AN EMPIRICAL INVESTIGATION OF THE BEHAVIOUR OF
THE BILATERAL REAL EXCHANGE RATE BETWEEN INDIA
AND THE USA

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Declaration

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

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Statement 1

This thesis is the result of my own investigation, except where otherwise stated. Other sources are acknowledged by footnotes giving explicit references, a bibliography is appended.

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Abstract

Using annual data for the period 1959-2001 for India and USA, we examine the effect of productivity differential on bilateral real exchange rate between India and US (the Harrod Balassa Samuelson effect) in a non-linear framework. We find significant evidence of non-linear mean reversion towards the long run equilibrium. We get the evidence that rate of mean reversion is much faster with larger shock. We find that relatively higher productivity growth in US is accompanied by its real exchange rate appreciation vis-à-vis India. Then we explain the behaviour of Indian real exchange rate (US Dollar/ Indian Rupees) using Real Business Cycle model and Overlapping Wage Contract model for the period 1966-1997. We find that Indian real exchange rate appreciates as a result of a deterministic productivity growth shock and then falls back to a lower equilibrium, producing a business cycle and the expected simulation properties. In the Balassa Samuelson model we find that the long run real exchange rate appreciates with rise in productivity. But in the RBC model the short run real exchange rate appreciates as a result of a deterministic productivity growth shock and then goes back to an equilibrium lower than the previous equilibrium in the long run. We discuss the mechanisms of the two models in details in the thesis. We test the validity of the models by method of bootstrapping and we conclude that we can use both Balassa Samuelson model and RBC / OLW model to explain the behaviour of the bilateral real exchange rate between India and US.
PREFACE

We examine the behaviour of the bilateral real exchange rate between India and US in Balassa Samuelson framework and Real Business Cycle framework in this thesis. Chapter 1 reviews the Indian economic situation for the last few decades. In Chapter 2 we examine the behaviour of the real exchange rate in Balassa Samuelson Framework- we test for the presence of real effects on the equilibrium real exchange rate (the Harrod Balassa Samuelson effect) in a non-linear framework. We find significant evidence of non-linear mean reversion towards the long run equilibrium. In Chapter 3 we review Real Business Cycle (RBC) literature. In Chapter 4 we describe the behaviour of real exchange rate and also some other important macroeconomic variables like output, price level, investment, consumption etc. using the RBC model (Minford, Sofat 2004). We also describe the simulation and calibration of the model in this chapter. Then we incorporate an overlapping wage contract equation into the RBC model (Minford, Sofat 2004) for monetary shocks to have real effects and note the differences in the behaviour of the variables in the two models. In Chapter 5 we explore the ability of the RBC and the overlapping wage contract (OLW) model to account for the behaviour of the real exchange rate, using Indian data. We find that a deterministic productivity growth shock causes the real exchange rate to appreciate and then fall back to a lower equilibrium, producing a simulation property which in turn generates the business cycle properties of the real exchange rate. We show that the RBC and the OLW model can reproduce the univariate properties of the real exchange rate. In chapter 6 we use a Full Information Maximum Likelihood (FIML) estimator to estimate our dynamic stochastic general equilibrium (DSGE) model outlined in chapter 4 using annual
data for India. The FIML algorithm solves the model repeatedly in full for all sample periods for set of parameters, choosing the likelihood maximising set. Following Minford and Webb (2004) we use the simultaneous estimator FIML designed for a multivariate normal error distribution. Given that the errors are approximately multivariate normal, the method used is an approximation to maximum likelihood. We start our bootstrapping procedure with our sample residuals obtained by FIML. We also present the confidence limits using the parameter distributions as related to the FIML estimate around which they are generated. We use the gap between the FIML estimate and bootstrap mean as an estimate of bias and correct the bias accordingly. Bias adjustment is important as FIML is generally biased. The final chapter concludes.
Chapter 1. Introduction: Review of Indian Economy

1.1 Introduction
To understand the current economic performance, we have to analyse the evolution of Indian policies over the past decades. Prior to Indian economic liberalisation of early 1990s India has followed a more conservative economic policy. In this chapter we critically analyse the chronological development of Indian economic policy and performance during 1966-2001. Section 1.2 describes the pre-reform development while section 1.3 explains the need for the reform and the features of the reform. Section 1.4 examines the achievements and the weakness of the reform. We describe the plan of the chapters in section 1.5.

1.2.1 Pre-reform Planning of Indian Economy since 1966
After independence, a Soviet-style planning system was developed, but without the state having a monopoly of control over the resources. Capitalism was allowed to flourish, but a large bureaucracy was nurtured. Huge investments were made in basic industries but at the same time several sectors were protected as belonging to small-scale sector. The guiding principles of India's five-year plans are provided by the basic objectives of growth, employment, self-reliance and social justice. Apart from these basic objectives, each five-year plan takes into account the new constraints and problems faced during the period and attempts to make the necessary directional changes and emphasis.

It was the fourth plan that was supposed to be implemented in 1966. But the plan was abandoned on account of the pressure exerted on the economy by two years of drought, devaluation of rupee and the inflationary recession. Instead of a five-year plan three annual plans described as “Plan Holiday” were implemented. India
learnt a bitter lesson during Indo-Pakistan war when its so-called allies refused to supply essential equipment and raw materials for its economic development. The fourth plan set two principal objectives of growth with stability and progressive achievement of self-reliance. The fourth plan aimed at an average of 5.5% rate of growth in national income and provision of national minimum for the weaker section of the community – the latter came to be known as the objectives of "growth with justice" and "removal of poverty".

The fifth plan (1974-79) was introduced at the time when the country was reeling under a vertible economic crisis arising out of a run-away inflation, fuelled by the hike in oil prices since September 1973 and failure of government take-over of wholesale trade in wheat. But the Indian planners were concerned with the slogans of "removal of poverty" and "growth with justice". The fifth plan emphasised that the main causes of abject poverty were open unemployment, underemployment and low resource base of a very large number of producers in agriculture and service sectors. The elimination of poverty could not be attained simply by acceleration in the rate of growth of economy alone but the strategy should be to launch direct attack on the problems of unemployment, underemployment and massive low-end poverty. But this approach was eventually abandoned and emphasis was given on removal of poverty and attainment of self-reliance through promotion of higher rate of growth, better distribution of income and a very significant step-up in domestic rate of saving. The fifth plan was terminated by the Janata party at the end of fourth year of the plan in March 1978.

There were two sixth plans. The Janata party sixth plan (1978-83) openly praised the achievements of planning in India but held the Nehru model of growth responsible for growing unemployment, for the concentration of economic power
in the hands of few powerful business and industrial families, for widening of inequalities of income and wealth and for mounting poverty. The Janata sixth plan sought to reconcile the objectives of higher production with those of greater employment so that millions of people living below poverty line could benefit from it. The focus of Janata sixth plan was enlargement of employment potential in agriculture and allied activities, encouragement to household and small industries producing consumer goods for mass consumption and to raise the incomes of the lowest income classes through a minimum needs programme.

When the new sixth plan (1980-85) was introduced by the Congress party, the planners rejected Janata approach and brought back Nehru model of growth by aiming at a direct attack on the problem of poverty by creating conditions of an expanding economy.

The seventh five-year plan (1985-90) has been introduced since April 1985, after the country has enjoyed a reasonable rate of growth during the sixth plan. The seventh plan seeks to emphasise policies and programmes that will accelerate the growth in food grains production, increase employment opportunities and raise productivity.

1.2.2 Achievements under the five-year plans

Increase in national and per capita income

One of the basic objectives of economic planning in our country is to increase national and per capita incomes. As a direct consequence of economic planning, India's national income and per capita income rose, though not as rapidly as the planners planned and anticipated. India has experienced three major conflicts with
its neighbouring countries, serious famine conditions, sometimes run-away inflationary conditions under the influence of national and international forces, shortages and bottlenecks etc. In spite of these difficulties, national income increased by 19 percent during the first plan, 20 percent during the second plan, 12 percent during the third plan, 18 percent during the fourth plan, 29 percent during the fifth plan and 23 percent during the sixth plan.

The per capita income in real terms had, however increased at much lower rates indicating that part of the increase in real national income had been eaten up by the increase in population. While national income had increased by 242 percent between 1951 and 1985, per capita income had increased only by 66 percent during this period.

Fulfilment of the employment target

One of the major objectives of economic planning in India has been progressive reduction in unemployment. All our plans have assumed that growth of economy would provide the capacity for absorbing the backlog of unemployment and under-employment and a substantial proportion of additions to the labour force. The planners provided supplemental programmes for specific target groups for employment creation, income generation and poverty alleviation. The sixth plan had anticipated a net addition to labour force of order of 34 million during 1980-85. The lower rate of growth in manufacturing industries during sixth plan and poor agricultural crops were responsible for significant shortfall in the plan's employment target.
Price situation
At the outset of sixth plan, the price situation was rather serious with inflationary pressures building up dangerously. In the first year of the sixth plan, the whole sale price index rose by about 17 percent over the previous year. But by using strict price control as well as monetary and fiscal measures, the government was able to keep the prices in check, despite the fact that this period witnessed heavy deficit financing. The average rise of price level was nearly 9.2 percent per annum (compound) during the sixth plan period and this had resulted in a serious erosion of real wages of the working classes.

1.3.1 Economic reform in 1991
The most serious problem faced by Indian economy during the seventh plan (1985-90) was the increase in average trade deficit. The average trade deficit jumped from Rs. 5930 crores during 1980-85 to Rs. 10840 crores during 1985-90. There was also decline in the receipts on invisible account from Rs. 19070 crores during the sixth plan to Rs. 15890 crores during the seventh plan. Consequently, the country was faced with a serious balance of payment crisis. Thus India was forced to approach the World Bank and IMF to provide a huge loan of the order of about $ 7 million to bail India out of the crisis. While agreeing to provide assistance to India, the World Bank –IMF insisted that government must put its economy back on the rails.

The Congress (I) Government adopted a number of stabilisation measures as reforms in 1991. Those stabilisation measures were designed to restore internal and external confidence. Monetary policy was tightened further through increase
in interest rates, the exchange rate of rupee was adjusted by 22 percent and major simplification and liberalisation of trade policy was announced. The Government adopted a programme to bring about reduction in fiscal imbalance to be supported by reforms in economic policy that were essential to impart a new element of dynamism to the growth process in the economy. India adopted a more cautious and step-by-step approach to reforms and liberalisation than most other similarly placed emerging economies.

The prevailing perception among international financial bodies and the most influential section of Indian economists was that the proximate cause of payment crisis was the faulty macroeconomic policies the country had been pursuing during 1980s. More fundamentally inefficiency and non-competitiveness of the country’s products due to subversion of market forces through a plethora of controls and quantitative restrictions contributed to the problem. The problems faced by the country were accordingly sought to be solved through two sets of policies, the first macroeconomic, the second efficiency promoting.

1.3.2 Macro stabilisation programme

The thrust of macro stabilisation programme consisted of the following measures:

a) reduction in fiscal deficit with curbs on government expenditure including subsidies; privatisation along with priority accorded to profitability in running public enterprises and tax reforms for imparting buoyancy to revenue receipts.

b) sharp devaluation at the initial stage followed by

i) a fairly rapid transition to a more or less market driven exchange rate system and
ii) encouragement of the inflow of foreign capital through opening up avenues for foreign institutional investment and considerable relaxation of controls on foreign direct investment.

c) significant scaling down of net central bank credit to government to meet its financing requirement and the large measure of autonomy granted to the Reserve Bank of India (RBI) for maintaining the country’s external and internal balance.

Similarly, all financial sector measures have both efficiency and macroeconomic implications.

a) a cutback in the Statutory Liquidity Ratio (SLR) from 38.5% to 25%, along with moves to make interest rates on government securities market determined (so that there could be a level playing field for private sector borrowers and the government)

b) adoption of international best practice (Basle) norms relating to capital adequacy, income recognition, asset classification and provisioning and these were sought to enforced by strengthening of the RBI’s regulatory –cum-supervisory role

c) gradual abolition of the system of fixing bank’s lending and borrowing rates of interest except for the rate on short-term (savings) deposit.

While adoption of Basle norms and strengthening of the regulatory and supervisory role of RBI were intended to impart stability or strengthen the shock absorptive capacity of the financial and of the macro-economic system, steps like cutbacks in SLR, removal of controls on interest rates, entry of private firms in the financial
market, etc were designed to raise allocative efficiency through greater flows of funds to the relatively productive lines of activity in the economy. Such measures were complemented by policies relating to de-reservation so that private entrepreneurs were now permitted to set up production units in any sector except for a few defence or strategic industries.

1.3.3 Exchange control in pre-reform period

As a member of IMF, India had adopted multilateral payments but actually it had followed strict exchange control. The RBI was entrusted with a comprehensive system of exchange controls under the Foreign Exchange Regulation Act (FERA), with the purpose of conserving India’s foreign exchange resources. Imports were rigidly controlled and imports of all unnecessary items were prohibited. All external payments had to be made through authorised dealers controlled by RBI and foreign exchange was rationed out strictly according to availability. Exporters who acquired foreign exchange had to surrender their earnings to authorised dealers and get rupee in exchange. Besides, purchase and sale of foreign securities were strictly controlled. The purpose of strict exchange control was to prevent wrong use of available foreign exchange resources and utilise them for importing machinery and raw materials that are essential for India’s economic development.

The foreign exchange controls and enforcement of FERA led to serious difficulties to both importers and exporters and to general public. As part of liberalisation of Indian economy and doing away with the licensing and control regime, the

1.3.4 Dual Exchange Rate System in 1992-93

During 1992-93 the Government of India introduced a dual exchange rate system. Under this system:

a) The Government of India accepted the existence of two exchange rates in the country- the official rate of exchange that was controlled and the market rate of exchange which was free to move or fluctuate according to market conditions.

b) All foreign exchange remittances into India-earned through export of goods and services or through inward remittances-were allowed to be converted in the following manner:

- 60 percent of export earnings to be converted at the free market determined rate; this amount could be used freely for current account transactions and payments that is for import of goods, for travel and for remittances abroad and the balance
- 40 percent of the earnings should be sold to RBI through authorised dealers at the official rate of exchange; this amount of foreign exchange would be made available by RBI for financing preferred imports, bulk imports, etc.

This system of dual exchange rate of rupee enabled the exporters to convert at least 60 percent of their export earning at the market rate of exchange. The Government hoped that this would be sufficient incentive to promote exports and increase foreign exchange reserves. The performance of the external sector was
indeed quite spectacular between 1991-92 and 1994-95. Foreign exchange reserves increased from $ 5.8 billion to $ 25.2 billion during this period.

1.3.5 Full convertibility of the rupee on current account

The existence of dual exchange rate hurt exporters and Indians working abroad who had to surrender 40 percent of their earnings at the official rate which was lower than the market rate of exchange. To remove this defect, the government adopted full convertibility of the rupee on trade account – Indian exporters and Indian workers abroad could convert 100 percent of their foreign exchange earnings at the market rates. As the next step, Dr. Man Mohan Singh, the Finance Minister introduced the convertibility of the rupee on the current account, that is, liberalise the access to foreign exchange for all current business transactions including travel, education, medical expenses, etc – the objective of the government was to eliminate reliance upon illegal channels for such transactions. Dr. Man Mohan Singh justified his decisions for convertibility of the rupee on current account because of his success in the international sector: spectacular rise in forex reserves, increase in exports, but stagnancy in imports in dollar terms and improvements in balance of payments on current account. If this step succeeded, the next step would be convertibility of rupee on capital account also.

It is generally agreed that full convertibility of the rupee, both on current account and capital account is necessary for closer integration of the Indian economy with the global economy. The major difficulty with the committee on capital account convertibility (Tarapore Committee) was that it would like the capital account convertibility to be achieved in a three-year period- 1998 to 2000. The period was
too short and the preconditions and the macro-economic indicators could not be achieved in such a short period. Finally, the committee failed to appreciate the political instability in the country and the complete absence of political will and vision to carry forward the process of economic reforms and economic liberalisation. As a result, Government of India failed to introduce full convertibility of the rupee on capital account also.

1.3.6 External value of the rupee and exchange control in the post reform period

The rupee has been depreciating in value in recent years because of high rate of inflation within India and a consequent decline in internal purchasing power of the rupee. The depreciation of the rupee in terms of US dollar and other major currencies before 1991-92 was due to mounting deficit in balance of payments which was worsened by liberal import policy and gulf-war, rapid fall in foreign exchange reserves and the devaluation of rupee forced on India by IMF as one of the conditionality clauses for helping India. The rupee has continued to depreciate since 1991-92 even though there has been improvement in India’s balance of payments positions and foreign exchange reserves. But this depreciation of the rupee was gradual and did not call for any intervention by Reserve Bank of India (RBI). Towards the end of 1997, there was tremendous speculation in foreign exchange market and the rupee depreciated rapidly. There were differing perceptions about the depreciation of rupee against dollar since the last quarter of 1997-98.
One theory was so-called “contagion effect”. Many Asian countries like Malaysia, Indonesia and South Korea had come under serious economic crisis and their currencies fell by 50 percent or more against dollar. The rupee also suffered in sympathy with other Asian currencies, this was called the “contagion effect”. RBI attempted to intervene in the foreign exchange market to check speculation and prevent the fall in the external value of the rupee, but it failed.

The Finance Ministry of the Government of India and RBI argued that they did not subscribe to the “contagion effect theory”. According to them, the basic cause of Asian crisis was heavy current account deficit of Asian countries - between 5 to 7 percent of GDP. But in India, the current account deficit was around 1 percent of GDP. Large current account deficit and external short term borrowings put heavy pressure on the rate of exchange of Asian currencies and led to serious turmoil in their foreign exchange markets. As these conditions did not prevail in India, the Government allowed the rupee to find its own level.

Now we can say that India is an interesting situation with a capital account that is significantly open, a pegged exchange rate and a quest for retaining autonomous monetary policy. According to the RBI, the Indian Rupee is a ‘market determined exchange rate’, in the sense that there is a currency market and the exchange rate is not administratively determined. However, RBI actively trades on the market, with the stated goal of ‘containing volatility’, and influencing the market price.

Theory of ‘Impossible Trinity’ (Mundell 1961) says that no country can simultaneously have an open capital account, a fixed exchange rate, and monetary policy independence. Specifically, once the capital account is open and
the exchange rate is fixed, monetary policy is solely driven by the need to uphold the fixed exchange rate.

The conceptual framework of the Impossible Trinity implies that as liberalisation of the current account and the capital account came about, India should have steadily faced constraints whereby monetary policy came to be strongly influenced by the compulsions of maintaining the currency regime (Joshi 2003). In this context, it is worth mentioning that India is not at a polar extreme of a completely open capital account, nor is India at the polar extreme of a fixed exchange rate. The present policy framework comprises partial controls on capital, a pegged (but not fixed) exchange rate with extremely low volatility and an attempt at having autonomy of monetary policy. (A graphical representation of the Indian real exchange rate – US dollar/Indian rupees and Indian Rupees/US Dollar for the period 1959-2001 is given in Figure 1.1 and 1.2 in the appendix of this chapter.)

1.3.7 Tax Reforms

While progress has been made in the area of tax reforms, the tax structure in India still remains very complicated with the high rates of taxation with regard to both direct and indirect taxes. In the area of personal income tax, the reforms have succeeded in establishing a regime of moderate tax rates, which compare well with other countries. The maximum rate of personal income tax has come down from 56 per cent at the start of the reform to 30 percent. The rate of corporation tax on Indian companies, which varied from 51.75 percent to 57.5 percent in 1991-92, depending upon the nature of the company, has been unified and reduced to 35 percent. Corporate tax rates are still quite high in India despite the reduction announced in the union budget 2002/03 for foreign companies. Despite
these reductions in rates, revenues from personal and corporate taxes have remained buoyant as indicated by the continuing increase in these revenues as a percentage of GDP. The share of direct taxes in GDP is still too low, but it has increased steadily over time. On the whole, this appears to be an area where the strategy of reform seems to be working fairly well and needs to be continued, with special emphasis on means to broaden the base of income tax payers and much stronger enforcement on tax collection.

Experience in other countries shows that a shift to value added tax (VAT) would help improve revenue generation but this is not possible in India under the present constitutional division of powers, whereby excise duties at the production stage are levied by the Central Government while sales taxes at the wholesale and retail level are levied by the States.

In the absence of VAT, India had introduced a modified VAT (called Modvat) under which credit is given for excise taxes paid on inputs against excise taxes due on outputs thus avoid cascading of excise duties. This system was extended and rationalised during the reforms in various ways. The tax credit facility (Modvat facility) was earlier not available for all products, but now has universal coverage. Earlier credit was given only for duties paid on inputs but since 1995 it has been extended to duties paid on capital goods. These efforts at rationalising the excise duty structure were expected to lead to a rising share in the ratio of excise revenue to GDP but in fact excise revenues have declined steadily.
1.3.8 Reforms in Public Sector Policy

Public sector reforms have done little in the cases of units that have been loss making. Many sick public sector units have been referred to the Board for Industrial and Financial Reconstruction (BIFR) for rehabilitation or, where necessary, for winding up. The latter option has been rarely exercised. The public sector is still stuck by government and bureaucratic controls.

An important area where domestic liberalisation has made very little progress is the policy of reserving certain items for production in the small-scale sector. India is unique in adopting reservation for small-scale producers and the policy obviously entails efficiency losses and imposes costs on consumers. An Expert Committee on Small Enterprises set up in 1995 has recommended that reservation should be completely abolished and efforts to support small scale producers should focus on positive incentives and support measures. None of the governments in the post reform period has been inclined to accept a drastic re-orientation of policy along these lines.

1.4 An evaluation of the economic reform

The most striking feature of the Indian economy during 1990s was that unlike most other developing nations which had adopted IMF sponsored structural adjustment programmes, India did not have to undergo a prolonged period of downturn in GDP and capital accumulation. After a dip in the GDP growth rate along with a sharp absolute decline in aggregate investment during 1991-92, there was a smart spurt in both the variables from 1992-93 onwards. (Some statistical figures on growth rate of GDP, savings, investment, consumption and inflation
are also given in Table 1.1 in the appendix of this chapter.) The important positive features of the post reforms era are:

a) a considerable fall in the poverty ratio, from 36.0 per cent to 26.1 percent between 1993-94 to 1999-2000.

b) a sharp cut-back in monetised deficit from 2.7 percent of GDP in 1990-91 to (-0.2) percent in 1999-2000.

c) a decline in the inflation rate from double–digit figures in the early 1990s to less than 5 percent during the closing years of the decade.

The performance of the economy in the external sector was quite remarkable. Considerable improvements in the balance of payments over the period 1990-91 to 2000-01 were attested by

a) a reduction, as percentage of GDP, in external indebtedness and current account balance from 28.7 percent and 3.1 percent to 22.3 percent and 0.5 percent respectively

b) increase in foreign exchange reserves from US Dollar 5.8 billion to US Dollar 42.2 billion

c) a sharp decline in short term debt as percentage of forex reserves from 146.5 percent to 8.2 percent

d) a fall in debt service payments as percentage of current receipts from 35.3 percent to 17.1 percent

The economy also became much more open during 1990s. The merchandise trade and invisible trade-GDP ratios indicate spectacular increases from 14.6 percent and 4.8 percent to 22.8 percent and 12 percent respectively. The economy
experienced a rise in the country’s share of exports in world trade from below 0.5 percent to nearly 0.6 percent.

After managing a rapid recovery, Indian economy lost its growth momentum within a few years. The average GDP and per capita income growth during 1997-01 were significantly lower than in 1992-97, but they were also less than the corresponding averages in 1980s. In fact, on the basis of the behaviour of growth rates of investment and other relevant macro variables, it appears reasonable to identify 1996 or even 1995 (rather than 1997) as the turning point when the prolonged period of deceleration was set in.

Extremely low growth of overall employment was another serious problem faced by Indian economy during 1990s. Between the periods 1993-94 and 1994-00 the average annual growth rate of total employment recorded a steep fall from 2.04 percent to 0.98 percent.

Financial sector reforms have been much more comprehensive than reforms in other areas. However this sector also has displayed a few negative developments. The most important of these are:

a) decline in bank credit to commercial sector and increasing bank holding of SLR securities far in excess of the stipulated minimum

b) failure of the capital market, especially during 1995-2000, to provide finance for domestic capital formation.

Fiscal scenario constitutes the weakest area in India’s reform programme as supported by the fact the country’s credit rating was downgraded by an international agency. During 1990s India experienced a declining trend in total revenue-GDP ratio and tax-GDP ratio. Again as ratios of GDP, government...
consumption, revenue deficit, fiscal deficit, primary deficit and public debt showed a declining trend during the initial phase of the reform programmes, but the trend has been completely reversed since mid 1990s. The focus of successive budgets over the last 5 to 6 years has been on the reduction of these ratios; but the problem has proved intractable and their actual figures have tended to be consistently higher than the budget estimates.

All the weakness of the economy may not be attributed to the reforms programme. Trade balance and exports constitute even now a relatively small fraction of GDP so that their behaviour cannot account for major changes in the country’s GDP or other macro variables. Again loss of growth momentum and declining trend started well before the Asian crisis, not to speak of the recent slowdown of the world economy.

The main sources of weakness of the reform programme seem to be the following:

a) Abdication of the government’s responsibility to ensure adequate effective demand, confirmed both by past policies and the Fiscal Responsibility and Budget Management bill lying before the Parliament.

b) Reforms, especially in the financial sector, without first promoting and ensuring well functioning markets so that there could be adequate credit delivery systems for small and medium borrowers and investors were not taken for a ride by unscrupulous companies or financial institutions.
Despite the weakness of the reform, it brought some favourable changes to the Indian economy, especially in the field of information technology. It is also important to note that India was an important beneficiary of the information technology (IT) revolution of 1990s. The reforms of early 1990s and the rise of IT in United States seem to be going in favour of Indian economy. IT is one sector in which India has actually benefited from brain-drain. So this increased demand for Indian computer personnel should be treated as a blessing and government should simply work hard to educate a large number of Indians appropriately.

The concern about IT sector is that is that IT sector is not labour-intensive enough. Currently, there are approximately half a million people employed in this sector. This is not a small number. According to the NASSCOM-McKinsey study, the employment will grow to 3 million by the year 2010. Most importantly, these calculations ignore the fall-out on the other sectors. India has an IT-enabled sector (such as call centres and data-processing units) that is growing more rapidly than the IT sector itself. The IT-enabled sector currently employs 100000 people but has enormous potential.

Another concern about the IT sector pertains to the supply side in India. With the large flight of computer professionals out of India, there is going to be a supply bottleneck forming in India’s software production. This problem is likely to be exacerbated with the recent increases in the US quota of H1B visas. Approximately 50000 computer professionals will leave India for the US each year.
Though India's achievement in the IT sector is remarkable, Indian economy has been experiencing deceleration in growth rates of GDP, investment, employment and showing signs of fiscal fragility. One worry for India's development prospect concerns the recent movements in the savings rate (and investment rate). After rising sharply through the 1970s and slowly through the 1980s and early 1990s, this has declined suddenly in the late 1990s. It was 22% in 1998-99, having climbed to over 24% in mid 1990s. There is a long run relationship between savings and investment rates and growth. A nation needs high investment and savings to have sustained high growth. Looking at cross-country experience we can see that East Asian economies achieving growth rate of 8% while saving over 30%. As Montek Ahluwalia (2002) points out, to have a growth rate of 8%, it will be essential to have an investment rate of between 29 and 30%. Even if one achieves a part of this from foreign direct investment, India can hardly afford to fall back on the savings front. It is difficult to point out all the reasons behind India's declining savings rate. But it is certain that the worsening fiscal situation and declining public sector savings are the contributory factors.

To achieve fast growth, India should develop market for mortgage and investment loans which in turn, are the key ingredients of fast growth. There is enough strength in the Indian economy for it to benefit from globalisation. Though there are still innumerable important reforms to undertake, the fundamentals of the Indian economy are strong enough for it to be able to implement and benefit from another round of market reform and further opening up of the economy. At the same time, globalisation has some disadvantages in a
country like India (where 35% of the people are illiterate) as globalisation is likely to bring prices of weaker economies into alignment with prices in the industrialised nations. Given that the price of illiterate labour is close to zero in industrialised nations, this means that the illiterate population of developing nations will tend to become extremely impoverished if there is globalisation without complementary government intervention. It is important to recognise this and to prepare for and benefit from it.

Significant reduction of fiscal deficit is very important for the growth of Indian economy. Lower fiscal deficits will help move towards a regime of low interest rates, which, with efficient financial intermediation, can give a boost to private sector investment. Reforms to further opening up of the economy to trade and foreign direct investment are crucial if India is to sustain high rates of economic growth.

Privatisation of India’s state-owned loss-making firms are also very important. These firms tend to be protected by grants of state monopoly, especially in areas of finance, such as commercial banking and insurance and infrastructure, in areas such as telecommunications, port facilities and road building. An end to the state monopolisation of these sectors is crucial to permit new, privately owned firms to introduce competition and higher productivity into these sectors. Privatisation of these enterprises is also desirable in most cases, since the government has no particular comparative advantages in running these enterprises.

One of the important problems with the Indian economic reform was that the targets in the agenda were set without critical examination of structural and other characteristics of the Indian economy- how the various targets are related to one
another; whether they are consistent; or how far they promote the basic objectives. Therefore, India would have been more benefited if the policy implementation could be made keeping in view the macro-economic linkages of the Indian economy.

In this context we shall examine the behaviour of the bilateral real exchange rate between India and US by using Balassa Samuelson model and a real business cycle (RBC) model. After liberalisation in the early 1990s India moved towards a market economy (a lot of controls have been abolished). Though clearly the latter period after liberalisation is more naturally regarded as suitable for an RBC approach, nevertheless we assume here that in spite of the economy's distortions the same approach can succeed for the earlier period. We argue that the economy's basic mechanisms do not change but rather they are merely more distorted in the earlier period.
APPENDIX

Table 1.1 Performance of Indian Economy
(Source: Reserve Bank of India, Report on Currency & Finance, 2000-01;

(in percentage)

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<td>Per capita GDP Growth rate</td>
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<td>Investment as % of GDP</td>
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<td>WPI inflation</td>
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<td>Services Share in GDP</td>
<td>34.40</td>
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<td>Industry Share in GDP</td>
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<td>Agriculture Share in GDP</td>
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Figure 1.1 Real Exchange Rate of India (US Dollar / Indian Rupees)

Figure 1.2 Real Exchange Rate of India (Indian Rupees / US Dollar)
Chapter 2. Productivity Differential and Bilateral Real Exchange Rate between India and US

2.1 Introduction

This Chapter examines the effects of productivity differential on bilateral real exchange rate between India and United States of America (Harrod Balassa Samuelson effect) and also investigates the purchasing power parity dynamics of the bilateral real exchange rate between India and USA. The general implication of this effect is that if a country experiences relatively higher productivity growth, then it will experience an appreciation of its exchange rate in real terms. We shall discuss about the Balassa Samuelson effect in details in section 2.3.2.

The work is novel in the sense that unlike most existing studies on India, we fit a non-linear exponential smooth transition auto-regressive (ESTAR) model to the real exchange rate data and also examine the underlying dynamics of the process. Here we focus on two issues in particular. The first is the effect of real variables (such as differences in relative productivity growth) on equilibrium levels of real exchange rates over the long run. The specific focus here is the productivity differential – Harrod Balassa-Samuelson effect. The second issue is the possibility of non-linear adjustment of real exchange rates to their long run equilibria.

Finally, in view of the emphasis of the on-going economic reform in India on boosting productivity since early 1990s, this study enables us to investigate the relationship between productivity growth and the bilateral real exchange rate between India and US. It is worth-examining as United States is one of the most important trading partners of India.
The chapter is developed as follows. Section 2.2 reviews the literature while section 2.3 explains the methodology. Section 2.4 discusses and analyses the results, section 2.5 describes the non-linear impulse response functions. In section 2.6, we generate the sampling variability within the model by the method of bootstrapping the model’s estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. We conclude in the final section.

### 2.2.1. Existing Literature

There has been a huge literature on the behaviour of the real exchange rate expressed as a measure of the deviation from purchasing power parity (PPP)

\[ y_t = s_t + p_t - p^*_t \]

where \( s_t \) is the logarithm of the nominal exchange rate and \( p_t \) and \( p^*_t \) denote the logarithms of the home and foreign price levels respectively, all at time \( t \).

It is now universally accepted that PPP certainly does not hold for major currencies continuously. A necessary condition for PPP to hold in a long run sense, is that \( y_t \) should be stationary.

### 2.2.2. Long-run PPP and its Dynamics

Researchers have generally tested for long run PPP by testing the null hypothesis that the process generating the real exchange rate series is linear and borderline
non-stationary in the sense that it has a unit root, while the alternative hypothesis is that all of the roots of the process lie within the unit circle. Thus the maintained hypothesis in the conventional framework effectively assumes a linear autoregressive representation for the real exchange rate, which means the adjustment is both continuous and of constant speed, regardless of the size of the deviation from PPP.

Equilibrium models of the exchange rate determination in presence of the transaction costs have been proposed by Benninga and Protopapadakis (1998), Dumas (1992) and Sercu, Uppal and Van Hulle (1995). As a result of cost of trading goods, persistent deviations from PPP are implied as an equilibrium feature of these models. Deviations are left uncorrected as long as they are small relative to the cost of trading.

A significant insight into the nature of PPP deviations is provided by Dumas (1992) who analyses the dynamic process of the real exchange rate in spatially separated markets under proportional transaction costs. Deviations from PPP are shown to follow a non-linear process that is mean reverting, with the speed of adjustment toward equilibrium varying directly with the extent of deviation from PPP. Within the transaction band, when no trade takes place, the process is divergent so that exchange rate spends most of the time away from parity. This implies that deviations from PPP last for a very long time, although they certainly do not follow a random walk.
2.2.3. Econometric analysis of PPP dynamics

In recent years, there has been a surge of papers econometrically analysing the nature of PPP dynamics. For example, using data for recent float alone, Taylor, Peel and Sarno (2001) provide strong confirmation that four major real bilateral dollar exchange rates are well characterised by non-linearly mean reverting processes over the floating rate period since 1973. They estimate smooth transition auto-regressive (STAR) models – for dollar-mark, dollar-sterling, dollar-yen and dollar-franc. Their estimated models in each case imply an equilibrium level of real exchange rate in the neighbourhood (of which the behaviour of the log level of the real exchange rate is close to a random walk), become increasingly mean reverting with the absolute size of deviation from equilibrium. Because of the non-linear nature of the estimated models, the half-lives of shocks to the real exchange rate vary both with size of the shock and with initial conditions. For dollar-mark and dollar-sterling in particular, even small shocks of one to five percent have a half life under three years. For larger shocks, the speed of mean reversion is even faster. Non-linear impulse response functions derived from ESTAR models show that whilst the speed of adjustment for small shocks around equilibrium will be highly persistent, larger shocks mean revert much faster than the glacial rates reported for linear models (Rogoff 1996). This property of ESTAR model provides some solution to the PPP puzzle described in Rogoff (1996) - namely how to reconcile the vast short run volatility of real exchange rates with the apparent mean reversion observed in the data.

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1 Rogoff (1996) notes that, even if we were to take the results of the long span or panel-data studies as having provided evidence of significant mean reversion in the real exchange rate, these studies typically point to a half life of deviations from PPP in the range of three to five years. If we take as given that real shocks cannot account for the major part of the short run volatility of real exchange rates (since it seems incredible that shocks to real factors such as tastes and technology could be so volatile) and that nominal shocks can only have strong effects over a time frame in which nominal wages and prices are sticky, then the apparently high degree of persistence in the real exchange rate becomes something of a puzzle.
exchange rates with the glacially slow rate of 3-5 years at which shock appears to damp out- in linear models.

Canzoneri, Cumby, Diba (1998) test the implications of the Balassa Samuelson model that the relative price of non-traded goods in each country should reflect the relative productivity of labour in traded and non-traded goods sector and the purchasing power parity holds for traded goods in the long run by using data from a panel of OECD countries. Their results support the fact that the relative price of non-traded goods in each country should reflect the relative productivity of labour in traded and non-traded goods sector. The evidence on purchasing power parity in traded goods is considerably less favourable. For US dollar exchange rates, PPP does not appear to hold for traded goods, even in the long run. For Deutsche Mark exchange rates, PPP appears to be somewhat better characterisation of traded goods prices.

Some attempts have been made recently to incorporate the determinants of the equilibrium real exchange rate in linear models of the adjustment of real exchange rates. Lothian and Taylor (2000), for example, incorporate linear and non-linear deterministic trends as proxy variables. Engel and Kim (1999) employ data on relative per capita real incomes motivated by the models of Balassa (1964), Samuelson (1964) and Lucas (1982). Lothian and Taylor (2000) found that adjustment speeds were much faster in the linear auto-regressive model embodying time trends than in a model which excludes them. Similar results were reported by Peel and Venetis (2003) when a non-linear adjustment mechanism was employed.
Lothian and Taylor (2004) have examined the long run behaviour of the real dollar-sterling exchange rate in a non-linear framework employing relative per capita income as a proxy for the equilibrium rate. Their results suggest that long run real dollar-sterling exchange rate mean reverts, in a non-linear manner, to a changing real equilibrium rate.

The high persistence of both PPP deviations and proxy variables for the equilibrium rate might create a problem of spurious coefficient significance. Paya and Peel (2004) investigate the possibility of spurious regression within non-linear models of PPP. Using Monte Carlo experiments they show that standard critical values are not appropriate in such a context. They used real dollar-sterling exchange rate for the period 1871-1994 for their empirical work. Due to change in exchange rate regimes for many times over the sample period they employ a bootstrap methodology that preserves the original structure of the estimated residuals and obtain new critical values of the coefficient estimates. A non-linear ESTAR process with a time varying equilibrium proxied by relative wealth and relative income per capita seems to parsimoniously fit their data. Their results provide the evidence for the non-linear model with a shifting equilibrium and the implied speed of adjustment is found to be faster than previously reported in literature.

2.2.4. Studies on India

In contrast to studies for developed countries like the UK, USA, Japan, there have been limited studies on India. To a large extent, most of these studies have examined the behaviour of real exchange rate of India in a linear framework. For example, Weliwita (1998) tests the validity of long run purchasing power parity
hypothesis by applying the Engle and Granger two step co-integration procedure and Johansen and Juselius multivariate co-integration technique to price and exchange rate from six developing countries in Asia. The results of both methods reject the existence of long run purchasing power parity for India, Pakistan, Indonesia, Malaysia, Sri Lanka and Thailand. This finding is confirmed by the unit root tests performed on real exchange rates. Unit root test results reveal that, in each case, the real effective exchange rate follows a random walk. One possible reason for failure of the PPP is that in these countries service sectors are playing important role, the domestic supply and demand conditions for services change over time relative to their trading partners, thus resulting in departures from PPP.

Kohli (2002) tests for mean reversion in real exchange rates for India during recent float period. Using unit root tests with improved power, they test for stationarity of real exchange rate, using several definition of the real exchange rate. They also conduct co-integration and variance ratio tests to complement the evidence from unit root tests. They find evidence of mean reversion in real exchange rate series constructed with the consumer price index as deflator, as well as for a series constructed using the ratio of wholesale and consumer price indices to proxy for shares of tradable and non-tradable goods. Though the evidence is based on a very short sample, the results indicate the sources of disturbance to real exchange rate are monetary. The evidence regarding trend stationarity of the WPI/CPI real exchange rate series is significant. Noting that PPP holds only with the inclusion of the trend term, it may reflect the presence of Balassa-Samuelson effect.
In an earlier paper, Kohli (2000) modelled a central bank reaction function to test for exchange rate management strategy, which yields evidence of a policy of real exchange rate stabilisation by the authorities. When the authorities are targeting the real exchange rate, they are attempting to move the real exchange rate towards equilibrium that is PPP, by varying nominal exchange rate to restore equilibrium. When a floating exchange rate is managed via this policy rule, short run deviation from PPP and mean reversion in the rupee’s real exchange rate vis-à-vis the dollar may not be surprising.
2.2.5. **Summing up**

The incompleteness of PPP to fully account for exchange rate movements is well recognised (Frankel, 1976) as it holds only when price disturbances are originating from monetary shocks. A monetary shock like an expansion in domestic money stock, will cause both the domestic price level and the exchange rate to increase in the same proportion. After accounting for the dynamics of all adjustments, an increase in money stock will reduce purchasing power of money, both in terms of domestic basket of goods, as in terms of foreign basket of goods. The proportionality between the price level and the exchange rate will not be maintained when a real shock impacts the exchange rate.

Within the Balassa-Samuelson framework the present chapter aims to examine the effects of productivity differential on the bilateral real exchange rate between Indian and United States. Empirical analysis is based on the data primarily obtained from the International Financial Statistics. We use annual data for the period 1959-2001 and unlike most existing Indian studies, movements in real exchange rates for our case is modelled in terms of a non-linear exponential smooth transition auto-regressive (ESTAR) model.

2.3.1. **Methodology**

One particular statistical characterisation of non-linear adjustment which appears to work well for exchange rates is the smooth transition auto-regressive (STAR) model.

A smooth transition model has the form

\[ y_t = \sum_{i=1}^{p} \alpha_i y_{t-i} + (1 - F_t) \left( \sum_{i=1}^{r} \delta_i y_{t-i} \right) + u_t \ \ldots \ldots(2.1) \]
Where \( a_t \) and \( \delta_t \) are constants and \( F_t \) is the continuous transition function, which is usually specified to be bounded between zero and unity. In the STAR model adjustment takes place in every period but speed of adjustment varies with the extent of the deviation from parity.

Ozaki (1978) and Hagan and Ozaki (1981) introduced the exponential autoregressive model known more widely these days as the exponential STAR (ESTAR) model.

In the ESTAR model

\[
F_t = e^{-\gamma(y_t - \delta)^2} \quad \ldots \ldots \ldots (2.2)
\]

Where \( d \) is the delay, \( \gamma \) is a positive constant and \( \lambda \) is a constant.

The transition function for the ESTAR model is symmetric in deviations of \( y_{t-d} \) from \( \lambda \). The transition function determines the degree of mean reversion.

The parameter \( \gamma \) determines the speed of transition process between two extreme regimes. We observe from (2.1) that as \( F_t \) varies between zero and one we obtain an infinite number of different autoregressive processes each corresponding to a different state.

An interesting special case of ESTAR which illustrates some of the possibilities is the model

\[
y_t = y_{t-1}e^{-\gamma(y_{t-1})^2} + u_t \quad \ldots \ldots \ldots (2.3)
\]

When the deviation \( y_{t-1} \) is large (2.3) will give approximately \( y_t = u_t \)

Conversely when \( y_{t-1} \) is small (2.3) will give approximately \( y_t = y_{t-1} + u_t \)

Or \( y_t - y_{t-1} = u_t \) so that \( y \) exhibits behaviour which varies from where \( y \) is approximately random to where changes in \( y \) are approximately random.
We can observe the non responsiveness of $y_t - y_{t-1}$ to $y_{t-1}$ near zero, which is the equilibrium value in this model. This type of adjustment was employed by Michael, Nobay and Peel (1997) to model deviations of real exchange rates from equilibrium. It captures the idea that there is little response of real exchange rates to deviations from equilibrium when they are small but adjustment is proportionately faster the greater the deviation. The idea is that there is some threshold related to the transactions costs of arbitrage across goods markets below which arbitrage is not worthwhile and above which it is. This threshold may be different for different goods and so there is a range across which the reaction may increase in a nonlinear way.

2.3.2. Allowing for Harrod Balassa Samuelson Effect

We start our discussion by indicating the primary assumptions and implications of this model. Suppose a country experiences productivity growth primarily in its traded goods sector and the law of one price (LOP) holds among traded goods and the nominal exchange rate remains constant. Productivity growth in traded goods sector will lead to wage rises in that sector without necessity for price rises. Hence traded goods prices can remain constant and LOP can continue to hold with the unchanged nominal exchange rate. But workers in the non-traded goods sector will also demand comparable pay rises and this will lead to a rise in the price of non-tradeables and hence an overall rise in the consumer price index (CPI). Since the LOP holds among traded goods, and by assumption, the nominal exchange rate has remained constant, this means the upward movement in domestic CPI will not be matched by a movement in the nominal exchange rates so that, if PPP initially held, the domestic currency must now appear overvalued on the basis of comparison made using CPI expressed in a common currency at
the prevailing exchange rate. The important assumption is that productivity growth is much higher in the traded goods sector. Relative price of non-tradeables may rise even in the case of balanced growth of the two sectors of the economy, as long as the non-traded goods sector is more labour intensive relative to the traded goods sector.

The Harrod Balassa Samuelson condition is that relatively higher productivity growth in the tradeables sector will tend to generate a rise in relative price of non-tradeables. The percentage change in the relative price of non-tradeables is determined only by production side of the economy, while the demand factors do not affect the real exchange rate in the long run. If the degree of capital intensity is the same across the traded and non-traded sectors, then the percentage change in relative prices is exactly equal to the productivity differential between the two sectors. If the non-traded sector is less capital intensive than the traded sector, then even in the situation of balanced productivity growth in the two sectors, the relative price of non-tradeables will rise.

In other words, the Harrod Balassa Samuelson model suggests that the long run equilibrium real exchange rate should depend on the productivity of tradeables and non-tradeable sectors in home and foreign economies. Given perfect labour mobility, changes in relative productivity across sectors lead to changes in relative prices. Since technological innovation is most likely to be concentrated in the tradeable goods sector, countries with higher long-run growth rates should have higher relative prices of non-tradeable goods as well as higher valued currencies. If we make approximation that productivity growth in non-tradeable sector is zero, then equilibrium real exchange rate will vary over time in response
to variations in tradeable sector productivity in the two economies. In fact, under this assumption, variations in productivity in the economy as a whole will depend largely upon variation in productivity in tradeable sector. Ideally, therefore, one would like to obtain data on tradeable sector output and employment over the sample period in order to measure tradeables sector productivity. These data are not available for India for the sample period. So we should take the central implications of the Harrod-Balassa-Samuelson model that if a country experiences relatively higher productivity growth, then it will experience an appreciation of its exchange rate in real terms.

Given the data availability, we measure the productivity term as the ratio of total national output, real GDP, to total population.

Now we briefly outline the essential features of the basic model as developed in Balassa (1964). We consider a small open economy that uses capital and labour to produce tradable goods (T) priced in world markets and non-tradable (N) priced in the domestic market. Both capital (K) and labour (L) are perfectly mobile across sectors domestically. Labour is immobile between countries whereas capital is perfectly mobile internationally.

\[ Y_T = \theta_T L_T^\alpha K_T^{1-\alpha} \quad \text{......(1)} \]

\[ Y_N = \theta_N L_N^\beta K_N^{1-\beta} \quad \text{......(2)} \]

\[ w = \theta_T \alpha \left( \frac{K_T}{L_T} \right)^{1-\alpha} = \theta_T \alpha (k_T)^{1-\alpha} \quad \text{......(3)} \]

\[ w = s \theta_N \beta \left( \frac{K_N}{L_N} \right)^{1-\beta} = s \theta_N \beta (k_N)^{1-\beta} \quad \text{......(4)} \]

\[ i = \theta_T (1 - \alpha) \left( \frac{K_T}{L_T} \right)^{-\alpha} = \theta_T (1 - \alpha) k_T^{-\alpha} \quad \text{......(5)} \]

\[ i = s \theta_N (1 - \beta) \left( \frac{K_N}{L_N} \right)^{-\beta} = s \theta_N (1 - \beta) k_N^{-\beta} \quad \text{......(6)} \]
\[ L_T + L_N = \tilde{L} \] \ldots (7)

Demand: \[ Y_N = s^{-\sigma} \left( Y_N + Y_T \right) \] \ldots (8)

We need the marginal productivity conditions to get the real exchange rate rising with relative traded sector productivity.

By using (3)-(6) we have

\[ \Delta \log w = \Delta \log \theta_T + (1 - \alpha) \Delta \log k_T \] \ldots (3)

\[ 0 = \Delta \log \theta_T - \alpha \Delta \log k_T \Rightarrow \Delta \log k_T = \frac{1}{\alpha} \Delta \log \theta_T \] \ldots (5)

\[ 0 = \Delta \log s + \Delta \log \theta_N - \beta \Delta \log k_N \] \ldots (6)

\[ \Delta \log w = \Delta \log s + \Delta \log \theta_N + (1 - \beta) \Delta \log k_N \] \ldots (7)

From (3) & (5) we get

\[ \Delta \log w = \left( 1 + \frac{1 - \alpha}{\alpha} \right) \Delta \log \theta_T = \left( \frac{1}{\alpha} \right) \Delta \log \theta_T \]

From (7) we get

\[ \Delta \log s = \Delta \log w - \Delta \log \theta_N - (1 - \beta) \Delta \log k_N \]

From (6) we get

\[ \Delta \log k_N = \frac{1}{\beta} \Delta \log s + \frac{1}{\beta} \Delta \log \theta_N \]

\[ \Delta \log s = \Delta \log w - \Delta \log \theta_N - \frac{1 - \beta}{\beta} \Delta \log s - \frac{1 - \beta}{\beta} \Delta \log \theta_N \]

\[ \therefore \left( \frac{1}{\beta} \right) \Delta \log s = \frac{1}{\alpha} \Delta \log \theta_T - \frac{1}{\beta} \Delta \log \theta_N \]
A rise in traded sector productivity raises capital-labour ratio in the traded sector. The two of them then raise real wage set in the traded sector by international competition. The real exchange rate is then set by the twin effect of rising wages driven by the traded sector and a rising capital-labour ratio in the non-traded sector.

2.4 Model and Empirical Results

Now we model PPP deviations as a non-linear ESTAR process incorporating the proxy for the equilibrium rate. The real exchange rate is modelled as an ESTAR process in which we model the equilibrium real exchange rate as dependent upon differences in real income per capita. We assume that the true data generating process for the purchasing power parity deviations \( y_t \) modified for equilibrium real determinants has the following form of ESTAR reported in Michael, Nobay and Peel (1997) and Killian and Taylor (2003)

\[
y_t = \alpha + \delta x_t + e^{-\gamma(y_t-a-\delta x_t)} \left[ \sum_{i=1}^{n} \beta_i (y_{t-i} - \alpha - \delta x_{t-i}) \right] + u_t, \quad \text{(2.4)}
\]

where \( y_t \) is the real exchange rate, \( s_t \) is the logarithm of the spot exchange rate (Indian rupees / US dollar), \( p_t \) is the logarithm of US price level, \( p_t^* \) is the logarithm of Indian price level, \( \alpha \) is a constant, \( x_t \) are the determinants of the equilibrium level of real exchange rate, \( \gamma \) is a positive constant, the speed of
adjustment, $\beta_i$ are constants and $u_i$ is a random disturbance term. Here $x_i$ is defined as productivity differential while productivity differential is measured as log difference of real GDP per capita of the two countries.

The starting point of our analysis is to check the stationarity property of the real exchange rate ($y_t$).

### 2.4.1 Unit root test
We perform two sets of unit root tests, namely, Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests using annual data for the period 1959-2001. These test results are summarised in Table 2.1 for $y_t$. First, using Augmented Dickey Fuller unit root test, we note that null of unit root cannot be rejected in levels for $y_t$. Phillips Perron tests show the similar results. When we do the unit root test for the first difference of $y_t$ using ADF and PP test, we find that the series is stationary. Now we check the non-stationarity property of productivity differential ($x_t$) using Augmented Dickey Fuller test and Phillips Perron tests. First, using Augmented Dickey Fuller unit root test and PP test, we note that null of unit root cannot be rejected in levels for $x_t$. When we do the unit root test for the first difference of $x_t$ using ADF and PP test, we find that the series is stationary. The results are reported in table 2.2.

We know that unit root tests have low power if the true data generating process is ESTAR (Taylor et al, 2001), so the results have to be interpreted with that caveat.
2.4.2 Plot of the data
Plot of the real exchange rate series and productivity differential suggest that there is some non-linearity in these series (Figure 2.1 and 2.2). Figure 2.3 shows the upward trend of \((s_t + p_t)\) and \(p^*_t\) over the whole sample period.

2.4.3 Results of the ESTAR model
Finally, we estimate equation (2.4). This is based on annual data (obtained from International Financial Statistics) for the period 1959–2001 as monthly data for GDP are not available for India. The results of the estimation are summarised in Table 2.3. This clearly suggests that the coefficient of \(x_t\) is positive. The latter in turn provides support to the central B-S hypothesis in that higher productivity growth in US is accompanied by the appreciation of its real exchange rate vis-à-vis India.

The reported tests for serial correlation and heteroskedasticity for equation (2.4) suggest that serial correlation and heteroskedasticity do not exist in the residual and the distribution of the residual is normal. The results are given in table 2.4A and 2.4B.

2.5 Non-linear impulse response function
In this section we calculate the half-lives of PPP deviations within the non-linear framework. We must take into account that a number of properties of the impulse response functions of the linear models do not carry over to the non-linear
framework. In particular, impulse responses produced by non-linear models are history dependent, so they depend on initial conditions. They are dependent on the size and sign of current shock and they depend on future shocks as well. Koop, Pesaran and Potter (1996) introduced the generalised impulse response functions (GIRF) for non-linear models. The GIRF is defined as the average difference between two realisations of the stochastic process \((y_{t+h})\) which start with identical histories up to time \(t-1\) (initial conditions) The first realisation is hit by a shock at time \(t\) while the other one is not:

\[
\text{GIRF}_h(h, \varphi, m_{t-1}) = E(y_{t+h} \mid u_t = \varphi, m_{t-1}) - E(y_{t+h} \mid u_t = 0, m_{t-1}) \quad (2.5)
\]

where \(h = 1, 2, \ldots\) denotes horizon, \(u_t = \varphi\) is an arbitrary shock occurring at time \(t\) and \(m_{t-1}\) defines the history set of \(y_t\). Given that \(\varphi\) and \(m_{t-1}\) are single realisations of random variables, (2.5) is considered to be a random variable. In order to obtain sample estimates of (2.5), we average out the effect of all histories \(m_{t-1}\) that consist of every set \((y_{t-1}, \ldots, y_{t+p})\) for \(t \geq p+1\) where \(p\) is the auto-regressive lag length and we also average out the effect of future shocks \(u_{t+h}\). We set \(\varphi = 3\%, 5\%, 10\%, 20\%\). The different values of \(\varphi\) would allow us to compare the persistence of very large and small shocks.

Table 2.5 reports the half-lives of shocks, that is time needed for \(GIRF_h < (1/2)\varphi\). We show the graphs of the non-linear impulse response in figure 2.4. Non-linear impulse response functions illustrate well the non-linear nature of the estimated real exchange rate models, with larger shocks mean reverting much faster than smaller shocks. In particular, for shocks of five per cent or three per cent, the half life is two years, while larger shocks like ten per cent or twenty per cent have a half life of one year. These results therefore accord broadly with those reported in Taylor, Peel and Sarno (2001) and shed some light on Rogoff’s
(1996) PPP puzzle. Once non-linearity is allowed for, even small shocks of one to five percent have a half-life of two years or less, conditional on average history and for larger shocks the speed of mean reversion is even faster.

2.6 Testing the model

We fit an integrated auto-regressive (AR(1)) model to the productivity differential series and bootstrap the model to get 500 pseudo sample for the real exchange rate. Our objective is to check whether we can generate the facts of the real exchange rate such as we find them, assuming that our model and its error processes are true. We generate the sampling variability within the model by the method of bootstrapping the model’s estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. Having established the non-stationarity of the real exchange rate series we now estimate the best fitting ARIMA process to the real exchange rate, using annual data from 1966 to 1997. The results in table 2.6 indicate that an ARIMA (1,1,1) best describes the data. We report the 95% Confidence limits generated by bootstrapping in table 2.7 and the best fitting ARIMA regression results in table 2.8. The results in Table 2.7 validate the hypothesis that real exchange rate (Indian Rupees/US Dollar) behaviour is explicable within the Balassa Samuelson framework. The ARIMA parameters lie within the 95% confidence intervals.

2.7 Concluding comments

This chapter has examined the effects of productivity differential on bilateral real exchange rate (Harrod Balassa Samuelson effect) between India and US, two
important trading partners and also the possibility of non-linear adjustment of real exchange rates to their long run equilibria.

We find the evidence of Harrod Balassa Samuelson effect using annual data for the period 1959-2001. When US experiences relatively higher productivity growth, its real exchange rate appreciates vis-à-vis India. At the same time we find evidence of significant non-linear mean reversion.

The impulse response functions for shocks of various magnitudes to the real exchange rates suggest that the half-lives for large shocks of twenty per cent or ten percent are only one year. For small shocks like three per cent and five percent they are two years. Our results support the earlier empirical findings of Lothian and Taylor (2004) and Paya and Peel (2004) that the rate of mean reversion is much faster with larger shocks.

The plot of nominal exchange rate and price level of US \( (s_t + p_t) \) and Indian price level \( (p_t^*) \) over the whole sample period (figure 2.3) provides strong visual evidence that PPP appears to play a strong role in driving real exchange rates. The price levels have quite similar trends. In contrast, the real exchange rate shows no marked trend movement. Therefore, the major implications of purchasing power parity are visually confirmed that national price levels expressed in a common currency move together closely over the long term and the real exchange rate is highly stable in the long term in comparison to the nominal data.

We can conclude whether our model could be consistent with the facts by asking whether it could have generated the patterns we find in the actual real exchange rate data. To do this we generate the sampling variability within the Balassa Samuelson model by the method of bootstrapping the model’s estimated
residuals. This allows us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. The AR and MA coefficients estimated with the actual data lie within the 95% confidence limits generated by the method of bootstrapping. This validates our hypothesis that the behaviour of the real exchange rate (Indian Rupees/US Dollar) is explicable within the Balassa Samuelson framework.

Table 2.1. Unit root test for $y_t$ (Using annual data 1959-2001)

<table>
<thead>
<tr>
<th></th>
<th>5% critical value</th>
<th>10% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF test statistic (level)</td>
<td>-3.1648</td>
<td>-3.5207</td>
</tr>
<tr>
<td>ADF test statistic (first difference)</td>
<td>-8.6182</td>
<td>-3.5236</td>
</tr>
<tr>
<td>Phillips Perron test statistics (level)</td>
<td>-3.0436</td>
<td>-3.5207</td>
</tr>
<tr>
<td>Phillips Perron test statistics (first difference)</td>
<td>-8.9744</td>
<td>-3.5236</td>
</tr>
</tbody>
</table>

Null Hypothesis: $Y$ has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic based on SIC, MAXLAG=9)

<table>
<thead>
<tr>
<th></th>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-3.164825</td>
<td>0.1054</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-4.192337</td>
<td></td>
</tr>
<tr>
<td>5% level</td>
<td>-3.520787</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-3.191277</td>
<td></td>
</tr>
</tbody>
</table>


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 11/13/06  Time: 17:50
Sample (adjusted): 1960 2001
Included observations: 42 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(-1)</td>
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<td>0.111603</td>
<td>-3.164825</td>
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<tr>
<td>C</td>
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<td>0.237284</td>
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<td>0.0033</td>
</tr>
<tr>
<td>@TREND(1959)</td>
<td>0.009945</td>
<td>0.003030</td>
<td>3.282716</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

R-squared 0.218510 Mean dependent var 0.020777
Adjusted R-squared 0.178434 S.D. dependent var 0.101817
S.E. of regression 0.092287 Akaike info criterion -1.859079
Sum squared resid 0.332158 Schwarz criterion -1.734960
Log likelihood 42.04066 F-statistic 5.452349
Durbin-Watson stat 2.303575 Prob(F-statistic) 0.008166

Null Hypothesis: D(Y) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic based on SIC, MAXLAG=9)

Augmented Dickey-Fuller test statistic -8.618271 0.0000
Test critical values: 1% level -4.198503
5% level -3.523623
10% level -3.192902


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(Y,2)
Method: Least Squares
Date: 11/13/06 Time: 17:50
Sample (adjusted): 1961 2001
Included observations: 41 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
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<tbody>
<tr>
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<td>0.0000</td>
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<tr>
<td>C</td>
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<td>0.032668</td>
<td>-0.016144</td>
<td>0.9872</td>
</tr>
<tr>
<td>@TREND(1959)</td>
<td>0.001318</td>
<td>0.001317</td>
<td>1.000813</td>
<td>0.3232</td>
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</table>

R-squared 0.661547 Mean dependent var 0.002454
Adjusted R-squared 0.643733 S.D. dependent var 0.165980
S.E. of regression 0.099070 Akaike info criterion -1.715622
Sum squared resid 0.372966 Schwarz criterion -1.590239
Log likelihood 38.17025 F-statistic 37.13772
Durbin-Watson stat 2.106214 Prob(F-statistic) 0.000000
Null Hypothesis: \( Y \) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 2 (Newey-West using Bartlett kernel)

<table>
<thead>
<tr>
<th>Phillips-Perron test statistic</th>
<th>Adj. t-Stat</th>
<th>Prob.*</th>
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</thead>
<tbody>
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<td>-3.043633</td>
<td>0.1330</td>
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Test critical values:
- 1% level: -4.192337
- 5% level: -3.520787
- 10% level: -3.191277


Residual variance (no correction) 0.007909
HAC corrected variance (Bartlett kernel) 0.006455

Phillips-Perron Test Equation
Dependent Variable: D(Y)
Method: Least Squares
Date: 11/13/06 Time: 17:51
Sample (adjusted): 1960 2001
Included observations: 42 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(-1)</td>
<td>-0.353203</td>
<td>0.111603</td>
<td>-3.164825</td>
<td>0.0030</td>
</tr>
<tr>
<td>C</td>
<td>0.742300</td>
<td>0.237284</td>
<td>3.128320</td>
<td>0.0033</td>
</tr>
<tr>
<td>@TREND(1959)</td>
<td>0.009945</td>
<td>0.003030</td>
<td>3.282716</td>
<td>0.0022</td>
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</tbody>
</table>

R-squared 0.218510 Mean dependent var 0.020777
Adjusted R-squared 0.178434 S.D. dependent var 0.101817
S.E. of regression 0.092287 Akaike info criterion -1.859079
Sum squared resid 0.332158 Schwarz criterion -1.734960
Log likelihood 42.04065 F-statistic 5.452349
Durbin-Watson stat 2.303575 Prob(F-statistic) 0.008166

Null Hypothesis: D(Y) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 3 (Newey-West using Bartlett kernel)

<table>
<thead>
<tr>
<th>Phillips-Perron test statistic</th>
<th>Adj. t-Stat</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8.974466</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

Test critical values:
- 1% level: -4.198503
- 5% level: -3.523623
10% level -3.192902


Residual variance (no correction) 0.009097
HAC corrected variance (Bartlett kernel) 0.007212

Phillips-Perron Test Equation
Dependent Variable: D(Y,2)
Method: Least Squares
Date: 11/13/06 Time: 17:51
Sample (adjusted): 1961 2001
Included observations: 41 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(Y(-1))</td>
<td>-1.322336</td>
<td>0.153434</td>
<td>-8.618271</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>-0.000527</td>
<td>0.032668</td>
<td>-0.016144</td>
<td>0.9872</td>
</tr>
<tr>
<td>@TREND(1959)</td>
<td>0.001318</td>
<td>0.001317</td>
<td>1.000813</td>
<td>0.3232</td>
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</tbody>
</table>

R-squared 0.661547 Mean dependent var 0.002454
Adjusted R-squared 0.643733 S.D. dependent var 0.165980
S.E. of regression 0.099070 Akaike info criterion -1.715622
Sum squared resid 0.372966 Schwarz criterion -1.590239
Log likelihood 38.17025 F-statistic 37.13772
Durbin-Watson stat 2.106214 Prob(F-statistic) 0.000000

Table 2.2: Unit root test for x_t (Using annual data 1959-2001)

<table>
<thead>
<tr>
<th></th>
<th>5% critical value</th>
<th>10% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF test statistics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(level)</td>
<td>2.0431</td>
<td>-3.5207</td>
</tr>
<tr>
<td>ADF test statistics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(first difference)</td>
<td>-7.0566</td>
<td>-3.5236</td>
</tr>
<tr>
<td>Phillips Perron</td>
<td></td>
<td></td>
</tr>
<tr>
<td>test statistics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(level)</td>
<td>-2.0431</td>
<td>-3.5207</td>
</tr>
<tr>
<td>Phillips Perron</td>
<td></td>
<td></td>
</tr>
<tr>
<td>test statistics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(first difference)</td>
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<td>-3.5236</td>
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</table>

Null Hypothesis: X has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 0 (Newey-West using Bartlett kernel)

Adj. t-Stat Prob.*
Phillips-Perron test statistic  -2.043150  0.5614
Test critical values:
1% level -4.192337
5% level -3.520787
10% level -3.191277


Residual variance (no correction)  0.012137
HAC corrected variance (Bartlett kernel)  0.012137

Phillips-Perron Test Equation
Dependent Variable: D(X)
Method: Least Squares
Date: 11/13/06 Time: 17:52
Sample (adjusted): 1960 2001
Included observations: 42 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(-1)</td>
<td>-0.200503</td>
<td>0.098134</td>
<td>-2.043150</td>
<td>0.0478</td>
</tr>
<tr>
<td>C</td>
<td>0.571391</td>
<td>0.266056</td>
<td>2.147635</td>
<td>0.0380</td>
</tr>
</tbody>
</table>
@TREND(1959)    | 0.010690    | 0.005025   | 2.127280    | 0.0398 |

R-squared 0.103982 Mean dependent var 0.051327
Adjusted R-squared 0.058032 S.D. dependent var 0.117796
S.E. of regression 0.114327 Akaike info criterion -1.430765
Sum squared resid 0.509753 Schwarz criterion -1.306645
Log likelihood 33.04606 F-statistic 2.262955
Durbin-Watson stat 2.044463 Prob(F-statistic) 0.117537

Null Hypothesis: D(X) has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 3 (Newey-West using Bartlett kernel)

<table>
<thead>
<tr>
<th>Adj. t-Stat</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phillips-Perron test statistic</td>
<td>-7.040998</td>
</tr>
</tbody>
</table>
Test critical values:
1% level -4.198503
5% level -3.523623
10% level -3.192902


Residual variance (no correction) 0.013459
HAC corrected variance (Bartlett kernel) 0.014135
Phillips-Perron Test Equation
Dependent Variable: D(X,2)
Method: Least Squares
Date: 11/13/06 Time: 17:53
Sample (adjusted): 1961 2001
Included observations: 41 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<td>0.4725</td>
</tr>
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R-squared    0.567228
Adjusted R-squared 0.544451
S.E. of regression 0.120507
Sum squared resid 0.551829
Log likelihood 30.13934
F-statistic 24.90306
Durbin-Watson stat 1.952077
Prob(F-statistic) 0.000000

Null Hypothesis: X has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic based on SIC, MAXLAG=9)

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-2.043150</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-4.192337</td>
</tr>
<tr>
<td>5% level</td>
<td>-3.520787</td>
</tr>
<tr>
<td>10% level</td>
<td>-3.191277</td>
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</tbody>
</table>


Augmented Dickey-Fuller Test Equation
Dependent Variable: D(X)
Method: Least Squares
Date: 11/13/06 Time: 17:54
Sample (adjusted): 1960 2001
Included observations: 42 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
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<td>0.005025</td>
<td>2.127280</td>
<td>0.0398</td>
</tr>
</tbody>
</table>
Null Hypothesis: D(X) has a unit root  
Exogenous: Constant, Linear Trend  
Lag Length: 0 (Automatic based on SIC, MAXLAG=9)

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-7.056628</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-4.198503</td>
</tr>
<tr>
<td>5% level</td>
<td>-3.523623</td>
</tr>
<tr>
<td>10% level</td>
<td>-3.192902</td>
</tr>
</tbody>
</table>


Augmented Dickey-Fuller Test Equation  
Dependent Variable: D(X,2)  
Method: Least Squares  
Date: 11/13/06 Time: 17:54  
Sample (adjusted): 1961 2001  
Included observations: 41 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(X(-1))</td>
<td>-1.132032</td>
<td>0.160421</td>
<td>-7.056628</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>0.031806</td>
<td>0.040055</td>
<td>0.794056</td>
<td>0.4321</td>
</tr>
<tr>
<td>@TREND(1959)</td>
<td>0.001159</td>
<td>0.001597</td>
<td>0.725651</td>
<td>0.4725</td>
</tr>
</tbody>
</table>

R-squared 0.567228  Mean dependent var -0.000464  
Adjusted R-squared 0.544451  S.D. dependent var 0.178543  
S.E. of regression 0.120507  Akaike info criterion -1.323870  
Sum squared resid 0.551829  Schwarz criterion -1.198487  
Log likelihood 30.13934  F-statistic 24.90306  
Durbin-Watson stat 2.044463  Prob(F-statistic) 0.117537
Table 2.3: Estimation of ESTAR model (equation 2.4)

Estimated model: \( y_t = \alpha + \delta_t x_t + e^{-y_t(\alpha - \delta_x, x_t)}^{(y_{t-1} - \alpha - \delta_x x_{t-1})} + u_t \) [using annual data from 1959-2001]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \delta_t )</th>
<th>( \gamma )</th>
<th>( R^2 )</th>
<th>Standard error of regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.55</td>
<td>1.53</td>
<td>0.96</td>
<td>0.06</td>
</tr>
<tr>
<td>(0.18)</td>
<td>(0.04)</td>
<td>(0.07)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figures in first brackets are standard errors.

Estimation results of equation 2.4

Dependent Variable: \( Y \)
Method: Least Squares
Date: 02/02/06 Time: 21:29
Sample (adjusted): 1960 2001
Included observations: 42 after adjustments
Convergence achieved after 11 iterations

\( Y = C(1) + C(2) \times X + C(3) \times (Y(-1) - C(1) - C(2) \times X(-1)) \times \exp(Y(-1) - C(1) - C(2) \times X(-1))^2 \)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>0.502210</td>
<td>0.187888</td>
<td>2.672924</td>
</tr>
<tr>
<td>C(2)</td>
<td>0.557686</td>
<td>0.047449</td>
<td>11.75331</td>
</tr>
<tr>
<td>C(3)</td>
<td>1.537044</td>
<td>0.072799</td>
<td>7.377107</td>
</tr>
</tbody>
</table>

R-squared 0.961083 Mean dependent var 2.668947
Adjusted R-squared 0.959088 S.D. dependent var 0.343259
S.E. of regression 0.069430 Akaike info criterion -2.428236
Sum squared resid 0.188002 Schwarz criterion -2.304117
Log likelihood 53.99296 Durbin-Watson stat 1.543619

Table 2.4A Test for serial correlation in the residual of equation 2.4

Breusch-Godfrey Serial Correlation LM Test:

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>Prob. F(2,37)</th>
<th>Obs*R-squared</th>
<th>Prob. Chi-Square(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.161571</td>
<td>0.324146</td>
<td>2.477197</td>
<td>0.289790</td>
</tr>
</tbody>
</table>

62
Table 2.4B. Test for heteroskedasticity in the residual of equation 2.4

White Heteroskedasticity Test:

<table>
<thead>
<tr>
<th></th>
<th>F-statistic</th>
<th>Prob. F(6,35)</th>
<th>Obs*R-squared</th>
<th>Prob. Chi-Square(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.836022</td>
<td>0.550531</td>
<td>5.264814</td>
<td>0.510325</td>
</tr>
</tbody>
</table>

Table 2.5. Estimated half-lives shocks in years for Indian Rupees/US Dollar (1959-2001)

<table>
<thead>
<tr>
<th>Estimated γ</th>
<th>3% shock</th>
<th>5% shock</th>
<th>10% shock</th>
<th>20% shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.53</td>
<td>6</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2.6. Best Fitting ARIMA

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.5494</td>
<td>0.0039</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.1544</td>
<td>0.7600</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.9800</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.9411</td>
<td>0.9466</td>
</tr>
</tbody>
</table>

Table 2.7. Confidence Limits for Real Exchange Rate ARIMA

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Limits</th>
<th>95% Confidence Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower (generated by</td>
<td>Upper (generated by</td>
</tr>
<tr>
<td></td>
<td></td>
<td>bootstrapping)</td>
<td>bootstrapping)</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.5494</td>
<td>-0.77486</td>
<td>0.676537</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.9800</td>
<td>-1.44681</td>
<td>0.484147</td>
</tr>
</tbody>
</table>
Table 2.8. Best Fitting ARIMA Results for Real Exchange Rate (Indian Rs./US Dollar)

Dependent Variable: D(Y)
Method: Least Squares

Sample: 1966 1997
Included observations: 32

Backcast: 1965

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.028539</td>
<td>0.004886</td>
<td>5.840519</td>
<td>0.0000</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.549438</td>
<td>0.174988</td>
<td>3.139860</td>
<td>0.0039</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.980000</td>
<td>0.144319</td>
<td>-6.910857</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.213464  Mean dependent var 0.031194
Adjusted R-squared 0.159220  S.D. dependent var 0.112039
S.E. of regression 0.102733  Akaike info criterion -1.624306
Sum squared resid 0.306068  Schwarz criterion -1.486893
Log likelihood 28.98889  F-statistic 3.935263
Durbin-Watson stat 2.175182  Prob(F-statistic) 0.030755

Inverted AR Roots .55
Inverted MA Roots .98
Figure 2.1. Plot of productivity differential ($x_t$)

Figure 2.2. Plot of real exchange rate ($y_t$)

Real Exchange rate of India (Indian Rupees / US Dollar)
Figure 2.3. Real Exchange Rate, Nominal Exchange Rate & Price Levels

Figure 2.4 Non-Linear Impulse Response Functions
Chapter 3. Literature Review for Real Business Cycle Models

3.1. Introduction
The objective of this chapter is to familiarise the reader with the underlying concepts of models embodying a real business cycle (RBC) framework which is used in subsequent chapters of the thesis to evaluate the impact of alternative monetary and fiscal shocks. Haberler (1963) defines the business cycle in the general sense as 'an alternation of periods of prosperity and depression, of good and bad trade.' Lucas (1977) developed the idea 'One exhibits understanding of business cycles by constructing a model in the most literal sense: a fully articulated artificial economy which behaves through time so as to imitate closely the time series behaviour of actual economies'. According to Long and Plosser (1983) 'The term 'business cycle' refers to joint time-series behaviour of a wide range of economic variables such as prices, output, employment, consumption and investment. In actual economies this behaviour seems to be characterised by at least two broad regularities: (i) Measured as deviations from trend, the ups and downs in individual series exhibit a considerable amount of persistence, (ii) measures of various economic activities (e.g. outputs in different sectors) move together.'

The literature was an outgrowth of the equilibrium strategy for business cycle analysis initiated by Lucas (1972, 1973, 1975) and extended by Barro (1976, 1981), but differs from them in two critical aspects. First, RBC models place much more emphasis on mechanisms involving cycle propagation, that is spreading over time of the effects of a shock. Second, RBC models emphasize the
extent to which shocks that initiate the cycles are real - as opposed to monetary – in origin.

The greatest advantage of the RBC approach is that the structural equations of the model have been derived via an optimisation, so that the parameters of the model such as preferences and technology are truly ‘structural’. It is an equilibrium model to analyse how key macroeconomic variables are likely to respond to known economic shocks or changes in the economic structure and to identify the economic shocks and changes in economic structure underlying the observed movements in economic data. Both of these functions are important in central banks’ economic analysis. In the RBC framework alternative policies can be compared on the basis of measures of the utility benefits or costs, rather than on the basis of ad hoc objectives. It also allows for the analysis of policy and other shocks in the dynamic-stochastic context of a fully specified system, as called for by rational expectations reasoning.

The chapter is organised as follows. Section 3.2 describes the background of the real business cycle theory, it then explains the concept of business cycles. Section 3.3 describes the pioneering work of Kydland and Prescott (1982), Long and Plosser (1983) and others. We discuss the role of government and money in RBC framework in Section 3.4 and 3.5 respectively. We describe the open economy extensions of RBC model in Section 3.6 and the final section concludes.

3.2 Background of the Real Business Cycle Framework

In the 1950s and 1960s many macroeconomists believed that their understanding of the economy was nearly complete with the IS-LM framework providing the theory of aggregate demand and Phillips curve providing a useful explanation of
how wages and prices adjust over time. But new classical macroeconomists came with different ideas in 1970s. They argued that the problem with Keynesian interpretation of macroeconomic phenomena was the absence of a consistent foundation based on the choice-theoretic framework of microeconomics. This led them to build models in which markets clear and agents always optimise. This research evolved in 1980s into real business cycle theory.

In the early 1980s we witnessed one of the most striking development in macroeconomics- the emergence of a substantial body of literature devoted to the real business cycle approach to the analyse economic fluctuations. There is a difference between business cycle literature during 1960s and recent models of business cycles.

Business Cycle literature during 1960s concentrated mainly on the internal dynamics of the capitalistic economies. In sharp contrast, recent models of business cycles rely more on exogenous factors and much less on internal dynamics of the system. The theory of business cycles mainly deals with approaches that explain fluctuations in major economic aggregates. The main task therefore was to establish necessary and sufficient conditions for the existence of more or less regular oscillations in a model economy.

Frisch (1933) and Slutsky (1937) developed linear economic models which produce more or less damped cycles. Exogenous disturbances or random shocks were required in order to generate persistent fluctuations in their models. The rational expectation hypothesis advanced by Muth (1961) and subsequently developed by Lucas and Sargent, has made it clear that stochastics cannot only be superimposed on existing models, but may also constitute an essential element in explaining individual behaviour. When agents take future events into
consideration, which may influence current decision making, they are forced to form expectation about future magnitudes of importance. Thus expectations play an important role in models that rely on intertemporal optimisation.

In the post-war period we find the tradition of modelling cycles using a log-linear specification in which the propagation is through random or autocorrelated shocks (Mullineux and Peng, 1993). In this tradition the pioneering work was that of Kydland and Prescott (1982).

### 3.3 Early Real Business Cycle Models

We begin this section with Kydland and Prescott’s (1982) work. They have only one good in their model and investment takes place several periods. Long and Plosser (1983) analysed a multi-sector version with technology shock being modelled as an AR (1) process. Hansen’s (1985) model with indivisible labour accommodates unemployment in this framework.

It is important to note that it is not generally possible to compute the solutions for these models analytically. This led Kydland and Prescott and subsequent researchers to consider a structure for which analytical solution was possible. One such structure is a quadratic objective function with linear constraints and exogenous disturbances driven by a first-order Markov process (see Hansen and Prescott, 1995). Kydland and Prescott (1982) linearise the non-linear first order

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2 For more on alternative approaches especially non-linear business cycle modelling and Chaos theory see Mullineux and Peng, 1993. Chaos theory offers economists the possibility of modelling economic fluctuations without random shocks, as a chaotic process exhibits random like fluctuations, even though it is totally deterministic.

3 There is no generally agreed procedure for solving these models. As Campbell (1994) notes, the difficulty arises due to fundamental non-linearity present in these models that arise mainly due to interaction between multiplicative elements such as Cobb-Douglas production function and additive elements present in the law of motion for capital stock. This non-linearity tends to disappear only in very unrealistic cases where capital depreciates fully in a single period and agents possess logarithmic utility function (as in Long and Plosser, 1983 and McCallum, 1989). In such special cases, one can
conditions derived from the Lagrangian using an approximation method prior to analysing the dynamic behaviour of the model and its empirical implications. These necessary conditions are the stochastic analogue of the well-known Euler equations. They analysed the empirical relevance of their model by way of calibration and simulation. Using quarterly US data they showed that several business cycle correlations can be mimicked reasonably well with an equilibrium model in which there is no role for money or government policy. It can account for much of the variability in gross national product and it can correctly predict that consumption is less variable than income while investment is more variable. The reason for calibrating the model was that the (discounted) dynamic programming problem that they posed to explain the decision rules of the representative agent, was too complicated to allow for a closed-form solution to be obtained.  

Long and Plosser (1983) show how certain ordinary economic principles lead maximising agents to choose consumption and production plans that exhibit some important features of observed business cycles. The authors argue that although their model is not capable of explaining all of the observed regularities, it provides a useful benchmark to assess the relative importance of various kinds of disturbances. The authors suggest that in case of present and future consumption being normal goods, the effects of any unanticipated wealth increment due to changes in productivity in a multi-sector model would force agents to allocate their incremental savings in such a way that leads them to increase their

---

obtain a closed form solution for the model. In all other cases, some form of linear approximation is required.

Calibration involves quantitative research in which theoretical model is taken very seriously rather than a particular technique for estimating the parameters of the model (Pagan, 1994). The approach was to use established empirical results to assign values to the behavioural parameters of the decision problem.
consumption of many different goods. Given constant relative prices, this means that persistence and co-movements which are the two main characteristics of business cycles are outcomes of desired consumption plans. The authors conclude that persistence and co-movement inherent in their model are not welfare-reducing as they signify the optimal response of agents to wealth effects. By implication efforts to stabilise the economy can only make consumers worse off.

Hansen (1985) considers a one-sector stochastic growth model with shocks to technology. Hansen's model differs from similar models in that a non-convexity (indivisible labour) is introduced and is assumed to be a property of preferences. Non-convexity in individual preferences could arise if utility function exhibits decreasing marginal utility of leisure at certain low levels of leisure and increasing marginal utility at higher levels of the same. Preference ordering of this form according to the author reflects fixed costs associated with working each period. The model is able to account for large fluctuations in hours worked relative to productivity without relying heavily on intertemporal substitution of leisure. Indivisible labour is modelled by assuming that agents can either work some given positive number of hours or not at all. Unlike previous models, fluctuations in aggregate employment in Hansen's model result from the agents entering and leaving the job market, rather than continuously employed agents adjusting the number of hours worked. The individuals in the model are forced to enter and exit the labour force in response to a stochastic technology shock. When one considers technology, individuals could face a production function which is convex at first and then becomes concave that is marginal productivity of work effort could increase during the beginning of the day / week and then
gradually subside owing to ‘warm up’ time required for becoming productive again.

According to McGrattan (1994) the introduction of non-convexity allows Hansen’s otherwise prototype RBC model to better mimic the variability of total hours worked than other models. However it cannot capture the observed variability in consumption and productivity.

As mentioned earlier, the goal of modern business cycle research in general and RBC school of thought in particular has been to develop models that mimic the cyclical patterns of aggregate data. In the models of Kydland and Prescott (1982), Long and Plosser (1983) and Hansen (1985) there are no externalities, taxes, government expenditure or monetary variables. All agents are assumed to be alike and each is infinite-lived. Consequently in these models, competitive equilibria have the property of Pareto optimality.

### 3.4 Role of Government in a Real Business Cycle Model

Incorporation of government expenditure (fiscal policy) into the RBC framework introduces a potential source of demand side disturbance to the basic model which is otherwise governed by supply side disturbances. Mankiw (1989) notes that an increase in government expenditure shifts the IS curve upwards i.e., increases the demand for goods. In order to achieve equilibrium in the market for goods, the real interest rate must rise. As pointed out earlier, business cycle theory assigns a pivotal role to intertemporal substitution of goods and leisure. An increase in the real interest rate would lead individuals to relocate leisure across time. At higher real interest rates, working today becomes more lucrative than working in future. This temporary increase in supply of labour leads to a rise in equilibrium output and employment. Although Keynesian theory like the RBC
theory predicts a rise in the real interest in response to an increase in government consumption, the impact of real interest rate on labour supply is not given prominence.

Some influential papers emphasising a role for government in a RBC set-up have been that of Christiano and Eichenbaum (1992) and McGrattan (1994). Christiano and Eichenbaum (1992) modify a prototypical RBC model by allowing a government expenditure shock to influence labour market dynamics. The authors claim that existing RBC models grossly overstate the correlation between hours worked and returns to working while Keynesian models greatly understate it. The authors argue that, by not assigning a role for shock to government expenditure in explaining labour market dynamics, existing RBC models implicitly assume that public and private consumption are perfect substitutes in the agents' utility function. The authors relax this assumption. Since government spending is modelled as a close substitute for private consumption in the utility function, an increase in government expenditure entails a negative wealth effect which shifts labour supply. Using Generalised Method of Moments (GMM) procedure for estimation, they conclude that when fiscal shocks were combined with shocks to technology, the model's performance in the case of the US is substantially enhanced.

McGrattan (1994) compares his model which includes fiscal disturbances to that of Prescott (1986) who estimates that technology shocks account for nearly 75% of the fluctuations in the post war US data. The author finds that a significant amount of variation in aggregate consumption, investment, output, employment and capital stock can be explained by innovations in labour and capital taxes and government consumption.
Braun (1994) investigates the macroeconomic effects of cyclical fluctuations in marginal tax rates and argues that the main ingredient missing in RBC models is the factors which shift the labour supply schedule. In his paper both shifts in labour demand and labour supply schedules (as a result of taxation), reduce the strong positive correlation between wages and hours worked. A fall in the effective tax rate on capital income increases the after-tax return (on interest rate) and leads agents to work more today. Similarly, fluctuations in the wage tax induces a large substitution effect that influences leisure / labour supply decision.

3.5 Money in a Real Business Cycle Model

In an ideal world under the assumption of perfect competition, instantaneous adjustment of prices / wages, rational expectations and neutral money, there exists a case for real business cycles mainly due to shocks to technology. Early equilibrium business cycle models were greatly influenced by the monetarist tradition and the empirical findings of Friedman and Schwartz (1963). Walsh (1986) notes that standard monetary theories of business cycles argue that changes in the stock of money are a major source of instability in real economic activity.

Influential papers emphasising a role for money in a RBC model include that of King and Plosser (1984) and Cooley and Hansen (1989). King and Plosser (1984) integrate money and banking into real business cycle theory. Their model has two productive sectors with one intermediate and one final good. The output of the final good industry serves either as a consumption good or as an input for further production. The output of financial industry (demand for which arises because it economises the transactions cost of carrying out exchange) serves as an
intermediate good used both by firms in the final goods industry and by households. Financial services in this set-up enter the representative firms’ production function. The production of financial services is depicted by an instantaneous production structure with constant returns to scale. Shifts to this structure take place as a result of innovations in this industry. Deposits and real currency are treated as close substitutes. If banks are forced by law to hold non-interest bearing reserves, the cost of bank-supplied intermediate goods goes up and that in turn raises the demand for currency.

An important implication of this model is that bank or inside money has no role in determination of prices in an unregulated banking environment. Banks (financial industry) influence price level determination indirectly through variations in the cost of financial services which alters the relative demands for inside and outside money. By implication, broad measures of money are likely to be pro-cyclical due to endogeneity of bank money which responds to a productivity shock. A rise in output due to a productivity shock causes both the demand for money as well as the interest rate to rise. As interest rates rise, financial institutions try to reduce their holdings of excess reserves (which earn no interest), by making new loans or by purchasing government securities. Given that all such loans ultimately end up as deposits at some financial institution, the broad measure of money expands in response to a productivity shock i.e., a reverse causation. The authors conclude that in empirical work it is important to distinguish inside from outside money.

Cooley and Hansen (1989) incorporate money by using cash-in-advance constraint in a RBC model. The cash-in-advance constraint in their model is binding only in case of the consumption good while leisure and investment are
treated as credit goods. Anticipated inflation in this set up would cause agents to substitute away from activities that require cash to ones that do not require them i.e., leisure and investment. Hence, if agents in this economy choose to reduce cash holdings in response to higher anticipated inflation (due to an increase in money growth), they can only do so by reducing their consumption expenditure. In this framework when money supply follows a constant growth rate rule individuals substitute leisure for goods and both output and investment fall. However, when money supply process is erratic the cyclical behaviour of real variables is altered: consumption becoming more variable relative to income and price level becoming volatile as well.

It is necessary to introduce some form of price / wage rigidity for monetary shocks to have real effects. If prices are fixed, a variety of real effects readily follows as the central bank’s power of a monopoly over the supply of currency affects relative prices. As Plosser (1990) documents, one can classify the attempts to explain monetary non-neutrality into three broad categories: (a) sticky prices; (b) nominal wage contracts; (c) imperfect price information

3.6 Open Economy Extensions of the RBC Framework

Quantitative studies on closed economies suggest that a stochastic growth model with a single aggregate technology shock can replicate the magnitude of fluctuations, relative to output, in consumption and investment and the correlation of these with output. In the analogous world economy, economies experience imperfectly correlated shocks to their technologies. The magnitude and character of aggregate fluctuations can be substantially influenced by interaction between technology shocks and the ability to borrow and lend internationally. According
to Backus, Kehoe and Kydland (1992), a country’s consumption and investment decisions are no longer constrained by its own production in open economies. The opportunity to share risk across countries would lead to equilibrium consumption paths that are less variable and less closely related with the domestic output compared to a closed economy. On the other hand, domestic investment will tend to be more volatile as capital will be allocated to the country with a more favourable technology shock. The most distinguishing feature of an open economy is that it can borrow and lend in international markets by running trade surpluses and deficits.

Mendoza (1991) extended the basic RBC framework of Kydland and Prescott (1982) to the case of a small open economy to explore the interaction of domestic physical capital and foreign financial assets as an alternative vehicle for savings in an economy where domestic productivity and world’s real interest rate are affected by stochastic disturbances. The novelty of this approach is that it explores real business cycles in a framework in which trade in foreign assets finances trade imbalances and plays a crucial role in explaining the dynamics of savings and investment. In contrast with the RBC framework the rate of time preference is endogenously determined.

Correia, Neves and Rebelo (1995) summarise the main features of business cycles in the small open economy-Portugal- and discuss the extent to which these can be rationalised on the basis of a simple dynamic stochastic general equilibrium model. The structure of their economy is very similar to Mendoza (1991) and Lundvik (1992) in which there is a single asset that can be traded with the rest of the world- an international bond that yields a real rate of return that is viewed as exogenous by agents in the economy. The crucial finding of the paper
is that the ability of small open economy models to mimic business cycles depends heavily on the form of momentary utility. If they adopt momentary utility function proposed by Greenwood, Hercowitz and Huffman (1988), then the relative variability and co-movement patterns found in the data for the components of the national income identity can be replicated. According to them it is the adoption of the Greenwood, Hercowitz and Huffman (1988) momentary utility function that is fundamental to the good performance of the models of Mendoza (1991) and Lundvik (1992).

In another paper Mendoza (1995) uses an inter-temporal general equilibrium framework to explore the effects of random shocks to productivity and terms of trade. The stylised fact is that movements in real exchange rate are pro-cyclical. Observed terms of trade shocks are largely pro-cyclical but persistent. Less developed countries tend to have larger cycles but both developed and developing economies have similar variability ratios, auto-correlation and gross domestic product (GDP) correlation. The paper shows that the terms of trade shocks are the main driving force behind almost half of the GDP and real exchange rate variability.

McCurdy and Ricketts (1995) extend the basic RBC model by employing stochastic international model with money. Using a sample of US and Canadian data they show that changes in the rate of growth of money causes fluctuations in consumption and investment. It is also shown that monetary fluctuations are transmitted to other economies via exchange rates and terms of trade adjustments.

Obstfeld and Rogoff (1995) launched a new wave of research. They introduced nominal rigidities and market imperfections into a dynamic general equilibrium.

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5 These functions imply that labour effort is determined independently of the inter-temporal consumption-savings choice.
model with well specified micro-foundations. The presentation of explicit and profit maximising problems provides clarity compared to the models of the past which used ad-hoc assumptions to generate features of the data such as inflation / real exchange rate persistence. It allows us to conduct welfare analysis and policy evaluation. The presence of nominal rigidities and market imperfections alters the transmission mechanism for shocks and also provides a more potent role for monetary policy.

There is a growing literature on open economy real business cycle models. Balsam and Eckstein (2001) describe small open economy real business cycle model with non-traded goods. Their analysis starts with a small open economy with one traded good and endogenous labour supply as in Correia, Neves and Rebelo (1995) (CNR)\(^6\). They have used quarterly data (1970-1997) of Israel, for their analysis. The purpose of their research is to provide a simple extension of the real business cycle model of a small open economy that can fit the following facts of the Israeli economy. First, aggregate consumption is twenty percent more volatile than output and is pro-cyclical. Second, the trade balance is much more volatile than output and is pro-cyclical. Third, investment is almost five times more volatile than output. Fourth, the auto-correlation in output is low. They find that the observed phenomenon that volatility of consumption is twenty percent higher than the volatility of output is compatible with a simple specification of a two-sector model of a small economy. They show that when the two sector economy converges to a one sector economy, the volatility in consumption becomes a small fraction of output volatility whereas other parameters stay the same. The puzzle that this observation implied is explained by the result that with

\(^6\) For an explanation of the differences in consumption volatility due to utilities functions specifications in open small economies, see CNR (1995)
the three parameters of CES utility function and the share of non-traded goods in government expenditure, one can get almost any volatility in consumption holding constant the production side parameters. The openness of the economy enables the representative consumer to react in different ways to shocks in traded and non-traded goods, depending on her preferences. By lending and borrowing, consumers can change their consumption of traded goods while non-traded goods consumption can be smoothed in response to shocks in non-traded output.

Sunghyun Henry Kim and M. Ayhan Kose (2003) examine the dynamic implications of different preference formulations in open economy business cycle models with incomplete asset markets. They study two preference formulations: a time separable preference formulation with a fixed discount factor and a time non-separable preference structure with an endogenous discount factor. They analyse moment implications of two versions of an identical open economy model- one with a fixed discount factor and the other with an endogenous discount factor and study impulse responses to productivity and world real interest rate shocks. They find that business cycle implications of the two models are quite similar under conventional parameter values.

Barro and Tenreyro (2004) describe closed and open economy models of business cycles with marked up and sticky prices. They show that shifts in the extent of competition, which affect mark-up ratios, are possible sources of aggregate business fluctuations. They begin with a real model in which mark-up ratios correspond to the prices of differentiated intermediate inputs relative to the price of undifferentiated final product. If the nominal prices of the differentiated goods are relatively sticky, unexpected inflation reduces the relative prices of
intermediates and mimics the output effects from an increase in competition. In an open economy, domestic output is stimulated by reductions in the relative price of foreign intermediates and, therefore, by unexpected inflation abroad. The models imply that relative output prices are more counter-cyclical the less competitive the sector.

Minford and Sofat (2004) construct a dynamic general equilibrium open economy model based on optimising decisions of rational agents, incorporating money, government and distortionary taxes. Their model is different from other open economy RBC models in the sense that the interaction with the rest of the world comes in the form of uncovered interest parity and current account both of which are explicitly micro founded. First order conditions of firm’s and household’s optimisation problem are used to derive the structural equations of the model.

In a prototype RBC model with complete markets and in the absence of any form of externality there is no role for the government. Still one could think of a government providing public goods from the tax revenue it collects, although there is no stabilisation role for the government. Incorporation of government expenditure (fiscal policy) into RBC framework by Minford, Sofat (2004) introduces a potential source of demand side disturbance to the basic model which is otherwise governed by supply side disturbances. It is also assumed in their model that money has value in exchange (e.g., see Lucas, 1980, 1987). In order to give value to money trading in decentralised markets is introduced. Here, to motivate the use of money a subset of consumption goods must be paid for with currency acquired in advance. The cash-in-advance model is a convenient way for representing the aspects of classical monetary theory in the context of an inter-temporal model.
The choice of exchange rate regime is a special case of the general issue of optimal monetary policy in an open economy. Researchers have been using the stochastic general equilibrium paradigm to investigate the performance of alternative open economy monetary policy rules, for e.g. Benigno and Benigno (2001), the analysis of the alternative exchange rate regimes in terms of macroeconomic and welfare properties, see Devereux and Engel (2001), Collard and Dellas (2002), Dellas and Tavlas (2002), and the welfare implications of different degrees on international policy coordination, Corsetti and Pesenti (2001a, 2001b). The message from this literature regarding the value of the exchange rate instrument is mixed. The results depend on the currency denomination of trade, the structure of financial markets, the type of policy rule considered and the difference in size across countries.

3.7 Conclusions
In this chapter we have traced the birth of real business cycle theory. We have also traced the development in the RBC literature – from closed economy models to open economy models. Open economy RBC models are vital for our understanding of international effects of business cycles. We also get an advantage from the RBC framework that is alternative policies can be compared on the basis of measures of the utility benefits or costs, rather than on the basis of ad hoc objectives. In this thesis, we use Minford, Sofat (2004) model which is micro-founded general equilibrium open economy model based on optimising decisions of rational agents, incorporating money, government and distortionary taxes.
Chapter 4. AN OPEN ECONOMY REAL BUSINESS CYCLE

MODEL

OF THE INDIAN ECONOMY

4.1 INTRODUCTION

In Chapter 2 we have examined the effects of productivity differential on real exchange rate in the context of Harrod-Balassa-Samuelson effect. In this chapter, we extend this analysis into a dynamic general equilibrium open economy framework in terms of a real business cycle model (hereafter RBC model). This enables us to study the effect of productivity shock not only on real exchange rate, but also on output, price levels, consumption, investment, export, import and other relevant macroeconomic variables.

An RBC model exhibits an understanding of the business cycle by constructing a fully artificial model which behaves through time so as to imitate closely the time series behaviour of the economy (Lucas, 1977). RBC models originated from the pioneering work of Kydland and Prescott (1982) and Long and Plosser (1983). These first generation models place more emphasis on the cycle propagation that is spreading the effects of a shock over time. Moreover most of these models were developed for closed economy set up that assumed away externalities, taxes, government expenditure and monetary variables. These essentially assumed a competitive theory of economic fluctuations with pareto optimal equilibria. Since then there has however been various extensions introducing role of government (e.g., Mankiw, 1989), role of money (King and Plosser, 1984), role of distortionary taxes (Braun, 1994) into these first generation RBC models.
For our purpose we however focus on the open economy extension of the RBC models. This is because in reality most economies exhibit well defined empirical regularities not only in terms of domestic, but also international indicators. The common theme of this new wave pioneered by Obstfeld and Rogoff (1995) is the introduction of nominal rigidities and market imperfections into dynamic general equilibrium model. This generates a rigorous analytical framework for policy analysis, especially monetary policy.

The choice of the exchange rate regime is a special case of optimal monetary policy in an open economy framework. There is a sizeable literature in this respect including among others Benigno and Benigno (2000), Devereux and Engel (2001), Collard and Dellas (2002) and Dellas and Tavlas (2002).

In this chapter we make use of a micro-based dynamic general equilibrium open economy model (Minford and Sofat, 2004) for a medium sized economy incorporating money, government expenditure and distortionary taxes. Interaction with the rest of the world comes in the form of uncovered interest rate parity and the current account. The model has obvious relevance for studying the case of India over the period 1966-1997. This is particularly because this model allows us to incorporate the intrinsic characteristics of the Indian economy. For example, the assumptions of a medium sized economy, open economy (export and import) and the prevalence of distortionary taxes, money and government expenditure are clearly pertinent for the Indian economy over this period. We also calibrate this model using annual data for the country over this period. This exercise proves to be useful for us as it allows us to simulate the time-series behaviour of the real exchange rate and also a wide range of other economic variables for the Indian economy. This kind of micro-based optimising macroeconomic analysis
in real business cycle framework has not been done in the Indian context. This kind of analysis is also extremely useful for policy analysis. The greatest advantage of the RBC approach is that the structural equations of the model have been derived via optimisation, so that the parameters of the model such as preferences and technology are truly 'structural'. It is an equilibrium model to analyse how key macroeconomic variables are likely to respond to known economic shocks or changes in the economic structure and to identify the economic shocks and changes in economic structure underlying the observed movements in economic data. Both of these functions are important in central banks’ economic analysis. In the RBC framework alternative policies can be compared on the basis of measures of the utility benefits or costs, rather than on the basis of ad hoc objectives. We use the simulation results for both demand and supply shocks with calibrated parameters to assess the overall properties of the model.

The chapter is developed as follows. Section 4.2 explains the model. Section 4.3 describes calibration and solution algorithm, section 4.4 describes the data, section 4.5 describes the simulation results. In section 4.6 we add an overlapping wage contract equation to the RBC model (Minford, Sofat 2004) and compare the simulations of the two models (with and without overlapping wage contract). The final section concludes.

4.2 Model
In this section we describe the characteristic features of an open economy RBC model as developed by Minford and Sofat (2004). This is important for our purpose as this helps us to generate the structural equations of the model to be
calibrated for the Indian economy. The existence of representative household, representative firm, government and foreign sector of Minford, Sofat (2004) model are very much pertinent to the Indian economy as to the UK economy given that both are open economies of moderate size that cannot however affect world variables. Consider an economy populated by identical infinitely lived agents who produce a single good as output and use it both for consumption and investment. To simplify the notation we abstract from population growth and represent all variables in per capita terms. We assume that there are no market imperfections i.e. no frictions or transaction costs. At the beginning of each period t, the representative agent chooses a) the commodity bundle necessary for consumption during the period, b) the total amount of leisure that she would likely to enjoy during the period and c) the total amount of factor inputs necessary to carry out production during the period. All of these choices are constrained by fixed amount of time available and the aggregate resource constraint that agents face. During the period t, the model economy is influenced by various random shocks. In an open economy goods can be traded but for simplicity it is assumed that these do not enter in the production process but are only exchanged as final goods. The consumption, \( C_t \) in the utility function below is composite per capita consumption, made up of agents’ consumption of domestic goods, \( C^d_t \) and their consumption of imported goods \( C^f_t \). The composite consumption function can be represented as an Armington aggregator of the form

\[
C_t = \omega (C^d_t)^{-\rho} + (1 - \omega) (C^f_t)^{-\rho} \left( \frac{1}{\rho} \right)
\]

(4.1)

where \( \omega \) is the weight of home goods in the consumption function and \( \sigma \), the elasticity of substitution is equal to \( \frac{1}{1 + \rho} \).
The consumption-based price index that corresponds to the above specification of preference, denoted \( P_t \), is derived as

\[
P_t = \left[ \frac{1}{\omega^{1+\rho}} \left( P_t^d \right)^{\frac{\rho}{1+\rho}} + (1 - \omega) \frac{1}{1+\rho} \left( P_t^F \right)^{\frac{\rho}{1+\rho}} \right] \tag{4.2}
\]

where \( P_t^d \) is domestic price level and \( P_t^F \) is the foreign price level in domestic currency.

Given the specification of the consumption basket, the agent’s demand for home and foreign goods are a function of their respective relative price and composite consumption:

\[
C_t^d = \left( \frac{P_t^d}{\omega P_t} \right)^{\frac{1}{1+\rho}} C_t \tag{4.3}
\]

\[
C_t^f = \left( \frac{P_t^F}{(1 - \omega) P_t} \right)^{\frac{1}{1+\rho}} C_t \tag{4.4}
\]

In a stochastic environment a consumer is expected to maximise her expected utility subject to her budget constraint. Each agent’s preferences are given by

\[
U = \max_{\mathbb{E}_0} \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \quad 0 < \beta < 1 \tag{4.5}
\]

where \( \beta \) is the discount factor, \( C_t \) is consumption in period \( t \), \( L_t \) is the amount of leisure time consumed in period \( t \) and \( \mathbb{E}_0 \) is the mathematical expectational operator. The essential feature of this structure is that agent’s tastes are assumed to be constant over time and is not influenced by exogenous stochastic shocks.

The preference ordering of consumption sub-sequences \([C_1, L_1), (C_{t+1}, L_{t+1}), \ldots]\) does not depend on \( t \) or on consumption prior to time \( t \). We assume that \( u(C, L) \)
is increasing in \((C,L)\) and concave \(u'(C,L) > 0, \ u''(C,L)<0.\) We also assume that \(u(C,L)\) satisfies Inada type conditions: \(u'(c) \to \infty\) as \(c \to 0\) and \(u'(c) \to 0\) as \(c \to \infty, u'(L) \to \infty\) as \(l \to 0\) and \(u'(L) \to 0\) as \(l \to \infty\).

### 4.2.1 The Representative Household

The model economy is populated by a large number of identical households who make consumption, investment and labour supply decisions over time. Each household's objective is to choose sequence of consumption and hours of leisure that maximises its expected discounted stream of utility. We assume a time separable utility function of the form

\[
U(C_t,1-N_t) = \theta_0 (1-\rho_0)^{-1} C_t^{(1-\rho_0)} + (1-\theta_0)(1-\rho_2)^{-1}(1-N_t)^{(1-\rho_2)}
\]

where \(0 < \theta_0 < 1\) and \(\rho_0, \rho_2 > 0\) are substitution parameters. This sort of functional form is used for example by McCallum and Nelson (1999a). The advantage of using this specification is that it does not restrict elasticity of substitution between consumption and leisure to unity. Barro and King (1984) note that time-separable preference ordering of this form would not restrict the sizes of intertemporal substitution effects. However time separability constrains the relative size of various responses such as those of leisure and consumption to relative price and income effects.

Individual economic agents view themselves as playing a dynamic stochastic game. Changes in expectation about future events would generally affect current decisions. Individual choices at any given point of time are likely to be influenced by what agents believe would be their available opportunity set in the future. Each agent in our model is endowed with a fixed amount of time, which she spends on leisure \(L_t\) and / or work \(N_t\). If total endowment of time is normalised to
unity, then it follows that $N_t + L_t = 1$ or $L_t = 1 - N_t$ ......(4.7)

Let us assume $I$ is the normal amount of leisure which is necessary for an agent to sustain her productivity over a period of time. If an agent prefers more than normal amount of leisure say $U_t$, she is assumed to be unemployed ($U_t = (1 - N_t)$ $-I$ ) in this framework. An agent who chooses $U_t$ is entitled to get an unemployment benefit $\mu_t$. It is assumed that $\mu_t < v_t$ (i.e. the consumer real wage as defined below) so that there is an incentive for the agent to search for a job. With the introduction of unemployment benefit substitution between work and leisure is higher.

The representative agent’s budget constraint is

$$\left(1 + \phi_t\right)C_t + \frac{b_{t+1}}{1 + r_t} + \frac{Q_t b_{t+1}'}{1 + r_t'} + \frac{p_t S_t^p}{P_t} =$$

$$(1 - \tau_{t-1})N_{t-1} + \mu_{t-1}\left[(1 - N_{t-1}) - I\right] + b_t + Q_t b_t' + \frac{(p_t + d_t)S_t^p}{P_t} \quad (4.8)$$

where $p_t$ denotes present value of share, $v_t = \frac{W_t}{P_t}$ is real consumer wage, $w_t = \frac{W_t}{P_t}$ is producer real wage. Consumption and labour income are taxed at rates $\phi_t$ and $\tau_t$ respectively, both of which are assumed to be stochastic process.

Also $(1 + \phi_t)C_t^d = \frac{M_t^{d\phi}}{P_t^{d\phi}}$ that is representative agent’s real demand for domestic money is equal to consumption of domestic goods inclusive of sales tax. In a similar way, the agent’s real demand for foreign money is equal to consumption
of foreign goods inclusive of sales tax \( (1 + \phi_f)C_t^f = \frac{M_t^{fp}}{P_t^f} \). This follows from the fact that consumption in this framework is treated as a cash good i.e. cash-in-advance constraint is binding only in the case of consumption. Investment is treated as a credit good. \( b_t^f \) denotes foreign bonds, \( b_i \) domestic bonds, \( S_t^p \) demand for domestic shares and \( Q_t \) is the real exchange rate.

In a stochastic environment the representative agent maximises her expected discounted stream of utility subject to her budget constraint. The Lagrangian associated with this problem is

\[
U = \max E_0 \left\{ \sum_{n=0}^{\infty} \beta^n \left[ \theta_0 (1 - \rho_0)^{-1} C_t^0 \right] + (1 - \theta_0) \left( (1 - N_t) \right) \right\} + \lambda \left\{ \left( 1 - \tau_{t-1} \right) N_{t-1} + \mu_t \left[ (1 - N_{t-1}) - 1 \right] + b_t + Q_t b_t^f + \frac{(P_t + d_t)S_t^p}{P_t} - (1 + \phi_f)C_t \right\}
\]

\[\lambda = \frac{b_{t+1}}{1 + r_t} - \frac{Q_t b_{t+1}^f}{(1 + r_t^f)(1 - rp)} - \frac{P_t S_t^p}{P_t}\] 

\[\ldots (4.9)\]

Where \( \lambda \) is Lagrangian multiplier, \( 0 < \beta < 1 \) is the discount factor and \( E(.) \) is the mathematical expectations operator.

First order conditions of household's optimisation problem are given in the equation appendix of chapter 4.
4.2.2 The Representative Firm

Firms rent labour and buy capital inputs from households and transform them into output according to a production technology and sell consumption and investment goods to households and government. The interaction between firms and household is crucial, as it provides valuable insights for understanding the fluctuations of macroeconomic aggregates such as output, consumption and employment.

The technology available to the economy is described by a constant returns to scale production function:

\[ Y_t = Z_t f(N_t, K_t) \]

\[ Y_t = Z_t N_t^a K_t^{1-a} \]

where \( 0 < \alpha < 1 \), \( Y_t \) is aggregate output per capita, \( K_t \) is capital carried over from previous period (t-1), and \( Z_t \) reflects the state of technology.

It is assumed that \( f(N,K) \) is smooth and concave and it satisfies Inada-type conditions i.e., the marginal product of capital (or labour) approaches infinity as capital (or labour) goes to 0 and approaches 0 as capital (or labour) goes to infinity.

The capital stock evolves according to \( K_{t+1} = (1 - \delta) K_t + I_t \)

Where \( \delta \) is the depreciation rate and \( I_t \) is the gross investment. In a stochastic environment the firm maximises present discounted stream, \( V \), of cash flows, subject to the constant returns to scale production technology, i.e.
$MaxV = E_t \sum_{i=0}^{T} d_t^i \left( Y_t - K_t (r_t + \delta) - w_t N_t^d \right)$ subject to $Y_t = Z_t f (N_t, K_t)$. Here $r_t$ and $w_t$ are rental rates of capital and labour inputs used by the firm, both of which are taken as given by the firm. Output of the firm depends not only on capital and labour inputs but also on $Z_t$. First order conditions of the firm’s optimisation problem are given in equation appendix of chapter 4. The relevant equations about the government and the foreign sector are also given in the equation appendix.

4.2.3 Behavioural equations of the model

First order conditions from the household’s and firm’s optimisation problem are used to derive the following behavioural equations of the model.

i) Consumption $C_t$; solves for $r_t$

\[
(1 + r_t) = \frac{1}{\beta} \left( \frac{C_t}{E_t [C_{t+1}]} \right)^{-\rho_t} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right)
\]

\[
r_t = \frac{1}{\beta} \left( \frac{C_t}{E_t [C_{t+1}]} \right)^{-\rho_t} \left( \frac{1 + \phi_{t+1}}{1 + \phi_t} \right) - 1
\]

where $C_t = \left[ \omega (C_t^d)^\rho + (1 - \omega) (C_t^f)^\rho \right]^{\frac{1}{\rho}}$

ii) Money supply $\tilde{M}_t^d$; solves for $P_t^d$

\[
\tilde{M}_t^d = (1 + \phi_t) C_t^d P_t^d + \tilde{G}_t P_t^d
\]
\[
P_t^d = \frac{\tilde{M}_t}{(1 + \phi)C_t^d + \tilde{G}_t}
\]

where \( P_t = \left[ \frac{1}{\omega^{\frac{1}{1+\rho}}} \left( P_{t}^a \right)^{\frac{\rho}{1+\rho}} + (1 - \omega)^{\frac{1}{1+\rho}} \left( P_{t}^k \right)^{\frac{\rho}{1+\rho}} \right] \)

iii) Demand for shares \( S_{t+1}^p \)

\[
S_{t+1}^p = \tilde{S}_t; \quad b_{t+1} = b_{t+1}^p \text{ implied}
\]

iv) Present value of share

\[
P_t = E_t \sum_{i=1}^{\infty} \frac{d_{t+i}}{(1 + r_t)^i} \left( \frac{P_t}{P_{t+i}} \right)
\]

v) Production function \( Y_t = Z_t N_t^a K_t^{(1-\alpha)} \)

vi) Demand for labour

\[
N_t^d = \left( \frac{\alpha Z_t}{w_t} \right)^{\frac{1}{1-\alpha}} K_t
\]

vii) Capital

\[
K_t = (1 - \alpha) \frac{Y_t}{r_t + \delta}
\]

viii) GDP identity, \( Y_t \); solves for \( C_t \)

\[
Y_t = C_t + I_t + G_t + NX_t
\]

Where \( NX_t \) is net exports

ix) Investment

\[
K_{t+1} = (1 - \delta) K_t + I_{t+1}
\]

x) Wage \( w_t \):

\[
w_t = w_t^*
\]
xi) Evolution of $b_t$; government budget constraint:

$$ b_{t+1} = (1 + r_i) b_t + P D_t - \frac{\Delta M_t}{P_t} $$

xii) Equilibrium wage, $w^*_t$; $w^*_t$ is derived by equating demand for labour, $N^d_t$, to the supply of labour $N^s_t$, where

$$ (1 - N^s_t) = \left\{ \frac{\theta_0 C_i^{-\rho_0} \left[ (1 - \tau_t) \exp \left( \log w^*_t - \frac{(1 - \omega)^\frac{1}{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^\frac{1}{1+\rho}} \log Q_t \right) - \mu_t \right]^{-1}}{(1 - \theta_0)(1 + \phi_t)(1 + r_i)} \right\}^{-\frac{1}{\rho}} $$

where $Q_t$ is the real exchange rate, $(1 - \omega)^\frac{1}{1+\rho}$ is the weight of domestic prices in the CPI index.

xiii) Dividends are surplus corporate cash flow:

$$ d_t S_t = Y_t - N^s_t w_t - K_t (r_t + \delta) $$

$$ d_t = \frac{Y_t - N^s_t w_t - K_t (r_t + \delta)}{S_t} $$

xiv) Primary deficit $PD_t$

$$ PD_t = G_t + \mu_t \left( 1 - N^s_t - i \right) - \tau_{t-1} v_{t-1} N^s_{t-1} - \phi_{t-1} C_{t-1} - T_{t-1} $$
xv) Tax $T_t$

$$T_t = T_{t-1} + \gamma^g (PD_{t-1} + b_t r_t) + \varepsilon_t$$

xvi) Exports

$$\log EX_t = \sigma_1 \log(1 - \omega^f) + \log C_t + \sigma_1 A^f \log Q_t$$

where $A^f = \frac{\left(\omega^f\right)^{1+\rho}}{\left(\omega^f\right)^{1+\rho} + (1 - \omega^f)^{1+\rho}}$

xvii) Imports $IM_t$

$$\log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma A \log Q_t$$

where $A = \frac{\left(\omega^{1+\rho}\right)^{1+\rho}}{\left(\omega^{1+\rho}\right)^{1+\rho} + (1 - \omega)^{1+\rho}}$

xviii) UIP condition

$$r_t = r_t^f + E \Delta \log Q_{t+1} + \varepsilon_{UIP}$$

where $r_t^f$ is the foreign real interest rate

xix) Net exports

$$NX_t = EX_t - IM_t$$

xx) Evolution of foreign bonds $b_t^f$

$$b_{t+1}^f = \left(1 + r_t^f\right)b_t^f + NX_t$$

xxi) Nominal exchange rate, $S_t$

$$\log S_t = \log Q_t - \log P_t^f + \log P_t$$

xxii) Evolution of household debt $D_{t+1}$

$$D_{t+1} = (1 + r_t)D_t - Y_{t-1} + (1 + \phi)C_t + \tau_t \nu_tN_t^f + T_t$$
4.3 CALIBRATION

Having explained the structural equations, we can now proceed to calibration for the Indian economy. In doing so, numerical values need to be assigned to the structural parameters of the models. These values such as the output elasticity of the production factors, the degree of risk aversion or the elasticity of intertemporal substitution are taken from some casual empirical characteristics pertaining to the Indian economy.

The exogenous stochastic processes should also be calibrated. However it is hard to find information from a real economy concerning the stochastic structure of technology shocks, shocks to preferences, error of controlling money growth or tax revenues or the correlations among them. For this purpose, persistence properties in actual time series data can be used to calibrate some aspects of the model. For instance, in the simplest business cycle model, an AR(1) model is assumed for productivity shocks, with the coefficients generally chosen so that the simulated output series exhibits persistence similar to GNP series in actual economies.

The value of alpha is taken from the averages in Indian data and the values for the rest of the coefficients used in the model have been calibrated from the paper of Minford and Sofat (2004) and then these coefficients are estimated in chapter 6 by Maximum Likelihood Method (MLM) – choosing values of all parameters by

\[ Y_{t-1} - r_t D_t - \phi_t C_t - \tau_t v_t N_t^s - T_t = C_t \]
optimising a given criterion—the likelihood of the Indian data. We note at that later stage that the estimated parameters come out reasonably close to the ones chosen here.

### 4.3.1 Model Solution and Algorithm

In solving our model, we are forced by its complexity and non-linearity to use a computer algorithm. A well-behaved rational expectations model has a unique solution. To obtain this solution, the solving procedure sets the terminal condition that beyond some terminal date $N$, all the expectational variables are set to their equilibrium values. It is necessary for the terminal date to be large, in order to reduce the sensitivity of the model to the variations in the terminal date. The justification for the terminal condition is that non-convergent behaviour of the system would provoke behaviour by government or other economic agents (transversality condition) different from that assumed as normal in the model and that would eliminate the divergence. (Minford, 1979). As pointed out by Matthews and Marwaha (1979), the actual value of the terminal condition can be derived from the long run equilibrium condition of the model. In some cases, the steady-state properties of the model can be used to choose the terminal conditions of the model, although several other methods can be easily used.

There are several iterative methods, but the most common is the Gauss-Seidel method. This iterative method is built in the program developed by Matthews (1979) and Minford et al (1984) called RATEXP which has been used to get the model solution. The computer program typically uses a backward-solving (dynamic programming) technique. However, unlike the classical dynamic programming, the solution vector is approached simultaneously for all $t = 1, 2, ...$
T, but convergence follows a backward process. The problem lies in that the model must firstly obtain a dynamic solution for a given time span using initial guess values of the expectational variables. These initial values are then adjusted in an iterative manner until convergence is obtained. After checking for equality between expectations and solved forecasts, the initial expectations set is gradually altered until convergence is obtained. In effect this endogenises the expectational variables in that period. Our model is highly non-linear, consequently a larger number of iterations are required as compared to linear models. It should be noted that in general a non-linear model does not have a unique reduced form. When a non-linear model is solved in a deterministic manner the solution values of the endogenous variables are not in general equal to their expected values. A correct solution requires stochastic simulation.

In order to understand how the algorithm works, consider a set of simultaneous non-linear structural equations written in implicit form

\[ F\{y(t), y(t-1), x(t), u(t)\} = 0 \quad \text{(i)} \]

Where, \(y(t)\) is a vector of endogenous variables, \(y(t-1)\) is a vector of lagged endogenous variables, \(x(t)\) is a vector of exogenous variables and \(u(t)\) is a vector of stochastic shocks with mean zero and constant variances. \(F(.)\) represents a set of functional form. Setting the disturbance terms equal to their expected values and solving for the reduced form, we have

\[ y_t = H\{x(t), y(t-1)\} \quad \text{(ii)} \]

where \(H(.)\) is the reduced form functional form. Partitioning equation (ii) so as to distinguish between endogenous variables on which expectations are formed \(y(2)\) and the others \(y(1)\), we have

\[ y1(t) = h1\{x(t), y1(t-1)\} \quad \text{……(iii)} \]
\[ y_2(t) = h_2(x(t), y_2(t-1), E[y_2(t+j)/t]) \]  
\[ \text{....(iii)} \]

where \( E[y_2(t+j)/t] \) denotes the rational expectation of \( y(2) \) formed in period \( t+j \) based on information available at \( t \). Our program uses starting values for the vector \( E[y_2(t+j)/t] \) which, together with values for the fully exogenous variables, are assumed to extend over the whole solution period. The algorithm ensures that the expectational values stored in the vector \( E[y_2(t+j)/t] \) converge to the values predicted by the model for \( y(2) \) in period \( t+j \).

For simplicity, let us assume that the solution period extends from \( t = 1, \ldots, T \) and that expectations are formed for one period ahead only.

The convergence of the expectational values towards the model’s predicted values follows a Jacobi algorithm, which can be described as

\[ E[y_2(t,k+1)/t-1] = E[y_2(t,k)/t-1] + q(y_2(t,k)/t-1) \]  
\[ \text{....(iv)} \]

\[ 0 < q < 1, \ t = 1,2, \ldots, T \]

for the \( k^{th} \) iteration, with the objective of minimising the residual vector \( R(t) \), defined as

\[ R(t) = \text{abs}(y_2(t) - E[y_2(t)/t-1]) < L, \ t = 1,2, \ldots, T \]  
\[ \text{....(v)} \]

where \( q \) is the step length and \( L \) is some pre-assigned tolerance level.

Since \( E[y_2(t)/t-1] \) is stored in period \( t-1 \), the end period expectational variable remains undetermined. We require a value for \( y_2(T+1) \) which lies outside the domain of the solution period. The technique used in our program consists of imposing a set of terminal conditions on the rationally expected variables. In a rational expectations model, the forward expectations terms tend to induce unstable roots. The use of terminal conditions has the effect of setting the starting
values of the unstable roots to zero asymptotically, thereby ruling out unstable paths.

In sum, the complete algorithm can be described in the following series of steps:

Step 1. Solve the model given initial values for the expectational variables.

Step 2. Check for convergence.

Step 3. Adjust expectational variables.

Step 4. Re-solve the model given new iterated values of the expectational variables.

4.3.2 Steady State Equations of the Model

The steady state of an economy is its rest point when the variances of all shocks are zero and the levels of consumption, labour, stock of capital and inventories are constant. The study of steady state is important as it characterises the long run features of the economy. The steady state equations of the model are given below.

\[
\beta = \frac{1}{1 + r_i}
\]

\[
\bar{M} = (1 + \phi)CP + GP
\]

\[
p = \frac{d}{1 + r}
\]

\[
Y = ZN^\alpha K^{1-\alpha}
\]

\[
N = \frac{\alpha Y}{w}
\]
\[
K = \frac{(1 - \alpha)Y}{r + \delta}
\]

\[
Y = C + I + G + NX
\]

\[
I = \delta K
\]

\[
w = w^*
\]

\[
br = -PD + \frac{\Delta M^d}{P^d}
\]

\[
1 - N = \left( \frac{\theta_0 C^{-\rho_0}}{(1 - \tau) \exp \left[ \log w^* - \frac{1}{\omega^{\rho_0}} \log Q \right] - \mu} \right)^{\frac{1}{\rho_2}}
\]

\[
d = \frac{Y - w^*N - (r + \delta)K}{S}
\]

\[
PD = G + \mu \left( 1 - N - \tilde{l} \right) + \pi^d M^d - \tau v N - \phi C - T
\]

\[
\log EX = \sigma_1 \log (1 - \omega^f) + \log C^F + \sigma_1 \left( \frac{\omega^{\rho_0}}{\omega^{\frac{1}{\rho_0}} + (1 - \omega^{\rho_0})^{\frac{1}{\rho_0}}} \right) \log Q
\]

\[
\log IM = \sigma \log (1 - \omega) + \log C - \sigma \left( \frac{1}{\omega^{\rho_0}} + (1 - \omega)^{\frac{1}{\rho_0}} \right) \log Q
\]
\[ r = r' + \Delta \log Q \]

\[ r' b' = NX \]

\[ \log S = \log Q - \log P' + \log P \]

\[ Y - (1 + \phi)C - \tau v N - T = rD \]

4.4 DATA

Data on Indian interest rate (money market rate), domestic price level (consumer price index), output (gross domestic product), capital, consumption, investment, government expenditure, primary deficit, exports, imports, and nominal exchange rates have been taken from International financial Statistics (IFS). Data on real exchange rate have been generated by using the following equation.

\[ \log S = \log Q - \log P' + \log P \]

Data on labour supply have been collected from Economic Survey of India. The source for the wage data (manufacturing) is the United Nations Yearbook. We divide the wage data by consumers’ price index (obtained from IFS) to get consumers’ real wage and by producers’ price index (obtained from IFS) to get producers’ real wage. We subtract government revenue (obtained from IFS) from government expenditure (obtained from IFS) to get data on domestic bond. Data on foreign bonds have been generated by using the foreign bond evolution equation. Data on lump sum tax, labour income tax and consumption tax are not
easily available. So we take the ratio of tax revenues to non-agricultural GDP to 
get a proxy for the lump sum tax as agricultural income is tax free in India. We 
use the ratio of revenues from personal income tax (obtained from Economic 
Survey of India) to non-agricultural GDP (obtained from Economic Survey of 
India) as a proxy for the labour income tax. We take the ratio of indirect tax 
revenue (obtained from Economic Survey of India) to consumption (obtained 
from Economic Survey of India) as the proxy for consumption tax. In India 
unemployed people do not get any unemployment benefit from the government. 
They stay with their families. We assume that a family spends 20% of the 
consumer real wage for an unemployed person (as we assume that 20% of the 
consumer real wage is sufficient for an unemployed person to live at subsistence 
level in India.) So we take 20% of the consumer real wage as the proxy for 
unemployment benefit. The data on household debt have been generated by using 
the household debt evolution equation. Productivity is calculated as solow 
residual. Data of foreign (US) consumption, foreign (US) interest rate (Federal 
Fund Rate) and foreign (US) price (Consumer price index) have been taken from 
IFS.
4.5 SIMULATIONS

Once the model has been solved numerically, one can analyse the characteristics of the transition of the model to its steady state. This may arise either because initially the economy is outside steady state or because some structural change is introduced (it could be a policy intervention) altering the steady state. This type of analysis is crucial to evaluate the possible effects of change in policy rules, i.e. of policy interventions and to assess the overall properties of the model.

Standard simulations methods consist of comparing the solution of the model with one where one or more of the exogenous variables are perturbed. Comparing the base and the perturbed solutions gives an estimate of the policy multipliers if the exogenous variables perturbed is a policy instrument. In other words, comparing the results of the simulation experiments with those obtained in the base run provides valuable information regarding the effects of policy changes on the economy.

There is also question of selection of the length of the simulation period. The period should be long enough for the effect of changes to work through the model. This is especially important in models which contain long lags or slow rate of adjustment. Darby et al. (1999) lists two advantages of having a long simulation period. First, when solving non-linear rational expectation models it is important to ensure that the terminal date for the simulation is sufficiently far in the future so that the simulation is unaffected by the choice of the terminal date. Second, simulating the model over a long period makes it easier to observe the long run solution of the model.

Our simulation starts in 1968 and end in 1997 using annual Indian data. Results of our simulation exercise are reported in graphical form in the simulation
appendix. The graphs show the percentage deviation of a particular variable-real output, price level and so on- from the base line path.

4.5.1. Results
The effects of both demand and supply shocks on the behaviour of output, consumption, capital stock, investment, employment, price level, real wage, real interest rate, imports, exports and real exchange rate is examined by deterministically simulating the calibrated model using the extended path method discussed earlier.
In addition to providing quantitative input to policy analysis these deterministic simulations provide useful insights into the dynamic properties of the model.
For the baseline simulation- that is, the simulation with no change in policy instruments-the endogenous variables are set so as to track the actual historical values perfectly. This is done by adding residual to each equation. The residuals are computed using the future expectations of the endogenous variables generated by the model using an overlapping forecast.

4.5.1.1. Five Percent Permanent increase in money supply
We consider the case of an unanticipated 5% permanent increase in the level of money supply relative to the historical baseline. Although increase in money supply is unanticipated at the time of the initial increase, the entire path of the money supply is assumed to be incorporated into agent’s forecasts as of the first period of the simulation. Now people know that the increase in money is permanent. In the very first period of the simulation, price level and nominal exchange rate rise by 5%. Real interest rate, output, consumption, investment,
real exchange rate, exports, imports are not affected by the increase in money. We do not have any nominal rigidities in this model to enable agents to make instantaneous adjustments. Money is neutral here.

4.5.1.2. Five Percent permanent productivity growth shock
Although unanticipated at the time of initial increase, entire path of the productivity rise is assumed to be incorporated into agent's forecasts as of the first period of simulation. The predictions of the model for the case of an increase in productivity are shown in Figure S-4.1. The productivity growth raises permanent income and stimulates a stream of investments to raise the capital stock in line. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily. Thus URIP can be established consistently with a higher real interest rate. As real interest rates fall with the arrival of stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium.

4.5.1.3. Five Percent Permanent increase in labour income tax
We consider the case of an unanticipated permanent increase in labour income tax. The predictions of the model are shown in Figure S-4.2. An unanticipated permanent increase in labour income tax reduces labour supply which leads to a fall in the level of output and an increase in the price level from the first simulation period. As output falls, investment reduces, leading to a fall in capital
stock. The initial fall in interest rates causes a real and nominal depreciation which leads to an improvement of the trade balance. As wages start to decline, after the initial jump in the light of increment in labour income tax, labour supply, output and investment expand. This increases the real interest rates, leading ultimately to a real appreciation and a deteriorating trade balance.

An increase in income tax produces a substitution effects which outweighs the income effect of the tax. The increase in tax tends to discourage labour supply because of the wedge it creates between pre and post-tax return on labour. Further, the impact of changes in income tax depends on the perception of the agent. If a tax increase in the current period makes the agents react with the expectation that current disposable income would be lesser and thus alter consumption plans, an inward shift in the household budget constraint is expected.

4.5.1.4. Five Percent Permanent increase in consumption tax

We consider the case of an unanticipated 5% permanent increase in consumption tax. The predictions of the model are shown in Figure S-4.3. An unanticipated permanent increase in consumption tax raises the level of unemployment and reduces the level of output. With the fall in output, investment in the economy declines and hence capital stock also reduces. In case of increase in consumption tax and labour income tax, labour supply falls as a result of substitution effect outweighing income effect. Consumption tax discourages spending and hence we observe a fall in the price level. As price level falls, the currency appreciates.
4.5.1.5. Five Percent Permanent increase in the foreign interest rate

The predictions of the model for an unanticipated 5% permanent rise in the foreign interest rate are shown in Figure S-4.4. An unanticipated permanent rise in the foreign interest rate, through the uncovered interest rate parity leads to the depreciation of the real as well as nominal exchange rate. This leads to a fall in imports and rise in exports as domestic goods are now more competitive in the world markets. As domestic interest rates catch up with the foreign rates, real and nominal exchange rates appreciate leading to a small deterioration in the trade balance. With the increase in the cost of capital, investment falls and then gradually coming back to equilibrium. Capital stock falls, output falls and price level increases.

4.5.1.6. 5% Permanent increase in foreign price level

We consider the case of an unanticipated 5% permanent increase in the foreign price level. An unanticipated permanent rise in foreign prices does not have any effect on output, labour supply, capital, consumption, investment, real wage, export, import etc.

4.6. Significance of an overlapping wage contract in a rational expectation framework

According to Friedman, suppliers of labour at the beginning of an inflationary period underestimate the price level that will prevail over the period of the work contract, accordingly overestimate the real wage and offer a greater supply of labour at the prevailing nominal wage than they would if expectations were correct. The result is employment in excess of equilibrium level and a trade-off
between output and unanticipated inflation. However the expectational errors cannot persist so that employment returns to its equilibrium level and unemployment returns to its natural rate as expectations adjust to reality.

The widespread use of adaptive expectations suggested that an ever-accelerating rate of inflation could maintain an unemployment rate below the natural rate. The accelerationist version of natural rate hypothesis had two important consequences. First, by making the short run trade-off depend on expectational errors it brought to the fore the question of the optimality of the natural rate. Second, the reliance of the accelerationist hypothesis on expectational errors made it possible that some expectations mechanism other than adaptive expectations would imply that there is no trade-off usable by policy makers. Rational expectation is that hypothesis.

Under rational expectations, the expected price level will change in a manner dependent on the monetary supply rule. If monetary policy accommodates inflationary shocks, the expected price level will rise. If monetary policy counteracts inflationary shocks, the expected price level may be lower than the level expected for this period. The only way in which monetary policy can affect output is by creating a difference between the actual price level and the expected price level. If monetary authority has superior information to private economic agents, say because it receives data more rapidly than they do, it can affect the behaviour of output.

One of the assumptions required for anticipated monetary policy to have no effect on output in a rational expectation framework is that agents are able to act on their information set. If private agents cannot respond to new information by changing their consumption, wage-price decisions, etc., as quickly as the public...
sector can change any (at least one) of its controls, then scope emerges for systematic stabilisation policy to have real effects. This insight was developed by Fischer (1977a, b) and Phelps and Taylor (1977) in the context of multi-period non-contingent wage or price contracts.

Suppose all wage contracts run for two periods and the contract drawn in period t specifies nominal wages for period t + 1 and t + 2 (in the manner of Fischer). At each period of time, half the labour force is covered by a pre-existing contract. As long as the contracts are not contingent on new information that accrues during the contract period, this creates the possibility of stabilisation policy. Firms respond to changes in their environment like unpredictable changes in demand which were unanticipated at the time of pre-existing contract, by altering output and employment at the pre-contracted wage. Only contracts which are up for renewal can reflect prevailing information. If the monetary authorities can respond to new information that has accrued between the time two-period contract is drawn up and the last period of operation of the contract, then systematic stabilisation policy is possible. The overlapping wage contract equation which we add to the RBC model (Minford, Sofat 2004) is the following.

\[
\log w_t = \frac{1}{2} \left( E_{t-1} \log w_{t-1} + E_{t-2} \log w_{t-2} \right) - \frac{1}{2} (\log P_t - E_{t-1} \log P_{t-1}) - \frac{1}{2} (\log P_t - E_{t-2} \log P_{t-2})
\]

Adding an overlapping wage contract equation to the RBC model (Minford, Sofat, 2004) makes sense in the Indian context as we find evidence of wage contracting in India for the sample period. Some researchers have argued that employers resorted to hiring casual workers and / or contracting in response to liberalisation (Deshpande and Deshpande, 1998). The employment data reveals that the share of casual workers in heavy manufacturing increased during 1990s.
This process could conceivably raise wages of casual workers (though below the wage of regular workers). In this model we assume that the wage contract runs for two years as a check on the robustness of our analysis: contract for two years represents the longest contract we may observe and therefore test the robustness most effectively. We attach a table for the distribution of adult male workers by employment status (%)

**Distribution of adult male workers by employment status (%)**

<table>
<thead>
<tr>
<th>Rural</th>
<th>Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regular</td>
</tr>
<tr>
<td>1983</td>
<td>10.38</td>
</tr>
<tr>
<td>1993</td>
<td>10.58</td>
</tr>
<tr>
<td>1999</td>
<td>10.80</td>
</tr>
</tbody>
</table>

Notes: Calculation from NSS surveys for adult male workers aged 15-65 years.

We do not have data on actual length of wage contract, only this data is on the extent of contract coverage. Hence our strategy here is to have two versions of the model, one with complete wage flexibility and one with two year wage contract; between the two extremes we check for sensitivity. As can be seen our results are indeed rather robust to the contract element.

**4.6.1 Five percent permanent shock to money supply in Overlapping Wage Contract model**

We consider 5% permanent increase in money supply. The predictions are shown in Figure S-4.5. 5% increase in money supply gives rise to a 5% increase in price level. Price rises, real wage falls and employment rises (as forced by the wage contract). Output rises and investment and consumption also rises. We can see the
rise in real and nominal exchange rate as well. After two periods, the wage contract gets over, so we do not have any forced increment in labour supply after two periods. When the wage contract gets over, the model behaves like the RBC model where real wage is equal to equilibrium real wage ($w = w^*$).

4.6.2 Five percent permanent shock to productivity in Overlapping Wage Contract model

We consider 5% permanent increase in productivity. The predictions are shown in Figure S-4.6. 5% increase in productivity increases income, stimulates investment and capital stock. Output cannot be increased without increase in labour supply and capital which takes some time. Thus real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates Uncovered Real Interest Parity (URIP) which must be restored by a rise in real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. Real exchange rate moves back to equilibrium as real interest rates fall with the rise in capital and output. Rise in output is more in the overlapping wage contract model as compared to the RBC model due to 5% permanent rise in productivity. The reason is that in the overlapping wage contract model employment rises more as forced by the wage contract. Investment also rises more in OLW model as compared to the RBC model. When the wage contract gets over, the model behaves like the RBC model where real wage is equal to equilibrium real wage ($w = w^*$).
4.7 CONCLUSION

In this chapter we model the Indian economy by using a micro founded general equilibrium open economy model based on optimising decisions of rational agents, incorporating money, government and distortionary taxes. The first order conditions of the household and firm optimisation problem are used to derive the behavioural equations of the model. We calibrate the model using annual data for India for the period 1966-1997. We also discuss the simulation results for both demand and supply shocks with calibrated parameters which are used to assess the overall properties of the model. The results are consistent with our theoretical priors. Then we extend this analysis by including nominal wage contracts for monetary shocks to have real effects and compare the simulations of the two models (with and without overlapping wage contracts). If the monetary authorities can respond to new information that has accrued between the time two-period contract is drawn up and the last period of operation of the contract, then systematic stabilisation policy is possible. We can have more increment in output in overlapping wage contract model as compared to the real business cycle model as a result of permanent rise in productivity.
Table 4.1 Names of the variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Real interest rate</td>
</tr>
<tr>
<td>$Q$</td>
<td>Real exchange rate</td>
</tr>
<tr>
<td>$S$</td>
<td>Nominal exchange rate</td>
</tr>
<tr>
<td>$P$</td>
<td>Price level</td>
</tr>
<tr>
<td>$S^p$</td>
<td>Demand for shares</td>
</tr>
<tr>
<td>$G$</td>
<td>Government expenditure</td>
</tr>
<tr>
<td>$b$</td>
<td>Domestic bonds</td>
</tr>
<tr>
<td>$K$</td>
<td>Capital</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment</td>
</tr>
<tr>
<td>$N^d$</td>
<td>Demand for labour</td>
</tr>
<tr>
<td>$PD$</td>
<td>Primary deficit</td>
</tr>
<tr>
<td>$EX$</td>
<td>Export</td>
</tr>
<tr>
<td>$IM$</td>
<td>Import</td>
</tr>
<tr>
<td>$NX$</td>
<td>Net exports</td>
</tr>
<tr>
<td>$C$</td>
<td>Composite consumption</td>
</tr>
<tr>
<td>$Y$</td>
<td>Output</td>
</tr>
<tr>
<td>$T$</td>
<td>Tax (lump sum)</td>
</tr>
<tr>
<td>$D$</td>
<td>Household debt</td>
</tr>
<tr>
<td>$w$</td>
<td>Producer real wage</td>
</tr>
<tr>
<td>$v$</td>
<td>Consumer real wage</td>
</tr>
</tbody>
</table>
Equilibrium real wage

Foreign bonds

Foreign interest rate

Foreign Price Level

Money

Productivity

Present value of shares

Dividend per share

Labour income tax

Consumption tax

Supply of shares

Population

Foreign consumption

Table 4.2 Exogenous processes in the RBC model

1) $\Delta \ln Z_t = \varepsilon_{1,t}$

2) $\Delta \tau_t = \varepsilon_{2,t}$

3) $\Delta \phi_t = \varepsilon_{3,t}$

4) $\Delta \mu_t = \varepsilon_{4,t}$

5) $\Delta \ln \bar{M}_t = \varepsilon_{5,t}$

6) $\Delta \ln P_t^f = \varepsilon_{6,t}$

7) $\Delta \ln C_t^F = \varepsilon_{7,t}$
8) \( \Delta \ln r_i = \varepsilon_{8,i} \)

Table 4.3 Initial value of \( \varepsilon_i \)

\( \varepsilon_i \) are parameters for defining exogenous random processes;

<table>
<thead>
<tr>
<th>( \varepsilon_i )</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_1 )</td>
<td>0</td>
</tr>
<tr>
<td>( \varepsilon_2 )</td>
<td>0</td>
</tr>
<tr>
<td>( \varepsilon_3 )</td>
<td>0</td>
</tr>
<tr>
<td>( \varepsilon_4 )</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \varepsilon_i )</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_5 )</td>
<td>0</td>
</tr>
<tr>
<td>( \varepsilon_6 )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.4 Values of Coefficients

<table>
<thead>
<tr>
<th>Output elasticity of production</th>
<th>( \alpha )</th>
<th>0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>( \beta )</td>
<td>0.97</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta )</td>
<td>0.0125</td>
</tr>
<tr>
<td>Fraction of elasticity of goods substitution</td>
<td>( \rho_0 )</td>
<td>1.20</td>
</tr>
<tr>
<td>Parameter</td>
<td>Symbol</td>
<td>Value</td>
</tr>
<tr>
<td>--------------------------------------------------------</td>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>Money multiplier</td>
<td>$\theta_0$</td>
<td>0.50</td>
</tr>
<tr>
<td>Degree of seignorage</td>
<td>$\gamma$</td>
<td>0.05</td>
</tr>
<tr>
<td>Fraction of elasticity of goods substitution</td>
<td>$\rho_2$</td>
<td>1.00</td>
</tr>
<tr>
<td>Weight of home goods in consumption function</td>
<td>$\omega$</td>
<td>0.70</td>
</tr>
<tr>
<td>Fraction of elasticity of goods substitution</td>
<td>$\rho$</td>
<td>-0.50</td>
</tr>
<tr>
<td>Weight of foreign goods in consumption function</td>
<td>$\omega'$</td>
<td>0.70</td>
</tr>
<tr>
<td>RER sensitivity to demand for labour</td>
<td>$h$</td>
<td>0.80</td>
</tr>
<tr>
<td>Fraction of elasticity of goods substitution</td>
<td>$\rho_3$</td>
<td>-0.50</td>
</tr>
<tr>
<td>Elasticity of import substitution</td>
<td>$\sigma$</td>
<td>2.0</td>
</tr>
<tr>
<td>Elasticity of export substitution</td>
<td>$\sigma_1$</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Note: Value of alpha is taken from the averages in Indian data and the values for the rest of the coefficients used in the model have been calibrated from the paper of Minford and Sofat (2004) and then these coefficients are estimated in chapter 6 by Maximum Likelihood Method (MLM) – choosing values of all parameters by optimising a given criterion—the likelihood of the Indian data.
Figure S-4.1 5% Permanent Shock to Productivity in RBC Model

Real interest rate

Price level
Figure S-4.2. 5% Permanent Shock to Labour Income Tax in RBC Model

Real interest rate

Price level
Nominal exchange rate

0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0

Nominal exchange rate
Figure S-4.3. 5% Permanent Shock to Consumption Tax in RBC Model

- Real interest rate
- Price level
Real exchange rate

Nominal exchange rate
Figure S-4.4. 5% Permanent Shock to Foreign Interest Rate in RBC Model

Real interest rate

Price level
Figure S-4.5. 5% Permanent Shock to Money in Overlapping Wage Contract Model

![Graph of Real Interest Rate and Price Level](image)

- Real interest rate
- Price level
Figure S-4.6. 5% Permanent Shock to Productivity in Overlapping Wage Contract Model and RBC Model

Real interest rate in RBC and OLW model

Price Level in OLW and RBC model
Output in OLW and RBC model

Employment in OLW and RBC model

Capital Stock in OLW and RBC model
Consumption in OLW and RBC model

Investment in OLW and RBC model
Export in OLW and RBC model

Import in OLW and RBC model
Real Exchange Rate in OLW and RBC model

Nominal Exchange rate in OLW and RBC model
Chapter 5. Real Exchange Rate Overshooting

5.1 Introduction

In this chapter we explore the ability of a Real Business Cycle (RBC) and the overlapping wage contract (OLW) model to account for the behaviour of the real exchange rate, using Indian data.

First, we find that a deterministic productivity growth shock causes the real exchange rate to appreciate and then fall back to a lower equilibrium, producing a simulation property which in turn generates the business cycle properties of the real exchange rate.

Second, we can show that the RBC and the OLW model can reproduce the univariate properties of the real exchange rate.

The chapter is organised as follows. In section 5.2 we discuss the relationship between purchasing power parity and real exchange rate. Section 5.3 establishes the facts of the real exchange rate: it is integrated of order 1, the best fitting univariate process being an ARIMA (1,1,1). Section 5.4 explains the main features of the model. In Section 5.5 we calibrate the model to Indian annual data and show the results of a deterministic productivity growth shock, which is very much encouraging to the idea that the behaviour of the real exchange rate is explicable within the RBC context. Then we repeat the same procedure for the OLW model. In section 5.6 we formally test our model and evaluate statistically whether our calibrated model is consistent with the real exchange rate data, using bootstrapping procedure. The final section concludes.
5.2 Purchasing power parity and real exchange rate

According to Lothian and Taylor (2005) ‘The purchasing power parity (PPP) exchange rate is the exchange rate between two currencies that would equate the two relevant national price levels if expressed in a common currency at that rate, so that the purchasing power of a unit of one currency would be the same in both economies. If the nominal exchange rate is defined simply as the price of one currency in terms of another, then the real exchange rate is the nominal exchange rate adjusted for relative national price level differences. When PPP holds, the real exchange rate is a constant, so the movements in the real exchange rate represents deviations from PPP.’

The validity of PPP has been sought in the empirical literature by examining whether the real exchange rate tends to settle down at a long run equilibrium level-to check whether the time series appears to have been generated by a mean reverting process.

Dornbusch (1976a, 1976b) and Mussa (1976) explain the short run fluctuations in exchange rate by assuming that domestic nominal prices are temporarily fixed. So the prices of goods available to agents in one country change relative to prices of the same goods in another country and monetary shocks can cause a change in the exchange rate even if real supplies and demand for goods are unaffected. Dornbusch’s overshooting model provided some respite to PPP by providing rationale for short run deviations. However the empirical evidence against PPP was overwhelming.
As noted by Rogoff (1996), the growing empirical literature on PPP has arrived at a surprising degree of consensus on some basic facts. i) there is evidence that real exchange rates tend towards PPP in the very long run and ii) short run deviations from PPP are large and volatile.

Corsetti et al. (2004) point out that the expectations of persistent productivity growth raise domestic consumption and investment much more than the domestic supply. Forward looking consumers increase consumption due to expectations of higher future income and higher productivity increases expected future profits, raising investment demand. Now, in order to clear the market, a higher international price is needed to ‘crowd-out’ net exports. This would explain the appreciation of the currency.

It is quite clear that there is no single factor to determine the exchange rate. In general equilibrium, the exchange rate responds to many shocks - including productivity. It is a well-established empirical fact that a burst in productivity leads to an appreciation of the currency. According to the ‘conventional’ view, if a country becomes more productive, a higher world supply of its good should result in a relative price reduction; this occurs also here in the RBC model in the long run once capacity and demand have reached steady state growth but in the short run the surge in demand exceeds capacity growth causing appreciation as above. The Balassa Samuelson hypothesis can explain the appreciation following a burst in productivity in the tradable sector, thereafter the real exchange rate remains higher hence though consistent with the RBC model in the short run it differs in its long run prediction. It also fails to explain the cycles that we observe in actual real exchange rate data- but of course it is not a model of cyclical behaviour.
The objective of this chapter is to establish that RBC alone can reproduce the univariate properties of the real exchange rate. Then we are going to compare the univariate properties of the real exchange rate reproduced by the RBC and the overlapping wage (OLW) contract model.

5.3 Data patterns

Let us begin by looking at the bilateral real exchange rate between India and US (US Dollar / Indian Rupees). The path of the real exchange rate is presented in Figure 5.1. The univariate final form equation is in fact best described by a ARIMA (1,1,1) process; the series therefore is not actually mean reverting but integrated of order 1. Our main aim is to see whether our calibrated RBC model can generate the same univariate behaviour.

In this section we perform Augmented Dickey Fuller test and Phillips Perron test to check stationarity of the real exchange rate series. Using both the Augmented Dickey Fuller test and Phillips Perron test, we find that India’s real exchange rate vis-à-vis US (US $ / Indian Rupees) is an I(1) series. In chapter 2, we have also done some unit root tests for the bilateral real exchange rate between India and US. But we used Indian rupees/ US $ data for the unit root tests in chapter 2 and find that Indian Rupees / US $ is also an I(1) series. Table 5.1 reports the results. The real exchange rate series in levels fails to reject the null hypothesis of non-stationarity at 5% significance level, using both ADF and PP test statistics When we perform the test with the first difference of the series we can easily reject the null at 5 percent.

Table 5.1: Test for Non-stationarity of the Real Exchange Rate

<table>
<thead>
<tr>
<th>Unit root test (with trend and intercept)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

151
<table>
<thead>
<tr>
<th></th>
<th>Level</th>
<th>First difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF Test Statistic</td>
<td>-3.4223</td>
<td>-6.7618</td>
</tr>
<tr>
<td>PP Test Statistic</td>
<td>-3.4168</td>
<td>-7.2648</td>
</tr>
</tbody>
</table>

Having established the non-stationarity of the series we now estimate the best fitting ARIMA process to the real exchange rate, using annual data from 1966 to 1997. The results in Table 5.2 indicate that an ARIMA (1,1,1) best describes the data. The best fitting ARIMA regression results are shown in Table 5.3.

**Table 5.2 Best Fitting Real Exchange Rate ARIMA**

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>-0.6671</td>
<td>0.0024</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.1544</td>
<td>0.7600</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.5809</td>
<td>0.0329</td>
</tr>
<tr>
<td>MA(2)</td>
<td>-0.9411</td>
<td>0.9466</td>
</tr>
</tbody>
</table>

**Table 5.3 Best Fitting ARIMA Results for Rupees Real Exchange Rate (US Dollar/Indian Rupees)**

Dependent Variable: D(Q)
Method: Least Squares

Sample: 1966 1997
Included observations: 32
Convergence achieved after 17 iterations
5.4 Model
We use micro-founded stochastic general equilibrium open economy model on optimising decision of rational agents – the model of Minford and Sofat (2004) (described in chapter 4). The first order conditions of households’ and firms’ optimisation problems are used to derive the behavioural equations of the model. We are modelling Indian economy, taking the world economy as given. The interaction with the rest of the world comes in the form of uncovered real interest rate parity and the current account, both of which are explicitly micro-founded.

5.5 Calibration and simulation
In order to carry out model simulations, numerical values should be assigned to the structural parameters of the model. Once the model has been solved numerically, one can analyse the characteristics of the transition of the model to its steady-state and also test if the model is consistent with the facts.
5.5.1 Calibration and simulation of the RBC model

Our simulations start in 1968 and end in 1997 using annual Indian data. We begin by running a deterministic productivity growth shock through our model calibrated to annual data for India. This deterministic simulation is done in order to establish the basic order of magnitude and shape of the response function of the real exchange rate to the workhorse RBC technology shock.

The response profile is attractive in exhibiting a pronounced cycle. We consider a deterministic productivity growth shock - a 5-year rise of productivity growth rate by 1% per annum to check the effect of a wave of technological growth. Productivity grows at 1% in the first year, 2% in the second year, 3% in the third year and so on till the fifth year when it grows at 5%. After that it is permanently 5% above the base. Results of our simulation exercise are reported in graphical form in figure S-5.2 in the simulation appendix.

The productivity growth raises income and stimulates a stream of investments to raise the capital stock in line. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival of stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium. This gives us a business cycle in the real exchange rate. This new equilibrium represents a real depreciation on the
previous steady state since output is now higher and must be sold on world markets by lowering its price.

5.5.2 Calibration and Simulation of the OLW model

We consider a deterministic productivity growth shock to the OLW model – a 5-year rise of productivity growth rate by 1% per annum. Productivity grows at 1% in the first year, 2% in the second year, 3% in the third year and so on till the fifth year when it grows at 5%. After that it is permanently 5% above the base. Results of our simulation exercise are reported in graphical form in figure S-5.2 in the simulation appendix.

Growth in productivity increases output, stimulates investment to raise capital stock. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily, so enabling URIP to be established consistently with a higher real interest rate. Real exchange rate moves back to equilibrium as real interest rates fall with the rise in capital and output. Rise in output is more in the overlapping wage contract model as compared to the productivity shock in the real business cycle model. The reason is that in the overlapping wage contract employment rises more as forced by the wage contract. Investment also rises more in OLW model as compared to the RBC
model. When the wage contract gets over, the model behaves like the RBC model where real wage is equal to equilibrium real wage \( w = w^* \).

5.6 Bootstrapping

Our objective is to check whether we can generate the facts of the real exchange rate (US $ / Indian Rupees) such as we find them, assuming that our model and its error processes are true. We want to find the sampling variability implied by the model - to find the 95% confidence limits around the real exchange rate ARIMA parameters. One approach is to linearise the model, which would allow us to map it to a VARMA and in principle compute reduced form standard errors for each parameter. However the reliability of the standard errors would be open to question given our small sample size. Analytical computation of the confidence limits is also not possible given the non-linear nature of the model.

Comparison of our model with the ARIMA we have estimated on the actual data cannot be done via deterministic simulation. We want to replicate the stochastic environment to see whether our estimated ARIMA equations could have been generated within it. We do this via bootstrapping the RBC model with its error processes. In the RBC model, the error in the UIP equation is basically the risk premium. In the equations that are identities we have residuals either due to measurement errors or due to approximations made in the model. They are treated as fixed elements and are not bootstrapped. Having obtained the residuals, we determine the best fitting data generating process for them, to obtain the i.i.d. shocks in our error processes. In our model we also have the exogenous

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processes- productivity, labour income tax, consumption tax, money supply, government expenditure, foreign prices, foreign consumption and foreign interest rates. To replicate the real exchange rate with its unit root we need unit root drivers in the system which are coming from the exogenous processes all of which have been modelled as random walks.

We generate the sampling variability within the model by the method of bootstrapping the model’s estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. The idea is to create pseudo data samples (here 500) for the real exchange rate. We draw the vectors of i.i.d shocks in our error processes with replacement, by drawing vectors for the same time period we preserve their contemporaneous cross-correlations; we then input them into their error processes and these in turn into the model to solve for the implied path of real exchange rate over the sample period.

We run ARIMA regressions on all the samples to derive the implied 95% confidence limits for all the coefficients. Finally we compare the ARIMA coefficients estimated from the actual data to see whether they lie within these 95% confidence intervals. The comparison informs us whether the data rejects the model.
Table 5.4 Confidence Limits from our Model for Real Exchange Rate ARIMA

<table>
<thead>
<tr>
<th></th>
<th>Estimated</th>
<th>95% Confidence Limits</th>
<th>95% Confidence Limits</th>
<th>95% Confidence Limits</th>
<th>95% Confidence Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lower (derived from RBC)</td>
<td>Upper (derived from RBC)</td>
<td>Lower (derived from OLW)</td>
<td>Upper (derived from OLW)</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.6671</td>
<td>-0.75486</td>
<td>0.656537</td>
<td>-0.79034</td>
<td>0.63692</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.5809</td>
<td>-1.46681</td>
<td>0.664147</td>
<td>-1.46672</td>
<td>0.683383</td>
</tr>
</tbody>
</table>

The results in Table 5.4 validate the hypothesis that the Indian real exchange rate behaviour is explicable within the RBC framework and the OLW framework for the period 1966-1997. The ARIMA parameters lie within the 95% confidence intervals. In chapter 2 also we have used the technique of bootstrapping to test the validity of Balassa Samuelson model in Indian context for the period 1959-2001 and find that the AR and MA coefficients estimated from the actual data (Indian Rupees/ US $) lie within the 95% confidence intervals generated by the method.
of bootstrapping. So we can use both RBC / OLW and Balassa Samuelson model to explain the behaviour of the bilateral real exchange rate between India and US.

5.7 Conclusions

We find that the real exchange rate appreciates as a result of a deterministic productivity growth shock and then falls back to a lower equilibrium, producing a business cycle and the expected simulation properties. We can conclude whether our model could be consistent with the facts by asking whether it could have generated the patterns we find in the actual real exchange rate data. To do this we generate the sampling variability within the RBC and OLW model by the method of bootstrapping the model’s estimated residuals. This allows us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. The AR and MA coefficients estimated with the actual data lie within the 95% confidence limits generated by the method of bootstrapping. This validates our hypothesis that the real exchange rate behaviour is explicable within RBC and OLW framework. In chapter 2 also we have used the technique of bootstrapping to test the validity of Balassa Samuelson model in Indian context for the period 1959-2001 and find that the AR and MA coefficients estimated from the actual data (Indian Rupees/ US $) lie within the 95% confidence intervals generated by the method of bootstrapping. So we can use both RBC / OLW and Balassa Samuelson model to explain the behaviour of the bilateral real exchange rate between India and US.
Appendix

Figure 5.1. Bilateral Real Exchange Rate between India and US (US Dollar/Indian Rupees)

Table 5.5 ADF Test of Real Exchange Rate in Levels with Trend and Intercept

<table>
<thead>
<tr>
<th>t-Statistic</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller test statistic</td>
<td>-3.422353</td>
</tr>
<tr>
<td>Test critical values: 1% level</td>
<td>-4.284580</td>
</tr>
</tbody>
</table>
Table 5.6 ADF Test of Real Exchange Rate in First Difference with Trend and Intercept

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(-1)</td>
<td>-0.477063</td>
<td>0.139396</td>
<td>-3.422353</td>
<td>0.0019</td>
</tr>
<tr>
<td>C</td>
<td>0.869276</td>
<td>0.254734</td>
<td>3.412490</td>
<td>0.0020</td>
</tr>
<tr>
<td>@TREND(1966)</td>
<td>-0.017853</td>
<td>0.005048</td>
<td>-3.536529</td>
<td>0.0014</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.311888</td>
<td>Mean dependent var</td>
<td>-0.024927</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.262737</td>
<td>S.D. dependent var</td>
<td>0.114579</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.098382</td>
<td>Akaike info criterion</td>
<td>-1.708156</td>
<td></td>
</tr>
<tr>
<td>Sum squared resid</td>
<td>0.271011</td>
<td>Schwarz criterion</td>
<td>-1.569383</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>29.47642</td>
<td>F-statistic</td>
<td>6.345534</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.030437</td>
<td>Prob(F-statistic)</td>
<td>0.005336</td>
<td></td>
</tr>
</tbody>
</table>

Null Hypothesis: D(Q) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic based on SIC, MAXLAG=7)
Table 5.7 PP Test of Real Exchange Rate in Levels with Trend and Intercept

Null Hypothesis: Q has a unit root
Exogenous: Constant, Linear Trend
Bandwidth: 1 (Newey-West using Bartlett kernel)

<table>
<thead>
<tr>
<th>Phillips-Perron test statistic</th>
<th>Adj. t-Stat</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.416840</td>
<td>0.0674</td>
<td></td>
</tr>
</tbody>
</table>

Test critical values:

- 1% level: -4.284580
- 5% level: -3.562882
- 10% level: -3.215267

Sample (adjusted): 1967 1997  
Included observations: 31 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q(-1)</td>
<td>-0.477063</td>
<td>0.139396</td>
<td>-3.422353</td>
<td>0.0019</td>
</tr>
<tr>
<td>C</td>
<td>0.869276</td>
<td>0.254734</td>
<td>3.412490</td>
<td>0.0020</td>
</tr>
<tr>
<td>@TREND(1966)</td>
<td>-0.017853</td>
<td>0.005048</td>
<td>-3.536529</td>
<td>0.0014</td>
</tr>
</tbody>
</table>

R-squared 0.311888  Mean dependent var -0.024927
Adjusted R-squared 0.262737  S.D. dependent var 0.114579
S.E. of regression 0.098382

Table 5.8 PP Test of Real Exchange Rate in First Difference with Trend and Intercept

Null Hypothesis: D(Q) has a unit root  
Exogenous: Constant, Linear Trend  
Bandwidth: 3 (Newey-West using Bartlett kernel)

<table>
<thead>
<tr>
<th>Phillips-Perron test statistic</th>
<th>Adj. t-Stat</th>
<th>Prob.*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-7.264824</td>
<td>0.0000</td>
</tr>
<tr>
<td>Test critical values:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1% level</td>
<td>-4.296729</td>
<td></td>
</tr>
<tr>
<td>5% level</td>
<td>-3.568379</td>
<td></td>
</tr>
<tr>
<td>10% level</td>
<td>-3.218382</td>
<td></td>
</tr>
</tbody>
</table>


Residual variance (no correction) 0.011260
HAC corrected variance (Bartlett kernel) 0.007351

Phillips-Perron Test Equation  
Dependent Variable: D(Q,2)  
Method: Least Squares  
Date: 04/27/06  Time: 22:31  
Sample (adjusted): 1968 1997  
Included observations: 30 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>D(Q(-1))</td>
<td>-1.220875</td>
<td>0.180553</td>
<td>-6.761866</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>-0.014415</td>
<td>0.044000</td>
<td>-0.327624</td>
<td>0.7457</td>
</tr>
</tbody>
</table>
@TREND(1966)  -0.001343  0.002390  -0.561755  0.5789

R-squared  0.630116  Mean dependent var -0.006413
Adjusted R-squared  0.602717  S.D. dependent var  0.177463
S.E. of regression  0.111855  Akaike info criterion -1.448582
Sum squared resid  0.337813  Schwarz criterion -1.308462
Log likelihood  24.72873  F-statistic  22.99791
Durbin-Watson stat  2.165381  Prob(F-statistic)  0.000001

Figure S-5.2. 1% Per Annum Productivity Growth Shock in RBC and OLW Model

Real Interest Rate in OLW and RBC model

Price Level in OLW and RBC model
Output in OLW and RBC model

Employment in OLW and RBC model

Capital Stock in OLW and RBC model
Consumption in OLW and RBC model

Investment in OLW and RBC model

Equilibrium Real Wage in OLW and RBC model
Export in OLW and RBC model

Import in OLW and RBC model
Real Exchange Rate in OLW and RBC model

Nominal Exchange rate in OLW and RBC model
Chapter 6. Estimation and Bootstrapping

6.1 Introduction

In this chapter we use a Full Information Maximum Likelihood (FIML) estimator to estimate our dynamic stochastic general equilibrium (DSGE) model outlined in chapter 4 using annual data for India. The FIML algorithm solves the model repeatedly in full for all sample periods for set of parameters, choosing the likelihood maximising set. The justification for using FIML is that it reduces the bias on the estimates in comparison to both single equation and multi-equation simultaneous system estimations.

This Chapter is organised in the following way. Section 6.2 discusses alternative techniques to estimate models. Section 6.3.1 explains the concept of bootstrapping and section 6.3.2 explains bootstrapping in the context of simultaneous equation systems. Section 6.4 explains the procedure used and also the maximisation of the likelihood. In section 6.5 we discuss the issues relating to convergence of bootstraps and non-cointegration. We report our results in section 6.6. The final section concludes.

6.2 Estimation

There are basically two approaches to estimate structural equations, namely, single-equation methods, also known as limited information methods and system methods, also known as full information methods. In single equation method, each equation in the system is estimated individually, taking into account any restrictions placed on that equation such as exclusion of some variables, without worrying about restrictions on the other equations in the system, hence the name
limited information method. In the system methods, all the equations in the model are estimated simultaneously, taking account of all restrictions on such equations by the omission or absence of some variables, hence the name full information methods.

Many economic relationships are of single-equation type. In such models there is an implicit assumption that the cause and effect relationship, if any, between the variables is unidirectional: the explanatory variables are the cause and the dependent variable is the effect. However, the relationship may not be unidirectional. For example in our model interest rates are determined to a large extent by consumption. However, interest rates also have an impact on consumption demand. This leads us to consider simultaneous equation models where there is more than one regression equation, one for each interdependent variable.

To preserve the spirit of our simultaneous-equation model, we should use full-information maximum likelihood method (FIML). Such methods are not commonly used for the following reasons. First, the computational burden is enormous. Second, the systems methods, such as FIML, lead to solutions that are highly non-linear in the parameters and are therefore difficult to determine. Third, the difficulty with all system estimation techniques is that individual parameter estimates-by construction-are sensitive to the specification of the entire system of equations. A serious specification error in one equation can substantially affect the parameter estimates in all equations of the model. Thus the decision to use system estimation involves a trade-off between the gain in efficiency and the potential costs of specification error.
Nowadays the three commonly used single-equation methods are ordinary least squares (OLS), indirect least squares (ILS) and two-stage least squares (2SLS). Although OLS is inappropriate in a simultaneous equation set up, it can be applied to recursive models-where there is a definite but unidirectional cause and effect relationship among endogenous variables. Many argue that even though OLS is inappropriate in case of simultaneous models it can still be used-if only-as a benchmark of comparison. The method of ILS is suitable for just or exactly identified equations and 2SLS is especially designed for over-identified equations. Both ILS and 2SLS estimates are consistent in the sense that as the sample size increases indefinitely, the estimates converge to their true population values. But the estimates may not satisfy small-sample properties such as unbiasedness and minimum variance.

The dynamic stochastic general equilibrium model- whose parameters are tastes and technology and so attempts to avoid Lucas' (1976) Critique, has sufficient restrictions and simultaneity implying that FIML estimation, which exploits the parameters interdependence in explaining the data, should undoubtedly yield gains in efficiency over single equation methods such as 2SLS. Here we use an FIML estimator developed by the Liverpool Research Group to estimate our model using annual data for India. The FIML algorithm solves the model repeatedly in full for all sample periods for sets of parameters, choosing the likelihood maximising sets.
6.3 Bootstrapping

6.3.1 Concept

In statistics bootstrapping is a method for estimating the sampling distribution of an estimator by resampling with replacement from the original sample. Bootstrap technique was invented by Bradley Efron (1979, 1981, 1982) and further developed by Efron and Tibshirani (1993). ‘Bootstrap’ means that one available sample gives rise to many others by re-sampling. Classical parametric tests compare observed statistics to theoretical sampling distributions. Re-sampling is a revolutionary methodology because it departs from theoretical distributions. Rather the inference is based upon repeated sampling within the same sample and so the school is called re-sampling. While the original objective of cross validation\textsuperscript{7} is to verify replicability of results and that of jackknife\textsuperscript{8} is to detect outliers.

In bootstrap, the original sample can be duplicated as many times as the computing resources allow and then this expanded sample is treated as a virtual population. Then samples are drawn from this population to verify the estimators. Obviously, the source for re-sampling in bootstrap could be much larger than that in the other two. Unlike cross validation and jackknife, the bootstrap employs sampling with replacement.

\textsuperscript{7} In cross validation a sample is randomly divided into two or more subsets and test results are validated by comparing across sub-samples.

\textsuperscript{8} Jackknife is a step beyond cross validation. In Jackknife the same test is repeated by leaving one subject out each time. Thus this technique is also called leave one out. This procedure is especially useful when the dispersion of distribution is wide or extreme scores are present in the data set.
6.3.2 Bootstrapping in simultaneous models

In a linear model parameter estimates have the dimension of a mean and for a sample with reasonable size normality of parameter distribution is a persuasive assumption. But this assumption does not apply in non-linear simultaneous models especially with our sample size. Even if we assume that multivariate error is normally distributed, parameter estimates generated are likely to be biased. The bias would not disappear even if the sample size tends to infinity. Hence Monte Carlo methods\textsuperscript{9} must be used to reach an assessment for a non-linear model, given an estimate of the error distribution. In traditional Monte Carlo studies, errors are drawn from a theoretical distribution such as normal distribution. In many cases, the asymptotic results are not reliable in small samples because the errors are not believed to be distributed normally. To deal with this problem, one must find a way of drawing errors more representative of the unknown actual error distribution. Bootstrapping is a method for doing this. It is computationally intensive, non-parametric technique for making inference about the population characteristic. Since we have no a priori knowledge of the error distribution, the natural starting point is an estimate of them from the sample itself.

Following Minford and Webb (2004) we use the simultaneous estimator FIML designed for a multivariate normal error distribution. Given that the errors are approximately multivariate normal, the method used is an approximation to maximum likelihood. We start our bootstrapping procedure with our sample residuals obtained by FIML. We use the distribution of the sample residuals as an estimate of their true distribution. The bootstrap can then be regarded as a Monte Carlo simulation researchers make up data and draw conclusions based on many possible scenarios (Lunneborg (2000)). Today Monte Carlo simulations are widely used by statisticians to study .

\textsuperscript{9} In Monte Carlo simulation researchers make up data and draw conclusions based on many possible scenarios (Lunneborg (2000)). Today Monte Carlo simulations are widely used by statisticians to study .
Carlo study to find the true distribution of the parameter estimates on the assumption that the sample residual distribution and the parameter estimates are both the true ones. We can then use the bootstrap results to generate the confidence limits from the parameter distributions yielded around the true parameters. In general, to produce a confidence interval one finds the standard error of estimate, multiplies it by a suitable critical value taken from either the normal distribution or students’ t distribution tables and then adds and subtracts the results from the estimate. This procedure can be inaccurate when the errors are not normally distributed. By bootstrapping the entire procedure, the actual sampling distribution can be estimated, allowing an appropriate confidence interval to be produced.

6.4 Bootstrap Procedure

In this section we describe a method for FIML estimation of simultaneous equation models developed by the Liverpool Research Group over several years\textsuperscript{10}. At the centre of this method lies a solution programme used now for over two decades to solve the Liverpool UK and multi-lateral world models\textsuperscript{11}. This programme is capable of solving large non-linear macro models in a wide variety of forecast situations—with forecasting being done quarterly for the first decade and monthly. This programme has a wide variety of traps built into it to prevent the solution algorithm stopping its search, in effect these traps restart the algorithm. The solution establishes convergence in the now-standard manner, as exemplified in Fair and Taylor (1983).

\textsuperscript{10} The discussion in this section is based on the exposition in Minford and Webb (2004)
\textsuperscript{11} See Minford, Marwaha, Matthews and Sprague (1984) and Minford, Agenor and Nowell (1986).
For FIML estimation along the lines described above we use a new hill-climbing algorithm to search over the parameter space. Starting from a set of parameters-calibrated or estimated by single equation methods, the algorithm varies each parameter in turn by plus or minus some percentage of its initial value, this reference value being held constant. Whichever parameter movement generates the biggest improvement in the likelihood is adopted. The operation is then repeated for the newly altered parameter set. The search process begins with + or - 10% variations in parameter values; once any improvement with such a step variation is exhausted, it tries 5% variations; then 2 and then 1%, after which it stops.

As with all algorithms there is no guarantee that it will find the global maximum. However, the algorithm will climb any slope it finds itself on locally. The algorithm checks on this aspect in standard ways such as allowing the initial parameter set to be varied randomly and at random points in the search process making random perturbations in the set of parameters reached. At the end of the search for the global maximum, the algorithm restarts the search from a different, randomly chosen initial parameter values and checks that it reaches the same maximum again. In the final stage of our procedure we use bootstrap to compute confidence intervals.

One of the problems in our bootstrap is the presence of auto-correlation in certain residuals. This implies that the errors can not be regarded as random and are therefore unsuitable for re-sampling in any order. The errors used in this stage have all been purged off auto-correlation at the original FIML estimation stage by the inclusion of appropriate auto-correlation parameters in the model itself. This is done by generating pseudo-samples from the vectors of estimated errors, with
replacement on a model consisting of estimated parameters; these are used to
produce new sets of FIML estimates. For our FIML estimation we use annual
Indian data from 1966 to 1997 and we estimate 11 parameters.

6.4.1 Maximising the likelihood

Our model is non-linear and so we represent it generally as

\[
y_t = f(y_t, (L)y_t, x_t, (L)x_t, y_t^e, (L)y_t^e; \pi) + u_t \quad t = 1, 2, \ldots, n \quad (6.1)
\]

\[
x_t = A(L)x_t + \varepsilon_t, \ldots \ldots \ldots (6.2)
\]

Where \(y_t\) is the vector of G endogenous variables and \(x_t\) is a vector of
exogenous variables assumed to follow a linear univariate time-series process. \(L\)
is the lag operator, \(y_t^e\) is the vector of rational expectations projections of the
model based on information at \(t\), \(\pi\) is the vector of parameters and \(u_t\) is the
vector of structural equation errors. We can also focus on the reduced form error
vector, \(v_t\). This is the error that is created when in (6.1) \(y_t\) on the right hand side
is replaced by its model prediction, \(\hat{y}_t\).

\[
y_t = f(\hat{y}_t, (L)y_t, x_t, (L)x_t, y_t^e, (L)y_t^e; \pi) + v_t \quad t = 1, 2, \ldots, n \quad (6.3)
\]

Hence here we see how the model will solve for \(y_t\), given its solution for the
other \(y_t^e\). It clearly follows that \(v_t\) is a combination of \(u_t\). However, in the
analysis below we focus on (6.1) as there is simultaneity built into it, given the
importance of expectations in our model.
We assume that the errors in either case follow a multivariate normal distribution with expectation of zero and variance-covariance matrix of $\Sigma$ and that estimation has eliminated any auto-correlation. This implies that the likelihood is

$$p(u_1, u_2, \ldots, u_n) = (2\pi)^{-nG/2}(\det \Sigma)^{-n/2} \exp \left( -\frac{1}{2} \sum_{i=1}^{n} u_i' \Sigma^{-1} u_i \right) \quad \cdots \cdots \cdots \cdots (6.4)$$

We evaluate the likelihood (6.1) directly. That is to say, for each trial set of parameters $\hat{\pi}$ we obtain an implied set of $u_i$ for which (6.4) yields the likelihood. This is maximised by the hill-climbing algorithm described earlier. The bootstrap procedure takes the $u_i$ generated by the maximum likelihood parameter set $\hat{\pi}$. It then re-samples these vectors with replacement and inserts the sample with $\hat{\pi}$ into equation (6.1) to generate the bootstrap samples of $y_t$. The exogenous variables are held constant across bootstraps. Then the above estimation procedure is repeated on each bootstrap sample.

The bootstrap procedure can be explained in the following series of steps;

For a simple linear model with 't' observations

$$y_t = \beta x_t + \varepsilon_t \quad (6.5)$$

The residual re-sampling bootstrap procedure to compute standard errors for $\hat{\beta}$ is

1) estimate $\hat{\beta}$ by OLS or 2SLS.

2) Calculate $\hat{Y}$ and the residual $\hat{\varepsilon}$ and store them.

3) Re-scale the residuals as described below.

4) For the 'B' bootstrap samples, do the following:

   (a) Draw a sample of size 't' with replacement from the set of re-scaled residuals $\hat{\varepsilon}$.
(b) Construct new dependent variables $Y_i^b$ with the formula

$$Y_i^b = Y_i^* + \varepsilon$$

That is, for each element of $Y$, draw an adjusted residual randomly with replacement and add it to generate a new $Y$ variable.

(c) Regress $Y_i^b$ on $X_i$ in (6.5) and save the estimated coefficients.

5) Repeat this procedure several thousand times.

6) Compute a 95% confidence interval i.e., compute the standard error of $\hat{\beta}$ from the sample of bootstrapped $\hat{\beta}$'s with the standard formula.

$$\sigma_{\hat{\beta}} = \sqrt{\frac{1}{B-1} \sum_{i=1}^{B} \left\{ (\beta^i)^2 - \left( \hat{\beta} \right)^2 \right\}^2}$$

Where $\hat{\beta} = \frac{1}{B} \sum_{i=1}^{B} \beta^i$

As already noted, it is not appropriate merely to use the residuals. When $\varepsilon_i$ is i.i.d., the OLS residuals of equation (6.5) has variance

$$E(\varepsilon_i^2) = (1 - h_i) \sigma^2$$

Where $h_i$ is defined as

$$h_i = X_i (X' X)^{-1} X_i'$$

Instead, one can re-scale the residuals in this manner:

$$\varepsilon_i^* = \frac{\varepsilon_i^\wedge}{\sqrt{1 - h_i}} - \frac{1}{N} \sum_{s=1}^{N} \frac{\varepsilon_s^\wedge}{\sqrt{1 - h_s}}$$
The re-scaled residuals are sometimes called the standardised residuals, and ‘$h_i$’ is the $i^{th}$ diagonal element of the hat matrix. The second term in our rescaled residual is there to ensure that the mean of the resulting residual remains zero.

6.5 Convergence, Unit Roots and Non-Cointegration

6.5.1 Convergence of Bootstrap

How many bootstraps should be used? The number of bootstrap to be used is regarded as a matter of experience from previous studies, which indicate how the margin of error around estimates of parameter distributions is reduced by extra bootstraps. The answer depends on whether an additional bootstrap provides any extra information on the parameter distributions. Therefore we compute measures of the currently estimated distributions from the bootstraps up to a point and then check whether these measures alter with extra bootstraps. Once they fail to change we conclude that further bootstraps add no useful information. As four aggregate measures the programme uses the first four moments of all the parameters, viewed as a group. As a further check we consider the upper and the lower 95% confidence limit for every parameter separately. When none of these show any change-within some specified low tolerance limit-as the number of bootstraps increases we declare convergence.

From these convergence tests we find the extra information starts to fall off at a number of bootstraps in the low hundreds rather than the thousands generally thought of as necessary- the actual number in our case is only 130.
6.5.2 Unit roots and non-cointegration

Now researchers check for cointegration - stationarity of errors-between any I(1) variables they wish to include in a regression. If it exists, they are protected from spurious regression. We would ideally like all our long run relationships between I(1) variables postulated in the model to exhibit cointegration between them. It cannot be said with confidence that the error terms in these relationships are stationary within our sample. Nevertheless we have faith in our long-run relationships as they have strong theoretical underpinnings. So we assume that there are one or more omitted variables and model the residual as a proxy for these, usually with some autoregressive process, with the possibility open for a unit root in this. Typically we estimate stationary process but with high roots.

Suppose that these errors or some of them are non-stationary, then the question comes- are the estimates endangered by non-stationarity of errors? Does it seriously affect the estimates or the distribution of the parameters. In our model we found no evidence of non-stationarity in the equation errors-the autocorrelation roots found are comfortably less than unity. However we are interested to know whether the presence of unit roots in the errors will be detected reliably by our autocorrelation estimates and whether when this is done the main structural parameters are estimated well.

To answer this question, we do a Monte Carlo bootstrap on our model assuming that the true parameters are those estimated by the FIML. Then we assumed that all the equation errors we are using in our bootstrap, instead of being random variables as estimated, are random walks. This would simulate the absence of cointegration - each equation now has an I(1) process which in each sample will
drive the dependent variable cumulatively away from the set of right hand side variables.

The results are reassuring. We find that if the errors are assumed to be non-stationary neither the estimates of the behavioural parameters nor their standard errors change. This suggests that if there is non-stationarity in the errors it should be absorbed in the coefficients of autocorrelation and should not unduly affect the main parameter estimates or their distributions. This is a feature of a simultaneous model where the cross equation restrictions are created by a rational expectation model and which when exploited by a systems estimator produce a high degree of stability in estimation.

6.6 Results

We now present the FIML estimates of the model parameters based on 130 bootstraps. We find from these convergence tests that the extra information starts to fall off at a number of bootstraps in the low hundreds rather than the thousands generally thought of as necessary; the actual number is 130 in our case. The extra bootstraps tend to gravitate around sets of potential parameters for which the model solves sensibly; once these have been located the same ones crop up again and again.

We can see from the table 6.1 that

1) The system estimates are reasonably close to the limited information starting point. In this sense there is little obvious gain from system estimation.

2) We also present the confidence limits using the parameter distributions as related to the FIML estimate around which they are generated. In the table L95 and U95 are the lower and upper 95% confidence limits, i.e. 2.5% of the
distribution lies above 95% and below 95%. These are the correct measures of
dispersion for hypothesis testing because the distributions are not generally
normal or t-shaped and hence standard errors are not useful. Bias adjustment is
important as FIML is generally biased. Table 6.1 reports the estimates of the
model parameters.

3) We use the gap between the FIML estimate and bootstrap mean as an estimate
of bias. Table 6.2 reports the bias-adjusted FIML estimates and their
confidence limits.

The bias correction procedure using the bootstrap is the following.

1) Compute the mean value for each coefficient and let \( \hat{\beta} \) denote the vector of
mean values. Let \( \theta = \beta - \hat{\beta} \) denote the vector of estimated biases, where \( \beta \)
is the estimated coefficient vector.

2) After the coefficient vector \( \hat{\beta} \) is estimated by FIML, it has \( \theta \) subtracted
from it to correct for bias to give \( \tilde{\beta} \) the vector of FIML bias adjusted
coefficients.

The confidence limit for the bias corrected coefficients \( \tilde{\beta} \) is calculated as follows.

1) Compute \( \theta_1 = ( U95 \text{ or } L95) - \tilde{\beta} \) the limit band for the bias adjusted FIML
parameters

2) After computing \( \theta_1 \) add it to \( \tilde{\beta} \) to calculate the U95 or L95 limits for the bias
adjusted coefficients.
Table 6.1. FIML Estimates of the Model Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibrated</th>
<th>FIML estimates</th>
<th>Bootstrapped mean</th>
<th>L95</th>
<th>U95</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.66442</td>
<td>0.6125</td>
<td>0.72187</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.97</td>
<td>0.95</td>
<td>0.971889</td>
<td>0.9306</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0125</td>
<td>0.016667</td>
<td>0.012689</td>
<td>0.011174</td>
<td>0.1724</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1.2</td>
<td>1.2</td>
<td>1.227453</td>
<td>1.06667</td>
<td>1.44</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>0.5</td>
<td>0.489583</td>
<td>0.516628</td>
<td>0.458333</td>
<td>0.572222</td>
</tr>
<tr>
<td>$\gamma^G$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.050234</td>
<td>0.043958</td>
<td>0.05125</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>1</td>
<td>0.97619</td>
<td>0.957919</td>
<td>0.833333</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.5</td>
<td>-0.493</td>
<td>-0.49305</td>
<td>-0.5</td>
<td>-0.475</td>
</tr>
<tr>
<td>$\omega^i$</td>
<td>0.7</td>
<td>0.7</td>
<td>0.772871</td>
<td>0.583333</td>
<td>0.875</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>2.01</td>
<td>1.998576</td>
<td>1.708333</td>
<td>2.222222</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>2</td>
<td>2.083333</td>
<td>2.113935</td>
<td>1.941667</td>
<td>2.494334</td>
</tr>
</tbody>
</table>
Table 6.2. FIML Bias Corrected Estimates

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Calibrated FIML estimates</th>
<th>Bias – corrected FIML estimates</th>
<th>Bias – corrected L95</th>
<th>Bias – corrected U95</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.7</td>
<td>0.7</td>
<td>0.73558</td>
<td>0.68366</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.97</td>
<td>0.95</td>
<td>0.928111</td>
<td>0.886822</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.0125</td>
<td>0.016667</td>
<td>0.020645</td>
<td>0.01913</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>1.2</td>
<td>1.2</td>
<td>1.172547</td>
<td>1.011761</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>0.5</td>
<td>0.489583</td>
<td>0.462538</td>
<td>0.404243</td>
</tr>
<tr>
<td>( \gamma^G )</td>
<td>0.05</td>
<td>0.05</td>
<td>0.049766</td>
<td>0.04349</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>1</td>
<td>0.97619</td>
<td>0.994461</td>
<td>0.869875</td>
</tr>
<tr>
<td>( \rho )</td>
<td>-0.5</td>
<td>-0.493</td>
<td>-0.49295</td>
<td>-0.4999</td>
</tr>
<tr>
<td>( \omega^f )</td>
<td>0.7</td>
<td>0.7</td>
<td>0.627129</td>
<td>0.437591</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>2.01</td>
<td>2.021424</td>
<td>1.731181</td>
</tr>
<tr>
<td>( \sigma_1 )</td>
<td>2</td>
<td>2.083333</td>
<td>2.052731</td>
<td>1.880463</td>
</tr>
</tbody>
</table>

\( \omega \), the share of home goods in home consumption is a known fact and so it is fair enough to fix it in the estimation.
We use the gap between the bootstrapped mean and the FIML estimates as an estimate of bias and we subtract this gap from the FIML estimates to get the bias-corrected FIML estimates. We also calculate the 95% confidence limits for the bias-corrected FIML estimates following the formula described earlier in this section. The bias corrected parameters are fairly close to the calibrated values we originally chose and indeed the biases are not severe. This appears to result from the fact that these models only behave sensibly for values close to the ones we chose: such parameters as labour's share in production function for example do not allow for much variation from the actual share in the economy. In FIML estimation the whole model is simulated with the parameters under trial; the likelihood is then closely related to the model's overall simulation performance since if it simulates wildly it will create a poor fit to the data.

6.7 Conclusion

In this chapter we use a Full Information Maximum Likelihood (FIML) estimator to estimate our dynamic stochastic general equilibrium (DSGE) model outlined in chapter 4 using annual data for India. The FIML algorithm solves the model repeatedly in full for all sample periods for set of parameters, choosing the likelihood maximising set. We do bootstrapping with our sample residuals obtained by FIML. We find that the system estimates are reasonably close to the limited information starting point, it would seem because the model's simulation performance greatly affects the likelihood and the parameter values for best performance seem to lie in quiet a narrow range. In this sense there is little obvious gain from system estimation. On the other hand it is reassuring that to find that systems estimation confirms the
original calibrations choices to a fair extent. We present the confidence limits using
the parameter distributions as related to the FIML estimate around which they are
generated. We use the gap between the FIML estimate and bootstrap mean as an
estimate of bias and we correct the bias accordingly, though this turns out not to be
severe (again because the model’s simulation performance will not tolerate big
deviations from chosen values.
Figure 6.1 Simulation for 1% Per Annum Productivity Growth Shock in RBC Model with Calibrated Co-efficient and FIML bias corrected Co-efficient

Real interest rate

Price levels
With Calibrated coefficients
With FIML bias corrected coefficients
Output

Labour supply
With calibrated coefficients

With FIML bias corrected coefficients

Capital stock

With calibrated coefficients

With FIML bias corrected coefficients

Consumption
With calibrated coefficients
With FIML bias corrected coefficients

Investment

Equilibrium real wage
With calibrated coefficients

With FIML bias corrected coefficients

Exports

Imports

Real exchange rate
Nominal exchange rate
Chapter 7. Conclusion

In this thesis we examine the behaviour of the bilateral real exchange rate between India and US. In chapter 1, we review the Indian economic situation for the last few decades.

In chapter 2, using annual data for the period 1959-2001 for India and USA, we examine the effect of productivity differential on bilateral real exchange rate between India and US (the Harrod Balassa Samuelson effect) in a non-linear framework. We find significant evidence of non-linear mean reversion towards the long run equilibrium. We find that relatively higher productivity growth in US is accompanied by its real exchange rate appreciation vis-à-vis India. We test the validity of the Balassa Samuelson model in this context by using the following procedure.

We fit an integrated autoregressive (AR(1)) model to the productivity differential series and bootstrap the model to get 500 pseudo sample for the real exchange rate. Our objective is to check whether we can generate the facts of real exchange rate such as we find them, assuming that our model and its error processes are true. We generate the sampling variability within the model by the method of bootstrapping the model’s estimated residuals; this permits us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. Having established the non-stationarity of the series we estimate the best fitting ARIMA process to the real exchange rate, using data from 1966 to 1997. We find the 95% Confidence limits generated by bootstrapping. The ARIMA parameters estimated with the actual real exchange rate data lie within the 95% confidence intervals. This validates the hypothesis that real exchange rate (Indian Rupees/US Dollar) behaviour is explicable within the Balassa Samuelson framework. In
other words, we find that higher productivity growth in US is accompanied by its real exchange rate appreciation vis-à-vis India.

In Chapter 3, we review the Real Business Cycle literature and in chapter 4 we calibrate the RBC model (Minford, Sofat (2004) using annual data for India and run the simulations of the important macroeconomic variables including real exchange rate for various deterministic shocks in the exogenous variables. Then we add an overlapping wage contract equation to their RBC model and calibrate the model using Annual data for India and run the simulations for various deterministic shocks in exogenous variables. In chapter 5 we explain the behaviour of Indian real exchange rate (US Dollar/ Indian Rupees) using RBC model and OLW model. We find that real exchange rate appreciates as a result of a deterministic productivity growth shock and then falls back to a lower equilibrium, producing a business cycle and the expected simulation properties. We can conclude whether our model could be consistent with the facts by asking whether it could have generated the patterns we find in the actual real exchange rate data. To do this we generate the sampling variability within the RBC and OLW model by the method of bootstrapping the model’s estimated residuals. This allows us to find the 95% confidence limits around the real exchange rate ARIMA regression parameters. The AR and MA coefficients estimated with the actual data lie within the 95% confidence limits generated by the method of bootstrapping. This validates our hypothesis that real exchange rate behaviour is explicable within RBC and OLW framework.

In both RBC and Balassa Samuelson models we find that real exchange rate appreciates with rise in productivity. But the mechanism is different in the two models. In Balassa Samuelson model relatively higher productivity growth in the
tradeables sector will tend to generate a rise in relative price of non-tradeables. The percentage change in the relative price of non-tradeables is determined only by production side of the economy, while the demand factors do not affect the real exchange rate in the long run. If the degree of capital intensity is the same across the traded and non-traded sectors, then the percentage change in relative prices is exactly equal to the productivity differential between the two sectors. If the non-traded sector is less capital intensive than the traded sector, then even in the situation of balanced productivity growth in the two sectors, the relative price of non-tradeables will rise.

In other words, the Harrod Balassa Samuelson model suggests that the long run equilibrium real exchange rate should depend on the productivity of tradeables and non-tradeable sectors in home and foreign economies. Given perfect labour mobility, changes in relative productivity across sectors lead to changes in relative prices. Since technological innovation is most likely to be concentrated in the tradeable goods sector, countries with higher long-run growth rates should have higher relative prices of non-tradeable goods as well as higher valued currencies. If we make approximation that productivity growth in non-tradeable sector is zero, then equilibrium real exchange rate will vary over time in response to variations in tradeable sector productivity in the two economies. In fact, under this assumption, variations in productivity in the economy as a whole will depend largely upon variation in productivity in tradeable sector.

In the RBC model, the productivity growth raises income and stimulates a stream of investments to raise the capital stock in line. Output cannot be increased without an increase in labour supply and capital which takes time. Thus the real interest rate must rise to reduce demand to the available supply. The rising real
interest rate violates uncovered real interest parity (URIP) which must be restored by a rise in the real exchange rate relative to its expected future value. This rise is made possible by the expectation that real exchange rate will fall back steadily enabling URIP to be established consistently with a higher real interest rate. As real interest rates fall with the arrival of stream of sufficient capital and so output, the real exchange rate also moves back to equilibrium. This gives us a business cycle in the real exchange rate. This new equilibrium represents a real depreciation on the previous steady state since output is now higher and must be sold on world markets by lowering its price. In the RBC model real exchange rate appreciates as a result of deterministic productivity growth shock in the short run and it depreciates in the long run. But in the Balassa Samuelson model real exchange rate appreciates as a result of productivity growth in the long run.

We have found that both the Balassa Samuelson model and RBC / OLW model explain the behaviour of the bilateral real exchange rate between India and US in their different ways, presumably because both is being used to deal mainly with short run fluctuations in the data. The two models as we have seen however have different implications for the long run behaviour of the real exchange rate; but to distinguish these and reject one or other of them requires further work.
Some important equations to understand the RBC model (Minford, Sofat 2004) described in chapter 4.

The first order conditions of household’s optimisation problem:

The first order conditions of household’s optimisation problem with respect to $C_t$, $N_t$, $b_t$, $b_t'$, $S_t$ are:

\[
(1 - \rho_0)\theta_0 (1 - \rho_0)^{-1} C_t^{-\rho_0} = \lambda_t (1 + \phi_t) \quad \text{.........(4.10)}
\]

\[
(1 - \rho_2)(1 - \theta_0)(1 - \rho_2)^{-1} (1 - N_t)^{-\rho_2} = \beta E_t \lambda_{t+1} \left[ (1 - \tau_t) v_t - \mu_t \right] \quad \text{.........(4.11)}
\]

\[
\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} \quad \text{.........(4.12)}
\]

\[
\frac{\lambda_t Q_t}{1 + r_t} = \beta E_t \lambda_{t+1} Q_{t+1} \quad \text{.........(4.13)}
\]

\[
\frac{\lambda_t p_t}{P_t} = \beta E_t \lambda_{t+1} \left( \frac{p_{t+1} + d_{t+1}}{P_{t+1}} \right) \quad \text{.........(4.14)}
\]

The first of the above equations (equation 4.10) equates the marginal utility of domestic consumption to shadow price of output. Sales tax impinges on this equation. The second equates the marginal disutility of labour to labour’s marginal product—the real wage. The marginal product of labour is affected both by tax on labour and the unemployment benefit. From the representative household’s first order condition we know that supply of labour is positively related to the net-of-tax real wage and negatively related to the unemployment benefit. If the after-tax real wage is temporarily high, substitution effect overpowers the income effect. The increase in work effort raises employment and output. On the other hand unemployment benefit negatively impinges upon
supply of work effort. These equations which are stochastic analogue of the well
known Euler equations, which characterises the expected behaviour of the
economy, determine the time path of the economy’s values of labour,
consumption and investments (in financial assets).

Substituting equation (4.8) in (4.6) yields

\[(1 + r_t) = \left( \frac{1}{\beta} \left( \frac{C_t}{C_{t+1}} \right)^{-\rho} \left( 1 + \phi_{t+1} \right) \right) \left( 1 + \phi_t \right) \] \hspace{1cm} (4.15)

Substituting (4.6) and (4.8) in (4.7)

\[(1 - N_t) = \left( \frac{\theta \phi C_t^{-\rho} \left[ (1 - \tau_t) v_t - \mu_t \right]}{(1 - \theta) \left( 1 + \phi_t \right) \left( 1 + r_t \right)} \right)^{1/\rho_2} \] \hspace{1cm} (4.16)

where \( v_t \) (consumer real wage) enters labour supply equation so that

\[
\log v^*_t = \log w^* - \left[ \frac{1}{\omega^{1+\rho}} \log P^d + \frac{1}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \log P^c \right]
\]

Also given that \( \log W_t = \log w_t + \log P^d \), (producer real wage) and using

\[
\log Q_t = \log P^c - \log P_t, \quad \text{then}
\]

\[
\log v^*_t = \log w^*_t - \frac{(1 - \omega)^{1+\rho}}{\omega^{1+\rho} + (1 - \omega)^{1+\rho}} \log Q_t
\]

Therefore (4.12) becomes
\[ (1 - N_t) = \theta_0 C_i - \rho_i \left[ (1 - \tau_i) \exp \left( \log w_i^* - \frac{(1 - \omega)}{(1 + \omega)^{r_i}} \log Q_t - \mu_i \right) \right] \frac{1}{\rho_i} \]

.....(4.17)

If each household can borrow an unlimited amount at the going interest rate, then it has an incentive to pursue a Ponzi game. The household can borrow to finance current consumption and then use future borrowing to roll over the principal and pay all of the interest. To prevent the household from playing a Ponzi game it is further assumed that the household’s decision rule is subject to a transversality condition

\[ Y_{t-1} - r_T D_T - \phi_T C_T - \tau_T v_T N_T^+=T_T = C_T \] (4.18)

Substituting (4.8) in (4.10) gives

\[ p_t = \left( \frac{p_{t+1} + d_{t+1}}{1 + r_t} \right) \frac{P_t}{P_{t+1}} \] (4.19)

Using \( p_{t+1} = \frac{p_{t+2} + d_{t+2}}{1 + r_{t+1}} \) in above yields

\[ p_t = \left( \frac{p_{t+2} + d_{t+2}}{(1 + r_t)(1 + r_{t+1})} \right) \left( \frac{P_t}{P_{t+2}} \right) + \left( \frac{d_{t+1}}{1 + r_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \] (4.20)
using the arbitrage condition and by forward substitution the above gives

\[ p_t = \sum_{i=1}^{n} \frac{d_{t+i}}{(1 + r_t)} \left( \frac{p_{t+i}}{p_{t+i}} \right) \]  \hspace{0.5cm} (4.21)

Equation (4.21) states that the present value of share is simply discounted future dividends.

In small open economy models the domestic real interest rate is equal to the world real interest rate which is taken as given. Further it is assumed that the economy has basically no effect on the world rate because, being a small part of the world, its affect on the world savings and investment is negligible. These assumptions imply that the real exchange rate for the small open economy is constant. However we are modelling a medium sized economy. In our set up the economy is small enough to continue with the assumption that world interest rates are exogenous but large enough for the domestic rate to deviate from the world rate. In our model real exchange rates are constantly varying. To derive the uncovered interest parity condition equation (4.12) is substituted into (4.13)

\[ \frac{1 + r_t}{1 + r_t^f} = \frac{Q_{t+1}}{Q_t} \]  \hspace{0.5cm} (4.22)

In logs this yields to

\[ r_t = r_t^f + E_t \Delta \log Q_{t+1} \]  \hspace{0.5cm} (4.23)
The Government

In this framework it is assumed that the government spends current output according to a non-negative stochastic process that satisfies $G_t \leq Y_t$ for all $t$. The variable $G_t$ denotes per capita government expenditure at $t$. It is also assumed that government expenditure does not enter the agents objective function. In case of equilibrium business cycle models embodying rational expectations, output is always at the desired level. Given the information set, agents are maximising their welfare subject to their constraints. Since there are no distortions in this set-up government expenditure may not improve welfare through its stabilisation program. This is why government expenditure has been excluded from the representative agent's utility function. The state also pays out unemployment benefits $\mu_t$ which leads to higher substitution between work and leisure.

The government finances its expenditure by collecting taxes on labour income $\tau$, and taxes on consumption $\phi$, which are assumed to be stochastic processes. Also it issues debt, bonds $b_t$ each period which pays a return next period. Then it collects seigniorage, i.e. $\frac{M^{d}_{t+1} - M^{d}_{t}}{P^{d}_t}$ which is assumed to act as a lump-sum tax, leaving real asset prices and allocation unaltered and is assumed to be a stochastic process.

Since tax on labour income reduces the after-tax return accruing to an agent from supplying labour in market, it is likely to affect her choice as to how much of labour to supply at a given point of time. By reducing the take-home wage, the labour income tax reduces the opportunity cost of leisure, and there is a tendency to substitute leisure for work. This is the substitution effect and it tends to
decrease labour supply. At the same time tax reduces the individual’s income. Given that leisure is a normal good, this loss in income leads to a reduction in consumption of leisure, ceteris paribus. The income effect tends to induce an individual to work more. It is the relative strengths of the income and substitution effects which would ultimately determine whether an agent would work more or less.

Tax on consumption are similar to income tax in the sense that they are imposed on flows generated in the production of current output. However income tax is imposed on the net income received by agents whereas sales tax is imposed by the state on the sales of business firms.

The government budget constraint is

\[ G_t + b_t + \mu \left[ (1 - N_t) - I_t \right] = \tau_{t-1}N_{t-1} + \phi_{t-1}C_{t-1} + \frac{b_{t+1}}{1 + r_t} + \frac{M^{d}_{t+1} - M^{d}_t}{P^d_t} \]

(4.24)

where \( b_t \) is real bonds and \( P^d_t \) is the domestic price level. Note that \( \tau_{t-1}N_{t-1} + \phi_{t-1}C_{t-1} \) is the total tax revenue collected by the state. Also the government faces a cash-in-advance constraint i.e.,

\[ P^d_t G_t \leq M^{dg}_t \]

(4.25)

where \( M^{dg}_t \) is government’s demand for domestic money. Here we assume that the government has some bias, i.e. it consumes only domestic goods.

**Firm’s Optimisation Problem**

The technology available to the economy is described by a constant-returns to scale production function:

\[ Y_t = Z_t f(N_t, K_t) \]

(4.26)
The capital stock evolves according to $K_{t+1} = (1 - \delta)K_t + I_t \quad (4.27)$

$$\text{Max } V = \sum_{i=0}^{\tau} d_i^u \left( Y_i - K_i \left( r_i + \delta \right) - w_i N_i^d \right)$$

Subject to (4.26)

The firm optimally chooses capital and labour so that marginal products are equal to price per unit of input. The first order conditions with respect to $K_t$ and $N_i^d$ are as follows

$$K_t = \frac{(1 - \alpha)Y_t}{r_t + \delta} \quad (4.29)$$

$$N_i^d = \left( \frac{w_i}{\alpha Z_t} \right)^{\frac{1}{(\alpha - 1)}} K_t \quad (4.30)$$

The non-negativity constraint applies i.e. $K_t \geq 0$. Firms own the capital stock and choose investment and domestic labour.

**The Foreign Sector**

The response of trade balance to shocks on the terms of trade has preoccupied trade theorists for decades. In open economies a country’s investment and consumption plans are no longer constrained by its own production frontier. As in Armington (1969), demands for products in this framework are distinguished not only by their kind but also by their place of production. The Armington assumption that home and foreign goods are differentiated purely because of their origin of production has been workhorse of empirical trade theory.
In a stochastic environment the representative agent maximises her expected discounted stream of utility subject to her budget constraint. In order to derive the real exchange rate and hence the balance of payments explicitly from microfoundations we take into account the consumption constraint on agent

\[ P_tC_t = P'_tC'_t + P''tC''_t \quad (4.31) \]

Consumption function is an Armington aggregator of the form

\[ C_t = \left[ \omega (C'^d_t)^{-\rho} + (1 - \omega) (C'^f_t)^{-\rho} \right]^{-\frac{1}{\rho}} \quad (4.32) \]

where \( C_t \) is composite per capita consumption, made up of \( C'^d_t \), agents consumption of domestic goods and \( C'^f_t \), their consumption of imported goods and \( \omega \) is the weight of home goods in the consumption function. The utility based price index corresponding to the above consumption function is of the form

\[ P_t = \left[ \omega \left( P'^d_t \right)^{-\rho} + (1 - \omega) \left( P'^f_t \right)^{-\rho} \right] \quad (4.33) \]

Now the Lagrangian associated with the agent’s maximisation subject to the budget as well as consumption constraint is

\[ U = \max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \theta_0 (1 - \rho_0)^{-1} C'^{\left(i - \phi_1 \right)}_t + (1 - \theta_0) (1 - \rho_2)^{-1} (1 - N_t)^{\left(i - \phi_2 \right)}_t \right] \right\} + \]

\[ \lambda_i \left\{ (1 - r_{i,t}) N_{t-1} + \mu_{t-1} \left[ (1 - N_{t-1}) - l \right] + b_t + \frac{(p_t + d_t) S^p_t}{P_t} = \left( 1 + \phi \right) C_t - \frac{b_{t+1}}{1 + r_i} \right\} \]

\[ - \frac{Q_t b'_{t+1}}{1 + r'_t (1 - rp)} - \frac{P_t S^p_t}{P_t}. \]
+ \lambda t \{ P_t^d C_t^d + P_t^F C_t^f - P_t C_t \} \}

(4.34)

The first order conditions with respect to $C_t^d$ and $C_t^f$ are

\[
\frac{\partial C_t}{\partial C_t^d} - \lambda t (1 + \phi) \frac{\partial C_t}{\partial C_t^d} = \lambda t^d P_t^d + \lambda t^f P_t \frac{\partial C_t}{\partial C_t^f}
\]

(4.35)

\[
\frac{\partial C_t}{\partial C_t^f} - \lambda t (1 + \phi) \frac{\partial C_t}{\partial C_t^f} = \lambda t^c P_t^c + \lambda t^f P_t \frac{\partial C_t}{\partial C_t^f}
\]

(4.36)

Dividing equation (4.36) by equation (4.35) we have

\[
\frac{P_t^c}{P_t^d} = \frac{\partial C_t}{\partial C_t^f} \frac{\partial C_t}{\partial C_t^d}
\]

(4.37)

or

\[
\frac{P_t^c}{P_t^d} = \frac{1 - \omega}{\omega} \left( \frac{C_t^d}{C_t^f} \right)^{1+\rho}
\]

(4.38)

Now we can write equation (4.38) as

\[
Q_t = \frac{1 - \omega}{\omega} (F)^{1+\rho}
\]

(4.39)

where $Q_t = \frac{P_t^c}{P_t^d}$ and $F = \frac{C_t^d}{C_t^f}$
Elasticity of substitution between home goods and imported foreign goods is given by

$$\sigma = \left( \frac{\partial F}{\partial Q} \right) \left( \frac{Q}{F} \right) = \frac{1}{1 + \rho} \quad (4.40)$$

Substituting (4.40) in (4.39) we have real exchange rate

$$Q_t = \frac{1 - \omega}{\omega} \left( \frac{C_t^d}{C_t^f} \right)^{\frac{1}{\sigma}} \quad (4.41)$$

To the extent that home and imported goods are not perfect substitutes, $\sigma$ will take some finite value. The lower the estimated $\sigma$ means the less the substitution between the two goods. In other words the greater the degree of product differentiation, the smaller is the elasticity of substitution between the products.

From the real exchange rate equation, we can derive import equation for our economy. Taking logs of equation (4.41) we have

$$\log IM_t = \sigma \log \left( \frac{1 - \omega}{\omega} \right) + \log C_t^d - \sigma \log Q_t \quad (4.42)$$

Note that $IM_t = C_t^f$

To derive the import function we need to substitute out for $\log C_t^d$ from household's expenditure minimisation we know

$$C_t^d = \left( \frac{P_t^d}{\omega P_t} \right)^{\frac{1}{1 + \rho}} C_t \quad (4.43)$$

Taking logs
\[ \log C_i^d = \sigma \log \omega + \sigma \log P_t - \sigma \log P_t^c + \log C_t \quad (4.44) \]

Now substituting equation (4.44) in equation (4.42), we have

\[ \log IM_t = \sigma \log(1 - \omega) + \log C_t - \sigma A \log Q_t \quad (4.45) \]

\[ A = \frac{\frac{1}{\omega^{1+\rho}}}{\frac{1}{\omega^{1+\rho}} + \frac{1}{(1 - \omega)^{1+\rho}}} \quad (4.46) \]

The equation states that imports into the country are positively related to the total consumption in the home country and negatively related to the real exchange rate, i.e. as \( Q_t \) increases (i.e., the currency depreciates), import demand falls.

Now an Armington aggregator consumption function and a corresponding real exchange rate equation exists for the foreign country as well.

\[ C_i^{f} = \left[ \omega^f \left( C_i^{ff} \right)^{-\phi_5} + (1 - \omega^f) \left( C_i^{ff} \right)^{-\phi_5} \right]^{-\phi_5} \quad (4.47) \]

\[ Q_i^{f} = \left( \frac{1 - \omega^f}{\omega^f} \right)^{\frac{1}{\phi_5}} \quad (4.48) \]

where \( C_i^{f} \) is the composite consumption of the foreign country, \( C_i^{ff} \) is the foreign country’s consumption of own goods, \( C_i^{ff} \) is foreign country’s consumption of home goods, \( \omega^f \) is the weight of foreign country’s own goods in its composite consumption function, \( Q_i^{f} \) is the real exchange rate for the foreign country and \( Q_i^{f} = \frac{1}{Q_t} \).
\[ \sigma_i = \frac{1}{1 + \rho_3} \] is the elasticity of substitution between home goods, i.e. home exports and foreign country’s own goods.

Taking logs of equation (4.48)

\[ \log EX_i = \sigma_i \log \left( \frac{1 - \omega f}{\omega f} \right) + \log C^i_{df} + \sigma_i \log Q_i \tag{4.49} \]

Note that \( EX_i = C^i_{df} \) and \( Q'_i = \frac{1}{Q'_i} \).

To derive the export function we need to substitute out for \( \log C^i_{df} \). As before, from the foreign household’s expenditure minimisation we know

\[ C^i_{df} = \left( \frac{P^i_{df}}{\omega f P^*_i} \right)^{\frac{1}{1 + \rho_3}} C^F_i \tag{4.50} \]

where \( P^*_i \) is the foreign CPI of the form

\[ P^*_i = \left[ \omega f \left( P^i_{df} \right)^{\rho_3_{1}} \left( 1 - \omega f \right)^{\rho_3_{2}} + \left( 1 - \omega f \right)^{\rho_3_{2}} \left( P^D_i \right)^{\rho_3_{2}} \right] \]

where \( P^i_{df} \) is the foreign country’s own price level and \( P^D_i \) is the domestic price level in foreign currency.

Taking logs of equation (4.50)

\[ \log C^i_{df} = \sigma_i \log \omega f + \sigma_i \log P^*_i - \sigma_i \log P^i_{df} + \log C^F_i \tag{4.52} \]

Substituting equation (4.52) in equation (4.49)
\[
\log EX_i = \sigma_i \log(1 - \omega^f) + \log C_i^f + \sigma_i A^f \log Q_i \tag{4.53}
\]

\[
A^f = \frac{\left(\omega^f\right)^{1/\tau + \rho}}{\left(\omega^f\right)^{1/\tau + \rho} + \left(1 - \omega^f\right)^{1/\tau + \rho}} \tag{4.54}
\]

The equation states that export of the home country is a positive function of the total consumption in the foreign country and also a positive function of the real exchange rate. If \(Q_i\) increases, i.e. home currency depreciates then exports will increase. In the model home and foreign agents need foreign and home money respectively, in order to transact with each other. The foreign agents need home money to buy our exports, but get home money for imports as well as our purchase of foreign bonds. So their net supply of foreign money is equal to net exports plus sales of foreign bonds i.e. balance of payments surplus. This surplus is equal to home agents net demand for foreign money, who get foreign money from firms exporting to foreign agents and need foreign money for imports and purchases of foreign bonds. So if home agents adjust their sales of foreign bonds, then all balances. In equilibrium it is assumed that exports and imports are equal and hence the agents would have no tendency to change their asset position. In disequilibrium the changes between domestic and foreign bonds will depend upon net exports.

\[
NX_i = EX_i - IM_i \tag{4.55}
\]

Foreign bonds thus evolve over time according to the following equation.

\[
b^f_{i+1} = \left(1 + r^f_i\right)b^f_i + NX_i \tag{4.56}
\]


137. Patnaik, I. "India's experience with a pegged exchange rate.", in "The India Policy Forum 2004,"


