Money Aggregates, Monetary Transmissions, and the Business Cycle

by

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Economics section of Cardiff Business School, Cardiff University

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ABSTRACT

This thesis formulates, calibrates, and simulates Dynamic Stochastic General Equilibrium (DSGE) models to investigate the monetary transmission mechanism and the real impacts of monetary policy in a Cash-in-Advance (CIA) economy. The contributions of this thesis include: a resolution of the liquidity effect puzzle, which is a negative correlation between nominal interest rates and money growth rate, through the banking sector; an examination of the real impacts of monetary policy, with various nominal interest rates, under a CIA framework; an evaluation of a simple CIA economy and monetary banking models with business cycle facts; and an emphasis of the contribution of a banking sector and a Stockman (1981) CIA constraint.

The first chapter reviews a theoretical explanation of the liquidity effect puzzle. It includes the limited participation monetary shock from Lucas (1990) and market segmentation from Alvarez, Lucas and Webber (2001). It discusses the interaction between nominal and real economy with different monetary transmission channels (such as nominal wage contract, sticky price and monetary misperceptions). Chapter 2 resolves the liquidity effects puzzle with a banking sector. By generating liquidity effect on nominal interest rates, the model is able to replicate the economic fluctuations observed in the data. Chapters 3 and 4 explain the real impacts of monetary aggregates under a flexible price framework. The model is able to account for both the nominal interest rate behaviour and the business cycle facts without sticky price/wage and limited participation monetary shocks. Chapter 5 evaluates both simple and banking CIA models with business cycle facts, and emphasises the contribution of productive banks and Stockman’s (1981) CIA constraint. It focuses on the interaction between nominal and real variables and concludes that by integrating banking production function and Stockman CIA constraint into CIA economy, the model is able to examine the certain business cycle facts. Chapter 6 concludes the contributions of the thesis.
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Non-technical summary

This thesis explains the interactions between the nominal and real economy, and examines the monetary transmission mechanism and real impacts of monetary aggregates within a DSGE framework. It resolves the liquidity effect puzzle by assuming that money injections are received by financial firms instead of households. It explains the real impacts of monetary aggregates through various monetary transmissions by extending the monetary RBC model with the functions of financial intermediates and productive banks. This thesis also investigates the interactions between nominal and real variables in a Cash-in-Advance (CIA) economy.

There is long debate on the interactions between nominal and real side of economy and how monetary policy affects real activity. The thesis in chapter 5 starts with standard Lucas (1980) and/or Svensson (1985) type of CIA model, which has been discussed in Walsh (2003) and extends with Stockman (1981) CIA constraint to discuss the ability of the model to match the business cycle facts. The second part of chapter 5 extend Lucas (1980) CIA economy with banking sector, where productive bank produce exchange credit service for good market transaction. It is similar to Benk et al (2005a) monetary banking model and also extends with Stockman (1981) CIA constraint. The model is able to explain the pro-cyclical behaviour of nominal interest rate and the negative correlations between money growth rate and real activities, such as output, consumption, labour and investment. At mean time, both standard and banking CIA models fail to produce the positive responses of output and employment subject to positive monetary shock. Therefore, the models which had been developed in chapter 3 and 4 try to generate the positive responses of output and employment subject to monetary expansion through cost channel of monetary policy under the flexible price framework. Chapter 4 employs McCandless (2008) working capital CIA model to explain the increasing in output and employment with monetary expansion. Since the model fails to generate the consumption behaviour subject to monetary innovation, the Stockman CIA constraint will be applied to the model in order to overcome the negative correlation between consumption and output and the negative response of consumption subject to monetary innovation, which had been founded
in standard McCandless (2008) working capital CIA model. The crucial assumption of the chapter 4 is that financial intermediates receive money injections from monetary authority instead of households and firms have to issue the corporative bond to finance the wage payment before any goods been produced. Alternative approach to generate the increasing output and employment with monetary expansion has been discussed in chapter 3. The chapter 3 extends Benk et al (2005a) monetary banking model with functions of financial intermediate to explain the positive responses of aggregate output and employment subject to monetary innovation. Although the model also assumes that firms are issuing corporative bond to borrow way bill in advanced, it extends CIA economy with productive banks and requires households receive money injections rather than financial sector in chapter 4. The chapter 2 is modelling one of monetary transmission mechanism, which is liquidity effect of money growth rate or lower nominal interest rate with monetary expansion through capital bond market and evaluates the model with the business cycle facts. The model in chapter 2 does not include the cost channel of monetary policy to generate real impacts of monetary policy, which is the crucial assumption to chapter 3 and 4.

The liquidity effect on the nominal interest rate describes a decrease in nominal interest rates with monetary expansion. It is an important feature in many theories of the monetary transmission mechanism. There are strong positive correlations between the money growth rate and the nominal interest rates in most monetary RBC models (such as Lucas Island, Cash-in-Advance, nominal wage, and sticky price); however, the major failing of the monetary RBC models is the negative correlation which is found in the data. This negative correlation means that positive monetary innovations should reduce the nominal interest rate, instead of increasing it which happens in most monetary RBC models. Chapter 2 of this thesis generates a liquidity effect on the nominal interest rate by extending the CIA economy with a productive banking sector. It extends two the exchange technologies of the monetary banking model with a government bond capital market. This thesis also assumes that money injections are received by banks instead of households in order to generate a decreasing nominal interest rate with monetary expansion. To evaluate the liquidity effect model, this thesis replicates the real effects of monetary aggregates which has been
observed in the data through the liquidity effect on nominal interest rate within the model.

Without a liquidity effect on nominal interest rate, the flexible monetary business cycle model has difficulty replicating the real effects of a monetary shock. Modern monetary RBC models explain the real impacts of monetary aggregates by including sticky wages and/or prices setting. The sticky price/wage is the major transmission mechanism of monetary aggregates in these models. Chapter 3 extends the standard CIA model with functions of productive banks and financial intermediates. The model at this point does not require a sticky price/wage and limited participation monetary shock as a monetary propagation mechanism in order to explain the real impacts of monetary aggregates through various nominal interest rates. Chapter 4 employs the CIA economy and extends it with the function of financial intermediates in order to generate the real effectiveness of monetary policy. It particular emphasise the contribution of Stockman's (1981) CIA constraint to explain the consumption movement with monetary expansion. Chapter 3 includes two representative agents in the financial sector (i.e. productive banks and financial intermediates) and does not request that the money injections are received by the financial sector in order to explain the real impacts of monetary aggregates. Chapter 4 does not include the productive banks in financial sector and it has to request that financial intermediates receive money injections in order to generate the real effects of monetary policy.

The assumption which firms have to borrow fund to pay wage bill in advanced is crucial to generate the real effects of monetary policy for the models in chapter 3 and 4. The firms' borrowing in both models has to be considered as one period bond rather than loan contract from financial intermediates. For chapter 4, this is due to households own the financial intermediates through saving deposit and the return of saving deposit can be considered as a kind of investment return. Before the money injections occur, the return of investment in financial firms is equal to the nominal interest rate on corporative bond. After money injections, return of investment in financial firm increases and nominal interest rate on

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1 This is explained in more detail in Chapter 3&4.
2 Following Fuerst (1992), the function of financial intermediates is to receive money injections and saving funds, and to make loans to firms which they use to make their wage payments.
A corporative bond is decrease. It creates a kind of liquidity premium due to only firms not households are able to access the money injections. For chapter 3, the households own the financial sector through cost of exchange credit. The difference between the marginal cost of productive bank and the return on corporative bond can be considered as risk premium due to money has to be held for good market CIA constraint.

Chapter 5 focuses on the ability of monetary RBC models to examine the interaction between real and nominal variables through evaluating a simple CIA economy and banking monetary models in the light of monetary business cycle facts. The nominal interest rate is the only monetary transmission channel in simple CIA economy. Since the nominal interest rate increases with the expected inflation effect of the money growth rate through a Fisher relation, the economy concludes that there are negative effects of monetary expansion on real activity. The banking monetary model which has been developed by Benk, Gillman and Kejak (2005a) extended the simple CIA economy into a two exchange technologies framework through integrating productive banks into their model. Their model has endogenous velocity through the banking sector and includes a deposit rate or marginal cost of exchange technology as an additional monetary transmission channel, which has a positive effect on real activity. There are two types of exchange technology constraint in the CIA economy, which are: standard and Stockman constraints. In this research project the simulation of the model’s moments indicates that there is a contribution from the banking sector and Stockman constraint to explain the monetary business cycle facts in a CIA framework.

In conclusion, this thesis investigates the contribution of the banking sector and Stockman exchange technology constraint on the interaction between nominal and real economy. It focuses on the contribution of the financial sector to the short run positive relation between monetary aggregates and real activity, and explains the liquidity effect puzzle and replicates the real effects of monetary policy without a sticky price/wage and a cost channel of monetary aggregates.

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3 This has been discussed in Cooley and Hansen (1989) and has been denoted as ‘inflation tax’ effect.
Chapter 1 - Introduction

This chapter reviews some of the previous explanations of the liquidity effect puzzle and looks at the interactions between nominal and real variables. The first part of the chapter summarises the relevant literature on the solution of the liquidity effect puzzle. There are two main approaches which have previously been used to solve the liquidity effect puzzle under a cash-in-advance environment. The first is limited participation monetary shock, which was developed by Lucas (1990) and followed by Fuerst (1992), and Christiano and Eichenbaum (1992). The second is market segmentation, which was developed by Monnet and Weber (2001), and Alvarez, Lucas, and Weber (2001), and followed by Occhino (2004).

The second part of this chapter concludes the analysis of the relevant literature by looking at the interaction between the nominal and the real side of economy in generating an equilibrium framework with rational expectations. For decades economists have explored the ways in which changes in money stock can influence real economic activity. Some of the mechanisms which they have developed include: monetary changes cause confusion making it hard to differentiate relative price changes from average price level changes (i.e. monetary misperceptions); prices are slow to adjust (i.e. sticky prices); wages are set in nominal terms (i.e. nominal wage contracting); households and firms change their portfolios at different frequencies (i.e. limited participation).

1.1 The Liquidity Effects Puzzle: A Theoretical Review

Monetary economists are concerned about the relationship among money stocks, interest rates, inflation, and real activity. The monetary transmission mechanism is a central topic to our understanding of how monetary aggregates affect real economic activity. Fisher’s

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4 Sticky prices mainly include Fisher (1977), Taylor (1979) and Calvo (1983) types of price and/or wage setting behaviour.
5 Limited participation model comes from Lucas (1990) limited participation monetary shock
equation states that the nominal interest rate equals the real rate plus the expected rate of inflation. If real rates are determined by ‘fundamentals’ in the long run, and are independent to the rate of inflation, then nominal interest rates should be positively related to expected inflation. This means that monetary injections increase nominal interest rates via the anticipated inflation effect, and indicates a positive relation between nominal interest rates and money growth rate. Fisher’s equation concludes that the exogenous increasing money supply should raise expected inflation and nominal interest rates.

In contrast, the liquidity effect on nominal interest rate, which refers to a negative relationship between nominal interest rate and money supply growth rate, is a structural element in both traditional Keynesian (Tobin, 1947) and monetarist macroeconomics models (Friedman, 1968 and Cagan, 1972). The liquidity effect argues that an exogenous increase in money supply would lower nominal interest rates and lead to a positive effect on real economic activity.

To interpret this conflicting result of the effectiveness of money growth rate on nominal interest rate, the thesis argues that only unanticipated increases in the money supply growth rate lower interest rates: which creates a liquidity effect on nominal interest rates. In this view an anticipated money growth rate produces only the expected inflation effect.

Friedman (1968) interpreted the facts of the liquidity effect as a trade off between the effects of partial and general equilibrium. He argued that real interest rates are determined by ‘fundamentals’ in the long run. This includes both the rate at which households discount the future and average productivity growth. Therefore, the long term real interest rates are relatively stable and are unaffected by transitory monetary disturbances. Long term nominal interest rates are considered to be equal to this stable real rate plus expected inflation. In the short term, real and nominal interest rates are both volatile and positively correlated, and nominal interest rates and expected inflation are negatively correlated. Suppose that a monetary authority increases the money supply by conducting an unexpected outright purchase of bonds. In the short term, nominal interest rates fall so that households are willing to hold a smaller quantity of bonds and a larger quantity of money.
However, this is only a partial equilibrium effect. As households spend their increased money holdings on goods, the price level increases and so real balances do not rise as fast as nominal balances. This general equilibrium effect mitigates the need for the nominal interest rate to fall. Therefore, if households spend money so ‘fast’ that the general equilibrium price level effect can completely overturn the partial equilibrium effect, it indicates that there is a positive relation between nominal interest rates and money growth rate: which represents the Fisher effect on the nominal interest rate. If households spend money so ‘slowly’ that the generated equilibrium price level effect cannot overturn the partial equilibrium effect, then this implies a negative relation between nominal interest rates and money growth rate: which indicates the liquidity effect on the nominal interest rate.

In the studies of Lucas (1982) and Svensson (1985) monetary injections are distributed proportionately to all agents in a representative agent Cash-in-Advance (CIA) economy. Thus, a proportional rise in the price level leaves all agents with the same level of real money balances as they had previously. This model assumes that agents have perfect flexibility in responding to shocks and that all decisions are made after recognising the shocks. It also assumes that the households’ allocation decisions can completely reflect the current period surprise in the change of money growth or technology. These decisions include those households deciding how to divide their money holdings between consumption and loans, and how to split their time between labour and leisure. It also includes those firms deciding how much labour to hire, and how much to expand their plant and equipment. This leads to a nominal interest rate increase with monetary expansion, and reflects the expected inflation effect. This model is able to introduce the expected inflation effect but it fails to generate the liquidity effect on nominal interest rates.

By considering inventory transaction costs of money from Baumol (1952) and Tobin (1956), the liquidity effect on nominal interest rate was first captured in the general equilibrium model under standard CIA environment from Grossman and Weiss (1983) and
Rotemberg (1984). The assumption of Grossman and Weiss (1983) and Rotemberg (1984) is that at any time an economy's money is distributed over distinct locations, or markets, and that it takes time to move funds from one location to another. The implication is that an unanticipated change in the excess demand for cash in any one market will have different effects on prices and interest rates, depending on the way cash is distributed when the change occurs. They further assume that the households do not go to the bank together, which introduces the distribution of money demand. Money injections must through a financial intermediary to affect real economy activity. Because the households do not go to the bank at same time, monetary shock cannot affect all agents at same time. Monetary shock cannot be fully translated into inflation under a flexible price system. The extra money supply creates a liquidity effect through Fisher's equation. The key to the monetary transmission mechanism, or to generate the liquidity effect in this type of model, is the asymmetry of the monetary injections, or the representative agent changes their portfolio at different frequencies. Both models had allowed monetary policy to affect the size of withdrawals and the pattern of spending between withdrawals, but not the times of the withdrawals themselves. Romer (1987) extended the work of Grossman and Weiss (1983) and Rotemberg (1984) by allowing agents to choose the timing of trips to the bank within an overlapping generation framework, and found that the economy's responses to a nominal interest rate shock exhibits large cycles and differs dramatically (both qualitatively and quantitatively) from its response when the timing of the trips is fixed.

Lucas (1990) modified the standard CIA model with the timing of monetary shock to generate the liquidity effect on the nominal interest rate. In this model there are two uses of cash, which are: buying goods in the market and investing bonds in an asset market. Monetary shock occurs after a household has made an allocation decision of cash between its two uses. Therefore, when monetary shock occurs a household cannot adjust their cash position between the goods market and the capital market; this creates a liquidity effect of monetary shocks. Lucas (1990) noted that the liquidity effects are a source of non-Fisherian or 'excess' volatility of nominal interest rates. The lower nominal interest rate

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with monetary expansion following the negative liquidity effect dominates the positive expected inflation effect on nominal interest rate.

Figure 1.1: Limited participation capital market

Figure 1.1 illustrates the supply and demand of bonds in the capital market in Lucas’s (1990) limited participation CIA economy. Clearly, there is a vertical supply of bonds subject to monetary shock since it is pre-determined by the amount of households’ saving which has been allocated before the monetary shock occurs. The monetary innovation here is an increase in the demand for bonds. A vertical supply curve gives an increase in the price of bonds and a lower return on the bonds.

Fuerst (1992) argued that there is extra term, which is called ‘liquidity effect’, in the Fisherian fundamental. The positive or negative ‘liquidity effect’ depends on the difference of value of cash in goods and credit markets. If the credit market is relatively liquid then the value of cash in the credit market is less than the value of cash in the goods market (i.e. the liquidity effect is negative), this leads to a nominal interest rate which is lower relative to Fisherian fundamentals. When the goods market is relatively liquid then the value of cash in the goods market is less than the value of the credit market (i.e. the liquidity effect is positive), this leads to a nominal interest rate which is higher relative to Fisherian fundamentals.
Fuerst (1992) further integrated functions of financial intermediates with Lucas’s (1990) limited participation CIA economy, and explained both the liquidity effect and the real impacts of monetary shock. The model assumes that financial intermediates have two functions, which are: to receive saving funds from households and money injections from central bank, and to give loan to firms for wage payment. Since the monetary injections happen after the cash-in-advance constraint is satisfied only borrowers have direct access to the newly injected cash, and this leads to a fall in nominal interest rates with money injections. If firms are borrowers then monetary injections are increased in both current and future real activities.

Christiano (1991) found that for plausible parameter values the liquidity effect which was introduced by Lucas (1990) and Fuerst (1992) is not sufficiently strong enough to dominate the anticipated inflation effect. In order to allow the liquidity effect to dominate expected inflation effect, Christiano (1991) adjusted Lucas-Fuerst (1992) model by assuming that both the household portfolio decisions and firms’ investment decisions must be made before the current value of a shock is known. The assumption that investment cannot respond instantly to shock is intended to capture the real world fact that investment decisions require at least some advance planning. All other decisions in the model are assumed to respond perfectly flexibly to a shock. The model is able to produce a liquidity effect which is stronger than the anticipated inflation effect. The weakness of this model is that it no longer adequately accounts for some non-monetary features of U.S business cycles.

Christiano and Eichenbaum (1992) added adjustment costs onto the goods market CIA constraint to focus on the persistence of the liquidity effect. The adjustment cost of cash in the goods market means that households cannot reallocate their cash position in the next period. Due to adjustment costs, households need to adjust their cash position slowly (subject to monetary shock), and this creates a persistence liquidity effect. The adjustment cost of cash in the goods market is the key to generating persistence of the liquidity effect.

Monnet and Weber (2001) studied the relationship between money and interest rates
through panel data. They find that data since 1960 for about 40 countries supports the Fisher equation view that money growth rate and nominal interest rates are positively correlated. If expectations are taken into account then this supports the liquidity effect view that the variables are negatively correlated. Monnet and Weber (2001) argued that which view applies at any point in time depends on when the change in money occurs and how long the public expects it to last. A surprising money change that is not expected to change future money growth moves interest rates in the opposite direction, while one that is expected to change future money growth moves interest rates in the same direction. Furthermore, Monnet and Weber (2001) also find that monetary policy as a rule for interest rates rather than money does not change the relationship between variables.

Monnet and Weber (2001) further introduced market segmentation into the CIA economy and argued that the nominal interest rate at any point in time is determined by current and expected future money growth rates. A surprising increase in the current rate of money growth causes the nominal interest rate to fall if the public expects the surprising increase to be temporary. If a surprising increase in current money growth is interpreted by public as permanent then nominal interest rate will rise. In conclusion, a surprising increase only in expected future money growth rates will also raise the nominal interest rate. Therefore, the changes in the money stock that affect interest rates depends not only on what is happening to money today, but also on what is expected to happen to money in the future.

Alvarez, Lucas, and Weber (2001) developed a segmented market CIA model to explain how an unexpected increase in nominal interest rate leads to a persistent decrease in the money growth rate and persistence increase in the real interest rate. The crucial set-up of this type of model is that there are two types of household, which are called traders and non-traders. The difference between traders and non-traders is that traders can purchase and hold bonds while non-traders cannot. With an endowment economy this means that non-traders only have consumption decisions and do not have portfolio decisions. In contrast, traders can have both consumption and portfolio decisions. In this type of model monetary shock not only affects the distribution of money (like Grossman and Weiss (1983)) but it also affects the distribution of consumption. With an endowment economy
the real interest rate is determined by the trader's Euler equation. The weight of the trader in the economy is crucial for the model's response to monetary shocks.

Occhino (2004) modelled the dynamic response of the nominal interest rate, the money growth rate, and the real interest rate to monetary policy shocks through a market segmentation CIA model. This model assumes that the markets are segmented in the sense that some households are permanently excluded from the market in government securities. The endogenous process of the money growth rate and real interest rate crucially depends on the degree of market segmentation. This model is able to replicate the persistent decrease in the money growth rate, and the persistent increase in the real interest rate, which follows an unexpected increase in the nominal interest rate.

The central feature of a segmented markets CIA model is that some households are permanently excluded from the market in government securities. Open market operations affect the distribution of money and consumption expenditures across households. The distribution of consumption expenditures affects the real interest rate, since equilibrium real interest rate is determined by an inter-temporal marginal rate of substitution of the subset of households participating in the securities market. Unlike limited participation representative agent models, open market operations affect the real interest rate through this mechanism even when perfectly anticipated. Provided that the monetary policy variable is serially correlated, a monetary policy shock affects persistently both the monetary policy variable and the real interest rate.

Lucas's (1990) limited participation monetary shock assumes that the supply of the capital market is determined by the households' saving fund and does not respond to money injections. An increase in the money supply increases the demand of the capital market, and with a fixed supply this lowers the nominal interest rates of bonds. The weakness of Lucas's (1990) limited participation monetary model is that because households are able to adjust their consumption-saving portfolio in every period, the liquidity effect is entirely driven by serially uncorrelated expectation errors. The model only generates a transitory liquidity effect on nominal interest rates, even when monetary shocks are persistent. To
explain the persistence of liquidity effect on nominal interest rate Alvarez, Lucas, and Webber's (2001) type of market segmentation approach relies upon traders who have consumption and portfolio decisions and non-traders who have only consumption decisions. The weakness of this approach is that with a production economy both trader and non-trader can have consumption and portfolio decisions. The liquidity effect on the nominal interest rate disappears since non-traders have the same behaviour as traders.

In addition to the limited participation monetary shock and market segmentation approach, there are some other approaches which have been developed to explain the empirically plausible liquidity effect. For example, Edge (2007) developed a sticky price monetary business cycle model with investment gestation lags and habit-persistence in consumption to capture the liquidity effect. In another example, Li (2000) employed general equilibrium model with explicit financial sector to generate the liquidity effect when money injections occur through the financial sector.

In conclusion, under a flexible price framework there are two major approaches to generate the liquidity effect on nominal interest rate, which are: limited participation monetary shock and market segmentation. To generate the liquidity effect, limited participation models assume that households cannot adjust their consumption-saving decision before recognising a monetary shock, they also assume that money injections have to be received by the financial sector instead of households. However, these limited participation models are not able to generate the persistence of liquidity effect, even with persistence of the monetary shock and have to assume ‘working capital’ in the model economy to obtain the liquidity effect on nominal interest rate. Meanwhile, the segmented market approach not only generates a liquidity effect on nominal interest rate, but they also generate the persistence of liquidity effect with persistence of monetary policy shock. The weakness of the approach is that the liquidity effect disappears with a production economy since both traders and non-traders behave the same.
1.2 Monetary Business Cycles: An Overview

Can changing nominal variables (such as money aggregates or the nominal interest rate) affect real economic activity? Classical economic doctrine has argued that there is no interaction between the nominal and the real side of the economy. Change in nominal money stock is considered to affect only nominal variables (particular on a price level), and are thought to have no impact on real variables (such as output, consumption, investment and employment).

Sidrauski (1967) employed Ramey’s (1928) growth model and assumed that money yields affect utility by incorporating money balances directly into the utility functions of agents. The paper concluded that long run capital stock is independent to the rate of monetary expansion. In the model a rise in the rate of monetary expansion results in an equal absolute increase in the rate of change in prices and reduces the stock of real cash, but it does not affect steady state consumption. Therefore, the higher the rate of monetary expansion then the lower the steady state level of utility will be. In short run, an increase in the rate of monetary expansion is equivalent to a rise in government transfers to the private sector, which results in an increase in consumption and a fall in the rate of capital accumulation. Furthermore, Fischer (1979) showed that the transition paths are independent of the money supply when log separate utility function is applied because the marginal rate of substitution between leisure and consumption is independent of real money balances.

The real-nominal interactions in the general equilibrium framework with rational expectation was first used by Lucas (1972, 1973), who stressed the idea that there are informational frictions that result in the inability of agents to distinguish changes in relative prices from changes in the absolute price level. The key feature of Lucas’s model is that the agent’s expectations are rational, and so anticipated changes in the price level or money supply are entirely neutral; however, unanticipated movements in money can significantly affect business cycles. This explains the nominal real interaction through the short-run Phillips curve. Lucas (1980) employed Clower’s (1967) cash-in-advance
constraint into the general equilibrium economy, and argued that not only does unanticipated movement in monetary aggregates have real effects but that anticipated changes in money supply also generates negative real effects under flexible price.

The real effectiveness of unanticipated money growth rate, or inflation tax, in stochastic CIA economy has been studied by Cooley and Hansen (1995) who conclude that the movement of output is not really generated from a money growth rate shock. They argued that changes in money growth rate affects real variables only to the extent that they signal changes in the inflation tax. That is, increases in the money growth rate leads agents to expect higher inflation in the future. In response to this, agents substitute away from activities that involve the use of cash in favour of activities that do not require cash. Although they found that monetary shocks do increase somewhat the standard deviation of consumption and lowers its correlation with output, they also find that it has almost no effect on output or hours and it has a sizeable and quite small effect on consumption and investment. Except for consumption, the model economy displays very little correlation between money growth and real variables. The cyclical behaviour of nominal variables which are simulated in the model economy is quite different from the behaviour of nominal variables which are observed in the U.S. economy. Therefore, they concluded that monetary growth shocks do not contribute greatly to the fluctuations in the real variables which are displayed by a basic neoclassical growth model when money is introduced by requiring cash-in-advance.

The Cooley and Hansen (1995) CIA economy is extended with limited participation monetary shock, and includes the liquidity effect to explain the monetary business cycle. Fuerset (1992) assumed that firms have to borrow cash from financial intermediaries in order to pay their workers at each period. After a positive monetary shock, the nominal interest rate decreases so that firms find it optimal to borrow the unexpected increase in money balances. This is an increase in the firms’ labour demand and aggregate output. Thus, limited participation monetary models are consistent with the commonly held view that positive monetary shocks have a positive, albeit temporary, effect on output.
Christiano and Eichenbaum (1995) integrated Lucas's (1990) timing assumption on monetary shock, and Fuerest's (1992) function of financial intermediates, into a real business cycle framework to account for the key aspects of the macroeconomic effects of monetary policy shocks. They introduced the liquidity and real effects of monetary policy by modifying a basic CIA model to distinguish between households, firms, and financial intermediaries. The model argues that households allocate resources between bank deposits and money balances that are used to finance consumption. Financial intermediaries lend out their deposits to firms that borrow to finance purchases of labour services from households. After households have made their choice between money and bank deposits, the financial intermediaries receive lump-sum money injections. In this model only firms and intermediaries interact in financial markets after the monetary injection. If the injection initially affects only the balance sheets of the financial intermediaries, a new channel is introduced by which employment and output will be affected. As long as the nominal interest rate is positive, financial intermediaries will wish to increase their lending in response to a positive monetary injection. To induce firms to borrow additional funds, the interest rates on loans must fall. This generates the liquidity effect, which is a decline in interest rates in response to a positive monetary injection. The lower nominal interest rate decreases the marginal cost of labour, and increases employment and output.

King and Watson (1996) evaluated three monetary business cycle models to explain the link between money, prices, interest rates and economic fluctuation. They documented the key empirical aspects of these relationships, and asked how well the three quantitative rational expectations macroeconomic models work (i.e. a real business cycle model with endogenous money, real business cycle model with sticky price, and limited participation model). They concluded that out of all the prominent macroeconomic models, those which stress a single set of economic mechanisms have substantial difficulties matching the core features of nominal and real interactions. None of the models are found to capture the post-war U.S business cycle finding that a high real or nominal interest rate in the current quarter predicts a low level of real economic activity two to four quarters in the future. All of models are capable a forecasting role for money relative to real economic activity, similar to that found in the U.S data.
Cooley and Hansen (1997) re-examined the work of Lucas (1972, 1975) using the methods of quantitative equilibrium business cycle theory. They showed that the confusion between aggregate monetary shock and Island, or individual monetary, shock could have a significant effect on real activity. Agents confuse changes in the economy wide price level with changes in a market-specific relative price. This happens because individuals are only able to observe the market price and they cannot directly observe the economy-wide average price level. This leads perfectly rational agents to confuse money stocks with market-specific demand shocks, and so respond to the former as though they were the latter. The confusion between aggregate and individual monetary shock can have a significant effect on real economic activity.

Christiano, Eichenbaum, and Evans (1997) compared the limited participation model and sticky price model with monetary business cycle facts from an identified VAR model. They conclude that the sticky price model cannot account for the fact that profits fall after a negative monetary policy shock. They also find that the limited participation model cannot account for the fact that prices do not immediately respond to monetary shock with plausible labour supply elasticity.

Cooley and Hansen (1998) compared a nominal wage contract model with Lucas and Stocky's (1987) cash-credit good model and Lucas's Island model. They find that an economy with labour contracting and misperception of monetary shocks have similar cyclical properties. None of the models can capture the phase shift found in the correlation of money growth with real variables. Also, weak correlation between the cyclical component of money growth and prices (or inflation) is puzzling, as is the negative correlation between money growth rate and nominal interest rates. Neither can the models account at the same time for the observed counter-cyclical level price level and the pro-cyclical rate of inflation.

Gavin and Kydland (1999) employed endogenous money supply rules in a shopping time type of monetary business cycle model. They argued that changing the money supply rules
has almost no effect on the cyclical behaviour of real variables. In contrast, they find that the cyclical nature of the nominal variables can be highly sensitive to small changes in the decision rule governing the money supply; however, such changes have almost no impact on the cyclical behaviour of the real variables. But the changes in the money supply process have significant effects on both variability of price level, and size and sign of the correlation between the nominal variables and output.

Freeman and Kydland (2000) asked whether monetary facts may result from endogenously determined fluctuations in the money multiplier, rather than a causal influence of money on output. They assume that consumption goods can be purchased using either currency or bank deposits. The cost of acquiring money balances determine the demand for money and make endogenous the velocity of money. The fixed cost of using deposits determines the division of money balances into currency and interest-bearing deposits. In this model households make decisions by facing these two costs and determining the velocity of money and money multiplier. The model is able to account for the observed correlations of nominal variables and real output by imposing no rigidity in prices or agent choices.

Ireland (2003) estimated an RBC model with endogenous money supply, and showed that nominal rigidity over and above endogenous money plays a role in accounting for key features of correlations between nominal and real variables in post war US data.

Benk, Gillman and Kejak (2005a) explained the monetary business cycle facts with three exchange technology models, which are: standard CIA economy with cash only exchange technology, shopping time model, and CIA economy with cash and exchange credit two exchange technologies. They concluded that the CIA model with cash and exchange credit improves the ability of CIA economy to explain the pro-cyclical movement of monetary aggregates, inflation, and nominal interest rate.

Bruckner and Schabert (2006) considered non-separable real money balance in shopping time function within a new Keynesian framework, and concluded that money matters for real activity.
Dow (1995) specified a monetary model with sticky price and financial market frictions, with one period price and savings contract decisions made before observing the monetary policy shock. This model is able to generate the appropriate responses for output, consumption, investment and nominal interest rate through both nominal price adjustment and saving behaviour. Keen (2004) extended the Dow (1995) model to allow for partial adjustment of prices and savings to current period monetary policy shock. This model avoids the infinite transaction costs on goods and capital markets and can generate the appropriate responses from output, consumption, investment, price level and nominal interest rate to monetary policy shock.

In conclusion, the studies on the monetary business cycle in the RBC framework have concluded that the model economy has to include the sticky price/wage setting and/or financial frictions to generate significant effects of monetary aggregates. The chapter 3, 4 and 5 do not require the stick price/wage setting, non-separate money in utility function or endogenous money supply rule to obtain the real effectiveness of monetary aggregates. It explains the effects of monetary policy through the functions of financial firm (which includes financial intermediates and banks) under Cash-in-Advance economy.
Chapter 2 - Explaining the Liquidity Effects Puzzle and Real Activity

Abstract

This chapter examines the liquidity effects puzzle (i.e. lower nominal interest rates with monetary expansion) and real economic activity by extending the Cash-in-Advance (CIA) economy with the function of productive banks. It employs Benk, Gillman and Kejak (2005) monetary banking model and assumes that money injections are received by banks instead of households to generate liquidity effect on nominal interest rates and replicate the business cycle facts which have been observed from the data. There are five representative agents, they are: household consumers, firms, productive banks, monetary authority, and government in the economy. It includes two types of exchange technology: real money balance and exchange credit. Both cash and exchange credit are able to be used by households for goods market transactions. Competitive banks produce exchange credit through Cobb-Douglas type of production function with labour, capital and deposit. For each unit of exchange credit has been produced, the banks request the equal number of government bonds from capital market. This indicates that the demand for government bonds is equal to the number of exchange credits produced by the banks. The model assumes that households recognise a monetary shock after their exchange technology portfolio decision has been made. This indicates that the supply of government bonds in the capital market is determined by the amount of exchange credit collected by households, and does not vary with money injections. Supposing money injections are received by banks instead of households, and are used for purchasing government bonds at capital market, then this means that money injections are increased with the demand of the capital market through banks. Pre-determined supplies of bonds lower the nominal interest rate or create a liquidity effect. The liquidity effect on nominal interest rate raises consumption and employment through intra-temporal leisure consumption substitution. Using the Stockman (1981)
CIA constraint (which argues that output and exchange technology are complementary goods) the model is able to replicate the key business cycle facts which have been observed in the data.

2.1 Introduction

Fisher’s equation implies that the nominal interest rate is equal to the real return plus the expected inflation rate, and it indicates that money injections raise nominal interest rates via an anticipated inflation effect. In contrast, decreasing nominal interest rates with monetary expansion (which is referred to as the liquidity effect) has been recognised as an important monetary transmission feature in both traditional Keynesian (Tobin, 1947) and monetarist (Friedman, 1968 and Cagan, 1972) macroeconomic models. To examine the liquidity effect on the nominal interest rate and the real effects of monetary policy, this chapter employs two exchange technologies monetary RBC model from Benk, Gillman and Kejak (2005), assumes that money injections are received by banks instead of households and supposes the limited participation monetary shock from Lucas (1990).

The Cash-in-Advance (CIA) models which were developed by Lucas (1982) and Svensson (1985) include the expected inflation effect on nominal interest rate and generate negative relations between monetary aggregates and real economic activities through an ‘inflation tax’ effect.7 The cash-credit goods CIA model that was developed by Lucas and Stocky (1987) has been simulated by Cooley and Hansen (1995) with indivisible labour supply, it assumes that: agents have perfect flexibility in responding to monetary shock, that all decisions are made after recognising the money injections, and that all allocations completely reflect the current period surprise in the change of the money growth rate. Although it generates the expected inflation effect, it ignores the liquidity effect on the nominal interest rate. The monetary innovations have negative effects on real economic activity through an expected inflation effect on the nominal

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7 This will be discussed in more detail in Chapter 5.
interest rate.

The liquidity effect on nominal interest rate was first captured in a CIA economy by Grossman and Weiss (1983) and Rotemberg (1984). By considering the inventory transaction costs of money demand from Baumol (1952) and Tobin (1956), an open market purchase of a bond for fiat money could drive down nominal and real interest rates, leading to a delayed positive price response; it can also damp the persistent effects on both prices and nominal interest rates. Lucas (1990) modified the standard CIA model by assuming that a limited participated monetary shock generates a liquidity effect. It can be argued that there are two uses of cash, which are: buying goods in the market and investing in bonds in the asset market. In this model monetary shocks are assumed to occur after households have made an allocation decision of cash between either of its two uses. This means that households cannot adjust their cash position between the goods and capital market subject to money injections. This creates a liquidity effect of money growth rate as money injections only affect capital market, and not the goods market.

Fuerst (1992) integrated the functions of financial intermediaries into the standard CIA economy with Lucas's (1990) timing of the monetary shock assumption, and so generated the real effects of money growth rate through the cost channel of monetary policy. The key innovation of Fuerst (1992) is that firms have to borrow cash from financial intermediaries in order to pay the wage bill. Monetary injections occur after goods market CIA constraint has been satisfied. Money injections will lower the nominal interest rate because the size of the market is pre-determined by firms’ wage payment. After a positive monetary shock, the nominal interest rate decreases so that firms find it optimal to borrow to meet the unexpected increase in money supply. The result is an increase in firms’ labour demand and a rise in aggregate output. This model is consistent with the commonly held view that positive monetary shocks have a positive and temporary effect on output. The main contribution of Fuerst’s (1992) approach is that it provided a general equilibrium explanation of a liquidity effect of monetary injections on nominal interest rates and real activity by introducing a cost
channel of monetary policy.

Monnet and Weber (2001) argue that the changes in the money stock which affect interest rates depends not only on what is happening to money today, but also on what is expected to happen to money in the future. They argued that if the money stock is changed today but the future money growth rates are not expected to change then interest rates will move in the opposite direction to the money stock, which is the liquidity effect. However, if the money stock is changed today and future money growth rates are expected to move in the same direction then interest rates will move in the same direction, which is the Fisher effect.

Alvarez, Lucas, and Weber (2001) developed a simple segmented market approach to generate a persistent decrease in the money growth rate, and a persistence increase in the real interest rate, after an unexpected increase in the nominal interest rate. There are two types of households in this model, which are called traders and non traders. The difference between traders and non traders is that traders can purchase and hold bonds while non-traders cannot. With an endowment economy, this means that non-traders only have consumption decisions and do not have portfolio decisions. In contrast, traders can have both consumption and portfolio decisions. With a segmented market approach, monetary shocks not only affect the distribution of money (as in Grossman and Weiss (1983)) but they can also affect the distribution of consumption. The weight of traders in the economy is crucial for the model’s response to monetary shocks.

Occhino (2004) presented a segmented market model to generate the dynamic response of the nominal interest rate, the money growth rate, and the real interest rate to a monetary policy shock. In this model markets are segmented in the sense that some households are permanently excluded from the market in government securities. The endogenous processes of the money growth rate and real interest rate crucially depend on the degree of market segmentation. The model is able to replicate both the persistent decrease in money growth rate, and the persistent increase in the real interest rate, which follows an unexpected increase in the nominal interest rate.
In a word, the central feature of segmented markets models is that some households are permanently excluded from the market in government securities. Open market operations can affect the distribution of money and consumption expenditures across households. The distribution of consumption expenditures affects the real interest rate because the equilibrium real interest rate is determined by the inter-temporal marginal rate of substitution of the subset of households participating in the securities market. Unlike limited participation representative agent models, open market operations affect the real interest rate through this mechanism even when perfectly anticipated. Provided that the monetary policy variable is serially correlated, a monetary policy shock affects persistently both the monetary policy variable and the real interest rate. The weakness of a market segmented approach is that with a production economy both trader and non trader are able to make consumption and portfolio decision. Consequently, the liquidity effect disappears since non traders have the same behaviour as traders.

In conclusion, Lucas-Fuerst's (1992) type of limited participation CIA models argued that there is an inflation effect and a liquidity effect on nominal interest rate with monetary innovations. If the inflation effect dominates the liquidity effect, then nominal interest rates will increase with monetary innovation. If the liquidity effect dominates the inflation effect then the nominal interest rate will decrease with monetary shock. This model generates real effects of monetary aggregates through the cost channel of monetary policy. Notice that the liquidity effect on nominal interest rate which is generated from Christiano's (1991) limited participation model does not result from consumption behaviour since it does not generate the positive response of consumption subject to monetary shock. By allowing firms' wage payment in the cash in advance constraint (such as in Christiano and Eichenbaum (1992)) the model is able to generate the positive response of consumption to money injections. Monnet and Weber (2001) have argued that only unanticipated increases in the money supply can lower nominal interest rates, and an anticipated money growth rate only produces an expected inflation effect. Furthermore, the liquidity effect on the nominal interest rate which is created by

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8 See Fuerst (1992), and Christiano and Eichenbaum (1992)
monetary innovation has a positive effect on real economic activity, and the inflation effect on nominal interest rate has a negative effect on macro aggregates.

This chapter formulates, calibrates, and simulates a Dynamic Stochastic General Equilibrium (DSGE) model that incorporates two exchange technologies and government bond trading at the capital market to investigate the liquidity effects on nominal interest rate. By extending the model with Stockman's (1981) CIA constraint, this model is able to generate the real effects of monetary policy without cost channel, which have usually been assumed in Lucas-Fuerst (1992) type of limited participation monetary models. The main findings of this chapter are that it firstly explains the liquidity effect on nominal interest rate and generates real effectiveness of monetary policy without cost channel, and, secondly, that it accounts for long run monetary effects on both nominal interest rate and real activity.

This chapter is going to be organised into 8 sections, the first of which is this introduction. Section 2 describes the empirical evidence on monetary transmissions and real impacts of money growth rate. Section 3 presents a theoretical two exchange technologies DSGE model with a banking sector. Section 4 explains the procedure of calibration. Section 5 discusses how the model’s steady state is affected by changing money growth rate. Section 6 examines the model’s dynamic and simulations. Section 7 has a detailed discussion on the model's properties and policy implications. And, finally, Section 8 concludes the chapter.

2.2 Empirical Evidence of the Fisher and Liquidity Effects on Nominal Interest Rate

This section presents some the empirical evidence of the relationship among money growth rate and varies nominal interest rates. Fisher's equation implies that nominal interest rates are positively related with expected inflation because real returns are independent to the rate of inflation. An increase in the money growth rate is followed by
rising nominal interest rate through expected inflation effect. This indicates that there is a positive relation between nominal interest rate and money growth rate. Figure 2.1 represents the relationship between the quarterly narrow money growth rate and short and long term interest rates from 12 countries, at least, over 19 years. The countries and time periods have been included in Table 2.1. This information shows that the correlation between quarterly short term interest rate and narrow money growth rate is 0.562, and the quarterly long term interest rate and narrow money growth rate is 0.566. Therefore, the data from these 12 countries shows that there is a positive relationship between the money growth rate and the nominal interest rate.

Figure 2.1: The Fisher effect on nominal interest rates

In contrast, the liquidity effect view argues that there is negative relation between the money growth rate and nominal interest rate. It states that the positive monetary shocks push nominal interests down, and that the negative shocks push them up. One of the supports of the evidence of the liquidity effect on the nominal interest rate is the Hodrick-Prescott (HP) time series of the correlations between money growth rate and nominal interest rates. Cooley and Hansen (1995) concluded that the monetary business cycle facts through the U.S data which have been taken logged and detrended by a HP filter. They found that in the U.S data, between 1954Q1 and 1991Q2, there are negative correlations between the quarterly M1 growth rate and both ten year U.S Treasury bond
yields and one month U.S Treasury bill yields. Therefore, the negative correlation between money and interest rates can be seen as evidence of a liquidity effect. Table 2.2 concludes the correlations among the quarterly M1 growth rate and nominal interest rates for U.S economy from 1959Q1 to 2004Q2. It indicates that there is negative correlation between money growth rate and nominal interest rate, which represents the liquidity effect view on the relation between money and nominal interest rate.

Table 2.1: The countries and time periods in Figure 2.1

<table>
<thead>
<tr>
<th>Country</th>
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<th>Time Period</th>
<th>Country</th>
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<tbody>
<tr>
<td>Australia</td>
<td>1975.2-2010.2</td>
<td>Canada</td>
<td>1957.1-2010.2</td>
<td>Denmark</td>
<td>1991.2-2010.2</td>
</tr>
<tr>
<td>Finland</td>
<td>1988.1-2010.2</td>
<td>France</td>
<td>1978.2-2010.2</td>
<td>Germany</td>
<td>1980.2-2010.2</td>
</tr>
<tr>
<td>Italy</td>
<td>1980.2-2010.2</td>
<td>Netherland</td>
<td>1983.2-2010.2</td>
<td>New Zealand</td>
<td>1977.3-2010.2</td>
</tr>
<tr>
<td>Sweden</td>
<td>1963.1-2010.2</td>
<td>UK</td>
<td>1969.4-2010.2</td>
<td>US</td>
<td>1959.2-2010.2</td>
</tr>
</tbody>
</table>

Table 2.2: Cross-correlation among quarterly money growth rate and various nominal interest rates, from 1959 Q1 to 2004 Q2

<table>
<thead>
<tr>
<th></th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 growth rate</td>
<td>-0.02</td>
<td>0.21</td>
<td>0.15</td>
<td>1.00</td>
<td>0.15</td>
<td>0.20</td>
<td>-0.02</td>
</tr>
<tr>
<td>Federal policy rate</td>
<td>-0.06</td>
<td>-0.12</td>
<td>-0.26</td>
<td>-0.40</td>
<td>-0.29</td>
<td>-0.22</td>
<td>-0.14</td>
</tr>
<tr>
<td>Discount rate</td>
<td>-0.03</td>
<td>-0.09</td>
<td>-0.25</td>
<td>-0.37</td>
<td>-0.32</td>
<td>-0.27</td>
<td>-0.22</td>
</tr>
<tr>
<td>Treasury bill rate</td>
<td>-0.05</td>
<td>-0.12</td>
<td>-0.32</td>
<td>-0.43</td>
<td>-0.32</td>
<td>-0.24</td>
<td>-0.17</td>
</tr>
<tr>
<td>Bank lending rate</td>
<td>-0.02</td>
<td>-0.10</td>
<td>-0.20</td>
<td>-0.39</td>
<td>-0.35</td>
<td>-0.27</td>
<td>-0.21</td>
</tr>
<tr>
<td>20 years bond rate</td>
<td>0.03</td>
<td>-0.11</td>
<td>-0.33</td>
<td>-0.42</td>
<td>-0.35</td>
<td>-0.27</td>
<td>-0.18</td>
</tr>
</tbody>
</table>
The empirical evidence from the Vector Auto Regression (VAR) models also seems to qualitatively support the existence of a liquidity effect on the nominal interest rate. The identified monetary shocks in the VAR model are that part of the policy variables which cannot be explained given the information set available at the time. Christiano,
Eichenbaum, and Evans (CEE) (1999) examined the effects of monetary shock with three identification schemes and found that an expansionary monetary policy shock causes the nominal interest rate to fall, output to rise, and the price level to increase slowly. In addition, there are many empirical studies (such as Bernanke and Blinder (1992), Christiano and Eichenbaum (1992)) which have found negative contemporaneous correlation between monetary shocks and nominal interest rates, and monetary aggregates and real activity tend to be positively correlated over the business cycle.

Figure 2.2 represents the impulse responses of nominal interest rates to monetary innovation in four variables (real GDP, price level, nominal interest rate, and M1) reduce VAR model for the U.S economy with data period from 1959 Q1 to 2004 Q2. All variables are in log form and the number of lags of VAR model is selected by using Akaike Information Criterion (AIC) and Schwarz Information criterion (SIC). In order to obtain the reasonable impulse responses function, the reduce VAR model is transformed to a recursive VAR model through the use of a generalized impulse. In this model there is evidence of liquidity effects on nominal interest rate. A decline in nominal interest rate that is qualitatively consistent with the findings of empirical work is found by Leeper et al (1996), CEE (1999), and Bernanke and Mihov (1998).

In other words, there is empirical evidence of both inflation and liquidity effects on the nominal interest rate. A similar result has been found by Monnet and Weber (2001), who have concluded two contradictory monetary phenomena between money growth rate and interest rates. The data since 1960, for about 40 countries, supports the Fisher equation view that these variables are positively related. After taking expectations into account, this supports the liquidity effect view that they are negatively related. Monnet and Weber (2001) developed a segmented market CIA model to explain the liquidity effect on the nominal interest rate. They argue that if the money stock is changed today, but future money growth rates are not expected to change, then interest rates move in

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9 Pesaran and Shin (1998) construct an orthogonal set of innovations that does not depend on the VAR ordering.
the opposite direction to the money stock - which is a liquidity effect view. But if the money stock is changed today, and the future money growth rates are expected to move in the same direction, then interest rates move in same direction - which is a Fisher effect view. This part of chapter demonstrates that anticipated changing in money growth rate has only inflation effect on nominal interest rate and unanticipated changing in money growth rate has both inflation and liquidity effects on nominal interest rate. The movement of nominal interest rate (increase or decrease) with monetary expansion depends on the persistence of money growth rate. The more persistence on money growth rate leads the stronger inflation effect on nominal interest rate; the less persistence on money growth rate introduces the stronger liquidity effect on nominal interest rate.

2.3 The Structure of the Economy

This part of chapter introduces the structure of economy and displays the problems which are solved by households, firms and banks. It describes the behaviour of monetary policy authority and government. There are three sources of uncertainty in the model, which are: exogenous structure shocks in goods producing firms, banks, and money growth rate.

Figure 2.3 describes the structure of a decentralised economy. The economy includes five infinitely lived representative agents (i.e. goods producer firms, household consumers, productive banks, a central bank or monetary authority, and government) and two exchange technologies (i.e. cash and exchange credit) with two types of CIA constraints. Since the representative agents are infinitely live and have been assume to make decisions period by period basis. It is able to ruling out multi-period borrowing and lending in capital market. Therefore, goods producing firms employ labour and rent capital stocks to produce investment, consumption goods and pay labour, and capital bills. Households collect exchange credit from productive banks for goods market transaction before recognising money injections. This determines the supply of
government bonds. The government finances their expenditure by collecting tax from households and issuing bonds with an interest rate of $R_t$ to banks at the capital market. Competitive banks produce exchange credit through Constant Return to Scale (CRS) production function with labour, capital stocks, and deposit. For each unit of exchange credit which is produced, banks have to demand government bonds to backup the exchange credit. This creates the demand for government bond. Money injections from the central bank are received by banks and are invested in capital market with an amount of exchange credit. This increases the demand for government bonds. Since the supply of government bonds is determined by the exchange credit collected by households and does not vary with money injections, the increasing demand for government bonds raises the price of bonds and lowers the return on the bonds. With a lower nominal interest rate, the model generates the real effects of monetary shock through intra-temporal substitution between leisure and consumption instead of the cost channel of monetary policy.

Figure 2.3: The structure of the model economy
Figure 2.4 represents the behaviour of the capital market. Without money injections, the equilibrium of the capital market is determined by the supply and demand of government bonds. The supply of government bonds is equal to the demand of exchange credit from households. The demand of government bonds is determined by the supply of exchange credit from banks. The key assumption of this model is that it assumes that money injections are received by banks instead of households, and that the size of the capital market or supply of government bond does not increase with money injections. The monetary expansion increases the demand for government bonds through banks. It raises the prices of bonds, lowers the return of bonds, and creates the liquidity effect on the nominal interest rate.

Figure 2.4: The supply and demand of government bonds at the capital market

There are two types of Cash-in-Advance (CIA) constraint which have been employed by the model economy: standard and Stockman constraints. The standard CIA constraint was first employed by Lucas (1982) and Svensson (1985) in a general equilibrium economy, both argued that households hold exchange technologies for purchasing consumption. The Stockman CIA constraint was employed by Stockman (1981) in a
CIA economy. Stockman argues that exchange technologies are held by households, not only use for purchasing consumption but also for investment. This explains the opposite of the Tobin effect and indicates that the marginal utility of consumption substitution is affected by both the real interest rate and the nominal interest rate. The model with Stockman constraint generates a stronger liquidity effect on the nominal interest rate and replicates the monetary business cycle facts which have been observed in the data.

In other words, by assuming money injections are received by banks instead of households, the model is able to generate a liquidity effect on the nominal interest rate. In so doing it explains the real effects of monetary aggregates through intra-temporal substitution between leisure and consumption instead of the cost channel of monetary policy.

2.3.1 Banks

King and Plosser (1984) have pointed out that if the CRS assumption is made using just labour and capital as inputs, and then there is a flat marginal cost curve of exchange credit supply. With an alternative of money for making exchanges and a marginal shadow cost that is flat at nominal interest rate, then there is no unique equilibrium between money and credit use. In order to have an upward slopping marginal cost curve subject to credit-deposit ratio, competitive banks have been assumed to produce exchange credit through Cobb-Douglas production function from Clark (1984) with labour, capital stocks, deposits. The share of labour, capital and deposits are $\gamma_1, \gamma_2, 1-\gamma_1-\gamma_2$.

$$f_t = A q e^{\alpha}(l_t^f)^{\gamma_1} (s_t^f k_{t-1})^{\gamma_2} d_t^{1-\gamma_1-\gamma_2}$$ (1)

Where $f_t$ represents number of exchange credit; $l_t^f$ and $s_t^f$ stand for the share of labour and capital in banks, $d_t$ indicates aggregate deposits from households. An exogenous banking sector technology shock is assumed to follow AR (1) process with

\[10\] This will be discussed in more detail in Chapter 4 & 5.
autoregressive parameter $\rho_q$ and structure innovation $\epsilon^q_i$.

$$q_i = \rho_q q_{i-1} + \epsilon^q_i, \quad \epsilon^q_i \sim (0, \sigma^2_q), \quad 0 < \rho_q < 1 \quad (2)$$

Banks sell exchange credit to households, receive money injections from the central bank, and purchase government bonds. The cost of exchange credit for households equals labour, capital, and dividend income from the banking sector. In other words, households have to pay labour, capital, and deposit for collecting exchange credit from banks. The lifetime budget constraint for exchange credit production which has been faced by banks is represented by equation (3).

$$r^d_t d_t = p_t f_t - w_t l_t - r_t s_t k_{i-1} \quad (3)$$

$$w_t = p_t \gamma_1 \frac{f_t}{l_t} \quad (4)$$

$$r_t = p_t \gamma_2 \frac{s_t}{k_{i-1}} \quad (5)$$

$$r^d_t = p_t (1 - \gamma_1 - \gamma_2) \frac{f_t}{d_t} \quad (6)$$

Where $p_t$ represents the price of exchange credit, $r^d_t$ stands for the interest rate for deposit, $w_t$ and $r_t$ represents the real wage and real interest rate, respectively. Banks maximise the exchange credit production function subject to a lifetime budget constraint. Equilibrium conditions of the banking sector (which are the marginal costs of labour, capital and deposit) have been indicated by equation (4)-(6). These equations imply that households collect exchange credit from banks with labour, capital, and deposit costs. For each unit of exchange credit is obtained by households; there are labour, capital and deposit costs deducted from households' aggregate income.

Following Benk, Gillman and Kejak (2005) setup, the model assumes that the amount of deposit is equal to the amount of exchange technology which is held by households for goods market transaction.

$$d_t = \frac{M_{t-1}}{P_t} + f_t \quad (7)$$

Where $P_t$ represents the price level, $M_{t-1}$ represents initial money holding.
2.3.2 The Capital Market

Limited participation monetary shock was developed by Lucas (1990) in a CIA economy, and it argues that money injections happen after the goods market exchange technology constraint have been satisfied. The model employs the limited participation monetary shock by assuming a cash-exchange credit portfolio for goods market transactions has been determined by households before money injections occur. This means that the supply of government bond does not vary with money injections. Since the money injections are received by banks instead of households, the demand for government bonds is indicated by equation (8).

\[ f_i + T_i = b_i \quad (8) \]

Where \( T_i \) represents money injections from central bank, and \( b_i \) represents the number of government bonds.

In contrast to the standard limited participation model, where the size of the capital market is determined by firms’ wage payments, this model assumes that the government is the only borrower at the capital market and the cost of exchange credit or banking sector determines the size of capital market, which is represented by equation (9). Without money injections, the equation indicates a nominal interest rate which is equal to the marginal cost of banking. With money injections, the equation implies that the size of the capital market is determined by the cost of exchange credit. With the amount of exchange credit being chosen by households, the monetary shock has a negative effect on the nominal interest rate since the demand for government bonds exceeds the supply.

\[ R_i b_i = R_i^f f_i \quad (9) \]

Where \( R_i^f = 1 + p_i^f \) stands for the gross marginal cost of banking sector; \( R_i \) represents the gross nominal interest rate on government bonds.
2.3.3 Government

The government finances their expenditure by issuing within period bonds and taxing households. The government’s budget constraint is represented by equation (10). It indicates that: the government consumes consumption, investment goods; collects taxes from households; and sells bonds to banks and pays a gross interest rate $R_t$.

$$
\tau_t - G_t = \frac{R_t b_{t-1}}{\pi_t} - b_t
$$

(10)

Where $G_t$ represents the government’s expenditure, $\tau_t$ represents lump-sum tax. The government’s budget constraint implies that the government surplus/deficit varies with the net payout on bond at capital market.

2.3.4 Household Consumers

Representative households maximize their expected log utility function (11), which includes: consumption goods $c_t$ and leisure $x_t$, and a discount factor $\beta \in (0,1)$. The households allocate their timing endowment among leisure, labour in goods production sector $l^g_t$, and the labour in banking sector $l^f_t$.

$$
U = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t)
$$

(11)

$$
1 - x_t = l^g_t + l^f_t
$$

(12)

$$
1 = s^g_t + s^f_t
$$

(13)

Aggregate output, which includes consumption and investment goods, has been produced by goods producing firms. The next period’s physical capital stocks are accumulated through law of motion equation, with a quarterly discount rate $\delta$. The share of capital stocks within goods and banking sectors are $s^g_t$ and $s^f_t$, respectively.

$$
y_t = c_t + i_t
$$

(14)

$$
i_t = k_t - k_{t-1}(1 - \delta)
$$

(15)
The model assumes that households hold exchange technology before attending the goods market. There are two types of exchange technology for household consumers: cash and exchange credit. Households decide the amount of cash and exchange credit for the goods market before recognising monetary shock. The exchange technology constraint which has faced by households at the goods market is given by equation (16). The amount of cash purchase good is represented by equation (17).

\[
\frac{M_{t-1}^c}{P_t} + f_t = c_t + \Omega i_t 
\]  

(16)

\[
\frac{M_{t-1}^e}{P_t} = a_t(c_t + \Omega i_t) 
\]  

(17)

Where \(a_t\) represents fraction of cash purchase goods. When \(\Omega = 0\), the standard constraint is applied. This indicates that households hold exchange technology for consumption purchase only. When \(\Omega = 1\), the Stockman constraint is applied. This implies that both consumption and investment goods are purchased using exchange technology. Household consumers supply labour and rent capital stock to the real sector and receive labour \(w_t l_t^w\) and capital \(r_t k_{t-1}\) incomes. The aggregate income is spent on consumption and investment goods. The lifetime budget constraint which has been faced by households is indicated by equation (18).

\[
\frac{M_t}{P_t} = M_{t-1}^c + r_t^d d_t + w_t (1 - x_t) + r_t k_{t-1} - c_t - k_t + (1 - \delta) k_{t-1} - (R_t - 1) b_t 
\]  

(18)

The equilibrium conditions of households are represented by equations (19)-(22), from maximising households’ expected log utility function subject to lifetime and cash-in-advance constraint for the goods and capital markets.

\[
\frac{\mu_t}{\lambda_t} = R_t - 1 
\]  

(19)

\[
\frac{x_t}{\Psi c_i} = \frac{R_t - r_t^d}{w_t} 
\]  

(20)

\[
\beta E_i \left[ \frac{R_{t+1}^c \lambda_{t+1}}{\pi_{t+1}} \right] = 1 
\]  

(21)

\[
\beta E_i \left[ \frac{(r_{t+1} + 1 - \delta) + (1 - \delta) \Omega (\frac{\mu_{t+1}}{\lambda_{t+1}} - r_{t+1}^d)}{\lambda_t} \right] = 1 + (\frac{\mu_t}{\lambda_t} - r_t^d) \Omega 
\]  

(22)
Where $\lambda_t$ and $\mu_t$ represent the shadow prices of lifetime and CIA constraints. Equation (19) indicates that the marginal cost of money is equal to the nominal interest rate on bonds. The marginal utility of consumption is affected by the shadow price of lifetime budget constraint, the nominal interest rate, and the deposit rate. The intra-temporal substitution relation is represented by equation (20), which argues that the substitution between marginal utility of consumption and leisure is influenced by nominal interest rate, the deposit rate, and the real wage. The lower nominal interest rate has a negative effect on leisure-consumption substitution and increases consumption. Equation (21) indicates the Fisher relation of the nominal interest rate. When the liquidity effect dominates the expected inflation effect on nominal interest rate, the Fisher relation indicates a decrease in the nominal interest rate with money injections. When the expected inflation effect dominates the liquidity effect on nominal interest rate, the Fisher relation implies an increase nominal interest rate with monetary shock. The households' inter-temporal substitution is represented by the model's Euler relation, which is indicated by equation (22). If $\Omega = 0$ then the households hold exchange technology for purchasing consumption goods only. This implies that exchange technology and investment are perfect substitution goods in the economy. The Euler relation states that the current and future shadow price of lifetime substitution is determined by the real interest rate.

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (r_{t+1} + 1 - \delta) \right] = 1 \quad (23)$$

If $\Omega = 1$ then the households hold cash and exchange credit for purchasing consumption and investment goods. This implies that the exchange technology and output are complementary goods in the economy. The Euler equation from equation (24) indicates that the current and future marginal utility of consumption substitution is determined by real interest rate discounted by the difference between nominal interest rate and deposit rate. Therefore, under CIA framework, the Euler equation indicates that the decreasing (increasing) on nominal interest rate has positive (negative) effect to current consumption. In contrast, the rise (lower) expected real interest rate has negative (positive) effect on current consumption. With Stockman constraint, the expected real interest rate effect on current consumption is discounted by the decreasing (or
increasing) in nominal interest rate. It leads the negative effect on current consumption stronger compare with standard CIA constraint. Therefore, there is a hump shape response of consumption to monetary shock in stockman CIA economy.

\[ \beta E_t \left[ \frac{c_t}{c_{t+1}} \left( \frac{r_{t+1}}{R_{t+1}} - r^d_{t+1} + 1 - \delta \right) \right] = 1 \]  

(24)

2.3.5 Firms

Aggregate output which has been produced by representative firms is indicated by equation (25) through Cobb-Douglas production function with labour, capital stocks and exogenous technology. The shares of labour and capital stocks are \(1 - \alpha\) and \(\alpha\) respectively.

\[ y_t = e^{\alpha} (l^g_t)^{1-\alpha} (s^f k_{t-1})^\alpha \]  

(25)

The exogenous good sector Total Factors Productivity (TFP) is assumed to follow AR (1) process with autoregressive parameter \(\rho_z\) and structure shock \(\varepsilon_t^z\).

\[ z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad \varepsilon_t^z \in (0, \sigma^2_z) \quad 0 < \rho_z < 1 \]  

(26)

Representative firms have an income from the goods market and pay wages and capital bills to households. This maximises goods production function subject to firms’ lifetime budget constraint, which is represented by equation (27). Real prices or the marginal costs of firms, (which are real interest rate and real wage) have been indicated by equation (28) and (29).

\[ r_t s^f k_{t-1} + w_t l^g_t = y_t \]  

(27)

\[ w_t = (1 - \alpha) \frac{y_t}{l^g_t} \]  

(28)

\[ r_t = \alpha \frac{y_t}{s^f k_{t-1}} \]  

(29)

The real price equations indicate that the firms are willing to employ labour and rent capital stocks from households until the marginal cost of labour and capital is equal to the real wage and real interest rate. The ratio between the capital and labour income is
constant at rate \( \frac{\alpha}{1 - \alpha} \) across equilibrium. To generate the real effects of money growth rate, the Lucas-Fuerst (1992) type of limited participation model requests the marginal costs of firms to be varied with the nominal interest rate. This is achieved by requiring firms to borrow wage payments before aggregate output has been produced. The lower nominal interest rate with money injections is decreased with the marginal cost of labour, increased employment, and aggregate output. In contrast to Lucas-Fuerst's (1992) type of limited participation model, this model does not request cost channels of monetary policy which assumes that firms borrow in advanced to generate the real effects of monetary expansion since the lower nominal interest rate increases consumption and decreases leisure through intra-temporal substitution.

2.3.6 Monetary Policy

Monetary policy which has been implemented by the central bank is represented through money supply rule in equation (30). This indicates central bank conduct monetary policy by influencing the next period of nominal money stocks. As nominal money demand is always equal to nominal money supply at the money market, the money supply rule indicates that short term interest rates will adjust money demand to equal money supply at equilibrium.

\[
M_t = M_{t-1} + T_t 
\]

(30)

The monetary expansion equation is represented by equation (31). It indicates that monetary injections from the central bank depends on constant money growth rate \( \Theta^* \), monetary shock \( e^m_t \), and initial money stocks.

\[
T_t = (\Theta^* + e^m_t - 1)M_{t-1} 
\]

(31)

The deviation of money growth rate will be considered to follow AR (1) process with autoregressive parameters \( \rho_m \) and structure shock \( \varepsilon^m_t \).

\[
u_t = \rho_m u_{t-1} + \varepsilon^m_t \quad \varepsilon^m_t \in (0, \sigma_m^2) \quad 0 < \rho_m < 1
\]

(32)
2.3.7 Competitive Equilibrium

Competitive equilibrium of this economy consists a set of feasible allocations \( \{ y_t, c_t, k_t, M_t, l^g_t, l^f_t, x_t, s^g_t, s^f_t, f_t, d_t \} \), a set of prices \( \{ r_t, w_t, r^d_t, R_t, R^f_t \} \), exogenous shocks \( \{ z_t, q_t, u_t \} \), and aggregate outcomes such that:

- Given \( r_t, w_t, r^d_t, R_t \), allocation \( c_t, k_t, M_t, x_t, f_t \) solves the households' problem;
- Given \( r_t, w_t, r^d_t, R^f_t \), allocation \( l^f_t, s^f_t, f_t, d_t \) solves the banks' problem;
- Given \( r_t, w_t \), allocation \( l^g_t, s^g_t, y_t \) solves the firms' problem; and,
- The goods, labour, credit and money markets are clear.

Nominal variables are divided by \( P_t \) in order to transform into a stationary model.

2.4. Calibration

The procedure of calibrating the model's deep structure parameters is mapping the model economy into observed features of the data. This means that the value of variables particular for the great ratios in the model, which are implied by the deep structure parameters, can be observed from the real economy. Table 2.3 summarises the value of base line deep structure parameters, which are implied by post war U.S data. Compare with Cooley and Hansen (1995), who had a quarterly depreciation rate \( \delta \) equal to 0.019, the data from Gomme and Rupert (2007), with duration from 1959 Q1 to 2004 Q2, implies a quarterly depreciation rate 0.024 and investment-output ratio 0.26. With a given depreciation rate and investment output ratio then the capital-output and consumption-output ratios will be 10.8 and 0.74 at steady-state. Furthermore, there are two sources of households' income, which are: labour and capital incomes. The shares of capital and labour income are calibrated by using post-war U.S data from 1959 Q1 to 2004 Q2. It indicates that the share of wage income is equal to 0.6, which means that \( \alpha = 0.4 \). This is consistent with the findings of Cooley and Hansen (1995). The steady-state working hours from the goods producing sector and leisure implied by post-war U.S. data are 1/3 and 2/3, respectively. This requires deep structure parameters \( \Psi \) equal
to 1.6.

With a given capital share, depreciation rate, and investment-output ratio then the steady-state real interest rate is equal to 0.013 after depreciation. This further implies that the time preference parameter $\beta$ is equal to 0.987 through steady-state Euler's relation. The money supply rule implies that quarterly inflation is equal to the money growth rate at steady-state. The U.S data from 1959 Q1 to 2004 Q2 indicates the fraction of cash purchase consumption equal to 0.67, and the M1-output is 0.5. Since the quarterly deposit rate is equal to the real rate at stationary-state, the marginal cost of deposit equation at stationary-state implies that the sum of labour and capital shares in banks is 0.16. The model assumes that the ratio of labour and capital cost is the same across sectors, which are equal to $3/2$. This indicates that the shares of labour and capital stocks in the banking sector are 0.096 and 0.064.

Table 2.3 also concludes the behaviour of technology, credit and monetary shocks. Following the standard RBC framework, the steady-state technology shock is normalized equal to one. Autoregressive process and variation of technology shock follows Cooley and Hansen (1995), which has a persistence parameter of 0.95 and variance of structure shock of 0.7%. The model assumes the symmetric process of credit shock to technology shock. Cooley and Hansen (1995), and Benk, Gillman and Kejak (2005), estimated that the money supply equation had a quarterly steady state money growth rate 1.3% and 1.23% and auto correlation parameter 0.49 and 0.58, respectively. The persistence and variance of the money growth rate will be estimated from following the M1 money growth rate regression which has been employed by Cooley and Hansen (1995), and Benk, Gillman and Kejak (2005), with time duration from 1959 Q1 to 2004 Q2. This indicates 1.2% quarterly money growth rate at steady state with persistence 0.64, and the variance of monetary shock from regression is 0.9%. This is close to the number of Cooley and Hansen (1995), and Benk, Gillman and Kejak (2005), which are 0.89% and 1%, respectively.

$$
\Delta \log M_t = 0.0045 + 0.64\Delta \log M_{t-1} \quad \sigma_m = 0.9\% \\
(0.0009) (0.0545)
$$

47
Table 2.3: Baseline parameters

<table>
<thead>
<tr>
<th>Preferences</th>
</tr>
</thead>
</table>
| $\beta$ | 0.987  
| $\psi$ | 1.6  
| **Goods Production** |  
| $\alpha$ | 0.4  
| $\delta$ | 0.024  
| $e^z$ | 1  
| **Banking sector** |  
| $\gamma_1$ | 0.096  
| $\gamma_2$ | 0.064  
| **Monetary authority** |  
| $\Theta^*$ | 1.2%  
| **Shock's processes** |  
| **Autocorrelation parameters** |  
| $\rho_z$ | 0.95  
| $\rho_q$ | 0.95  
| $\rho_m$ | 0.64  
| **Standard Deviation of Shock Innovations** |  
| $\sigma_z$ | 0.7%  
| $\sigma_q$ | 0.7%  
| $\sigma_m$ | 0.9%  

Discount factor  
Leisure weight  
Capital share in goods sector  
Capital stock depreciation rate  
Good sector productivity parameter  
Labour share in credit production  
Capital share in credit production  
Quarterly money growth rate  
Goods sector productivity  
Banking productivity  
Money growth rate  
Goods sector productivity  
Banking productivity  
Money growth rate
Table 2.4: Target values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{ss}$</td>
<td>1.2%</td>
<td>Quarterly inflation rate</td>
</tr>
<tr>
<td>$i^{ss}/y^{ss}$</td>
<td>0.26</td>
<td>Investment-output ratio</td>
</tr>
<tr>
<td>$k^{ss}/y^{ss}$</td>
<td>10.7</td>
<td>Capital-output ratio</td>
</tr>
<tr>
<td>$x^{ss}$</td>
<td>2/3</td>
<td>Leisure</td>
</tr>
<tr>
<td>$m^{ss}/c^{ss}$</td>
<td>0.67</td>
<td>Money-consumption ratio</td>
</tr>
<tr>
<td>$m^{ss}/y^{ss}$</td>
<td>0.5</td>
<td>Money-output ratio</td>
</tr>
</tbody>
</table>

2.5 Fisher and the Long Run Effectiveness of Monetary Policy

This section of the chapter discusses the monetary properties of the model at a stationary-state with varying money growth rate and evaluates the real effects of monetary aggregates. Table 2.5 concludes that a permanent increase in the money growth rate has positive effects on the nominal interest rate and negative effects on real economic activity. The money supply rule indicates that inflation is equal to the money growth rate at a stationary-state. This reflects the long run relationship between inflation and money growth rate. The stationary-state Fisher relation which is represented by equation (33) implies that the money growth rate has a positive effect on nominal interest rate through the rate of inflation. This reflects the expected inflation effect on the nominal interest rate. This is consistent the work of Monnet and Weber (2001), which anticipated that the money growth rate would only have an expected inflation effect and would not include a liquidity effect. With a given fraction of cash and credit purchase goods at a stationary state, the demand for money and exchange credit ratio which has been indicated by equation (34) rises with the rate of inflation.

\[
R^{ss} = \frac{\pi^{ss}}{\beta} \tag{33}
\]

\[
\frac{m^{ss}}{f^{ss}} = \frac{c^{ss}}{1 - c^{ss}} \pi^{ss} \tag{34}
\]

With a given proportion of exchange credit purchase goods, equation (35) implies that
there is a positive relationship between exchange credit price and inflation rate, and that
the deposit rate is moving with the exchange credit price at stationary-state. Therefore,
with a given fraction of cash and purchase goods at a stationary-state, the increase in the
money growth rate will raise the inflation rate, the nominal interest rate, the exchange
credit price, the deposit rate, and the money-credit ratio.

\[ R_{f}^{ss} = \frac{\pi^{ss}}{\beta} (1 + \alpha^{ss} (\pi^{ss} - 1)) \]  

(35)

The equation (35') is obtained by combining the goods and capital markets CIA
constraint. It indicates that the number of government bonds at a stationary-state is
determined by the velocity, or the fraction, of exchange credit purchase goods. With a
government budget constraint, the deficit-output ratio is equal to the net bond-output
ratio at a steady-state.

\[ \frac{b^{ss}}{c^{ss} + \Omega^{ss}} = (1 - a^{ss}) \]  

(35')

For the real effectiveness of the money growth rate at a stationary-state, \( \Omega = 0 \) implies
that only consumption goods are purchased using exchange technology. The Euler
relation at stationary-state which is represented by equation (36) indicates that the real
rate is independent of the money growth rate. This further implies that the real wage and
great ratios (such as investment-output ratio and capital-output ratio) are independent of
the stationary-state money growth rate. The consumption-leisure substitution which is
represented by equation (38) is affected by the money growth rate through the nominal
interest rate and the real deposit rate. Clearly, an increase in the nominal interest rate
and deposit rate has a negative and positive effect on leisure-consumption substitution,
respectively. Since the nominal interest rate dominates the deposit rate effects,
increasing the money growth rate raises leisure and lowers consumption through intra-
temporal substitution. Equations (39) and (40) imply that the banking-goods sector
labour and leisure-labour ratios increase with the money growth rate through an
expected inflation effect. This leads to a decrease in goods sector labour with an
increase in the money growth rate. Since the ratio of real price is independent of the
money growth rate, equation (41) states that the capital and labour ratio does not vary
with monetary aggregates. With lower labour supply, this implies that the capital stocks
decrease and investment is lowered through the law of motion equation. The decrease in employment and capital stocks is a lower aggregate output through the production function. Since the consumption-output ratio does not vary with the monetary aggregate at steady-state, this indicates that consumption decreases with an increase in the money growth rate.

\[
\alpha \frac{y^{ss}}{(s,k)^{ss}} = r^{ss} = \frac{1}{\beta} - 1 + \delta \tag{36}
\]

\[
w^{ss} = (1 - \alpha)\left(\frac{x^{ss}}{\alpha}\right)^{\alpha-1} \tag{37}
\]

\[
\frac{x^{ss}}{\Psi c^{ss}} = \frac{R^{ss} - r^{ss}}{w^{ss}} \tag{38}
\]

\[
\frac{x^{ss}}{l^{ss}} = \frac{c^{ss} \Psi (R^{ss} - r^{ss})}{y^{ss}} \frac{1 - \alpha}{(1 - \alpha)} \tag{39}
\]

\[
\frac{l^{ss}}{l^{ss}} = \frac{R^{ss} \gamma_1 f^{ss}}{(1 - \alpha) y^{ss}} \tag{40}
\]

\[
\frac{w^{ss}}{r^{ss}} = \frac{1 - \alpha}{\alpha} \frac{k^{ss}}{l^{ss}} \tag{41}
\]

In a word, with a standard CIA constraint, the model economy generates a negative effect of monetary aggregates on the level of real activity through an expected inflation effect. The increase in the money growth rate is lower than the level of real economic activities (such as aggregate output, consumption, investment, and employment).

When \( \Omega = 1 \), it indicates that both consumption and investment goods are purchased by exchange technology at the goods market. The Euler relation at steady-state which is represented by equation (42) indicates that there is a positive effect of the money growth rate on the real rate. This implies that the real wage has a negative correlation with the money growth rate. The consumption-leisure substitution is affected by the money growth rate through real wage, and the leisure-labour substitution moves with the consumption-output ratio. Therefore, besides the nominal interest rate and the deposit rate, the Stockman constraint introduces the third monetary transmission channel through the real prices (such as the real interest rate and real wage). The increasing
money growth rate has a positive effect on the leisure-consumption substitution since the real wage decreases. This indicates that the changes in the real prices have a negative effect on real activity. Therefore, with given steady state money growth rate, the model with stockman CIA constraint generates lower level of real economic activities compare with standard constraint. With the Stockman exchange technology constraint, equation (41) reflects the Stockman (1981) argument that a lower capital-labour ratio increases the money growth rate. This is the opposite of the Tobin effect (1965), where increases in the steady-state money growth rate will also increase the capital-labour ratio.

\[
\alpha - \frac{y^{ss}}{(s, k)^{ss}} = r^{ss} = \left(\frac{1}{\beta} - 1 + \delta \right) (R^{ss} - r_{ss}^{d})
\]  

Table 2.5: Long run effects of the money growth rate

<table>
<thead>
<tr>
<th></th>
<th>Model with standard constraint</th>
<th>Model with Stockman constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Theta^* = 1.2%$</td>
<td>$\Theta^* = 2%$</td>
</tr>
<tr>
<td>$R^{ss}$</td>
<td>1.0253</td>
<td>1.0334</td>
</tr>
<tr>
<td>$m^{ss}$</td>
<td>0.8163</td>
<td>0.8217</td>
</tr>
<tr>
<td>$f^{ss}$</td>
<td>0.3973</td>
<td>0.3968</td>
</tr>
<tr>
<td>$x^{ss}$</td>
<td>0.6661</td>
<td>0.6661</td>
</tr>
<tr>
<td>$c^{ss}$</td>
<td>1.2039</td>
<td>1.2024</td>
</tr>
<tr>
<td>$i^{ss}$</td>
<td>0.4203</td>
<td>0.42</td>
</tr>
<tr>
<td>$y^{ss}$</td>
<td>1.6241</td>
<td>1.6228</td>
</tr>
</tbody>
</table>

In other words, the steady state of the model's economy indicates that the increase in the money growth rate has a positive effect on inflation and the nominal interest rate, but has a negative effect on real activities (such as aggregate output, consumption, and investment). This increases the nominal variables and lowers the level of real economic activities. Table 2.5 above indicates that leisure does not vary with the money growth rate at a steady-state. This can be considered as the long run aggregate supply curve
relation in the economy.

2.6 The Model's Dynamic and Simulations

This section of the chapter discusses and evaluates the monetary transmissions and the real impacts of the money growth rate on real economic activity. There are two key findings, which have been summarised as follows. Firstly, it generates a decrease in the nominal interest rate with monetary expansion, which is a liquidity effect on nominal interest rate through the interaction between bank produce exchange credit and government bond at the capital market. Secondly, it does not request the cost channel of monetary policy to generate the effectiveness of monetary aggregates since the liquidity effect on the nominal interest rate affects real activity through the intra-temporal substitution.

2.6.1 Explaining the Liquidity Effect Puzzle

Because the model assumes that households cannot choose their cash and exchange credit portfolio before monetary shock occurs, and that the money injections are received by banks instead of households, it creates an additional constraint which is the capital market CIA constraint in the economy. The representative agents maximise their utility subject to the lifetime, goods, and capital market CIA constraint. Where \( b_t \) denotes the number of government bonds.

\[
\text{Max} E_0 \sum_{t=0}^\infty \left\{ \beta' (\ln c_t + \Psi \ln x_t) + \left[ \lambda_t \left( \frac{m_{t+1}}{\pi_t} + r_f k_{t-1} + w_t (1 - x_t) + r^d c_t - m_t \right) \right] + \eta_t (b_t - f_t - T_t) \right\} 
\]
Where \( \eta_t \) and \( \mu^g_t \) represent the shadow prices of capital and goods market CIA constraints, respectively. The first order conditions of representative agents are given by equations (43)-(45). By substituting equation (45) into (44), and combining it with equation (45), the Fisher relations of nominal interest rate are represented by equation (46). Clearly, the liquidity effect on the nominal interest rate depends on the term. If the negative effect on nominal interest rate from \( \eta_t - \mu^g_t \) dominates the expected inflation effect, then the nominal interest rate is decreased with money injections. If the expected inflation effect on nominal interest rate dominates the negative effect from \( \eta_t - \mu^g_t \), then the nominal interest rate is increased with monetary expansion.

\[
R_t = \frac{(\eta_t - \mu^g_t) + (\lambda_t + \mu^g_t)}{\beta E_t \left( \frac{\lambda_{t+1} + \mu^g_{t+1}}{\pi_{t+1}} \right)}
\]  

(46)

Since the model employs the limited participation monetary shock from Lucas (1990), the representative agents choose the number of government bonds subject to monetary innovations, instead of choosing the amount of exchange credit. If households are able to choose the amount of exchange credit subject to monetary shocks, then equation (47) implies that the liquidity effect term in the nominal interest rate is disappeared because households are able to re-allocate their exchange technology portfolio.

\[
f_t : \eta_t = \mu^g_t
\]  

(47)

Figure 2.5 reflects the liquidity effect on the nominal interest rate with and without the Stockman CIA constraint. First of all, the monetary innovations have a positive effect on rate of inflation through the money supply function. The increase in the rate of inflation with money injections leads to a positive response of the exchange credit price to monetary shock, since it only includes the inflation effect of the money growth rate. The
model assumes that households choose their cash and exchange credit portfolio before recognising the monetary shock, and that the money injections are received by banks instead of households who use it to purchase government bonds at the capital market. This indicates that the supply of government bonds does not change with money injections, and the demand for government bonds is increased with monetary expansion. Since the size of the capital market is pre-determined by the cost of exchange credit, the money injections lower the nominal interest rate through increasing the demand for government bonds. Therefore, the model is able to generate the liquidity effect on the nominal interest rate with and without the Stockman exchange technology constraint.

*Figure 2.5: The impulse response of the nominal rate to a 1% positive monetary shock* ($\rho_m = 0.64$)
Figure 2.6: The impulse response of nominal rate to a 1% positive monetary shock with different persistence of money growth rate when $\Omega = 0$

Figure 2.7: The impulse response of nominal rate to a 1% positive monetary shock with different persistence of money growth rate when $\Omega = 1$
Figures 2.6 and 2.7 represent the liquidity effect on the nominal interest rate, with the difference degree of persistence on the money growth rate. Clearly, the lower persistence of money growth rate implies that there will be a smaller expected inflation effect on nominal interest rate, and it generates a stronger liquidity effect. The expected inflation effect dominates the liquidity effect and increases the nominal interest rate when the persistence of money growth rate is 0.7 and 0.8 for the economy with standard constraint and Stockman constraint, respectively. Therefore, the degree of persistence of the money growth rate is the key for generating the liquidity effect on the nominal interest rate. The more the persistence on money growth rate is then the stronger the expected inflation effect will be, it also creates a weaker liquidity effect on nominal interest rates.

2.6.2 The Effectiveness of Monetary Policy

Walsh (2003) has argued that limited participation CIA models have to be evaluated from other model's implications to discuss whether a lower nominal interest rate is an important monetary transmission to real activity. Dotsey and Ireland (1995) have found that the interest rate effects of limited participation models do not account for actually observed in the data. Christiano, Eichenbaum, and Evans (1997) argued that, due to absence of labour market frictions in the limited participation models, it requests high labour supply elasticity and mark-up to match the evidence of the effects of monetary shocks on prices, output, real wages and profits. Furthermore, James M Nason and Timothy Cogley (1994) evaluated the limited participation economy with a structural VAR (SVAR) model and concluded that the predictions of models for real side variables are rejected, but that there is evidence that nominal side predictions of the models are not rejected. King and Watson (1996) have also found that in their version of the limited participation model, monetary shock may not have a significant contribution to business fluctuations. Therefore, this section of the chapter will discuss the real impacts of monetary aggregates through the liquidity effect on nominal interest rate. Since the
nominal interest rate affects real activity through intra-temporal substitution, it indicates that the liquidity effect on nominal interest rate has a positive effect on real activity.

Figure 2.8 indicates the impulse responses of real variables to 1% positive monetary shock in the economy, both with and without the Stockman constraint. Clearly, the liquidity effect on the nominal interest rate increases consumption, employment and decreases leisure through intra-temporal substitution. For the economy with a standard CIA constraint, the investment has a negative response to monetary shock since the consumption and investments are substitution goods. Increasing consumption with monetary innovation has implicated the lower investment with monetary expansion and it is decreasing the capital stocks through the law of motion equation. The aggregate output has a negative response to monetary shock since the positive effect from increasing consumption is dominated by the negative effect from decreasing investment. In other words, although the model is able to generate the liquidity effect on nominal interest rate, it is not able to replicate all responses of real activities (such as output and investment) subject to monetary expansion by employing a standard CIA constraint.

Figure 2.8: The impulse response of real variables to a 1% positive monetary shock
In the case of a model economy with a Stockman (1981) CIA constraint, Figure 2.8 concludes that the real activities have a positive response to monetary shock. First of all, the lower nominal interest rate is increasing consumption through intra-temporal substitution and has a negative effect on the current consumption through the discount factor of the Euler relation. With the trade off between the liquidity effect on consumption through inter-temporal and inter-temporal substitutions, the model is able to generate the hump shape response of consumption subject to monetary shock. Since the liquidity effect on the nominal interest rate decreases leisure and increases labour supply, it leads to an aggregate output which has a positive response (subject to monetary expansion) with given initial capital stocks. Since the Stockman exchange technology constraint indicates that output and exchange technologies (cash and exchange credit) are complementary goods. The raising aggregate output is increasing the exchange technologies. The raise in investment and capital stocks with monetary expansion due to output or exchange technology increase more than consumption. In other words, the model with a Stockman constraint not only generates the liquidity effect on the nominal interest rate but also replicates the positive response of real economic activities, subject to money injections, without the cost channel of money growth rate. In particular, it generates a hump-shape response of consumption subject to monetary expansion.

In conclusion, by employing a limited participation monetary shock and assuming that money injections are received by banks instead of households, the model is able to generate the liquidity effect on the nominal interest rate. The degree of liquidity effect on the nominal interest rate depends on the persistence of the money growth rate. The model with Stockman exchange technology constraint is able to generate the liquidity effect on the nominal interest rate, and can also examine the positive response of real activity, subject to monetary expansion, without the cost channel of monetary policy. The key failure of the model is that monetary shock has large impact on rate of inflation, in contrast to the empirical VAR approach which has founded that price is gradually increased after monetary expansion. Therefore, the model fails to replicate the response of inflation subject to monetary shock.
2.6.3 Other Shocks

Figures 2.9 and 2.10 represent the impulse responses of variables to technology and credit shocks. For the economy with a standard constraint, the technology shock is an increase in the real economic activity, and has a small positive effect on the nominal interest rate due to the real interest rate effect which dominates the inflation effect through Fisher’s relation. The credit shock is an increase in consumption and lower investment, which reflects the substitution of goods between consumption and investment. This generates the positive responses of output, employment and a negative response of the nominal interest rate subject to credit shock. For the economy with a Stockman constraint, a technology shock increases real economic activity and has a stronger positive effect on the nominal interest rate. At the beginning, the credit shock is an increase in investment and a decrease in consumption, which increases after couple of quarters since the investment decreases faster than the output.

Figure 2.9: Variables impulse response to a 1% technology shock
For the behaviour of the inflation rate, the technology shock is decrease the rate of inflation, which reflects the counter cyclical behaviour of the price level. This has been found in both RBC model and the post war U.S data. There is positive response of inflation subject to credit shock. This is due to the exchange technology rise more than aggregate output. It drives up the good market price, which is reflected on rate of inflation.

2.6.4 Model Simulation and Business Cycle Facts

The model economy is also quantitatively evaluated by comparing its implications of business cycle facts with observed U.S data. The finding is that the model is able to capture many of important features of U.S economy, such as the volatility of nominal interest rate and monetary aggregate; the pro-cyclical behaviour of inflation, velocity and nominal interest rate; the negative correlation between money growth rate and
nominal interest rate. Table 2.6 describes the cyclical behaviours of the U.S economy, the data is obtained from the detrended HP time series data with duration from 1959 Q1 to 2004 Q2. It summarises the relative standard deviation of macroeconomic aggregates to output, and concludes the correlations of real and nominal variables with real GDP. According to the U.S data, both consumption and capital stock has nearly half the volatility of aggregate output. Investment has about three times the volatility of output. Labour supply has a similar volatility with output. And nominal variables have less volatility with output, except monetary aggregates (M1) and income velocity. The monetary aggregates, inflation, income velocity, and nominal interest rate have positive correlations, or pro-cyclical behaviour, with aggregate output. The money growth rate and price level has negative correlations, or counter-cyclical behaviour, with aggregate output. There is a negative correlation between money growth rate and real activity; such as, output, consumption, and investment and working hours. The negative correlation between the money growth rate and the nominal interest rate reflects the liquidity effect on the nominal interest rate.

**Table 2.6: The model simulation and business cycle facts**

<table>
<thead>
<tr>
<th></th>
<th>Relative SD (%)</th>
<th>Correlations with aggregate output</th>
<th>Correlations with M1 growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Ω=0</td>
<td>Ω=1</td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.52</td>
<td>0.52</td>
<td>0.31</td>
</tr>
<tr>
<td>Investment</td>
<td>2.71</td>
<td>3.37</td>
<td>3.16</td>
</tr>
<tr>
<td>Labour</td>
<td>1.00</td>
<td>0.49</td>
<td>0.62</td>
</tr>
<tr>
<td>M1</td>
<td>1.42</td>
<td>1.82</td>
<td>1.79</td>
</tr>
<tr>
<td>Treasury rate</td>
<td>0.18</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>Price</td>
<td>0.77</td>
<td>2.60</td>
<td>3.07</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.29</td>
<td>1.96</td>
<td>2.61</td>
</tr>
<tr>
<td>Velocity</td>
<td>1.70</td>
<td>1.60</td>
<td>2.33</td>
</tr>
<tr>
<td>M1 rate</td>
<td>0.62</td>
<td>0.76</td>
<td>0.76</td>
</tr>
</tbody>
</table>
Table 2.6 also includes the simulated economy statistics with technology, credit and monetary shocks. The statistics of this simulation are computed from an artificial time series which consists of 182 periods, with 50,000 times. In order to compare with business cycle facts, it has been taken, logged, and detrended by a Hodrick-Prescott filter. Clearly, the model economy is able to examine the liquidity effect puzzle since it generates a negative correlation between money growth rate and nominal interest rate. It also generates the pro-cyclical behaviours of the nominal interest rate and income velocity, and the variation of monetary aggregates. When the economy has standard CIA constraint, it is not able to explain the variation of nominal interest rate and also cannot generate the pro-cyclical behaviour of monetary aggregates. Although it can examine the counter-cyclical behaviour of price level, it does not replicate the pro-cyclical behaviour of inflation simultaneously. By replacing the standard constraint with the Stockman CIA constraint, the model economy simultaneously explains the pro-cyclical behaviour of inflation and the counter-cyclical behaviour of price level. It also examines the variation of nominal interest rate and the pro-cyclical behaviour of monetary aggregates. The weakness of the model economy is that it cannot explain the variation of price level and rate of inflation.

2.7 Liquidity effect with price stickiness

Most of researchers have looked at the output and price effects of the monetary policy with price stickiness. The further evaluation on the sticky price model is not only the output and price effects of monetary policy, but also the liquidity effect on nominal interest rate. This part of chapter discusses the ability of sticky price model to generate the liquidity effect in both traditional IS-LM model and the RBC framework. Under traditional IS-LM model, the conventional money demand function is represented by equation (48).

\[
\ln\left(\frac{M_t}{P_t}\right) = \beta_0 + \beta_1 \ln(y_t) + \beta_2 \ln(R_t) + \epsilon_t^n
\]  

(48)

Where \( M_t \) represents nominal money demand; \( P_t \) is the pre-determined price level.
(price is stickiness); \(y_t\) stands for real aggregate output; \(R_t\) is nominal interest rate. Assuming nominal money supply which controlled by central bank is increasing 2 percent, since the price level is pre-determined and the nominal money demand equals nominal money supply at money market in equilibrium, it leads real money balance increase 2 percent with monetary expansion. If the real income rises less than 2 percent in response to the exogenous increase in nominal money supply, the nominal interest rate has to be decreased to equilibrate money supply and demand at money market. It is generating the liquidity effect. It is not difficult to generate a liquidity effect in traditional IS-LM model with sticky price since the empirical estimates of income elasticity is less than one, which reflects the changing in real income less than the exogenous change in real money balance.

The further research is continues to ask that can sticky price models generate liquidity effect with explicit inter-temporal substitution within the RBC framework. Ohanian and Stockman (1995) discussed one sector sticky price RBC model with CIA constraint and concluded that the nominal interest rate falls only in sticky price model when the decline in the real interest rate\(^{11}\) is large enough to offset the increasing in expected inflation. With separable utility preference, the sticky price model interprets the liquidity effect as the trade off between negative response of consumption growth rate (lower real interest rate) and positive response of expected inflation to monetary expansion. Therefore, the liquidity effect will be depended on the risk aversion parameter in the households’ utility function since it is able to affect the consumption growth rate through consumption inter-temporal substitution equation. The general form of inter-temporal substitution equation under monetary RBC economy has been represented by equation (49).

\[
R_t = E_t \pi_{t+1} \pm U_{c,t} - U_{c,t+1}
\]  \hspace{1cm} (49)

Where \(E_t \pi_{t+1}\) represents expected inflation; \(U_{c,t}\) is the marginal consumption of utility at period \(t\); \(U_{c,t+1}\) is the marginal consumption of utility at period \(t+1\); \(R_t\) is nominal interest rate. Clearly, according to equation (49), the behaviour of nominal interest rate

\(^{11}\) It reflects the negative consumption growth subject to monetary expansion.
subject to monetary shock is affected by the expected inflation effect and the inter-temporal substitution between marginal utility of consumption. In order to generate liquidity effect, the positive response of expected inflation has to be dominated by negative response of real interest rate subject to monetary expansion.

Javier Andres et al (2002) found that with separable preferences, which indicate the marginal utility of consumption is only driven by the dynamics of consumption, a positive monetary shock induces a fall in nominal interest rate only if the degree of inter-temporal substitution is low enough or the risk aversion parameter is high enough. They concluded that without capital stock and with separable preferences, the stick price model has to take a high risk aversion parameter (about 8-10) for decreasing the nominal interest rate with monetary expansion. They also found that the non-separable preferences with no capital accumulation, the sticky price model cannot produce a liquidity effect even with varies value of the risk aversion parameters since the consumption and labour movements induce opposite effects on the marginal utility of consumption.

Keen (2004) investigated liquidity effect in sticky price model without capital adjustment costs, which similar to Ireland (1997) and Kiley (2002) models and found that both nominal and real interest rates are increase with monetary expansion. He further argued that with the absence of other frictions, sticky price model itself cannot produce the liquidity effect. Kimball (1995) and Ireland (2001) included capital adjustment costs, which reduce the elasticity of investment demand to the real interest rate and lower the increase in investment demand caused by monetary shock into sticky price model to assist to generate the liquidity effect since the negative effect of real interest rate is able to dominate positive effect of expected inflation in model’s inter-temporal substitution equation.

Alternative approach to obtain the liquidity effect in sticky price model is introduced by Ohanian and Stockman (1995). They extended one sector sticky price model with two consumption goods X and Y. And argued that the price for consumption good X is
flexible, the price for consumption good Y is pre-determined (can be viewed as sticky price). With absence the capital stock, the model is able to generate the liquidity effect with small fraction of consumption good Y since the anticipated inflation effect getting smaller with the small degrees of price stickiness.

In conclusion, the limited participation models generate a liquidity effect by allowing restrictions on the adjustment of agents’ consumption-saving portfolios which break down the inter-temporal allocation of consumption (Fuerst, 1992; Christiano et al, 1997). The sticky price model interprets the liquidity effects as results of sluggish nominal price adjustment. The gradually increase in price level reflects increasing in expected inflation, to account for the liquidity effect, the standard sticky price model either generate a decline in the real interest rate large enough to offset the positive inflation expectations (such as Ireland 2001) or introduce the degree of stickiness to reduce the positive expected inflation effect subject to monetary shock (such as Ohanian and Stockman (1995)).

2.8 Policy Implications and Discussion

Limited participation monetary shock and functions of financial intermediates in a Cash-in-Advance (CIA) framework was developed by Lucas (1990) and Fuerst (1992). Both argued that the Fisherian fundamental includes an extra term that has been called a liquidity effect on nominal interest rate, which can be either positive or negative depending on the difference of value of cash in the goods and credit market. If the credit market is relatively liquid then the value of cash in the credit market will be less than the value of cash in the goods market. In this case the liquidity effect is negative and it leads to a nominal interest rate which is lower relative to Fisherian fundamentals. When the goods market is relatively liquid then the value of cash in the goods market is less than the value of the credit market. In this case the liquidity effect is positive and it leads to nominal interest rate which is higher relative to Fisherian fundamentals.

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12 This reflects the degree of price stickiness in the model
This chapter employs a monetary banking model with limited participated monetary shock from Lucas (1990) to force households who cannot adjust their cash-credit holdings to monetary innovations, and it assumes that money injections are received by banks. Since the government bonds are the only asset which has been traded in the capital market, and the supply of government bonds is pre-determined by the number of exchange credits held by households for goods market transaction before recognising monetary shock, and the size of the capital market is determined by the cost of exchange credit. The model assumes that banks have to demand government bonds for each exchange credit which has been produced, with receiving the money injections from a monetary authority. The increasing money supply raises the demand for government bonds through banks, increases the price of bonds, and lowers the return on bonds.

Furthermore, Fuerst (1992) employed a limited participation monetary shock from Lucas (1990), with a cost channel of money injections, to generate the real effect of money growth rate. Since only investors, and not shoppers, can have direct access to money injections, monetary injections lower the nominal interest rates. By assuming that firms are the only borrower at the capital market, money injections increase current and future real activities through firms' borrowing equation (which has been considered as a cost channel of monetary policy). In contrast to Lucas-Fuerst's (1992) type of limited participation models, this model does not include a cost channel of monetary aggregates to generate real effectiveness. The lower nominal interest rate with monetary innovation has a positive effect on consumption, and a negative effect on leisure, through intra-temporal substitution between consumption and leisure. It leads to an increase in consumption and employment, subject to monetary expansion. The increase in the labour supply from households will raise the aggregate output with a given initial capital stock. This generates the real impacts of monetary aggregates without the cost channel of monetary policy.

In conclusion, the model is able to account for the effectiveness of both expected and unexpected money growth rate on nominal interest rate and real economic activities.
(such as aggregate output, consumption, and employment). Instead, by using a cost channel of monetary policy to determine the size of capital market which limited participation models have often employed, the model argues that the size of capital market has to be determined by the cost of exchange credit. It allows the model is able to generate the liquidity effect on nominal interest rate. With decreasing nominal interest rate, consumption and employment rise through intra-temporal substitution equation. By extend the model with stockman CIA constraint, aggregate output also increase with monetary expansion. Therefore, by assuming a limited participation monetary shock, the model generates a lower nominal interest rate with monetary expansion and an increase in real economic activity without a cost channel of money growth rate.

2.9 Conclusion

The monetary transmission mechanism describes how a change in monetary policy has an impact on real variables (such as output, consumption, investment and employment). This chapter extends the two exchange technologies Cash-in-Advance (CIA) model of Benk, Gillman and Kejak (2005) to explain transmissions and impacts of expected and unexpected money injections on real economic activity in the context of a DSGE framework. The model assumes that households cannot adjust their exchange technology portfolio subject to monetary innovations, and that money injections have to be received by productive banks instead of households. The model economy predicts that an unanticipated change in monetary policy would lower the nominal interest rate and have a positive effect on real activity. An anticipated changing in money supply would increase the nominal interest rate and have a negative effect on real activity. The model includes both inflation and liquidity effect of money growth rate on nominal interest rate, and explains the real effects of monetary innovations.

The classical economists have argued that an unanticipated increase in the money supply would lower the nominal interest rate, which reflects the liquidity effect of
money growth rate. They also argued that an anticipated rise in the money growth rate increases the inflation rate and produce an expected inflation effect. With the quantity theory of money, they argue that an unexpected increase in the money growth rate has positive effects on the level of real economic activity, while an expected rise in the money growth rate has negative effects on the level of real economic activity.

Lucas (1990) explained the lower nominal interest rate with monetary expansion with the timing of monetary shock. It assumed that households use cash in both the goods and capital market. The cash allocations between goods and capital market were made by households before monetary innovations occurred. Monetary expansion flows into the capital market only. It increases demand of capital market and with given or fixed supply of capital market it will increase prices and lower the return of the capital market. Fuerst (1992) further argued that money injections are received by financial intermediates and lent out as firms wage payments to generate the real effects of monetary shock. Households make savings into financial intermediates, which can be considered as capital market cash. Including money injections from monetary authority, this is can be viewed as the demand of the capital market. Demand of the capital market is determined by the real sector, which is the firms’ wage bill payment. Households cannot adjust their cash portfolio within the capital and goods market subject to monetary innovations. This will increase the demand of the capital market and have a negative effect on the returns of the capital market. It further increases the aggregate output and employment through the lower marginal cost of labour. Monnet and Weber (2001) developed a segmented market approach and argued that the inflation effect comes from anticipated money injections, in this case an unanticipated monetary innovations cause a liquidity effect.

This chapter develops a cash-in-advance economy with five representative agents, who are: households, banks, government, firms, and monetary authority. The households choose exchange credit before recognising monetary shock. Competitive banks produce exchange credit, receive money injections, and purchase government bonds. The supply of government bonds is determined by the number of exchange credits collected by
households. The demand for government bonds is determined by banks, which includes both the number of exchange credits produced and money injections. Money injections which are received by banks instead of households increase the demand for government bonds, raise prices, and lower the returns on bonds.

Therefore, this model is able to examine the decreasing nominal interest rate with an unanticipated increasing money growth rate. And it accounts for the positive responses of real activity subject to positive monetary innovation with lower nominal interest rate. In contrast to model’s dynamic, at a stationary-state, the model generates an increasing nominal interest rate with an anticipated rising money growth rate and lowers the level of real activity (such as aggregate output and consumption). The evolution of the model is that it accounts for the effect of monetary shocks on the nominal interest rate (including both the liquidity and inflation effect). The model indicates that unexpected money injections have a liquidity effect of money growth rate. Anticipated money injections have an expected inflation effect on the money growth rate. The major contribution of this model is that it not only generates the liquidity effect of the money growth rate, but it also includes the positive effect of monetary policy to real activity without a cost channel.
References


Christiano, Lawrence J (1991) “Modelling the liquidity effect of a money shock”, Quarterly review, Federal Reserve Bank of Minneapolis, 15, 3-34


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Chapter 3 - The Transmission Mechanism and the Effectiveness of Monetary Aggregates: A Banking Approach

Abstract

This chapter examines the effectiveness of monetary aggregates through various monetary transmission mechanisms by integrating the financial sector into the Cash-in-Advance (CIA) economy. The model assumes that there are two types of representative agents in the financial sector, which are: productive banks and financial intermediates. The functions of productive banks and financial intermediates follow the standard banking literature. The productive banks supply a financial service (which is an exchange technology service) to household consumers and financial intermediates receive savings fund from savers and purchase corporative bond from firms. Money injections are received by household consumers instead of the financial sector (such as banks or financial intermediates). With functions of financial intermediates and productive banks, the banking costs are increased with monetary expansions through the rate of inflation. This leads households to use more exchange credit relative to cash at the goods market. The model further assumes that the number of savings funds is equal to the number of exchange credits used at the goods market. Therefore, money injections lower the nominal interest rate on bond or saving as the saving fund increases with exchange credit. By assuming that firms are the only borrowers at the capital market, a lower nominal interest rate on the saving fund reduces the marginal cost of labour and increases labour demand. Meanwhile, the increasing marginal cost of money through the expected inflation effect has a negative effect on labour supply. Both output and employment increase with monetary expansion since the effect on labour demand

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13 The working capital monetary RBC models usually assume that money injections are received by financial intermediates instead of households, such as the model in Chapter 4.
14 In last Chapter this has been referred to as the liquidity effect on nominal interest rate.
15 Firms have to borrow working capital before any goods have been produced.
dominating the effect on labour supply,

3.1 Introduction

The positive correlations among monetary aggregates and real economic activity are a key empirical fact about the macro economy. And the decreasing nominal interest rates with monetary expansion is also an important monetary transmission mechanism in both traditional Keynesian (Tobin, 1947) and monetarist (Friedman, 1968 and Cagan, 1972) macroeconomics models. The flexible price monetary RBC models (such as that of Cooley and Hansen (1989), (1995), and (1998), Cash-in-Advance (CIA), and Gavin and Kydland's (1999) endogenous money supply models) cannot account for both the nominal interest rates and the real impacts of the money growth rate. Although Benk, Gillman, and Kejak (2005) extend the standard CIA model to endogenous velocity through the function of productive banks and emphasises the contribution of the financial shock to business fluctuations. They still cannot explain the lower nominal interest rate and the increase in real economic activity with monetary expansion.

Lucas (1990) extended the standard CIA model with limited participation monetary shock, which assumes that households make their consumption-saving decision before recognising monetary innovation. This model indicates that money injections from the monetary authority enter the capital market instead of the goods market. The model is able to generate a liquidity effect on nominal interest rate because the size of the capital market cannot increase with monetary expansion. Fuerst (1992) linked the liquidity effect on the nominal interest rate to real activity by assuming that firms' borrow the wage bill before any goods have been produced, and in so doing explained both the liquidity and real effects of the money growth rate. In other words, the Lucas-Fuerst (1992) type of limited participation CIA models have to request that households cannot adjust their consumption-saving portfolio with monetary innovations, and that firms

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16 This is due to limited participation monetary shock (1990) assumption. The size of the capital market is pre-determined by the number of the savings fund.
have to borrow wage payment before any goods been produced, in order to generate a lower nominal interest rate and raise output with the money growth rate. In contrast to the flexible price monetary model, the new Keynesian economists employed Calvo’s (1983) type of price stickiness and combined it with a DSGE framework to examine the positive response of output to money injections. However, they failed to include the negative response of the nominal interest rate to monetary expansion.

Benk, Gillman, and Kejak’s (2005) monetary model has extended standard CIA model into two exchange technologies framework with productive banks. This model argues that there are cash and exchange credits which can be held by households for goods market transactions. The exchange credit is produced by productive banks through Cobb-Douglas production function, and held by households for goods market transaction besides the real money balance. Although this model did explain the behaviour of velocity through exchange credit production functions, it fails to explain the increasing economic activity with monetary expansion.

This chapter extends Benk, Gillman, and Kejak’s (2005) monetary banking model with financial intermediates, and assumes that households make savings to financial intermediates when they collect exchange credit from productive banks. For every unit of exchange credit which has been collected by households, financial intermediates receive an equal number of savings fund and they lend to borrowers at the capital market with a positive interest rate. Following Fuerst’s (1992) assumption, firms are the only borrowers at the capital market because they have to borrow the wage payments before any goods have been produced. Money injections from the monetary authority increases the marginal cost of money and credit price. With a higher marginal cost of money, households prefer to collect more exchange credit from banks and increase the saving fund of financial intermediates. Increasing savings funds with a fixed number of demands from firms’ wage bill will lower the capital market interest rate. It reduces the marginal cost of labour and leads firms to increase their borrowing from the capital market. The increasing borrowing from the capital market by firms indicates that there is an increase in firms’ labour demand. With a given initial capital stock, increasing
labour demand will raise aggregate output through the production function.

In a word, this chapter formulates, calibrates, and simulates the dynamic stochastic general equilibrium model which incorporates the functions of productive banks and financial intermediates to investigate whether the model is able to account for the relationship between monetary aggregates and real activity under a flexible price framework. The key finding of the papers are: firstly, that it generates a decreasing nominal interest rate with an increasing money supply with an absence of limited participation monetary shocks from Lucas (1990); and secondly, by allowing firms to borrow wage bills payment from financial intermediates, the model is able to examine the positive response of aggregate output subject to monetary expansion under flexible price framework.

This chapter is organised into eight sections, the first of which is this introduction. Section 2 presents the empirical evidence of money shock on real activity and nominal interest rate. Section 3 sets-up the theoretical two exchange technologies DSGE model. Section 4 explains the model’s calibration procedure. Section 5 discusses how the model’s steady state is affected by the money growth rate. Section 6 examines the model’s dynamic and findings. Section 7 has a detailed discussion of the model properties. Section 8 will conclude the chapter.

3.2 Empirical Evidence

Leeper et al. (1996) examined 13 variables VAR model and found that after an expansionary monetary policy shock both consumption and investment rise. They, therefore, argue that any plausible model of the monetary transmission mechanism should generate a rise in output, consumption, and investment. Christiano, Eichenbaum, and Evans (CEE) (1999) employed an identify VAR model and found that aggregate output declines in response to a negative monetary policy shock. This section of the
chapter employs a 5 variables\textsuperscript{17} recursive VAR (2) model\textsuperscript{18} to examine the positive effects of monetary innovation on real economic activity. Figure 3.1 indicates that federal fund rate and real activity are affected by the innovations on money equation of VAR model for U.S economy with time period from 1959Q1 to 2004Q2. The impulse responses of variables reflect the effect of changing in money equation error in VAR model. Clearly, money equation innovation has positive effect on real activity, which includes aggregate output, consumption and investment and negative effect on nominal interest rate, which reflects the liquidity effect of money growth rate.

\textit{Figure 3.1: Impulse responses of the nominal interest rate and real activity to M1 innovation}

\textsuperscript{17} The 5 variables include real GDP, consumption, investment, federal fund rate and M1. With the exception of the federal fund rate, all variables are in log form.

\textsuperscript{18} The number of lags are selected by using Schwarz criterion
3.3 The structure of economy

This part of chapter explains the model economy and displays the problems which are solved by banks, firms and households. It also describes the behaviour of financial intermediates and monetary authority. The model includes three sources of uncertainty which are total factor productivity shocks to firms, banks and money growth rate.

Figure 3.2: The standard CIA economy with a financial sector

Figure 3.2 reflects the structure of the economy. It consists of four infinitely lived representative agents, who are: financial firms, final goods sector, household consumers, and the monetary authority. There are two types of representative agents in the financial sector: ‘financial intermediates’ and ‘productive banks’. The function of financial intermediates is to receive savings from households and to purchase
corporative bond from firms. The firms issue the corporative bond to pay wage bill in advanced. With infinitely lived representative agents, the model further assumes that agents have to make their consumption and portfolio decisions period by period basis. It is ruling out the multi-period borrowing and lending decisions. Productive banks produce an exchange credit service to households through Cobb-Douglas type of production function. Both saving funds are supplied by financial intermediates, and exchange credit is provided by productive banks, which can be considered as an intra-temporal source of finance. The model does not include any inter-temporal finance between agents. Households receive money injections from the monetary authority and labour, capital incomes from firms. The exchange technology held by households for goods market transactions includes real money balance and exchange credit. Households collect exchange credit from banks and make savings with financial intermediates. The model assumes that the number of households' savings are equal to the number of exchange credits which been collected from banks. Firms have to issue corporative bond to borrow wage payments from financial intermediates before any final goods have been produced. The price of exchange credit has to be above the nominal interest rate on savings fund to allow money to be held by households. If the price of exchange credit is equal to the nominal interest rate, then households will only hold exchange credit for goods purchase and cash will be ruled out in the economy.

Therefore, the positive monetary shock increases the rate of inflation through the money supply equation. Rising inflation will increase the marginal costs of money and exchange credit price. An increasing marginal cost of money and exchange credit price has a negative effect on real activity. It also decreases the money demand and increases the exchange credit of purchase goods. This indicates that the consumption velocity increases with the money growth rate, and has a positive effect on real activities at the goods market. This has been considered as a velocity channel of money growth rate with two exchange technologies framework. An increasing exchange credit purchase goods implies that households have to increase savings as the marginal costs of money

\[ 9 \]

\[ 19 \]

The inter-temporal finance between agents has been discussed in Bernanke, Gertler and Gilchrist (1999).

\[ 20 \]

It will be discussed in detail at chapter five.
increase. This leads to financial intermediaries receiving more saving funds from households. The fixed number of demand of saving fund from real sector indicates that the nominal interest rate has to fall in order to allow financial intermediaries to lend extra savings at the capital market. When firms issue corporative bond to borrow to pay wage bill, this can generate an increasing employment and output with monetary expansion. This has been called a cost channel of money growth rate. In other words, the model indicates employment and output are rising with unanticipated increasing money growth rate without sticky price/wage and limited participation monetary shock.

3.3.1 Productive Banks

The competitive banks in the economy follow Benk, Gillman, and Kejak’s (2005) banking sector specification. Exchange credit $f_t$ is produced by banks using Cobb-Douglas production function with constant returns-to-scale in labour $l_t'$ and households’ deposit $d_t$. This is indicated by equation (1). The shares of labour and deposit are $\gamma$ and $1-\gamma$, respectively.

$$f_t = A_t e^\delta (l_t')^\gamma d_t^{1-\gamma} \quad \text{(1)}$$

Assuming that exogenous exchange credit technology are following an AR (1) process with autoregressive parameter $\rho_q$ and structure shock $\epsilon_t^q$.

$$q_t = \rho_q q_{t-1} + \epsilon_t^q \quad \epsilon_t^q \in (0,\sigma_q^2) \quad 0 < \rho_q < 1 \quad \text{(2)}$$

Each unit of exchange credit is sold by banks at price $p_t$. The banks have to pay the wage bill $w_t l_t'$ and transfer dividends $r_t d_t$ to households. The lifetime budget constraint that has been faced by banks is represented by equation (3). The constraint indicates that households collect exchange credit from banks with labour and deposit costs. For each unit of exchange credit which is obtained by households, there is a dividend and labour income deducted from households’ aggregate income. The marginal cost of labour and deposit are indicated by equations (4) and (5), which come from maximising the exchange credit production function subject to banks’ lifetime budget.
constraint. They indicate that the shares of wage bill and dividend from banks are equal to the shares of labour and deposits in an exchange credit production function.

\[ r^d_i d_i = p_i f_i - w_i t_i \]  
\[ r^d = p_i (1 - \gamma) \frac{f_i}{d_i} \]  
\[ w_i = p_i \gamma \frac{f_i}{t_i} \]

Where \( w_i \) represents real wages and \( r^d_i \) represents dividend payment per deposit. Following Benk, Gillman and Kejak (2005) setup, the model assumes that the amount of household deposits in banks is equal to the amount of exchange technologies, which include both cash and credit which has been used in the goods market.

\[ d_i = \frac{M_{t-1}}{P_i} + f_i + T_i \]

Where \( M_{t-1} \) represents the initial nominal money stock holding, \( P_i \) stands for price level, and \( T_i \) represents money injections from monetary authority. With the relationship represented in equation (6), the deposit rate in the equation (4) can be considered as the marginal cost of exchange technology.

### 3.3.2 Financial Intermediates

In contrast to Lucas-Fuerst's (1992) limited participation CIA models, this model assumes that monetary innovations happen before households make a consumption-saving decision. It also assumes that money injections are received by households rather than financial intermediates or banks. This indicates that households are able to adjust their consumption-saving portfolio subject to money injections, such as in the standard CIA economy of Lucas (1982) and Svensson (1985). The only function of financial intermediates is to receive savings funds from households, which happen after the exchange credit has been collected, and to purchase corporative bond from firms in the capital market. The amount of savings funds from households has been supposed to be equal to the amount of exchange credit which has been collected by households from
banks. This is indicated by equation (7). Where \( b_t \) stands for the amount of bank loans to firms in the capital market.

\[
f_t = b_t
\]  
(7)

By assuming that financial intermediaries have zero profit, the income of bond is received by financial intermediaries from capital market equal to the savings payout to households. This indicates that the nominal interest rate on the saving fund, which has been denoted as \( R_t \), will be equal to the corporative bond interest rate across the equilibrium. Due to the marginal cost of money and exchange credit price, which has a positive response to monetary shock, an increasing money growth rate raises the share of exchange credit purchase goods. This leads to more exchange credit being collected by households, and increases the saving funds paid to financial intermediaries. With a positive interest rate, financial intermediaries have an incentive to invest extra savings funds to corporative bond in the capital market. If the demand for savings funds (supply of corporative bond) at the capital market has been determined by the real sector, then financial intermediaries have to lower nominal interest rate in order to lend an extra saving fund to the capital market. This leads to a decreasing nominal interest rate with money injections.

### 3.3.3 Household Consumers

Representative households maximize their expected log utility function (8) (including consumption \( c_t \), leisure \( x_t \), with a discount factor \( \beta \in (0,1) \)) and allocate their time endowment among leisure, labour in goods production sector \( l_t^g \), and labour in banking sector.

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t)
\]  
(8)

\[
1 = x_t + l_t^g + l_t^f
\]  
(9)

Aggregate output \( y_t \) includes consumption and investment \( i_t \) goods and is produced by firms. The next period’s physical capital stocks \( k_t \) have been accumulated through the
law of motion equation (11), with quarterly deprecation rate $\delta$.

$y_t = c_t + i_t$  \hspace{1cm} (10)

$i_t = k_t - k_{t-1}(1 - \delta)$ \hspace{1cm} (11)

Goods market exchange technology constraint has been represented by equation (12), which implies that households can either choose cash or exchange credit to purchase consumption goods. The amount of cash and exchange credit which is held by households for the goods market has to depend on the marginal cost of money and exchange credit. Equation (13) represents the amount of cash purchase consumption.

Where $a_t$ denotes the fraction of cash purchase goods. Therefore, with quantity theory of money, the fraction of exchange credit purchase consumption can be considered as velocity of money growth rate.

$\frac{M_{t-1} + T_c}{P_t} + f_t = c_t$ \hspace{1cm} (12)

$\frac{M_{t-1} + T_l}{P_t} = a_t c_t$ \hspace{1cm} (13)

The households receive labour $w_t l_t^p$ and capital $r_t k_{t-1}$ income from goods producing firms; obtain dividend $r_t d_t$ and labour $w_t l_t^f$ income from production banks; receive net saving $R_t f_t$ from financial intermediate. The aggregate income of households will be spent on consumption, investment and exchange credit. Equation (14) represents the next period's money holding for households.

$\frac{M_t}{P_t} = \frac{M_{t-1} + T_c}{P_t} + r_t d_t + w_t (1 - x_t) + r_t k_{t-1} + R_t f_t - c_t - p_t f_t - k_t + (1 - \delta) k_{t-1}$ \hspace{1cm} (14)

Equilibrium conditions of households is represented by equations (15)-(19), which come from households maximising the expected log utility function subject to lifetime and goods market CIA constraint.

$\frac{\mu_t}{\lambda_t} = p_t^f - R_t$ \hspace{1cm} (15)

$1 + \frac{\mu_t}{\lambda_t} = R_t^e$ \hspace{1cm} (16)

85
Where $\lambda_t$ and $\mu_t$ represents the shadow prices of lifetime and goods market CIA constraint; $R_t^e$ stands for the marginal cost of money or credit. Equation (15) indicates that the marginal cost of holding money has to be equal to the marginal cost of exchange credit. The marginal cost of money can be explained by the relative shadow prices between good market CIA constraint and lifetime budget constraint, which is indicated by equation (16). The marginal cost of credit is the difference between the unit exchange credit price $p_t^e$, and the return from saving $R_t$, due to the exchange credit being used. The marginal utility of consumption has to be influenced by the marginal cost of money, the shadow price of lifetime budget constraint, and the deposit rate, or marginal cost of exchange technology. The substitution between marginal utility of consumption and leisure is affected by the marginal cost of money, the deposit rate and the real wage, and is represented by equation (17). Equation (19) represents the standard RBC type of Euler equation. When combined with equation (18), it indicates that the standard Fisher relation, which is the marginal cost of money, is equal to the real interest rate plus the expected rate of inflation.

\[
\frac{\Delta x_t}{\Psi C_t} = \frac{R_t^e - r_t^d}{w_t} 
\]

(17)

\[
\beta E_i \left( \frac{p_t R_{t+1}}{P_{t+1} \lambda_{t+1}} \frac{\lambda_t}{\lambda_t} \right) = 1 
\]

(18)

\[
\beta E_i \left( \frac{\lambda_{t+1}}{\lambda_t} (r_{t+1} + 1 - \delta) \right) = 1 
\]

(19)

According to equation (13), the equation (4) can be re-written as the equation (20). It indicates the deposit rate is influenced by the fraction of exchange credit goods and credit price. This implies that there is a velocity effect of money growth rate on marginal utility of consumption and leisure substitution. Furthermore, according to firms' labour costs equation, the real wage is a negative correlation with the nominal interest rate at the capital market. This means that the nominal interest rate at the capital market is able to affect consumption-leisure substitution through the real wage.
3.3.4 Firms

Aggregate output is produced by representative firms using the Cobb-Douglas production function, which includes exogenous technology \( e^z \), capital stocks, and labour. The shares of capital stocks and labour are \( \alpha \) and \( 1-\alpha \), respectively.

\[
y_i = e^z (I^g_i)^{1-\alpha} k_{t-1}^\alpha
\]

(21)

An exogenous Total Factors Productivity (TFP) shock has been assumed to follow AR (1) process with autoregressive parameter \( \rho_z \) and structure shock \( e^z_i \).

\[
z_t = \rho_z z_{t-1} + e^z_i \quad e^z_i \in (0, \sigma^2_z) \quad 0 < \rho_z < 1
\]

(22)

In order to generate the real impacts of the money growth rate, the model follows Fuerst’s (1992) assumptions: firms are the only borrowers at the capital market and have to issue corporative bond to borrow working capital in order to pay the wage bill before aggregate goods have been produced. This creates an additional CIA constraint that is faced by firms at the capital market, which is indicated by equation (23). This shows that the amount of exchange credit which would be collected by households is equal to the firms’ cost of labour demand.

\[
b_t = w_t I^g_t
\]

(23)

Firms have sales income from the goods market, and borrow income from the capital market. They need to pay the wage and capital bill to households, and transfer the interest rate payment to financial intermediates in the capital market. The marginal cost of labour and capital comes from the maximised production function, subject to lifetime and capital market constraints.

\[
(1 + R_t) w_t I^g_t + r_t k_{t-1} = y_i
\]

(24)

\[
(1 + R_t) w_t = (1 - \alpha) \frac{y_i}{I^g_t}
\]

(25)

\[
r_t = \alpha \frac{y_i}{k_{t-1}}
\]

(26)

Where \( R_t \) represents the net bank loan rate at the capital market,\(^{21} \) \( r_t \) represents the real

\(^{21} \) This is equal to nominal interest rate on saving fund.
interest rate. The model follows Fuerst’s (1992) assumption that firms must borrow to fund their wage bill. Consequently, the appropriate marginal cost of labour to the firm in equation (25) is the real wage times the gross rate of interest on bank loans. This reflects the cost channel of monetary policy. The interest rate decline which is generated by the liquidity effect lowers the marginal cost of labour. At each real wage, the labour demand increases and equilibrium employment and output rise.

3.3.5 Monetary Policy

Monetary policy which has been implemented by central bank through money supply rule is represented by equation (27). This indicates that the central bank conducts monetary policy through its influence on the next period of nominal money supply. In the money market, the nominal money demand is equal to the nominal money supply. The money supply rule indicates that the short term interest rate will adjust money demand equal to money supply at equilibrium.

\[ M_t = M_{t-1} + T_t \]  

Money injections from monetary authorities are represented by equation (28). It indicates that monetary expansion from central bank depends on constant money growth rate \( \Theta^* \), monetary innovation \( e^n \) and the initial money stock.

\[ T_t = (\Theta^* + e^n - 1) M_{t-1} \]  

The deviation of money growth rate is assumed to follow the AR (1) process, with autoregressive parameter \( \rho_m \) and structure shock \( \varepsilon_t^m \).

\[ u_t = \rho_m u_{t-1} + \varepsilon_t^m \quad \varepsilon_t^m \in (0, \sigma_m^2) \quad 0 < \rho_m < 1 \]  

3.3.6 Competitive Equilibrium

Competitive equilibrium of this economy consists a set of feasible allocations \( \{ y_t, c_t, k_t, M_t, l^x_t, l^f_t, x_t, f_t, d_t \} \), a set of prices \( \{ r_t, w_t, r^d_t, R_t, p^f_t \} \), exogenous

\( \{ y_t, c_t, k_t, M_t, l^x_t, l^f_t, x_t, f_t, d_t \} \), a set of prices \( \{ r_t, w_t, r^d_t, R_t, p^f_t \} \), exogenous
shocks \{ z_t, q_t, u_t \} and aggregate outcomes, such that:

- Given \( r_t, w_t, r^d_t, R_t, p_t^f \), allocation \( c_t, k_t, M_t, x_t, f_t \) solves the households’ problem;
- Given \( w_t, r^d_t, p_t^f \), allocation \( l^f_t, f_t, d_t \) solves the banks’ problem;
- Given \( r_t, w_t, R_t \), allocation \( l^x_t, k_t, y_t \) solves the firms’ problem;
- The goods, labour, credit and money markets are clear;

And nominal variables are divided by \( P_t \) in order to transform into a stationary model.

### 3.4 Calibration

The procedure of calibrating deep structure parameters is to map the model economy into observed features of data. It implies that the steady-state value of the model can be indicated by deep structure parameters. With given deep structure parameters, great ratios are predicted by the model’s steady state - which can be directly observed from the data.

Table 3.1 below summarises base line deep structure parameters which are implied by U.S post war data. Compare this with the results of Cooley and Hansen (1995) that had a quarterly depreciation rate \( \delta = 0.019 \). The data set, which comes from Gomme and Rupert (2007), with duration from 1959 Q1 to 2004 Q2, indicates a quarterly depreciation rate \( \delta \) which is equal to 0.024. This also implies that the investment-output ratio is 0.26. With a given depreciation rate and investment output ratio, the steady state capital-output ratio is 10.8. Capital and labour income shares are calibrated by using U.S data from 1959 Q1 to 2004 Q2. The results here are the same as those of Cooley and Hansen (1995), in that it indicates that the share of wage and capital income is 0.6 and 0.4 respectively. With a capital share of 0.4, depreciation rate 0.024, and capital-output ratio 10.8 the steady-state Euler equation implies that \( \beta \) is equal to 0.987; it further indicates that the quarterly real interest rate is equal to 0.013, after depreciation rate. Furthermore, the U.S data indicates that the steady-state working
hours from the goods producing sector and leisure are 1/3 and 2/3, respectively. This requires deep structure parameters $\Psi$ which are equal to 1.61.

**Table 3.1: Baseline parameters**

<table>
<thead>
<tr>
<th>Preferences</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.987</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>1.61</td>
<td>Leisure weight</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Goods Production</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>Capital share in goods sector</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.024</td>
<td>Capital stock depreciation rate</td>
</tr>
<tr>
<td>$e^z$</td>
<td>1</td>
<td>Good sector productivity parameter</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Banking sector</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.21</td>
<td>Labour share in credit production</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monetary authority</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta$</td>
<td>1.2%</td>
<td>Quarterly money growth rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shocks processes</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.95</td>
<td>Goods sector productivity</td>
</tr>
<tr>
<td>$\rho_q$</td>
<td>0.95</td>
<td>Banking productivity</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>0.64</td>
<td>Money growth rate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Deviation of Shock Innovations</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_z$</td>
<td>0.7%</td>
<td>Goods sector productivity</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>0.7%</td>
<td>Banking productivity</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.9%</td>
<td>Money growth rate</td>
</tr>
</tbody>
</table>

Table 3.1 also concludes the behaviours of technology, exchange credit and monetary innovations. The steady-state technology shock has been normalised to one. The autoregressive process and variation of technology shock follow the work of Cooley and Hansen (1995). By assuming a symmetric process between technology and
exchange credit shock, the model has the same autoregressive parameter and standard deviation of exchange credit shock with technology innovation. The monetary shock process is estimated by following regression with time duration from 1959 Q1 to 2004 Q2. This indicates that there is a 1.2% money growth rate per quarter at steady-state with persistence 0.64. This result compares with those of Cooley and Hansen (1995), who had steady-state money growth rates of 1.3% with persistence 0.49, and Benk, Gillman and Kejak (2005), who found 1.23% steady state money growth rate with persistence 0.58. The variance of monetary shock from M1 regression is 0.9%, which is close to the results of Cooley and Hansen (1995) and Benk, Gillman and Kejak (2005), which are 0.89% and 1% respectively.

\[ \Delta \log M_t = 0.0045 + 0.64 \Delta \log M_{t-1} \quad \sigma_m = 0.9\% \]  
\[ (0.0009) \quad (0.0545) \]

There are three deep structure parameters within the model: \( A_q, a, \) and \( \gamma \). Given one set of calibrated parameter values, the other two can be implied by the model’s steady-state. The model employs the degree of diminishing return in the credit sector which is set \( \gamma \) equal to 0.21, which borrows from Gillman and Otto’s (2002) estimate for the U.S.

**Table 3.2: Target values**

<table>
<thead>
<tr>
<th>( \pi^{st} )</th>
<th>1.2%</th>
<th>Quarterly inflation rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k^{st} / y^{st} )</td>
<td>10.76</td>
<td>Capital-output ratio</td>
</tr>
<tr>
<td>( i^{st} / y^{st} )</td>
<td>0.26</td>
<td>Investment-output ratio</td>
</tr>
<tr>
<td>( x^{st} )</td>
<td>2/3</td>
<td>Leisure</td>
</tr>
</tbody>
</table>
3.5 The Transmissions and Real impacts of Monetary Aggregates at Steady-State

This section of the chapter discusses the properties of the model’s steady state with different levels of the stationary-state money growth rate. For the nominal side of the economy, the money supply rule implies that the rate of inflation is determined by the money growth rate at a stationary-state. The marginal cost of money can be examined by time preference and inflation rate, which is indicated by equation (30). Equation (31) indicates that the marginal cost of money is equal to the difference between the exchange credit price and the nominal interest rate.

\[ R_{ss}^* = \frac{\pi_{ss}}{\beta} \]  

\[ (R_{ss}^* - 1) = p'^* - R^{ss} \]  

The Euler relation indicates that the real interest rate is independent to the rate of inflation, and the money growth rate is at a stationary-state. It further implies that great ratios are independent to the nominal side of the economy.

\[ \alpha \frac{\nu^{ss}}{k^{ss}} = r^{ss} = \frac{1}{\beta} - 1 + \delta \]  

With great ratios are independent to money growth rate, equation (33) implies that the nominal interest rate has a negative relation with the fraction of exchange credit purchase consumption. The increasing fraction of exchange credit used by households at the goods market is decreasing the nominal interest rate at stationary-state. Equation (34) implies that real wages have a negative relation with the nominal interest rate. A lower nominal interest rate has a negative effect on real wages. Therefore, with a given fraction of cash purchase goods at a stationary state, both the nominal interest rate and the real wage are independent to the rate of inflation. In other words, increasing the money growth rate at a stationary-state raises the nominal variables (such as inflation, marginal cost of money and credit price). Real prices, great ratios, and nominal interest rate are independent to the rate of inflation when the fraction of cash purchase consumption is given at stationary-state.

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Equation (35) indicates that the deposit rate, or marginal cost of exchange technology moves with the exchange credit price. With a given fraction of exchange credit consumption, the money growth rate has a positive effect on the deposit rate through the exchange credit price.

\[ r_{ss}^{d} = p_{ss}'(1-\gamma)(1-a^{ss}) \]  

(35)

The effectiveness of monetary expansion at stationary state is indicated by equation (36). This reflects the model's stationary state transmission channel between money growth rate and the real economic activity. By assuming independent cash purchase consumption to inflation rate, the deposit rate indicates that there is a negative relation between the money growth rate and leisure-labour in goods sector substitution. In contrast, the marginal cost of money implies that there is a positive effect of money growth rate on leisure-labour in goods sector substitution. With a positive effect from marginal cost of money, which dominates the negative effect from marginal cost of exchange technology, rising money growth rate at steady-state increases leisure-labour substitutions, which indicates a rise in leisure and lower labour at steady-state with money growth rate.

\[ \frac{\pi_{ss}}{l_{ss}} = (\frac{R_{ss}^{e} - r_{ss}^{d}}{1-a^{ss}}) \]  

(36)

\[ \frac{l_{ss}}{l_{ss}} = (\frac{\pi_{ss}}{\beta} - 1)\gamma \]  

(37)

Labour supply substitution between sectors is represented by equation (37), which indicates that the substitution between sectors only depends on the rate of inflation at stationary-state. It further implies the positive effect of money growth rate on banking sector labour supply. With equation (36) and (37), increasing the money growth rate lowers labour in the goods sector and has a negative effect on aggregate output through the production function.
In conclusion, table 3.3 concludes that with a given fraction of cash purchase consumption, increasing money growth rate at stationary state will increasing nominal prices (such as inflation, exchange credit price) and lower real activities (such as output, consumption, investment and labour supply) through the rate of inflation and exchange credit price. This indicates that there is negative effect of money growth rate on the economy at a stationary-state. Therefore, increasing the money growth rate will have a negative effect on real activity.

<table>
<thead>
<tr>
<th>$\Theta^* = 1.2%$</th>
<th>$\Theta^* = 2%$</th>
<th>$\Theta^* = 1.2%$</th>
<th>$\Theta^* = 2%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{s^s}$</td>
<td>0.0428</td>
<td>0.0509</td>
<td>1.1932</td>
</tr>
<tr>
<td>$k^s / y^s$</td>
<td>10.7610</td>
<td>10.7610</td>
<td>0.4155</td>
</tr>
<tr>
<td>$c^s / y^s$</td>
<td>0.7417</td>
<td>0.7417</td>
<td>1.6087</td>
</tr>
</tbody>
</table>

3.6 The Model's Dynamic and Findings

The following section of the chapter discusses the transmissions and impacts of monetary innovation on real economic activity. It explains the inflation and liquidity effects on nominal interest rate with monetary expansion, and generates a positive response of output subject to monetary innovations through three monetary channels (marginal cost of money, nominal interest rate, and cost of exchange credit). It also examines the effect of technology and credit shocks on monetary transmissions and real economic activity.

3.6.1 The Effects on Nominal Interest Rates

By integrating financial intermediates with households' problem, the model introduces an additional CIA constraint which is faced by a representative agent. In contrast to
those limited participation models where the representative agent can only choose the number of bonds and is not saving subject to monetary innovation, this model allows households to choose both the number of bonds and exchange credit, subject to monetary innovations. Therefore, equations (39)-(42) represents those households who maximise their expected log utility subject to lifetime, the goods market, and capital market CIA constraints.

\[
\text{Max } E_0 \sum_{t=0}^\infty \left\{ \beta^t \left[ (\ln c_t + \psi \ln x_t) + [\lambda_t \left( \frac{m_t}{\pi_t} + r_t k_{t,-1} + w_t (1-x_t) + R_t b_t + r_t c_t - m_t \right] \right. \\
\left. - c_t - k_t - p_t f_t + (1-\delta)k_{t,-1} + \mu_t \left( \frac{m_t}{\pi_t} + f_t - c_t \right) + \eta_t (f_t - b_t) \right\} \}
\]

\[
c_t : \frac{1}{c_t} + r_t \lambda_t = \lambda_t + \mu_t \tag{39}
\]

\[
f_t : \lambda_t p_t = \eta_t + \mu_t \tag{40}
\]

\[
b_t : \lambda_t R_t = \eta_t \tag{41}
\]

\[
m_t : \beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} + \mu_{t+1} \right) = \lambda_t \tag{42}
\]

Where \( \eta_t \) represents the shadow prices of capital market CIA constraint. According to equation (39)-(42), the Fisher relations for nominal interest rate and cost of exchange credit have been represented by the following equations:

\[
1 + R_t = \frac{(\eta_t - \mu_t) + (\lambda_t + \mu_t)}{\beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} + \mu_{t+1} \right)} \tag{43}
\]

\[
1 + p_t = \frac{(\eta_t - \lambda_t) + (\lambda_t + \mu_t)}{\beta E_t \left( \frac{\lambda_{t+1}}{\pi_{t+1}} + \mu_{t+1} \right)} \tag{44}
\]

Clearly, equation (44) indicates there is only an inflation effect on the cost of exchange credit. It implies that banking costs always increase with monetary shock as it alone has an expected inflation effect. In contrast, the nominal interest rate on savings funds includes both inflation and a liquidity effect terms, and the liquidity effect will depend on the \( \eta_t - \mu_t \) term. If \( \eta_t - \mu_t < 0 \) there is a liquidity effect on the nominal interest rate, if \( \eta_t - \mu_t > 0 \) there is no liquidity effect on the nominal interest rate. Whether the
nominal interest rate decreases with monetary expansion has to depend on the size of liquidity and inflation effect on the nominal interest rate. If the inflation effect dominates the liquidity effect then the nominal interest rate has a positive response to positive monetary innovation. In contrast, if the liquidity effect dominates inflation effect then the nominal interest rate has a negative response to a positive monetary shock.

Figure 3.3 indicates the effect of monetary innovations on the capital market. The number of bonds which have been traded at the capital market can be considered as a firms' borrowing (corporative bond) for wage payments. Firms are issuing within period claims for wage payments, which determines the supply of bonds. Households which collect exchange credit and make savings can be considered as the demand on the number of bonds. Monetary innovations increase the demand on the number of bonds through raising households' exchange credit. When the supply of the numbers of bonds is determined by real economic activity, increasing demand for bonds raises the price of bonds and lowers their return, which is the nominal interest rate on saving fund.

Figure 3.3: The supply and demand of corporate bonds
3.6.2 The Transmission Mechanism and Effectiveness of Monetary Policy

Figure 3.4 represents the responses of the monetary transmission mechanism, which includes the nominal interest rate at the capital market and the cost of the banking sector to positive monetary innovation. It shows that the model is able to generate a negative response of nominal interest rate to monetary shock, and indicates an increase in real economic activity with monetary expansion.

First of all, monetary expansions have a positive pressure on the rate of inflation through the money supply function. The increasing rate of inflation with money injections leads the banking costs and the marginal costs of money to increase. When the cost of holding money increases, households substitute from real money balance to exchange credit for goods market transactions. This means that the share of exchange credit purchase goods rises, and the cash purchase consumption falls. With an increasing banking cost, banks have an incentive to supply more exchange credit to households. This leads financial intermediates to receive more savings funds from households as more exchange credits have been used for goods market transaction. The supply of bonds is determined by firms’ wage payment. Demand for bonds increase through rising in banks’ exchange credit, which has a negative effect on the nominal interest rate of the capital market. With an increasing exchange credit holding by households, financial intermediates have to lower nominal interest rate on savings in order to lend extra savings funds to firms. Therefore, monetary expansion increases the banking cost and marginal costs of money through an expected inflation effect, and decreases the nominal interest rate on savings.

Figure 3.4 also summarises (for the effectiveness of monetary aggregates) the responses of output, investment, consumption, and labour supply to monetary shock. Clearly, with a decreasing nominal interest rate and a cost channel of monetary policy assumption, the
model is able to generate a positive response of real activity (except for consumption) to monetary expansion without sticky price/wage and a sticky consumption-saving portfolio. A decreasing nominal interest rate in the capital market has a positive effect on the marginal cost of labour and increases firms’ labour demand. The lower marginal cost of labour to monetary innovation leads to an income and substitution effect on labour supply. The labour supply in the goods sector will increase, and leisure will decrease, with a positive monetary shock due to the income effect on marginal cost of labour, which dominates the substitution effect. With a given initial capital stock, rising labour supply increase the aggregate output through the production function. Increasing output with money growth rate has a positive effect on the real interest rate through the marginal cost of capital equation because the initial capital stock has been given. A change in the real interest rate introduces income and substitution effects on capital stock. Figure 3.4 shows that the capital stock positive response to monetary shock is due to the income effect of real rate, which dominates the substitution effect. Furthermore, through the law of motion equation, investment moves in the same direction with capital stock (subject to monetary expansion). The model does not explain the behaviour of consumption subject to monetary innovation because the consumption is affected by the marginal cost of money, rather than nominal interest rate at the capital market.

By extending the monetary banking model of Benk, Gillman, and Kejak (2005) with the function of financial intermediates, this model is able to generate a lower nominal interest rate on saving, subject to monetary expansion, without the limited participation monetary shock assumption from Lucas (1990). In this model monetary innovations increase exchange credit, and decrease money demand, through the expected inflation effect of the money growth rate. This encourages households to use more exchange credit in the goods market and decrease their proportion of cash purchase consumption. For every unit of exchange credit which is used by households, the model further assumes that households have to make saving funds at financial intermediates to back-

---

22 This refers to limited participation monetary shock assumption.
23 Leeper et al (1996) found that consumption also increases with monetary expansion.
up the exchange credit which has been collected from productive banks. Therefore, savings funds increase with exchange credit, and increase the demand of bond at capital market. With a pre-determined firm's wage bill, financial intermediates have the incentive to lend savings funds at a lower interest rate and decrease marginal cost of labour. The decreasing marginal cost of labour rise employment and output through production function.

Figure 3.4: Variables response to a 1% positive monetary shock

In other words, the model examines decreasing nominal interest rate with monetary expansion by integrating the functions of productive banks and financial intermediates into a cash-in-advance framework. By assuming that firms have to borrow working capital before any goods have been produced, it explains the positive relation between monetary aggregates and real activity without sticky price/wage and limited participation monetary shock.
3.6.3 Other shocks

Figures 3.5 and 3.6 represent the response of the model economy to technology and credit shocks. Clearly, the real economy activities (such as output, consumption, investment and labour supply) have a positive response to technology innovation. The nominal interest rate on savings and banking costs increases with the technology shock, while the marginal cost of money has a negative response to technology shock. This happens because of the nominal interest rate, which increases more than the banking costs. For the credit shock, monetary transmissions (such as the marginal cost of money, the nominal interest rate and banking costs) are a negative response to the credit shock. Real economy activity, except for consumption, has a positive response to banking sector innovation.

*Figure 3.5: Variables response to a 1% technology shock*
In conclusion, the main function of financial intermediaries is to receive savings and invest corporative bond at capital market. The number of savings funds from households occurs when the exchange credit has been used for goods market transaction. This determines the demand for bonds in the capital market. Following Fuerst's (1992) assumption, the supply of bonds in the capital market is determined by the amount of goods producing firms who have to borrow their wage payments. In this model monetary policy has been implemented by the central bank through the money supply rule. The positive monetary shock increases the marginal cost of money through an expected inflation effect on the money growth rate, and has a negative effect on labour supply. With a rising marginal cost of money households will increase the exchange credit which has been used in the goods market, and this creates more savings for financial intermediaries. This indicates that velocity has a positive response to monetary shock, and lowers nominal interest rate. Both the positive response of velocity and the negative response of nominal interest rate have positive effects on labour demand. The velocity and liquidity effects, which dominate the inflation effect on the labour market, indicate that households increase the labour supply and decrease leisure
with a positive monetary shock. With a given initial capital stock, an increasing labour demand raises aggregate output through the production function. Therefore, this model is able to explain the negative response of the nominal interest rate to money injections without limited participation monetary shock, and generates positive responses of real activities subject to monetary expansion under a flexible price framework.

3.6.4 Business Cycle Facts

This section of the chapter concludes some of the observed features of monetary business cycle facts which are replicated with model’s simulations. In other words, the model economy is quantitatively evaluated by comparing its implications of business cycle facts with observed U.S data. The finding is that the model is able to capture many of important features of U.S economy, such as the negative correlation between nominal interest rate and money growth rate, the pro-cyclical behaviour of inflation and Treasury bill rate and the volatility of the nominal interest rate. Table 3.4 describes the cyclical behaviours of the U.S economy, which are obtained from the detrended HP time series data, with duration from 1959 Q1 to 2004 Q2. It also summarises the simulated economy statistics with technology, credit, and monetary shocks.

Table 3.4: Simulated monetary economy with technology, credit and monetary shocks

<table>
<thead>
<tr>
<th></th>
<th>Relative SD (%)</th>
<th>Correlations with output</th>
<th>Correlations with M1 growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data Model</td>
<td>Data Model</td>
<td>Data Model</td>
</tr>
<tr>
<td>Output</td>
<td>1.00 1.00</td>
<td>1.00 1.00</td>
<td>-0.09 0.06</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.5163 0.4295</td>
<td>0.79 0.92</td>
<td>-0.05 -0.24</td>
</tr>
<tr>
<td>Investment</td>
<td>2.7078 2.7852</td>
<td>0.92 0.98</td>
<td>-0.10 0.19</td>
</tr>
<tr>
<td>Hours</td>
<td>1.0036 0.4087</td>
<td>0.82 0.95</td>
<td>-0.24 0.23</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.2854 1.9809</td>
<td>0.22 0.15</td>
<td>-0.25 0.70</td>
</tr>
<tr>
<td>Treasury rate</td>
<td>0.1809 0.2588</td>
<td>0.24 0.79</td>
<td>-0.43 -0.42</td>
</tr>
<tr>
<td>M1 rate</td>
<td>0.6205 0.8302</td>
<td>-0.09 0.06</td>
<td>1.00 1.00</td>
</tr>
</tbody>
</table>

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Although for the real side of the economy the model is able to explain the relative volatilities of consumption and investment, it cannot explain the relative volatility of working hours. For the nominal side of the economy, the model well explains the volatility of the nominal interest rate, but it falls to examine inflation volatility. For nominal real variables interaction, the model is able to generate the pro-cyclical behaviour of inflation and nominal interest rate, but it fails to generate the negative correlation between the money growth rate and real economic activity (except for consumption). The important contribution of this model is that it can generate a negative correlation between the M1 growth rate and the nominal interest rate, which has been interpreted by Cooley and Hansen (1995) as a liquidity effect of the money growth rate. Therefore, Table 3.4 concludes that the model is able to explain the pro-cyclical behaviour of nominal variables (such as inflation and the nominal interest rate). It well explains the behaviour of the nominal interest rate from relative volatility and correlation. Although the model does explain the pro-cyclical behaviour of inflation, it fails to examine the relative volatility of inflation compare with data.

3.7 Discussion

Cooley and Hansen (1995) have argued that the nominal interest rate (which has been referred to as an inflation tax effect of the money growth rate) is the only monetary transmission channel in Lucas and Stocky’s (1987) cash-credit goods CIA model. They simulated the model with monetary business cycle facts and concluded that monetary shock does not contribute much to the economic fluctuations in the real variables displayed by a basic neoclassical growth model when money is introduced by requiring a cash-in-advance constraint. Therefore, combining a CIA constraint with a simple RBC structure model cannot account for either the observed cyclical behaviour of nominal variables or the interaction between real and nominal variables. This suggests that in order to successfully account for the interaction between real and nominal variables in the data we need to introduce more sources of non-neutrality than the inflation tax.
alone.

Benk, Gillman, and Kejak (2005) extended Cooley and Hansen's (1995) CIA model with productive banks to emphasise the contribution of the exchange credit shock to business cycle fluctuations. At mean time, they introduced an additional monetary transmission channel, which they called a velocity effect of the money growth rate (this will be discussed in more detail in the chapter five). Lucas-Fuerst's (1992) type of limited participation CIA models with single exchange technology assumes that households cannot adjust their cash-saving portfolio subject to monetary innovations, and that money injections from the central bank are received by financial intermediates to generate lower nominal interest rate with an increasing money growth rate. This chapter extends Benk, Gillman, and Kejak's (2005) two exchange technologies CIA model with the functions of financial intermediates, which does not request a sticky price/wage or a sticky consumption-saving portfolio to examine the transmissions and impacts of the money growth rate to real economic activity.

Monetary innovation is an increase of the marginal cost of money, and of the proportion of exchange credit purchase goods, through an inflation effect of the money growth rate. With the function of financial intermediates, the model assumes that the number of savings funds received by financial intermediates is equal to the number of exchange credits which have been used by households in the goods market. Raising the exchange credit from productive banks can lead to either an increase in the savings funds or in the demand for bonds. When the supply of bonds is determined by real activity, which is the cost of labour demand, increasing the savings funds within financial intermediates lowers the nominal interest rate. By employing the cost channel of monetary policy, decreasing the nominal interest rate with money injections has a positive effect on labour demand and increases real economic activity.

The policy implication of this model is that it introduces the interaction between monetary policy and exchange credit. The monetary authority can increase the money growth rate to allow households to collect more exchange credits from banks and
increase the savings funds through raising exchange credit. This leads to more savings from households being made with financial intermediaries, which decreases the nominal interest rate of the capital market. A lower nominal interest rate with monetary expansion increases the labour demand from firms and raises aggregate output.

### 3.8 Conclusion

The flexible monetary RBC models (such as endogenous money supply and cash-in-advance (CIA) models) find it difficult to account for the real impacts of monetary aggregates through existing monetary transmissions. In contrast, the New Keynesian economists employ sticky price/wage setting with RBC models to explain the effectiveness of monetary policy.

This chapter extends Benk, Gillman, and Kejak's (2005) two exchange technologies monetary banking model with the function of the financial sector, and does not request a sticky price/wage or a sticky consumption-saving portfolio to examine the impacts of monetary aggregates with various monetary transmission channels in a Dynamic Stochastic General Equilibrium (DSGE) framework. There are two types of 'banks' in this model financial sector, which are: productive banks and financial intermediates. Productive banks provide exchange credit services to households for goods market transactions, and financial intermediates receive savings funds from households and supply loans to goods producing firms. The model assumes that financial intermediates receive savings funds after an exchange credit has been used for a goods market transaction and representative firms, which are the only borrowers in the economy, have to finance their wage payments in advance.

In contrast to Lucas-Fuerst (1992) type of limited participation models, money injections from the central bank in this model are received by households instead of financial intermediaries. Monetary shock raises both the marginal cost of money and the price of exchange credit through an expected inflation effect. This lowers the real
money balance and increases exchange credit. The increasing exchange credit indicates that there is an increasing demand for bonds through financial intermediates, and it lowers the nominal interest rates because the supply of loanable funds (which is a cost of labour) has been determined by real economic activity. With a lower marginal cost of labour demand, firms have an incentive to employ more labour and increase aggregate output. Therefore, the model is able to explain the real effects of money growth rate through varying the nominal interest rates without sticky price/wage or limited participation monetary shocks under flexible price.

References


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Chapter 4 - Monetary Policy, Financial Intermediates, and Business Cycles

Abstract

This chapter explains and evaluates the transmissions and effectiveness of monetary policy shock in a simple Cash-in-Advance (CIA) economy with financial intermediates. By assuming a limited participation monetary shock and the function of financial intermediates, Lucas-Fuerst's (1992) type of limited participation CIA models have explained decreasing nominal interest rates and increasing real economic activity with monetary expansion. Although Calvo's (1983) type of sticky price model examines the real effects of money injections through firms' price setting behaviour, it fails to generate a negative correlation between nominal interest rates and money growth rate (which has been observed in the data). This chapter employs McCandless (2008) financial intermediates CIA model to explain the transmissions and impacts of monetary shocks. The model does not request limited participation monetary shock from Lucas (1990), or a Keynesian type of sticky price/wage, to examine the lower nominal interest rate and increasing real economic activity with monetary expansion. By extending the model with Stockman's (1981) CIA constraint, it is able to account for the positive response of consumption subject to monetary innovations (which has been found in Leeper et al. (1996)) and it generates a positive correlation between output and consumption in the data.

\[24\] Lucas (1990) assumes that households cannot adjust their consumption-saving portfolio before recognising monetary shock.

\[25\] Fuerst (1992) adds two functions of financial intermediates into the CIA economy, which are: to receive savings funds and to make loans to borrowers, and to receive money injections from the central bank.
4.1 Introduction

The increasing real activity with monetary expansion and the negative correlation between short-term nominal interest rates and money growth rate have been recognised as an important monetary transmission channel and real impact of the money growth rate in both traditional Keynesian (Tobin, 1947) and monetarist (Friedman, 1968 and Cagan, 1972) macroeconomics models. The general equilibrium CIA models with flexible price from Lucas (1982) and Svensson (1985) indicate positive correlations between money injections and the nominal interest rates. By affecting leisure-consumption substitution, the model generates negative effects on real economic activity. In contrast, sticky price models emphasise the degree of price stickiness to explain the positive effects of monetary aggregates, and ignore the negative correlation between nominal interest rates and money growth rate. Limited participation CIA models were developed by Lucas (1990), and followed by Fuerst (1992). Christiano and Eichenbaum (1992), (1995), and Christiano, Eichenbaum and Evan (1997) are able to explain the negative response of nominal interest rates, and the positive response of real activity to monetary innovation under a flexible price framework.

Lucas-Fuerst’s (1992) limited participation CIA models assume that households cannot adjust their consumption-saving portfolio subject to money innovations, and that money injections from the central bank have to be received by financial intermediates as part of the savings funds. With a positive interest rate, financial intermediates have an incentive to lend extra saving funds in the capital market. By assuming that the supply of government bonds has been determined by the amount of savings from households before a monetary shock occurs, the money injections increase the demand for government bonds and decrease the return of bonds. This negative response of bond rate subject to monetary expansion has been referred as the liquidity effect on nominal interest rates. Christiano (1991) argued that the introduction of a liquidity effect into the model may not be enough to generate a lower nominal interest rate. He suggests that the

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26 Cooley and Hansen (1989) referred to the inflation tax of money growth rate on real activity.
27 Most types of limited participation model use firms’ wage payment as the demand for bonds.
liquidity effect must be sufficiently strong to dominate the anticipated inflation effect. This, in turn, depends on the precise relationship among its variables or the values of its parameters.

In order to include the real impacts of money aggregates through the liquidity effect of money growth rate, Fuerst (1992) supposed that firms have to borrow from the capital market to pay their wage bills. With a lower borrowing rate because of a limited participation monetary shock, firms are more willing to increase their borrowing from the capital market and employ more labour. With a given initial stock capital, an increasing labour demand raises the aggregate output through the production function. This allows limited participation CIA models to explain the positive response of real activity to money injections, without sticky price, through the cost channel of monetary policy. However, the weakness of this model is that it fails to examine the increasing consumption with money injections, and it cannot generate the positive correlation between output and consumption which been observed in the data. Christiano and Eichenbaum (1992), (1995), and Christiano, Eichenbaum and Evan (1997) modified the CIA constraint, which allows aggregate consumption to be equal to the next period of money demand. This modification generates the behaviour of consumption that is observed in the data.

This chapter employs McCandless (2008) financial intermediates CIA model to explain the transmissions and impacts of monetary innovation to real activity. The model includes two nominal interest rates which reflect the price of money at the goods and capital markets, respectively. Although this model is able to account for the positive correlation between output and monetary aggregates through two nominal interest rates, it fails to examine the behaviour of consumption subject to monetary expansion (such as the response of consumption to monetary shock and correlation with output). After extending the model with Stockman’s (1981) CIA constraint (which indicates that both consumption and investment have to be purchased by households using real money balance) the model is able to explain the positive response of consumption subject to monetary shock and also the replicate positive correlation between output and
consumption in the data. Stockman's (1981) CIA constraint is crucial to explain the
behaviour of consumption and velocity subject to monetary innovations. It is also able
to generate the positive correlation between output and consumption that is observed in
the data.

This chapter is organised into seven sections, the first of which is this introduction.
Section 2 presents the theoretical dynamic stochastic general equilibrium (DSGE) CIA
model. Section 3 explains the procedure of model calibrations. Section 4 discusses how
the model's steady state is affected by the money growth rate. Section 5 examines the
model's dynamic and findings. Section 6 has detailed discussion of model properties.
Section 7 is a conclusion.

4.2 The Model Economy

This section of the chapter introduces the model economy and shows how the problems
are solved by households and firms. It also describes the behaviour of financial
intermediates and monetary policy authority. There are two sources of uncertainty
which come from the exogenous process of a firms' technology and money growth rate.

Figure 4.1 describes the structure of the model economy. The economy includes four
infinitely lived representative agents, who are: household consumers, firms who
produce final goods, financial intermediates, and the monetary authority. Money
injections from the monetary authority are received by financial intermediates instead of
households.\textsuperscript{28} Financial intermediates receive savings from households, money
injections from the central bank in the form of savings funds, and they purchase
corporative bond from firms at the capital market. Firms are borrowing their wage
payments with issuing corporative bond from financial intermediates before any goods
have been produced and they pay wages, and capital and loan bills. Households have to

\textsuperscript{28} The standard CIA model allows households to receive money injections instead of financial
intermediates.
divide their real money balance into goods market transaction and savings which are paid to financial intermediates. With infinitely lived representative agents, the model further assumes that agents have to make their consumption and portfolio decisions period by period basis. It is ruling out the multi-period borrowing and lending decisions.

Figure 4.1: The structure of a CIA economy with financial intermediates

There are two nominal interest rates, which are: the interest rate on the savings funds and the interest rate on firms’ corporative bond in the model economy. The interest rates are equal without money injections. Money injections lower the interest rate on firms’ borrowing with excess demand in the capital market, and increase the interest rate on savings funds through an expected inflation effect. The lower interest rate at capital market has positive effect on real activity through firms’ cost channel. The increasing interest rate on saving has negative effect on real activity through leisure-consumption substitutions. Since the income effect from cost channel dominates the substitution effect, the employment and aggregate output increase with monetary expansion. The
consumption also rises with positive monetary innovation if the Stockman (1981) constraint is applied.

4.2.1 Household Consumers

Representative households maximise their expected log utility function (1) (which includes consumption $c_t$ and leisure $x_t$) with discount factor $\beta \in (0,1)$, and allocate their time endowment between leisure and working hours $h_t$.

$$U = E_0 \sum_{t=0}^\infty \beta^t (\ln c_t + \Psi \ln x_t)$$  \hspace{1cm} (1)

$$1 = x_t + h_t$$  \hspace{1cm} (2)

Aggregate output $y_t$ includes consumption and investment $i_t$ goods. The next period’s physical capital stocks $k_t$ have to be accumulated through the law of motion equation, with a constant quarterly depreciation rate $\delta$.

$$y_t = c_t + i_t$$  \hspace{1cm} (3)

$$i_t = k_t - (1 - \delta)k_{t-1}$$  \hspace{1cm} (4)

At the beginning of each period, the initial real money balance is held by households and it has to be divided into the amount of savings and goods market transaction. Households have to deposit their saving funds into financial intermediates, and they receive a gross interest rate $R^n_t$. Cash is the only exchange technology which can be used for goods market transactions. Equation (5) represents the exchange technology constraint that is faced by households in the goods market. The model assumes that money injections are received by financial intermediates instead of households. This means that money injections from the central bank do not enter a CIA constraint at the goods market. Where $n_t$ represents households savings, $P_t$ represents price level, and $M_{t-1}$ represents the initial nominal money stock.

$$\frac{M_{t-1}}{P_t} - n_t = c_t + \Omega i_t$$  \hspace{1cm} (5)
When $\Omega = 0$ the exchange constraint implies that the real money balance is used to purchase consumption goods only. When $\Omega = 1$ the exchange technology constraint represents that both consumption and investment goods can be purchased by real money balance, which is indicated by Stockman’s (1981) CIA constraint.

The next period’s money holding includes labour $w_t k_t$, capital $r_t k_{t-1}$ and saving incomes $R^n_t n_t$, which are received from firms and financial intermediates. The next period’s expenditures are on consumption, investment, and savings. The lifetime budget constraint which is faced by households is represented in equation (6).

$$\frac{M_l}{P_t} = \frac{M_{t-1}}{P_t} + w_t (1 - x_t) + r_t k_{t-1} + R^n_t n_t - n_t - c_t - k_t + (1 - \delta) k_{t-1}$$  \hspace{1cm} (6)

Equilibrium conditions of households are represented by equations (7)-(11), which come from maximising the expected log utility function subject to lifetime and good market CIA constraints.

$$\frac{1}{c_t} = \lambda_t + \mu_t \Rightarrow \frac{\lambda_{t+1}}{\lambda_t} = \frac{c_t R^n_t}{c_{t+1} R^n_{t+1}}$$  \hspace{1cm} (7)

$$\frac{x_t}{\Psi c_t} = \frac{R^n_t}{w_t}$$  \hspace{1cm} (8)

$$1 + \frac{\mu_t}{\lambda_t} = R^n_t$$  \hspace{1cm} (9)

$$\beta E_t \left( \frac{P_t c_t - R^n_t}{P_{t+1} c_{t+1}} \right) = 1$$  \hspace{1cm} (10)

$$\beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} - (r_s + 1 - \delta) + (1 - \delta) \Omega \frac{\mu_{t+1}}{\lambda_{t+1}} \right) = 1 + \frac{\mu_t}{\lambda_t} \Omega$$  \hspace{1cm} (11)

Where $\lambda_t$ and $\mu_t$ represent the shadow price of lifetime and goods market CIA constraint. Equation (9) implies that the marginal cost of money is equal to the saving rate across equilibrium. Fisher’s relation, which states that the nominal interest rate on the savings fund depends on expected inflation and current-future consumption substitution, has been indicated by equation (10). The nominal interest rate on the savings fund has a positive effect on leisure-consumption substitution, which is
indicated by equation (8). This indicates that the expected inflation effects of money growth rate have a negative impact on real activity. Equation (7) shows that the marginal utility of consumption depends on the lifetime shadow price and the marginal cost of money.

The model's Euler equation is represented by equation (11). When \( \Omega = 0 \) the exchange constraint is represented by equation (5), this implies a standard CIA constraint in which money demand and investment are perfect substitution goods. The Euler relationship which is represented by equation (12) indicates that the current and future consumption substitution is affected by the real interest rate and the movement of the nominal interest rate on the savings fund. Monetary injections raise the nominal interest rate on savings through an expected inflation effect. This has a negative effect on money holding, lowers consumption through the CIA constraint and increases investment.

\[
\beta E_t \left[ \frac{c_t R^m}{c_{t+1} R^m_{t+1}} (r_{t+1} + 1 - \delta) \right] = 1
\]  
(12)

If \( \Omega = 1 \) then the exchange constraint which is represented by equation (5) indicates that Stockman's CIA constraint is applied, which indicates that money demand and output are complementary goods. The Euler relationship which is represented by equation (13) indicates that substitution between current and future marginal consumption of utility depends on a real interest rate which is discounted by the marginal cost of money. Therefore, the behaviour of consumption subject to money injections depends on the real interest rate and the marginal cost of money. Monetary expansion increases the marginal cost of money through an inflation effect of the money growth rate, and has a negative effect on consumption. Real interest rates rise with money injections as output increases, and have a positive effect on consumption. Therefore, consumption increases with money injections as the real interest rate dominates the expected inflation effect.

\[
\beta E_t \left[ \frac{c_t}{c_{t+1}} \left( \frac{r_{t+1}}{R^m_{t+1}} + 1 - \delta \right) \right] = 1
\]  
(13)
households from immediately adjusting their cash-saving portfolio subject to monetary shocks. Extra money supply from the central bank, together with household savings received by financial intermediates as a deposit. With a positive interest rate, financial intermediaries have an incentive to invest their deposits in the capital market. Fuerst (1992) further assumed that the size of the capital market was determined by wage bills. The only way to invest extra money out at the capital market is to lower the interest rate of bond and it has positive effect on employment and output. The assumption follows that of Fuerst (1992) and McCandless (2008) in that money injections received by financial intermediaries instead of households and firms have to corporative bond to borrow wage payment before any goods been produced. The flow constraint which has been faced by financial intermediates is represented by equation (14).

\[ n_t + T_t = b_t \]  

Where \( T_t \) stands for money injections from the central bank, and \( b_t \) denotes the demand of the capital market. By assuming that financial intermediaries have zero profit, the gross return of the saving fund which is received by households is equal to the interest of financial intermediates from the capital market; this is indicated by equation (15).

\[ R^m n_t = R_t b_t \]  

Where \( R_t \) stands for the nominal interest rate on borrowing at the capital market, money injections from the central bank which are received by financial intermediaries increase the loan able funds for the capital market. They have a negative effect on the borrowing rate in order to increase the loan able money supply of the capital market. This indicates that there is a lower nominal interest rate on the capital market.
4.2.2 Firms

Aggregate output is produced by representative firms through the Cobb-Douglas production function; with labour, capital stock, and exogenous technology $e_t$. The shares of labour and capital are $1 - \alpha$ and $\alpha$, respectively.

$$y_t = e_t h_t^{1-\alpha} k_t^\alpha$$  \hspace{1cm} (16)

The exogenous Total Factors Productivity (TFP) is assumed to follow an AR (1) process with autoregressive parameter $\rho_z$ and structure shock $\varepsilon_i$.

$$z_t = \rho_z z_{t-1} + \varepsilon_i$$  \hspace{1cm} (17)

Following Fuerst's (1992) assumption, goods producing firms are the only borrowers at the capital market and they have to issue corporate bond to borrow cash from capital market for wage payment $w_t h_t$ before any goods have been produced. This creates an additional CIA constraint which is faced by firms at capital market, represented by equation (18). Therefore, a lower nominal interest rate on borrowing with monetary expansion reduces the marginal cost of labour and encourages firms to employ more labour and raise aggregate output. This has been referred to in the literature as the cost channel of monetary policy.

$$h_t = w_t h_t$$  \hspace{1cm} (18)

The aggregate incomes of firms include capital market borrowing and goods market sales. The capital market income is used to pay wages. Goods market income is used for renting capital and borrowing payments. Equation (19) indicates that a lifetime budget constraint has been faced by firms. Representative firms maximise their production function subject to their lifetime budget and capital market CIA constraint to obtain the marginal cost of labour and capital, which are indicated by equations (20) and (21).

$$R_t w_t h_t + r_t k_{t-1} = y_t$$  \hspace{1cm} (19)

$$R_t w_t = (1 - \alpha) \frac{y_t}{h_t}$$  \hspace{1cm} (20)

$$r_t = \alpha \frac{y_t}{k_{t-1}}$$  \hspace{1cm} (21)
Where $w_t$ represents the real wage, and $r_t$ represents the real interest rate. Equation (20) indicates that the marginal cost of labour is varied with the borrowing rate at the capital market. This further implies that an increasing borrowing rate has a negative effect on labour demand, while a decreasing borrowing rate has a positive effect on labour demand. In contrast to a standard CIA model, there is a gap between household labour income and firms' labour costs in this model. The gap, which is represented by $(R_t - 1)w_t h_t$, can be considered as a financial friction due to money injections. It allows the borrowing rate to be varied with money injections. Monetary expansion affects real economic activity through the labour demand equation as the marginal costs of labour are varied with the nominal interest rate on borrowing.

### 4.2.3 Monetary Policy

The monetary policy that has been implemented by the central bank through the money supply rule is represented by equation (22). This equation indicates that the central bank conducts monetary policy through its influence on the next period of nominal money supply. As nominal money demand is always equal to the nominal money supply at the money market, the money supply rule indicates that the short term interest rate will adjust money demand equal to money supply at equilibrium.

$$M_t = M_{t-1} + T_t$$  (22)

Monetary expansion has been represented by equation (23). It indicates that money injections from the central bank depend on steady-state money growth rate $\Theta^*$, monetary shock $\epsilon^{u_t}$, and the initial money stock.

$$T_t = (\Theta^* + \epsilon^{u_t} - 1)M_{t-1}$$  (23)

The deviation of the money growth rate was assumed to follow AR (1) process, with autoregressive parameters $\rho_m$ and structure shock $\epsilon^{m_t}$.

$$u_t = \rho_u u_{t-1} + \epsilon^{m_t} \quad \epsilon^{m_t} \in (0, \sigma^2_m) \quad 0 < \rho_m < 1$$  (24)

---

29 Household labour income and firms' labour costs are the same in the standard CIA model and are not directly affected by the nominal interest rate.
4.2.4 Competitive Equilibrium

Competitive equilibrium of this economy consists of a set of feasible allocations \( \{ y_t, c_t, k_t, M_t, h_t, n_t, x_t \} \), a set of prices \( \{ r_t, w_t, R_t^m, R_t \} \), exogenous shocks \( \{ z_t, u_t \} \), and aggregate outcomes, such that:

- Given \( r_t, w_t, R_t^m \), allocation \( c_t, k_t, M_t, x_t, n_t \) solves the households' problem;
- Given \( r_t, w_t, R_t \), allocation \( h_t, k_t, y_t \) solves the firms' problem;
- The goods, labour, and money markets are clear;

And nominal variables are divided by \( P_t \) in order to transform into a stationary model.

4.3 Calibration

The procedure of calibrating deep structure parameters is to map the model economy into the observed features of data. This means that the steady-state value of the variables can be implied by the deep structure parameters. With given deep structure parameters, the model's steady-state can generate great ratios which can be observed directly from data.

Table 4.1 concludes the behaviour of technology and monetary shocks. Steady-state technology shock has to be normalised equal to one, which indicates \( z = 0 \). Autoregressive process and variation of technology shock follows Cooley and Hansen (1995), which has a persistence parameter of 0.95 and a variance of structure shock of 0.7%. The persistence and variance of money growth rate will be estimated from following the M1 money growth rate regression, with time duration from 1959 Q1 to 2004 Q2. The results indicate that there is a 1.2% money growth rate per quarter at steady-state, with a persistence of 0.64. This compares with Cooley and Hansen (1995) who had steady-state money growth rates of 1.3%, with a persistence of 0.49. It also compares with Benk, Gillman and Kejak (2005) who found a 1.23% steady-state money
growth rate, with a persistence of 0.58. The variance of monetary shock from M1 regression is 0.9%, which is also close to the results of Cooley and Hansen (1995) and Benk, Gillman and Kejak (2005) (which are 0.89% and 0.1%, respectively).

\[
\Delta \log M_t = 0.0045 + 0.64 \Delta \log M_{t-1} \quad \sigma_m = 0.9% 
\]

(0.0009) (0.0545)

Table 4.1: Baseline parameters

<table>
<thead>
<tr>
<th>Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
</tr>
<tr>
<td>( \psi )</td>
</tr>
</tbody>
</table>

Goods Production

| \( \alpha \)  | 0.4 Capital share in goods sector |
| \( \delta \)  | 0.024 Capital stock depreciation rate |
| \( \varepsilon \) | 1 Good sector productivity parameter |

Monetary authority

| \( \Theta^* \) | 1.2% Quarterly money growth rate |

Shocks processes

Autocorrelation parameters

| \( \rho_z \)  | 0.95 Goods sector productivity |
| \( \rho_m \)  | 0.64 Money growth rate |

Standard Deviation of Shock Innovations

| \( \sigma_z \)  | 0.7% Goods sector productivity |
| \( \sigma_m \)  | 0.9% Money growth rate |

Table 4.1 summarises the base line deep structure parameters, which can be implied from two groups of U.S data. Firstly, the data set from Gomme and Rupert (2007) with duration from 1959 Q1 to 2004 Q2 indicates that the quarterly depreciation rate \( \delta \) is
0.024, which compares with the results of Cooley and Hansen (1995) which found that $\delta = 0.019$. It also indicates that the investment-output ratio is 0.26. With a given depreciation rate and investment output ratio, the steady-state capital-output ratio is going to be 10.8.

There are two sources of income for households, which are: labour and capital income. Capital and labour income shares are calibrated in the model by using U.S data from 1959 Q1 to 2004 Q2. The data implies that the share of wage income is equal to 0.6, and which compares with the results of Cooley and Hansen (1995) which were that $\alpha = 0.4$.

With a capital share of 0.4, a depreciation rate 0.024, and a capital-output ratio of 10.8, the model implies that the real interest rate is equal to 0.037 before depreciation rate. Because the quarterly steady-state inflation is equal to the money growth rate, the steady-state Euler equation implies that the preference parameter $\beta$ is equal to 0.987. The U.S data also indicates that the steady-state working hours from the goods producing sector and leisure are 1/3 and 2/3, respectively. This requires deep structure parameters $\Psi$ which are equal to 1.58. With Stockman's (1981) CIA constraint, the model's Euler equation and great ratios imply that $\beta$ is equal to 0.988, and that $\Psi$ is equal to 1.59.

### Table 4.2: Target values

<table>
<thead>
<tr>
<th></th>
<th>Quarterly inflation rate</th>
<th>Investment-output ratio</th>
<th>Capital-output ratio</th>
<th>Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{ss}$</td>
<td>1.2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$i^{ss}/y^{ss}$</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k^{ss}/y^{ss}$</td>
<td>10.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x^{ss}$</td>
<td>2/3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.4 Monetary Transmissions and Real Activities at Steady-State

This section of the chapter compares monetary properties of model at stationary-state with a varying money growth rate, and evaluates the impacts of monetary aggregates on real activity.

With a standard CIA constraint the model includes two monetary transmission channels (which are saving and borrowing rates at stationary-state) and generates a positive relationship between monetary aggregates and real activity. By extending the model with Stockman’s (1981) CIA constraint, the model includes three monetary transmission channels (which are saving, borrowing and real interest rates) and it generates a negative relationship between the money growth rate and real activity. Therefore, the money growth rate at a stationary-state has a positive effect on real activity when a standard CIA constraint is applied and a negative effect when a Stockman CIA constraint is applied.

First of all, the inflation effect of the money growth rate at a stationary-state is given by equation (25), which indicates a positive correlation between the nominal interest rate on saving and the rate of inflation. By combining the households’ and firms’ steady-state CIA constraint, equation (26a and 26b) reflects a negative relationship between borrowing and inflation rates at a stationary-state. In other words, the stationary state money growth rate is positively correlated with the saving rate, and negatively correlated with the borrowing rate.

\[
R_m^s = \frac{\pi^s}{\beta} \tag{25}
\]

\[
R^s = \frac{1}{\beta} \left( \frac{(\frac{m}{c})^s - \pi^s}{(\frac{m}{c})^s - 1} \right) \tag{26a}
\]
\[ R^* = \frac{\left( \frac{m^*}{y^*} - \pi^* \right)}{\beta \left( \frac{m}{y} \right)^* - 1} \]  \hspace{1cm} (26b)

With a standard CIA constraint, the model's stationary state Euler relation, which is represented by equation (27a), indicates that the real interest rate is independent of the rate of inflation and leads to great ratios (such as investment-output, consumption-output and capital-output ratios) which are independent to the rate of inflation or the monetary growth rate at a stationary-state.

With Stockman's (1981) CIA constraint, the model's stationary state Euler relation, which is implied by equation (27b), indicates that the real interest rate varies with inflation or money growth rate, and leads both real price and great ratios to vary with the money growth rate. This reflects the third monetary transmission channel at stationary-state, which has been called a 'real interest rate' effect of the money growth rate. In other words, with a standard CIA constraint, the model implies that monetary aggregates affect real activity through 'liquidity' and 'inflation tax' effects of the money growth rate. With a Stockman CIA constraint, besides the liquidity and inflation tax effects of monetary aggregates, the model includes the third monetary transmission channel, which has been called a 'real interest rate' effect of the money growth rate.

\[ \frac{\alpha \delta}{i^*} \frac{\pi^*}{y^*} = r^* = \frac{1}{\beta} \left( \frac{1}{1 + \delta} \right) \]  \hspace{1cm} (27a)

\[ \frac{\alpha \delta}{i^*} \frac{\pi^*}{y^*} = r^* = \frac{1}{\beta} \left( \frac{1}{1 + \delta} \right) \frac{\pi^*}{y^*} \]  \hspace{1cm} (27b)

By assuming that the firms' wage bill varies with the lending rate, both leisure-consumption and leisure-labour substitutions are affected by the saving rate, and the borrowing and real interest rates. The effects of saving, borrowing, and real interest rates on leisure-consumption and leisure-labour substitutions at a stationary state will be considered as inflation, 'liquidity', and real interest rate effects of the money growth rate.
With a standard CIA constraint, the real interest rate effects of the money growth rate is zero because real prices are independent of the money growth rate. Therefore, equation (29) indicates that leisure-labour substitution is affected by two nominal interest rates, which are saving and borrowing rates. The saving rate increases the leisure-labour substitution and has negative effect on employment and output; the borrowing rate decreases the leisure-labour substitution and has positive effect on employment and output. Since the effect from saving rate is dominated by the effect from borrowing rate, the leisure-labour ratio decrease with monetary expansion and employment and output rises.

With a Stockman CIA constraint, the real interest rate increases with inflation. Equation (28)-(29) indicates that the money growth rate has a negative effect on leisure-labour substitution through the borrowing rate. It also indicates a positive effect on leisure-labour substitution through saving rate, and a positive effect on leisure-labour substitution through the real interest rate. Since the negative effect from the borrowing rate is dominated by the positive effect from the saving and real interest rate, then the model concludes that there is increase in leisure and decrease labour supply with the money growth rate. The monetary expansion has negative effect on employment and output.

\[
\frac{i^s}{y^s} = \frac{\alpha \delta}{r^s}
\]  

(28)

\[
\frac{x^s}{h^s} = \frac{\Psi R^s R^c c^s}{(1-\alpha)y^s}
\]  

(29)

With standard CIA constraint, equation (30) indicates lower capital-labour ratio with monetary aggregates due to cost channel. This is the opposite of the Tobin effect (1965), where increases in the steady state money growth rate will raise the capital labour ratio. And it consistent with Stockman (1981) argument, where it needed the Stockman CIA constraint and the model does not request it.

\[
\frac{k^s}{h^s} = \frac{\alpha R^w w^s}{(1-\alpha)r^s}
\]  

(30)
In conclusion, the financial intermediate CIA model with standard constraint indicates both a real interest rate and great ratios which are independent to the money growth rate at stationary-state. By assuming that firms borrow funds from the capital market to pay the wage bill in advance, real wages are varied with the borrowing rate and this leads to a leisure-labour substitution which is affected by both the saving rate from Fisher relation and by the bond rate from the wage equation. A rising stationary state money growth rate will increase the goods sector labour demand, the capital stock, and the level of real activities (such as output, consumption and investment). Household utility will increase with an increasing money growth rate. This concludes the positive effect of the money growth rate on real activity at stationary-state.

By extending the model with Stockman’s (1981) CIA constraint, the model indicates that both the real interest rate and the consumption-output ratio are increased with the stationary state money growth rate. This leads leisure-labour substitution to affect the saving rate, bond rate, and consumption-output ratio. Increasing the stationary state money growth rate will decrease goods sector labour demand, capital stock, and the level of real activities (such as output, consumption and investment). Household utility will decrease with an increasing money growth rate. This concludes the negative effect of the money growth rate on real activity at stationary-state.

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Table 4.3: the real effectiveness of money growth rate at stationary-state

<table>
<thead>
<tr>
<th></th>
<th>$\Omega = 0$ implies standard CIA constraint</th>
<th>$\Omega = 1$ implies Stockman CIA constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Theta^* = 1.2%$</td>
<td>$\Theta^* = 2%$</td>
</tr>
<tr>
<td>$c^{&quot;}$</td>
<td>1.2055</td>
<td>1.2071</td>
</tr>
<tr>
<td>$i^{&quot;}$</td>
<td>0.4197</td>
<td>0.4203</td>
</tr>
<tr>
<td>$y^{&quot;}$</td>
<td>1.6252</td>
<td>1.6274</td>
</tr>
</tbody>
</table>
4.5 The Model’s Dynamic and Simulation

This section of the chapter analyses the dynamic behaviour of monetary transmissions, discusses the effectiveness of monetary policy shock on real economic activity through various monetary transmission channels, and evaluates the effectiveness of the model economy with business cycle facts.

4.5.1 The Effects on Nominal Interest Rates

The effectiveness of monetary aggregates on nominal interest rates is investigated through the centralised economy as follows. The representative agents’ problem under a centralised economy can be considered as maximising the expected utility subject to lifetime, goods market CIA constraints, and capital market CIA constraints.

\[
\{ \beta' (\ln c_t + \Psi \ln x_t) + [\lambda_t \left( \frac{m_{t-1}}{\pi_t} + r_t k_{t-1} + w_t (1 - x_t) + (R_t^m - 1)n_t - m_t \right] \\
\text{Max}_{E_0} \sum_{t=0}^{\infty} -c_t - k_t - (R_t - 1)b_t + (1 - \delta)k_{t-1}) + \mu_t \left( \frac{m_{t-1}}{\pi_t} - n_t - c_t \right) \\
+ \eta_t (b_t - w(1 - x_t)) \}
\]

\( n_t : \lambda_t (R_t^m - 1) = \mu_t \) \hspace{1cm} (31)

\( b_t : \lambda_t (R_t - 1) = \eta_t \) \hspace{1cm} (32)

\( m_t : \beta E_t \left( \frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}} \right) = \lambda_t \) \hspace{1cm} (33)

Where \( \eta_t \) represents the shadow prices of capital market CIA constraint. According to equations (7), (31), (32) and (33), the Fisher relations on nominal interest rates for the borrowing and saving funds have been represented by following equations, where \( \eta_t - \mu_t = (R_t - R_t^m)\lambda_t \).
Clearly, equation (34) and (35) indicate that there is only an inflation effect on the saving rate and both inflation and liquidity effect on the borrowing rate. The liquidity effect on borrowing rate depends on the term $\eta_i - \mu_t$. With money injections are received by financial intermediates as part of investment fund and zero profit made by financial intermediates assumptions, the equation (36) states the borrowing rate is less than the saving rate with positive money injections from the central bank, which implies that the liquidity effect term $(\eta_i - \mu_t < 0)$ is negative. This is because the value of cash in the capital market is less than the value of cash in the goods market as money injections are received by financial intermediates and loaned to firms in the capital market.

$$R_i = \frac{(\eta_i - \mu_t) + (\lambda_i + \mu_t)}{\beta \varepsilon_i (\lambda_{i+1} + \mu_{i+1})} \pi_{i+1} \quad (34)$$

$$R_i^m = \frac{(\lambda_i + \mu_i)}{\beta \varepsilon_i (\lambda_{i+1} + \mu_{i+1})} \pi_{i+1} \quad (35)$$

4.5.2 Monetary Transmissions and Real Effects

Figures 4.2 and 4.3 reflect the monetary transmissions and real variables response to 1% positive technology and monetary shocks in the model economy, both with and without a Stockman CIA constraint. A negative response of inflation to technology shock reflects the counter-cycle behaviour of the price level. Real economic activities (such as output, consumption, investment and labour supply) are increased with a technology shock. The monetary transmissions (which include savings and borrowing rates) have a positive response to a technology shock. The responses of velocity to a technology shock reflect the relationship between the exchange technology constraint and the quantity theory of money. The Stockman type of exchange technology constraint implies that velocity does not move with a technology shock due to the effectiveness of
technology shock on output which has fully reflected by real money demand. The
standard type of exchange technology constraint indicates that only the effectiveness of
technology shock on consumption is reflected on real money demand. Because the
output increases more than consumption with a technology shock, the quantity theory of
money requests a velocity increase with output.

Figure 4.2: Variables response to a 1% positive technology shock

For the model with a standard CIA constraint, figure 4.3 indicates that the positive
response of the saving rate to monetary expansion reflects the expected inflation effect
of money growth rate. This has been referred to by Cooley and Hansen (1995) as an
‘inflation tax effect’ and it has a negative effect on real economic activity. Since the
money injections are received by financial intermediates instead of households, and
have to be loaned to firms for wage payments in the capital market, it creates excess
demand in the capital market. Since the supply of the capital market is determined by
real activity (which is firms’ wage bills) financial intermediates have to lower the
borrowing rate in order to allow the extra savings funds to be loaned to firms at the
capital market. It is lower interest rate at capital market subject to monetary expansion.
The negative response of the borrowing rate has a positive effect on real activity through the marginal cost of labour. In other words, there are two monetary transmission channels when the standard CIA model is extended with function of financial intermediates, which are: saving and borrowing rates. The money injections increase leisure, decrease the labour supply, and have a negative effect on real activity through the saving rate channel. They decrease the leisure, increase labour demand, and have a positive effect on output through the borrowing rate channel. Figure 4.3 concludes that both employment and output are increased with monetary expansion because the positive effect from the borrowing rate dominates the negative effect from the saving rate.

Figure 4.3: Variables response to a 1% positive monetary shock

According to figure 4.3, the working capital CIA model with standard constraint fails to generate the behaviour of consumption subject to monetary shock since the consumption is decreased instead of increased with monetary expansion. This indicates

---

30 The function of financial intermediates refers to the reception of money injections and savings funds, and the making of loans to borrowers at the capital market.
that the positive effect from the borrowing rate cannot dominate the negative effect from the saving rate on consumption. Increasing aggregate output with money injections raises money demand and velocity through the quantity theory of money. Since the money injections do not appear at the goods market, an increase in the money demand has a negative effect on the goods market price. This leads the price level or inflation rate to move with monetary shock gradually and not immediately.

Beside the borrowing and saving rates, the working capital CIA model with a Stockman constraint introduces an additional monetary transmission channel through the real interest rate in the Euler equation, and has positive effects on real activity (particularly on consumption). This allows the model to overcome the weakness on consumption behaviour. Figure 4.3 shows that the consumption does increase with monetary expansion since the positive effect from borrowing is able to dominate the negative effect from the saving rate, with a lower substitution rate\(^{31}\) of consumption due to the Stockman CIA constraint. The velocity has negative response to monetary shock since money demand increases more than aggregate output. Therefore, the CIA model which employs the Stockman CIA constraint, and assumes that money injections are received by financial intermediates instead of households, is able to examine the behaviour of real activity subject to monetary expansion (such as output, consumption, investment, capital stock and labour supply) through the cost channel of monetary policy under a flexible price framework, without a limited participation monetary shock.

### 4.5.3 Business Cycle Facts

Table 4.4 describes contemporaneous correlations with aggregate output from the log detrended time series data with duration from 1959 Q1 to 2004 Q2. It also summarises the simulated economy statistics with both technology and monetary shocks. The statistics of simulation are computed from an artificial time series consists of 182

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\(^{31}\) The substitution rate is the real interest rate discounted by saving rate, which is indicated by the model's Euler equation.
periods with 50,000 times simulations. In order to compare these results with real data statistics, it has to be taken logged and detrended by a Hodrick-Prescott filter. When a standard CIA constraint is applied (where $\Omega = 0$), the model is able to generate pro-cyclical behaviour on rate of inflation and nominal interest rate; however, it fails to replicate the positive correlation between aggregate output and consumption since consumption has negative response to monetary expansion. Therefore, the model does not imply the consumption behaviours which have been observed in the data. By extending the model with Stockman’s (1981) constraint (where $\Omega = 1$), the model is able to overcome the weakness of the model economy (i.e. consumption behaviour cannot explained). It is not only generate positive correlation between nominal interest rate and aggregate output, but also replicates the positive correlation between output and consumption which has been observed in the data. The weakness of the model is that it cannot explain the pro-cyclical behaviour of the inflation rate.

<table>
<thead>
<tr>
<th>Table 4.4: Contemporaneous correlations with aggregate output</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Output</td>
</tr>
<tr>
<td>Consumption</td>
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<tr>
<td>Investment</td>
</tr>
<tr>
<td>Employment</td>
</tr>
<tr>
<td>Treasury bill rate</td>
</tr>
<tr>
<td>Inflation rate</td>
</tr>
</tbody>
</table>

### 4.6 Discussion

Lucas-Stocky’s (1987) cash-credit goods CIA model has been simulated by Cooley and Hansen (1995), and they have concluded that money injections raise the nominal interest rate and have a negative effect on the real economic activity. Lucas-Fuerst’s (1992) limited participation models generate the negative relation between the money growth rate and the nominal interest rate, and are able to examine the positive impacts
of monetary aggregates on real activity. This chapter integrates the CIA economy with the function of financial intermediates, and allows money injections to be received by financial intermediates instead of households. It is, therefore, able to explain the transmission and real impacts of monetary aggregates. It generates the real impacts of monetary policy without sticky price/wage and limited participation monetary shock.

For the model with a standard CIA constraint, money injections increase the saving rate through an expected inflation effect, and have a negative effect on labour supply. At the same time, it lowers the borrowing rate at the capital market and has a positive effect on labour demand through the marginal cost of labour. Because the positive effect on labour demand dominates the negative effect on labour supply, employment rises with monetary expansion and increases output through the production function. However, the model with a standard CIA constraint is unable to explain the behaviour of consumption because consumption has a negative response to monetary shock, and it leads a negative correlation between consumption and output.

By replacing the standard CIA constraint with a Stockman constraint, the model’s Euler equation indicates that the substitution of current and future marginal utility of consumption is affected by the real interest rate discounted by saving rate. This means that the real interest rate is affected by the money growth rate through Euler equation. This has been considered as a third monetary effectiveness channel (which is referred to as a ‘real interest rate’ effect in this chapter). An increasing real interest rate with a positive monetary shock has a positive effect on current consumption and negative effect on investment. When combined with a lower borrowing rate at the capital market, they are able to dominate the expected inflation effect on consumption and replicate the consumption behaviour which has been observed in the data. Stockman’s (1981) CIA constraint plays a major role to obtain a positive response of consumption to monetary expansion, and a positive correlation between output and consumption is observed in the data.
4.7 Conclusion

In a standard CIA framework (such as that of Cooley and Hansen (1995)) monetary shocks happen before households make a consumption-saving decision, and this allows households to receive money injections and take this into the goods market. The amount of output, or consumption goods, is determined by real activity. The extra money supply in the goods market raises the price of goods, and has a negative effect on output and employment through an expected inflation effect on the nominal interest rate. However, it does not explain the real effectiveness of the money growth rate which can be observed from the data. At mean time, the sticky price models are able to account for positive correlations between monetary aggregates and real activity through Calvo’s (1983) price setting behaviour.

A limited participation model was developed by Lucas (1990), who argues that monetary injections happen after households’ consumption-saving decisions are made and that households cannot take extra money into the goods market because their savings decisions have been made. The extra money supply from the monetary authority is transferred into the capital market. When the numbers of bonds have been determined by household savings, extra money at the capital market increase the price of bonds and decrease the return of bonds. Fuerst (1992) further assumes that the demand of savings funds is equal to firms’ wage bills, and this creates financial friction and generates real impacts of monetary aggregates. This argues that the marginal cost of labour is varied with the nominal interest rate. A lower nominal interest rate on borrowing decreases the marginal cost of labour and increases firms’ borrowing and labour demand. The increasing working hours with a given initial capital stock raises both aggregate output and the real interest rate. In other words, Lucas-Fuerst’s (1992) type of limited participation models employ timing of monetary shock and financial friction created by firms’ borrowing to examine both the lower nominal interest rate and an increasing output with monetary expansion.
This chapter employs McCandless (2008) working capital CIA model by replacing monetary transactions services with Stockman’s (1981) CIA constraint, which exchanges the technology used for both consumption and investment, to explain the real effects of monetary shock and the monetary transmission mechanism. The model does not request limited participation monetary shock from Lucas (1990), but it does assume that money injections are received by financial intermediates instead of households to generate the real impacts of monetary aggregates. With standard CIA constraint, the model is able to explain the employment and output behaviour subject to monetary innovation. But it does not well explain the behaviour of consumption and generate negative correlation between consumption and output. By adding the Stockman constraint, the model is able to explain the cyclical behaviour of consumption and velocity. It generates a positive response of consumption and a negative response of velocity to monetary innovation, and a positive correlation between output and consumption. Therefore, the financial intermediate CIA model with Stockman’s (1981) CIA constraint not only generates the positive response of output, consumption, and investment to monetary expansion but also the negative response of the nominal interest rate to monetary shock. It explains both monetary transmissions and the impacts of monetary shock to real activity.

References


Christiano, Lawrence J (1991) “Modelling the liquidity effect of a money shock”, Quarterly review, Federal Reserve Bank of Minneapolis, 15, 3-34

Proceedings of the Hundred and Fourth Annual Meeting of the American Economic Association, 82(2), (May), 346-53


Chapter 5 - Money and the Business Cycle: The Contribution of Banks and Stockman CIA constraint

Abstract

This chapter studies the interaction between nominal and real variables in both a simple Cash-in-Advance (CIA) economy and a monetary banking economy (where money or exchange credit has been introduced as an exchange technology for goods market transactions). The model economies are evaluated with two types of CIA constraint, which are: a standard constraint and a Stockman (1981) CIA constraint, separately. In the simple CIA economy, monetary aggregates distort allocations because of the tax associated with inflation and the affect leisure-consumption substitution. In the monetary banking economy, monetary aggregates affect real activity through two monetary transmission channels, which are the nominal interest rate and the deposit rate. The first part of this chapter simulates the standard and Stockman CIA models, and evaluates them with monetary business cycle facts. The main finding is that a CIA economy indicates nominal and real interactions at both stationary state and transition period, both with and without a Stockman constraint. The Stockman constraint does not improve the ability of simple CIA model to examine the interaction between nominal and real variables. The second section of this chapter evaluates the monetary banking model with monetary business cycle facts, and it finds that there is a contribution from the banking sector and the Stockman CIA constraint which can be used to examine certain facts on the interaction between nominal and real variables.

32 Money is the only exchange technology for goods market transaction, which comes from Lucas (1982) and Svensson (1985)
33 There are two exchange technologies, which are: money and exchange credit for goods market transaction. This has been discussed in Benk, Gillman and Kejak (2005)
34 Standard CIA constraint refers to exchange technology used for purchasing consumption goods. It indicates that consumption and investment are substitution goods subject to exchange technology.
35 A Stockman CIA constraint argues that both consumption and investment goods have to be purchased by exchange technology. It implies that consumption and investment are complementary goods subject to exchange technology.
5.1 Introduction

The contribution of exogenous monetary shock to economic fluctuations in a CIA economy had been discussed by Cooley and Hansen (1995). They employed Lucas and Stocky's (1987) cash-credit goods CIA model with an RBC framework, and argued that changes in money growth rate affect real variables only to the extent that they signal changes in inflation tax. They concluded that monetary shock does not contribute much to the economic fluctuations in the real variables when money is introduced by requiring a Cash-in-Advance constraint. Combining a CIA constraint with simple a RBC structure model cannot account either for the observed cyclical behaviour of nominal variables or for the interaction between real and nominal variables. Furthermore, Cooley and Hansen (1997) re-examined the Lucas Island model by using the methods of quantitative equilibrium business cycle theory, and concluded that the confusion between aggregate and individual monetary shock could have a significant effect on real economic activity. Cooley and Hansen (1998) simulated three monetary business cycle models, and summarised that neither of three models can capture the phase shift which is found in the correlation of money growth with real variables. The weak correlation between the cyclical component of money growth and prices (or inflation) is puzzling, as is the negative correlation between the money growth rate and nominal interest rates. The models cannot account for the observed counter-cyclical price level and the pro-cyclical rate of inflation simultaneously.

Benk, Gillman and Kejak (2005a) extended Lucas (1982) and Svensson (1985) monetary CIA economy into two exchange technologies framework through the banking sector, and allowed the velocity to fluctuate with the nominal interest rate. They concluded that monetary business cycle facts (with the exception of the liquidity effect) can be explained by adding a banking sector TFP shock into the exchange economy.

36 They are: Cash-in-Advance, staged nominal wage contract, and unanticipated inflation effect models in Cooley and Hansen (1998)
This chapter investigates the real impacts of monetary shock in a simple CIA economy which is drawn from Lucas (1982), where money is the only exchange technology that has been introduced for goods market transactions and the two exchange technologies monetary banking economy from Benk, Gillman and Kejak (2005a), where both money and exchange credit has been introduced as exchange technologies for goods market transactions. It emphasises the transmission, and contribution of monetary shock to output fluctuation. In other words, the chapter formulates, calibrates and simulates dynamic stochastic general equilibrium CIA economy to investigate whether the models are able to account for business cycle facts. The main findings of the chapter are: firstly, both the standard and Stockman's simple CIA models imply the interaction between nominal and real variables, but they cannot account for the business cycle facts; and, secondly, by extending the model with productive banks and a Stockman CIA constraint, the CIA model is able to generate certain economic fluctuation facts.

This chapter has five sections, the first of which is this introduction. Section 2 presents the stylised facts of the business cycle. Section 3 sets up simple CIA economy, discusses the nominal and real interaction on the stationary state, and evaluates the model economy with business cycle facts. Section 4 builds a monetary banking DSGE model, interprets the nominal and real interaction on stationary state, and evaluates the model economy with monetary facts. Section 5 concludes the chapter.

5.2 Style Facts

This part of the chapter reviews some of the monetary features of business cycles through post-war quarterly US data, and compares the features of the model economy which are computed from the artificial economy with the data. Figure 5.1 below
describes the behaviour of M1 (money supply), CPI (price) and GDP (real output) from 1959Q1 to 2004Q2 for the U.S economy. The data has been detrended by a Hodrick-Prescott (HP) filter and expressed in logarithms. Lucas (1977) defined business cycles as the deviations of aggregate real output from trend. This chapter employs an HP trend as long term activity trend, and defined the business cycles as the deviations of economic activity from HP trend.

Figure 5.1: The relations among money, price and output

Clearly, there is co-movement between monetary aggregates and real output before the 1980s. With rapid change in the financial sector after 1980s, monetary aggregates become more volatile than aggregate output. However, there is a positive correlation between monetary aggregates and real output. The right of the figure reflects the counter-cyclical behaviour of price level. This indicates the negative correlation
between price level and aggregate output. This has been discussed in Kydland (1989), Cooley and Ohanian (1991) and Kydland and Prescott (1991). The bottom of the figure indicates that there is negative relation between the deviation of money aggregates and price level. This reflects the contrary spirit, or leaning against the wind of monetary policy. Monetary policy is change in response to changes in inflation and real GDP. For example, if the central bank “Learned Against The Wind,” by easing money market conditions in response to lower inflation or declines in production, and by tightening money market conditions in response to higher inflation or an increase in production. This is reflected through the negative correlation between the money growth rate and the price level negative correlation between money growth rate and output.

Table 5.1 describes the cyclical statistics of the U.S economy, with duration from 1959Q1 to 2004Q2. Following Cooley and Hansen (1995), the statistics of the time series data are obtained from log detrend by a HP filter. This indicates the relative standard deviation of macroeconomic aggregates to output and concludes correlations of real and nominal variables with real GDP.

There are several important characteristics which can be concluded as business cycle facts. Firstly, it indicates that both consumption and capital stock have nearly half of the volatility of aggregate output. It also indicates that investment has about three times the volatility of output, and that labour supply has similar volatility with output. It also concludes that nominal variables have less volatility with output, except for monetary aggregates. Secondly, monetary aggregates, inflation, and the nominal interest rate have positive correlations with aggregate output. In contrast, the money growth rate and price level have negative correlations with aggregate output. Furthermore, there are negative correlations between the money growth rate and real economic activity. The money growth rate is also negatively correlated with the nominal interest rate, which has been interpreted as a liquidity effect of the money growth rate.
Table 5.1: Descriptive statistics of cyclical behaviour of series from 1959Q1 to 2004Q2

<table>
<thead>
<tr>
<th>Relative SD (%)</th>
<th>Cross-correlation with output</th>
<th>Corr with M1 growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>0.44</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.5163</td>
<td>0.43</td>
</tr>
<tr>
<td>Investment</td>
<td>2.7078</td>
<td>0.47</td>
</tr>
<tr>
<td>Capital stock</td>
<td>0.4114</td>
<td>-0.13</td>
</tr>
<tr>
<td>Hours</td>
<td>1.0036</td>
<td>0.14</td>
</tr>
<tr>
<td>M1</td>
<td>1.4206</td>
<td>0.19</td>
</tr>
<tr>
<td>Price level</td>
<td>0.7686</td>
<td>-0.69</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.2854</td>
<td>-0.21</td>
</tr>
<tr>
<td>Treasury rate</td>
<td>0.1809</td>
<td>-0.41</td>
</tr>
<tr>
<td>Velocity</td>
<td>1.7021</td>
<td>-0.21</td>
</tr>
<tr>
<td>M1 growth rate</td>
<td>0.6205</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Figure 5.2 describes the cross correlations between the nominal variables and aggregate output. Clearly, it indicates that future nominal interest rates and inflation are more highly correlated with the current output than contemporaneous nominal interest rates and inflation. It also indicates that current output is more highly correlated with lagged values of money aggregates and lagged price level. A similar finding has been concluded by both Kydland and Prescott (1990) and Cooley and Hansen (1995). There is no evidence that M1 leads the business cycle. Indeed, rather than leading, M1 lags have a stronger relation with aggregate output. Therefore, there is a pronounced phase shift relation between nominal variables (i.e. nominal interest rate, inflation, price level monetary aggregates) and output.
In conclusion, there are three categories of business cycle facts which can be considered as model evaluation, they are: relative volatilities of macroeconomic aggregates, contemporaneous correlations, and pronounced phase shift relation. Following Cooley and Hansen (1995), this chapter has concluded that the monetary business cycle facts are:

- Consumption and capital stocks have half the volatility of output and hours has similar volatility with output.
- Nominal variables, except for M1 and velocity, have lower volatility when compared with output.
- Monetary aggregates, inflation, nominal interest rate and velocity are pro-cyclical.
- Money growth rate and price level are counter-cyclical.
- M1 growth rate and nominal interest rate are negatively correlated, which reflects the existing liquidity effect of money growth rate.
- Lag price level and monetary aggregates have stronger relations with aggregate...
5.3 A simple Cash-in-Advance (CIA) economy

This section of the chapter studies the stationary state and dynamic behaviour of a simple Cash-in-Advance economy when money has been introduced as exchange technology to purchase consumption goods (or both consumption and investment goods). It emphasises the ability of the models to explain the certain facts of economic fluctuations, particularly on the interaction between nominal and real variables. The principle conclusion is that although these simple CIA models are able to imply the interaction between nominal and real variables at both stationary state and transition dynamics, they cannot explain the business cycle facts which have been found in the observed data. This finding is consistent with that of Cooley and Hansen (1995). Even when the simple CIA economy is extended with a Stockman constraint, the model still cannot examine certain of the business cycle facts.

5.3.1 The Structure of the Model Economy

This part of chapter introduces the model economy and displays the problems which are solved by households and firms. It also describes the behaviour of the monetary policy authority. There are two sources of uncertainty in the model, which are: exogenous structure shocks from firms and the money growth rate. Figure 5.3 outlines the structure of a Cash-in-Advance economy. The economy includes three infinitely lived representative agents, which are: households, firms, and the monetary authority. Households receive labour and capital incomes, and exchange these for investment and/or consumption goods. Competitive firms produce final goods, and pay wages and capital bills. Money injections from the monetary authority are received by households for goods market transaction. Households have to hold cash when they purchase consumption and/or investment goods at the goods market. This introduces the rule of
money demand for representative agents and creates an additional constraint for households. There are two types of CIA constraint, which are Lucas's (1982) CIA constraint (where households use money to purchase consumption goods) and Stockman's (1981) CIA constraint (where households use real money balance to purchase both consumption and investment goods). Lucas's (1982) CIA constraint implies that the perfect substitution between exchange technology (real money balance only in simple CIA economy) and investment goods and Stockman's (1981) CIA constraint implies that output and exchange technology are 'complementary goods.' Finally, with infinitely lived representative agents, the model further assumes that agents have to make their consumption and portfolio decisions period by period basis. It is ruling out the multi-period consumption and investment decisions.

Figure 5.3: The structure of a CIA economy

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t) \]

5.3.1.1 Representative households

Household consumers maximize their expected log utility function (1) (which includes consumption \( c_t \) and leisure \( x_t \), with a discount factor \( \beta \in (0,1) \)) and allocate their timing endowment between leisure and working hours \( h_t \).

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t) \]  

(1)
1 = x_t + h_t \quad (2)

Aggregate output \( y_t \) includes consumption and investment \( i_t \) goods, and the next period of capital stock \( k_t \) is accumulated through the law of motion equation (3), with quarterly depreciation rate \( \delta \).

\[ y_t = c_t + i_t \quad (3) \]

\[ i_t = k_t - k_{t-1}(1-\delta) \quad (4) \]

Nominal money stock is the only exchange technology which can be used by households for goods market transaction. Households receive money injections from central bank. The goods market Cash-in-Advance constraint and quantity theory of money are indicated by equation (5) and (6), respectively. When \( \Omega = 0 \), the standard constraint is applied. This indicates that households hold real money balance for consumption purchase only. When \( \Omega = 1 \), the Stockman constraint is applied. This implies that both consumption and investment goods are purchased using real money balance.

\[ M_{t-1} + T_t = P_t(c_t + \Omega i_t) \quad (5) \]

\[ M_t v_t = P_t y_t \quad (6) \]

Where \( P_t \) represents the price level, \( T_t \) stands for money injections from monetary authority, and \( v_t \) represents money velocity respect to output. The next period’s money stock \( M_t \) and bond \( B_t \) holdings are equal to the initial money stock and bond holding plus money injections from the central bank. The money market equilibrium condition is represented by equation (7), where the gross return of bonds is denoted as \( R_t \).

\[ \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{M_{t-1} + T_t}{P_t} + \frac{R_t B_{t-1}}{P_t} \quad (7) \]

Where \( M_{t-1} \) and \( B_{t-1} \) represent the initial money stock and bond holding, respectively.

The sources of household income are labour supply and capital stock, and this is exchanged for consumption and investment goods. Equation (8) implies the lifetime households’ income constraint.

\[ w_t h_t + r_t k_{t-1} = c_t + i_t \quad (8) \]
When the money market condition is combined with household's lifetime income constraints, the lifetime budget constraint which has been faced by household consumers is indicated by equation (9).

\[
\frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t} + \frac{R_t B_{t-1}}{P_t} + w_t h_t + r_t k_{t-1} - c_t - k_t + (1 - \delta)k_{t-1}
\]  

(9)

The equilibrium conditions of household consumers are represented by equations (10)-(14), which come from maximising the expected log utility function subject to lifetime and CIA constraints. Where \( \lambda_t \) and \( \mu_t \) represent the shadow prices of lifetime and good market CIA constraints.

\[
\frac{x_t}{\Psi c_t} = \frac{1 + \mu_t}{\lambda_t} \quad \text{(10)}
\]

\[
\beta E_t \left[ \frac{P_t}{P_{t+1}} \lambda_{t+1} (1 + \frac{\mu_{t+1}}{\lambda_{t+1}}) \right] = \lambda_t
\]  

(11)

\[
\beta E_t \left( \frac{P_t}{P_{t+1}} \lambda_{t+1} R_{t+1} \right) = \lambda_t
\]  

(12)

\[
\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} ((r_{t+1} + 1 - \delta) + (1 - \delta) \Omega \frac{\mu_{t+1}}{\lambda_{t+1}}) \right] = 1 + \frac{\mu_t \Omega}{\lambda_t}
\]  

(13)

The first order Taylor expansion of equilibrium conditions imply that the marginal cost of money, which is represented by relative price between goods market and lifetime constraint, is equal to the nominal return of bonds. The intra-temporal substitution between consumption and leisure that is indicated by equation (10) shows that the marginal cost of money, or nominal interest rate, has a positive effect on leisure and a negative effect on consumption. When \( \Omega = 0 \), it implies that only consumption goods are purchased by household consumers in the goods market. The Euler equation which is implied by equation (14) states that the marginal product of capital stock or real interest rate is independent to nominal interest rate and rate of inflation and it is determined by the current and future shadow price of lifetime budget constraint. It further indicates that the nominal interest rate, or marginal cost of money, moves with expected inflation.
When $\Omega = 1$, it indicates that household consumers hold money stock for both consumption and investment goods transactions. The Euler equation which is indicated by equation (15) concludes that inter-temporal substitution between current and future marginal utility of consumption determines the marginal product of capital stock, or real interest rate, discounted by the nominal interest rate. Therefore, under CIA framework, the Euler equation indicates that the increasing (decreasing) nominal interest rate has negative (positive) effect on current consumption. In contrast, the lower (rise) expected real interest rate has positive (negative) effect on current consumption. With Stockman constraint, the expected real interest rate effect on current consumption is discounted by the increase (or decreasing) nominal interest rate. It leads the positive effect on current consumption stronger compare with standard CIA constraint. In a word, consumption is decrease less in stockman CIA economy with same monetary innovation.

\[
\beta E_t \left[ \frac{c_t R_t}{c_{t+1} R_{t+1}} (r_{t+1} + 1 - \delta) \right] = 1
\]  

(14)

\[
\beta E_t \left[ \frac{c_t r_{t+1}}{c_{t+1} R_{t+1}} + 1 - \delta \right] = 1
\]  

(15)

5.3.1.2 Firms

Aggregate output is produced by representative firms using a Cobb-Douglas production function with exogenous technology $e^{z_t}$, labour, and capital stock. The shares of labour and capital stock are $1 - \alpha$ and $\alpha$, respectively.

\[
y_t = e^{z_t} h^{1-\alpha} k^{\alpha}
\]  

(15)

Assuming that exogenous Total Factors Productivity (TFP) follows AR (1) process with autoregressive parameter $\rho_z$ and structure shock $e_i^z$.

\[
z_t = \rho_z z_{t-1} + e_i^z, \quad e_i^z \sim (0, \sigma_z^2) \quad 0 < \rho_z < 1
\]  

(16)

Firms have sales income from the goods market and pay wages and capital bills to households. The lifetime budget constraint which is faced by firms has been indicated by equation (17). The marginal cost of labour and capital are indicated by equations (18) and (19), which are obtained from maximising the production function subject to
lifetime budget constraint.

\[ y_t = w_t h_t + r_t k_{t-1} \]  

(17)

\[ w_t = (1 - \alpha) \frac{y_t}{h_t} \]  

(18)

\[ r_t = \alpha \frac{y_t}{k_{t-1}} \]  

(19)

Where \( w_t \) represents the real wage, and \( r_t \) stands for the real interest rate. Real price equations indicate that firms employ labour and rent capital from households until the marginal cost of labour and capital are equal to the real wage and real interest rate. This implies that the ratio between share of capital and labour income is constant at rate \( \frac{\alpha}{1 - \alpha} \) across equilibrium.

### 5.3.1.3 Monetary policy

Monetary policy which has been implemented by the central bank through the money supply rule is represented by equation (20). It indicates that the central bank conducts monetary policy through influencing the next period of nominal money supply. With the nominal money demand equal to the nominal money supply at the money market, the money supply rule implies that the short term interest rate has to adjust the money demand equal to the money supply at equilibrium.

\[ M_t = M_{t-1} + T_t \]  

(20)

The exogenous money supply rule is indicated by equation (21). It implies that money injections from the central bank have to depend on steady-state money growth rate \( \Theta^* \), monetary shock \( \epsilon^w \), and initial money stock.

\[ T_t = (\Theta^* + \epsilon^w - 1)M_{t-1} \]  

(21)

The deviation of the money growth rate has been assumed to follow AR (1) process with autoregressive parameters \( \rho_m \) and structure shock \( \epsilon_i^m \).

\[ u_t = \rho_m u_{t-1} + \epsilon_i^m \quad \epsilon_i^m \sim (0, \sigma_m^2) \quad 0 < \rho_m < 1 \]  

(22)
### 5.3.1.4 Competitive Equilibrium

The competitive equilibrium of this economy consists a set of feasible allocations \( \{ y_t, c_t, k_t, M_t, B_t, h_t, x_t \} \), a set of prices \( \{ r_t, w_t, R_t \} \), exogenous shocks \( \{ z_t, u_t \} \), and aggregate outcomes, such that:

- Given \( r_t, w_t, R_t \) allocation \( c_t, k_t, M_t, x_t \) solves the households’ problem;
- Given \( r_t, w_t \) allocation \( h_t, k_t, y_t \) solves the firms’ problem;
- The goods, labour, and money markets are clear;

Nominal variables are divided by \( P_t \) in order to transform into stationary model.

### 5.3.2 Calibration

The procedure of model calibration follows Cooley and Hansen (1995), and emphasises the point that calibrating deep structure parameters is mapping the model economy into observed features of data. This means that the steady-state value of the variables can be implied by deep structure parameters. With given deep structure parameters, the great ratios are generated by the model’s steady-state that is able to be observed directly from the data.

Table 5.2 summarises the base line deep structure parameters of the model economy, which have been indicted by two groups of U.S data. Firstly, the data set from Gomme and Rupert (2007), with duration from 1959 Q1 to 2004 Q2, indicates that quarterly depreciation rate \( \delta \) and investment-output ratio are 0.024 and 0.26, respectively. With a given depreciation rate and investment output ratio, the model implies that the steady-state capital-output ratio is 10.8. The U.S data from 1959 Q1 to 2004 Q2 implies that the share of wage income is equal to 0.6, and further indicates \( \alpha = 0.4 \). This is consistent with the findings of Cooley and Hansen (1995). With a given capital income share equal to 0.4, depreciation rate is 0.024 and the capital-output ratio is equal to 10.8.
The stationary-state Euler equation indicates that the discount factor $\beta$ is equal to 0.987 (and 0.988 if Stockman constraint applied) and implies a real interest rate equal to 0.013, after the depreciation rate. The U.S data also shows that the steady-state working hours from the goods producing sector and leisure are $1/3$ and $2/3$, respectively. It, therefore, requires deep structure parameters $\Psi$ equal to 1.58.

Table 5.2: Baseline parameters

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<thead>
<tr>
<th>Preferences</th>
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<tr>
<td>$\beta$</td>
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<tr>
<td>$\Psi$</td>
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<table>
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<th>Goods Production</th>
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<td>$\alpha$</td>
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<tr>
<td>$\delta$</td>
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<td>$e^\gamma$</td>
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<th>Monetary authority</th>
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<td>$\Theta^*$</td>
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<tr>
<td>$\rho_z$</td>
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<td>$\rho_m$</td>
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<th>Standard Deviation of Shock Innovations</th>
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<td>$\sigma_z$</td>
<td>0.7%</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>0.9%</td>
</tr>
</tbody>
</table>

Table 5.2 also indicates the behaviour of technology and monetary shocks. The steady-state technology shock has been normalised to one. The autoregressive process and variation of technology shock follow the work of Cooley and Hansen (1995), which is equal to 0.95 and 0.7%. M1 growth rate has been considered as a monetary shock process. The persistence and variance of money growth rate has been estimated from
following regression with time duration from 1959 Q1 to 2004 Q2. This indicates that the steady-state money growth rate is 1.2%, with persistence 0.64. This compares with the results of Cooley and Hansen (1995), and Benk, Gillman and Kejak (2005), who found that steady-state money growth rates are 1.3% and 1.23% per quarter, with persistence 0.49 and 0.58 respectively. The variance of monetary shock from following regression is 0.9%, which is also close to the results of Cooley and Hansen (1995), and Benk, Gillman and Kejak (2005), which are 0.89% and 1%, respectively.

\[ \Delta \log M_t = 0.0045 + 0.64 \Delta \log M_{t-1} \quad \sigma_m = 0.9\% \]

\[ (0.0009) (0.0545) \]

Table 5.3: Target values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^{ss} )</td>
<td>1.2%</td>
</tr>
<tr>
<td>( k^{ss} / y^{ss} )</td>
<td>10.8</td>
</tr>
<tr>
<td>( i^{ss} / y^{ss} )</td>
<td>0.26</td>
</tr>
<tr>
<td>( x^{ss} )</td>
<td>2/3</td>
</tr>
<tr>
<td>Quarterly inflation rate</td>
<td>Capital-output ratio</td>
</tr>
</tbody>
</table>

5.3.3 The Model’s Steady-State

This section of the chapter discusses the monetary properties of the model at a stationary-state. The money supply rule indicates that the rate of inflation equals the money growth rate at stationary-state. The long run Fisher relation, which is represented by equation (23), implies that the nominal interest rate is determined by the rate of inflation. Therefore, an increase in the money growth rate is followed by a rise in the inflation and nominal interest rates at stationary-state.

\[ R^{ss} = \frac{\pi^{ss}}{\beta} \]  

(23)

When \( \Omega = 0 \), the CIA constraint implies that households hold cash for consumption goods only. The stationary-state Euler relation is indicated by equation (24), which implies that the real rate is independent to the money growth rate or the rate of inflation.
Equations (25)-(27) further indicate that real wages and great ratios (such as investment-capital, investment-output, and consumption-output ratios) are independent to the money growth rate at a stationary-state. The effectiveness of the monetary growth rate to real economy activity at steady-state comes from intra-temporal substitution between consumption and leisure, which is referred by Cooley and Hansen (1989) as an inflation tax effect. According to equation (28) and (29), an increase in the money growth rate raises leisure-consumption and leisure-labour supply ratios when at a steady-state. When real prices and great ratios are independent to the rate of inflation, increasing the money growth rate increases leisure and decrease the labour supply. Furthermore, since the real prices are independent to the rate of inflation, equation (30) indicates that capital stocks decrease with lower labour supply. The decreasing capital stock lowers the level of investment through the law of motion equation at a steady-state. The level of aggregate output has to decrease with the rate of inflation because of the fall in the supply of labour and capital stock. When the consumption-output ratio is independent to the money growth rate, decreasing the level of output implies a lower level of consumption.

\[
\alpha \frac{Y^\ast}{k^\ast} = r^\ast = \left(\frac{1}{\beta} - 1 + \delta\right) \quad (24)
\]

\[
w^\ast = (1 - \alpha) \left(\frac{r^\ast}{\alpha}\right)^{a-1} \quad (25)
\]

\[
\frac{i^\ast}{y^\ast} = \frac{\delta \alpha}{r^\ast} \quad (26)
\]

\[
\frac{c^\ast}{y^\ast} = (1 - \frac{\delta \alpha}{r^\ast}) \quad (27)
\]

\[
\frac{x^\ast}{\Psi c^\ast} = \frac{R^\ast}{w} \quad (28)
\]

\[
\frac{x^\ast}{h^\ast} = \frac{R^\ast \Psi}{(1 - \alpha) y^\ast} \quad (29)
\]

\[
\frac{w^\ast}{r^\ast} = \frac{1 - \alpha}{\alpha} \frac{k^\ast}{h^\ast} \quad (30)
\]

When \( \Omega = 1 \), the CIA constraint indicates that both consumption and investment goods
have to be purchased by households using exchange technology. The stationary-state Euler relation is indicated by equation (31). It implies that the real interest rate varies with the money growth rate, or rate of inflation, at stationary-state. With equation (25)-(27), real wage and great ratios (such as investment-capital, investment-output, and consumption-output ratios) are varied with the money growth rate at steady-state. The effectiveness of monetary aggregates at stationary-state not only includes the inflation tax effect on leisure-consumption substitution, but also the changing of real prices with nominal interest rate. The increase in the real interest rate with the nominal interest rate lowers the real wage and has a positive effect on leisure-consumption substitution. With an inflation tax effect, it introduces the stronger negative effect on labour supply when compared with a standard constraint CIA economy.

\[
\alpha \frac{y^*}{k^*} = r^* = \left( \frac{1}{\beta} - 1 + \delta \right) R^* \\
(31)
\]

Tobin (1965) argued that higher inflation would induce a portfolio substitution toward capital that would increase the steady-state capital-labour ratio. In contrast, according to equation (30), the simple CIA economy with Stockman constraint indicates that there is a decrease in the capital-labour ratio at steady-state because the real rate increase with nominal rate and real wage decreases with the nominal rate. This is has been called a 'Stockman effect.'

In conclusion, the CIA economy with standard constraint implies that an increase in the money growth rate cannot affect real prices and great ratios at stationary-state. The expected inflation effect is increase the nominal interest rate. The negative effect of monetary aggregates on the level of real variables (such as output and consumption) comes from the intra-temporal substitution equation through the nominal interest rate. The CIA economy with Stockman constraint indicates that real prices and great ratios vary with the money growth rate at stationary-state. The effectiveness of monetary aggregates on the level of real variables comes from both intra-temporal and inter-temporal equations at stationary-state.

5.3.4 The Model’s Dynamic and Findings

This section of the chapter discusses the response of monetary transmission and real activity to technology and monetary innovations. Figure 5.4 reflects the variables response to a 1% technology shock. Clearly, technology innovation can increase the real economic activities, such as: output, consumption, investment and employment. For the response of nominal interest rate, a technology shock has more effect on nominal interest rate when the economy employs a Stockman CIA constraint. A goods sector productivity shock has no effects on the nominal interest rate when the economy employs a standard CIA constraint since the inflation effect cancels out with the real interest rate effect through Fisher’s relation.

Figure 5.4: Variables response to a 1% technology shock

Figure 5.5 reflects the variables response to a 1% monetary shock. First of all, the nominal interest rate has a positive response to monetary innovation through an

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38 The nominal interest rate is the only monetary transmission in simple CIA economy.
expected inflation effect. With an increasing marginal cost of money, the households lower their real money balance and decrease goods market transaction. For the economy with a standard CIA constraint, consumption has a negative response to monetary shock through a goods market exchange technology constraint. Since the standard CIA constraint indicates that investment and exchange technology are perfect substitution goods. The negative effect of monetary shock on consumption implies a positive response of investment to monetary innovation. Through the law of motion equation, this leads the capital stock to rise with the money growth rate. With a given initial capital stock, the response of aggregate output to monetary innovation depends on the real interest rate via the marginal cost of capital and labour supply, through the Cobb-Douglas production function. The negative response of consumption with monetary shock indicates a lower real interest rate through Euler equation. A leisure-consumption substitution equation increases leisure and decreases the labour supply. Therefore, with a given initial capital stock, the negative responses of labour supply and real interest rate to monetary shock indicate an aggregate output decrease with monetary expansion.

Figure 5.5: Variables response to a 1% monetary shock
For the economy with a Stockman CIA constraint, since output and real money demand is ‘complementary goods’, the decreasing money demand with a monetary shock will lower aggregate output through an exchange technology constraint. The negative response of aggregate output to a monetary shock lowers real interest rate with given capital stock. The decreasing real rate has positive effect on current consumption and negative effect on investment. It leads lower investment with monetary expansion. Since the money demand decrease more than investment, the exchange technology constraint requests lower consumption with positive monetary innovation. In other words, the monetary transmission in a simple CIA economy is the nominal interest rate. A positive monetary shock raises the inflation rate through a money supply function, and increase the nominal interest rate through a Fisher relation. This leads money injections to have negative effects on real economic activities, which reduces employment, consumption, and aggregate output.

5.3.5 The Model’s Simulations

Figure 5.6: Money, price and output in a simple CIA economy when $\Omega = 0$
Figures 5.6 and 5.7 reflect the simulation relationship among money, price and output in the economy, both with and without a Stockman CIA constraint. Clearly, both models generate the positive relations between monetary aggregates and price level which has been implied by the money supply rule. The two figures also reflect the negative correlations between deviation of aggregate output and price level which indicates a counter-cyclical behaviour of price level. However, the models fail to generate short run positive co-movement between monetary aggregates and real output. Tables 5.4 and 5.5 summarise the simulated economy’s statistics. The statistics of simulation shown in the tables are the averages of statistics computed from 50,000 simulations. Each simulation consists of 182 periods, which is equal to the number of quarters in the U.S sample used in constructing Table 5.3. The statistics are computed from an artificial time series that has been taken, first logged and then detrended, by using a Hodrick-Prescott filter. Clearly, according to table 5.4, the standard and Stockman CIA models are not able to explain the relative volatility for nominal variables (except for monetary aggregates), such as price level and inflation. Both models are able to generate the interaction between nominal and real variables. But they cannot explain the pro-cyclical or counter-cyclical behaviour of nominal variables, except for price level. The CIA economy with a Stockman constraint is able to explain the correlations between money growth rate and
real activities, such as consumption and investment.

**Table 5.4: Relative standard deviations and contemporaneous correlations**

<table>
<thead>
<tr>
<th></th>
<th>Relative SD (%)</th>
<th>Correlation with $Y_t$</th>
<th>Correlation with M1 growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>$\Omega = 0$</td>
<td>$\Omega = 1$</td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.52</td>
<td>0.54</td>
<td>0.32</td>
</tr>
<tr>
<td>Investment</td>
<td>2.71</td>
<td>3.31</td>
<td>3.18</td>
</tr>
<tr>
<td>Capital stock</td>
<td>0.41</td>
<td>0.27</td>
<td>0.26</td>
</tr>
<tr>
<td>Hours</td>
<td>1.00</td>
<td>0.50</td>
<td>0.61</td>
</tr>
<tr>
<td>M1</td>
<td>1.42</td>
<td>1.82</td>
<td>1.80</td>
</tr>
<tr>
<td>Price level</td>
<td>0.77</td>
<td>1.96</td>
<td>2.07</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.29</td>
<td>1.07</td>
<td>1.18</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>0.18</td>
<td>0.49</td>
<td>0.40</td>
</tr>
<tr>
<td>M1 growth</td>
<td>0.62</td>
<td>0.76</td>
<td>0.77</td>
</tr>
</tbody>
</table>

**Table 5.5: Cross correlations with aggregate output**

<table>
<thead>
<tr>
<th></th>
<th>$\Omega = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3</td>
</tr>
<tr>
<td>M1</td>
<td>0.02</td>
</tr>
<tr>
<td>Price level</td>
<td>0.01</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.04</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\Omega = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3</td>
</tr>
<tr>
<td>M1</td>
<td>0.10</td>
</tr>
<tr>
<td>Price level</td>
<td>-0.03</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.14</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Table 5.5 concludes that the cross correlations of nominal variables with aggregate output are generated by the model economy, both with and without a Stockman constraint. Clearly, there is no phase shift relation between nominal variables and output in the model economy. The fact that the model fails to explain the phase shift relation between nominal variables and output reflects the failure of the Euler equation in the monetary RBC model.

5.3.6 Discussion

This section of the chapter has discussed the stationary state and dynamic behaviours of both standard and Stockman CIA models without a banking sector. At a stationary state, the models imply that there are positive relations among money growth, inflation and nominal interest rates. Monetary aggregates are able to influence real economic activity due to leisure-consumption substitution, which is affected by money growth rate through an expected inflation effect on nominal interest rate. The increasing money growth rate lowers the level of labour supply, consumption, investment and aggregate output when at a stationary-state. For the stochastic version of models, there is a positive response of the inflation rate subject to monetary shock. The nominal interest rate is increased with monetary innovation through an expected inflation effect. The increasing nominal interest rate has a negative effect on consumption, labour supply and it has a positive effect on leisure. This indicates lower real activities (such as lower aggregate output, consumption, and employment) with monetary expansion.

Although the CIA economy is able to generate the interaction of nominal and real variables at steady state and transition period, the standard and Stockman CIA models have difficulty in explaining certain facts on nominal real variables interaction. Therefore, adding a cash-in-advance constraint to a simple RBC structure model cannot account for either the observed cyclical behaviour of nominal variables or the interaction between real and nominal variables. This suggests that in order to successfully account for the interaction between real and nominal variables in the data
we need to introduce more sources of non-neutrality than just the inflation tax. One of the results of an extension of a simple CIA model is that velocity can fluctuate with nominal interest rate. This has been achieved by adding additional exchange technology through the productive banking sector into a Cash-in-Advance economy, which will be discussed in more detail in the following section.

5.4 A Cash-in-Advance Economy with Productive Banks

The following section of the chapter extends the simple CIA economy with productive banks, and includes in addition the monetary transmission channel which does not appear in the simple Cash-in-Advance economy. The major conclusion of this extension is that both standard and Stockman monetary banking models imply the interaction between nominal and real variables, and the Stockman monetary banking model is able to generate some of the business cycle facts.

5.4.1 The Structure of the Model Economy

This section of the chapter introduces the model economy and displays the problems which are solved by households, firms, and banks. It also describes the behaviour of the monetary policy authority. There are three sources of uncertainty in the model, which are: exogenous structure shocks from firms, banks and money growth rate. Figure 5.8 describes the structure of a CIA economy with productive banks. The economy includes four infinitely lived representative agents, who are: household consumers, productive banks, good producing firms and central banks. There are two exchange technologies (i.e. cash and exchange credit) which can be held by households for goods market transaction. And two types of CIA constraints (i.e. standard and Stockman CIA constraints) which can be employed by the economy. With infinitely lived representative agents, the model further assumes that agents have to make their consumption and

portfolio decisions period by period basis. It is ruling out the multi-period consumption and investment decision.

Households collect exchange credit from banks by deducting labour, capital, and deposit income from their lifetime income. Households receive labour and capital income from good producing firms, and consume consumption and investment goods. Firms produce final goods, and pay wage and capital bills. Banks sell exchange credit to households and pay labour, capital, and dividends. Money injections from the central bank are received by household consumers. The function of productive banks is to produce exchange credit to households for goods market transaction. This kind of intra-temporal banking service contrasts with that used by Bernanke, Gertler and Gilchrist (1999). Therefore, the model does not include a ‘financial accelerator’, which has been more commonly used with banking RBC models.

Figure 5.8: The structure of a CIA economy with productive banks

The monetary banking models are very similar with Benk, Gillman and Kejak (2005a) two exchange technologies CIA model. There are two innovations of the monetary

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banking model: the first is that there are two exchange technologies which can be used for goods market transaction, and the second is that velocity is endogenously determined through banking sector and varied with the nominal interest rate. At the beginning of each period, households can either choose cash or credit as their goods market transaction technology. The amount of cash or credit held by households depends on the marginal cost of cash and credit. Velocity is endogenously determined by households' optimal choice between cash and credit holding. Monetary innovations have a positive effect on the nominal interest rate and velocity through an expected inflation effect and credit production in the banking sector. Besides the nominal interest rate, the economy includes an additional monetary transmission channel, which is the velocity of money growth rate through the deposit rate, or the marginal cost of exchange technology. Therefore, the real impacts of monetary aggregates come from both inflation tax and the deposit rate, which contrasts the effectiveness of Cooley and Hansen (1995) CIA model from inflation tax effect only. In other words, the monetary banking model not only includes the nominal interest rate but also includes the velocity channel of monetary aggregates.

5.4.1.1 Banks

Competitive banks produce exchange credit \( f_t \) through a Cobb-Douglas production function, which comes from Clark (1984), with labour, capital, and deposits. The shares of labour, capital, and deposits are \( \gamma_1 \), \( \gamma_2 \), and \( 1 - \gamma_1 - \gamma_2 \), respectively.

\[
f_t = A_q e^\theta (l_t^f)^{\gamma_1} (s_t^f k_{t-1})^{\gamma_2} d_t^{1 - \gamma_1 - \gamma_2} \tag{1}
\]

Where \( l_t^f \) and \( s_t^f \) represents labour and the share of capital stocks in the banking sector, and \( d_t \) stands for deposits from households. It is assumed that an exogenous credit technology shock follows AR (1) process with autoregressive parameter \( \rho_q \) and structure innovation \( \varepsilon_t^q \).

\[
q_t = \rho_q q_{t-1} + \varepsilon_t^q, \quad \varepsilon_t^q \in (0, \sigma^2_q), \quad 0 < \rho_q < 1 \tag{2}
\]

Households pay labour, capital, and deposits to collect exchange credit from banks.
Banks sell exchange credit to households at price $p^d_i$. The lifetime budget constraint which is faced by banks is represented by equation (3). It includes exchange credit income and labour, capital stocks, and deposits cost.

$$r^d_i d_i = p^d_i f_i - w_i l^d_i - r_i s_i k_{i-1}$$

(3)

$$w_i = p^n y_1 f_i \frac{l_i}{l^d_i}$$

(4)

$$r_i = p^n y_2 f_i \frac{s_i}{s_i k_{i-1}}$$

(5)

$$r^d_i = p^n (1 - y_1 - y_2) \frac{f_i}{d_i}$$

(6)

Where $w_i$, $r_i$, and $r^d_i$ represent real wage, real interest rate, and deposit rate, respectively. The marginal cost of labour, capital, and deposits that have been indicated by equations (4), (5), and (6) are obtained by maximising credit production function (1) subject to lifetime budget constraint (3). The equations imply that households collect exchange credit from banks with labour, capital and deposits cost. For each unit exchange credit is obtained, and there are labour, capital and deposit incomes deducted from households' aggregate income. The model assumes that the amount of households' deposits at banks is equal to the amount of exchange technologies, which includes both cash and exchange credits which have been used at the goods market. This has been represented by equation (7).

$$d_i = \frac{M_{i-1}}{P_i} + T_i + f_i$$

(7)

Where $P_i$ represents price level, $M_{i-1}$ stands for initial money holding, and $T_i$ represents money injections from the central bank. With equation (6), equation (7) indicates that the deposit rate is the marginal cost of exchange technology.

5.4.1.2 Household Consumers

Representative households maximise their expected log utility function (8), which
includes consumption \( c_t \), leisure \( x_t \), with discount factor \( \beta \in (0,1) \). And they allocate their time endowment among leisure, firms, banks and capital stocks between firms and banks.

\[
U = E_t \sum_{s=0}^{\infty} \beta^s (\ln c_t + \Psi \ln x_t)
\]  
\[
1 = x_t + l_t^x + l_t^f
\]  
\[
1 = s_t^x + s_t^f
\]

Where \( l_t^x \) and \( s_t^x \) represents labour and capital shares in the goods market. Aggregate output \( y_t \), includes consumption and investment goods and the law of motion of capital stocks, have been written as equation (11) and (12). Physical capital stocks have a quarterly depreciation rate \( \delta \).

\[
y_t = c_t + i_t
\]  
\[
i_t = k_t - k_{t-1}(1-\delta)
\]

Where \( i_t \) represents investment, \( k_t \) indicates the next period's capital stocks. Compare this with a simple CIA economy which assumes that the real money balance is the only exchange technology for goods market transaction. The models considered that there are two exchange technologies which are used for goods market transaction, they are: cash and exchange credit. Both of the models consider money injections from central bank as a lump sum transfer to households, and can be added into the goods market. The CIA constraint which has been faced by households at the goods market is represented by equation (13). The equation (14) indicates the amount of exchange credit purchase goods and can be considered as money velocity with quantity theory of money equation. The equation (15) reflects the relation between fraction of credit purchase goods and marginal cost of exchange technology.

\[
\frac{M_t}{P_t} + T_t + f_t = c_t + \Omega i_t
\]  
\[
f_t = (1-a_t)(c_t + \Omega i_t)
\]  
\[
r_t^d = p_t' (1 - \gamma_1 - \gamma_2)(1-a_t)
\]

Where \( a_t \) represents the fraction of cash purchase goods. When \( \Omega = 0 \), the exchange
technology constraint implies that households hold a real money balance and exchange credit for consumption. If $\Omega = 1$ indicates that exchange technologies are held by households for both consumption and investment, and this has been called a Stockman (1981) CIA constraint in the literature. The next period’s money stocks and bond holding is equal to the initial money stocks and bond holding plus money injections from central bank, with a gross return of bonds $R_t$. Equation (16) represents the next period portfolio holdings for households at the money market.

$$\frac{M_t + B_t}{P_t} = \frac{M_{t-1} + T_t + R_t B_{t-1}}{P_t} \quad (16)$$

Where $B_{t-1}$ represent the initial bond holding. The sources of households’ income include labour and capital stock income, and these are spent on consumption and investment. The lifetime income constraint which is faced by households has been indicated by equation (17).

$$w_t l^e_t + r_t (s^e_t k_{t-1}) = c_t + i_t \quad (17)$$

By combining money market and households’ lifetime income constraints, the next period portfolio holding by households is indicated by equation (18). It indicates that there are three sources of households’ income, which are: labour income $w_t l^e_t$, capital income $r_t k_{t-1}$, dividend income $r_t^d d_t$ and the income which households’ have to be spent on consumption, investment and the cost of exchange credit.

$$\frac{M_t + B_t}{P_t} = \frac{M_{t-1} + T_t + R_t B_{t-1}}{P_t} + \frac{M_{t-1} + T_t + R_t B_{t-1}}{P_t}$$

Model equilibrium conditions of households are indicated by equation (19)-(23), which come from maximising the expected log utility function subject to lifetime and good market CIA constraint.

$$\frac{M_t}{\lambda_t} = p^f_t \quad (19)$$
Where \( \lambda_i \) and \( \mu_i \) represent the shadow prices of lifetime and good market CIA constraints. Equilibrium conditions have implied an exchange credit price which is equal to the marginal cost of money across equilibrium. The marginal utility of consumption not only depends on the shadow price of lifetime budget constraint and the marginal cost of money, but is also influenced by the deposit rate: which can be interpreted as a velocity effect of the money growth rate. Equation (20) indicates that the substitution between leisure and consumption is affected by the nominal interest rate and deposit rate. Equation (22) indicates the long run Fisher relation where the nominal interest rate is equal to the inflation rate plus the real interest rate at the model’s steady-state. If \( \Omega = 0 \) implies that only consumption goods were purchased by exchange technologies, then this indicates that the current and future marginal utility of consumption substitution is influenced by the real interest rate, the current and future nominal interest rate, and the deposit rate.

\[
\frac{x_i}{\Psi c_i} = \frac{1 + \mu_i - r^d}{\lambda_i} \frac{w_i}{w}
\]

(20)

\[
\beta E_i \left[ \frac{P_{r+1} - \lambda_{r+1}}{P_{r+1} \lambda_{r+1}} (1 + \frac{\mu_{r+1}}{\lambda_{r+1}}) \right] = \lambda_i
\]

(21)

\[
\beta E_i \left( \frac{P_{r+1} - \lambda_{r+1} R_i}{P_{r+1}} \right) = \lambda_i
\]

(22)

\[
\beta E_i \left[ \frac{\lambda_{r+1}}{\lambda_i} \left( (r_{r+1} + 1 - \delta) + (1 - \delta) \Omega \left( \frac{\mu_{r+1}}{\lambda_{r+1}} - r^d_{r+1} \right) \right) \right] = 1 + \left( \frac{\mu_i}{\lambda_i} - r^d_i \right) \Omega
\]

(23)

If \( \Omega = 1 \) implies that both consumption and investment goods were purchased by money and exchange credit, then this indicates that the rate which is determined by the current and future marginal utility of consumption is the real interest rate discounted by the difference between nominal interest rate and deposit rate.

\[
\beta E_i \left[ \frac{c_i}{(R_i - r^d_i)} (1 - \delta) \right] = 1
\]

(24)

\[
\beta E_i \left[ \frac{r_{r+1}}{1 + p_{r+1} - r^d_{r+1}} + 1 - \delta \right] = 1
\]

(25)
5.4.1.3 Firms

The aggregate output that has been produced by goods producing firms is indicated by equation (26), through a Cobb-Douglas production function with labour and capital stock. The shares of labour and capital are $1 - \alpha$ and $\alpha$, respectively.

$$y_i = e^z_i (l_t^x)^{1-\alpha} (s_t^x k_{t-1})^{\alpha}$$  \hspace{1cm} (26)

It is assumed that exogenous Total Factors Productivity (TFP) follows AR (1) process with autoregressive parameter $\rho_z$ and structure shock $\epsilon_t^z$.

$$z_t = \rho_z z_{t-1} + \epsilon_t^z, \quad \epsilon_t^z \in (0, \sigma^2) \quad 0 < \rho_z < 1$$  \hspace{1cm} (27)

Firms have sale income from the goods market, and pay wages and the capital bill to household consumers. The lifetime budget constraint which is faced by firms has been indicated by equation (28). The marginal cost of labour and capital which is indicated by equation (29) and (30) are obtained from maximising the production function subject to lifetime budget constraint.

$$r_l (s_t^x, k_{t-1}) + w l_t^x = y_t$$  \hspace{1cm} (28)

$$w_t = (1 - \alpha) \frac{y_t}{l_t^x}$$  \hspace{1cm} (29)

$$r_c = \alpha \frac{y_t}{s_t^x k_{t-1}}$$  \hspace{1cm} (30)

The real price equations indicate that firms employ labour and rent capital from households until the marginal cost of labour and capital is equal to the real wage and the real interest rate. The share between capital stock and labour demand cost is constant at rate $\frac{\alpha}{1 - \alpha}$ across equilibrium.

5.4.1.4 Monetary policy

The monetary policy that has been implemented by the central bank is represented through the money supply rule in equation (31). This indicates that the central bank conducts monetary policy through influencing the next period of nominal money supply.
With nominal money demand equal to the nominal money supply at money market, the money supply rule indicates that the short term interest rate will adjust money demand equal to money supply at equilibrium.

\[ M_t = M_{t-1} + T_t \]  

(31)

Monetary expansion is represented by equation (32). It indicates that money injections from central bank depend on a steady-state money growth rate \( \Theta^* \), monetary shock \( e^m \), and initial money stock.

\[ T_t = (\Theta^* + e^m - 1) M_{t-1} \]  

(32)

The deviation of the money growth rate is assumed to follow AR (1) process with autoregressive parameter \( \rho_m \) and structure shock \( \varepsilon^m \).

\[ u_t = \rho_m u_{t-1} + \varepsilon^m, \quad \varepsilon^m \in (0, \sigma_m^2), \quad 0 < \rho_m < 1 \]  

(33)

5.4.1.5 Competitive Equilibrium

Competitive equilibrium of this economy consists a set of feasible allocations \{ \( y, c, k, M, B, l^g, l^f, x, s^g, s^f, f, d \} \), a set of prices \{ \( r, w, p^d, p^f, R \) \}, exogenous shocks \{ \( z_t, q_t, u_t \) \}, and aggregate outcomes, such that:

- Given \( r, w_t, r^d_t, p^f_t, R_t \) allocation \( c, k, M_t, B_t, x_t, f_t \) solves the households’ problem;
- Given \( r_t, w_t, r^d_t, p^f_t \) allocation \( l^g_t, s^g_t, f_t, d_t \) solves the banks’ problem;
- Given \( r_t, w_t \) allocation \( l^g_t, s^g_t, y_t \) solves the firms’ problem;
- The goods, labour, credit, and money markets are clear;

And nominal variables are divided by \( P_t \) in order to transform into stationary model.

5.4.2 Calibration

The procedure of calibrating deep structure parameters is to map the model economy
into the observed features of data. This means that the steady-state value of the model economy should be implied by deep structure parameters. In other words, with given deep structure parameters, the great ratios which are indicated by the model's steady state are observed directly from real data.

Table 5.6 summarises the value of the deep structure parameters which have been implied by U.S data. The data set from Gomme and Rupert (2007), with duration from 1959 Q1 to 2004 Q2, indicates that quarterly depreciation rate $\delta$ is 0.024 and the investment-output ratio is 0.26. With a given depreciation rate and investment output ratio, the steady-state of this economy implies that the capital-output ratio is 10.8. Capital and labour income shares are calibrated by using U.S data from 1959 Q1 to 2004 Q2. The data shows that the share of wage income is equal to 0.6, and it further indicates $\alpha = 0.4$, that is same result as that of Cooley and Hansen (1995). With capital income share equal to 0.4, the depreciation rate is 0.024 and the capital-output ratio is equal to 10.8. The stationary state Euler equation indicates that discount factor $\beta$ is equal to 0.987 (0.988 if a Stockman constraint is applied), which further implies that the real interest rate is equal to 0.013 after depreciation rate. U.S. data shows that steady state working hours and leisure is 1/3 and 2/3, respectively. It requires deep structure parameters $\Psi$ to be equal to 1.6.

Table 5.6 concludes the behaviour of technology, credit, and monetary shocks. A steady-state technology shock will be normalised to be equal to one. Autoregressive process and variation of technology shock follow Cooley and Hansen (1995), which is equal to 0.95 and 0.7%, respectively. The M1 growth rate has been considered as a monetary shock process. The persistence and variance of money growth rate has been estimated from the following regression, with time duration from 1959 Q1 to 2004 Q2. It indicates that the steady-state money growth rate is 1.2%, with persistence 0.64. This result compares with that of Cooley and Hansen (1995) and Benk, Gillman and Kejak (2005), which found that steady-state money growth rates are 1.3% and 1.23% per quarter, with persistence 0.49 and 0.58. The variance of monetary shock from the regression is 0.9%, which compares with Cooley and Hansen (1995) who found that it is 0.89%, and Benk,
Gillman and Kejak (2005) who found that it is 1%.

\[
\Delta \log M_t = 0.0045 + 0.64 \Delta \log M_{t-1} \quad \sigma_m = 0.9\% \quad (e)
\]

(0.0009) (0.0545)

Table 5.6: Baseline parameters

<table>
<thead>
<tr>
<th>Preferences</th>
<th>( \beta )</th>
<th>0.987/0.988</th>
<th>Discount factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Psi )</td>
<td>1.6</td>
<td>Leisure weight</td>
<td></td>
</tr>
<tr>
<td>Goods Production</td>
<td>( \alpha )</td>
<td>0.4</td>
<td>Capital share in goods sector</td>
</tr>
<tr>
<td></td>
<td>( \delta )</td>
<td>0.024</td>
<td>Capital stock depreciation rate</td>
</tr>
<tr>
<td></td>
<td>( e^z )</td>
<td>1</td>
<td>Good sector productivity parameter</td>
</tr>
<tr>
<td>Banking sector</td>
<td>( \gamma_1 )</td>
<td>0.096</td>
<td>Labour share in credit production</td>
</tr>
<tr>
<td></td>
<td>( \gamma_2 )</td>
<td>0.064</td>
<td>Capital share in credit production</td>
</tr>
<tr>
<td>Monetary authority</td>
<td>( \Theta^* )</td>
<td>1.2%</td>
<td>Quarterly money growth rate</td>
</tr>
<tr>
<td>Shocks processes: autocorrelations and standard deviation of innovations</td>
<td>( \rho_z )</td>
<td>0.95</td>
<td>Goods sector productivity</td>
</tr>
<tr>
<td></td>
<td>( \rho_q )</td>
<td>0.95</td>
<td>Banking productivity</td>
</tr>
<tr>
<td></td>
<td>( \rho_m )</td>
<td>0.64</td>
<td>Money growth rate</td>
</tr>
<tr>
<td></td>
<td>( \sigma_z )</td>
<td>0.7%</td>
<td>Goods sector productivity</td>
</tr>
<tr>
<td></td>
<td>( \sigma_q )</td>
<td>0.7%</td>
<td>Banking productivity</td>
</tr>
<tr>
<td></td>
<td>( \sigma_m )</td>
<td>0.9%</td>
<td>Money growth rate</td>
</tr>
</tbody>
</table>

The U.S. data from 1959 Q1 to 2004 Q2 implies that money-consumption and money-output ratios are 0.67 and 0.5. For banking sector calibration, with an exchange technology constraint, it requests that the marginal cost of exchange technology is equal
to the marginal cost of capital. This means that the deposit rate is equal to the real rate at stationary-state. It implies \( \gamma_1 + \gamma_2 = 0.16 \) through the marginal cost of deposit equation in the banking sector. Since the sectors have labour-capital ratios which equal 3/2, it indicates that \( \gamma_1 = 0.096 \) and \( \gamma_2 = 0.064 \), respectively.

\begin{table}[h]
\centering
\caption{Target values}
\begin{tabular}{|c|c|}
\hline
\( \pi^{**} \) & 1.2\% & Quarterly inflation rate \\
\hline
\( k^{**} / y^{**} \) & 10.8 & Capital-output ratio \\
\hline
\( i^{**} / y^{**} \) & 0.26 & Investment-output ratio \\
\hline
\( m^{**} / c^{**} \) & 0.67 & Money-consumption ratio \\
\hline
\( m^{**} / y^{**} \) & 0.5 & Money-output ratio \\
\hline
\( x^{**} \) & 2/3 & Leisure \\
\hline
\end{tabular}
\end{table}

5.4.3 The Model’s Steady-State

This part of the chapter discusses the steady-state properties of the model economy. First of all, the money supply rule indicates that inflation is equal to money growth rate at stationary-state. The long run Fisher relation, which is represented by equation (34), implies that the nominal interest rate is determined by the rate of inflation and households’ time preference. Equation (35) shows that the marginal cost of exchange credit equals the marginal cost of money at stationary-state. Therefore, an increase in the money growth rate raises the nominal interest rate and the cost of exchange credit through an inflation effect of the money growth rate.

\[
R^{**} = \frac{\pi^{**}}{\beta} \tag{34}
\]

\[
p^{**}_c = R^{**} - 1 \tag{35}
\]

The stationary-state cash-credit ratio which is represented by equation (36) reflects the fraction of cash and credit purchase goods. Since the deposit rate is affected by the fraction of credit purchase goods, it can be considered as a velocity channel of the
money growth rate at steady-state.

\[
\frac{m^{ss}}{f^{ss}} = \frac{a^{ss}}{1-a^{ss}} \tag{36}
\]

\[
r_{ss}^d = \left(\frac{\pi^{ss}}{\beta} - 1\right)(1-\gamma_1-\gamma_2)(1-a^{ss}) \tag{37}
\]

For the real side of the economy, when \( \Omega = 0 \) then the Euler equation at stationary-state implies that the real rate is independent of the money growth rate, and further indicates that real wages and great ratios are independent of monetary aggregates. This is consistent with a simple CIA economy.

The effectiveness of monetary aggregates at steady-state has been reflected in the leisure-consumption substitution equation. According to equations (40) and (41), increasing the nominal interest rate with constant consumption-output ratio has a positive effect on the leisure-labour ratio. The difference from a simple CIA economy is that the velocity effect of the money growth rate, which is represented by the deposit rate, appears in a leisure-consumption substitution. It has a negative effect on the leisure-labour ratio when at a steady-state. Since the ‘inflation tax’ effect dominates the velocity effect, the goods sector labour supply is decreased with increasing money growth rate when at a steady-state. According to equation (42), labour in the banking sector is increased with the money growth rate when at a steady-state.

\[
\alpha \frac{y^{ss}}{(s,k)^{ss}} = r^{ss} = \frac{1}{\beta} - 1 + \delta \tag{38}
\]

\[
w^{ss} = (1-\alpha)(\frac{r^{ss}}{\alpha})^{a-1} \tag{39}
\]

\[
\frac{x^{ss}}{\Psi e^{ss}} = \frac{(R^{ss} - r_{ss}^d)}{w^{ss}} \tag{40}
\]

\[
\frac{x^{ss}}{l_k^{ss}} = \frac{(R^{ss} - r_{ss}^d)\Psi c^{ss}}{(1-\alpha) y^{ss}} \tag{41}
\]

\[
\frac{l_f^{ss}}{l_k^{ss}} = \frac{(R^{ss} - 1)\gamma_1 (1-a^{ss}) c^{ss}}{(1-\alpha) y^{ss}} \tag{42}
\]

When \( \Omega = 1 \), the Stockman constraint is applied. The model’s long run Euler relation
indicates that the real return is affected by the nominal interest rate and deposit rate. Since the nominal interest rate dominates the velocity effect, the real interest rate increases with the money growth rate and lowers real wages. This leads real prices and great ratios to vary with the money growth rate and have a negative effect on the level of real activity when at a steady-state. Therefore, with same proportion increase in money growth rate, the model with a Stockman constraint has a stronger negative effect on the level of real activity because it includes the negative effect from real prices or great ratios.

\[
\alpha \frac{y''}{(s_k k)''} = r'' = \frac{1}{\beta} (1 + \delta)(R'' - r_d'')
\]

In conclusion, the monetary banking model with standard constraint generates positive relations among money growth rate, inflation rate, and the nominal interest rates at steady-state. Since leisure-labour substitution is affected by monetary aggregates through nominal interest rates and deposit rates, monetary expansion has a negative effect on the level of consumption, output, and goods sector labour supply at steady-state with the nominal interest rate effect dominates the money velocity effect. Furthermore, the leisure-labour substitution is not only affected by nominal interest rate and deposit rate, but it is also influenced by real prices or great ratios when the model employs a Stockman constraint. Since both real prices and inflation tax effects have a negative effect on real activity, the monetary model with Stockman constraint indicates that there is a negative relation between real activity and monetary aggregates.

5.4.4 The Model's Dynamics and Findings

This section of the chapter discusses and analysis the impacts of technology, credit and monetary shocks on monetary transmissions (which are nominal interest rate and deposit rate) and real economic activity.

Figures 5.9 and 5.10 illustrate the variables response to 1% technology and credit shocks of the model, both with and without a Stockman constraint. Clearly, the real
variables increase with technology shock, and monetary transmissions (which include nominal interest rate and deposit rate) have a stronger response to technology shock when a Stockman CIA constraint is applied. The credit shock increases aggregate output and employment, and lowers the nominal interest rate. The deposit rate has a stronger negative effect when a Stockman constraint is applied. The credit shock increases consumption and lowers investment through nominal interest rate when a standard constraint is applied. The consumption is decreased in first period and increases after one year, due to the positive effect of credit shock on investment which decreases faster than output. The deposit rate does not varied with technology and credit shock since the negative effect of nominal interest rate is cancelled with positive effect of money velocity.

*Figure 5.9: Variables response to a 1% technology shock*
Figure 5.10: Variables response to a 1% credit shock

Figure 5.11 indicates the impulse responses of a variable to 1% positive monetary shock. First of all, the nominal interest rate is increased with monetary innovation through an expected inflation effect. The deposit rate also raises with monetary shock since the fraction of credit purchase goods increases with the nominal interest rate. When compared with a simple CIA economy, there are two monetary transmission channels (i.e. nominal interest rate and deposit rate) for the real effects of monetary banking model. An increase in the nominal interest rate and the deposit rate has negative and positive effects on labour supply through intra-temporal substitution equation. Since the nominal interest rate dominates the deposit rate effects on labour supply, employment in the goods sector decreases with monetary expansion. With a given initial capital stock, output is decreased with labour supply through a Cobb-Douglas production function.

The increasing in the nominal interest rate lowers exchange technology holdings since the marginal cost of exchange technology increases. It decreases consumption through an exchange technology constraint. When a standard constraint is applied, exchange
technology and investment are substitution goods. With an increasing marginal cost of exchange technology, the households substitute away from holding exchange technology for consumption and towards investment goods. This increases investment subject to a monetary shock. With a given initial capital stock, the increase in investment raises the next period’s capital stocks through the law of motion equation. Aggregate output is decreased at the beginning period with monetary expansion since the labour supply decreases, and it moves back to equilibrium faster due to an increase in the next period’s capital stock.

Figure 5.11: Variables response to a 1% monetary shock

The lower exchange technology holding with monetary expansion has a negative effect on money and exchange credit demand. The increasing marginal cost of exchange credit with nominal interest rate has a positive effect on exchange credit supply through the banking sector. With a supply effect which dominates the demand effect, the exchange credit increases with monetary expansion. Therefore, the monetary expansion in the monetary banking model with standard constraint increases the nominal interest rate and deposit rate, lowers the exchange technology and consumption, increases investment
and capital stocks, and decreases employment and output.

For the monetary banking model with a Stockman constraint, figure 5.11 indicates that the lower exchange technology with nominal interest rate decreases aggregate output through a goods market exchange technology constraint. This lowers both consumption and investment goods. Therefore, in contrast to a standard constraint, the Stockman constraint in a monetary banking model, which leads not only to consumption but also to investment, has a negative response to a monetary shock. There is an additional monetary transmission channel which is created by extending the simple CIA economy with an intra-temporal banking sector. The real effectiveness of monetary shock depends on the nominal interest rate and the deposit rate. Figure 5.11 shows that the monetary aggregate lowers employment and output because the inflation tax effect through the nominal interest rate dominates the money velocity effect through the deposit rate. Even with a Stockman constraint, the monetary banking models are not able to generate the positive response of aggregate output subject to monetary expansion.

5.4.5 The Model’s Simulations and Business Cycle Facts

This section of the chapter analyses the interaction between real and nominal variables that are obtained from the simulated economy, and compare these with the monetary business cycle facts which have been observed in the data. Figures 5.12 and 5.13 represent the relationships among monetary aggregates, price levels, and output in a banking CIA economy, both with and without a Stockman constraint. Clearly, both economies imply that there is a significant positive co-movement between monetary aggregate and price level because the money supply rule has been employed and it also can generate the counter-cyclical movement of price levels. However, there is no clear co-movement between monetary aggregates and output in the monetary banking model, which compares with the significant negative relation that has been generated by a simple CIA economy and the positive correlation that has been observed in the data.
Tables 5.8, 5.9 and 5.10 summarise the simulated economy statistics for a two exchange technology CIA economy; where $\Omega = 0$ reflects the exchange technology is only used for consumption, and $\Omega = 1$ shows that both consumption and investment have to be purchased by households using exchange technology. The statistics of this simulation...
are computed from an artificial time series consists of 182 periods, with 50,000 times simulations. In order to compare these results with the real data statistics, they have to be taken, logged, and detrended by using a Hodrick-Prescott filter. Table 5.8 argues that the models may not be good at explaining the relative volatility of monetary aggregates and price level, but they are able to examine the relative volatility of nominal interest rate and income velocity.

Table 5.8: Relative standard deviation (%)

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Ω = 0</th>
<th>Ω = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.5163</td>
<td>0.3287</td>
<td>0.3164</td>
</tr>
<tr>
<td>Investment</td>
<td>2.7078</td>
<td>3.1700</td>
<td>3.1200</td>
</tr>
<tr>
<td>Capital stocks</td>
<td>0.4114</td>
<td>0.2700</td>
<td>0.2600</td>
</tr>
<tr>
<td>Hours</td>
<td>1.0036</td>
<td>0.4900</td>
<td>0.4800</td>
</tr>
<tr>
<td>M1</td>
<td>1.4206</td>
<td>1.7745</td>
<td>1.8939</td>
</tr>
<tr>
<td>Price level</td>
<td>0.7686</td>
<td>2.1878</td>
<td>2.5737</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.2854</td>
<td>1.4900</td>
<td>1.7800</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.1809</td>
<td>0.2400</td>
<td>0.1400</td>
</tr>
<tr>
<td>Velocity respect to income</td>
<td>1.7021</td>
<td><strong>1.1700</strong></td>
<td><strong>1.2200</strong></td>
</tr>
<tr>
<td>M1 growth rate</td>
<td>0.6205</td>
<td>0.7600</td>
<td>0.7800</td>
</tr>
</tbody>
</table>

The interactions between aggregate output and nominal variables have been implied by Table 5.9. When a standard constraint is applied (Ω = 0), the monetary banking model explains the negative correlations between money growth rate and real activities, except for capital stocks and investment. It also generates the positive correlations between aggregate output and real variables, and the negative correlations between output and nominal variables (except for velocity). Clearly, although the monetary banking model is able to explain both the volatility and pro-cyclical behaviour of income velocity and the counter-cyclical behaviour of price level, it fails to generate a negative correlation between nominal interest rates and money growth rate (which is a liquidity effect on the nominal interest rate) and it cannot generate the pro-cyclical behaviour of nominal
interest rate and rate of inflation. When a Stockman constraint is applied \((\Omega = 1)\), the monetary banking model is able to explain the pro-cyclical behaviour of the nominal interest rate, but it still fails to generate the positive correlation between aggregate output and the rate of inflation. It does, however, explain the negative correlations between money growth rate and real economic activities (particularly on the investment and capital stock) which the standard constraint cannot fully explain.

Table 5.9: Contemporaneous correlation with aggregate output and \(M1\) growth rate

<table>
<thead>
<tr>
<th></th>
<th>Correlation with aggregate output</th>
<th>Correlation with (M1) growth rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data (\Omega = 0) (\Omega = 1)</td>
<td>Data (\Omega = 0) (\Omega = 1)</td>
</tr>
<tr>
<td>Output</td>
<td>1.00 1.00 1.00</td>
<td>-0.09 -0.02 -0.05</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.79 0.84 0.89</td>
<td>-0.05 -0.33 -0.00</td>
</tr>
<tr>
<td>Investment</td>
<td>0.92 0.99 0.99</td>
<td>-0.10 0.08 -0.06</td>
</tr>
<tr>
<td>Capital stocks</td>
<td>0.34 0.35 0.35</td>
<td>-0.14 0.02 -0.01</td>
</tr>
<tr>
<td>Hours</td>
<td>0.82 0.98 0.97</td>
<td>-0.24 -0.05 -0.15</td>
</tr>
<tr>
<td>(M1)</td>
<td>0.17 0.07 -0.00</td>
<td>0.21 0.21 0.21</td>
</tr>
<tr>
<td>Price level</td>
<td>-0.64 -0.09 -0.32</td>
<td>-0.11 0.59 0.59</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.22 -0.09 -0.16</td>
<td>-0.25 0.80 0.74</td>
</tr>
<tr>
<td>Treasury rate</td>
<td>0.24 -0.01 0.27</td>
<td>-0.43 1.00 0.90</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.16 0.50 0.15</td>
<td>-0.28 0.80 0.86</td>
</tr>
<tr>
<td>(M1) growth rate</td>
<td>-0.09 -0.02 -0.05</td>
<td>1.00 1.00 1.00</td>
</tr>
</tbody>
</table>

Table 5.10 concludes the cross-correlations between nominal variables and aggregate output. Clearly, monetary banking models are not able to replicate the phase shift relations of nominal variables with aggregate output. This reflects the failure of the Euler equation of the models. In conclusion, the Stockman constraint helps the monetary banking model to explain the pro-cyclical behaviour of the nominal interest rate and the negative correlations between the money growth rate and real activities. However, only extending the CIA economy with an inter-temporal banking sector and a Stockman CIA constraint cannot explain the phase shift relations between nominal variables and aggregate output.
Table 5.10: Cross correlations with aggregate output

<table>
<thead>
<tr>
<th></th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.07</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
</tr>
<tr>
<td>Price level</td>
<td>0.06</td>
<td>0.03</td>
<td>-0.02</td>
<td>-0.09</td>
<td>-0.08</td>
<td>-0.07</td>
<td>-0.06</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.04</td>
<td>-0.05</td>
<td>-0.07</td>
<td>-0.09</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.01</td>
<td>-0.00</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.05</td>
<td>0.04</td>
<td>0.03</td>
<td>-0.00</td>
<td>-0.02</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>Price level</td>
<td>-0.03</td>
<td>-0.10</td>
<td>-0.20</td>
<td>-0.32</td>
<td>-0.25</td>
<td>-0.17</td>
<td>-0.11</td>
</tr>
<tr>
<td>Inflation</td>
<td>-0.07</td>
<td>-0.10</td>
<td>-0.13</td>
<td>-0.16</td>
<td>0.12</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>0.10</td>
<td>0.15</td>
<td>0.21</td>
<td>0.27</td>
<td>0.19</td>
<td>0.12</td>
<td>0.06</td>
</tr>
</tbody>
</table>

5.4.6 Discussion

This section of the chapter discusses and evaluates the steady-state and dynamic behaviours of both the standard and Stockman monetary banking models with business cycle facts. At a stationary-state, the monetary banking models imply the positive relations among money growth, inflation and nominal interest rates. Monetary aggregates are able to influence real economic activity through leisure-consumption substitution. A monetary banking model includes an additional monetary transmission channel (which is a deposit rate) when compared with a CIA model of the banking sector. The money growth rate affects leisure-consumption substitution through both an expected inflation effect on nominal interest rate and a velocity effect on the deposit rate. The increasing money growth rate lowers the level of labour, consumption, investment, and aggregate output at stationary-state because the expected inflation effect dominates the velocity effect.
For the stochastic version of the models, there is a positive response of the inflation rate subject to monetary shock. The nominal interest rate and deposit rates increase with an expected inflation effect and a velocity effect of monetary shock. The monetary banking models are able to explain the relative volatility and pro-cyclical behaviour of the velocity. The increasing nominal interest rate has a negative effect on consumption and labour supply, and positive effect on leisure. The rising deposit rate has a positive effect on consumption and employment. Since the ‘inflation tax’ effect dominates the velocity effect, the monetary banking models generate the negative effectiveness of monetary expansion on real activities (such as lower aggregate output, consumption, and employment). The Stockman constraint leads the current and future substitution behaviour to be affected by nominal the interest rate and deposit rate through the real interest rate. This allows the model to be able to explain the negative correlations between money growth rate and aggregate output, and it also explains the pro-cyclical behaviour of the nominal interest rate.

5.5 Conclusion

This chapter evaluates the interaction between nominal and real variables, in both a simple CIA economy and in a monetary banking model, with monetary business cycle facts. It emphasises the contribution of both the banking sector and of the Stockman (1981) CIA constraint. It also discusses the effectiveness of technology, credit, and monetary shocks on monetary transmission and real activities.

For a simple CIA economy, the nominal interest rate is the only monetary transmission channel from monetary aggregate to output fluctuation which has been denoted by Cooley and Hansen (1989) as an ‘inflation tax’ effect of the money growth rate. Monetary aggregates have a positive expected inflation effect on nominal interest rate through a Fisher relation, and they have a negative effect on aggregate output, consumption, and labour supply because the leisure-consumption substitution is affected by the nominal interest rate. Although the models are able to generate the interaction
between nominal and real variables, they cannot generate the counter-cyclical and pro-
cyclical behaviour of nominal variables (except for price level). They also fail to
generate the expansion of real activity with positive monetary innovation. In other
words, the simple CIA economy cannot explain the monetary business cycle facts
through the inflation tax effect alone.

For the monetary banking economy, because households can either choose cash or
credit as their goods market transaction technology, and the amount of cash or credit
held by households depends on the marginal cost of cash and credit, it means that the
velocity is endogenously determined through households’ optimal choice between cash
and credit holding and varies with the money growth rate through a credit production
function. This allows the monetary banking models to explain the behaviour of velocity,
which a simple CIA economy cannot do. Since the velocity effect on deposit rate is able
to influence households’ leisure and consumption decision, the real effectiveness of a
monetary shock in the monetary banking model comes not only from an expected
inflation effect on nominal interest rate but also includes the velocity effect on the
deposit rate. With a Stockman constraint, the monetary banking model is able to account
for the negative correlation between money growth rate and real activities, and for the
pro-cyclical behaviour of the nominal interest rate. In other words, the intra-temporal
banking sector and a Stockman constraint with CIA economy are able to explain the
monetary business cycle facts.

However, neither the simple CIA model nor the monetary banking model can examine
the short-run positive relations between monetary expansion and real activities that have
been observed from the data. Therefore, the chapters 3 and 4 extend the CIA economy
with the function of financial intermediates to generate the positive responses of real
activity subject to monetary shock through the cost channel of monetary policy.
References


Chapter 6 - Conclusion

This chapter concludes the contribution of this research project. This thesis has emphasised the contribution of the financial sector and Stockman's (1981) constraint to explain the monetary transmission mechanism and economic fluctuation in a Cash-in-Advance (CIA) economy. It has discussed the interaction between nominal and real variables under the context of CIA framework, and concluded that there are interactions between nominal and real variables. It is able to generate certain business cycle facts by integrating productive banks and employing Stockman's (1981) CIA constraint with neoclassical growth model. It examines the effectiveness of monetary aggregates through various nominal interest rates with productive banks and/or financial intermediates. It resolves the liquidity puzzle on nominal interest rate and accounts for a number of business cycle facts.

Chapter 2 resolves the liquidity effect puzzle and explains the real impacts of monetary shocks by assuming that money injections are received by the financial sector in a banking CIA economy. Although the liquidity effects puzzle arises as a positive correlation between nominal interest rate and money growth rate is generated by monetary RBC models, a negative correlation has been found in the data. This chapter employs the banking CIA model that was developed by Benk, Gillman and Kejak (2005a) and assumes that money injections are received by productive banks instead of households. With the aggregate cost of banking service determined before money injections happen, the model is able to replicate a lower nominal interest rate with the monetary expansion that is observed in the data. By extending the model with a Stockman (1981) CIA constraint, it is able to examine the effectiveness of monetary aggregates and generate a hump shaped response of consumption subject to monetary innovations.

For the last few decades, economists have explored the ways in which changes in money stock can have a short-term positive effect on real economic activity. The
mechanism that they have developed includes: prices that are slow to adjust; wages which are set in nominal terms; monetary changes which cause confusion, making it hard to differentiate relative price changes from average price level changes; and, households and firms which change their portfolios at different frequencies. Chapters 3 and 4 explain the real impacts of monetary aggregates through various nominal interest rates with a financial sector. Without sticky prices and/or wages, the flexible price monetary model has difficulty explaining the real economic activity subject to monetary innovation. Chapter 3 extends the standard CIA economy with functions of productive banks and financial intermediates, and does not request sticky price/wage or limited participation monetary shock to examine the effectiveness of monetary aggregates through the cost channel of monetary policy. The model is able to explain the increase in real activities (such as output and employment) subject to monetary innovation. Chapter 4 extends the standard CIA model with the function of financial intermediates, which assumes that money injections are received by financial intermediates instead of households, in order to explain the real impacts of monetary aggregates. With Stockman's (1981) CIA constraint, the model further explains the behaviour of consumption subject to monetary expansion and the correlation between output and consumption which has been observed in the data.

The ‘classical dichotomy’ argues that the real and nominal variables are independent at both steady-state and in a transition period. There is no interaction between nominal and real economic activities. Since the data we observed indicates that there is an interaction between nominal and real variables, there are various approaches which have been developed to break up the classical dichotomy in the DSGE framework (such as non-separable households’ utility function, endogenous money supply, or sticky price/wage setting). They are able to explain the interaction between nominal and real variables. Chapter 5 formulates, calibrates, and simulates a CIA economy with a banking sector and a Stockman (1981) CIA constraint to examine the interaction between real-nominal variables, and investigates whether the model is able to generate business cycle fluctuations. The conclusion is that there is a contribution of the banking sector, and a Stockman constraint to explain the business cycle behaviour.
Bibliography


Christiano, Lawrence J (1991) “Modelling the liquidity effect of a money shock”, Quarterly review, Federal Reserve Bank of Minneapolis, 15, 3-34


Fischer, S (1979) “Capital accumulation on the transition path in a monetary optimizing model,” Econometrica, 47(6), 1433-39


Li, V (2000) “Household Credit and the Monetary Transmission Mechanism,” Journal of Money, Credit and Banking, 32(3), 335-56


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Appendix: Data Description

This Appendix contains a description of the data series which have been used in the thesis.

**Fisher effects on nominal interest rate (Figure 1)**

- Nominal money stocks for twelve countries (Narrow money):

  Australia: Summary currency plus bank current deposits of the private non-bank sector. Source: RESERVE BANK OF AUSTRALIA.

  Canada: Summary currency in circulation plus bank reserves. Source: CANSIM - STATISTICS CANADA.

  Denmark: Summary MO refers to all circulating banknotes and coins issued by Danmarks National bank. Copyright DANMARKS NATIONAL BANK.

  Finland: Summary MO. Source: BANK OF FINLAND.

  France: Summary A “narrow” monetary aggregate that comprises currency in circulation and overnight deposits. Source: BANQUE DE FRANCE.

  Germany: Summary MO. Source: THOMSON REUTERS AND NATIONAL SOURCE.

  Italy: Summary M1 the sum of currency in circulation, excluding banknotes and coins in lire and other Euro-area currencies held by Italian MFIs, and overnight deposits. Source: BANCA D'ITALIA

  Netherland: Summary currency in circulation comprises the outstanding guilder and
Euro coins issued by the Dutch Mint as well as the outstanding guilder and euro banknotes in circulation. Source: DE NEDERLANDSCHE BANK N.V.

New Zealand: Summary notes and coin held by the public plus cheque able deposits, minus inter-institutional cheque able deposits, and minus central government deposits. Source: RESERVE BANK OF NEW ZEALAND.

Sweden: Summary M0 consists theoretically of the Swedish public's holdings of banknotes and coins issued by the Riksbank. Source: BANK OF SWEDEN.

United Kingdom: Summary sterling notes and coin in circulation outside the Bank of England (including those held in banks' and building societies’ tills). Source: OFFICE FOR NATIONAL STATISTICS.

United States: Summary monetary base, not break-adjusted and not seasonally adjusted. Source: FEDERAL RESERVE.

- Money growth rate: the log difference of nominal money stock:

U.S Business Cycle


- Discount Rate: Refers to the rate at which the Federal Reserve Bank of New York discounts eligible paper and makes advances to member banks. Source:
International Monetary Fund Financial Statistics.

- Treasury bill Rate: Refers to the weighted average yield on multiple price auctions of 13-week treasury bills. Source: International Monetary Fund Financial Statistics.

- Bank Lending Rate: The rate is used by banks to price short term business loans. Source: Federal Reserve.

- Bond Rate: Treasury yield adjusted to constant maturity - 20 years. Source: Federal Reserve.

- Nominal Money Stock (M1): Consists of (1) currency outside the U.S. Treasury, Federal Reserve Banks, and the vaults of depository institutions; (2) traveller’s checks of non-bank issuers; (3) demand deposits at commercial banks (excluding those amounts held by depository institutions, the U.S. government, and foreign banks and official institutions) less cash items in the process of collection and Federal Reserve float; and (4) other checkable deposits (OCDs), consisting of negotiable order of withdrawal (NOW) and automatic transfer service (ATS) accounts at depository institutions, credit union share draft accounts, and demand deposits at thrift institutions. Seasonally adjusted M1 is constructed by summing currency, traveller’s checks, demand deposits, and OCDs, each seasonally adjusted separately. Source: Federal Reserve

- Output, $Y_t$: Is the market value of goods and services produced by labour and property in United States. Source: BUREAU OF ECONOMIC ANALYSIS.

- Consumption, $C_t$: Is the goods and services purchased by persons. Source: BUREAU OF ECONOMIC ANALYSIS.
- Investment, $I_t$: Source: Gomme and Rupert (2007)


- Hour, $l^e_t$: The expected or actual period of employment for the week, expressed in numbers of hours. Source: BUREAU OF LABOR STATISTICS.

- Labour Income: Is the wage and salary disbursement. Source: BUREAU OF ECONOMIC ANALYSIS.

- Personal Income: The income is received by persons from all sources. Source: BUREAU OF ECONOMIC ANALYSIS.

- CPI, $P_t$: The Consumer Price Index is a measure of the average change over time in the prices of consumer-items, -goods, and -services that people buy for day-to-day living. Source: BUREAU OF LABOR STATISTICS.

- Inflation, $\pi_t$: Is the log difference of CPI.