

5. It follows that calculation of the value of any real options created by a project are an integral part of a proper NPV calculation and are not a substitute for the NPV method.

6. The outputs of many government projects are not traded in markets and decisions about the exercise of options are generally made at specific review dates. Hence the valuation of real options created by government projects requires a different methodology to those techniques that can be employed when the principal output of a project is traded in a competitive market and decisions about the exercise date of the option may be made continuously.

7. The construction of a decision tree should lead the measurement of the value of real options and the calculation of any NPV measure. A decision tree is a useful tool for setting out business cases for interventions. It sets out what actions are available to which parties at each decision date, what information is likely to be available to them at each date, and the likely payoffs of the different possible decisions.

8. Full NPV calculations can be expensive. For example eliciting individual valuations by survey or other methods can be very costly. We propose that the NPV calculation be done in two stages. A pilot NPV using expert valuations in place of elicited valuations is undertaken at the first stage. If the probability that the NPV is positive is sufficiently high then resources are invested in a full scale NPV calculation, possibly including a pilot project from which informed consumer valuations can be obtained.

9. In principle if MCA is deemed the appropriate tool for evaluating a particular investment, and real options arise, they can be evaluated using the
appropriate MCA scores for flexibility and learning in place of market valuations.

10. Whenever expert valuations are employed, whether in NPV or MCA it must be recognised that experts often have a vested interest in the outcome of the calculation. Industry experts may have a financial interest and academic experts may have “pet” projects that they wish to see implemented.

11. Risk, including optimism bias, that is not resolved in the successive stages of a project should be factored in to the NPV values (e.g. the equivalent of expected cash-flows) of the real options analysis.

12. The risk-neutrality generally prescribed for government projects implies DTI project portfolio selection should ignore risk profiles and focus only on expected net present values or equivalent measures. However if projects do not satisfy the assumption that benefits are spread over the whole population then other selection criteria, such as some form of risk pooling and risk trade-offs may be appropriate.

13. The principles underlying government project appraisal require that in projects with mixed funding the same public sector approach to discounting and risk assessment should be employed to appraise private fund commitments. If this appears to rule out private funding then attempts should be made to understand and resolve the divergence between private and public sector assessments of risk. Private funding decisions should be factored in so as to predict the likelihood of project continuation when public funding is withdrawn or reduced.

14. The impact of regulations also needs to be evaluated on a case by case basis. Individuals are the best judges of the merits of legislation and hence a conventional NPV is normally appropriate. When regulations also have a direct impact on businesses, these should be taken into account but avoiding any potential double-counting. It may be possible to make the case for an MCA approach in specific situations where the benefits can be quantified but not easily priced.

15. The optimal allocation of investment over time dictated by real option analysis may be in conflict with the incentives set up by the need to work within annual budgets. Annual budgets on the other hand introduce a possibility of learning from past experiences and revising decisions. It would be desirable to formulate departmental budgets in such a way that they approximate as closely as possible the best allocation of investment over time.
Introduction

This report examines the scope and limitations of methods of appraisal for projects, programmes and policies of particular concern for the Department of Trade and Industry. It addresses approaches to assessing the value of flexibility in decisions over time, suitable for joint public and private sector funded projects, and those appropriate for policies, programmes and projects with multiple objectives.

DTI undertakes a very wide range of activities. Of particular interest to DTI is the government’s concern to promote investment in science and innovation which are fundamental contributors to productivity growth. Such projects are especially subject to uncertainty and may well be of a sequential or cumulative nature. Projects in this area often consist of a series of rounds. Frequently the early stages are primarily public funded and the later stages primarily privately funded. Yet the public and the private sectors have different approaches to project appraisal. DTI are interested in whether and how they might need to take account of private sector appraisal in evaluating these projects. The grounds for doing so would be that if the private sector judges the projects unprofitable the later stages will not go ahead, however much net benefit the public sector appraisal establishes. This is just one of many anomalies that arise when the public sector discount rate is less than the private sector discount rate.

Another significant area for DTI is the appraisal of regulations. The Department are therefore interested in alternative techniques that might be useful in this field. The detailed evaluation of regulations will be highly case specific and the costs and benefits will be often much harder to quantify than in conventional investment projects with somewhat more tangible outputs. Nevertheless the general principles of NPV should offer a conceptual framework for the evaluation of regulations. The ultimate objective of regulation must be to raise the welfare of society and it should normally be assumed that it is the individuals’ monetary valuation of the gain that is the relevant measure of benefit, either directly or through its impact on business. The costs -and often the benefits- of regulation will normally have a monetary valuation. If real options are created then real option analysis may also be added to the analytical tools. For example legal regulation is likely to be more expensive than a voluntary industry code but its effectiveness can be uncertain. It might be appropriate to delay an investment in formal regulation until the success of voluntary industry agreements can be seen. It should be also noted that the possibility of introducing legal regulation may sharpen industry incentives to develop a code that meets government requirements.

In order to assist the choice between conventional net present value (NPV) appraisal and multi-criteria analysis (MCA) the following section sets out the reasons for favouring the market and quasi-market valuations as an ideal. Section 2 then discusses some of the paradoxes and tensions that can arise with the weighting and scoring that underpin MCA. The central sections, 3 to 6, concern real option appraisal. They explain the type of project appraisals for which real option theory is helpful, and set out the information required and the preferred approach, as well as the strengths of the technique. Section 7 discusses how other types of risk should be incorporated into a real option approach, concerned primarily with risk that is resolved with the passage of time or by earlier project phases. Section 8 addresses the discount rate for appraisal, with particular reference to the possible conflicts between public and
private sector approaches that arise in jointly funded projects. Section 9 considers the problem of appraising, and in particular real option appraisal, with the capital rationing that is typical of government departments.

1. NPV v MCA

The expected NPV approach of formal cost-benefit analysis and MCA are alternatives, each of which may be appropriate in different appraisal circumstances. A key to choosing is the practicality of obtaining market or quasi-market valuations of the benefits and costs of the ultimate consumer, user or target group. If this is not practical for a large fraction of a project’s output then quantification of outputs by experts in the MCA framework should be applied.

We argue that NPV is the preferred technique unless a strong argument to the contrary can be made in a particular case. Only if a large fraction of the project’s inputs and outputs cannot be reasonably and economically measured by market or quasi-market monetary valuations should MCA ranking dominate an NPV evaluation. Even in this case the NPV calculation should be included as one criterion in the MCA calculation. We first discuss principles and then turn to practical examples relevant to the business of the DTI.

1.1 Principles: The ‘Quasi-market’ Valuation Ideal

At the heart of the NPV approach is the assumption that costs and benefits ultimately accrue to individuals and that projects should be accepted if the discounted sum of net benefits to individuals are positive. Individuals affected should normally be assumed to be the best judges of the costs and benefits. Their valuations are revealed in markets where markets exist and otherwise must be obtained by indirect methods as discussed in the Green Book. In general it should be presumed that this is correct. Nevertheless in some cases it will be impractical to elicit honest valuations and in other cases individuals may not be well informed about the consequences for them of the project outputs. In either case valuation by experts may be preferred. Under these circumstances, if expert valuations can be reduced to monetary values and incorporated in an NPV then this is the preferred method. If outputs can be quantified, but not readily valued in monetary units by experts, then MCA may be the appropriate framework.

The key principle of welfare economics that underpins the NPV approach is that individuals affected are the best judges of the value of a project. An important criticism levelled against the use of quasi-market valuations by individuals is that they may not be well informed about the value to them of public sector investments. For example individuals may not be the best judge of the ultimate cost of increased CO₂ emissions or the benefits of a hospital building programme. Where novel services are introduced, with which the consumer is unfamiliar, pilot studies offer an important mechanism for obtaining consumer valuations which can then be used to measure the

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1 In practice a large part of MCA and related approaches (e.g. de Neufville 2000) is concerned with the process of appraisal that should be common to what we have called the NPV approach as well; the identification of options and constraints for the project of concern by discussion with, and the participation of, informed and interested parties. Here we focus only on what we regard as the critical difference.
NPV of a full scale service. When outputs are complex and not well understood by the user or recipient, and pilot studies are impractical, the merits of a technique built up from individuals’ own valuations vanish, and assessment by experts may be preferred.

In some cases individuals’ valuations are the appropriate measure of benefit but they cannot be elicited because of the well-known public good problem. If individuals know that a project is to be financed out of general taxation they have a private incentive to overestimate the value of the outputs to them. Much effort has been expended trying to devise mechanisms for eliciting honest valuations, but ultimately there remain circumstances where true preferences are not revealed.

If individuals’ valuations are imperfectly informed, or not honestly revealed, then in some cases valuation by experts may be used to measure the value of outputs to individuals. Expert valuations may then still be aggregated in an NPV. An example might be a hospital building programme where methods to measure the monetary value of improvements in health care have been developed.

It is when even experts cannot reasonably put a monetary value on the projects outputs than MCA should be considered. For example when choosing a site for the disposal of nuclear waste there is no generally agreed method for putting a price on the different environmental and risk factors. Different sites may vary in safety, many dimensions of environmental impact/damage, and size of local population affected. An MCA offers a systematic framework for ordering the relative merits of different sites. Even though a unique ranking might be impossible for reasons set out in the appendix, the coherent enumeration of points in an MCA can provide a valuable aid to rational decision making.

Even with familiar outputs that are well understood by individuals, laboratory experiments with individuals reveal choice behaviour inconsistent with the underlying assumptions of traditional welfare economics. This, it might be contended, casts doubt on the claim that market values reflect willingness to pay. On the other hand paradoxically laboratory market experiments do seem to show rapid convergence to the price equilibrium of welfare economics (Loomes et al 2003). If market values do not reflect some aggregate of willingness to pay they may still be used for valuation in appraisal. Sugden (2004) proposes an “opportunity criterion” for assessing resource allocation systems. When rational arbitrageurs compete to make profits by trading with non-rational consumers the resulting equilibrium satisfies the opportunity criterion, the maximising of opportunity and responsibility.

Whether expert evaluations are used in NPV or MCA it needs to be recognised that expert judgements may be tempered by self interest. It is hard to find experts who do not have some stake in related industries since they are experts by virtue of the fact that they are already working in the area. Academics, for instance, may have founded a reputation on a particular approach and may favour projects that they particularly wish to see initiated. It is for these reasons that the affected individuals’ own valuations are to be preferred if they are available.
1.2 NPV versus MCA: Examples to illustrate when each should be used.

In this section we consider some examples of DTI projects and review which technique seems more appropriate for each.

1.2.1 Projects where NPV is to be preferred

- Launch Investment Aid should be evaluated by NPV. Costs and revenues can be measured in monetary units and the monetary value of any further environmental factors should also be included.
- The impact of employment regulation, for example maternity leave, working time, minimum wage, is primarily in a form where individuals affected should be the best judge of the costs and benefits. For example survey work and pilot studies should reveal the value individuals place on maternity leave or a working hours regulation, and the costs of these proposals to industry can be estimated.
- Consumer Direct should be evaluated by NPV. Consumers should be the best judge of the value of the advice given to them, and the costs of providing the service is principally measured in market costs. This would be a new service and valuations about a hypothetical service will be unreliable. There is a strong case for a pilot study so that valuations can be obtained from those who have experience of the service.
- Grants for collaborative Research and Development will normally lead to outputs which should be directly valued by consumers and so this ultimate value, weighted by the probability of success, should form the core of the NPV calculation.
- Space-related projects ultimately deliver outputs where consumers should be the best judges of value. If the programme gives rise to technical equipment that is used by individuals, then those individuals should be the best judges of its value. Many of these benefits take the form of real options and must be valued as discussed below.
- The Business Support Projects should be valued by the NPV approach. Whether support is given for innovation, to encourage best practice, or for regional investment, the benefits accrue to firms or individuals in a form that can be best assessed by them. For example to the extent that regional investment reduces regional unemployment then this clearly can be valued in monetary units, and the value individuals place on innovations is normally the value of those innovations to society. Again innovations create value best measured as a real option.

1.2.2 Projects where MCA may be preferred

- Choice of a site for disposal of nuclear waste. The different aspects of the environmental issues are not well understood by the individuals affected so their valuations cannot be relied upon. Experts may be able to quantify and rank sites on the many dimensions of impact but cannot plausibly put a monetary value on the costs to individuals. Forcing experts to come up with a
monetary value will inevitably introduce substantial arbitrariness. This is a clear case for MCA. Of course one criterion will be the costs at each site.

- National Measurement System projects. These concern the integrity and efficiency of national weights and measures system. There are no market valuations of their usually highly technical outputs and because their impacts are diffused throughout the economy it is hard to see how quasi-market valuations could be obtained. Expert assessment of effectiveness of alternative projects therefore cannot be based upon monetary valuation of benefits. Instead some form of ranking or scoring, characteristic of MCA, is necessary.

1.2.3 Projects where a mix of techniques may be required

- Funding and grants for renewable energy. Although some outputs can be measured in market prices, such as the energy produced, the important benefits are the conservation of fossil fuels and in many cases a less adverse environmental impact. The issue here is to ensure that the government pound sterling is spent on the project which delivers the greatest fossil fuel saving or least environmental disadvantage. Aggregation of these diverse benefits should be in the context of an MCA model. Rather analogously with the idea that a fatality prevented should be given a standard value, so renewable energy projects which offer the greatest MCA score per pound should be financed.

1.3 Practice: Measurement of Costs and Benefits

The standard approach for central government project appraisal is laid down in HM Treasury’s (2003) *Green Book*. The central idea is that where possible all benefits and costs should be adjusted for their risk and discounted at the Treasury-determined discount rate (3.5 percent real).

The valuation of benefits, and sometimes of costs as well, can be extremely challenging. In an ideal world generally HMT prefers ‘willingness to pay’ or ‘to accept’ valuations of benefits and costs, if market values are not available, simply because they are the values of those whom the project or programme affect. What an individual is prepared to give up for something is an indicator of the value they place on it. Moreover the measuring rod of money allows the comparison and adding up of multiple diverse outputs.

Eliciting quasi-market valuations (numbers of ‘clients’, value of what they receive) can be problematic, and in some circumstances, excessively costly. The DLTR/ODPM’s Multi-Criteria Analysis manual refers to

‘social and environmental impacts that may be quantified but not valued (such as the number of houses suffering specified increases in noise) or assessed only in qualitative terms (such as impacts on landscape).’

Nonetheless a great deal of ingenuity has been expended on making precisely such valuations practicable (e.g. Bateman et al 2002, DTI and, ODPM 2002). To hold down appraisal costs, the Green Book advocates ‘benefit transfer’, the use of quasi–market valuation obtained in one context in another, as a way of economising on appraisal costs.
One reason that it may be difficult to elicit honest valuations is where the consumer is unfamiliar with the outputs of the proposed project, for example the benefits of Consumer Direct. There is a strong case for the use of pilot projects in these cases.

When appraisal costs are high, perhaps because of the costs of eliciting individual’s valuations, there is a case for a two-stage NPV calculation. In the first stage a rough “back of the envelope” or “pilot” NPV calculation is done. Here plausible valuations by experts are used in place of precise valuations obtained by survey or other costly methods. The more costly detailed NPV, possibly including pilot projects, is only undertaken if it looks reasonably likely from the pilot NPV that the project will be positive NPV after a full scale study.

An example of a project where MCA might be preferred to NPV because a large part of the output cannot be practically or reliably valued by those directly affected is the choice of site for the disposal of nuclear waste. Expert valuations may be more reliable than consumer’s own valuations, and MCA is well suited to handling the diverse aspects of the disposal decision.

1.4. The Role of Government Policy in Project Evaluation

There is a tension between full cost-benefit analysis according to the tenets of welfare economics and the right and the obligation of the elected government to form policy and redistribute income. The case for the NPV approach is underpinned by Paretoian welfare economics which lays down a programme for a government of undertaking all projects, rectifications of market failures and missing markets, where social benefits exceed social costs,

- where all individuals count equally,
- where income distribution is irrelevant and
- where the gainers could compensate the losers and still be better off even though they do not pay compensation in practice.

One reason that government is the ultimate arbitrator on the decision to proceed with a project that has been evaluated by NPV is that the ‘winners’ do not in practice compensate the ‘losers’. Distributional considerations mean that an NPV calculation should not be the only investment criteria and responsible political judgement may overturn the NPV verdict. An NPV calculation can allow for the relevance of income distribution by including explicit income weights (as the Green Book does). Nevertheless a uniform formal mechanism may be unable to capture the nuances of distributional issues, for example those intended to impact on minorities or regions. The elected government is the ultimate evaluator of social benefit when a project has a significant impact on specific groups. For instance, by ring fencing resources to a programme that targets a specific group a government articulates its distributional preferences.

If there are different possible alternatives, the NPV calculation is always necessary even when a project is primarily motivated by distributional issues. It provides an important measure of the costs of proceeding with a project on distributional criteria. There may be choice of projects that meet a distributional objective and the NPV should serve as a mechanism for selecting the most cost effective means of meeting that objective. When MCA evaluation must be employed, for reasons already outlined, then it is the role of the government to judge the relative importance of the
different criterion, and select the project with the relatively high scores on those
criteria it judges to be the most important.

1.5 Regulatory Impact Assessment

In principle a full cost benefit analysis using an NPV calculation should be the
standard method. It may be potentially technically demanding and costly to perform
Regulatory Impact Assessments but the types of problems involved do not seem to
raise any issues of principle. Valuation can usually be addressed by benefit transfer
methods so that employment of Multi-Criteria Analysis for such purposes appears
unnecessary except in special cases. Equally most regulations do not involve
successive stages in which uncertainty is dispelled. An important exception is where a
voluntary scheme of regulation might be considered in place of government
regulation. A voluntary scheme is more likely to succeed if the government retains the
option to introduce a compulsory scheme if it is not satisfied with the outcome of the
voluntary scheme.

An NPV calculation could rule out the adoption of the proposed regulation altogether
but it may be that ministers have nevertheless decided on the broad regulatory
objective. In this case the economist is required to advise on the most effective means
of achieving the specified objective. This calls for an NPV which only looks at costs
and takes the need to deliver the regulation as exogenous. This kind of calculation is
known as cost-effectiveness analysis (CEA). An important aspect of this measure is
the need to ensure that the burden on industry and households of additional regulation
imposed by policy initiatives was not excessive.

There is no single generic approach that can be recommended for appraisal of
regulatory intervention but rather the costs and benefits of specific regulations have to
be evaluated using all tools available on a case by case basis. MCA or real option
theory might be helpful in particular circumstances. While models implicit or explicit
are still necessary to identify what would happen with and without the regulation, the
non-market valuation problem is usually largely removed for CEA. For example, how
much business wage costs rise as a result of the minimum wage can be modelled with
standard labour market analysis.

2. Issues in the application of MCA

MCA draws upon weighting and scoring techniques and therefore may in some
circumstances suffer from certain drawbacks. When intensities of preferences are
allowed to count, as with market (or quasi-market) valuations, aggregated rankings
may differ from those where only preference rankings are taken into consideration.
Possibly arbitrarily chosen weights for aggregating attributes can also influence the
preferred option. Where alternative projects differ in their costs as well as in their
benefit scores, then the best choice may depend upon the value or weight assigned
to benefit scores

2 A possible exception is in appraising distributional impacts. This is covered in the Green Book but not
necessarily as comprehensively as would be helpful in all cases.
The basis of MCA is that the inputs and outputs are quantified by stakeholders or experts. They may or may not additionally be able to place a monetary value on what these outputs might be worth to individuals. When reliance must be placed on expert evaluation of the benefits of a project and these evaluations cannot easily be reduced to monetary units and incorporated in an NPV then MCA has important advantages.

- It imposes a disciplined and consistent approach upon the decision maker.
- It makes possible an evaluation of the sensitivity of project rankings to different weighting and scoring schemes.
- As with any weighting and scoring technique, it ensures that the decision-takers who have delegated powers for purposes of carrying out government business act transparently on behalf of the principal\(^\text{3}\).
- It allows the government department to readily see the aggregate contribution of its portfolio of projects to different non-market outputs.
- It provides a set of focal points for key players to discuss the merits and demerits of various options.
- It can reduce the list of options by quickly identifying dominated options.

A caveat is that experts may have a private interest and may also have private information that they choose not to reveal. An expert may both tell the truth and judiciously conceal information. Private interest may not be simply a financial interest but broader and nevertheless significant; for example the desire to see implemented favored projects in which the manager or expert has invested intellectual capital.

Although MCA in principle has merits in certain cases, there are important judgemental issues that arise in practical applications. Decision Conferencing uses participants both to assign values and to forecast—a variant of the Delphi technique. This may well be the best approach to scientific uncertainty where the participants are technical experts. In the case of intermediate future products, the valuations may be largely a technical matter, when the value is derived from what the new technology may be expected to do. Weighting and scoring may be an effective way of summarising expert opinion on these issues, providing expert opinion can be distinguished from ‘stakeholder’ opinion.

Many organisations have special interests in aspects of projects that are not necessarily identical with society’s interests, or with their expertise. Weighting and scoring by such stakeholders may then become an exercise in political economy, rather than in efficient resource allocation. For example, Neufville (2000) advocates such an approach and refers to the involvement of unions and employees in the appraisal of an instance of telecom deregulation in the US. Trade unions and employees are stakeholders likely to agree and to press for regulation that avoids such policies if they are seen lead to job losses. Two problems arise in this case: Firstly, it may be that not all the alleged job (and subsequent earnings) losses are genuine costs of the intervention. Secondly, the number of potential stakeholders can make a balanced assessment unmanageable and eventually dilute the role of the taxpayer’s

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3 For instance weighting and scoring can establish an audit trail showing that contracts have been allocated in accordance with the appropriate rules and without collusion between members of a panel.
stakeholder. Treasury guidance is very clear in this respect, warning about this problem and the risks it entails.\footnote{The Green Book states (para 5.79) ‘The risk that [weights given to factors by key players] are weighted towards acceptance of more expensive solutions by those who would enjoy the potential benefits should be tempered by at least one stakeholder representing the opportunities that an expensive solution would be foregone elsewhere.’}

Multi-objective organisations need to allocate resources within and between ring-fenced budgets and weighting and scoring is one method employed. Clearly it is useful because, regardless of the outcome, those involved in the process cannot deny that their opinions have been taken into account through an agreed process. They must be therefore more inclined to accept the outcome even when it runs counter to their interests. Moreover where those involved possess relevant legitimately devolved authority (as managers), the process and the outcome acquires legitimacy. Although it is obviously important that the decision making process is explicit and transparent the ultimate criterion must be the quality of decisions the process generates. For the exercise to involve efficient resource allocation, the opinions must have a basis in fact about the ultimate beneficiaries of the policy, and/or about the likely success of policies. That a project may make a large difference to a few interested parties must not obscure the individually small interests of a very large number of more distant parties. To ensure a balanced outcome the ‘stakeholders’ therefore must in this sense be independent ‘experts’ rather than advocates or managers. The ultimate ‘stakeholder’ is society at large.

The most critical issue in practical application of MCA is how to aggregate the opinions of different experts, or the contribution of projects to different objectives. The merit of NPV is that willingness to pay or accept are cardinal measures of benefit – they can be added and subtracted. Where money valuations are not readily available one MCA methodology is that experts or stakeholders are asked to give rankings to alternatives, which are then added to give a best option. It is easier to agree on ranking than on absolute numbers but when rankings differ on different outputs, or as evaluated by different experts, problems arise (numerical examples are given in the Appendix).

Consider first the case where several experts are consulted about four possible projects. The problem with ranking rather than the scoring of alternatives is that the intensity of opinion is lost in an ordinal ranking. One expert might only marginally favour one project over another, but the second expert might very strongly recommend the project perceived as a close runner up by the first. A ranking approach would treat the two as equally favoured, neglecting the fact that one expert is almost indifferent and the other has very strong views.

Similar considerations arise when projects rank differently in the contribution they make to different objectives. Assume all projects cost the same. In order to come up with an aggregate ranking of projects weights for summing a number of qualitatively different outputs need to be allocated. Inevitably plausible changes in the weights could influence the preferred option.
No project appraisal technique can sidestep this problem of quantifying a relative ranking of different non-market outputs. The choice of weights on the different outputs must ultimately be political. MCA has the important advantage of ensuring that political values are applied consistently across projects.

So far we have considered ranking projects with identical costs. When scoring is intended to rank the total benefits for projects with different costs, matters can be still more complicated. If project A has a higher benefit score than B for a lower Present Value of Costs (PVC), then accepting A is unproblematic. But when project C has a higher benefit score than B for a higher PVC, the correct choice is not obvious. A basis is required for assuming that the greater benefit scored for C over A does, or does not, more than compensate for the increase in costs. Assigning weights to the two characteristics of the projects (PVC and scores) potentially allows a systematic decision.

The more fundamental choice of whether to proceed at all also reflects an implied weight or valuation of the scores. The decision to go ahead requires that the maximum difference between weighted scores and costs yields a positive benefit.

Unless measures of contribution to different objectives like air quality, quiet, landscape, can be reduced to a common scale by relative weights for each, which in turn are related to money values, no ranking of projects can be treated as simple a “technical” decision. One solution is to infer implicit values or weights from decision-makers’ preferences after scoring.

There is a conflict here. If expert valuations can be reduced to monetary units then NPV can be applied. If they cannot then MCA decisions may be arbitrary and rather sensitive to the chosen weighting scheme. Nevertheless the use of MCA can make for consistent decision making across the government sector. It may be difficult to determine the value of non-market outputs, such as prevented fatalities, but standard values at least ensure the government pound is buying the same advantage in every project. Using standard values for air quality contribution across all government projects ensures at least that society—in a hypothetical example—is not spending £5 to improve air quality in pollution regulation when the same benefit could have been achieved for the cost of £1 in a forestry project.

3. Real Options and Flexibility in Appraisal

*Research and development, innovation and productivity growth are sequential processes at the heart of DTI’s concerns. Real option theory therefore offers a valuable addition to DTI’s appraisal toolkit. However the recommended approach differs considerably from the use of real option theory based on techniques developed in finance.*

3.1 What are Real Options?

Real options analysis is applicable whenever it is expected that more information about the value of an investment will arrive over time and investment decisions can be made at different dates. For example a primary project may make possible, or reduce
the costs of, other projects which may be undertaken in the future, if at that later date (in the light of more information) they are NPV positive. The creation of the possibility of later projects (or simply the reduction in costs of later projects) is part of the benefits of the primary project, but these later projects are contingent on information that will later be available. The primary project may contribute information (R and D explicitly, technical Know-H-ow, or indirect information about consumers preferences) that makes later projects possible, or it may contribute infrastructure or capital that reduces the start-up costs of later projects. Alternatively future decisions may be informed by more information that has arrived exogenously, for example growth in demand.

There are several rather different situations where the idea of real options may be applied. One is when a standard single stage project makes possible subsequent projects. For example building a road makes possible a string of potential new projects. In this case the real option values of the potential projects must be included as part of the NPV calculation for the first project.

A second case is when the first project has actually multiple decision stages, so that the first investment itself can be viewed as a real option. Rather than simply one output of a project being an option, the whole project is effectively a real option, for example an investment in R and D today creates the option, but not the necessity to go into production in the future.

Another important idea in the real option literature is the option to delay. The idea here is that if the decision to undertake the project is delayed into the future then the decision can be better informed. For example if demand turns out to be weaker than expected today then the project will not be initiated. Thus the NPV of investing today must be compared to the expected NPV if a decision is delayed. In calculating the latter it is allowed that investment in the future will only proceed if it still appears a positive NPV project in the light of the new information. This is effectively an inter-temporal version of the textbook case of mutually exclusive projects. Investing in a particular project today or in the future involves comparing mutually exclusive projects. The textbook recommends that when projects are mutually exclusive the project with the highest NPV should be chosen. In the same way investment should be delayed if the expected NPV from allowing the decision to be made in the future is greater than the NPV of the investment today.

In what follows, multi-stage projects, such as research and development followed by production, are considered. The key characteristic is that the decision to make an investment in the first stage today is not a commitment to future production in the later stages. Whether or not further investments will be made at later dates will depend on information that has become available at that later date. The relationship between NPV analysis and the valuation of real options in appraisal of such multi-stage projects is expounded. The general framework for project evaluation where the project is itself a real option is set out, before turning to numerical techniques and specific examples.

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5 Newton, Paxson and Widdicks (2004) have undertaken for DTI a thorough survey of the literature on real options for Research and Development. Detailed references on the real options literature are provided in that report.
3.2 **Relationship between real options and NPV analysis.**

A more complex than usual NPV appraisal problem arises when an initial investment does not directly lead to specific outputs, but instead creates the option to undertake production at a later date if that is profitable in the light of the additional information that will be available at the later date when the decision to enter production must be made. For example investment in R and D creates the option to produce later and without the R and D the later production is simply not possible. Calculation of the NPV of the R and D will include the present value of this real option as the benefit of the R and D. This highlights the point made above that real options are included as a benefit in an NPV calculation. For example the benefits in an NPV evaluation of Launch Investment will all be in the form of the valuation of the real option to build airliners that it creates.

Standard textbook NPV calculations are for a project where an investment decision has to be taken today and thereafter the project delivers a stream of net revenues. Critically there are no future investment or continuation decisions in these accounts. The possibility that the same investment might be undertaken in a future year in the light of more information is not considered. There may be investment expenditures implicit in the net revenue calculations but continuation/termination decisions are not usually explicitly allowed or discussed. However many real investment decisions have the same structure as the investment in an option in a financial market. Since a sophisticated theory of option pricing has been developed it seems natural to apply the same ideas to real investment.

Although the ideas in real option theory are applicable in general, the formal techniques that are appropriate to apply in financial markets are often not practical for the kinds of decisions that arise with real government investment. In particular the techniques applied in financial markets build on the assumption that the asset value today is observed in a market and subsequently will evolve as a continuous time process, often assumed for simplicity to be Brownian motion. This is a natural assumption for an option on a share. However it has no analogous easy interpretation for many real investments, particularly government projects. For many government projects it makes no sense to assume the project is an option on production where the current value and volatility of the underlying asset price is known. We review in more detail in Section 7 the problems of applying continuous time methods to government project appraisal. For example in the context of Launch Investment the value of the option to produce airliners will depend on the growth in demand for air travel, cost of oil, and competition from other companies. There is no single price, and associated volatility, that simply determines the likelihood of the option actually leading to production. Instead the large number of diverse relevant factors must be assessed and valued by experts. We set out next a framework for aggregating expert advice in such a way that an NPV can be calculated.

We develop here a simple general methodology for public sector decision making that builds on ideas in option theory and is applicable in a more general context where there are not markets for the outputs of the projects and hence where volatility of the value of the output market cannot be easily empirically estimated. Of course in those
cases where there is a single state variable that determines the value of the option, for example when the principal output of the government project is traded in a market where volatility can be measured, then the standard techniques from finance may be applied. An example would be investment in renewable energy technology where the benefits will largely depend on energy prices whose volatility can be reasonably estimated. We focus on methods to apply real options analysis in the more difficult general case where there is no single state variable, or market, from which to infer option values. For example, the Galileo project may create the option to introduce a new media distribution network but exactly what market would one look to in order to infer an option value using standard continuous time methods? The only way to proceed we believe is to work with a discrete decision tree and numerical techniques where the probabilities of different real options and associated values are estimated by experts. Dixit and Pindyck (1994) emphasize the need for thinking in discrete terms for many particular cases and advocate methods rather like the decision tree that we propose. These numerical techniques are discussed after a review of the basic ideas.

3.2 Real Options

Options in financial markets give the owner the right, but not the obligation, to make a future stock purchase at a specified price. Depending on the resolution of uncertainty over time it may or may not be desirable to exercise the option. Clearly this option is valuable since if future price exceeds the exercise price a future profit can be made. Option theory describes how the value is determined. Real option theory was developed in response to the fact that real investments today may similarly give the option, but not the obligation, to produce in the future. In order to make the correct investment decision today it is necessary to look forward to the decision one expects to make in the future.

For example if land comes up for sale next to an airport, purchasing it today gives the option to expand runways in the future. If it is sold for housing then there is no future opportunity to expand. Purchasing the land is an investment in a real option. Investment in the R and D effort to develop a super size aircraft gives the option to build one if it later appears profitable. Without the R and D today it will be impossible to catch up later if demand later materializes. In both of these cases expansion/production later is simply not possible if the investment is not undertaken today.

A more complex example, important in practice, is whether or not it is desirable to make “excessive” investment in infrastructure or fixed (irreversible) costs today, so that if subsequent expansion proves desirable (as uncertainty about demand is resolved), it can be undertaken more cheaply than if the extra infrastructure capacity had to be added at a later date. For example suppose it is likely that a four lane motorway is likely to be sufficient but there is some chance demand will increase so that two lanes will need to be added in the future. Also suppose laying foundations today for an extra two lanes today as the motorway is built is much cheaper than adding foundations at a later date. Real option ideas can be used to evaluate the marginal investment in the extra two lanes of foundations.

If there is no later real decision about later production or investment then the ideas of real option analysis are not relevant and basic one stage NPV is applied. It follows
therefore that ideas from real option analysis should be applied as part of a multi-stage NPV. The general principle is that the first step should be to establish whether a particular investment problem can be set out as a sequential decision tree. If it can, then ideas from real option analysis will be relevant. The next section outlines the basic framework when there are multiple decision stages.

3.3 Integration of NPV and real option analysis

NPV and real options are often presented as alternatives that give different verdicts on the desirability of an investment. This is not correct. They are complements and when an NPV calculation is done correctly it will always include the value of any options created. It is true that an NPV calculation that does not include the value of any embedded options will give the wrong answer. But that is not because the principle is wrong but because NPV has been incorrectly applied. It is always important to include externalities in a cost-benefit analysis and the creation of options is one example of an externality. When options are implicit in a multi-stage decision problem the NPV appraisal must be undertaken with care. The option to delay is evaluated by comparing the NPV of a project started today with the expected NPV if the decision is delayed until more information is available.

Reasoning based on real option theory is appropriate whenever there are real production and investment decisions that will be made at a later stage after some uncertainty has been resolved. If there is no uncertainty that will be resolved before future decisions must be made then effectively the investment can be evaluated as a single stage problem. Central to real option analysis is the fact that later production and investment decisions will be made in the light of more information that arrives after the initial investment stage.

The basic idea in a multi-stage problem, with the option to stop at a number of decision nodes, is that at each stage the forward looking NPV must be positive for continuation. (This is the same solution structure as sub-game perfection in a bargaining game.) The key to the solution of any multi-stage decision problem is that when a decision is made at any particular stage it must be assumed that decisions at subsequent stages will be optimal in the light of information available at that later date. In this context this is the requirement that NPV must be positive at each stage for continuation. Implementing this idea requires solving backwards from the last decision date.

We consider the investment decision in the final stage first and work backwards through preceding stages until we reach the current stage. This is described in detail in the next section.

4. Applying NPV with embedded real options: a discrete stage approach

The first step in appraising multistage decisions using the NPV rule is to identify the timing of each decision. The second step is to create a decision tree, with pay-offs to each branch and the perceived probability of each branch at the current date. This is analogous to setting out the extensive form of a game before a formal solution. It is
important before attempting quantification to be very precise about a) exactly what
decisions are made when and b) what the decision maker knows when each decision
is made.

Although later decisions will be made on the basis of later information, the likely
future value of the option at each future stage can only be evaluated using current
information. Future decisions will be made in the light of information that either
becomes available exogenously, or is generated internally by virtue of the initial
investment. An example of the latter is R and D, and the former is expansion of
consumer demand or a rival producer. In many cases investment today makes future
production possible, but if subsequently production appears on later calculation to
have a negative NPV then the project is aborted.

---

**Box 1 Computing NPV from time line and decision tree: two stage problem**

The two-stage problem assumes that initial R and D expenditure of RD1 is required in
stage 1, before investment in production can be made at a second stage, I2. This is a
classic real option because later production will not be possible unless the R and D is
undertaken first. Investment in R and D at stage 1 provides the option, but not the
obligation, to produce at stage 2.

The question is whether to invest in the first stage, RD1. This is solved by working
backwards. Risk neutrality is assumed so that the decision maker is only concerned
with expected values.

The timing of each decision and what is then known by the decision taker is as
follows:

**Decision date 1:** RD1 is known. NPV2 is not known with certainty, but there is a
known distribution over likely values for NPV2. Invest RD1 if NPV1>0.

**Decision date 2:** NPV2 is learnt. Therefore invest if NPV2>0.

Now solve ‘backwards’:

**Stage 2.**

Today’s decision can only use today’s probabilities and expected revenues. One must
estimate, using today’s probabilities, a distribution over possible values for NPV2.
Since this is a decision date the requirement is NPV2>0 for continuation. Therefore
only the area under the part of the distribution where NPV2 is positive need be
considered in deriving E[NPV2].

**Stage 1.**

If E[NPV2]>RD1 then invest.
Box 2 Computing NPV from timeline and decision tree: three stage problem

A third stage introduces a new concept, an option to buy an option, but once three stages are clear, introducing further stages involves nothing conceptually new. Investing at stage 2 is also buying an option so, formally in a three stage problem, investing in the first stage is itself an option to buy an option at stage 2. The stage 2 option cannot be bought unless stage 1 investment is made.

Consider investing in R and D for a super size aircraft. Demand is uncertain today and production at stage 3 will only be undertaken if the demand is sufficiently high then that expected net revenue exceeds I3. Assume for simplicity (this can easily be relaxed) that demand and net revenue are known exactly once stage 3 is reached. Initial R and D expenditure of RD1 is required. This R and D will let us learn what R and D costs will be for the second stage of R and D, RD2. It will reveal RD2 which will be required before actual investment in production, I3. Assume for simplicity that no new information arrives about demand for the aircraft until stage 3.

Investment at both stages 1 and 2 creates the option, but not the obligation, to produce at stage 3. Whether RD1 should be invested is established by calculating NPVs backwards, this time from stage 3. Probabilities and expected values for each branch of the decision tree must be introduced.

### Stage 3.
This is analogous to the final stage of the two stage problem. Using today's probabilities, a distribution over possible values for NPV3 must be estimated. Since this is a decision date the requirement is NPV3>0 for continuation. Therefore only the area under the section of the distribution where NPV is positive need be considered in deriving E[NPV3].

### Stage 2.
RD2 has been learnt from the first stage investment. We will invest at stage 2 if NPV2>0, where NPV2=E[NPV3]-RD2. Note E[NPV3] is a scalar. RD2 is not known at stage 1. The distribution of RD2 must be estimated and from this constructed the distribution of E[NPV3]-RD2. E[NPV2] is calculated only from the area under the distribution where NPV2>0.

### Stage 1.
If NPV1 = E[NPV2]-RD1>0 then invest.

<table>
<thead>
<tr>
<th>Stage 3.</th>
<th>Decision date 1: RD1 is known. Neither RD2 nor NPV3 is known with certainty, but there is a known distribution over likely values for both. Invest RD1 if NPV1&gt;0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision date 2: RD2 is learnt. Invest RD2 if NPV2&gt;0</td>
<td>Decision date 3: NPV3 is learnt. Invest if NPV3&gt;0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision date 1: RD1 is known. Neither RD2 nor NPV3 is known with certainty, but there is a known distribution over likely values for both. Invest RD1 if NPV1&gt;0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision date 2: RD2 is learnt. Invest RD2 if NPV2&gt;0</td>
</tr>
<tr>
<td>Decision date 3: NPV3 is learnt. Invest if NPV3&gt;0</td>
</tr>
</tbody>
</table>

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**Stage 1.**
If NPV1 = E[NPV2]-RD1>0 then invest.
4.1 Introducing Risk Aversion

A risk-averse decision maker will be concerned with the distribution of outcomes and not just expected values. Risk aversion must be modelled consistently at each stage and between stages. For example risk aversion at the third stage would actually reduce the likelihood of going ahead and hence the expected net present value at stage 3. It would be inconsistent simply to add risk aversion into the stage 1 decision. The key point is that if risk aversion reduces the uptake of later projects then this will have a negative effect on the benefits of the primary project.

4.2 Relationship between Real Options and MCA

Real options and MCA are always complements. If MCA numbers replace NPVs for the stages of a real options analysis then flexibility can be valued in MCA index units instead of discounted constant price money. For example if the primary project to reduce emissions creates a real option to invest in a second pollution reducing project then the probability of this second project going ahead, coupled with the pollution units saved, should be included in the MCA score of the first project. The (substitute) alternative is to value flexibility as another criterion in MCA and weight and score accordingly. However, this entails introducing an unnecessary layer of arbitrariness into the analysis; stakeholders’ must be asked to value flexibility when there is a procedure for doing so building on their assessment of other concepts already included in the appraisal – probabilities and payoffs.

4.3 Investment in Real Option Appraisal

As with most appraisals, a vital element of real options appraisal is accurate assessments of payoffs and risks, which are likely to be more challenging the longer the duration of the project. Spending on appraisal and eliciting expert opinion as to payoffs and risks should be proportionate to the possible commitment of public and private funds. Again we propose a pilot real option calculation before investing in a full scale calculation.

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Box 3. Numerical example of two stage problem

To motivate this problem consider a simple practical example of the value of options and flexibility. Airlines often prefer to lease aircraft rather than to purchase them. Opening up a new route (project) may have a positive NPV if aircraft are leased so exit is cheap if later demand turns out to be lower than expected. However if aircraft have to be purchased so it is more expensive to exit if demand is low, then the new route may no longer have a positive NPV. This is a specific example of the model set out in this section.

Table 1 sets out a numerical example of a two stage problem. It serves to bring out the exact value of an option to stop. We assume discount rates are zero to keep calculations as clear as possible. The discount rate plays no conceptual part in the analysis and only serves to scale each number. Clearly in applications the values at
each stage need to be appropriately discounted back to the decision stage where they enter the numerical calculations.

### Table 1: Two-stage real options investment

<table>
<thead>
<tr>
<th></th>
<th>PV Invest</th>
<th>PV gross revenue</th>
<th>NPV and ENPV of future stages</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability=0.5</td>
<td>1-Probability =0.5</td>
<td></td>
</tr>
<tr>
<td>Stage 1</td>
<td>RD1 = 4</td>
<td>0</td>
<td>NPV1=5-4=1</td>
</tr>
<tr>
<td>Stage 2</td>
<td>PROD2 = 20</td>
<td>0</td>
<td>ENPV2=0.5*(30-20)+0.5*zero=5</td>
</tr>
</tbody>
</table>

If RD1=4 is invested, production is *possible* in a second stage if in addition PROD2 =20 is invested at stage 2. However if NPV2, perceived from date 2 when uncertainty is resolved, is negative the project can be halted. If RD1 is not invested, production cannot take place in the second stage. This occurs when R and D is a necessary condition for later production.

Revenue at stage 2 is uncertain at stage 1 but with p=0.5 for the ‘good’ state and revenue=30 and with (1-p)=0.5 for the ‘bad’ state and revenue =zero. However uncertainty is resolved before PROD2 is invested.

There are two questions that can be asked:

a) At stage 1, should investment be undertaken?

b) What is the value of the option to stop before investing ‘PROD2’ at stage 2?

This second question might arise with purchase of specialized capital goods. Perhaps money could be saved by purchasing a capital good today, but part of the cost would be giving up the option to stop the project at the beginning of stage 2. In terms of the airline example, leasing is more expensive than buying, but the option to stop is lost.

**a) Invest at stage1?**

As before work backwards and apply the NPV rule.

**At stage 2** To decide whether to invest PROD2, calculate NPV2. At this stage whether the good or the bad state has occurred is revealed. Either NPV2 =30-20=10, and therefore proceed, or  NPV2 = 0-20=−20 and terminate. The implications of thinking in terms of using real options is that -20 is replaced with zero because the option to proceed will not be exercised. The lowest possible NPV is therefore zero. This is the point that the forward looking NPV must be positive at every stage and it is why the problem must be solved backwards. No rational decision maker will choose a course of action which promises a non-positive NPV; consequently such alternatives can be ruled out when appraising their decisions at the time they make them.

As set out above E[NPV2]=0.5*10+0.5*zero=5.

**At stage 1**
Ignoring discounting $E[\text{NPV}_2]=5$
Investment $\text{RD}_1=4$
Hence $\text{NPV}_1=5-4=1$
Therefore $\text{RD}_1$ should be invested.

The decision tree of figure i below offers another illustration of this problem. The bottom left branch is the worse outcome; the box at the branch end is the worse payoff from investment in the production stage (2). It is therefore ignored in the appraisal at stage 2; the stop option can be exercised and the payoff set to zero. The appraisal at stage 1 then follows the thickened branch. The box at the end of this branch is the payoff to investment in stage 2 production, in the better ‘state of the world’. The NPV at stage 1 is thus the probability of this payoff minus the costs of the R&D investment at stage 1, $(0.5*10-5)=5$.

![Figure i Real Options Analysis of a Two Stage Project](image)

**b) Suppose 2 could be saved by abandoning the option to stop. Should it be given up?**

In the context of this example suppose that the capital costs only 18 if bought at stage 1. Capital has no scrap value so should flexibility to stop be abandoned to save 2 at stage 1? To answer this question the project must be considered without the option to stop. Note with the obligation to proceed then $\text{NPV}_2$ would have been $0.5*30+0.5*0=15$. And hence $\text{NPV}_1=15-18-4=-7$.

Since $\text{NPV}_1=1$ with the option to stop, but -7 without the option to stop, in this example the option to stop should not be abandoned to save 2. Options to stop are valuable and this shows the principles that are applied to calculate this value.
Box 4. Numerical example of a three-stage R and D problem

This example is summarised in Table 2. For simplicity the discount rate will be set to zero. Suppose R and D has been undertaken for a satellite system to provide a rival service to the US GPS. There are two stages of R and D necessary to produce a rival. However they do not commit finally to offer a service. That will be done if the service looks profitable when the time comes. For example whether the service is profitable will depend on whether a foreign country also produces a competitor, and what is learned about costs in the course of the R and D.

After the first stage R and D (RD1), discovery that US costs cannot be matched will lead to abandonment of the project. Assume that the probability of this =0.5. Initial R and D gives the option to produce later, but not the obligation. There will be no continuation to stage 2 if costs are found to be high, and no proceeding to stage 3 if foreign competition emerges by then. Hence the initial R and D is buying an option.

Stage1 RD1=500
Stage 2 RD2=200,
Stage 3 Invest I3=500 if revenue, R, sufficiently high that NPV3>0
NPV3=R-500

Assume with p=0.5 that the final PV of revenue is 2000 because foreign competition does not enter. And p=0.5 for PVR=300 because the competition has entered. Should RD1 be invested?

Failure of stage 1 R and D to unlock stage 2 R and D implies that subsequent NPV=0 and the initial RD1=500 is lost. By stage 2 assume something more is learned about the likely commercial success at stage 3; with p=0.5 stage 3 becomes possible and with p=0.5 RD2 is simply lost. To solve the problem backwards ask, viewed from stage 1 and evaluated with probabilities known at stage 1, what is the expected net present value of stage 3, ENPV3?

With p=0.5 NPV3= 2000-500=1500
With p=0.5 NPV3=300-500<0. But since this outcome terminates the project (by the rule NPV>0 at each stage) the negative NPV is replaced by zero. This gives ENPV3 as viewed from stage 1, =750

Viewed from stage 1 and evaluated with stage 1’s probabilities what is NPV2?
ENPV2 = (0.5*ENPV3) - 200 = 175
because there is a fifty percent chance of successful R and D at stage 2 that will lead on to stage 3. But the R and D must certainly be spent regardless of outcome, unlike stage 3, where we know whether competition has emerged before we invest.

So if the first stage R and D is successful, which it is with p=0.5, investment should always take place. If the first stage R and D fails then the project is terminated and NPV2 if aborting =0.
Should 500 be invested at stage 1?
NPV1= ENPV2 -500 = 175-500 = -325<0
This is a negative NPV project and therefore unacceptable.
Table 2 Real Option Analysis of a Three Stage Project:

<table>
<thead>
<tr>
<th>Decision: Whether to Invest following sums</th>
<th>PV gross revenue</th>
<th>NPV and ENPV of future stages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability=0.5</td>
<td>1-Probability =0.5</td>
<td></td>
</tr>
<tr>
<td>Stage 1 RD1 = 500</td>
<td>0</td>
<td>NPV1=ENPV2-500 = 175-500 = -325&lt;0</td>
</tr>
<tr>
<td>Stage 2 RD2=200</td>
<td>0</td>
<td>ENPV2=(0.5*ENPV3)-200 =175</td>
</tr>
<tr>
<td>Stage3 I=500</td>
<td>2000</td>
<td>ENPV3=0.5(2000-500)+0.5*zero=750</td>
</tr>
</tbody>
</table>

Figure ii is a decision tree for this three stage project. The diagram should be read from the bottom to the top, although the perspective is that of the decision-taker in the base period. Viewed from the beginning of stage 1 there is a fifty percent chance that foreign competition will have emerged by the beginning of stage 3, allowing gross revenue of only 300. This revenue would be insufficient to cover costs so the 500 investment for production (I3) would not be committed. Hence the expected net present value of stage 3 (ENPV3) viewed from the beginning of stage 1 is 750.

Viewed from the beginning of stage 1 Stage 2 R&D (RD2) of 200 has a fifty percent chance of unlocking stage 3 technology and investment. The expected net present value of RD2 (ENPV2) is therefore (0.5*ENPV3) - 200. The decision to invest RD1 at stage 1 then turns on whether the expected payoff at stage 2 exceeds the outlays, here assumed to be 500.

With the payoffs and probabilities assumed the project should not go ahead. But if the third stage gross revenue payoff in the absence of foreign competition is increased by 50 percent to 3000, then the project would have a positive expected NPV. Similar sensitivity analysis can be conducted with the assumed probabilities.
The complexity can be introduced at stage 2 that more will be learned about the commercial viability. This will affect the stage 2 decision but it will not affect the stage 1 NPV calculation. The stage 1 decision can only be made on basis of probabilities known or assumed at stage 1.

5. Application of techniques from the financial option literature to real options

Following seminal work of Dixit and Pindyck (1994) it has been recognised that when there is uncertainty about future demand then an important choice about when to undertake investment must be made. For example there is a trade-off between investing early and getting the (uncertain) benefit of the investment, and delaying until there is more information so that when bad states turn out then the investment can be avoided. Another example is where the investment by a firm today in extra capacity gives the firm the option to expand in the future if at some later date price is
sufficiently high that expansion is profitable. The basic idea is that just as the value of a stock option depends on the volatility of the stock price, so the value of an investment in the option to produce, say, depends on the level and volatility of a single “real” state variable. If the primary outputs of the project are traded in one market where volatility can be measured then the option value can be directly calculated using standard real option methods that build on techniques developed for financial options. Crucial to these methods is an empirical measure of the volatility of an output price. If, as with many government projects, many outputs are not traded in markets, then there is no observable single measure of volatility measure to insert in standard formulas.

It could be argued that in the same way that experts are asked to estimate the value for outputs not traded in markets, they could also be asked to estimate the time series volatility of the estimated value. But this imposes excessively heavy demands upon expertise about the future that are unnecessary in a discrete time framework. Discrete time analysis is more transparent than continuous time techniques, which will inevitably be a “black box” to most reviewers. An analysis that relies on techniques that are invisible to most readers cannot easily have its assumptions checked and errors detected.

An example where there is no simple observable current market value (i.e. state variable) for the project that the government is considering is where the government purchases land to give the option of later building an airport. There is no obvious market price that determines the real option value. Instead experts will be asked to determine the likely growth of airline traffic and the variance of this forecast. It is the variance of their forecast that will determine the real option value. We contend that if option values are obtained in this way there is no benefit in translating the experts’ estimate of variance into a parameter of a continuous time process. It is better to work with an explicit discrete model and explicitly in terms of the variance of forecasts than transforming variance estimates into a parameter of a time series of model.

Similarly, the government might support initial basic R and D in anticipation that if it is successful private finance will be available for product development. But later development could be undertaken by any of a number of firms and consequently it is unclear as to which asset value it is that follows a Brownian motion or any other standard stochastic process. The general principle is that for most real options that will be of interest to the government there is no relevant observable market price from which to infer the value of the real option the government may be considering. Experts could be asked the likely value of the project, but that value itself can only be obtained from the type of analysis set out here. For most government projects there is no easy shortcut, based on measures of current price and volatility, to the fundamental analysis that covers the estimates of the variance of future output values, we have described.

6. Other applications of real option theory

6.1 The value of an option.
Situations often arise where a commitment today can make a saving. The two-stage example showed that keeping options open could be valuable. A practical example would be where for an extra cost capital could be designed for multiple uses so that it could be resold if the project is terminated. This is buying the option to exit and the value of this option must be compared to the extra cost of capital. Another example would be holding land for a potential airport to keep open the option to build an airport later. The price of that option is the forgone value of the land for housing and the benefit is the expected present value of its use as an airport. If a decision about the use of an asset is irreversible (once the land is used for housing it cannot be used for airport expansion) then leaving assets idle is essentially buying the option to make a decision about their use later.

6.2 Over-investing in fixed capacity to give an option to expand cheaply in the future

In many cases it is much cheaper to put in the fixed capital for expansion at the time the project is built than to add it later. For example the incremental cost of building a bridge to accommodate six lanes rather than four lanes is very much less than expanding the bridge later to an extra two lanes. Although six lanes are not needed today it may be worth buying the option to expand cheaply later. The benefit is as always the area under the likely NPV of the extra two lanes, in the region where the NPV is positive and this has to be compared to the incremental cost today.

<table>
<thead>
<tr>
<th>Box 5. Investing to reduce uncertainty</th>
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Option theory emphasises that uncertainty means that options are valuable. Reducing uncertainty itself is valuable and some projects may reduce uncertainty. For example market research may at cost MR reduce the likelihood of embarking on a loss making project. Suppose that investment in a new service costs 120. The belief is that the probability of a new service being profitable is 0.5, where profitable is 200. Then ENPV1=0.5*200-120=100 and the project is not viable.

Now suppose investment is deferred to a second stage and first stage investment to reduce uncertainty, MR, is evaluated. Assume for investment MR=10 one will learn that true probability of success is either 0.1 or 0.9. If p=0.9 then NPV2 will be 0.9*200-120=180-120=60=NPV2>0. Then if MR shows the probability is 0.9 this will be a positive NPV investment at date 2 and it will go ahead. If p=0.1 it will not go ahead and there is no further payoff. i.e. NPV2=0.

Learning p=0.9 will happen with p=0.5. Therefore the NPV of investing in market research, NPV1, is

NPV1=0.5NPV2 +0.5*zero-MR
=30-10>0

Therefore it is worth investing in market research. It gives the option to produce if the MR reveals this service is likely to be successful. Without the extra information that is contributed this service would not have a positive NPV.

It will be apparent from the discussion of this section that projects subject to real option appraisal should be sequential in the sense that earlier stages both can be
separated from later stages (later stages can be stopped) and substantially contribute to information about the payoffs of later stages. Because of the present scope of government activity, which does not now usually involve large capital-intensive, advanced technology projects, the opportunity for using this approach will be more limited than in the private sector. However the development of science and innovation, roads and certain aspects of defence, appear to be major exceptions to this generalisation.

It might be contended that many government programmes involve renewals or rounds that are informed by earlier phases. But the potential for credibly quantifying such learning is also critical to the application of real options analysis. If plausible estimates of future payoffs and their likelihoods cannot be made then real options appraisal could become a Pandora’s Box, with possible future benefits justifying almost any present expenditure.

Avoidance of this eventuality, as with most appraisals, requires investment in accurate assessments of payoffs and risks. Multi-stage projects are likely to be of greater duration than most others and therefore the assessments will be more challenging and expensive for real option appraisal. But it is a well established principle that spending on appraisal and eliciting expert opinion should be proportional to the possible commitment of public and private funds, and multi-stage projects will almost certainly involve large sums of money.

7. Risk

Risk, including optimism bias, that is not resolved in the successive stages of a project should be factored in to the NPV values of the real options analysis.

Real option analysis is a way of addressing a certain type of risk; risk that will be resolved or reduced by delaying action until it is resolved. Practical appraisal employing a real options approach cannot however ignore more conventional risks, those that are not resolved or reduced in successive stages of a project. Standard NPV appraisal identifies three broad classes of risk: idiosyncratic or project-specific; market or systematic; and appraisal risk or the risk of ‘expert advice bias’. One example of expert advice bias is over-optimism, but as noted above expert advice may also be influenced by interests experts may have in the outcome of the decision, known as moral hazard. This is a risk because the strength of their interest is not always known and hence the extent of the bias may be unknown. It should at least be possible in many cases to identify the likely direction of expert bias and this alone is useful additional information.

For present purposes the ‘optimism bias’ of the Green Book – which appears to be based on project champion enthusiasm or moral hazard (Flyvberg et al 2002)- can be grouped with project-specific risk. Both are asymmetric risks; the downside is greater than the upside. This has the consequence that the expected value of the distribution, the figure that should be used in conventional NPV analysis, is less than the most likely value, the figure that is commonly used (fig iii). Private sector appraisal often addresses this asymmetry by adding into costs a sum for ‘contingencies’ and the same principle can be applied to benefit realisation. Both adjustments are likely to be appropriate for the NPVs of real options based appraisals in practice.
The Arrow-Lind theorem (Arrow and Lind 1970) concerns risk spreading of projects for the public benefit. By contrast risk pooling is a way of reducing the impact of project-specific risks when the risks of those in the pool are independent. Risk pooling uses the insurance or portfolio principle. An example is that British government buildings are not insured commercially, on the grounds that the ownership of large numbers of buildings allows government to pool risks as effectively as insurance companies without contributing to their profits. The chance of any one government building burning down is independent of the chances of any of the others going up in flames. Therefore if one building is ‘unlucky’ the others will almost certainly not be. The costs of replacement could be notionally recouped from a small charge levied on all the surviving buildings.

Risk spreading by contrast, presupposes that the failure or success of any one public project would be spread over so many beneficiaries that the loss or gain to any individual would be negligible. If a major project undertaken for the National Measurement System is written off, the costs are spread over the 57 million British residents who continue to benefit from the projects that are successful. With risk pooling specific risk is reduced as the number of ‘projects’ increases whereas with risk spreading specific risk is lowered as the number of beneficiaries (or taxpayers) rises.

From the viewpoint of any individual government department such as the DTI, even with only one ‘project’ in the portfolio the risks could be spread conceptually over many people so that specific risk is virtually eliminated for the individual. Hence although the individuals they represent may be risk averse, the government should be risk neutral.

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Such an approach does presuppose that all government projects are undertaken to benefit the British population as a whole. Questions may be raised about a government financed technology development project where a firm ends up with a valuable patent as a result. There certainly is a reasonable question as to whether such a firm should charge royalties or exclude competitors altogether for the use of such a patent, but insofar as some private resources went into the project the firm could make a case for doing so. To the extent that British technology is advanced and therefore British people as a whole are made better off than they would have been had the resources been employed in other uses, the ultimate benefits are widely diffused and the assumption may be approximately satisfied.

Systematic or covariant risk is not eliminated by risk spreading (or pooling). It should therefore be factored in to the NPVs of the payoffs in a real options analysis. For most government projects the assumption has widely been made that such risk is small.

8. The Discount Rate

Private and public sectors typically discount at different rates. If their risk-adjusted flows of costs and benefits are similar then they may reach different conclusions about a given project’s viability and desirability. In practice there could well be a short-termist bias in private sector appraisal. This may create tensions in joint public-private funded projects that can only be resolved by an agreed assessment of risks and how they should be covered.

In 2001, when public sector projects were appraised at a real discount rate of 6 percent, that portion of British manufacturing industry appraising investment with a required rate of return on average looked for 11.3 percent real or 13.5 percent nominal returns (Godden 2001)⁷. Subsequently, with the reduction of the public sector discount rate to 3.5 percent real, the divergence between the two sectors is likely to have widened.

The principles underling government project appraisal require that in projects with mixed funding the same public sector approach to discounting and risk assessment should be employed to appraise private fund commitments. If this appears to rule out private funding then attempts should be made to understand and resolve the divergence between private and public sector assessments of risk.

Current UK government practice is to use a close to risk-free discount rate- albeit a rate higher than that on index-linked government bonds. This has been criticised as unduly favouring public over private sector projects (Grout 2002) because the private sector typically discounts at their cost of capital which includes an element of systematic or market risk⁸, and is therefore higher than the public sector discount rate.

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⁷ The majority of these targets were pre-tax, although not for larger companies. Moreover the most common quantitative method of appraisal was the pay back period, averaging 3.6 years and 2.5 years for the largest firms, implying a very high discount rate.

⁸ The balance of empirical evidence is either that the private sector does not price market or systematic risk (beta fails to predict stock market returns) or that appropriate measurement techniques have not yet
The counter-argument is that for public sector projects, market risk is typically small and anyway should be included in the numerator of an NPV calculation along with the more important project-specific risks. Taking account of the second risk in the discount rate, as the private sector is inclined to do, is in danger of creating short-termist and anti-investment biases in appraisal. It is unlikely that market risks or most project-specific risks typically compound with time in the same way as does the pure time discount factor\(^9\).

Should the public and the private sector discount at the same rate? It has been contended that the true cost of capital is the same in the private and public sectors, in the absence of tax distortions and with complete capital markets (Brealey et al 1997; Grout 1997)\(^{10}\). Certainly there is little difference between rates for AAA bond rates and for gilts. The divergence arises with the equity premium, since there is no such thing for government.

Equity requires a very much higher rate of return, partly to cover the risk that the project or enterprise will go wrong and partly to cover the tax on the profits that constitute the returns to equity. The opportunity cost of capital to the private sector is a combination of the bond and the equity rates that constitutes the weighted average cost of capital. Government can pass losses on to taxpayers (rather than equity holders) so there is virtually no default risk involved in lending to them, unlike the private sector. This only demonstrates that the ‘true’, corrected, government borrowing rate, or ‘shadow interest rate’ is higher than the gilt rate, which is generally accepted. But this is not the same as conceding that private and public sector rates are identical.

A number of considerations militate against the equality hypothesis. The first is that the public sector restricts itself to different types of activities, and therefore different risks, from the private sector. The second is that capital markets are unlikely to be complete (savings and investment for every term and risk are matched in a market somewhere). But this is a condition for interest rates to be identical in the two sectors. The third is that taxation may be an efficient way of funding collective goods, with lower transaction costs than the stock market but with similar or greater scope for spreading risks (Spackman 2002). In which case, a lower public sector discount rate would be appropriate.

Where DTI’s support for science and innovation is concerned it might be contended that this spending is not typical of the public sector as a whole, for the returns are extremely uncertain. On the other hand, though the specific returns may be uncertain, there is some evidence that there have been quite widely diffused returns in the past, and therefore the outlays are for a classic public good of the type covered by the Arrow-Lind theorem.

The tension in jointly funded projects can be illustrated by the EC Galileo programme (PWC 2001). For Galileo

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\(^9\) In addition the public sector is in practice capital rationed so that in principle projects need to earn a higher social rate of return at the margin than the public sector discount rate.

\(^{10}\) In the context of the London Underground investment David Currie (2000) adopts the same position.
‘After taking account of capital expenditure the projected Internal Rate of Return of the Deployment and Operations phases of project is just 4.1 percent real. This is less than the cost of private capital and means that public sector support will be needed for capital expenditure on Deployment or in the Operating phase. Private sector finance can make a contribution to the costs but only with some continuing support.’

Given the present Treasury discount rate of 3.5 percent an Internal Rate of Return of 4.1 percent with conventional cash flows would show a positive present value, assuming the cash flows have been risk-adjusted to obtain expected values.

If the private sector does use high implicit or explicit discount rates to appraise investment then it inordinately discourages investment in general and especially in long duration projects (with conventional cash flows.) This is because it is unlikely that all or most risks increase with futurity in the same way as the time discount factor. It is however reasonable to adjust upwards the discount rate for bankruptcy or termination risk as is shown in the box below.

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**Box 6. Termination Risk and the Discount Rate**

Let \( p \) = probability of bankruptcy in each period and \( 1-p \) the probability of survival. The expected value is zero with a probability \( p \) and the best guess revenue \( R \) with a probability \( 1-p \). So under risk neutrality, net revenues are discounted by the factor \( (1-p) \). Survival through the previous period is necessary to go bankrupt in the present period. So in the next period the chances of being around at the beginning are \( (1-p)^2 \) and the probability of then going bankrupt \( p (1-p)^2 \).

For an \( n \) period project we have a discount factor \( (1-p) \) increasing at the same rate as the time discount factor:

\[
\text{Expected Present Value} = R_1(1-p)/(1+r) + R_2(1-p)^2/(1+r)^2 + R_3(1-p)^3/(1+r)^3 + \ldots = \sum R_i[(1-p)/(1+r)]^n
\]

When \( p = 1 \) percent and \( r=10 \) percent, the discount factors for the risk adjusted discount rate and the risk adjustment in the numerator are shown in Table 3 below. The table shows that adding on the bankruptcy risk probability to the discount rate would be a tolerable approximation for most purposes with these numbers.

**Table 3 Discount factors and termination risk adjustments**

<table>
<thead>
<tr>
<th>Period</th>
<th>( 1/(1+r+p)^n )</th>
<th>( [(1-p)/(1+r)]^n )</th>
<th>Difference, percentage point</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9009</td>
<td>0.9</td>
<td>0.09</td>
</tr>
<tr>
<td>2</td>
<td>0.8116</td>
<td>0.81</td>
<td>0.16</td>
</tr>
<tr>
<td>3</td>
<td>0.7312</td>
<td>0.729</td>
<td>0.22</td>
</tr>
<tr>
<td>4</td>
<td>0.6587</td>
<td>0.6561</td>
<td>0.26</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>20</td>
<td>0.124</td>
<td>0.1216</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Where \( r=10 \)%

\( p=0.01 \)

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**Box 7. Bias in the Typical Risk-Adjusted Discount Factor**  

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Where the project-specific risk factors do not involve termination, are independent and apply in each period, the discount rate adjustment approximation is very poor. Assume the probability distribution of outcomes is such that the most likely value of the cash flows or benefits, \( R_i \), should be adjusted downwards by a factor \( (1 - k) \) to obtain the expected value. Then as far as this adjustment is concerned

\[
\text{Expected present value} = \sum R_i \left( \frac{1}{1+r} \right)^n
\]

Table 4 below compares with this the risk adjusted discount rate expression for the expected present value \( \text{EPV} = \sum R_i / (1+r+k)^n \). The assumed expected value of \( R \) is only 80 percent of the most likely value and the risk premium added to the discount rate is 20 percent.

### Table 4 The discount rate and a constant risk adjustment factor

<table>
<thead>
<tr>
<th></th>
<th>( 1/(1+r+k)^n )</th>
<th>( (1-k)/(1+r)^n )</th>
<th>difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.769231</td>
<td>0.727273</td>
<td>-0.04196</td>
</tr>
<tr>
<td>2</td>
<td>0.591716</td>
<td>0.661157</td>
<td>0.069441</td>
</tr>
<tr>
<td>3</td>
<td>0.455166</td>
<td>0.601052</td>
<td>0.145886</td>
</tr>
<tr>
<td>4</td>
<td>0.350128</td>
<td>0.546411</td>
<td>0.196283</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>20</td>
<td>0.005262</td>
<td>0.118915</td>
<td>0.113653</td>
</tr>
<tr>
<td>r=10% k=20%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Whereas the discount factor with the adjustment in the numerator is larger in period 1, thereafter it becomes much smaller. For example in period 4 the interest rate adjustment gives a discount factor of 0.35, the constant risk adjustment in the numerator yields a discount factor nearer 0.55, almost two thirds as large again. The proportion difference between the discount factors becomes even greater after period 4; by period 20 with the interest rate adjustment the factor is so large that whatever happens in this period is irrelevant. On the other hand with the numerator risk discount factor of nearly 0.12, costs and benefits incurred could still be pertinent to a decision.

More generally, whenever risk does not increase with the passage of time, an equivalent percentage adjustment of the discount rate is wildly inappropriate. It can be shown that the divergence occurs whatever adjustment factor is chosen.

A general discount rate risk adjustment is likely to impose a short-termist and under-investment bias upon appraisal methods. This conclusion bears on DTI’s objective of raising productivity and the low and falling British investment ratio. It suggests that policy might usefully focus on methods of investment appraisal in British industry.

A practical difficulty arises from the two approaches to appraisal in joint public and private projects. In a real option appraisal of investment in science and technology, in which the early stages involve high proportions of public funding, it is essential to reach agreement between the public and private sector about the risks in the later stages that will not be resolved by the earlier stage investments. In principle identical magnitudes of risk can be covered either by numerator or denominator adjustments in the present value calculation, and ideally therefore the conclusion of an appraisal can be independent of the choice of risk adjustment.
I suggest including the following paragraph] It is also important to have an understanding of any differences of the costs and benefit flows between society as a whole and the specific private sector companies involved in the project. Although it is the former that really matters in a government appraisal, the figures for the latter would provide an indicator of the feasibility of the project in the presence and absence of support – and ultimately inform the degree of additionality from the intervention.

9. Capital rationing

A simple rule for DTI selection of projects when budgets are ring fenced is to order projects by the ratio of NPV or MCA to capital costs and select the higher ranks until the budget is exhausted.

Because much project selection takes place within ring-fenced budgets – including those using real options theory or MCA- the standard rule for project acceptance needs to be supplemented by others. The objective should be to maximise ideal net present value of social benefits subject to the departmental or divisional budget constraint.

Ordering projects by net present value (or MCA) and selecting those with the highest ranking until the budget is exhausted is a principle that may not correspond with the objective of maximising net benefits when there is a fixed budget. This is because the rule does not consider the relative sizes of net present value and initial cost of a project. Ranking projects by the ratio of net present value to costs and selecting the highest is more satisfactory providing that the budget is effectively allocated for a single period.

Where the capital rationing and disbursements of competing projects extend over several years – perhaps especially applicable to those projects for which NPV with real option appraisal is appropriate- then a linear programming solution may help. Here the principle is to maximise the sum of the net present values of all projects undertaken, subject to the budgets in each year not being exceeded. Hence there are as many constraints as there are budget-rationed years plus boundary constraints that investment should not exceed 100 percent of the project or be less than zero. The standard solution is likely to recommend partial funding of some projects. An integer programming approach can select the projects that maximise net present value or an MCA index by requiring either full funding or zero funding, with nothing in between.

The case for taking into account the risk profile of projects, as well as their expected net present values, does not appear to be strong as long as biases in the calculation of expected values are taken into account – e.g. optimism bias. The analogy that might seem appropriate is private sector portfolio balancing of risk and return. However the systematic risk that lies behind this approach is much weaker for almost all areas of government activity.

One of the consequences of capital rationing is that the implicit rate of return to optimum government investment will be higher than in the unconstrained case. Possibly this may reduce some of the conflict between appraisal recommendations.
stemming from the low discount rate used for public sector appraisal and the higher rate of the private sector.

10. Conclusion

DTI undertake a wide range of interventions, programmes and projects in the course of their departmental business, many of which can be appraised adequately with techniques that are well known. We have reservations about the employment of Multi-Criteria Analysis as a substitute for conventional cost-benefit or cost-effectiveness (or ‘net present value’) analysis, simply because of the arbitrary nature of some of the procedures. But the difficulty of obtaining quasi-market valuations, or distributional weights, is recognised to provide a justification in some circumstances.

The principal advantage of the NPV approach is that, by reducing costs and benefits to common units, money values, it allows the adding up of a wide variety of impacts. The basis of MCA is that the inputs and outputs are quantified, but not necessarily valued, by experts. The arbitrariness of MCA is the weighting of heterogeneous outputs or impacts to obtain a net benefit figure. Nevertheless MCA has the great merit that, although weights are debateable, the mechanism is transparent.

The field where newer techniques could add value is most likely long-term projects to encourage innovation and productivity. In particular the valuation of flexibility that real option appraisal provides for such multi-stage projects promises to be helpful. The information requirements though— in particular the payoffs from exploiting technologies many years in the future, and the likelihood of being able to do so—are substantial and require considerable investment in expert opinion. But since large volumes of resources may be at stake, such investment will often be worthwhile.

We have set out our recommended approach to real option appraisal. The proposed model uses information that might reasonably be available, and is transparent. In this respect it contrasts with some approaches based upon financial options. Moreover the recommended approach should be easy to assimilate because it uses concepts such as decision trees that will already be familiar to appraisers.

Real options and Net Present Value (NPV) are not alternatives in appraisal. NPV when used correctly is always appropriate. In sequential investment decisions, the case where real options arise, expected NPVs must be calculated from the most distant stages in the future (as viewed from the present) before the NPV of current investment can be calculated. Real options analysis emphasizes the value of the retaining flexibility to make investment decisions contingent on the most recent information and reflects it into an NPV calculation.

Closely related to the role of risk in appraisal is the discount rate. Public sector practice does not involve substantially risk adjusting the discount rate because the nature of public sector activity means that risk can be spread over a large number of beneficiaries or diversified across a wide variety of projects. Few private companies can diversify risk across their portfolio of projects in this way. It follows that, to the extent that private sector investment is influenced by risk-adjusted discount rates, it is likely to be short-termist relative to the public sector. The pressing concern for
appraisers of joint or sequentially public and privately funded projects however is that schemes that would be rejected by the private sector might be accepted by the public sector, because of the relatively low discount rate employed. The resulting distortion and its relationship with aspects of additionality and genuine intervention impact lie beyond the scope of this study.
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APPENDIX  Some Pitfalls in Scoring and Weighting

1. Sensitivity of decisions to cardinal and ordinal measures

In the example reported in table 1, two experts (alpha and beta) offer diametrically opposed rankings (ordinal scale measures) of four projects. Assume all projects cost the same. An unweighted sum of their rankings gives each project tied first or last (add columns (1) and (2) to give column (3)).

Suppose now that intensity of preferences are allowed to count and 10 marks can be allocated any way the experts choose (cardinal scale measures are employed). Alpha believes the intervals of superiority between each project in the ranking are equal (column 4). Beta believes project A is massively superior (column 5). This means in the joint ranking expert beta’s choice of A wins (column 7).

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</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>7</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>1.5</td>
<td>3.5</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>0.5</td>
<td>4.5</td>
<td>2</td>
</tr>
</tbody>
</table>

2. Sensitivity of decisions to weighting of project characteristics

Project A makes a large contribution to the landscape but very little to air quality (Table 2). Project D is the opposite. If the weights on the two objectives are equal and the marks are as indicated then the projects are equally attractive or ranked (adding columns (1) and (2)).

When the weights on the two characteristics are not equal then the relative advantages of the projects can shift. Suppose that the landscape characteristic is assigned a weight of 0.4 and air quality 0.6. Given the larger weight on air quality, the aggregate ranking reflects that of the air characteristic when rankings alone are used (column (3)). In this case D is preferred.

<table>
<thead>
<tr>
<th>Project</th>
<th>Characteristic air quality rank (1)</th>
<th>Characteristic landscape rank (2)</th>
<th>Ranks added: 0.6 weight on air quality, 0.4 weight on landscape (3)</th>
<th>Contribution to air quality (4)</th>
<th>Contribution to landscape (5)</th>
<th>Cardinal measure added air wt=0.6 landscape wt=0.4 (6)</th>
<th>Cardinal measure added air wt=0.7 landscape wt=0.3 (7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>1</td>
<td>2.8</td>
<td>1</td>
<td>7</td>
<td>3.4</td>
<td>2.8</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>2</td>
<td>2.6</td>
<td>2</td>
<td>1.5</td>
<td>2.2</td>
<td>1.85</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>3</td>
<td>2.4</td>
<td>3</td>
<td>1</td>
<td>2.2</td>
<td>2.4</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>4</td>
<td>2.2</td>
<td>4</td>
<td>0.5</td>
<td>2.6</td>
<td>2.95</td>
</tr>
</tbody>
</table>
With the same rankings but (cardinal scale measure) marks assigned to project characteristics (columns (4) and (5)), project A (with the poorest air quality contribution) is preferred (has the highest score, column (6)). This is because the magnitude of A’s landscape ‘output’ more than offsets the effect of the weighting in favour of the air characteristic. However with small changes in the weights, to 0.7 for air quality and 0.3 for landscape, the marks give the same preferred option as the ranking; project D achieves the highest score of 2.95 (column (7)).

3. Sensitivity of decisions to valuations of point scores

In the hypothetical case of Table 3 below, five projects each have different present values of costs (column (1)) and benefits as measured by point scores (column (2)). The correct choice of project is not obvious from these data. A basis is required for assuming that the greater benefit scored for C over A (200 compared with 30) does, or does not, more than compensate for the increase in costs (from 10 to 100). Assigning weights to the two characteristics of the projects (PVC and scores) potentially allows a systematic decision.

**Table 3a Hypothetical valuation of scores in choice of options: Project benefit scores and costs**

<table>
<thead>
<tr>
<th>project</th>
<th>PVC (£000)</th>
<th>Score (points)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td>D</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>E</td>
<td>200</td>
<td>230</td>
</tr>
</tbody>
</table>

**Table 3b: Hypothetical valuation of scores in option choice: Benefit (score) weights and net benefits**

<table>
<thead>
<tr>
<th>project</th>
<th>Weight:</th>
<th>1.00</th>
<th>2.00</th>
<th>0.75</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>20</td>
<td>50</td>
<td>12.5</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>100</td>
<td>300</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>100</td>
<td>350</td>
<td>37.5</td>
<td>-25</td>
</tr>
</tbody>
</table>

In table 3, some projects are clearly dominated, (achieve a lower score at higher cost). B is dominated by A and E by D. This permits B and E to be excluded from the choice.

Any of the three remaining projects can be justified, depending on the weight given to the score. If each point is worth the same units as costs (£1000 or weight 1.00) then decision-makers should be indifferent between C and D, so long as both are affordable. If each point is worth one half (that is weight 0.50 or £500) then A is the only project with a positive PV. If each point is assigned a weight of 0.75 or is worth £750, project C should be chosen.