Graph Theoretic Methods for Radio Equipment Selection

A thesis submitted in partial fulfilment of the requirement for the degree of Doctor of Philosophy

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Declaration

This work has not previously been accepted in substance for any degree and is not concurrently submitted in candidature for any degree.

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Abstract

In the 1970s and 1980s, a small group of American engineers recognised the importance of the graph-colouring ideas studied by mathematicians and the potential for these ideas to be used in practical radio frequency assignment procedures. Some groundbreaking work led to a long period of study in academia where many variants on the Frequency Assignment Problem have been considered and some advanced algorithms developed. This thesis has investigated the Frequency Assignment Problem for microwave fixed links and, taking account of the constraints experienced in professional practice, extended this to include the problem of Equipment Selection.

For a particular data-rate, standard radio equipment using relatively lower-or higher-order modulation schemes can be deployed by the fixed link operator. While the higher-order options use less bandwidth, they radiate at higher powers and require more protection in the radio interference environment. That is, they are more potent interferers and present a greater challenge to distant interferers. Therefore, when the assigner's objective is to minimise the span of frequencies used by a network, the higher-order modulation radio is not always the most spectrally efficient. The thesis has hypothesised that by doubling the bandwidth requirement on selected links, the assigner can actually reduce the overall span of frequencies used to support a frequency assignment for the entire network.

With a minimum span objective, fixed link deployment scenarios have been exposed to a standard IP Solver that gives exact solutions. Using graph-theoretic methods, equipment selection heuristics have been developed and tested in offline and online environments. This work has gathered significant evidence in support of the hypothesis.
Acknowledgements

Many thanks to Dr Stuart Allen and Professor Steve Hurley for the excellent supervision and support given throughout this study.
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## Glossary

The glossary focuses on some of the key radio engineering terms and parameters used in professional frequency assignment work and throughout this thesis.

<table>
<thead>
<tr>
<th>Term/parameter</th>
<th>Unit</th>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate interference</td>
<td>dBW</td>
<td>$\Sigma I$</td>
<td>The sum of interfering signal powers incident to a receiver.</td>
</tr>
<tr>
<td>Carrier-to-Noise plus aggregate interference</td>
<td>dB</td>
<td>$C/(N + \Sigma I)$</td>
<td>The ratio of carrier signal power to the sum of noise and aggregate interference.</td>
</tr>
<tr>
<td>Decibel</td>
<td>dB</td>
<td>$dB$</td>
<td>In radio engineering, a logarithmic unit that expresses the ratio of radio signal powers $P_1$ and $P_2$ given by $10 \cdot \log(P_1 / P_2)$.</td>
</tr>
<tr>
<td>dBW</td>
<td>dBW</td>
<td>$dBW$</td>
<td>A measure of radio signal power $P$ referenced to 1 Watt of power given by $10 \cdot \log(P/1)$. Therefore 1 Watt of signal power = 0 dBW.</td>
</tr>
<tr>
<td>dBi</td>
<td>dBi</td>
<td>$dBi$</td>
<td>A measure of antenna gain referenced to a theoretical isotropic radiator.</td>
</tr>
<tr>
<td>EIRP</td>
<td>dBW</td>
<td>$EIRP$</td>
<td>Effective Isotropic Radiated Power. A measure of radiated power referenced to a theoretical isotropic radiator. Specifically, the radiated power required from an isotropic radiator to equal the maximum EIRP delivered by an antenna.</td>
</tr>
<tr>
<td>Excess interference</td>
<td>dB</td>
<td>$e_{uv}$</td>
<td>The amount by which the interference threshold at a receiver $u$ associated with a single source of interference $v$ is breached.</td>
</tr>
<tr>
<td>Fixed link</td>
<td>-</td>
<td>-</td>
<td>A microwave communications link between two fixed points.</td>
</tr>
<tr>
<td>Frequency Division Duplex</td>
<td>-</td>
<td>-</td>
<td>A highly organised radio frequency channel plan supporting two-way communications where GO and RETURN signals are assigned frequencies separated by a constant frequency separation or <em>duplex spacing</em>.</td>
</tr>
<tr>
<td>Term/parameter</td>
<td>Unit</td>
<td>Notation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------------------------</td>
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<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$kTB$</td>
<td>dBW</td>
<td>$kTB$</td>
<td>Thermal noise in a receiver’s bandwidth. The product of Boltzmann’s constant $k$, ambient temperature $T$ and bandwidth $B$.</td>
</tr>
<tr>
<td>Net Filter Discrimination</td>
<td>dB</td>
<td>$NFD$</td>
<td>The discrimination (or advantage) obtained at a receiver, relative to a co-frequency interference scenario, when the interferer is offset in frequency.</td>
</tr>
<tr>
<td>Noise</td>
<td>dBW</td>
<td>$N$</td>
<td>In radio frequency assignment work, the total noise accounted for at a receiver. $N$ is the sum of $kTB$, the Noise Figure of the radio and fixed system losses that can be modelled as noise.</td>
</tr>
<tr>
<td>Noise-interference budget</td>
<td>-</td>
<td>-</td>
<td>A budget setting out the various signal powers and power ratios at a receiver. Can be used to derive a protection ratio.</td>
</tr>
<tr>
<td>Noise-limited frequency assign-</td>
<td>-</td>
<td>-</td>
<td>A frequency assignment method used in professional practice where the interfering signal powers incident to a receiver are maintained at levels below Noise.</td>
</tr>
<tr>
<td>Radiation Pattern Envelope</td>
<td>-</td>
<td>-</td>
<td>A graph, specified in the antenna manufacturing standards, that sets-out the maximum gain of an antenna at angles offset from the main beam.</td>
</tr>
<tr>
<td>Receiver Sensitivity Level</td>
<td>dBW</td>
<td>$RSL$</td>
<td>The smallest radio signal power required at a receiver in order that Quality of Service is maintained. Used in fixed link planning and frequency assignment procedures.</td>
</tr>
<tr>
<td>Single-entry interference thres-</td>
<td>dBW</td>
<td>$I_T$</td>
<td>A threshold at a receiver for the incident interfering signal power sourced from a single interferer.</td>
</tr>
<tr>
<td>Wanted-to-Unwanted ratio</td>
<td>dB</td>
<td>$W/U$</td>
<td>A ratio of wanted signal power to a single source of unwanted (interfering) signal power.</td>
</tr>
</tbody>
</table>

Table 1: Glossary of radio engineering terms and parameters
Chapter 1

Introduction

Historically, microwave fixed links have been used in telecommunications networks to facilitate high-capacity connections between important or remote nodes, for broadcast distribution and access connections direct to customer premises. In modern times they are also deployed in very large numbers to support base-station interconnection in mobile overlay networks. Heavy use within some frequency bands motivates the investigation of spectral efficiency questions.

This thesis aims to close the gap between professional practice and the advanced ideas that have been developed in the academic research. Despite the importance of the problem, coverage in the literature is often limited to more general and abstract problems of frequency assignment, neglecting practical considerations and some important features of fixed links planning. This study extends established graph-theoretic modelling of the frequency assignment problem to take account of equipment selection; a problem often neglected both by the professional frequency assigner and in the academic studies.

The radio equipment used in fixed link networks is highly standardised including the data rates supported. In Europe, for example, manufacturing standards for fixed links are specified by the European Telecommunications Standards Institute (ETSI). Network operators will require a specific data-rate to satisfy the traffic demand between the nodes at either end of the microwave fixed link, which can usually be satisfied by exactly two radio systems from the ETSI specifications, one using a relatively lower-
order modulation scheme and the other a relatively higher-order scheme. This feature allows for a very systematic approach to the equipment selection problem.

While the higher-order modulation option uses less bandwidth and is thus more spectrally efficient in isolation, it is required to radiate at a higher power and demands greater protection in the radio interference environment than the lower-order alternative. Therefore, it may not necessarily be optimal when the spectral efficiency of the entire fixed links network is considered.

With spectral efficiency questions often being posed sharply in relation to congested spectrum, there is a clear need for a detailed understanding of the equipment selection problem and a much closer engagement between professional engineers working in the frequency assignment sub-discipline and academic experts in frequency assignment. This thesis aims to address these questions.

1.1 Contributions to knowledge

The contributions to knowledge made by this study can be summarised as follows:

**Identification of the equipment selection problem.** This study discusses the spectral efficiency of the standard radio systems used on microwave fixed links and the inequalities between these systems in the radio interference environment. The selection of radio equipment for the fixed link is identified as a problem; this is an extension of the established Frequency Assignment Problem.

**Problem formulation.** Extending established graph theoretic methods and analysis, the thesis has set out the first precise mathematical description of the fixed links frequency assignment problem with equipment selection with a minimum span objective. This takes account of practical constraints and regulatory features including, in particular, consideration of real world radio system parameters and the use of highly organised channel plans.
Hypothesis. The thesis sets out a hypothesis in relation to the equipment selection problem. This proposes the existence of a equipment selection paradox which can be summarised as follows: *By doubling the bandwidth on selected fixed links, the overall spectrum required by a network of links can be reduced in practice. Alternatively, a doubling of the bandwidth on selected links can be accommodated, reducing interference in the network without increasing the overall spectrum requirement.*

Exact solutions The equipment selection problem is formulated as an Integer Program (IP), where exact solutions, using a standard IP Solver, provide very strong evidence that improvements in spectral efficiency can be made through equipment selection over a suite of benchmark problems.

Orderings for sequential frequency assignment Consideration of the ordering techniques often applied to the request queue in frequency assignment problems and the multi-raster environment familiar to professional frequency assignment engineers led the study to develop a novel ordering technique that takes account of the practical channel plans used in professional practice. A second novel approach to ordering developed in this study is a variation on the classical Generalised Largest First technique.

Offline heuristics While exact solution techniques support the hypothesis, these are neither tractable nor practical for real world problems, hence the study has also considered the development of heuristic approaches. A detailed analysis of the IP solver’s exact solutions is the basis for this work which, in order to allow for experimentation, is first of all constrained to an offline environment.

Online heuristics Using the experimental results from the development of offline heuristics, online equipment selection heuristics have been developed.

Publications Aspects of the problem formulation and exact solutions reported on in Chapters Three and Four of the thesis are discussed in an article published by
Wireless Personal Communications in 2012:


Material from Chapter Five is discussed in an invited conference paper for the URSI Commission F Triennial, Open Symposium on Radiowave Propagation and Remote Sensing, April 30th - May 3rd, 2013, Ottawa, Canada. Preparations are being made for submission to *Radio Science*:

Background and literature review

This chapter of the thesis discusses the relationship between modulation and spectral efficiency, the application of graph theory to the frequency assignment problem and explains how a frequency assignment service for microwave fixed links works in professional practice. A frequency assignment maps frequencies to radio transmitters whilst ensuring that specific objectives are attained with respect to spectrum usage and that the constraints required to mitigate the effects of interference at radio receivers are respected.

2.1 Modulation schemes and spectral efficiency

The digital modulation scheme used with a microwave radio system places data on the carrier signal and, for some bandwidth, determines the data-rate on the fixed link [1]. Data-rate is defined as the amount of information carried over the fixed link in one second and is normally expressed in Mbit/s. The engineering textbooks and regulatory literature, see [1] and [2] for example, often illustrate this using constellation diagrams as shown in Fig 2.1, explaining that while higher-orders of modulation are able to send more data per Hz of bandwidth, they operate with a shorter decision distance between signal states and so, in order to avoid error, require a greater ratio between the wanted signal and noise. This parameter is generally referred to as the signal-to-noise ratio and denoted by S/N.
2.1 Modulation schemes and spectral efficiency

Figure 2.1: Constellation diagrams for 4-QAM and 16-QAM modulation

When developing the frequency assignment criteria used in assignment procedures, radio engineers will, typically, characterise a radio system with respect to its modulation scheme, designing interference thresholds based on the core $S/N$ [3], [4].

Considering the equipment options set-out in the ETSI standards [5], it is well established that when planning to resolve a specific data-rate, the operator will often have a choice of exactly two radio systems. The lower-order modulation option will support the data-rate using twice the bandwidth of the higher-order radio and, on this basis, it could easily be assumed that the higher-order modulation radio is always more spectrally efficient since a higher data-rate per Hz is possible. Much of the established engineering literature is focused on this measure of spectral efficiency. Freeman [1], for example, sets out the following equation:

$$\eta = \frac{R_b}{W}$$  \hspace{1cm} (2.1)

where $\eta$ is a measure of spectral efficiency, $R_b$ is the bit rate (or data rate) and $W$ is the bandwidth utilised. Clearly, this approach evaluates the operation of the individual radio in relation to the amount of data sent per Hz, a measure sometimes referred to as the transmission efficiency [6]. However there is no consideration given here to questions of frequency re-use or the acquisition of spectrally efficient frequency assignments for the fixed link network. These questions are addressed by very few authors when dis-
cussing the modulation techniques used with fixed links radio systems, however, \cite{6}, \cite{7} and \cite{8} examine the radio system parameters associated with alternative modulation techniques and discuss spectral efficiency. We discuss these papers in more detail here.

Leuenberger \cite{7} notes that higher-order modulation schemes require a higher $S/N$ and, consequently, lower interference thresholds. He goes on to consider the use of higher-order modulation schemes in dense networks, deriving minimum angular spacings for link-ends (antennas) sharing a node. He explains that the higher $S/N$ requirement limits the spectral efficiency gain obtained through the use of higher-order modulation systems and, from a classical radio engineering perspective, that higher-performance antennas should be deployed with these systems; in general, this means antennas with higher gain in the main beam and less gain in the side-lobes (off-axis).

Farrar and Hinkle \cite{6} set out a more detailed discussion of the radio system parameters associated with different modulations schemes and spectral efficiency. They explain that bandwidth, $S/N$, the interference threshold and radiated power level are all functions of the modulation type and that all of these may be traded-off against each other. An interference threshold is a limit applied to the interfering signal power incident to a victim receiver and the radiated power level of a radio system is a measure of the signal power radiating from an antenna (see Chapter Three for a detailed explanation of these parameters). The authors develop an analysis which goes beyond a simple calculation for transmission efficiency. They set out an equation for a Spectrum Conservation Factor:

$$SCF = \frac{VC}{(T.A.B)}$$

where $VC$ is the number of voice channels accommodated by the communications link (at the time that their paper was written, microwave links were mainly used to carry voice traffic in the public telephone networks), $T$ is the proportion of time that the
system is in operation (fixed links transmit data constantly so assume a value of 1), \(A\) is the *denial area* in \(km^2\) and \(B\) is receiver bandwidth.

Evaluation of denial area is a function of *antenna type* as well as of parameters directly associated with the modulation scheme. The parabolic antennas used on fixed links are highly directional and are characterised by a *half-power beamwidth* which is given by the angular distance between points on either side of the antenna’s *main lobe* that are 3 dB lower than the *boresight gain* [9]. However, parabolic antennas will radiate some signal level in all directions and the characterisation of antenna performance is vital when modelling the fixed links interference environment. In his later paper, Hinkle [8] elaborates on this measure of spectral efficiency. He explains that both the amount of spectrum assigned and *spatial denial* are affected by the choice of modulation scheme.

Although Farrar and Hinkle’s work is incorporated in ITU literature [10], suggesting a fairly wide-ranging acceptance of the analysis within sections of the international radio engineering community, this type of paper is rare in modern times and rarely cited in the academic discussions. In general, they are written from a radio engineering perspective and so do not really address spectral efficiency or the *frequency assignment problem (FAP)* as understood by the Computer Science or Mathematics disciplines and defined by [11] and [12], for example. However, they do consider modulation and radio system parameters at the *planning* level and develop an analysis of spectral efficiency, on this basis, that takes the discussion well beyond a simple assessment of transmission efficiency for the *individual* fixed link solution. To a frequency assignment engineer, aiming to conserve spectrum, these papers certainly imply that sophisticated algorithms are required in order that the most spectrally efficient radio systems are selected for any particular deployment scenario. From this perspective, these papers make an important contribution to the discussion.

While papers such as [6], [7] and [8] are rare, there are other investigations that have been undertaken by radio engineers in the past that are even more obscure with respect to the *general availability* of the literature. A particularly notable contribution
2.1 Modulation schemes and spectral efficiency

here is that of Bacon [13] who reports (in a committee paper) on the results of simulations where a large set of 38 GHz fixed links data is exposed to a sequential frequency assignment algorithm with the objective of minimising the span of the assignment. Bacon is focused on the efficiency of the frequency assignment using the conventional low-end packing approach described in [14]. Here, the algorithm assigns the first frequency available, working up-band from the smallest frequency; this has the effect of packing the frequency assignments towards the band’s lower bound. The frequency assignment is run three times, once with lower-order modulation parameters selected for each link in the problem, a second time with higher-order equipment selected and a third time as per the original set of requests made by the fixed link operators (the requests were a mix of relatively lower or higher-order modulation systems). Although the lower-order radios used twice the bandwidth of the higher-order alternatives per link, the ratio of the spans obtained by the two frequency assignments was less than 2:1; in fact, the higher-order solution used 75% of the spectrum required by the lower-order solution. This early study of the problem, again, suggests that careful selection of radio equipment could lead to more spectrally efficient frequency assignments.

A previous work by the author of this thesis [3] investigated the parameters associated with alternative radio systems and the effect of these on interference calculations. Specifically, this work analysed the frequency assignment criteria used by the UK regulator. Further, the loss objective on the interference path, which can be defined as the loss required to satisfy the single-entry interference threshold at the victim receiver, was calculated for different radio system types, highlighting the enormous inequalities present in the radio interference environment when lower and higher-order modulation radios are present. This analysis is discussed in Section 3 of the literature review.

Although there is little in the academic literature that discusses modulation in relation to the FAP, the well understood trade-offs between radio system parameters suggests that links using higher-order modulation radios may, while utilising only half the bandwidth of a lower-order option, actually introduce greater assignment difficulty in some
scenarios. This poses a challenging problem to engineers and scientists aiming to use the spectrum resource more efficiently. Because higher orders of modulation operate with a higher $S/N$, these systems radiate at higher powers and require higher protection ratios and lower interference thresholds relative to lower-order alternatives. Therefore, when running frequency assignment procedures, the higher-order modulation radios will present a greater challenge to distant interferers and receivers than the lower-order systems [3].

2.2 Graph-theoretic methods: early developments

A graph, $G = V, E$, consists of a set of points or vertices denoted by $V$ and a set of lines, or edges, making connections between vertices and denoted by $E$. The number of edges incident with a vertex $v$ determines its degree and this can be denoted by $\text{deg}(v)$. If we label an edge with the value of some constraint, this is said to be its weight and the weighted degree of $v$ is the sum of these weights over all incident edges. These graphs can be used to model scientific and engineering problems [15] including frequency assignment problems.

With graph colouring problems, the objective, for the problem solver, is to assign a colour to each vertex in the graph using the minimum number of colours; vertices that are connected by a graph edge may not be assigned the same colour. The basic principles are set out in vintage papers such as [16] and variants of these classical problems are studied by mathematicians today [17]. See [18] for a comprehensive and contemporary survey of both classical and emerging graph colouring techniques, written from the mathematician’s perspective.
2.2 Graph-theoretic methods: early developments

2.2.1 Pioneering work by American Engineers

Metzger is credited by many, including Zoellner[19], as the first to understand that the graph colouring problems studied by mathematicians were analogous to frequency assignment problems with co-channel constraints. Metzger saw that the vertices of a graph could represent requests for frequency assignment and that the edges could define the co-channel constraint, where vertices connected by an edge cannot be assigned the same frequency. Metzger’s paper is now difficult to obtain but this early work is considered to be groundbreaking and was consolidated and extended by a number of American engineers in the 1970s and 1980s who were motivated by an understanding that spectrum resources were, or soon would be, under pressure and that more spectrally efficient frequency assignment procedures were required.

Zoellner gives a detailed description of Metzger’s work. Ordering frequency assignment requests according to assignment difficulty was already well understood and was established practise. He explains how graph decomposition techniques can be applied to order the vertices of the graph in a request queue. With co-channel problems, the degree of a vertex describes the number of constraints in-play and, on this basis, vertices, or nodes, are removed from the graph and positioned in the ordered list according to their degree, with the vertices of smallest degree being removed first of all. The colouring or frequency assignment procedures are then applied with the vertices of greatest degree (and so, greatest assignment difficulty) first in order. Here, the request queue is in node-degree order. Metzger developed more sophisticated de-composition techniques where sub-graphs of the main graph were identified and all of the vertices in the sub-graph placed on the ordered list (again, in some order). With this approach, the request queue is in node-colouring order.

Metzger set-out three colouring procedures, two of which are closely related: the frequency-exhaustive technique, the requirement-exhaustive technique and the uniform-assignment technique. These three techniques are illustrated in Figure 2.2. With the frequency-exhaustive technique, the assigner works through the set of available fre-
quencies, assigning the first frequency that satisfies constraints to the request under consideration. A slightly alternative approach here is the requirement-exhaustive technique; here, the assigner attempts to assign the first frequency, from the set of available frequencies, to each request in the queue. If some requests remain unsatisfied then the assigner considers the next frequency in the set and so on until the entire request queue is resolved. For a problem with co-channel constraints and with the request queue in the same order for both exercises, the frequency-exhaustive and requirement-exhaustive techniques will produce equivalent results.

The uniform-exhaustive technique attempts an assignment of the least used frequency to each request. This concept demands that the set of frequencies available to the assigner have already been assigned to earlier requests; however, if assignment is not possible from the set of previously assigned frequencies then an unused frequency is added to the pool (this must be the case for the first request considered by the assigner in virgin spectrum).

Again, it is important to note that the colouring techniques set down by Metzger, reflected established frequency assignment procedures already used in professional practice [19]. Metzger’s work allowed for frequency assignment problems to be defined formally in graph-theoretic terms, giving precise mathematical formulations. His node-colouring order brought about progress in the understanding of order in the frequency assignment request queue. The results obtained by Zoellner using these techniques show that node-colouring order gives a significant improvement in spectral efficiency over node-degree order.

Later, in an important and often referenced paper [20], Hale extended the work significantly by developing heuristic principles for frequency assignment. The paper sets out procedures for what are now termed sequential assignment methods [21]. Specifically, Hale set-out three components to the frequency assignment heuristic (figure 2.3) covering methods for ordering the transmitters in advance of channel assignment, methods to select transmitters for assignment and finally, methods for channel assignment.
Hale’s study laid the basis for some really significant effort in academia and, despite the development of sophisticated heuristics, the sequential frequency assignment methods that Hale discusses are still highly relevant today in professional practice. His work is discussed in some detail here.

Hale explains that the order of the requests as read into memory is termed the initial order. The task is to re-order the requests with respect to assignment difficulty, a task given the designation Ordering the Transmitters (OT) by Hale (in general this

Figure 2.2: Metzger’s three frequency assignment techniques
thesis refers to requests but Hale refers to transmitters in place of requests). A simple approach is to arrange the requests in decreasing order by degree, a measure of the number of vertices in the problem requiring co-ordination with the radio under consideration. This is termed the Largest First (LF) method. In graph-theoretic terms, LF orders $V$ in decreasing order of degree. Ties (equalities) are broken by using the initial order i.e. if the degree of $u$, termed $\text{deg}(u)$, is equal to $\text{deg}(v)$ and $u$ appears before $v$ in the initial order then $u$ appears before $v$ in the LF ordering of $V$.

The labels on the edges of the graph are its weight and the largest weight on the subset of edges connecting $u$ to all $v \in V$ is the maximal weight of $u$. The Generalised Largest First (GLF) technique orders the vertices of the graph by weighted degree; that is the sum of weights on edges connecting $u$ to all $v \in V$. Ties may be broken by listing the vertex of largest maximal weight first and then with reference to the initial order.
An alternative approach to ordering the request queue, also given by Hale, is the \textit{Smallest Last (SL)} technique. Here, the degree of each $v \in V$ is calculated, then the vertex of smallest degree is selected and placed in the request queue. The degree of each vertex is re-calculated at each iteration of the SL procedure so giving a different order from LF. Ties are broken using the initial order. With the \textit{Generalised Smallest Last (GSL)} technique, we select the vertex of smallest weighted degree from $V$. Ties are broken by i) selecting the vertex with smallest maximal weight and ii) using initial order.

Hale goes on to discuss methods for \textit{Selecting the Next Transmitter (SNT)} for channel assignment. The simplest approach is termed the \textit{sequential technique} which takes the next object in the list obtained from the OT module.

Saturation degree is defined as a measure of the number of radio channels denied to the candidate transmitter due to co-channel constraints with established assignments. The \textit{Saturation Degree Technique (SATD)} selects the transmitter with maximal saturation degree from the request queue obtained from the OT module; ties are broken with respect to the initial order. Some further refinements of this method were also considered.

Finally, Hale considers three techniques for \textit{Selecting and Assigning a Channel}:

The \textit{Smallest Acceptable (SA)} technique assigns $v$ to the smallest acceptable channel; that is: the first channel to meet the assignment criteria when evaluating the channel-set in ascending order from a start point where the candidate frequency is $f_0$.

With the \textit{Smallest Acceptable Occupied (SAO)} technique, $v$ is assigned the smallest acceptable occupied channel where possible; else the smallest acceptable channel is assigned.

The \textit{Smallest Most Heavily Occupied (SMHO)} technique assigns the smallest most heavily occupied channel that meets the assignment criteria, but if $HOA(v) = 0$, \text{...}
meaning that none of the occupied channels meet the assignment criteria, then the smallest acceptable channel is assigned.

Hale combines the different approaches used in each module to form a set of frequency assignment heuristics. He explains that these are used to tackle problems with the objective of minimising the order, or span of the frequency assignment, or both of these. Hale defines the order of the assignment as the number of channels used in the assignment process (really, equivalent to the chromatic number of a graph) and he explains that span is given by the highest frequency channel used in the assignment (assuming that the lowest frequency channel is channel 1, the extent or span of frequencies used is easily calculated).

Hale’s paper is very focused on the practical problems faced by frequency assignment engineers. His work suggests that while exact solutions are desirable from a research perspective, heuristics are necessary in order that real-world problems can be addressed. Even at this early stage, not all of the focus was on the spectral efficiency gain delivered by the heuristics. Hale discusses the use of these tools and techniques to investigate the relationship between spectral efficiency and equipment characteristics, for example; a topic that this study has returned to.

These pioneering studies by American engineers laid the basis for a more extensive and theoretically developed period of work in academia.

### 2.3 Later developments: Academia and optimisation

Later, the research effort was led by academia and by Mathematics and Computer Science Departments in particular. The academic work has developed these studies significantly. First of all, the FAP has been given a very precise mathematical description on the basis of a graph-theoretic analysis. Some specific problem types with different objectives have been defined in the literature and Mathematical programming
techniques have been used to find optimal solutions and provide benchmark solutions. Finally, highly sophisticated heuristics and meta-heuristics have been developed. Each of these points is elaborated on in the following subsections.

2.3.1 Mathematical description of the FAP

Many authors give a precise mathematical description of the FAP based on a graph-theoretic analysis, see \[12\]. In general, with a graph-theoretic representation of the FAP a constraint graph or interference graph \( G = (V, E) \) is specified. A set of vertices \( V \) represents the radio transmitters, or communications links, in the problem and an edge set \( E \) can be labelled precisely with the channel separation constraints, for example, required between vertices where edge \( uv \) is labelled with the pairwise constraint \( c_{uv} \) associated with vertices \( u \) and \( v \). That is, \( c_{uv} \) is an integer specifying the minimum number of discrete radio channels required to separate the frequency assignments at \( u \) and \( v \) in order that harmful or excessive interference between this pair is avoided. The frequency assignment associated with a vertex \( v \) is termed \( f(v) \) and the set of radio channels (or frequencies) available represented, for example, by \( D = \{0, 1, 2, \ldots K\} \). The frequency assignment method is required to perform a mapping \( f : V \mapsto \{0, 1, 2, \ldots K\} \) such that:

\[
| f(u) - f(v) | \geq c_{uv} \quad (2.3)
\]

is satisfied \( \forall u, v \in V \). \[1\]

\[1\]There is a split in the literature with respect to this labelling of the graph edges; in some cases the constraint is specified such that:

\[
| f(u) - f(v) | > c_{uv}. \quad (2.4)
\]

In this study, the constraints must be resolved according to (2.3).
There are some alternative formulations of the problem described in the literature. With \textit{t-colouring} as described in [20] and [22], for example, frequencies are assigned in order that $|f(u) - f(v)| \neq T_{uv}, \forall uv \in E$ where $T_{uv}$ is a set of \textit{tabu} channel separations for vertices $u$ and $v$. This approach is useful when the assigner is required to consider channels that must be rejected in order to avoid harmful interference from \textit{harmonic products} of the carrier centre frequency, for example. However, with fixed links, the channel plans are specified in order that these effects are mitigated through the use of \textit{duplex sub-bands} and so t-colouring is not the most appropriate method.

### 2.3.2 Problem types

Variants of the FAP with different aims are described in the literature. Koster [23] classifies models for the FAP according to the \textit{objective} as follows:

- **Minimum order frequency assignment**: where the number of frequencies used are minimised (in an interference free environment);

- **Minimum span frequency assignment**: where the difference in frequency between the highest and lowest frequency assignment is minimised (in an interference free environment);

- **Minimum blocking frequency assignment**: where the blocking probability of a network is minimised; this can be defined as a the probability of calls being lost in a telephone network.

- **Minimum interference assignment**: where some measure of the interference present in a network is minimised.

As stated in Hurley \textit{et al} [21], there are only two types of problem that are normally considered \textit{in practice}: the minimum span problem and the \textit{fixed spectrum problem}.
In general, with the minimum span problem, there is no predetermined set of frequencies available to the assigner. Rather, the assigner selects frequencies from a domain represented, for example, by the set of positive integers such that the frequency separation between the maximum frequency used $f_{\text{max}}$ and the minimum frequency used $f_{\text{min}}$ is at a minimum. Therefore $f_{\text{max}} - f_{\text{min}}$ gives the minimum span and this is denoted by $sp(G)$.

With the minimum interference assignment, or fixed spectrum model, the objective is to assign frequencies from a predetermined set of frequencies $\{f_1, f_2, ..., f_N\}$ while minimising some measure of interference. That is, the use of a predetermined frequency domain implies that not all of the constraints associated with a set of requests will, necessarily, be satisfied. Therefore, the objective is to run all requests while minimising interference (for example by satisfying as many constraints as is possible).

In general, the real-world frequency assignment problem for microwave fixed links is often a hybrid of these two models \[24\]. The spectrum is highly organised, often with more than one channel raster (set of frequencies) specified per frequency band and with requests mapped to the appropriate raster according to their bandwidth requirement. The aim is to resolve all requests while minimising span and satisfying all constraints. Generally, in professional practice, a zero-violation assignment is the required standard; requests that fail to satisfy constraints for at least one candidate frequency are simply rejected. Future work could include some investigation of problems where the demand for radio frequency channels where all constraints are satisfied cannot be satisfied.

### 2.4 The development of heuristics

Hale and others were clear that the ordering of requests had a significant impact on spectral efficiency and more recent studies have confirmed this and contributed new ideas and refinements including those completed by Hurley et al \[21\] who set out the
2.4 The development of heuristics

following alternative approaches.

With *Largest First 2 (LF2)* the vertices of the graph are organised in decreasing order by degree. At each iteration of the module, the vertex of greatest degree is placed on the ordered list and then removed, together with associated edges, from the remaining set. The degree of each remaining vertex is then re-calculated and, again, the vertex of greatest degree selected (and placed on the LF2 ordered list).

*Generalised Largest First 2 (GLF2)* follows the method for LF2 but takes account of *weighted degree* at each iteration of the module.

*Size of Domain (SD)* may be used in scenarios where a request \( v_i \) has a specified *frequency domain* \( D_i \). The vertices may be listed in increasing order according to values for \( |D_i| \) for all \( v_i \in V \). This approach allows vertices with fewer options for frequency assignment to be considered ahead of those with a greater number of options. This might be appropriate for fixed link problems where the fixed link radio has a tuning range \( < D \) (the radio is unable to use all of \( D \)) or the operator has specified a *preferred* channel, or channels, in the request.

While Hale developed 13 combinations of OT, SNT and SAC modules, Hurley *et al* discuss the use of 48 possible combinations; they indicate that results for span are problem specific, a heuristic may perform well on one problem but not so well on another and that running many sequential algorithms allows the user to select the best span available. For Hurley *et al* a *zero violation* sequential assignment is a start point for the more advanced minimum span algorithms developed in the 1990s including *tabu search* and *simulated annealing*. These sophisticated *meta-heuristics* are suitable for frequency assignment problems where the entire request queue is apparent at the outset and can be exposed to the meta-heuristic *en bloc*. For real-world problems where a new request queue is specified each working day and each request must coordinate with the established set of assignments with no opportunity to reassign these estab-
lished requests, professional practice continues to makes use of sequential assignment algorithms.

2.5 Mathematical programming techniques

Mathematical programming formulations give a more formal interpretation of the FAP. Integer Programming (IP) is used widely in academia and industry to formulate engineering and operations problems [25]. These IP formulations are highly standardised and, in general, the aim is to find values for decision variables while maximising or minimising an objective function whilst satisfying a set of linear constraints. The program is integer when one or more of the variables must be assigned integer values.

Although programming is used here to mean planning rather than the drafting of software code, IP formulations can be coded and exposed to solvers; these are software tools designed specifically to solve IP formulations of problems. Aadal et al [12] set out an excellent explanation of an IP formulation for the Minimum Span Problem, which is reproduced here.

If $Z_{\text{max}}$ is the largest frequency used and $Z_{\text{min}}$ the smallest, then the objective function is to minimise the difference between this pair while satisfying all frequency assignment requests and all constraints. The highest available frequency is denoted by $f_{\text{max}}$ and chosen to be much larger than needed.

The studies are often focused on problems where more than one frequency assignment is required at the vertices of the graph. This can be modelled using a multiplicity constraint, $m(u)$, where $m(u) = 1$ for problems where a single frequency assignment is required at each $u \in V$.

These frequency assignments are constrained by interference between vertices and penalty matrices, denoted by $P_{uv}$ (e.g. radio interference between $u$ and $v$), are used in
2.5 Mathematical programming techniques

conjunction with a pre-determined threshold, $P_{\text{max}}$, to ensure that the frequency assignments respect pairwise constraints.

Let $f \in F$ denote a frequency from the set of frequencies available to the assigner and let $f \in F(u)$ denote the set of frequencies that can be used with request $u$. Then the decision variables $x_{uf}$ and $y_f$ can be defined as follows:

$$x_{uf} = \begin{cases} 
1 & \text{if } f \in F(u) \text{ is assigned to } u \\
0 & \text{otherwise}
\end{cases}$$

$$y_f = \begin{cases} 
1 & \text{if } f \in F \text{ is used} \\
0 & \text{otherwise}
\end{cases}$$

and a formulation for the minimum span problem can be set out:

$$\text{Min } z_{\text{max}} - z_{\text{min}} \tag{2.5}$$

subject to

$$\sum_{f \in F(u)} x_{uf} = m(u) \quad \forall u \in V \tag{2.6}$$

$$x_{uf} + x_{vg} \leq 1 \quad \forall u, v \in E, f, g \in F, P_{uv} > P_{\text{max}} \tag{2.7}$$

$$z_{\text{max}} \geq fy_f \quad \forall f \in F \tag{2.8}$$

$$z_{\text{min}} \leq fy_f + f_{\text{max}}(1 - y_f) \quad \forall f \in F \tag{2.9}$$

$$x_{uf} \leq y_f \quad \forall u \in V, f \in F(u) \tag{2.10}$$

$$x_{uf} \in \{0, 1\} \quad \forall u \in V, f \in F \tag{2.11}$$

$$y_f \in \{0, 1\} \quad \forall f \in F \tag{2.12}$$

(2.5) is the objective function: to minimise the span of frequencies used. (2.6) ensures that $m(u)$ frequencies are assigned to $u$. (2.7) ensures that, where there is an edge
joining \( u \) and \( v \), frequency assignments are constrained in order that the threshold \( P_{\text{max}} \) is respected. \( x_{uf} + x_{vg} > 1 \) if frequencies \( f \) and \( g \) are assigned to this pair; the constraint blocks at least one of these assignments in cases where \( P_{uv} > P_{\text{max}} \). (2.8) ensures that \( Z_{\text{max}} \) is greater than or equal to \( fy_f \). If \( f \) is unused then \( y_f = 0 \), \( fy_f = 0 \) and \( Z_{\text{max}} > 0 \). If \( f \) is assigned to a vertex then \( y_f = 1 \), \( fy_f = f \) and \( Z_{\text{max}} \geq f \). (2.9) If \( f \) is assigned to a vertex then \( Z_{\text{min}} \leq f \) but if \( f \) is unused then \( fy_f = 0 \) and the constraint reduces to the trivial \( Z_{\text{min}} \leq f_{\text{max}} \). (2.10) If \( x_{uf} = 1 \) then \( y_f = 1 \) and \( x_{uf} = y_f \). If \( x_{uf} = 0 \) (\( f \) is not assigned to \( u \)) and \( y_f = 1 \) (\( f \) is assigned to some other \( u \in V \)) then \( x_{uf} < y_f \). (2.11) and (2.12) specify values of 1 or 0 for the decision variables \( x_{uf} \) and \( y_f \).

2.6 Professional practice

In this section we review the methods used in professional practice with particular focus on the approach taken by Ofcom\(^2\) in the UK.

2.6.1 Some definitions

Some definitions of the radio system and planning parameters used in the frequency assignment process are set-out here. See [26] for a comprehensive description of the noise-limited frequency assignment methodology.

A link-end is one end of a fixed link. Each link-end is associated with an item of radio equipment and an antenna. Because the two link-ends on a link radiate power at different frequencies, the assigner must consider interference sourced from and incident to both link-ends.

If a fixed link is viewed from the perspective of one of its link-ends, the second link-end is often referred to as the distant-end (in classical telecommunications engineering

\(^2\)Office of Communications; the UK regulatory body responsible for radio spectrum management.
2.6 Professional practice

terminology).

A request is an application for frequency assignment defined in technical terms. In a classical engineering model of the radio interference environment, the request is a fixed link with two link-ends. In a graph-theoretic model of the radio interference environment, the request is represented by a vertex of the graph.

The Equivalent Isotropic Radiated Power (EIRP) is the signal power radiated from one end of a fixed link via an antenna and normally expressed in dBW. In the UK, the frequency assignment process includes an EIRP assignment where the minimum EIRP required to resolve the link’s planning objectives is calculated using procedures set out in ITU-R Recommendation P.530 [27]. This is considered to be a spectrally efficient approach, since higher radiated power levels (assigned or selected arbitrarily) would result in lower frequency re-use across the spectrum space.

The following definitions for the various signal levels, including noise, and ratios of these signal levels seen from the perspective of the receiver, are illustrated in Figure 2.4 which, without any loss of generality, sets out a noise-interference budget.

The Receiver Sensitivity Level (RSL), denoted by $R_{\text{ref}}$ in Figure 2.4, specified in dBW, is the minimum wanted signal required at the receiver for normal operations. This value is normally associated with a Bit Error Rate of $10^{-6}$ and is the start point for calculation of EIRP at the distant-end of the fixed link i.e. at the other end of the fixed link.

Fade Margin $M$ is the margin (in dB) added to RSL to give a median wanted signal level $R_{\text{med}}$ (dBW). $M$ protects the fixed link from fading due to precipitation and multi-path effects and there are well established semi-empirical propagation models that allow for a very precise calculation. $M$ is dependent on the operator’s propagation availability requirement, this parameter is denoted here by $A$ and is normally expressed as the percentage of time in a year that the radio link will operate without experiencing outage; 99.99% is a typical availability requirement. In practical terms, this means that the link is planned such that wanted signal levels can be expected to fall below RSL.
2.6 Professional practice

for just 0.01% of time. A Minimum Fade Margin may be used where it is desirable that a minimum $R_{\text{med}}$ value associated with some bit error rate $< 10^{-6}$ is supported during normal operations. Use of a Minimum Fade Margin is appropriate on shorter link paths.

The carrier-to-noise plus aggregate interference ratio $C/(N + \Sigma I)$ is fundamental to the modulation scheme used by the radio system and describes the minimum distance required between wanted signal levels and noise at the receiver for error-free operation. This ratio is more commonly referred to in the literature as signal-to-noise $S/N$ but $C/(N + \Sigma I)$ is a more accurate notation when we account for degradation of noise due to interference. The noise plus aggregate interference threshold is denoted by $N + \Sigma I$.

$kT B$ (dBW) denotes the noise present in the receiver bandwidth where $k$ is Boltzmann’s constant, $T$ is temperature expressed in degrees Kelvin and $B$ is the bandwidth of the radio system. A radio system’s Noise Figure $NF$ is the thermal noise attributable to the radio equipment and fixed system losses $L$ are losses that can be modelled as noise e.g. branching losses (branching allows for signals to be split and directed along alternative waveguide paths). The total noise allowance $N$ is the sum of these contributions.

Degradation of $N$ due to the aggregate effects of interference can be accounted for by

Figure 2.4: Generalised noise-interference budget
setting an interference margin $M_I$; this parameter is often set equal to 1 dB in a noise-limited planning environment. A 1 dB degradation of $N$ accounts for an aggregate interference $\Sigma I$ that is 5.9 dB below $N$.

$I_T$ is the threshold for *single-entry interference* (a single source of interference); this threshold is set according to some assumptions as to the number of equal single-entry interferers contributing to $\Sigma I$.

Finally, the wanted-to-unwanted ratio $W/U$ compares the wanted (desirable) signal and unwanted interference from some distant transmitter in the spectrum space.

### 2.6.2 A strategic approach to frequency assignment

Here, aspects of professional practice in the UK are discussed where, in general, Ofcom (Office of Communications) are responsible for frequency assignments in the fixed link frequency bands.

In most frequency bands, the assigner operates in a multi-raster environment. That is, frequencies are assigned from more than one set of radio channels, each with a different channel spacing but all sharing the same spectrum space. With modern plans, these rasters tend to be subdivided such that one 56 MHz channel corresponds to two 28 MHz channels, four 14 MHz channels and so on, see [28] (used in this study), for example. Assignments on the higher rasters (with larger channel spacings) can often dominate the span of a frequency assignment [13], no matter how much effort is applied to requests on lower rasters.

These channel plans are designed to support *Frequency Division Duplex (FDD)* working where two *duplex sub-bands* are defined. This allows for a *duplex channel* to be assigned to a request such that the two transmitters on a fixed link operate at frequencies with a constant frequency separation known as *duplex spacing*. This mitigates any potential for *self-interference* at a link-end (the radiated signal at a link-end will not cause harmful interference at its own receiver). A hypothetical FDD plan is illustrated
Figure 2.5: Hypothetical FDD plan

in Figure 2.5. This shows how a duplex channel is actually two radio channels separated by the duplex spacing. A Centre Guard Band ensures that the duplex spacing between the complimentary channels, \( f_1 \) and \( f'_1 \), for example, is correct.

Clearly, the use of FDD plans implies that the span of frequencies required to support a network frequency assignment is doubled when both duplex sub bands are considered. If all links are two-way (the link is transmitting and receiving signals at both link-ends) then the span of frequencies utilised in the two duplex sub-bands are exactly equal.

In professional practice, an objective analysis of the fixed links frequency assignment problem, where the assigner sets out to obtain an optimal frequency assignment for the entire set of links, is not possible \[24\]. Rather, the assigner is required to process the request queue one request at a time. The queue is regularly updated with new requests, quite possibly every working day in the more popular frequency bands. The problem is to process the request queue, co-ordinating each candidate link with the established set of frequency assignments. The assigner has no advance knowledge of how the request queue will develop.

Historically, this scenario has resulted in engineers taking a strategic view of the frequency assignment problem rather than focusing on the development of algorithms that deliver optimal solutions. In the UK, a sequential frequency assignment algorithm is used \[4\]. For a request, the algorithm works up-band from the smallest frequency, assigning the first frequency that meets the assignment criteria (the specific conditions
required in the interference environment such that a designated radio channel may be
utilised by the candidate fixed link). This approach corresponds exactly to the *frequency exhaustive technique* set-out by Metzger and discussed in Zoellner’s paper [19]
and has the effect of packing assignments towards the lower bound of the frequency
band; as discussed in Section 2.1 of the literature review, this is sometimes referred to
as *low-end packing* [14].

If this method is followed strictly then, in the earlier stages of band exploitation, the
channels towards the upper bound of the frequency band are subject to lower *reuse* or
are unassigned, so reserving contiguous spectrum for future deployments and possibly
alternative use. Efficient packing increases the possibility that future demand will be
met.

However, this approach is complicated in some bands and with some radio equipment
because of *tuning* issues. The radios may only tune to a *sub-band* of the frequency
band or, with some legacy equipment, may even be tuned to a specific channel. Modern
equipment, however, is easily tunable across a number of frequency bands and
from remote locations; a development that could lead to a more flexible interference
environment in the future.

In order to ensure a fair approach, requests must be processed in the order that they are
received. The software and protocols used in professional practice will normally allow
for a single request to be processed at a time in the frequency band under consideration;
this is in order to avoid *contention* between requests under process.

In effect, the assigner’s objective is to resolve all requests while maximising packing;
that is, to maximise re-use on the frequencies already assigned and minimise the span
of frequencies used. In practice, with the well established frequency bands, it is nearly
always the case that all frequencies are utilised, most likely with higher re-use towards
the band’s lower bound.
2.6 Professional practice

2.6.3 Soft Boundary Frequency Assignment

A variant of the frequency assignment method is Soft Boundary Frequency Assignment (SBFA) used by Ofcom in the 18 GHz frequency band [3], [4]. Here, the objective is to allow distinct communities of frequency assignments to develop in the spectrum space. Specifically, the aim is to mitigate the inequalities that exist between lower and higher-order modulation radio systems by maintaining a frequency separation between them.

The sequential algorithm has alternative start points in the band and works up-band or down-band in order that the search for an assignable channel facilitates the separation, in frequency, of radio system types. The method is Soft Boundary because there is no formal segmentation of the band; that is, there is an expectation that the communities of links will blend at some point.

If an abstract theoretical interference path is considered where an interferer \( u \) with radio equipment \( s \) selected must coordinate with a victim \( v \) using radio equipment \( t \), then a loss objective on the interference path can be determined using:

\[
L_{\text{obj}}(u,v,s,t) = RSL(u) - I_T(t).
\]  

This equation delivers a loss objective based on radio system parameters, neglecting real-world planning considerations. In the frequency assignment procedure, the EIRP required at the link-end is calculated using the RSL at the distant-end of the link as a start point and taking account of all gains and losses on the wanted path. On this basis, the interferer is characterised here using the receiver parameter RSL. When calculating interference, the key parameter at the victim receiver is the single-entry interference threshold \( I_T \) and this is the parameter used to characterise the victim receiver here.

Using this approach, the somewhat abstract loss objective \( L_{\text{obj}}(u,v,s,t) \) can be determined for any pair of radio system types at the two ends of a theoretical interference path and
the difference between any pair easily be calculated using:

$$\Delta L_{obj} = |L_{obj}(u,v_{s,t}) - L_{obj}(u,v_{s,s})|.$$  \hfill (2.14)

While this level of abstraction means that $L_{obj}(u,v_{s,t})$ is unrealistic, the value $\Delta L_{obj}$ is a precise measure of inequality between radio system parameters and this is unaffected if real-world planning considerations are accounted for.

Applying this analysis to a model where a radio system is paired with all other radio system types, the hardest objective for each radio system type on the theoretical interference path can be calculated. It is then apparent that separation of system types in the spectrum space allows for a relaxation of the hardest objective for a sub-set of radio systems.

This approach is relevant to the study, since the fundamental question is one of inequalities between radio system types and the effect on frequency assignment difficulty. SBFA aims to mitigate inequalities in the spectrum space and while Ofcom has formalised this strategy, the approach was well understood by some radio engineers beforehand and SBFA was inspired by a variant of this procedure that had been used by British Telecom’s radio engineers previously.

### 2.6.4 Frequency assignment criteria

Each request is assigned a frequency when a predetermined set of conditions in the interference environment are satisfied. Specifically, frequency assignment criteria that protects receivers from excess interference are satisfied. In the UK, a classical noise-limited frequency assignment methodology is employed [4].

Here a derivation of the wanted-to-unwanted $W/U$ ratio specified in the frequency assignment criteria is given. The diagram in Figure 2.6 is an example of a noise-interference budget that illustrates the method. Fixed link radio systems can be de-
A noise-interference budget can be constructed for a particular radio system by first of all sourcing RSL and $C/(N + \Sigma I)$ from the regulatory literature [2], [5]. Subtracting $C/(N + \Sigma I)$ from RSL gives a value for $N + \Sigma I$. In general, a 1 dB interference margin, $M_I$ is used and subtracting $M_I$ from $N + \Sigma I$ gives the total Noise allowance $N$. Figure 2.6 shows how $N$ accommodates the Noise Figure $NF$ of the radio and fixed system losses $L$, in addition to the fundamental noise level in the the radio channel $kTB$. When $N$ is subtracted from $N + \Sigma I$, a value for $\Sigma I$ is obtained:

$$\Sigma I = 10\log\left(10^{(N+\Sigma I)/10} - 10^{(N/10)}\right).$$

(2.15)

The value $n$ is set such that a specified number of single-entry interferers contribute to $\Sigma I$ and then $I_T$ may be calculated using:

$$I_T = 10\log\left(\frac{10^{\Sigma I/10}}{n}\right).$$

(2.16)
In the UK, Ofcom has recently revised all of the frequency assignment criteria, setting \( n = 4 \) for all fixed link radio system types [4].

The wanted-to-unwanted ratio \( W/U \) is obtained by \( RSL - I_T \). This is the ratio, in decibels, of the wanted signal carrier, represented here in its fully-faded condition by \( RSL \), to single-entry interference.

The derived \( W/U \) values are used in subsequent frequency assignment procedures. Modelling interference from a request to each transceiver in the interference environment and \textit{vice versa}; a \( W/U \) must be satisfied at each receiver in the interference environment in order for the frequency under test to be assigned to the request.

Ofcom specify two interference tests, for long- and short-term interference scenarios. The first test, \( W/U \) Test 1, models long-term interference and \( W/U \) is applied to the \( RSL \) value as indicated in the derivation. Here the median interferer incident to the victim receiver is modelled using ITU-R Recommendation P.452 [29]; that is, the interfering signal level exceeded for 50% of time. This interfering signal level must be at least \( W/U \) below the fully faded wanted signal, represented by \( RSL \).

In \( W/U \) Test 2, the short-term interference is modelled and the enhanced interferer must be at least \( W/U \) below the median wanted signal; this is the unfaded wanted signal level given by \( RSL + M \) and the enhanced interferer is the signal exceeded for \( p\% \) of time where \( p = 100 - A \) at the victim receiver.

It is well understood that interference has an aggregate effect at the victim receiver [30]. However, modelling aggregate interference is complex and the reasoning behind the classical noise-limited approach or any method where the single-entry interferer is considered, is that individual sources of interference are maintained at or below thresholds such that the aggregate effects are acceptable [31].

There is some discussion in the literature of \textit{interference limited} frequency assignment criteria [32], [26]. An interference limited approach is characterised by \( M_I > 3 \text{ dB} \), meaning that \( \Sigma I > N \). If \( M_I \) is increased, a higher-level of interference is acceptable.
at the victim receiver, so reducing $W/U$. However, each increase in $M_I$ increases $RSL$ by an equal amount, since the core $C/(N + \Sigma I)$ must be maintained.

This study has adopted the single-entry interference thresholds associated with $W/U$ Test 1. Because these investigations are abstracted from real-world problems and are entirely focused on the mitigation of inequalities in the radio interference environment using graph-theoretic methods, the study has not set out to reproduce all of the tools or methods used in professional practice (or in a particular frequency assignment system).

The definitions and explanations here are the basis for an analysis of inequalities between radio system types and the development of algorithms that will select equipment in a spectrally efficient manner.

### 2.6.5 Improving spectral efficiency in professional practice

The frequency assignment procedures used in professional practice are underpinned by classical radio engineering and the efforts applied to improving spectral efficiency can be divided into three areas.

In one area, design and manufacturing effort is being applied to the development of radio equipment with ever higher orders of modulation and so, transmission efficiency; however, there is very little contemporary discussion of the trade-off between these higher-orders of modulation and assignment difficulty. Efforts are also applied to the design of more efficient, higher-performance, parabolic antennas.

The second area is propagation modelling. Here, considerable expertise is employed in the development of radio propagation models. Through refinement of ITU-R Recommendation P.530, planning engineers are able, with some confidence, to run procedures that calculate the minimum $EIRP$ required to resolve the fixed link. This means that radiated power can be limited to the levels necessary for successful radio communications, without the use of excessive margins. Likewise, the efforts that are applied to the development of ITU-R Recommendation P.452 allow for signal losses to be modelled
on the radio interference path much greater than those given by the free space path loss model. This allows for the modelling of non-conservative interfering signal levels incident to the victim receiver, so increasing the possibility that $W/U$ will be satisfied.

A third area is the development of spectrally efficient frequency assignment criteria. Here, efforts have been made to study and implement new criteria and this work continues.

These efforts by professional radio engineers are absolutely necessary as the communications environment moves towards a scenario where radio solutions are ubiquitous, but there is very clearly a gap in the contemporary literature. While academia has developed highly sophisticated algorithms in the recent period and an extensive literature on the FAP, there has been very little input or engagement with these ideas by the professional radio engineering community.
Chapter 3

The Fixed Links Frequency Assignment Problem with Equipment Selection

This chapter formulates a mathematical description of the fixed links frequency assignment problem with equipment selection. This includes consideration of practical engineering constraints, an analysis grounded in graph theory and the formulation of an Integer Program for the minimum span problem for fixed links accommodating radio equipment selection.

3.1 Problem formulation

Using a graph theoretic approach, it is possible to model fixed links as a set of vertices in a constraint graph where the potential for interference between these links is represented by the weighted edges of the graph. For the assigner, a vertex is an object in the request queue. Each vertex will be required to carry telecommunications between its link-ends and the data-rate specified at a vertex determines which radio equipment options and frequencies are available to the assigner.
3.1 Problem formulation

3.1.1 Radio Frequency Channels and Radio System Types

**Definition 1.** Let $G = (V, E)$ be a constraint graph. Let $V = \{v_1, v_2, \ldots, v_N\}$ represent a set of fixed link frequency assignment requests, where each request $v \in V$ has a data rate requirement $d(v)$ defined. Let $E$ be the set of weighted edges representing interference paths between requests.

A **frequency assignment** is a mapping of the request queue to the set of frequencies available.

**Definition 2.** Let $F$ represent the total set of channels available to the network. Then a frequency assignment for the entire request queue is a mapping: $f : V \mapsto F$.

Frequency and channel are often used interchangeably by the assigner but radio channels can vary in bandwidth and a precise definition of the channel is given here.

**Definition 3.** For a given radio frequency channel with carrier centre frequency $f$, let $f^{\text{max}}$ denote the upper limit of the radio frequency channel (in MHz) and let $f^{\text{min}}$ denote the lower limit. Then for radio frequency channels of bandwidth $b$ MHz:

$$f^{\text{max}} = f + \frac{b}{2}, \quad (3.1)$$

$$f^{\text{min}} = f - \frac{b}{2}, \quad (3.2)$$

There is often reference to scenarios where radios are co-frequency or co-channel; these terms are also somewhat interchangeable but a precise definition is useful.

**Definition 4.** Let a request $u$ be assigned a frequency $f(u)$ and let a request $v$ be assigned a frequency $f(v)$. Then $u$ and $v$ are co-frequency when $f(u) = f(v)$. Then if $u$ also operates in a radio channel with a bandwidth $b(u)$ and $v$ operates in channel with bandwidth $b(v)$ and $b(u) = b(v)$ then $u$ and $v$ are co-channel.
3.1 Problem formulation

In general, frequency assignments for fixed links are made using highly structured channel plans. A raster is a set of radio frequency channels with a common bandwidth $b$. Modern channel plans will often specify more than one available raster and, when this is the case, the assigner may be said to operate in a multi-raster environment.

The approach taken by the study to using channel plans is consistent with ITU literature and professional practice. Without any loss of generality, the study has considered the appropriate ITU structured channel plan in this study. Access to a specific raster is determined by the data-rate and radio system specified for the fixed link. That is, the data-rate on the link and the modulation scheme used by the radio equipment determine a bandwidth requirement that can be satisfied by exactly one raster.

**Definition 5.** Let $r_i = \{f_1^b, f_2^b, \ldots, f_n^b\}$ denote a raster of radio frequency channels with bandwidth $b_i$. Let a channel plan be an arrangement of $n$ channel rasters $\{r_1, r_2, \ldots, r_n\}$ in the spectrum space such that $b_i = 2 \cdot b_{(i-1)}$ for $i = \{2, 3, \ldots, n\}$.

Figure 3.1 shows a schematic for a hypothetical subdivided channel plan showing just one duplex sub-band for illustration. This plan supports four channel rasters $\{r_1, r_2, \ldots, r_4\}$ where each raster $r_i$ is a set of radio frequency channels defined by their carrier centre frequencies. The bandwidth of a channel on raster $r_i$ is indicated and is denoted by $b_i$. The arrows in the schematic indicate the position of the carrier centre frequency $f$ for each radio channel. Clearly, the radio channels are clearly defined by lower and upper limits.

An important observation from Figure 3.1 is that radio systems operating on alternative rasters cannot be co-frequency; there is a minimum separation between the carrier frequencies $f$ and $g$ of systems operating on rasters $r_i$ and $r_j$ when $b_i > b_j$. Then the minimum frequency separation between these assignments is given by:

$$\text{Min}_{f \in r_i, g \in r_j} |f - g| = b_j / 2,$$

(3.3)
3.1 Problem formulation

while for radios systems operating on the same raster:

$$\min_{f,g \in r_i} |f - g| = 0.$$  \hfill (3.4)

When requests on alternative rasters are subject to the minimum frequency separation given by (3.3), we say that they are near co-channel.

**Definition 6.** Let $u$ be a request operating on raster $r_i$, i.e. $f(u) \in r_i$ and $v$ a request operating on raster $r_j$ (where $f(v) \in r_j$). Then if $b_i > b_j$ and $|f(u) - f(v)| = b_j/2$ then $u$ and $v$ are near co-channel.

In this formulation of the fixed links frequency assignment problem, it is assumed that $d(v)$ can be resolved by exactly two radio systems, one using a relatively lower- and one a relatively higher-order modulation scheme; these two systems operate on adjacent rasters. The more general case (where more than two radio systems are available) is a trivial extension.

**Definition 7.** Let $S = \{s_1, s_2, \ldots, s_m\}$ be the set of radio systems available, where system $s_i$ supports a data rate $D(s_i)$ and $F^s_i$ is the set of radio frequency channels that...
can be used with system $s_i$, with each channel defined by its carrier centre frequency, $f$. Hence $F = F^{s_1} \cup F^{s_2} \cup \cdots \cup F^{s_m}$ represents the total set of channels available to the network.

Let $F(v) \subseteq F$ be the set of channels that can be assigned to link $v$ in order to satisfy the required data rate $d(v)$, such that:

$$F(v) = \bigcup_{s \in S \atop D(s) = d(v)} F^s.$$  \hspace{1cm} (3.5)$$

In this study the request’s data-rate requirement is constrained to be satisfied exactly but other formulations of the problem could be investigated. It would be possible, for example, to assign any radio system to a request that either satisfied the data-rate requirement exactly or exceeded this such that:

$$F(v) = \bigcup_{s \in S \atop D(s) \geq d(v)} F^s.$$  \hspace{1cm} (3.6)$$

However, this is not considered viable from the spectral efficiency point of view because systems able to support larger data-rates would require larger $C/(N + \Sigma I)$ ratios and so larger $RSL$ and lower $I_T$ values. They would be more potent interferers and would present a greater challenge to distant interferers.

3.1.2 Constraint Generation

In common with the majority of the frequency assignment literature, see [12] and [23] for example, the potential for links to cause interference is expressed through pairwise separation constraints, however here they also need to take into account the modulation used.
Definition 8. For a pair of fixed links \( u, v \in V \) making use of radio systems \( s, t \in S \), let \( C_{s,t}^{u,v} \) (in MHz) denote the smallest value such that if \( u \) is assigned a channel with centre frequency \( f \in F_s \) and \( v \) is assigned a channel with centre frequency \( g \in F_t \) and 

\[
|f - g| \geq C_{s,t}^{u,v},
\]

then the interference between the two links is acceptable in both directions and at all receivers.

These constraints can be generated in a variety of ways to reflect different published assignment criteria such as those used by Ofcom in the UK [4]. Without any loss of generality, a technique can be used that requires the calculation of excess interference for each pair of requests with the assumption that they are tuned co-channel or near co-channel. These excess interference values can then be mapped to pairwise separation constraints. Since there are two possible equipment selections for each request, each edge in the graph will require four such constraints. Sections 3.1.3 and 3.1.4 specify how these constraints are generated in this thesis using conventional spectrum engineering techniques.

### 3.1.3 Calculating excess interference

In professional frequency assignment procedures, excess interference is the proportion of interfering signal power incident to a receiver that is above the interference threshold specified for the receiver.

Definition 9. Excess interference, denoted by \( e_{uv} \) and expressed in dBW, is defined as the difference between the calculated interfering signal power \( I_{uv}^C \) incident to a link-end of request \( u \) sourced from a link-end at interferer \( v \) and the single-entry interference
3.1 Problem formulation

threshold $I_T(s)$ specified at $u$ and associated with radio system $s$, where:

$$I_{uv}^C = EIRP_v - D(\theta)_v - 20 \cdot \log(4\pi r/\lambda) + G_u - D(\theta)_u,$$

and $EIRP_v$ is radiated power at the interference source, $D(\theta)_v$ and $D(\theta)_u$ are the antenna discrimination available at the two ends of the interference path and $G_u$ is the boresight gain at the victim antenna.

Then $e_{uv}$ is given by:

$$e_{uv} = I_T(s) - I_{uv}^C. \quad (3.9)$$

In order to determine values for $e_{uv}$ at each edge-end in $G$, the Equivalent Isotropic Radiated Power (EIRP) required at the two link-ends on each $v \in V$ is calculated.

The Receiver Sensitivity Level (RSL) at a receiver is the minimum wanted signal level required to support a specified Bit Error Rate (BER); often an RSL for BER = $10^{-6}$ is used in practical planning applications.

**Definition 10.** Let $RSL(s)$ be the receiver sensitivity level associated with radio system $s$. Using $RSL(s)$ at the distant-end of the link as a start point, calculate the EIRP required at each link-end, for all of the radio equipment types available:

$$EIRP = RSL(s)_{\text{distant-end}} - G_u + 20 \cdot \log(4\pi r/\lambda) + M, \quad (3.10)$$

where $G_u$ is the antenna gain available at the distant-end of the link, the log term describes free space path loss, $r$ is path length, $\lambda$ is the wavelength of the radio signal and $M$ is the fade margin required to combat signal attenuation due to rain and atmospheric losses.
Once the EIRP values are determined for each link-end in the problem, the interference incident to a link-end from each link-end transmitting in the same duplex sub-band is calculated using (3.8). Figure 3.2 shows a simple interference scenario where a link-end at $u$ is a victim of interference sourced from a link-end at $v$.

Table 3.1 sets out a calculation for the signal on the interference path. Here, the interferer radiates at -40.43 dBW on boresight (delivering a wanted signal to the distant end of its own path). Antenna discrimination at the interference source $D(\theta)_v$ reduces the signal power incident to the interference path by 10 dB. Assuming a line-of-sight interference path of 0.7 km and an operational frequency of 38 GHz, a value of 120.94 dB free space path loss is calculated. The victim link-end at $u$ has an antenna gain of 40 dB on boresight which can be adjusted by taking account of the antenna discrimination $D(\theta)_u$ of 17 dB available at the incident angle. The sum of interfering signal power incident to the link-end at $u$ is -148.36 dBW.

Fixed links are normally two-way connections and on this basis fixed service channel plans specify duplex radio channels; [28] details the 38 GHz plan used by the telecommunications industry throughout Europe and in our study, supporting Frequency
3.1 Problem formulation

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</tr>
<tr>
<td>$I_{uv}^C$</td>
<td>dBW</td>
<td>-148.36</td>
</tr>
</tbody>
</table>

Table 3.1: Interference calculations

Division Duplex (FDD) working. With FDD, the transmitter at one end of the microwave link uses a frequency from a lower duplex channel set while the transmitter at the distant end of the link utilises the complementary channel from an upper duplex set, or vice versa. Therefore, each raster on the channel plan has two duplex sub-bands.

**Definition 11.** Let $F_{\text{sub-low}}$ denote a lower-duplex sub-band on some channel raster and let $F_{\text{sub-upper}}$ denote the upper-duplex sub-band. Then, if $f_n \in F_{\text{sub-low}}$ is a frequency in the lower sub-band and $f_{n'} \in F_{\text{sub-upper}}$ is the complimentary channel in the upper-duplex sub-band, the duplex spacing $S_d$ expressed in MHz can be calculated using:

$$S_d = |f_n - f_{n'}|,$$

(3.11)

for any duplex channel on the raster.

The duplex spacing ensures that a receiver tuned to $f_n$ and co-sited with a transmitter radiating at $f_{n'}$ will not be a victim of harmful interference.
Figure 3.3: Interference between fixed links $u$ and $v$

Figure 3.3 shows a diagram where the solid lines represent fixed links and the dashed lines illustrate interference paths between link-ends with the arrows pointing away from the source of interference.

Request $v$ is the source of two interferers incident to $u$ and vice versa; on this basis the magnitude of excess interference incident to $u$ via both interference paths is calculated when $u$ and $v$ are tuned co-channel or near co-channel if operating on alternative rasters. The worst-case interference sourced from $v$ and incident to $u$ is calculated.

**Definition 12.** Let the link-ends on requests $u$ and $v$ be denoted by $u_a, u_b, v_a, v_b$. If $f$ is the frequency assigned to $u$ and $g$ is the frequency assigned to $v$, let $|f - g|$ be minimal and let the excess interference at these link-ends be calculated using equation (8) and be denoted by $e_{u_a,v_b}$, $e_{u_a,v_b}$, $e_{u_b,v_a}$ and $e_{u_b,v_a}$ with arrows pointing away from the source of interference and towards the victim link-end. Let the worst-case interference incident to $u$ and sourced from $v$ be determined using:

$$
e_{uv} = \text{Max}(e_{u_a,v_b}, e_{u_b,v_a}).$$

(3.12)
and similarly

\[ e_{uv} = \text{Max}(e_{\rightarrow uv}, e_{\leftarrow uv}). \]  

(3.13)

In this model of the radio interference environment, interference between every pair of requests is calculated using this approach, obtaining the most potent single-entry interference incident to \( u \) and sourced from \( v \), for all \( v \in V \) that are constrained with \( u \).

### 3.1.4 Calculating the frequency separation constraint

Having obtained \( e_{\rightarrow uv} \) and \( e_{\leftarrow uv} \) for co-channel and near co-channel scenarios, the minimum frequency separation required between frequency assignments for \( u \) and \( v \) can be calculated using a conventional engineering approach described here. The aim is to arrive at a single worst-case value that can be utilised in the frequency assignment procedures, where just one constraint is required for each edge \( uv \in E \), ensuring that all excess interference is accounted for when these constraints are satisfied.

**Net Filter Discrimination (NFD)** is the discrimination available at a victim receiver when the interfering transmitter is offset in frequency. The interferer is characterised using the Out-of-Band (OOB) masks specified in the ETSI equipment standards [5] and there are well established methods for deriving the receiver characteristic [33]. These OOB masks describe the performance of the transmitter and receiver at frequencies adjacent to the carrier. ETSI define OOB masks that extend two and a half times the bandwidth of the radio system, either side of the carrier.

Figure 3.4 shows an example OOB transmitter mask from the ETSI equipment standard and a derived OOB receiver mask. The masks are defined in the frequency domain and the x-axis shows a measure of frequency offset (in MHz) from the carrier frequency at the very centre of the mask. These example masks are for a 34 in 14 radio system.
and the OOB domain extends 35 MHz (two and a half times the channel bandwidth) either side of the carrier. The y-axis denotes attenuation in dB relative to carrier signal power. Therefore the mask segment that extends over the bandwidth of the 34 in 14 radio system has zero attenuation; as the mask extends into adjacent channels, really significant attenuation of signal power is observed until, finally, the mask levels-off to a constant noise level.

The receiver OOB masks are not specified in the manufacturing standards but a simple derivation is possible where the first significant roll-off on the transmitter OOB mask is extended to the noise floor; otherwise, the receiver mask is identical to the transmitter mask. In practical calculations, often the transmitter mask is used to model both transmitter and receiver since these adjustments have a marginal effect.

The transmitter masks are specified by ETSI in order that emissions in the frequency domain are highly regulated and these masks, together with the derived receiver mask can be used by assigners to calculate NFD without any loss of generality (that is, without any knowledge of specific deployments or radiated power levels).

**Definition 13.** Let $f$ be the carrier centre frequency of a radio frequency channel and let $b$ be the bandwidth in MHz of the system. Then let the extent of the radio system’s Out-of-Band frequency domain be defined by lower and upper bounds:

\[
 f_{\text{lower OOB}} = f - 2.5 \cdot b, \quad (3.14)
\]

\[
 f_{\text{upper OOB}} = f + 2.5 \cdot b. \quad (3.15)
\]

The OOB masks can be sampled using numerical methods where contributions from both masks are summed when convoluted in the frequency domain. The convolution is performed using discrete frequency separations between the carrier centre frequencies positioned at the centre of each mask and each of these is a step in the convolution
process. At each step, the assigner obtains a ratio of the sum of contributions in a co-frequency scenario and in an offset scenario. It should be noted that the co-frequency configuration will not respect raster organisation when the interferer and victim operate on alternative rasters but this reference point is required to calculate the NFD available from the discreet frequency offsets specified in the channel plan.

**Definition 14.** Let $R_i$ denote the $i$th sample from a victim receiver’s OOB mask and let $T_i^{co}$ denote the $i$th sample of the unwanted transmitter’s OOB mask tuned co-frequency with the receiver. Let $T_i^{(co-k\cdot s)}$ denote a sample from the interferer’s OOB mask incident to the receiver’s OOB domain when the interferer is offset $k$ steps. Let $s$ denote the step size between the carrier centre frequencies of the interferer and victim. If the interferer has a bandwidth $b_p$ and the victim has a bandwidth $b_q$ and $p \neq q$:

$$s = \begin{cases} \frac{\text{Min}(b_p, b_q)}{2} & \text{if } k = 0 \\ \text{Min}(b_p, b_q) & \text{otherwise} \end{cases}$$
and if both interferer and victim operate on a raster \( r_p \):

\[
 s = \begin{cases} 
 0 & \text{if } k = 0 \\
 b_p & \text{otherwise}
\end{cases}
\]

\( k \) is a function of the two OOB masks and the number of steps required to achieve a complete separation of these masks in the frequency domain. NFD is calculated for \( k = 0, 1, ..., K \) steps and \( i = 1, 2, ..., n \) samples using:

\[
 NFD_k = 10 \cdot \log \left[ \frac{\sum_{i=1}^{n} 10^{(T_{co} i + R_i)}/10}{\sum_{i=1}^{n} 10^{(T_i(co-k-s)+R_i)}/10} \right] 
\] (3.16)

Figure 3.5 shows a receiver OOB mask and a transmitter OOB mask located in the receiver’s first adjacent channel. Here, it can be seen that NFD is very significant because only a segment of the interferer’s mask falls within the bandwidth of the victim receiver. In fact, 19.28 dB of NFD is obtained in this scenario.

In a practical implementation a simple Bandwidth Correction Factor might also be specified, allowing for the bandwidth advantage (discrimination) to be modelled at a receiver \( b_p \) that is a victim of interference sourced from an interferer with bandwidth \( b_q \) and \( q > p \).

Having constructed a look-up table of the NFD available for each non-trivial discrete separation in frequency between \( u \) and \( v \), the excess values are mapped to channel separation constraints for each pair of radio system types covering scenarios where each radio system is considered both as a victim and as an interferer.

**Definition 15.** Let the NFD available at step \( k \) in the calculation process be denoted by \( NFD_k \). For each step \( k = \{0, 1, ..., K\} \) let \( k \) be the designation for a bin of
bandwidth $b$. Let each bin be defined by an interval such that a one-to-one mapping of $-e_{uv} \mapsto NFD$ is possible and the intervals are given by:

$$[0, NFD_k], \text{ when } k = 0$$

(3.17)

$$\{NFD_{k-1}, NFD_k\}, \text{ when } k > 0.$$  

(3.18)

Therefore, $e_{uv}$ sits within a bin $k$ of bandwidth $b$ which is easily mapped to a channel separation constraint expressed in MHz, since, for a scenario where $b_p = b_q$:

$$C_{s,t}^{u,v} = k \cdot b_q,$$

(3.19)
3.1 Problem formulation

Figure 3.6: Required channel separations as a function of excess interference for $s = 8 \text{ in } 3.5$.

and for a like-to-unlike scenario where $b_p \neq b_q$ and $p > q$

$$C_{s,t}^{uv} = k \cdot b_q + b_q/2. \quad (3.20)$$

Figure 3.6 shows an example graph of $C_{s,t}^{uv}$ as a function of $e_{uv}$. Using the syntax $M\text{bit/s in MHz}$ to describe the data-rate and bandwidth of the radio system types for two scenarios:

- a like-to-like case where the wanted and unwanted systems, $s$ and $t$, are both $8 \text{ in } 3.5$ (higher-order modulation)
- a like-to-unlike case where the wanted system, $s$, is $8 \text{ in } 3.5$ (higher-order modulation) and the unwanted, $t$, is $8 \text{ in } 7$ (lower-order modulation).

It can be seen that the two functions depicted in the graph are monotonically non-decreasing and that non-uniform gaps persist between them. The non-decreasing features of each function indicate the range of excess values accommodated by bin $k$ in the
convolution process. The slopes on each function are the jump from one discreet channel separation to the next; these always extend exactly 3.5 MHz in the y-axis except where the like-to-unlike systems are near co-channel; in this case a minimum channel separation constraint of 1.75 MHz applies. The non-uniform separation between functions is explained by the different shapes of the spectrum masks used in the numerical analysis. When the wanted system $s = 8$ in 3.5 and the unwanted system $t = 8$ in 7, around 3 dB of excess interference can be accommodated by the minimum frequency separation between the carriers. For higher excess values, the like-to-unlike problem demands a greater frequency separation than the like-to-like.

The discipline of the duplex channel plan ensures that $f(u_b) = f(u_a) + S_d$ and when the frequency assignment for the most potent interference scenario between $u$ and $v$ satisfies $|f(u_a) - f(u_b)| \geq C_{s,t}^{u,v}$, then all excess interference incident to $u$ and sourced from $v$ is resolved.

This model of radio interference incident to a link-end is specific to the geometry, radiated power, equipment type and antenna discrimination in play. Therefore, the interference incident to two links $u$ and $v$, located at either end of an interference path, is likely to be asymmetric. On this basis, when constructing a matrix of channel separation constraints required for the frequency assignment process, the worst-case value for element $C_{s,t}^{u,v}$ is selected for each pair of vertices. Satisfying these frequency separation constraints when running the frequency assignment procedures allows for all of the excess interference identified in the tuned network to be mitigated; that is, the assigner can achieve a zero violation frequency assignment.

### 3.1.5 Optimisation Problem

The optimisation problem considered in this thesis is to assign frequencies (with an implied choice of radio system) to each link request which satisfy all of the interference constraints while minimising the span of the assignment.
FAP with equipment selection is modelled as a minimum span problem while recognising that, in contrast to established academic work, the assigner is constrained by a frequency plan with fixed sets of radio channels. That is, aspects of the fixed spectrum or minimum interference frequency assignment problem are also considered. While the IP formulation respects the discipline of the 38 GHz multi-raster channel plan, it is not necessary to constrain the IP solver to a specific range of frequencies. The solver may locate the solution anywhere in the frequency domain while respecting the frequency separation constraints associated with the channel plan and interference analysis.

With the minimum interference frequency assignment problem, a frequency is assigned to each request while minimising the sum of penalties associated with excess interference and a fixed set of frequencies is available to the assigner when considering a request. If $P_{vwfg}$ is the penalty associated with excess interference on edge $vw \in E$ when frequencies $f \mapsto v$ and $g \mapsto w$ and $x_{vf}, x_{wg}$ are decision variables, defined such that:

\[
\begin{align*}
    x_{vf} &= \begin{cases} 
        1 & \text{if } f \in F(v) \text{ is assigned to link } v \\
        0 & \text{otherwise} 
    \end{cases} \\
    x_{wg} &= \begin{cases} 
        1 & \text{if } g \in F(w) \text{ is assigned to link } w \\
        0 & \text{otherwise} 
    \end{cases}
\end{align*}
\]

then the objective function can be formulated:

\[
\text{Min } \sum_{vw \in E} \sum_{f \in F(v), g \in F(w)} P_{vwfg} x_{vf} x_{wg}. \quad (3.21)
\]

This formulation demands that, for each edge $vw \in E$, frequencies are assigned to the links at each edge-end in order that penalties are minimised. However, the problems here demands a zero violation solution, so rather than minimise interference, the objective is always to ensure that the inequality given in (3.7) is satisfied.
3.1 Problem formulation

In this formulation of the frequency assignment problem with equipment selection, the frequency domain available to the assigner is constrained such that \( f(u) \in F(u), \forall u \in V \), so respecting the fixed channel plan associated with the minimum interference problem. This is an important constraint because, with the minimum span problem, the assigner generally has access to a frequency domain with no practical upper bound on the number of channels selected. The frequency domain might be defined by the set of positive integer numbers, denoted here by \( \mathbb{Z}^+ \); effectively, an infinite channel set. Although the channel set is infinite, the assigner is normally constrained by an organised frequency plan (a channel raster). This can be defined as follows: If \( f_i \) is the smallest frequency assigned and \( b \) is the bandwidth of each radio channel available to the assigner, then \( f_{i+n} = (f_i + n.b) \) where \( n \in \mathbb{Z}^+ \). However, in professional practice (or with the fixed spectrum problem), the assigner is constrained by a finite channel set and the rejection of some requests can be expected if the assigner is also constrained to obtaining frequency assignments with zero violations.

However, the problems developed for this study are relatively small and can be resolved without accessing the entire frequency domain available to the assigner (the assigner is not constrained by the cardinality of the channel set); the focus, therefore, is on addressing the minimum span problem while respecting the real-world raster organisation adopted for the study.

Closer packing might be possible in a raster free frequency assignment, for example. Here, rather than selecting discreet frequencies from an organised channel raster, frequencies could be assigned to \( u \) and \( v \) such that:

\[
NFD(|f(u) - g(v)|) \geq e_{uv}. \tag{3.22}
\]

That is, the \( NFD \) available when \( f \mapsto u \) and \( g \mapsto v \) is greater than or equal to the excess interference obtained when \( u \) and \( v \) are tuned co-channel or near-co-channel. Of course, (3.22) must still be satisfied when the assigner is constrained by a channel plan.
Here, a set of discrete frequency separations between $u$ and $v$ are possible, suggesting that when satisfying (3.22), sometimes $NFD \gg e_{uv}$.

With existing approaches to the *minimum span* problem in frequency assignment, it is sufficient to count the number of radio channels between the highest and lowest assignments. In contrast, since channels of differing bandwidths are assigned in this study, a simple count of the channels assigned would not give any indication of the span obtained; this is illustrated in Figure 3.7 where the green coloured channels are assigned. It is possible for a channel on a low raster to have a higher number and a lower frequency than a channel on a higher raster. Therefore this problem is generalised so that span is considered in terms of the absolute extents of the channels assigned.

Let $f : F \mapsto V$ be a frequency assignment. Let $z_{\text{min}}$ denote the minimum limit of all frequencies assigned:

$$z_{\text{min}}(f) = \min_{u \in V} f_{\text{min}}(u)$$  \hspace{1cm} (3.23)

and $z_{\text{max}}$ denote the maximum limit:

$$z_{\text{max}}(f) = \max_{u \in V} f_{\text{max}}(u)$$  \hspace{1cm} (3.24)
Define the span of the assignment, \( sp(f) \), as:

\[
sp(f) = z_{max}(f) - z_{min}(f).
\] (3.25)

### 3.1.6 Integer Program

The problem is formulated as an integer program by defining decision variables:

\[
x_{uf} = \begin{cases} 
1 & \text{if } f \in F(u) \text{ is assigned to link } u \\
0 & \text{otherwise}
\end{cases}
\]

\[
y_f = \begin{cases} 
1 & \text{if } f \in F \text{ is assigned to at least one link} \\
0 & \text{otherwise}
\end{cases}
\]

\[
r_{us} = \begin{cases} 
1 & \text{if radio system } s \text{ is selected for use at link } u \\
0 & \text{otherwise}
\end{cases}
\]

The problem can then be expressed as follows:
3.1 Problem formulation

\[
\text{Min } z_{\text{max}} - z_{\text{min}} \tag{3.26}
\]

subject to

\[
\sum_{f \in F(u)} x_{uf} = 1 \quad \forall u \in V \tag{3.27}
\]

\[
z_{\text{max}} \geq f_{\text{max}} y_f \quad \forall f \in F \tag{3.28}
\]

\[
z_{\text{min}} \leq f_{\text{min}} y_f + (\max_{g \in F} g_{\text{max}})(1 - y_f) \quad \forall f \in F \tag{3.29}
\]

\[
x_{uf} \leq y_f \quad \forall u \in V, f \in F \tag{3.30}
\]

\[
r_{us} = \sum_{f \in F^s} x_{uf} \quad \forall u \in V, s \in S \tag{3.31}
\]

\[
x_{uf} + x_{vg} \leq 3 - r_{us} - r_{vt} \quad \forall u, v \in V, f, g \in F, s, t \in S, |f - g| < C_{u,v}^{s,t} \tag{3.32}
\]

\[
x_{uf} \in \{0, 1\} \quad \forall u \in V, f \in F \tag{3.33}
\]

\[
y_f \in \{0, 1\} \quad \forall f \in F \tag{3.34}
\]

\[
r_{us} \in \{0, 1\} \quad \forall u \in V, s \in S \tag{3.35}
\]

The objective function (3.26) minimises the total span; constraint (3.27) ensures that each link request is assigned a single radio frequency channel; therefore, the summation of \(x_{uf}\) equals 1 when summing across the channel set available to request \(u\). This condition applies to all requests. (3.28) and (3.29) set the maximum and minimum limits of the frequencies used. \(Z_{\text{max}}\) is greater than or equal to the upper bound of assigned radio channels; if the channel is unassigned, \(f_{\text{max}} y_f = 0\) and \(z_{\text{max}} > 0\). \(Z_{\text{min}}\) is less than or equal to the lower bound of assigned radio channels; if the channel is unassigned then \(Z_{\text{min}}\) is less than the highest possible upper bound on a radio channel in \(F\). These conditions apply to all frequencies in \(F\). (3.30) determines the set of radio channels used. If \(x_{uf} = 1\) then \(f\) is assigned to \(u\), \(y_f = 1\) and \(x_{uf} = y_f\); if \(x_{uf} = 0\), \(y_f\) can be 0 or 1 (1 if \(f\) is assigned to some other \(u \in V\), 0 otherwise) and \(x_{uf}\) is less than or equal to \(y_f\). This applies to all requests in \(V\) and all frequencies in \(F\). (3.31) determines which radio system is used by each link. Here, \(r_{us} = 1\) when \(x_{uf} = 1\) and \(f\) is from the set of frequencies available to radio system \(s\): if \(f \in F^s\), then \(r_{us}\) may be
selected at \( u \). This applies to all requests in \( V \) and all radio systems in \( S \). Finally, (3.32) ensures that the relevant channel separation constraint is satisfied. This is a mechanism to reject at least one of the candidate frequencies \( f \) or \( g \) when \( C_{s,t}^{u,v} \) remains unsatisfied.

### 3.2 Hypothesis: the equipment selection paradox

Although, the trade off between radio equipment parameters and spectral efficiency has been discussed in the radio engineering profession, there is often an assumption made that higher-order modulation equipment is more spectrally efficient than lower-order alternatives. The study challenges this assumption and a hypothesis has been formulated on this basis.

**Hypothesis 16.** *Selection of lower-order modulation radio equipment for a sub-set of requests can reduce interference and achieve smaller spans for a significant sub-set of problems relative to solutions obtained in the All higher-order environment.*

The hypothesis can be considered paradoxical because it suggest that when doubling the bandwidth required by some requests relative to the All higher-order modulation environment, it is possible to maintain or actually reduce the span of the frequency assignment.
Chapter 4

Exact Solutions

This chapter sets out results obtained when exposing a set of fifty randomly generated benchmark problems, representative of real world frequency assignment scenarios (from the assigner’s perspective), to an IP solver. The development of IP formulations for frequency assignment problems is a well established approach in computer science; using a solver allows for exact, optimal, solutions to be obtained for these problems. However, these formulations are only practical for relatively small problems. On this basis, the hypotheses outlined in Chapter Three can be tested with rigour.

To analyse the trade off between global and local spectral efficiency, solutions to a number of randomly generated problem instances are analysed.

4.1 Data sets

Fifty representative problems have been generated, each consisting of a set $V$ of fifty 38 GHz fixed link requests. This band is used extensively throughout Europe for base station interconnection in mobile cellular networks. Each request $u \in V$ represents a fixed link with two link-ends, $u_a$ and $u_b$. In order to generate a request $u_a$ is assigned a random position in a region with dimensions $0.25 \times 0.25$ km. A wanted signal angle incident to $u_b$ in the range $[0, 360)$ degrees is assigned randomly and a wanted path length in the range $[0.3, 0.5]$ km (so that the total simulation space has dimensions $1.25 \times 1.25$ km). The position of $u_b$ is calculated from these parameters. Each re-
quest is randomly assigned a standard data-rate $d(v)$, expressed in Mbit/s, from a set 
\{8, 16, 34, 51, 155\} with the assumption that these can be resolved by exactly two radio
system types from the ETSI standards. An antenna characteristic is specified at each
link-end and the \textit{EIRP} required on the link is calculated using equation (3.10), for both
of the equipment types available. Table 4.1 lists the standard ETSI radio system types
\cite{5} used in this study with values for the key parameters \textit{RSL} and \textit{I_T} specified. This
illustrates the inequalities in play between system types.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{Mbit/s in MHz} & \textbf{\textit{RSL}(dBW)} & \textbf{\textit{I_T}(dBW)} \\
\hline
8 in 3.5 & -105.5 & -138.4 \\
\hline
8 in 7 & -106.5 & -132.9 \\
\hline
2x8 in 7 & -99.5 & -132.4 \\
\hline
2x8 in 14 & -103.5 & -129.9 \\
\hline
34 in 14 & -96.5 & -129.4 \\
\hline
34 in 28 & -100.5 & -126.9 \\
\hline
51 in 14 & -95.5 & -129.0 \\
\hline
51 in 28 & -97.5 & -130.4 \\
\hline
155 in 28 & -90 & -128.9 \\
\hline
155 in 56 & -92.5 & -125.4 \\
\hline
\end{tabular}
\caption{Radio systems}
\end{table}

The fifty links are packed into a relatively small space with the aim of causing suf-
ficient interference \textit{clashes} to exercise the IP solver. If each pair of vertices is tuned
co-channel, or near co-channel when operating on alternative rasters and higher-order
modulation equipment is selected at all vertices, then on average each vertex is con-
strained with 3.6 other vertices due to excess interference (disregarding the constraints
due to raster organisation). These deployments are somewhat abstract relative to real-
world networks but as the networks evolve, smaller cell sizes and shorter path lengths on microwave links are expected. In professional practice, these short links might be assigned an \textit{EIRP} uplift in cases where the calculated \textit{EIRP} is lower than the minimum \textit{EIRP} delivered by the equipment and antenna combination. However, this very practical step was considered unnecessary in this study, where the aim is simply to bring about sufficient interference clashes to test the hypotheses. A worst-case antenna characteristic is deployed on boresite for the same reason.

Figure 4.1 shows the \textit{Radiation Pattern Envelope (RPE)} used to model the fixed link antenna; the RPE specifies the off-axis performance of the antenna only and the ETSI Class 2 RPE shown here is a worst-case specification for fixed links operating in the 38 GHz frequency band in Europe \cite{34}. This RPE was used in conjunction with an antenna gain of 45 dBi at boresite and extended to 1 degree off-axis with a straightforward linear interpolation for gain in the range 1 to 5 degrees off-axis. The aim here was to build an antenna pattern that would generate interference problems, rather than to obtain a really accurate model of the antenna characteristic at angles close to boresight. Vertical polarisation is assumed on all antennas.

To calculate the potential for interference between each pair of requests, the geometry of the radio interference path between each pair is calculated; these paths are the basis for an edge set \( E \) to be constructed. The interference incident to each vertex is calculated, generating the constraints described in Chapter 3.

\subsection{The equipment selection paradox}

For each request \( v \in V \), let \( s_L(v) \) and \( s_H(v) \) denote the available lower- and higher-order modulation radio systems respectively. Three sets of solutions \( IP \) are obtained for the 50 problems.

\footnote{The RPEs specified by ETSI start at 5 degrees off-axis and extend to 180 degrees off-axis; they are assumed to be symmetrical.}
4.1 Data sets

Figure 4.1: Radiation Pattern Envelope

**All higher-order.** All links are chosen to use higher-order equipment, hence $F(u) = F^{s_H(u)}$ for all $u \in V$. Denote the resulting minimum span by $sp_H$.

**All lower-order.** All links are chosen to use lower-order equipment, hence $F(u) = F^{s_L(u)}$ for all $u \in V$. Denote the resulting minimum span by $sp_L$.

**Equipment selection.** All links may use either lower- or higher-order equipment, hence $F(u) = F^{s_L(u)} \cup F^{s_H(u)}$ for all $u \in V$. Denote the minimum span by $sp$.

All solutions were obtained by solving the integer programs with the CPLEX MIP solver, run on a MacBook Pro 2.66 GHz Intel Core i7 with 4GB ram, and the run times across each of the 50 data sets were typically less than one minute. Figure 4.2 shows the spans delivered by the solver while Figure 4.3 shows the ratios $sp_L/sp_H$ and $sp_L/sp$ delivered by the three sets of solutions with the results ordered by the values for $sp_L/sp_H$. 
The graph provides some compelling evidence for the existence of an equipment selection paradox. Specifically, while using all lower-order systems doubles the spectrum requirement for each individual fixed link relative to all higher-order, on average the ratio of $sp_L / sp_H$ is only 1.77. This alone suggests that it is possible to select lower-order modulation equipment on a sub-set of links, reducing levels of interference in the radio environment and increasing the efficiency of spectrum use. We discuss these results further in Section 4.1.2.
4.1 Data sets

4.1.2 Optimising Equipment Selection

It is possible to determine whether optimal results are obtained when adjusting the IP formulation to constrain the numbers of links that are assigned lower-order modulation equipment.

Define a variable $h_v$ for $v \in V$ which indicates whether link $v$ has been assigned relatively lower-order equipment:

$$ h_v = \begin{cases} 1 & \text{if a relatively lower-order system is selected for use at link } v \\ 0 & \text{otherwise} \end{cases} $$

Then add constraints to ensure that the proportion of links assigned lower-order equipment is between a lower bound $H_L$ and an upper bound $H_U$ where $0 \leq H_L \leq H_U \leq 1$:

$$ \sum_{f \in F^{S_L(u)}} x_{vf} - h_v = 0 \quad \forall v \in V \quad (4.1) $$

$$ \sum_{u \in V} h_v \geq H_L|V| \quad (4.2) $$

$$ \sum_{u \in V} h_v \leq H_U|V| \quad (4.3) $$

Figure 4.3: Ratios of spans (problems ordered by $sp_L/sp_H$)
4.1 Data sets

Figure 4.4: Spans for Equipment Selection (ordered by \( sp \))

Figure 4.4 shows a graph of the spans obtained for Equipment Selection where \( H_U \) is set to 0.1 i.e. 10% of requests (and span is denoted by \( sp_{10} \)), 0.25 i.e. 25% of requests (\( sp_{25} \)) and with no constraints of this kind (\( sp \)).

When Running Equipment Selection (0-10%), we obtain lower spans than All higher-order in 4% of runs but, in general, the solutions are clearly sub-optimal. Equipment Selection (0-25%) performs well, obtaining lower spans than All higher-order for 36% of problems but is still sub-optimal and for some problems is out-performed by All higher-order. We may say that when running Equipment Selection and seeking optimal solutions there is no advantage in constraining the numbers of lower-order systems selected and this indicates that, in some case, a significant number of lower-order selections are required for an optimal solution.

Equipment Selection outperforms All higher-order, delivering a lower span for 46% of problems and an average reduction in span of 10.7% for those solutions with an improvement.

With unconstrained Equipment Selection (\( sp \)), lower-order equipment is selected on a sub-set of requests (for all fifty problems) with 16 lower-order selections made on
average across the problem set; the minimum number of lower-order selections is 6 and the maximum 27 i.e. over half of all links. Figure 4.5 shows the CDF of lower-order modulation selections made by the IP solver for each of the fifty problems.

### 4.1.3 Structure of optimised assignments

Figures 4.6 to 4.11 show frequency assignments for problems 3, 11 and 17. There are a pair of graphs for each problem, one showing the frequency assignment delivered when the solver is constrained to the All higher-order modulation environment and a second where the solver operates in a *Mixed modulation environment* where Equipment Selection is exercised. The x-axis shows frequency normalised to a measure of frequency offset from the lower-bound of channel one on the 56 MHz raster and the y-axis shows a count of frequency assignments. The graphs include a plot for each channel raster and these show how the number of wider bandwidth systems increases while the span of the frequency assignment reduces when the solver is instructed to
In general, the Mixed modulation environment produces frequency assignment graphs with less peaks and that the assignments on higher rasters tend to dominate the graph, enveloping, to some extent, the assignments on lower channel rasters.
4.1 Data sets

Figure 4.7: Frequency assignment for problem 3 with the solver using lower and higher-order equipment.

4.1.4 Calculating excess constraints

For a request $u$, $s^{H}_i$ and $s^{L}_i$ are the higher- and lower-order modulation radio systems where the data-rate supported by the radio is denoted by $D(s_i)$ and, in our model, this is exactly equal to data-rate required by the request $d(u)$. In general, selecting $s^{L}_i$ at $u$ in place of the higher-order option $s^{H}_i$ will reduce the excess interference $e_{uv}$ since, a few idiosyncrasies aside, the equipment has the characteristics:

$$RSL(s^{H}_i) > RSL(s^{L}_i),$$

(4.4)

$$I_T(s^{H}_i) < I_T(s^{L}_i).$$

(4.5)

That is, for a given data-rate requirement $d(u)$, higher-order modulation requires a
Figure 4.8: Frequency assignment for problem 11 with the solver constrained to a higher-order environment.

higher received signal strength, and is more sensitive to interference.

If a pair of links $u$ and $v$ are considered, revised excess values can be calculated if $u$ is switched from higher- to lower-order modulation. In the higher-order modulation environment, the excess value at $u$, sourced from $v$, is denoted by $e_{uv}^{-}$ and the excess at $v$, and sourced from $u$, by $e_{uv}^{+}$.

Selecting lower-order equipment at $u$, the revised excess values at $u$ and $v$ are calculated by taking account of the inequalities given in 4.1 and 4.2. These revised excess values are calculated using:

$$e_{uv}^{−′} = e_{uv}^{−} − [I_T(s_i^H) − I_T(s_i^L)], \quad (4.6)$$
and

\[ e_{uv}^{r} = e_{uv} + [RSL(s_i^H) - RSL(s_i^L)] , \]  

(4.7)

with obvious reformulation of these equations where lower-order equipment is selected at \( v \) or at both \( u \) and \( v \).

The subdivided raster organisation used in these examples is such that two requests with alternative radio equipment are subject to a minimum channel separation constraint given by:

\[ C_{s,t}^{\text{min}} = \begin{cases} 
  b_i/2 & \text{if } s \text{ and } t \text{ operate on rasters } b_p \text{ and } b_q \text{ and } p > q \\
  0 & \text{for two requests operating on the same raster}
\end{cases} \]
Figure 4.10: Frequency assignment for problem 17 with the solver constrained to a higher-order environment.

It may be deduced that values for $C_{u,v}^{s,t} > C_{s,t}^{\text{min}}$ are due to excess interference at $u$ or $v$ (or both $u$ and $v$) and on this basis an excess channel separation constraint is calculated using:

$$E_{u,v}^{s,t} = C_{u,v}^{s,t} - C_{s,t}^{\text{min}}.$$  \hspace{1cm} (4.8)

Clearly, $E_{s,t}^{u,v}$ identifies, precisely, the contribution to the channel separation constraint from interference in a co-channel or near co-channel scenario.

Consider an assignment assignment $f$ obtained from the IP solver, and let:

$$V_L = \{ u \in V : f(u) \in F_{s,t}^{u,(u)} \},$$

$$V_H = \{ u \in V : f(u) \in F_{s,t}^{u,(u)} \}.$$
4.1 Data sets

Figure 4.11: Frequency assignment for problem 17 with the solver using lower and higher-order equipment.

That is, $V_L$ and $V_H$ are the sets of links assigned lower and higher-order modulation schemes respectively. Define the weight of $u$, $W(u)$, given by the sum of excess channel separation constraints involving $u$ under the assumption that all links are assigned higher-order modulation equipment:

$$ W(u) = \sum_{v \in N(u)} E_{s_H(u),s_H(v)}^{u,v}. $$

(4.9)

Then let $W'(u)$ denote the sum of excess constraints under the assumption that $u$ is assigned lower-order modulation equipment (irrespective of the IP solver’s actual selection) and all other links are assigned the equipment specified by the optimal IP
solution. That is:

$$W'(u) = \sum_{v \in N(u) \cap V_L} E_{s_L(u), s_L(v)} + \sum_{v \in N(u) \cap V_H} E_{s_L(u), s_H(v)}.$$ \hspace{1cm} (4.10)

Finally, let $\Delta W(u)$ denote the difference between these two weights:

$$\Delta W(u) = W'(u) - W(u)$$ \hspace{1cm} (4.11)

to give a measure of the effect on the constraints of changing $u$ from higher to lower-order modulation equipment. Figure 4.12 shows values for $\Delta W(u)$, normalised to a count of 1.75 MHz bandwidth segments, for the fifty requests associated with problem 1. Here, the series $V_L$ gives the $\Delta W(u)$ values for requests where the solver selected $s_L^i$ while $V_H$ is a plot of $\Delta W(u)$ values for requests where the solver actually made the decision to select $s_H^i$ and so shows the impact on the weight of constraints incident to $u$ when a hypothetical selection $s_L^i$ is made. The solver has made lower-order selections at a vertex when this reduces weight or the increase in weight is modest. Some hypothetical selections obtain quite significant reductions in weight but, clearly, on average, $\Delta W(u)$ is a positive number for $V_H$.

Figure 4.13 plots the average of $\Delta W(u)$ over the sets $V_L$ and $V_H$ for all fifty problems. These show a clear distinction, with the average for $V_H$ across the entire problem set being 6.3, while the average for $V_L$ is -0.8. This confirms that the solver tends to select $s_L^i$ in cases where the weight of channel separation constraints is reduced.

The set of constraints handled by the IP solver includes elements for all possible equipment selections and the average $\Delta W(u)$ value can easily be obtained $\forall u \in V$, irrespective of the actual selection made by the solver. Table 4.2 shows a subset of the constraints exposed to the solver. For each pair of requests, $u$ and $v$, radio systems $s$ and $t$ are available and the problem file, a constraint matrix, sets out constraints for lower- and higher-order modulation equipment options at both vertices. $r(u)$ and $r(v)$
denote the rasters associated with \( s \) and \( t \) for \( u \) and \( v \) respectively. When \( r(u) = 3.5 \) and \( r(v) = 7 \) higher-order modulation equipment is selected at both vertices and \( C_{s,t}^{u,v} = 1.75 \). If lower-order equipment is selected at \( u \) and \( v \), \( C_{s,t}^{u,v} = 3.5 \). When \( u \) is lower-order and \( v \) is higher-order, \( C_{s,t}^{u,v} = 0 \) and when \( v \) is lower-order and \( u \) is higher-order, \( C_{s,t}^{u,v} = 1.75 \). In each case, \( C_{s,t}^{u,v} = C_{s,t}^{\min} \). Clearly, \( u \) and \( v \) can operate co-channel when on the same raster. The extract highlights the relative measure of lower- and higher-order modulation; when lower-order is selected at \( u \) and higher-order at \( v \), they operate on the same channel raster.

<table>
<thead>
<tr>
<th>( u )</th>
<th>( v )</th>
<th>( r(u) )</th>
<th>( r(v) )</th>
<th>( C_{s,t}^{u,v} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3.5</td>
<td>7</td>
<td>1.75</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>7</td>
<td>14</td>
<td>3.5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>7</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3.5</td>
<td>14</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Table 4.2: Example constraints for a pair of requests
4.2 Conclusion

This chapter of the thesis has investigated the equipment selection paradox and demonstrated that the benchmark IP formulation of the problem allows the solver to deliver results that confirm the hypotheses: that when doubling the bandwidth requirement on a sub-set of $V$ it is possible to maintain or actually reduce the span of the frequency assignment delivered in the all higher-order modulation environment.

The analysis has shown that the solver’s selection of lower-order modulation equipment is related to the $\Delta W(u)$ parameter. The results show that by reducing $\Delta W(u)$ on a sub-set of requests, it is possible to accommodate systems with larger bandwidths without increasing the span of the frequency assignment and in many cases obtaining closer packing in the frequency domain (reduced spans). This work lays the basis for some further investigations including the development of heuristic principles and

![Figure 4.13: Average $\Delta W$ ordered by results for $V_L$](image)

The column specifying the channel separation constraint required between $u$ and $v$ uses the notation $C_{u,v}^{s,t}$ generically but the extract shows that the constraints: $C_{s,s}^{u,v}$, $C_{t,t}^{u,v}$, $C_{t,s}^{u,v}$ and $C_{s,t}^{u,v}$ are all specified in the problem file for each pair $u, v \in V$. 

4.2 Conclusion
4.2 Conclusion

algorithms.
Offline heuristics

Chapter Four reported on exact solutions delivered by the IP solver providing evidence in support of the hypothesis that the use of lower-order modulation equipment on selected links can either be accommodated by a frequency assignment with no degradation of span or can actually reduce span. This chapter reports on the design of heuristics in an offline environment.

The IP Solver finds optimal solutions but is constrained to an offline environment. This can be defined as an abstract investigation of the frequency assignment problem that allows the assigner (or the assigning software program) to develop an objective analysis of an entire request queue. The assigner can take account of all constraints in the neighbourhood of a request, irrespective of the order in which they were made.

The online environment simulates the real world frequency assignment problem from a regulatory perspective where the assigner must handle requests sequentially, requests are regularly added to the queue, the assigner has no knowledge of how the queue will develop and the requests must be handled on a first-come-first-served basis and in a timely manner. There are no opportunities for the assigner to re-order the request queue or to apply network analysis or an optimal frequency assignment procedure. On this basis, there is a strong motivation for the development of heuristics that can be used on-line. However, there could also be opportunities for a greedy heuristic to be incorporated into a meta-heuristic; here the greedy heuristic either provides an initial solution that can later be refined [35] or allows solutions to be encoded by ordering [36]. Meta-heuristics could be used for planning special events or for a re-tuning
5.1 Scope of the design

The design tasks have been categorised as follows:

**Ordering techniques.** The study has investigated techniques for re-ordering the request queue. Random re-ordering is used to investigate the solution space (the range of solutions available) and allows for the benchmarking of more sophisticated ordering techniques. The study proposes a new technique that takes account of raster organisation.

**Equipment selection.** Equipment selection heuristics have been developed that make a choice between the use of lower- and higher-order modulation equipment for individual requests based on the analysis developed in Chapter Four.

**Frequency assignment.** The heuristics run a sequential frequency assignment procedure.
Using the IP Solver’s solutions as a benchmark, the approach has been to experiment with the ordering of the request queue and with equipment selection heuristics that use criteria based on an analysis of interference at all $v \in V$ as well as random selection techniques. The study has used a sequential frequency assignment throughout. There is some elaboration of the tasks involved with each of these categories here. The heuristics have ordered the request queue making use of the classical Generalised Largest First (GLF) technique, a new ordering technique Raster Hierarchical Ordering (RHO) and, for comparison, random orderings.

An important measure here was a comparison of the spans obtained when using a good ordering technique (GLF or RHO) and those obtained when the ordering technique was combined with an equipment selection heuristic. A re-ordering of the request queue can radically improve on span; the question was whether equipment selection could deliver an extra spectral efficiency gain.

RHO was developed in order to take account of raster organisation and on the basis that requests for links using higher channel rasters would be more challenging with respect to the minimum span objective.

Some of the equipment selection heuristics use random selection techniques; the results delivered by these heuristics are for benchmarking and provide a measure of performance across the problem search space when compared with results from heuristics that analyse the radio interference environment and use selection criteria based on an analysis of the IP solver’s solutions.

A sequential frequency assignment procedure is used with all of these heuristics. While the objective analysis allows for some experimentation with ordering techniques and the development of selection criteria, ultimately, the aim is to use these investigations to inform the design of heuristics for on-line use and the sequential frequency assignment has been retained on this basis.
5.2 Ordering techniques

The study uses three approaches to ordering the request queue:

**Generalised Largest First (GLF).** This classical technique orders requests according to assignment difficulty and uses the weight of channel separation constraints incident to a vertex to do this.

**Raster Hierarchical Ordering (RHO).** RHO orders the request queue by raster, placing requests on higher rasters at the top of the queue. Ties are broken using the GLF method.

**Random ordering.** The request queue is exposed to random orderings. This allows for some investigation of the solution space and provides some benchmark solutions.

A range of ordering techniques were reported on in Chapter Two and previous studies have investigated these thoroughly. The approach taken here was to select just one of the *classical* ordering techniques. The focus of this study was the development of exact solutions and equipment selection heuristics. While new ordering techniques have been developed, exposing the problem set to a comprehensive set of ordering techniques was not within the scope of these investigations.

Chapter Two outlined the GLF technique. The channel separation constraint on an edge is its *weight*. GLF sums the weights on edges incident to a request and orders the queue accordingly. Given the necessity for additional constraints when addressing the Frequency Assignment Problem with Equipment Selection, the definition is extended here.

**Definition 17.** Given an equipment selection $g : V \rightarrow S$ for the request queue, let the channel separation constraint on an edge $u, v$ be denoted by $C^u,v_g$ where:

$$C^u,v_g = C^u,v_{g(u),g(v)}. \quad (5.1)$$
5.2 Ordering techniques

The weighted degree of request \( u \), given an equipment selection \( g \), is the sum of these constraints over all incident edges:

\[
d_g(u) = \sum_{v \in N(u)} C_{g}^{u,v}. \tag{5.2}
\]

and \( w_{\text{max}}(u) \) denotes the maximal weight on edges incident to \( u \) where:

\[
w_{\text{max}}(u) = \max_{v \in N(u)} w_{g(u,v)}. \tag{5.3}
\]

and \( w_{g(u,v)} \) is the weight on edge \( u, v \).

Let the GLF order of the request queue be given by a listing of \( V \) according to the magnitude of \( d_g(u), \forall u \in V \), with the largest weight listed first in order; let ties be resolved according to \( w_{\text{max}}(u) \) and then the initial order of \( V \).

While GLF orders a request queue by weighted degree and so an estimate of assignment difficulty, there is no account taken of radio system bandwidth. This may be important when operating in a multi-raster environment and addressing the minimum span problem.

Figure 5.1 sets out a simple example where a pair of requests \( u \) and \( v \) are constrained. The two requests operate on alternative rasters: \( u \) on a 7 MHz raster and \( v \) on a 28 MHz raster. For this example, let \( C_{g}^{u,v} = 17.5 \) MHz. The sequential frequency assignment procedure selects the smallest frequency whilst satisfying \( C_{g}^{u,v} \) working up-band from \( f_1 \). If \( u \) appears before \( v \) in the request queue then \( u \) is assigned \( f_1 \), \( v \) must be assigned \( f_2 \) and there is a separation between the carriers of 38.5 MHz, satisfying \( C_{g}^{u,v} \) and obtaining a span of 56 MHz. If the order in which the two requests are handled is reversed then \( v \) can be assigned \( f_1 \) and \( u \) can be assigned \( f_5 \). This ensures a separation between carriers that satisfies \( C_{g}^{u,v} \) exactly and reduces the span to 35 MHz.
5.2 Ordering techniques

Figure 5.1: Ordering assignments by raster

The example illustrates the reasoning behind the RHO technique, where requests on higher rasters are listed first. Clearly, this results in a large number of ties and these are broken by use of the, in this case, subordinate, GLF procedures.

Definition 18. Let the Raster Hierarchical Ordering be a listing of $V$ by raster order such that requests mapped to the highest raster $r_{\max}$ are first in order with ties broken by weighted degree, maximal weight and then initial order.
5.3 Sequential Frequency Assignment

The study has first of all assessed these sequential algorithms in the All-higher-order modulation environment. This approach was taken in order to test the RHO technique against the well established GLF, to measure the performance of both RHO and GLF against random orderings and to establish some useful benchmarks for the performance of more complex heuristics with equipment selection procedures. Table 5.1 lists these heuristics.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Order of V</th>
<th>Equipment selection</th>
<th>Frequency assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{glf} )</td>
<td>GLF</td>
<td>higher-order</td>
<td>sequential</td>
</tr>
<tr>
<td>( h_{rho} )</td>
<td>RHO</td>
<td>higher-order</td>
<td>sequential</td>
</tr>
<tr>
<td>( h_{rand} )</td>
<td>random</td>
<td>higher-order</td>
<td>sequential</td>
</tr>
</tbody>
</table>

Table 5.1: Frequency assignment heuristics

\( h_{glf} \) maps all requests to higher-order modulation equipment, applies the GLF ordering technique to \( V \) and exposes \( V \) to a sequential frequency assignment.

\( h_{rho} \) maps all requests to higher-order modulation equipment, applies the RHO ordering technique to \( V \) and exposes \( V \) to a sequential frequency assignment.

\( h_{rand} \) maps all requests to higher-order modulation equipment, applies a random ordering and a sequential frequency assignment procedure to \( V \).

A sequential, greedy assignment procedure is used with the experimental heuristics. The objective is to minimise the span of the frequency assignment via a low-end packing approach. On this basis, the sequential assignment works up-band from the lowest channel on the appropriate raster, assigning the smallest frequency that satisfies all of the constraints associated with the request under consideration.

The problem set was exposed to the simple heuristics and the results are set out here. \( h_{rand} \) was configured to deliver one hundred random orderings of \( V \) and so one hundred
network frequency assignments; this configuration of the heuristic is denoted by $h_{100}^{\text{rand}}$.

Figure 5.2 shows, for each problem, a plot of the spans delivered by $h_{100}^{\text{rand}}$ using a box plot representation and with line graphs showing the exact spans delivered by $h_{\text{glf}}$ and $h_{\text{rho}}$. The data is ordered according to the median spans delivered by $h_{100}^{\text{rand}}$.

The vertical lines associated with each box plot indicate the range of spans delivered by $h_{100}^{\text{rand}}$, while the box indicates the range of spans between the 25th and 75th percentiles denoted here by $sp_{(25\%)}$ and $sp_{(75\%)}$. The horizontal lines that divide the boxes indicate the median value. In the one case where the box representation is collapsed, the median value is exactly equal to $sp_{(25\%)}$ and $sp_{(75\%)}$. In cases where the box is fully developed and the median value is not apparent, the median is exactly equal to $sp_{(25\%)}$ or $sp_{(75\%)}$.

$h_{\text{glf}}$ and $h_{\text{rho}}$ deliver spans that are less than or equal to the span given by $h_{100}^{\text{rand}}$ for 84% and 92% of problems respectively.

Clearly, the new RHO technique is able to outperform the well established GLF with $h_{\text{rho}}$ delivering a smaller span than $h_{\text{glf}}$ for 30% of the problem set. Conversely, $h_{\text{glf}}$ improves on the solutions given by $h_{\text{rho}}$ for just 4% of problems.
The results show that good ordering techniques are required for the heuristics to approach the exact solutions. However, in agreement with Hurley et al. [21], the results also indicate that the best heuristic for a particular problem is not easy to assess without experimentation and this heuristic is not always the best performing over all problems.

5.4 Heuristics with equipment selection

Using the heuristics operating in the All higher-order environment as the basis for further development, \( h_{glf}, h_{rho} \) and \( h_{rand} \) can be refined by adding a subordinate equipment selection heuristic allowing for a request to be mapped to either \( s_i^L \) or \( s_i^H \). Here, the heuristics operate in the Mixed modulation environment. A range of equipment selection criteria are used with the modified \( h_{glf} \) and \( h_{rho} \).

\[ h_{es(T)}, h_{glf} \] orders \( V \) using the GLF_E technique and the set of excess constraints. The heuristic is configured to run the subordinate equipment selection heuristic with a criterion for selection \( T \). The GLF ordering technique is applied to the request queue delivered by \( h_{es(T)} \) using the full set of constraints and the GLF ordered request queue is exposed to a sequential frequency assignment procedure.

\[ h_{es(T)}, h_{rho} \] orders \( V \) using the GLF_E technique and the set of excess constraints. The heuristic is configured to run the subordinate equipment selection heuristic with a criterion for selection \( T \). The RHO ordering technique is applied to the request queue delivered by \( h_{es(T)} \) using the full set of constraints and the RHO ordered request queue is exposed to a sequential frequency assignment procedure.

\[ h_{res(p)}, h_{rand} \] selects lower-order modulation equipment randomly according to a probability \( p \). The heuristic then applies a random ordering to the request queue and runs a sequential frequency assignment.

The pre-equipment selection ordering technique, GLF_E, calculates the GLF order of \( V \) by the average excess weight of \( u \) across lower- and higher-order modulation envir-
5.4 Heuristics with equipment selection

environments and orders $V$ accordingly. The technique first of all calculates excess weight in All-lower and All-higher-order modulation environments.

**Definition 19.** Let $g : V \rightarrow S$ denote an equipment selection for the request queue and let $g(u)$ denote an equipment selection for request $u$. Then let $g_L(u)$ and $g_H(u)$ denote lower- and higher-order modulation equipment selections for $u$ respectively. The excess weighted degree of request $u$ in these two distinct environments is the sum of the excess constraints over all incident edges:

\[
\begin{align*}
d^L_E(u) &= \sum_{v \in N(u)} E_{g_L(u,v)}, \\
d^H_E(u) &= \sum_{v \in N(u)} E_{g_H(u,v)}.
\end{align*}
\]

Then the mean excess weighted degree over lower- and higher-order modulation environments is denoted by $\overline{d}_E(u)$.

GLF$_E$ uses the classical GLF techniques to break ties and so requires the maximum weight incident to a vertex with both All-lower and All-higher-order modulation environments considered.

**Definition 20.** Let $E^L_{\text{max}}(u)$ and $E^H_{\text{max}}(u)$ denote the maximal weight on edges incident to $u$ in the All-lower and All-higher-order modulation environments where:

\[
\begin{align*}
E^L_{\text{max}}(u) &= \max_{v \in N(u)} E_{g_L(u,v)}, \\
E^H_{\text{max}}(u) &= \max_{v \in N(u)} E_{g_H(u,v)}.
\end{align*}
\]

Then let

\[
\text{Max}(E(u)) = \max(E^L_{\text{max}}(u), E^H_{\text{max}}(u)).
\]
Finally, GLF_E can be defined.

**Definition 21.** Let the GLF_E order of the request queue be given by a listing of V according to the magnitude of \( \overline{d}_E(u) \), \( \forall u \in V \), with the largest weight listed first in order; let ties be resolved according to \( \text{Max}(E(u)) \) and then the initial order of V.

With \( h_{\text{es}(T), h_{\text{glf}}} \) and \( h_{\text{es}(T), h_{\text{rho}}} \) the subordinate equipment selection heuristic works through the GLF_E ordered request queue calculating the excess weight of \( u \) in the All higher-order environment, then the revised excess weight when lower-order equipment is selected at \( u \) or at \( u \) and \( v \) (in cases where request \( v \) has already been processed and lower-order equipment has been selected). The difference between these two weights is calculated. If \( \Delta W(u) \) is less than or equal to the threshold \( T \), the heuristic selects lower-order equipment at \( u \). The subordinate equipment selection heuristic can be described using pseudo-code:

```
foreach request \( u \in V \) do
    \( W(u) = \sum_{v \in N(u)} E^{u,v}_{g_{\text{H}}(u,v)} \);
    \( W'(u) = \sum_{v \in N(u)} E^{u,v}_{g_{\text{L}}(u),g(v)} \);
    \( \Delta W(u) = W'(u) - W(u) \);
    if \( \Delta W(u) \leq T \) then
        set \( g(u) = g_{\text{L}}(u) \);
    else
        set \( g(u) = g_{\text{H}}(u) \);
    end
end
```

**Algorithm 1:** Pseudocode for the equipment selection heuristic subordinate to \( h_{\text{glf}} \) and \( h_{\text{rho}} \).

The heuristic \( h_{\text{res}(p), h_{\text{rand}}} \) is configured such that, on average, lower-order equipment selections are obtained for a proportion of the request queue given by \( p \cdot |V| \). For the
initial run, $p$ was set to a value of 0.5.

```plaintext
foreach request $u \in V$ do
    generate a random number $x \sim U([0, 1])$;
    set $p$;
    if $x \leq p$ then
        set $g(u) = g_L(u)$;
    else
        set $g(u) = g_H(u)$;
    end
end
```

Algorithm 2: Pseudocode for the equipment selection heuristic subordinate to $h_{\text{rand}}$

Once the equipment selection procedure has run, the request queue is ordered according to GLF or RHO procedures for $h_{es(T)}, h_{\text{glf}}$ and $h_{es(T)}, h_{\text{rho}}$ and a random ordering is generated for $h_{res(p)}, h_{\text{rand}}$. The request queue is then exposed to a sequential frequency assignment.

Again, with $h_{\text{rand}}$, one hundred orderings are applied to $V$ ahead of frequency assignment and this configuration of the heuristic with equipment selection is denoted by $h_{res(p=0.5), h_{\text{rand}}}^{100}$

### 5.4.1 The equipment selection criteria

Here, the specification of the equipment selection criteria used with $h_{es(T)}, h_{\text{glf}}$ and $h_{es(T)}, h_{\text{rho}}$ is analysed. If the excess weights are normalised to a count of 1.75 MHz bandwidth segments then the smallest possible constraint greater than zero is two, corresponding to $E_{s,t}^{u,v} = 3.5$ MHz. Therefore, all $\Delta W$ values are divisible by 2 and the equipment selection criteria has been developed on this basis. 10 criteria are specified where the threshold $T = -2 \cdot k$ for $k = \{1, 2, ..., 10\}$. Therefore, each run of the
heuristic delivers 10 results per problem and we denote these particular configurations using $h_{es(T)}^{10}$.Gl$ and $h_{es(T)}^{10}$.R$

The numbers of lower-order selections across the range of equipment selection criteria used in these runs is evaluated and the gain associated with each criterion calculated. Figure 5.3 shows a CDF of the lower-order selections made for both $h_{es(T)}^{10}$.Gl$ and $h_{es(T)}^{10}$.R together with the CDF of lower-order selections made by the solver. The x-axis is a count of requests where the frequency assignment method selects lower-order modulation equipment and the y-axis is the probability that this count will occur e.g. when $T = -2$, an analysis of the selections made over fifty problems gives the one hundredth percentile $p_{1.0}$ a value of 18, the ninetieth percentile $p_{0.9}$ a value of 15 and so on.

From a start point of $T = -2$ the criteria becomes progressively more conservative and adjustments have a radical effect on the number of lower-order selections made. With $T = -2$ then $p_{0.5} = 11$ but when $T = -20$, then $p_{0.5} = 0$. It is clear that the criteria is relatively conservative with respect to the numbers of lower-order selections made by the solver.

The number of problems where a specific criterion delivers a reduction in span can be counted and expressed as a gain in MHz. That is, the reduction in span when results are compared with those delivered by the complimentary heuristic operating in the All higher-order modulation environment. To investigate the nature of these gains, the mean gain delivered by a criterion is calculated.

**Definition 22.** Let $C_{es} = \{T_1, T_2, ..., T_{|C_{es}|}\}$ denote the set of equipment selection criteria available. Let $sp(h_{L,H})$ and $sp(k_H)$ denote the spans obtained from heuristics operating in the Mixed modulation and All higher-order modulation environments respectively. Then, for a pair of heuristics $h, k$ addressing a problem $i \in P$ and using a criterion $j \in C_{es}$, let the gain obtained by the equipment selection criterion be calculated using:
5.4 Heuristics with equipment selection

Figure 5.3: CDF of lower-order selections delivered by the equipment selection criteria.

\[ G_{ij}(h_{L,H}, k_H) = sp(h_{L,H}) - sp(k_H), \quad (5.9) \]

and, neglecting any reductions in span, let the mean gain obtained when using a criterion \( j \) in combination with a heuristic \( h_{L,H} \), evaluated over all of \( P \) be given by:

\[ \overline{G}_j(h_{L,H}, k_H) = \frac{1}{|P|} \cdot \sum_{i \leq |P|, \, sp(h_{L,H}) < sp(k_H)} G_{ij}(h_{L,H}, k_H). \quad (5.10) \]

Therefore, \( \overline{G}_j(h_{L,H}, k_H) \) denotes the mean gain over \( P \) for a criterion \( j \) where gain is considered in cases where \( sp(h_{L,H}) < sp(k_H) \) only.

The pairs \( h_{glf}, h^{10}_{es(T)}, h_{glf} \) and \( h_{rho}, h^{10}_{es(T)}, h_{rho} \) are defined and Figure 5.4 shows the
mean gain delivered by the equipment selection criteria over the problem set. The graph shows that $h_{glf}^{10}(T)$ delivers more gain than $h_{rho}^{10}(T)$ over the complimentary heuristic when the equipment selection threshold is from \{T_1, T_2, ..., T_6\} but gives a lower gain when the more conservative criteria \{T_7, T_8, ..., T_{10}\} is applied.

Earlier, it was established that the RHO technique outperforms GLF and these results for gain suggest that there is less potential for the equipment selection heuristic to reduce span when it is paired with a superior ordering technique. Figure 5.4 shows that the gains only begin to correlate when the equipment selection criteria is conservative. Further, this suggests that an on the fly equipment selection heuristic has the potential to obtain radical reductions in span because in an online environment any optimisation of the frequency assignment available from an ordering of the request queue must be neglected by the assigner.
5.4 Heuristics with equipment selection

5.4.2 Set covering analysis

A more precise measure of performance can be obtained using a set covering analysis\([37, 38]\) where the spans obtained by each pair of heuristics over the problem set \(P\) are compared and between each heuristic and the frequency assignment methods delivering exact solutions.

Using the approach taken by Hurley \textit{et al.} [21] and making full use of the offline environment, the best span delivered by each heuristic is selected. That is, in cases where the heuristic is configured to deliver more than one solution per problem, the smallest span from this set of solutions is taken: for \(h_{es(T)}.h_{glf}\) and \(h_{es(T)}.h_{rho}\), this is the smallest span from ten solutions while for \(h_{rand}^{100}\) and \(h_{res(p=0.5)}.h_{rand}^{100}\), the smallest span from one hundred solutions is taken.

Two configurations of the frequency assignment method associated with the IP Solver and exact solutions are included in the set covering analysis.

\textbf{Definition 23.} Let \(IP_H\) and \(IP\) denote configurations of the frequency assignment method delivering exact optimal solutions constrained to the All-higher-order modulation and Mixed modulation environments respectively. Let \(h\) and \(k\) denote a pair of frequency assignment methods from \(\{h_{glf}, h_{rho}, h_{rand}, h_{es(T)}.h_{glf}, h_{es(T)}.h_{rho}, h_{res(p=0.5)}.h_{rand}, IP_H, IP\}\). Let \(sp(h)\) denote the span delivered by \(h\) and let the set covering \(c(h, k)\) denote the percentage of problems where \(sp(h) \leq sp(k)\). Let \(\overline{c}(h)\) denote the average of \(c(h, k)\) for \(h\) over all other frequency assignment methods.

Table 5.2 sets out the results of the set covering analysis including the average cover afforded by a frequency assignment method over all other methods \(\overline{c}(h)\).

The frequency assignment methods can be ranked by the average cover.

Table 5.3 ranks \(IP\) first since the IP solver delivers a set of exact solutions. \(IP_H\) is ranked second. It is expected that \(IP_H\) will deliver a 100% cover for all other frequency
### 5.4 Heuristics with equipment selection

<table>
<thead>
<tr>
<th>Method</th>
<th>( h_{glf} )</th>
<th>( h_{rho} )</th>
<th>( h_{100 \ rand} )</th>
<th>( h_{es(T) \ glf} )</th>
<th>( h_{es(T) \ rho} )</th>
<th>( h_{res(p=0.5) \ 100 \ rand} )</th>
<th>( IP_H )</th>
<th>( IP )</th>
<th>( \tau(h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{glf} )</td>
<td>-</td>
<td>70</td>
<td>84</td>
<td>72</td>
<td>62</td>
<td>100</td>
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<td>( h_{rho} )</td>
<td>96</td>
<td>-</td>
<td>92</td>
<td>86</td>
<td>86</td>
<td>100</td>
<td>48</td>
<td>28</td>
<td>76.6</td>
</tr>
<tr>
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<td>( h_{es(T) \ glf} )</td>
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<td>78</td>
<td>90</td>
<td>-</td>
<td>84</td>
<td>98</td>
<td>50</td>
<td>28</td>
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</tr>
<tr>
<td>( h_{es(T) \ rho} )</td>
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<td>98</td>
<td>54</td>
<td>32</td>
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</tr>
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<td>( h_{res(p=0.5) \ 100 \ rand} )</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>4</td>
<td>6</td>
<td>-</td>
<td>2</td>
<td>2</td>
<td>4.6</td>
</tr>
<tr>
<td>( IP_H )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>94</td>
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<td>-</td>
<td>54</td>
<td>91.7</td>
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<tr>
<td>( IP )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>-</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 5.2: Set covering analysis for the frequency assignment methods

<table>
<thead>
<tr>
<th>ranking</th>
<th>method</th>
<th>( \tau(h) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( IP )</td>
<td>100.0</td>
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<tr>
<td>2</td>
<td>( IP_H )</td>
<td>91.7</td>
</tr>
<tr>
<td>3</td>
<td>( h_{es(T) \ 10 \ rho} )</td>
<td>78.6</td>
</tr>
<tr>
<td>4</td>
<td>( h_{rho} )</td>
<td>76.6</td>
</tr>
<tr>
<td>5</td>
<td>( h_{es(T) \ 10 \ glf} )</td>
<td>74.0</td>
</tr>
<tr>
<td>6</td>
<td>( h_{glf} )</td>
<td>63.4</td>
</tr>
<tr>
<td>7</td>
<td>( h_{100 \ rand} )</td>
<td>37.4</td>
</tr>
<tr>
<td>8</td>
<td>( h_{res(p=0.5) \ 100 \ rand} )</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Table 5.3: Ranking of the frequency assignment methods

Assignment methods operating in the All-higher-order modulation environment and the results confirm that this is the case. However, \( IP, h_{es(T) \ glf} \) and \( h_{es(T) \ rho} \), operating in the Mixed modulation environment, are able to improve on the spans delivered by \( IP_H \) for a sub-set of \( P \). Therefore, the cover afforded to these frequency assignment methods by \( IP_H \) is less than 100%. This is considered to be a very important result and strong evidence in support of an equipment selection paradox: Exact, optimal solutions have been obtained by \( IP_H \) while constrained to the All-higher-order modulation environment.
environment but these can be improved upon by a configuration of the method delivering exact solutions and heuristics (non-optimal) that double the bandwidth requirement for selected requests in the Mixed modulation environment.

We have described how the equipment selection heuristic works through a GLF ordered request queue and, for some criterion, makes precisely the same selections whatever ordering technique is applied to \( V \) ahead of frequency assignment. It can be concluded that the ranking of the heuristics \( h_{es(T)}^{10}, h_{glf}^{10} \) and \( h_{es(T)}^{10}, h_{rho}^{10} \) is determined by the ordering technique applied to the request queue ahead of frequency assignment. Further, both of the heuristics using RHO dominate the pair using GLF. The study has established that RHO can outperform GLF and this set covering analysis shows that a heuristic constrained to the All-higher-order modulation environment and using a superior ordering technique can outperform a heuristic using equipment selection combined with a less effective ordering technique.

\( h_{100}^{100}, h_{res(p=0.5)}^{100}, h_{rand}^{100} \) are ranked last. These results suggest that a random ordering of \( V \) in the All-higher-order modulation environment or a random equipment selection, with the number of lower-order selections set arbitrarily and combined with a random ordering of \( V \) in the Mixed modulation environment are not profitable. This underlines the importance of well designed equipment selection criteria.

## 5.5 Further analysis

This section of Chapter Five investigates the IP ordering of the request queue where the problem set is exposed to a sequential frequency assignment using the IP solver’s orderings and random equipment selections using tighter constraints on the number of lower-order equipment selections made.

The IP order of \( V \) can be obtained by ordering the frequency assignments delivered by the solver in frequency order. A variant of the simple heuristic has been developed
using these IP orderings and a sequential frequency assignment. The spans obtained from this approach are compared with those delivered by \( h_{glf} \), \( h_{rho} \) and the IP solver.

Section 5.4 showed how random equipment selections, with approximately half of the request queue assigned higher-order modulation equipment and half lower-order, produces spectrally inefficient frequency assignments. The study demonstrates how this approach can be refined by tightening the constraint on the numbers of lower-order selections made. This investigation is to clarify the importance of carefully selected equipment selection criteria and shows that, even with random selections, a refined criterion can deliver spectral efficiency gains.

### 5.5.1 IP ordering of the request queue

Spans delivered by a new variant of the simple heuristic operating in the All-higher-order modulation environment were investigated. The ordering is based on an analysis of the IP solver’s solutions. Here, \( V \) is ordered by frequency according to the assignments made by the solver with the smallest frequency listed first and with ties broken by the initial order.

**Definition 24.** Let \( h_{ip-order} \) denote a simple heuristic operating in the All-higher-order modulation environment where the order of \( V \) is determined by the frequencies assigned by the IP solver. Let the IP order of \( V \) be a listing of the request queue in frequency order with the smallest frequency listed first and with ties broken by the initial order of \( V \).

To summarise the variant: \( h_{ip-order} \) maps all requests to higher-order modulation equipment, applies the IP-ordering to \( V \) and exposes \( V \) to a sequential frequency assignment. Having obtained the IP order for \( V \) for all \( i \in P \), we expose \( P \) to \( h_{ip-order} \). Figure 5.5 compares the spans obtained by \( h_{ip-order} \), \( h_{glf} \), \( h_{rho} \) and the results given by the solver in the All-higher-order modulation environment \( IP_H \).
5.5 Further analysis

Figure 5.5: Spans delivered by the frequency assignment heuristics and IP solver in the All-higher-order modulation environment.

The results are ordered according to the spans delivered by the IP solver with the smallest span listed first in order. It can be seen by inspection that there is a close correlation of these results.

Returning to the set cover analysis (see the final set cover analysis in Table 5.4), it can be seen that $h_{ip-order}$ obtains $\bar{c}(h) = 81.1\%$, outperforming $h_{es(T)}.h_{rho} (77.6\%)$. While $h_{es(T)}.h_{rho}$ obtains higher set covers for $h_{glf}$, $h_{es(T)}.h_{glf}$ and $h_{rho}$, the new heuristic returns solutions that secure higher covers for $h_{rand}$, $h_{res(p=0.5)}.h_{rand}^{100}$, $IP$ and $IP_H$.

Again, it can be shown that a heuristic that addresses optimisation using order only can outperform those using both order and equipment selection.
5.5 Further analysis

5.5.2 Refining the random equipment selection heuristic

It has been established that random equipment selections, with the number of lower-order modulation assignments constrained to around 25 per problem on average, combined with random orderings of the request queue do not produce optimal results. This section of the thesis demonstrates how these solutions can be improved through a refinement of the subordinate equipment selection heuristic. The aim is not to optimise these results, only to demonstrate that some improvement is possible.

Section 5.4 explained how random equipment selections were made with a probability of 0.5 that lower-order modulation equipment would be selected. Here a new variant of the heuristic denoted by by \( h_{res(p=0.08)} \cdot h_{glf}^{100} \cdot h_{rand} \) is introduced. As before, a real number \( x \) is generated randomly from a standard uniform distribution but \( p \) is now set to 0.08 in order that the number of lower-order selections is constrained to around 4 per problem on average.

Having set \( p = 0.08 \) for this experimental run, some significant reductions in span are obtained. Figure 5.6 shows the spans delivered by the two configurations of the heuristic \( h_{res(p=0.5)} \cdot h_{glf}^{100} \cdot h_{rand} \) and by \( h_{res(p=0.08)} \cdot h_{glf}^{100} \cdot h_{rand} \).

Clearly, \( h_{res(p=0.08)} \cdot h_{glf}^{100} \cdot h_{rand} \) delivers significant gain. While the new heuristic gives a

<table>
<thead>
<tr>
<th>Method</th>
<th>( h_{glf} )</th>
<th>( h_{rho} )</th>
<th>( h_{res}^{100} \cdot h_{rand} )</th>
<th>( h_{res(p=0.5)} \cdot h_{glf}^{100} \cdot h_{rand} )</th>
<th>( IP_H )</th>
<th>( h_{ip-order} )</th>
<th>( h_{res(p=0.08)} \cdot h_{glf}^{100} \cdot h_{rand} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_{glf} )</td>
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<td>70</td>
<td>84</td>
<td>72</td>
<td>62</td>
<td>100</td>
<td>36</td>
</tr>
<tr>
<td>( h_{rho} )</td>
<td>96</td>
<td>100</td>
<td>92</td>
<td>86</td>
<td>100</td>
<td>48</td>
<td>28</td>
</tr>
<tr>
<td>( h_{res}^{100} \cdot h_{rand} )</td>
<td>38</td>
<td>32</td>
<td>100</td>
<td>32</td>
<td>30</td>
<td>98</td>
<td>20</td>
</tr>
<tr>
<td>( h_{res(p=0.5)} \cdot h_{glf}^{100} \cdot h_{rand} )</td>
<td>90</td>
<td>78</td>
<td>90</td>
<td>100</td>
<td>84</td>
<td>98</td>
<td>50</td>
</tr>
<tr>
<td>( h_{res(p=0.08)} \cdot h_{glf}^{100} \cdot h_{rand} )</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>4</td>
<td>6</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>( IP_H )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>94</td>
<td>94</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( IP )</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( h_{ip-order} )</td>
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<td>80</td>
<td>92</td>
<td>82</td>
<td>76</td>
<td>100</td>
<td>72</td>
</tr>
<tr>
<td>( h_{res(p=0.08)} \cdot h_{glf}^{100} \cdot h_{rand} )</td>
<td>20</td>
<td>14</td>
<td>52</td>
<td>16</td>
<td>16</td>
<td>94</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 5.4: Final set cover analysis for the frequency assignment methods
5.6 Conclusions

It has been shown that it is possible to reduce the span of a frequency assignment by taking account of raster in the ordering technique applied to the request queue. The higher span for 3 problems, there is a reduction in span for 33 problems with the sum of these reductions across the problem set $P$ equal to 1309 MHz.

The study could persist with further experimentation and refinement of these heuristics that use random equipment selections but this experimental run has already demonstrated very clearly that the use of a fairly conservative selection criterion can radically improve on the spans delivered. This particular investigation highlights the vital role of the equipment selection criteria and the importance of setting these correctly in order to obtain improvements in spectral efficiency.

**Figure 5.6:** Spans delivered by $h_{\text{res}(p=0.5)} \cdot h_{\text{rand}}^{100}$ and by $h_{\text{res}(p=0.08)} \cdot h_{\text{rand}}^{100}$
RHO technique has outperformed the well established GLF even when the heuristic using GLF runs a subordinate equipment selection heuristic and the heuristic using RHO is constrained to the All-higher-order modulation environment.

The investigations of the ordering given by the IP Solver's solutions, again, show how ordering techniques can optimise the frequency assignment and outperform heuristics using equipment selection. In this case, the ordering is entirely experimental; it is very unlikely that the IP Solver could be used to resolve real-world frequency assignment problems. It may be possible to apply RHO or other ordering techniques to particular real-world problems including, perhaps, re-tuning campaigns where there might be opportunities for the assigner to re-order the request queue. However, these ordering techniques are used here for experimental purposes, primarily to investigate the scope for using equipment selection heuristics.

Equipment selection heuristics can improve on the gains offered by heuristics operating in the All-higher-order modulation environment. This confirms our earlier analysis of the exact solutions delivered by the IP solver. It has been demonstrated, again, that the use of lower-order modulation equipment on specific links, while doubling the bandwidth requirement for these requests, can actually reduce the span of the (network) frequency assignment.

An analysis of the gain offered by these equipment selection heuristics shows that there is less potential for the heuristic when combined with a good ordering technique. While RHO outperforms GLF, the equipment selection heuristic combined with GLF achieves greater gain over its complimentary heuristic than the equipment selection heuristic combined with RHO. The findings, including the use of random orderings, suggest that there is a great deal of scope for an on-line equipment selection heuristic; an environment where there is little or no opportunity for the assigner to make use of ordering techniques.

The results achieved here are sometimes sub-optimal and there was no expectation that these experimental heuristics would cover all of the IP solver’s solutions. However,
the results are very encouraging and often the heuristics are able to achieve the spans delivered by the solver or achieve spans that are very close to these exact, optimal solutions.
Online heuristics

Chapter Five discussed the development of offline heuristics and showed how heuristics using equipment selection can achieve spectrally efficient results. Chapter Six reports on the design of online heuristics. Here, motivated by the real-world environment, the assigner works in a simulated online environment defined by the following features:

**First-come-first-served rule.** The assigner respects a *first-come-first-served rule* where requests are handled strictly in the same order that they appear in the request queue. There are no opportunities for the assigner to reorder requests or to anticipate the constraints associated with future requests.

**Constraints.** The assigner is required to satisfy all of the frequency separation constraints associated with the request under consideration and all established fixed links. No account is taken of future requests.

**Sequential frequency assignment.** The first available frequency is assigned to a request with the assigner working up-band from the smallest frequency.

The sequential frequency assignment procedure simulates an online service where the constraints associated with the request under consideration and those requests already processed are considered. The equipment selection procedures are also configured to work in this way online. Re-ordering of the request queue is not possible and so any objective analysis of the complete graph is impossible except when the final request in a problem is considered.
Informed by the techniques developed in the offline environment, this Chapter of the thesis sets out some designs for online heuristics. This reflects the constraints placed on the assigner in the real-world where greedy, sequential frequency assignment methods are used to handle the request queue.

6.1 The equipment selection heuristic

This section of Chapter Six sets out the design principles derived from the development of offline heuristics and discusses how these ideas can be applied in the online environment.

6.1.1 Design principles

In the offline environment, a range of equipment selection criteria can be used and the best span delivered by the heuristic can be selected for each problem $i \in P$. For experimental purposes, this approach can also be used in a simulated online environment but, ultimately, the task here is to design heuristics and specify criteria that is effective when a problem exposed to just one run of the equipment selection heuristic.

Because the offline heuristics selected the best span available from multiple runs, the problem of loss, by which we mean an increase in span, was noticeable but did not dominate the problem. Loss can be defined by results for span where the equipment selection heuristic delivers a larger span than its complimentary benchmark heuristic.

**Definition 25.** Let $sp_i(h_{ex})$ and $sp_i(k)$ denote spans delivered for problem $i$ by a heuristic using equipment selection and a complimentary heuristic constrained to the All-higher-order modulation environment respectively. Then if $sp_i(h_{ex}) > sp_i(k)$, $sp_i(h_{ex})$ is said to have delivered a loss with respect to the complimentary heuristic for problem $i$. 
Specifying efficient equipment selection criteria in the online environment where positive gains (reductions in span) are obtained for some problems while minimising loss across the entire problem set is very challenging. Although it is not possible to apply any ordering techniques in the online environment, it is possible to encourage the selection of lower-order modulation equipment on particular rasters through specification of the appropriate equipment selection criteria.

Alternative criteria can be applied to sub-sets of the request queue and each of these sub-sets can be defined as an epoch in the frequency assignment process. Further, the criteria can be adjusted for different equipment selections. That is, a set of migrations can be defined where each migration defines a lower-order modulation equipment selection in place of a higher-order selection on an adjacent raster. Each migration can be associated with a specific lower-order modulation equipment selection threshold and these can vary per epoch.

**Definition 26.** Let \( m_{r_p,r_q}(u) \) denote a migration of radio equipment from raster \( p \) to raster \( q \) for request \( u \). That is, a lower-order modulation equipment selection at \( u \) on raster \( q \) in place of a higher-order modulation selection on the adjacent raster \( p \) where \( p < q \).

**Definition 27.** Let an epoch be defined as a closed interval \([a, b]\) where \( a \) and \( b \) are positive real numbers representing requests for frequency assignment. Let the epoch be associated with a set of equipment selection criteria for each migration available to the assigner \( \{T(m_{r_1,r_2}), T(m_{r_2,r_3}), \ldots, T(m_{r_4,r_5})\} \).

### 6.1.2 Specification of online heuristics

The study has produced a number of online heuristics including some required to inform design and others for benchmarking.

\( h_{io} \) denotes a online heuristic operating in the All-higher-order modulation environment that applies a sequential frequency assignment to the request queue in its
6.1 The equipment selection heuristic

The equipment selection heuristic $h_{es(p)}$ is configured to select lower- or higher-order modulation equipment randomly with a probability $p$ that lower-order is selected. The request queue is exposed to a sequential frequency assignment with the initial order respected.

$h_{es^{10}}$ allows for a range of equipment selection criteria to be tested and is configured such that each problem is exposed to ten criterion and ten sequential frequency assignments with the initial order of the request queue respected.

$h_{es(m)}$ exposes problems to a set of criteria where each migration is given a specific threshold for the selection of lower-order modulation equipment. Problems are exposed to a sequential frequency assignment with the initial order of the request queue respected.

$h_{es(m,e)}$ combines the use of specific thresholds per migration type with the specification of epochs. Each problem is exposed to a set of criteria per migration per epoch. Problems are exposed to a sequential frequency assignment with the initial order of the request queue respected.

Some further elaboration of these heuristics and their purpose is given here. In all cases, the heuristics are run in a simulated online environment.

With $h_{io}$, the assigner exposes the request queue to a sequential frequency assignment in the All-higher-order modulation, online, environment. This heuristic provides a useful benchmark for the heuristics using equipment selection. Gain (reductions in span) can easily be calculated when $h_{io}$ is used as a complimentary frequency assignment method, or benchmark, to the heuristics using equipment selection.

While $h_{io}$ provides a very useful benchmark, it is unrealistic to assume that all real-world frequency assignments are made in the All-higher-order modulation environment. To the assigner, working online without equipment selection heuristics, requests for frequency assignments to links using lower-order equipment may appear entirely
random. On this basis the study has developed a second benchmark heuristic $h_{es(p)}$,$h_{io}$ where lower-order selections are made randomly.

In the online environment, running alternative criteria or heuristics over the entire problem set and then selecting the best results from each run is unrealistic. However, in order to inform design, the problem set is exposed to a range of equipment selection criteria using the $h_{es}^{10}$,$h_{io}$ configuration.

The assigner can tune the equipment selection criteria to the different migration types. In order to encourage migration from a raster $p$ to $q$ in particular, the criterion for this particular migration can be relaxed and the configuration $h_{es(m)}$,$h_{io}$ facilitates this.

The specification of epochs in the frequency assignment process allows for further refinement. Here, criteria can be more conservative in an early epoch and less conservative in a latter epoch, say. The criteria can be adjusted to encourage selection of wider bandwidth systems in the latter epochs where the interference graph is more developed and there is more information about the weight of constraints incident to the request under consideration; $h_{es(m,e)}$,$h_{io}$ is configured to support these techniques.

### 6.2 Results

The study first of all investigated the results delivered by $h_{io}$ and $h_{es}^{10}$,$h_{io}$. The purpose of these initial runs was to investigate the gains delivered by ten configurations of $h_{es}^{10}$,$h_{io}$, using the ten equipment selection criteria reported in Chapter Five, when compared with the spans given by the benchmark $h_{io}$.

In the real-world online environment, it is not credible to select the best spans available per problem $i \in P$ from multiple runs of the heuristics as practised in the offline studies. This approach tends to mask the losses associated with a specific criterion but loss is an important parameter to be considered in the online environment. On this basis, Table 6.1 reports on the gains and losses in span associated with each criterion.
j delivered on each run of $h_{es}^{10}, h_{io}$ and denoted by $G_{j}^{total}$ and $L_{j}^{total}$ respectively. The percentage of $P$ contributing to these gains and losses is given e.g. when $T = -2$, $G_{j}^{total} = 105$ MHz and this value is the sum of gains over the 10 % of $P$ where gains were obtained. In addition, the cover $c(h)$ afforded to $h_{io}$ by $h_{es}^{10}, h_{io}$ is set out here.

<table>
<thead>
<tr>
<th>T</th>
<th>$G_{j}^{total}$</th>
<th>%P</th>
<th>$L_{j}^{total}$</th>
<th>%P</th>
<th>$c(h_{io})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>105</td>
<td>10</td>
<td>990.5</td>
<td>54</td>
<td>46</td>
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<tr>
<td>-4</td>
<td>105</td>
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<td>-6</td>
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<td>10</td>
<td>759.5</td>
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</tr>
<tr>
<td>-8</td>
<td>105</td>
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<td>675.5</td>
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<td>56</td>
<td>2</td>
<td>140.5</td>
<td>6</td>
<td>94</td>
</tr>
</tbody>
</table>

Table 6.1: Initial results

The results show that the best gains are obtained when $T \in \{-2, -4, -6, -8\}$. However, the losses are significant with the smallest obtained from this sub-set of the criteria when $T = -8$. The best cover for $h_{io}$ and minimal loss are obtained when the threshold is conservative and set to -20. However, there is gain for just 2% of problems when this criterion is used.

Some further analysis gives the mean number of migrations per equipment selection criterion per problem. Table 6.2 sets out these results.
6.3 Refinements and further investigation

Using the initial results, the study has developed configurations of \( h_{es(m), hiio} \) allowing for specific migrations to be encouraged and others dampened. Because \( T_{s_L(u)} = -8 \) gave the highest positive gain combined with a relatively low loss, the study focused on configurations of \( h_{es(m), hiio} \) anchored around this criterion.

Tables 6.3 and 6.4 set out some results from these runs. Here, the lower-order equipment selection threshold is specified per migration using the syntax: \( \{ T_{m_{r_1,r_2}}, T_{m_{r_2,r_3}}, T_{m_{r_3,r_4}}, T_{m_{r_4,r_5}} \} \).

The results given in these tables show how the equipment selection criteria can be tuned in order to encourage or dampen selections for specific lower-order modulation equipment types and the effects these criteria have on gain and loss.

A further consideration is the application of specific criteria to particular epochs in the evolution of the request queue using the \( h_{es(m,e), hiio} \) heuristic. This approach was
6.3 Refinements and further investigation

Table 6.3: Results for the $h_{es(m)}$ heuristic

<table>
<thead>
<tr>
<th>$T$</th>
<th>$G_j^{total}$</th>
<th>$% P$</th>
<th>$I_j^{total}$</th>
<th>$% P$</th>
<th>$c(h_{io})$</th>
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</thead>
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<tr>
<td>${-8, -8, -8, -2}$</td>
<td>105</td>
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<td>54</td>
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<td>5</td>
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<td>54</td>
</tr>
<tr>
<td>${-8, -8, -8, -6}$</td>
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<td>5</td>
<td>759.5</td>
<td>20</td>
<td>60</td>
</tr>
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<td>5</td>
<td>675.5</td>
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</tr>
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<td>535.5</td>
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</tr>
<tr>
<td>${-8, -8, -12, -8}$</td>
<td>105</td>
<td>5</td>
<td>577.5</td>
<td>17</td>
<td>66</td>
</tr>
<tr>
<td>${-8, -8, -14, -8}$</td>
<td>77</td>
<td>4</td>
<td>577.5</td>
<td>17</td>
<td>66</td>
</tr>
<tr>
<td>${-8, -8, -16, -8}$</td>
<td>77</td>
<td>4</td>
<td>577.5</td>
<td>17</td>
<td>66</td>
</tr>
<tr>
<td>${-8, -8, -18, -8}$</td>
<td>77</td>
<td>4</td>
<td>577.5</td>
<td>17</td>
<td>66</td>
</tr>
<tr>
<td>${-8, -8, -20, -8}$</td>
<td>77</td>
<td>4</td>
<td>577.5</td>
<td>17</td>
<td>66</td>
</tr>
</tbody>
</table>

taken based on the following rationale: In the simulated online environment, the complete graph $G$ is only complete once the entire request queue has been processed. As the heuristic works through the request queue, edges are formed between the request
Table 6.4: Mean number of migrations per problem per criterion

<table>
<thead>
<tr>
<th>$T$</th>
<th>$m_{r_1,r_2}(u)$</th>
<th>$m_{r_2,r_3}(u)$</th>
<th>$m_{r_3,r_4}(u)$</th>
<th>$m_{r_4,r_5}(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${-8, -8, -8, -2}$</td>
<td>0.5</td>
<td>0.04</td>
<td>1.08</td>
<td>2.2</td>
</tr>
<tr>
<td>${-8, -8, -8, -4}$</td>
<td>0.5</td>
<td>0.04</td>
<td>1.08</td>
<td>2.10</td>
</tr>
<tr>
<td>${-8, -8, -8, -6}$</td>
<td>0.48</td>
<td>0.04</td>
<td>1.08</td>
<td>1.92</td>
</tr>
<tr>
<td>${-8, -8, -8, -8}$</td>
<td>0.48</td>
<td>0.04</td>
<td>1.08</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -8, -10}$</td>
<td>0.42</td>
<td>0.02</td>
<td>1.04</td>
<td>0.96</td>
</tr>
<tr>
<td>${-8, -8, -8, -12}$</td>
<td>0.42</td>
<td>0.02</td>
<td>1.04</td>
<td>0.92</td>
</tr>
<tr>
<td>${-8, -8, -8, -14}$</td>
<td>0.40</td>
<td>0.02</td>
<td>1.04</td>
<td>0.80</td>
</tr>
<tr>
<td>${-8, -8, -8, -16}$</td>
<td>0.40</td>
<td>0.02</td>
<td>1.04</td>
<td>0.76</td>
</tr>
<tr>
<td>${-8, -8, -8, -18}$</td>
<td>0.42</td>
<td>0.02</td>
<td>1.02</td>
<td>0.28</td>
</tr>
<tr>
<td>${-8, -8, -8, -20}$</td>
<td>0.42</td>
<td>0.02</td>
<td>1.02</td>
<td>0.28</td>
</tr>
<tr>
<td>${-8, -8, -8, -22}$</td>
<td>0.44</td>
<td>0.02</td>
<td>1.02</td>
<td>0.20</td>
</tr>
<tr>
<td>${-8, -8, -2, -8}$</td>
<td>0.50</td>
<td>0.06</td>
<td>1.76</td>
<td>1.84</td>
</tr>
<tr>
<td>${-8, -8, -4, -8}$</td>
<td>0.50</td>
<td>0.06</td>
<td>1.70</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -6, -8}$</td>
<td>0.48</td>
<td>0.04</td>
<td>1.22</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -8, -8}$</td>
<td>0.48</td>
<td>0.04</td>
<td>1.08</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -10, -8}$</td>
<td>0.44</td>
<td>0.04</td>
<td>0.14</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -12, -8}$</td>
<td>0.44</td>
<td>0.04</td>
<td>0.12</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -14, -8}$</td>
<td>0.44</td>
<td>0.04</td>
<td>0.02</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -16, -8}$</td>
<td>0.44</td>
<td>0.04</td>
<td>0.02</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -18, -8}$</td>
<td>0.44</td>
<td>0.04</td>
<td>0.00</td>
<td>1.82</td>
</tr>
<tr>
<td>${-8, -8, -20, -8}$</td>
<td>0.44</td>
<td>0.04</td>
<td>0.00</td>
<td>1.82</td>
</tr>
</tbody>
</table>

under consideration and the established vertices of the graph (requests to which an equipment type and frequency are already assigned). Therefore, the assigner has little or no knowledge of the risk involved with an equipment selection made in an early
epoch; a request will be connected to established vertices but the assigner will not be aware of future requests and their contributions to the weight of the request under consideration or of the established vertices.

As the heuristic works its way through the request queue and enters the latter epochs, $G$ develops and the risk associated with the equipment selections is reduced.

Three epochs are considered in the evolution of the request queue.

**First epoch.** Risk can be reduced by using a very conservative equipment selection criteria or by designating the first epoch as an All-higher-order modulation environment.

**Second epoch.** $G$ is more developed and this epoch is exposed to a relatively conservative equipment selection criteria.

**Third epoch.** In this final epoch, $G$ is highly developed and a less conservative criteria is used.

In the example given below (Tables 6.5 and 6.6), three epochs are defined: [1, 20], [21, 40], [41, 50]. Here, the equipment selection heuristic is left idle in the first epoch and while the criteria stays anchored around a threshold of -8 in the second and third epochs, the criteria are tuned for the $m_{r_4, r_5}(u)$ migrations. The second epoch uses a relatively conservative criteria (-100) and the third epoch a relatively non-conservative criteria (-18).

<table>
<thead>
<tr>
<th>$T$</th>
<th>$G_j^{total}$</th>
<th>$P$</th>
<th>$I_j^{total}$</th>
<th>$P$</th>
<th>$c(h_{io})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${-8, -8, -8, -100}, {-8, -8, -8, -18}$</td>
<td>84</td>
<td>4</td>
<td>70</td>
<td>4</td>
<td>96</td>
</tr>
</tbody>
</table>

Table 6.5: Results for the $h_{es(m,e)}, h_{io}$ heuristic

These results show that it is possible to tune the criteria in order that $G_j^{total} > I_j^{total}$ but the payoff is a reduced value for $G_j^{total}$.

Using $h_{io}$ as the *complimentary* heuristic when evaluating gain is extremely challenging when the objective is a minimum span frequency assignment. $h_{es(p)}, h_{io}$ allows
6.4 Validation

This section reports on some further testing of the heuristics in the online environment using an alternative problem set. These tests were run in order to unveil any idiosyncrasies in the problem data and to ensure consistent performance. Here, the $h_{es(m,e)}h_{io}$ heuristic is tested on a second set of fifty problems. The heuristic is run ten times per problem with the criteria adjusted per run for the $m_{r1,r2}(u)$ migration. The results are set out in Tables 6.8 and 6.9.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$m_{r1,r2}(u)$</th>
<th>$m_{r2,r3}(u)$</th>
<th>$m_{r3,r4}(u)$</th>
<th>$m_{r4,r5}(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${-8,-8,-8,-100}$</td>
<td>0.18</td>
<td>0.02</td>
<td>0.62</td>
<td>0.00</td>
</tr>
<tr>
<td>${-8,-8,-8,-18}$</td>
<td>0.16</td>
<td>0.00</td>
<td>0.20</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 6.6: Mean number of migrations per problem per criterion using the $h_{es(m,e)}h_{io}$ heuristic.

for a more realistic comparison and this heuristic can be configured to allow for lower-order equipment selections to be made with a probability $p$. These arrangements simulate the arrival of requests requiring the use of lower-order modulation equipment in the real-world online environment.

Having configured $h_{es(p)}h_{io}$ for $p = 0.2$, table 6.7 sets out values for $G_{total}^j$, $L_{total}^j$ and $c(h_{es(p)}h_{io})$ when $h_{es(p)}h_{io}$ is the complimentary heuristic to $h_{es(m,e)}h_{io}$.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$G_{total}^j$</th>
<th>$P$</th>
<th>$L_{total}^j$</th>
<th>$P$</th>
<th>$c(h_{es(p)}h_{io})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${-8,-8,-8,-100}, {-8,-8,-8,-18}$</td>
<td>1032.5</td>
<td>31</td>
<td>42</td>
<td>1</td>
<td>98</td>
</tr>
</tbody>
</table>

Table 6.7: Supplementary results for $h_{es(m,e)}h_{io}$ heuristic

Here, $h_{es(m,e)}h_{io}$ delivers a very significant positive gain over a complimentary heuristic that selects lower-order systems randomly. However, the extra gain afforded by $h_{es(m,e)}h_{io}$ over $h_{io}$ is limited to the gains reported in table 6.5.
Table 6.8: Results for the $h_{es(m)}\cdot h_{io}$ heuristic (alternative problem set).

<table>
<thead>
<tr>
<th>$T$</th>
<th>$G_j^{total}$</th>
<th>$% P$</th>
<th>$L_j^{total}$</th>
<th>$% P$</th>
<th>$c(h_{io})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${-8,-8,-2,-8}$</td>
<td>56</td>
<td>2</td>
<td>794.5</td>
<td>24</td>
<td>52</td>
</tr>
<tr>
<td>${-8,-8,-4,-8}$</td>
<td>56</td>
<td>2</td>
<td>766.5</td>
<td>23</td>
<td>54</td>
</tr>
<tr>
<td>${-8,-8,-6,-8}$</td>
<td>42</td>
<td>2</td>
<td>738.5</td>
<td>22</td>
<td>56</td>
</tr>
<tr>
<td>${-8,-8,-8,-8}$</td>
<td>42</td>
<td>2</td>
<td>710.5</td>
<td>21</td>
<td>58</td>
</tr>
<tr>
<td>${-8,-8,-10,-8}$</td>
<td>28</td>
<td>1</td>
<td>710.5</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>${-8,-8,-12,-8}$</td>
<td>28</td>
<td>1</td>
<td>710.5</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>${-8,-8,-14,-8}$</td>
<td>28</td>
<td>1</td>
<td>710.5</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>${-8,-8,-16,-8}$</td>
<td>28</td>
<td>1</td>
<td>710.5</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>${-8,-8,-18,-8}$</td>
<td>28</td>
<td>1</td>
<td>710.5</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>${-8,-8,-20,-8}$</td>
<td>28</td>
<td>1</td>
<td>710.5</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 6.9: Mean number of migrations per problem per criterion (alternative problem set).

<table>
<thead>
<tr>
<th>$T$</th>
<th>$m_{r_1,r_2}(u)$</th>
<th>$m_{r_2,r_3}(u)$</th>
<th>$m_{r_3,r_4}(u)$</th>
<th>$m_{r_4,r_5}(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${-8,-8,-2,-8}$</td>
<td>0.78</td>
<td>0.08</td>
<td>2.14</td>
<td>1.94</td>
</tr>
<tr>
<td>${-8,-8,-4,-8}$</td>
<td>0.78</td>
<td>0.08</td>
<td>1.92</td>
<td>1.94</td>
</tr>
<tr>
<td>${-8,-8,-6,-8}$</td>
<td>0.74</td>
<td>0.08</td>
<td>1.52</td>
<td>1.92</td>
</tr>
<tr>
<td>${-8,-8,-8,-8}$</td>
<td>0.74</td>
<td>0.08</td>
<td>1.34</td>
<td>1.92</td>
</tr>
<tr>
<td>${-8,-8,-10,-8}$</td>
<td>0.74</td>
<td>0.06</td>
<td>0.26</td>
<td>1.90</td>
</tr>
<tr>
<td>${-8,-8,-12,-8}$</td>
<td>0.74</td>
<td>0.06</td>
<td>0.24</td>
<td>1.90</td>
</tr>
<tr>
<td>${-8,-8,-14,-8}$</td>
<td>0.74</td>
<td>0.06</td>
<td>0.16</td>
<td>1.90</td>
</tr>
<tr>
<td>${-8,-8,-16,-8}$</td>
<td>0.74</td>
<td>0.06</td>
<td>0.12</td>
<td>1.90</td>
</tr>
<tr>
<td>${-8,-8,-18,-8}$</td>
<td>0.72</td>
<td>0.06</td>
<td>0.04</td>
<td>1.90</td>
</tr>
<tr>
<td>${-8,-8,-20,-8}$</td>
<td>0.72</td>
<td>0.06</td>
<td>0.04</td>
<td>1.90</td>
</tr>
</tbody>
</table>
Graphs of gain and loss are shown in figures 6.1 and 6.2 where the heuristic is run against both the main problem set used throughout the study and the alternative problem set.

The graphs of gains and losses follow similar trends for both problem sets with both decreasing as the criteria becomes more conservative for the \( m_{r_3,r_4}(u) \) migration, eventually settling to a non-decreasing graph.

While different problems may pose more or less of a challenge for the heuristics, this simple test confirms that the heuristic runs successfully on an alternative problem set and delivers results with trends that are consistent with the results obtained from the main problem set.

### 6.5 Conclusions

The development of these experimental online heuristics has demonstrated that it is possible to run equipment selection procedures in this very challenging environment and achieve some reductions in span.

In an experimental, simulated, environment, the minimum span objective is a very useful measure of packing efficiency and an indication of how careful selection of lower-order systems can reduce interference and increase the potential for frequency reuse.

Spectral efficiency questions in the real-world online environment are likely to be dominated by questions of assignment difficulty rather than span and in well established frequency bands the expectation is that all available frequencies are utilised. The key issues here are mitigation of excess interference and frequency re-use.

These results have indicated that while reductions in span can be difficult to achieve online, using equipment selection heuristics to reduce the weight of excess constraints on a graph while maintaining data throughput is possible.
6.5 Conclusions

Figure 6.1: Gains obtained over ten sets of equipment selection criteria

Figure 6.2: Losses obtained over ten sets of equipment selection criteria
Conclusions

7.1 General observations

This thesis has proposed the existence of an equipment selection paradox. Specifically, a paradoxical relationship between the selection of lower-order modulation equipment which doubles the bandwidth requirement on individual fixed links and the potential to actually reduce the overall span of frequencies required for a network frequency assignment when these selections are made.

The academic literature sets out excellent mathematical descriptions of the frequency assignment problem but these are often generalised and usually neglect the practical considerations and constraints that are accounted for by the assigner working in professional practice. While the study has not attempted to reproduce the tools used in practice, the mathematical description developed in the study is service specific; that is, particular to the microwave fixed links frequency assignment problem and extended to include an equipment selection procedure. Some important real-world constraints and features such as multi-raster channel plans are captured in the mathematical description of the fixed links frequency assignment problem with equipment selection.

A set of fixed links problem instances has been developed by the study and IP formulations of the frequency assignment problem with equipment selection have allowed for a set of exact solutions to be delivered by a standard IP solver. This provides some very strong evidence in support of the hypothesis. The solver has obtained minimum spans
for fifty problems with some significant spectral efficiency gains observed when results from an IP formulation using equipment selection and a formulation constrained to the All higher-order modulation environment are compared.

Experimental *offline* and *online* heuristics developed in the study are a useful step towards the provision of equipment selection methods that can be applied in a real-world environment. With these heuristics, the study has demonstrated that a mathematical analysis of the radio interference environment using graph-theoretic methods allows for spectrally efficient decisions to be made regarding equipment selection.

The study has developed the new RHO technique for ordering the request queue and, using this in the offline environment, demonstrated superior performance over the well established GLF technique.

The hypothesis, mathematical descriptions and problem formulations, exact solutions, new ordering technique and experimental heuristics are important contributions because, while industry is currently focused on the transmission efficiency delivered by a particular modulation scheme, a detailed discussion of the frequency assignment problem and efficient equipment selection is required if the spectrum resource available to fixed services is to be optimised.

American engineers such as Metzger, Zoellner, Beall and Hale recognised the significance of abstract mathematical graph-colouring problems in the 1970s and 1980s, pioneering an extended study of the frequency assignment problem. These studies have been led by academia in the main and an excellent body of work exists in the academic literature with some really advanced algorithms now being used in practice. However, professional frequency assignment practice for the fixed service and the radio engineering community in general have not embraced these ideas and most of the expertise in this vital sub-discipline still resides within the Higher Education Sector, often in Computer Science or Mathematics Departments. This study has addressed the frequency assignment and equipment selection problems with some practical aims in mind: specifically to encourage inter-disciplinary working and closer engagement
7.2 Future work

The use of IP formulations allows for exact solutions to be obtained for smaller problems and for hypotheses to be rigorously tested. There is scope for these formulations to be applied to a range of practical engineering problems including, for example, the optimised selection of antenna types at link-ends or the formulation of problems where the solver is constrained by a High-Low protocol.

High performance antennas are often more expensive than the lower performance alternatives and although it is very well established that ubiquitous use of high performance antennas across a network is spectrally efficient, this may not be economically viable for the network operator. Deploying these antennas at selected link-ends only could have a radical effect on spectrum utilisation while minimising the extra costs involved.

Consideration of the practical planning constraints experienced by professional planners and frequency assigners can help bring simulation work closer to real-world problems. The High-Low protocols used in professional practice are constraints designed to ensure that transmitters and receivers are not co-sited if operating in the same duplex sub-band; this is in order to avoid co-site interference. Sites are given the designation High or Low dependent on which duplex sub-band is used by the transmitters at that site and a pair of sites that are directly connected must not have the same designation. A real world problem is for the network planner to make use of existing sites whilst resolving all network connections and respecting the High-Low protocol. Because of the extra costs involved, new sites are added as a last resort. This particular problem could be formulated using a graph theoretic approach where the nodes of a graph represent sites, a set of requests represent fixed links connecting these sites and the
assigner has the task of labelling each site with a designation High or Low such that the High-Low protocol is respected and network connections are resolved with an objective function to minimise the addition of new sites (nodes). This formulation would deliver the optimal High-Low labelling across the network and could be configured to allow the solver to make decisions about aspects of the network architecture.

Equipment selection heuristics could be developed and refined for use in both the offline and online environments. There may be options for re-tuning campaigns or pre-planning of fixed link deployments in some spectrum and these efforts could include the use of a subordinate equipment selection heuristic. Some further research here could inform this type of planning work.

In a practical frequency assignment system, it is expected that some requests are rejected in cases where a frequency assignment without violations is impossible. Some research could be done to investigate rejections for different frequency assignment methods including some using equipment selection. Equipment selection procedures have the potential to reduce the number of rejections in an environment where assignment difficulty is very high. In simulation work, this might require the use of artificially small frequency domains; this is especially the case for IP formulations of the problem where the problem size must be kept small.

Evolving fixed link radio systems include options for the use of Adaptive Coding and Modulation. Here, the radio is able to change coding regimes and modulation technique according to the radio propagation environment e.g. higher-orders of modulation can be selected when propagation conditions are favourable and unused fade margin can be used to support a higher $C/(N + \sum I)$ and so a higher data-rate. With these systems, the fixed link operator nominates a reference mode (radio system) which is used by the assigner for planning purposes; the operator is free to change coding or modulation while respecting the frequency assignment, the spectrum mask associated with the reference mode and the assigned EIRP. The selections made for the reference mode across a network exposed to various propagation effects could be optimised
using an equipment selection heuristic and extending the research into this area could be profitable.

Professional practice will very likely be handling sequential frequency assignments for some time. Equipment selection heuristics could be used to optimise the spectrum available to assigners. There is enormous scope for variants of these heuristics to inform discussions on spectral efficiency and the technical policy and procedures associated with frequency assignment in professional practice.

Batching of requests by larger fixed link operators is common in professional practice. This is where a set of requests is submitted by the operator. An investigation into the use of batches where the requests cover a particular frequency band and geographical area could prove to be interesting. In particular, it would be useful to investigate a problem where an optimised frequency assignment is obtained for one batch and then a second batch that is constrained with the first. The results could be compared with a single larger batch covering all requests.

These suggestions for future research effort could provide some very good opportunities for collaboration between computer scientists, frequency assigners working in professional practice and radio propagation experts.
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[33] ETSI, (2005), Fixed Radio Systems; ETSI TR 101 854 V1.3.1 point-to-point equipment; derivation of receiver interference parameters useful for planning fixed service point-to-point systems operating different equipment classes and/or capacities, Jan. 2005.

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