

SHOULD CENTRAL BANKS SWITCH FROM INFLATION TO PRICE-LEVEL TARGETING?

QUANTIFYING THE BENEFITS FROM LONG-TERM PRICE STABILITY

by

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Summary

Economic researchers have not yet quantified the long-term benefits of price-level targeting. Consequently, central banks are unable to conduct a full cost-benefit analysis vis-à-vis inflation targeting. The primary contribution of this thesis is to quantify these benefits within a dynamic stochastic general equilibrium framework, thereby laying the foundations for a full cost-benefit analysis. The thesis focuses on three key areas: consumption volatility; social welfare; and inflation risk premia on long-term nominal contracts.

Conventional wisdom holds that the main benefit of price-level targeting is a reduction in long-term inflation risk. However, the current workhorse model for monetary analysis cannot be used to evaluate this benefit, because long-term inflation risk does not affect agents' welfare. This thesis therefore builds and simulates overlapping generations models in which long-term inflation risk matters for social welfare. In these models, consumers save over a long horizon for old age using indexed and nominal government bonds that offer imperfect insurance against inflation risk. Importantly, the extent of nominal indexation is chosen endogenously in response to monetary policy in order to address the Lucas critique; and to allow for heterogeneities across countries and over time, three separate models are simulated in which consumers have access to different assets.

Key findings are as follows. First, price-level targeting reduces long-term inflation risk substantially compared to inflation targeting, leading to an increase in social welfare and a reduction in consumption volatility. Second, price-level targeting reduces by an order of magnitude the inflation risk premium on nominal bonds. Finally, optimal indexation of government bonds is substantially lower under price-level targeting. Notably, there is considerable heterogeneity in results across model specifications. The estimated welfare gain from price-level targeting ranges from 0.01 to 0.17 per cent of aggregate consumption, and the estimated reduction in consumption risk ranges from 13 to 95 per cent.

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Introduction

Inflation targeting (IT) is currently the most popular monetary policy framework with central banks, having been adopted in 26 countries worldwide (Roger, 2010).¹ However, recent research in the monetary policy literature has explored alternative policy options and their performance against IT in simulated models of the economy. This thesis contains five separate chapters that focus on one such alternative policy prescription: price-level targeting.

Price-level targeting (PLT), or stabilisation of the economy's aggregate price index around a predetermined target price path, has been the subject of extensive theoretical research in recent years. It is also a timely topic from a policy perspective as the Bank of Canada is currently conducting a research programme to evaluate the costs and benefits of switching from IT to PLT (see Bank of Canada; 2006, 2009),² and has left open the option to change its regime after 2011. The steady increase in literature on PLT has been driven in part by research at the Bank of Canada, but also reflects a renewed interest from academics due to the publication of articles on PLT in leading economic journals. Moreover, other central banks are now following suit by investigating PLT for themselves (Gaspar *et al.* 2007; Bank of Finland, 2008; Kahn, 2009; Bundesbank, 2010).

The aim of this thesis is to quantify the long-term benefits of price-level targeting. This investigation is carried out in the context of a dynamic stochastic general equilibrium (DSGE) modelling framework. The models employed are therefore microfounded optimisation-based models of the macroeconomy of the kind developed to address the Lucas critique of policy evaluation (Lucas, 1976). Such models have become popular tools for monetary policy analysis in recent years and have transformed the conduct of both theoretical and applied macroeconomic research. In order to provide context for the contributions that follow, the first chapter reviews the PLT literature, beginning with a simple explanation of PLT before proceeding to discuss its potential advantages, potential drawbacks, and practical issues. The first

¹ Examples include the Bank of Canada, the Bank of England, the Swedish Riksbank and the Reserve Bank of New Zealand.

² The Bank of Canada began looking at PLT in the mid-1990s and the Review was announced in Bank of Canada (2006).

chapter is followed by chapters 2 to 4, which contain the main contributions of thesis, and whose content is discussed in greater detail below. Finally, Chapter 5 summarises key findings and policy implications, before putting in context the overall contributions of the thesis and discussing potential extensions for future research.

In a comprehensive review of PLT, Ambler (2009) identifies three main potential advantages of PLT over IT. Firstly, PLT can improve the economy's short-term response to disturbances in models where economic agents are rational and forward-looking. Secondly, PLT can reduce the probability of the economy hitting or remaining at the zero lower bound (ZLB) on nominal interest rates, potentially leading to improvements in social welfare. Thirdly, by eliminating 'base-level drift' in the price level, PLT can reduce inflation and price-level volatility over medium and long-term horizons, with consequent benefits for economic agents entered into contracts denominated in nominal terms or imperfectly indexed to prices. The first advantage has been explored extensively within the PLT literature (see Ambler, 2009), such that recent research on PLT has turned away from this topic to more pressing issues (Bank of Canada, 2009). Considerable research effort has also been devoted to the second topic in recent years, following advances in computing power and numerical solution methods. The third advantage, however, has been neglected in recent research – in part because the current workhorse model of monetary policy analysis is not well suited to a study of the long-term welfare impact of PLT. This important but under-researched topic is taken as the focal point in this thesis.

Interestingly, the long-term impact of PLT was originally considered to be its main potential advantage, and is responsible for the long-standing interest in PLT at the Bank of Canada (see Duguay, 1994). As Ambler (2009) points out, further study is desperately needed on this topic, in particular to assess the impact of PLT on long-term contracting behaviour and social welfare. Such research would enable central banks to accurately assess the costs and benefits of PLT vis-à-vis IT. For this reason, the Bank of Canada has openly called for more research on the long-term impact of PLT whilst emphasising that such analyses should be conducted within DSGE models of the economy (see Bank of Canada; 2006, 2009). This primary aim of this thesis is to address this need for further research.

The main analysis begins in Chapter 2 where a DSGE model of the economy is built in which the impact of PLT on long-term inflation volatility and social welfare can be quantified explicitly using consumer utility. The model put forth is an overlapping generations model of life-cycle saving in which young generations save for old age using indexed and nominal government bonds whose payoffs are vulnerable to long-term inflation risk. The model also includes money and capital as additional assets, and the degree of indexation – that is, the share of indexed bonds in the total supply of government bonds – is chosen optimally in response to monetary policy as part of an optimal commitment Ramsey problem. In order to allow for heterogeneities across countries, three different versions of this general model are simulated in which consumers have access to different assets.

Chapter 2 first investigates social welfare when the model includes only nominal bonds and money, before extending the menu of assets available to include productive capital whose rate of return is driven by stochastic shocks to productivity. These special cases of the model are presented first because they highlight the workings of the underlying life-cycle model in a straightforward way, and are applicable for some developed economies. Chapter 3 then extends the model to include indexed government bonds, thus providing a robustness check on the results in Chapter 2 for countries in which indexation plays an important role. Chapter 3 is also important in its own right because the degree of nominal indexation is endogenised with respect to monetary policy – a key recommendation made by Ambler (2009). As a result, the welfare results obtained should not be vulnerable to the Lucas critique, making them more reliable from a policy perspective. Furthermore, in contrast to past literature focusing on optimal indexation of bonds, the unrealistic assumption of perfect indexation is relaxed. Finally, in Chapter 4, the three models analysed in chapters 2 and 3 are used to estimate the reduction in the long-term inflation risk premium on nominal bonds under PLT.

In order to model inflation risk over a long-term horizon, each period in the life-cycle model is interpreted as lasting 30 years and (long-term) money supply shocks are not assumed constant across monetary policy regimes, but are instead built-up from a yearly horizon. Crucially, this methodology means that the model captures the impact of IT and PLT on long-term inflation volatility – that is, the model captures ‘base-

level drift' in the price level under IT and its absence under PLT. To the author's knowledge, this is the first attempt to formally model the long-term impact of PLT and its welfare implications within a DSGE framework. In order to capture asset risk-premia in the model and obtain accurate welfare results, the model is solved using a second-order approximation method.

Several key results are found. First and foremost, the welfare gain of PLT over IT is estimated to be positive and economically non-trivial: the gain in social welfare under PLT is equivalent to a permanent increase in aggregate consumption of 0.01 to 0.17 per cent. The intuition for PLT increasing social welfare is that, due to the elimination of base-level drift, PLT reduces long-term inflation risk substantially compared to IT, which in turn reduces real return risk on long-term bonds, making consumption less volatile for old generations. Since consumers are risk-averse, this reduction in consumption risk leads to an increase in social welfare.³

Secondly, PLT leads to a substantial reduction in aggregate consumption risk, though this reduction is driven entirely by a fall in consumption risk faced by the old, since only they benefit directly from the lower level of inflation risk under PLT. The reasoning is simply that inflation risk matters only when considering the payoffs of nominal assets over a long horizon – payoffs which matter only to the 'old' who rely on savings (in real terms) to fund their consumption in retirement. There is considerable variation in the impact of PLT on old generations' consumption risk across the three models, with a volatility reduction of only 13 per cent in the full model with indexed bonds, capital and endogenous indexation, compared to 95 per cent in the model with only nominal bonds. Interestingly, the percentage reductions in volatility far exceed the potential welfare gains, because consumption risk has only a second-order impact on social welfare. As noted by Rudebusch and Swanson (2008), such impacts are typically around 100 times smaller than those at first-order in DSGE models.

Thirdly, optimal indexation of government bonds is substantially lower under PLT. The intuition for this result is that PLT reduces the benefits to be had from indexed

³ Given that consumption variability across old generations is lessened, PLT also has separate redistributive implications. These implications are not discussed in this thesis.

bonds, because consumers have less need to protect themselves against inflation variations if the level of long-term inflation risk is relatively low. On the other hand, indexation is relatively more costly under PLT, because the imperfections of indexation (as determined by institutional or political factors outside the model) are a relatively more important source of consumption variations if inflation risk is low. The result that optimal indexation is substantially lower under PLT has been echoed in recent work in the context of optimal indexation of wages (Minford and Peel, 2003; Amano *et al.* 2007), though the reasoning there is slightly different: indexing wages is 'costly' even if indexation is perfect, because the burden of adjustment is pushed onto employment following real shocks, leading to an increase in output variability (Gray, 1976).

Fourthly, due to the reduction in long-term inflation risk under PLT, the long-term inflation risk premium on nominal government debt is substantially lower than under IT – typically by 90 per cent or more. An important implication of this result is that in order to finance government spending, higher taxes are necessary under IT. In turn, this means that society does not gain (or lose) from risk-premia in terms of average consumption: the gain in average consumption by the old from a higher risk-premium is offset by a reduction in average consumption by the young due to higher taxes. As a result, the optimal indexation share does not depend on risk-premia and is determined purely by a 'consumption insurance' motive. Similarly, social welfare does not depend on indirectly on inflation risk through bond risk-premia, but only on the direct impact of such risk on consumption volatility faced by old generations (i.e. pensioners).

Fifth, there is considerable heterogeneity in results across model specifications and to changes in key model parameters. For instance, introducing productive but risky capital into the model reduces the welfare benefit of PLT somewhat because the presence of real productivity risk makes consumption in old age somewhat more volatile, lessening the extent to which PLT can reduce overall consumption risk. Similarly, the potential welfare gain from PLT falls even further once indexed bonds are introduced, because consumers can then directly protect their savings against unanticipated inflation variations, making the stabilising impact of PLT on old generations' consumption somewhat less than if only nominal government bonds are

issued. In the case of inflation risk premia, there is substantial variation in the magnitude of inflation risk premia, and also in the absolute reduction in the inflation risk premium that can be attained under PLT.

The main policy implications arising from these results are as follows. Firstly, the consumption volatility and welfare impacts of PLT are positive and non-trivial, and should therefore be taken into account in a full cost-benefit analysis by central banks. However, before any specific conclusions can be drawn, further research is needed to investigate this long-term impact within more comprehensive economic models of the kind used by central banks for quantitative policy analyses. Secondly, as optimal indexation of government bonds differs substantially between IT and PLT and this influences social welfare, it is important for future research to endogenise indexation in order to obtain accurate results when evaluating macroeconomic performance under IT and PLT using simulated models of the economy. Research that does not do so will clearly be vulnerable to the Lucas critique.

Thirdly, the long-term impact of PLT on social welfare and consumption risk is likely to vary considerably across countries. For example, in the case of the UK and US, private sector pensions play an important role; indexed government bonds account for a non-trivial share of government bonds; and public sector pensions are indexed once in payment. Therefore, the model with capital and indexed and nominal government bonds is likely to be most applicable, suggesting a relatively small welfare gain from switching to PLT of 0.01 per cent of aggregate consumption. On the other hand, public sector pensions account for around 85 per cent of retirement income in Germany (Berkel and Börsch-Supan, 2004) and the share of indexed bonds is extremely small (Garcia, 2008), suggesting that the model without any capital or indexed bonds should be used to estimate the welfare gain from PLT. Doing so implies a welfare gain around 10 times higher, or a 0.1 per cent increase in aggregate consumption. Moreover, an important finding from sensitivity analysis is that the long-term welfare gain from PLT varies substantially with the extent of risk aversion and the level of nominal volatility. In particular, countries in which risk aversion is relatively high and where IT and performs poorly will have more to gain from switching from IT to PLT.

Fourthly, as the magnitude of inflation risk premia varies substantially with risk aversion, nominal volatility and model specifications, the importance of long-term inflation risk premia is likely to vary considerably across countries, being higher in countries in which risk aversion and nominal volatility are higher, and also in countries like Germany and Canada where nominal assets play an important role in retirement income provision. However, it should be noted that the result that PLT leads to a large *proportional reduction* in the inflation risk premium compared to IT is strongly robust, and that this result holds potentially important public policy implications because it implies that, other things being equal, risk-averse consumers will be willing to save more in nominal pensions under PLT than IT.

Finally, the result that the inflation risk premium is an order of magnitude lower under PLT is likely to have separate welfare implications of its own. For instance, a lower inflation risk premium may boost investment – potentially raising the sustainable rate of economic growth – and will lower the average cost of issuing government debt, thereby enabling government spending to be increased for given taxes, or for taxes (and thus distortions) to be lowered for a given level of expenditure. Both of these factors are potentially important given that the sustainability of public sector pensions is now a major public policy issue in most developed economies. However, the models presented in this thesis would need to be extended to include a separate financial sector and/or endogenous growth in order to analyse these implications formally. These would be potentially fruitful extensions of the model for future research. Other potentially useful extensions are discussed at the end of Chapter 5.

Chapter 1

Price-level targeting: A literature review

1.1 Understanding price-level targeting

Under a price-level targeting (PLT) regime, monetary policymakers attempt to stabilise the aggregate price level around a predetermined long run target price path. Consequently, the target price level in any period is unaffected by past economic shocks. Hence if the current price level is above the level implied by the target price path, below-average inflation is required next period in order to return the price level to target, and *vice versa*. In this respect, PLT is effectively ‘average inflation targeting’, where the average is taken over a long horizon. Useful reviews of PLT are given by Ambler (2009), Courmède and Moccero (2009), Kahn (2009), and Parkin (2009).

The PLT approach to monetary policy stands in contrast to the inflation targeting (IT) mandates pursued by many central banks worldwide. In the case of IT, the *rate of change* of prices (over a short horizon) and not the *level* of prices is the target of policy. Hence if inflation rises above target, this deviation should not be offset in the future: ‘bygones and bygones’ and policymakers continue to aim at the same inflation target in all future periods. Consequently, there is ‘base-level drift’ in the price level under an IT regime, whilst this is ruled out by successful implementation of PLT. The response of policy to past deviations from target is the defining feature of PLT. Indeed, in the absence of economic shocks that give rise to such deviations, PLT and IT will produce identical outcomes provided that the long run inflation target implied by the target price path is consistent with the short-term inflation target.⁴

In order to make distinction between IT and PLT clear, the next section draws on a simple comparison due to Minford (2004), in which the transmission mechanism of monetary policy is suppressed for simplicity. The PLT case is dealt with first. The implications of PLT for the level and volatility of the price level and inflation are explained using diagrams, impulse responses and equations. These results are then contrasted with a simple representation of IT. Each period in these examples should

⁴ For example, a long run inflation target of 60% over 30 years would be consistent (ignoring compounding) with an annual inflation target of 2%.

be interpreted as lasting either one quarter or one year; the former is more consistent with academic literature, whilst the latter is closer to empirical estimates of the 'monetary policy transmission lag'.⁵

1.1.1 Price-level targeting: A simple example

Suppose that in each period t a path for log prices $p_0 + \pi^*t$ is targeted by the central bank, where $p_0 > 0$ is the initial price level and $\pi^* > 0$ is the inflation target that is consistent with the target price path. It is assumed that deviations from the target price path are offset in full and without error in the following period, and that the central bank has perfect credibility. Therefore, this example describes the impact of PLT under the assumption that it is implemented successfully in a world where economic agents have rational expectations.

Under the above assumptions, the actual level of prices in period t will deviate from its target only if there is a current price shock. Therefore, assuming that price shocks are temporary and uncorrelated, the time- t price level will be given by

$$(1.1) \quad p_t = p_0 + \pi^*t + \varepsilon_t$$

where p_t is the log of the aggregate price level, and ε_t is an IID shock to prices with mean zero and variance σ^2 .⁶

Note that the time- t price level depends on only a single (current) price shock, since any past deviations from the target price path have been offset. In order to make this feature of PLT clear, Figure 1.1 shows the response of the price level to a one-off positive price shock $\varepsilon_1 > 0$ under the assumption that the long run inflation rate implied by the target price path is positive. The price shock in period 1 pushes the price level p_1 above the target level (denoted with a star), but in period 2 the price level is returned to the target price path.

⁵ For a short review of this literature, see Walsh (2003, Ch. 1).

⁶ Henceforth I drop the 'log' and 'aggregate' labels and just refer to the price level or prices.

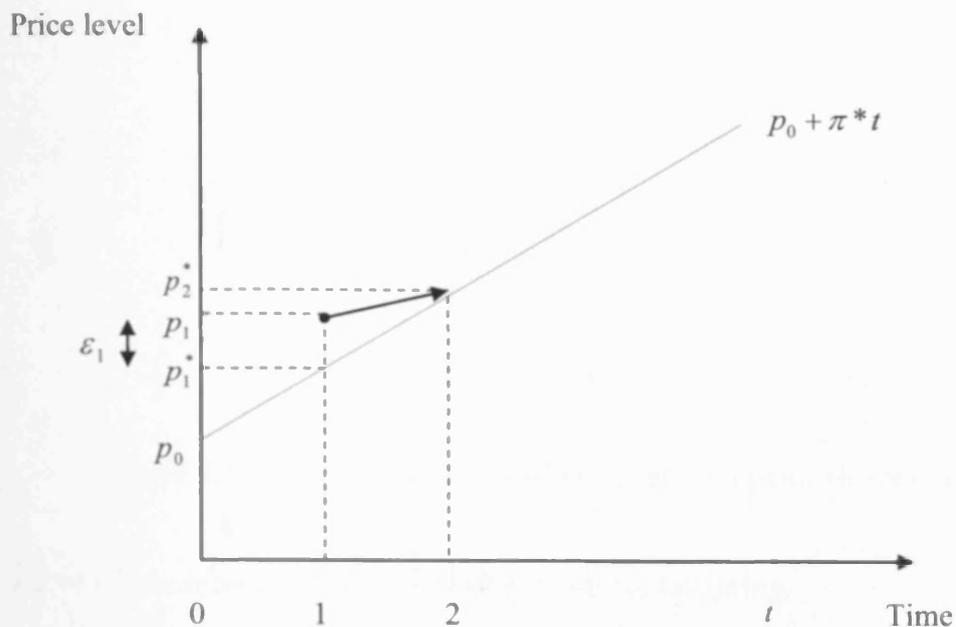


Figure 1.1 – Response of the price level following a price shock (PLT)

We can also use this simple example to look at the implied rate of inflation. Taking the first-difference of Equation (1.1) implies that inflation is given by

$$(1.2) \quad \pi_t = \pi^* + \epsilon_t - \epsilon_{t-1}$$

Equation (1.2) shows that inflation in period t is given by the inflation target implied by the target price path, plus the difference between the current shock to the price level and the shock in the previous period. Intuitively, the past shock to prices matters for current inflation because it is offset by the central bank in order to return the price level to its target path. For instance, in the face of an inflationary shock to the price level (inflation above target), inflation next period would need to undershoot its long run target in order to return the price level to target. Figure 1.2 shows this point explicitly by plotting the impulse response of inflation to a one standard deviation price shock. The standard deviation was set at $\sigma = 0.01$, since this implies an impulse response of one per cent on impact.

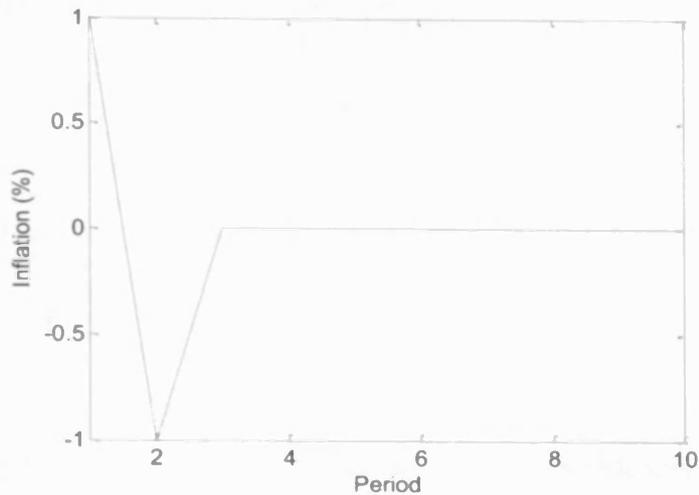


Figure 1.2 – Impulse response of inflation to a price shock (PLT)

1.1.2 Macroeconomic implications of price-level targeting

Firstly, note from Equation (1.2) that one-year-ahead inflation expectations are time-varying under PLT and given by

$$(1.3) \quad E_{t-1} \pi_t = \pi^* - \varepsilon_{t-1}$$

where E_{t-1} is the rational expectations operator conditional on information available in period $t-1$.

The intuition for this result is straightforward: past deviations from the inflation target are offset under PLT, and rational agents take this into account when forming their inflation expectations. Another way of thinking about this result is that the short-term inflation target becomes state-contingent. Specifically, the short-term inflation target is adjusted to ensure that, if the target is met, the price level will be returned to its target path. Since the PLT regime is perfectly credible, inflation expectations simply follow this ‘state-contingent inflation target’. Consider next the impact of PLT on price-level and inflation uncertainty.

Using Equation (1.1), the k -period-ahead price level is given by

$$(1.4) \quad p_{t+k} = p_0 + \pi^*(t+k) + \varepsilon_{t+k}$$

Therefore, the uncertainty associated with the price level k periods hence is given by

$$(1.5) \quad \text{var}_t(p_{t+k}) = \sigma^2$$

Equation (1.5) states that future price-level uncertainty is independent of the forecast horizon. This result means that the long-term purchasing power of money is preserved over time under PLT. The reason is that any shocks to prices between periods t and $t+k$ will be offset by policy in the intervening periods. That is, although the level of inflationary shocks cannot be forecast in advance, economic agents understand that they will be offset by a PLT central bank and therefore take this into account when forming their forecasts of future price-level uncertainty. As a result, the only uncertainty in the future price level under PLT comes from the last period of forecast horizon – the only period whose shock cannot be offset prior to the end of period $t+k$.

Now consider the implications of PLT for inflation measured over a k -period horizon, that is, the percentage change in prices between period t and period $t+k$. This measure of inflation would be relevant for consumers or firms who enter into medium or long-term nominal contracts like mortgages or long-term bonds.

Using equations (1.1) and (1.4), inflation over a k -period horizon is given by

$$(1.6) \quad \pi_{t \rightarrow t+k} = p_{t+k} - p_t = \pi^*k + \varepsilon_{t+k} - \varepsilon_t$$

where the subscript $t \rightarrow t+k$ indicates that inflation is measured over a horizon from period t to period $t+k$.⁷

⁷ Cochrane (2001) uses this notation to denote the holding period return on an asset held from period t to period $t+k$.

Consequently, the variance of k -period inflation is given by:

$$(1.7) \quad \text{var}_t(\pi_{t \rightarrow t+k}) = \sigma^2$$

Equation (1.7) shows that horizon- k inflation volatility is equal to k -period-ahead price-level volatility. Depending on the horizon chosen, k will refer to a short-term, medium-term, or long-term horizon. In the context of PLT, medium and long-term horizons are considered to be important due to the result that inflation uncertainty does not increase with the forecast horizon.

Finally, there is one last result regarding inflation volatility that should be noted. Using Equation (1.2), the unconditional variance of short-term inflation is given by

$$(1.8) \quad \text{var}(\pi_t) = 2\sigma^2$$

The intuition behind this result is straightforward. Since inflation depends on both a current shock and a past shock, unconditional inflation volatility is two times the price shock variance.⁸ It is important to note that unconditional variances are usually considered to be important in monetary policy analyses, because they allow alternative policies to be evaluated across all possible histories of shocks (see Damjanovic *et al.*, 2008).

The example used in this section is of course somewhat stylised, but deliberately so. Indeed, by abstracting from the monetary policy transmission mechanism, we have been able to clearly demonstrate main implications of PLT without the need to resort to a formal economic model. These implications are contrasted with the IT case in the next section.

1.1.3 Macroeconomic implications of inflation targeting

It is assumed that the central bank also has perfect credibility under IT, and that the short-term inflation target is consistent with the long-term one implied by the target price path under PLT (such that IT and PLT are directly comparable). Moreover, it is

⁸ The intuition for this result can be seen from the impulse response in Figure 1.2.

assumed that the price level is subject to an identical set of shocks, but that the central bank permits base-level drift in the price-level – as implied by the mandates of IT central banks.

Under this assumption the price level follows a random walk, with drift given by the short-term inflation target π^* :

$$(1.9) \quad p_t = p_{t-1} + \pi^* + \varepsilon_t$$

Consequently, the dynamic evolution of the price level is somewhat different under IT. This point is demonstrated clearly in Figure 1.3, which shows the response of the price level following a one-off positive price shock $\varepsilon_1 > 0$ in period 1. This price level response can be contrasted directly with the PLT case in Figure 1.1. Starting from period 0, the central bank aims to meet its inflation target π^* in year 1. However, the price shock frustrates its attempt to meet the inflation target: the actual price level is p_1 , and inflation in period 1 is above target. Under PLT, the price shock was offset to return the price level to the target path. However, there is no such response under IT: ‘bygones are bygones’ and central bank ignores the past deviation from the inflation target, with the aim of achieving the inflation target in every period henceforth. Hence, for instance, the inflation target is met in period 2 because the price level is p_2 which lies above p_1 by exactly π^* .

As shown by the upward shift in Figure 1.3, it is as though the IT central bank starts with a target path for prices in period 0 and subsequently revises this path permanently upwards by the shock in period 1. Furthermore, this point holds more generally: each price shock will have a permanent effect by causing the central bank to adjust its implied price path so that the inflation target can be met in each future period. The term ‘base-level drift’ is an appropriate description of the behaviour of the price level, because price shocks lead to drift in the forecast path for prices, with each shock implying a new ‘target path’ with a different base (or starting point).

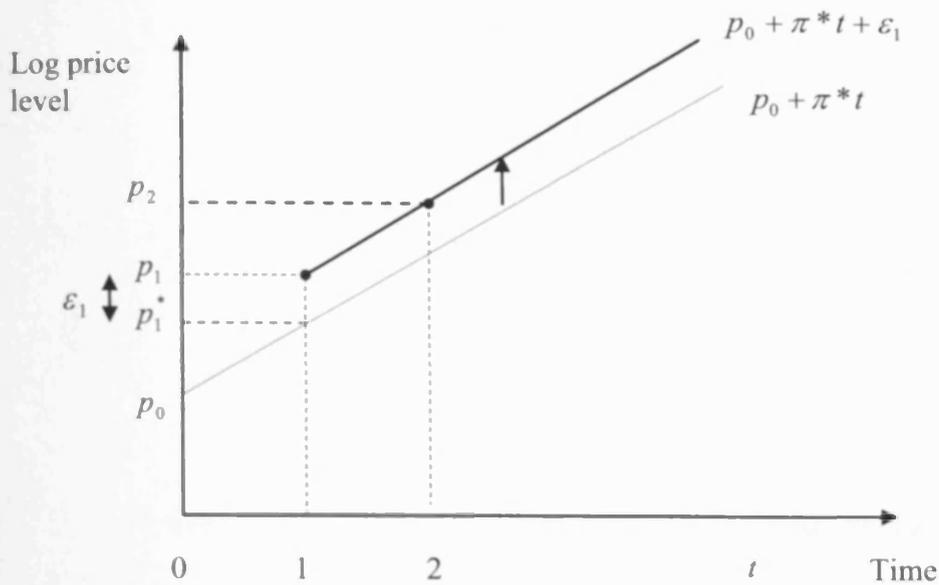


Figure 1.3 – Response of the price level following a price shock (IT)

Since past inflationary shocks are treated as bygones, inflation depends only on the current shock to prices. This point can be seen formally by subtracting the lagged price level on both sides of Equation (1.9) to get the following expression for short-term inflation:

$$(1.10) \quad \pi_t = \pi^* + \varepsilon_t$$

To provide a direct contrast with the PLT case dealt with above, Figure 1.4 shows the impulse response of inflation to a one standard deviation inflation shock, again assuming that $\sigma = 0.01$. Clearly, the impact of the shock on inflation is temporary: inflation rises by one per cent on impact but is returned to steady-state (i.e. to target) in the following period. By contrast, PLT offset the positive shock in period 2 by setting inflation one per cent below target.

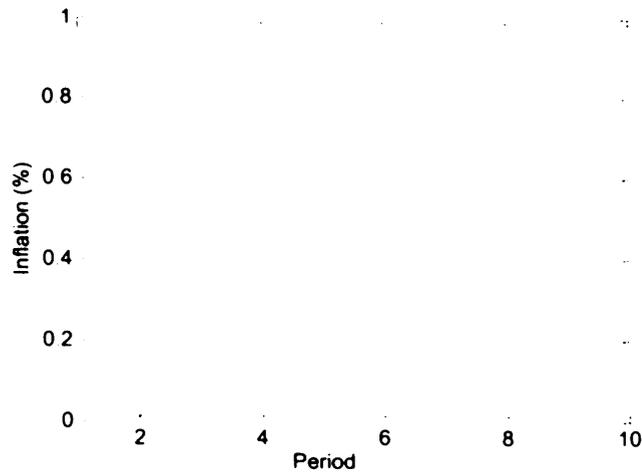


Figure 1.4 - Impulse response of inflation to a price shock (IT)

Now consider inflation expectations under IT. Using Equation (1.10), expected inflation under IT is given by

$$(1.11) \quad E_{t-1}\pi_t = \pi^*$$

Hence inflation expectations are constant at a level consistent with the inflation target. Intuitively, the central bank has perfect credibility and inflation depends only on the current shock to prices, whose expected value is zero. By contrast, expected inflation varies under PLT, because the central bank offsets past inflationary shocks in order to return the price level to its target path.

Finally, consider the implications of IT for price-level and inflation uncertainty. First, using Equation (1.9), the k -period-ahead price level is given by

$$(1.12) \quad p_{t+k} = p_{t+k-1} + \pi^* + \varepsilon_{t+k} = p_t + \pi^* k + \sum_{j=0}^{k-1} \varepsilon_{t+k-j}$$

Therefore, conditional on time- t information, uncertainty surrounding the k -horizon-ahead price level is given by

$$(1.13) \quad \text{var}_t(p_{t+k}) = k\sigma^2$$

Equation (1.13) states that under IT uncertainty regarding the future price level is proportional the forecast horizon k . Hence base-level drift means that past shocks to prices have a permanent effect on the current level of prices. Intuitively, since future price shocks are not known in advance and are not offset, shocks during the forecast horizon accumulate, with each one adding to forecast uncertainty. An important result that follows from Equation (1.13) is that price-level uncertainty is unbounded as the forecast horizon increases, that is, as $k \rightarrow \infty$. An equivalent expression of this result is that the unconditional variance of the period- t price level is given by $\text{var}(p_t) = t\sigma^2$, which is clearly not finite as t increases.

Now consider the implications of this result for inflation measured over a horizon of k periods. Using Equation (1.12), inflation over a k -period horizon is given by

$$(1.14) \quad \pi_{t \rightarrow t+k} = p_{t+k} - p_t = \pi * k + \sum_{j=0}^{k-1} \varepsilon_{t+k-j}$$

where the subscript $t \rightarrow t+k$ again indicates that inflation is measured over a horizon from period t to period $t+k$.

Consequently, the variance of horizon- k inflation, conditional on time- t information, is equal to the k -period-ahead price level variance:

$$(1.15) \quad \text{var}_t(\pi_{t \rightarrow t+k}) = k\sigma^2$$

Equation (1.15) demonstrates an important result: the variance of inflation is also unbounded as the forecast horizon increases. Inflation forecast uncertainty is likely to be important for economic agents who enter into medium or long-term nominal contracts, like long-dated bonds or mortgages. For instance, if these contracts are fixed in nominal terms for say k periods, then the value of $\pi_{t \rightarrow t+k}$ will determine the real value of these contracts in the period when they are paid-off or yield a return. Provided economic agents are risk-averse, they will also care about $\text{var}_t(\pi_{t \rightarrow t+k})$, since this characterises the uncertainty associated with the real value of a nominal contract

at time t when the contract is signed.⁹ In the PLT case, by contrast, both the future price level and inflation variances (k periods ahead) were finite. Moreover, the k -period-ahead variances were equal to σ^2 – the yearly price shock variance – since all past price shocks are subsequently offset.

Finally, consider the unconditional variance of short-term inflation under IT. Using Equation (1.10), the unconditional variance of inflation in any period is simply

$$(1.16) \quad \text{var}(\pi_t) = \sigma^2$$

Relative to the PLT case, this variance is halved. The intuition for this result is simple: under IT, inflation in any year or quarter will deviate from the inflation target only if there is a current shock, because past shocks to prices are treated as by-gones. Hence the inflation variance is simply given by the price shock variance. By contrast, inflation under PLT depends on a current price shock and also on the shock from the previous period which is actively offset by policy. As such, yearly inflation is twice as volatile under PLT. The result that short-term inflation volatility is increased under PLT was emphasised in early literature, and the relatively low level of short-term inflation variability under IT provides an explanation for its popularity with central banks. A second reason behind the popularity of IT is that it encourages agents' inflation expectations to converge on a constant inflation target, as highlighted by Equation (1.11).

It is notable, however, that recent literature has argued that short-term inflation volatility could actually be reduced under PLT. The reasoning is that, in forward-looking models with nominal rigidities, monetary policy can actually influence the volatility of 'price-level shocks': the size of such shocks will depend on the extent to which firms choose to 'pass-on' an increase (decrease) in costs through higher (lower) goods prices following cost-push shocks. In such models, prices are less sensitive under PLT than IT because firms expect the aggregate price level to be restored to the target price path in the near future, and therefore face a lower 'loss' from not

⁹ For instance, risk-averse economic agents will, *ceteris paribus*, be less willing to enter into nominal contracts the higher the level of inflation uncertainty.

adjusting prices in the face of cost-push shocks. The assumption made in the above example that price level shocks have the same volatility under IT and PLT is therefore not an innocuous one. In the context of the simple example above, we could allow for this difference by letting price-level shocks to have a lower variance under PLT. Short-term inflation volatility would then fall under PLT if price shock variance were less than one-half of the variance under IT.

Table 1.1 summarises the macroeconomic implications of IT and PLT explored in this section. Although the above comparison of IT and PLT is an extreme simplification of reality, it is powerful because of its transparency. In fact, somewhat remarkably, the simple comparison of this section provides the intuition for all the main results from the literature, and indeed all the key results in this thesis. If in doubt regarding the intuition behind any of the results that follow, the reader may therefore benefit from returning to this initial introductory section and the summary results in Table 1.1.

Table 1.1 – Summary of inflation and price-level targeting

	IT	PLT	Notes
Inflation	$\pi_t = \pi^* + \varepsilon_t$	$\pi_t = \pi^* + \varepsilon_t - \varepsilon_{t-1}$	With PLT, inflation depends on a current shock to inflation and also the past one which is offset.
Inflation expectations ($E_{t-1}\pi_t$)	π^*	$\pi^* - \varepsilon_{t-1}$	Constant under IT, but not under PLT since rational agents expect the price level to be stabilised after a shock.
Medium and long-term price/inflation uncertainty	$k\sigma^2$	σ^2	Increases one-for-one with the horizon k under IT due to base-level drift, but PLT eliminates base-level drift.
Short-term inflation volatility	σ^2	$2\sigma^2$	Doubled under PLT because past price shocks are offset. However, recent literature argues that volatility may in fact be reduced.

1.2 Potential benefits of price-level targeting

The literature has identified three main potential benefits from PLT. In this section, each of these benefits is discussed in turn. The second benefit discussed in this section – the impact of PLT on long-term inflation risk and the welfare of economic agents with long-term nominal contracts – provides the foundation for the later chapters of the thesis that contain the main contributions to the economic literature. The reader should therefore pay particular attention to this section and the results described therein.

1.2.1 Short-term response to economic disturbances¹⁰

A first benefit of PLT identified by the literature is a reduction in short-term macroeconomic volatility that shifts inwards the economic trade-off between inflation and output gap volatility (see Taylor, 1979).¹¹ In particular, PLT improves this trade-off when the expectations of economic agents are forward-looking and the central bank acts in a discretionary manner, re-optimising its decisions independently every period. The basic intuition for this result can be gleaned from one of the results in Section 1.1. In particular, it was shown that following an increase in inflation above target due to a price level shock, a credible PLT regime produces the expectation that inflation will be reduced below target in the following period. If current inflation depends positively on expected inflation – as it does in forward-looking models in which agents have rational expectations – then this expectation will reduce the extent to which inflation rises (falls) at times when inflationary pressure builds up (subsides), hence reducing deviations of inflation from target. These lower inflation deviations then feed through to lower output gap volatility via nominal rigidities.

A good starting point for understanding this result and the surrounding literature is the seminal paper by Svensson (1999). In this paper, Svensson reports a ‘free lunch’ to PLT: inflation volatility is reduced relative to IT for any given level of output gap volatility. This result means that PLT is desirable even if society has IT preferences, and this is the key point that Svensson emphasises. As in many papers in the monetary policy literature, Svensson assumes that social preferences take the form of a

¹⁰ This section draws heavily on the survey by Ambler (2009).

¹¹ To be clear, ‘short-term’ should be taken to mean a quarterly or yearly horizon.

quadratic ‘social loss function’ which states that inflation and output gap variations are costly for social welfare:¹²

$$(1.17) \quad L_t = E_t \sum_{i=0}^{\infty} \beta^i \{ \pi_{t+i}^2 + \lambda x_{t+i}^2 \}$$

where $0 < \beta < 1$ is social discount factor, λ is a constant indicating the relative importance of output gap fluctuations vis-à-vis inflation fluctuations, π_t is inflation between period $t-1$ and period t , and x_t is the output gap at time t , defined as the proportional difference between actual output and the level of output in an economy with perfectly-flexible prices.¹³

Aggregate supply in the economy is given by a New Classical Phillips curve:

$$(1.18) \quad \pi_t = E_{t-1} \pi_t + \varphi(x_t - \rho x_{t-1}) + e_t$$

where the parameter $0 < \rho < 1$ governs the extent of output gap persistence, $\varphi > 0$ is the Phillips curve slope parameter, and e_t is an IID supply shock with a mean of zero and constant variance.

The aim of the central bank is to minimise its loss function subject to the Phillips curve in Equation (1.18). It is important to note that the loss function in Equation (1.17) is *assumed* to be the correct representation of social preferences. However, in the workhorse New Keynesian model discussed later on, this same equation can be derived as a second-order approximation to the expected welfare loss of the representative household (see Woodford, 2001), thus giving a model-consistent representation of social preferences and laying the foundation for microfounded welfare analysis.

Svensson finds that in order to minimise the social loss function, it is better to delegate the central bank ‘PLT preferences’. In this context, PLT preferences can be

¹² For simplicity, it is assumed that the target inflation rate and target output gap are zero.

¹³ Each period in the loss function (1.17) should be interpreted as lasting one quarter or one year.

represented by a central bank loss function which specifies that price-level rather than inflation deviations are costly, or

$$(1.19) \quad L_t^{PLT} = E_t \sum_{i=0}^{\infty} \beta^i \left\{ (p_{t+i} - p^*)^2 + \lambda^{PLT} x_{t+i}^2 \right\}$$

where p_t is the log price level at time t , p^* is the constant target price level and λ^{PLT} is the relative weight that the central bank places on output gap versus price-level deviations.¹⁴

Svensson shows that delegating Equation (1.19) to the central bank leads to a lower social loss than delegating the social loss itself, because inflation volatility is reduced for any given level of output gap volatility.¹⁵ This result arises because the central bank re-optimises its decisions every period in a discretionary manner, such that policy is subject to ‘discretionary bias’.¹⁶ Indeed, under IT the central bank is not able to manipulate private sector inflation expectations when it lacks commitment, which in turn prevents it from dampening the impact of supply shocks on inflation. This is somewhat problematic because, if output gap deviations are strongly persistent, inflation will deviate from its target for substantial periods of time following supply shocks.

However, this same problem does not arise under PLT. The reason is that PLT creates the expectation that inflationary shocks will be undone in the future, which then reduces actual inflation deviations through the expectations term in the Phillips curve, partly offsetting the destabilising impact of a persistent output gap. Hence inflation variations can be reduced, whilst there is no impact on output gap variability itself since only ‘inflation surprises’ matter for output. In effect, this result shows that there is a critical value of output gap persistence above which extra fluctuations caused by offsetting past inflationary shocks under PLT are dominated by reduced volatility

¹⁴ A constant target price level is assumed because this is consistent with the assumed inflation target of zero. The relative weight on output gap deviations can potentially differ from the IT case under PLT, but it is important to note that Svensson’s result holds for any given relative weight on output gap deviations.

¹⁵ Equivalently, output gap volatility is lower for any given level of inflation volatility.

¹⁶ By contrast, delegating the IT loss function will minimise IT social preferences under commitment.

from stabilising inflation expectations. In fact, Svensson shows that an output gap persistence parameter of $\rho > 1/2$ is sufficient to deliver the free lunch result.

The free lunch result is viewed as important because it contradicts the conventional wisdom on PLT, and does so within a rational expectations setting. Moreover, it is important to note that this result is not limited to the case of endogenous output gap persistence, even if one considers a New Classical Phillips curve. For instance, Cover and Pecorino (2005) use the same model as Svensson but assume that the output gap is not persistent (i.e. $\rho = 0$) and that the central bank must choose its policy before knowing the current value of the supply shock. This change in timing makes it necessary to specify an IS curve for aggregate demand. Aggregate demand is assumed to depend negatively on the *ex ante* real interest rate, which in turn is negatively influenced by expected inflation through the Fisher equation. Consequently, an increase in expected inflation will reduce the *ex ante* real interest rate, stimulating aggregate demand and the output gap.

PLT performs well in this framework because, in contrast to IT, it automatically stabilises the output gap via impact of variations in expected inflation on the real interest rate. For example, following a demand shock both the output gap and inflation will increase, and under PLT this impact will be mitigated by a reduction in expected inflation for the next period, which then pushes up the *ex ante* real interest rate and stabilises output via the IS curve. This automatic stabilisation mechanism has become known as the ‘expectations channel’ of PLT.

Though these initial contributions are important for highlighting a key flaw in the conventional wisdom on PLT, they suffer from a major weakness: welfare analyses of monetary policy based on the New Classical Phillips curve are not microfounded, because the social loss function in Equation (1.17) cannot be derived as an approximation to the expected utility loss of the representative household.¹⁷ Furthermore, the New Classical Phillips curve has been widely criticised from an

¹⁷ Woodford (2003, Ch. 3) shows that a New Classical Phillips curve (with no output persistence) can be derived when a fraction of monopolistically competitive firms must set their output prices one period in advance, whilst the remaining firms have fully flexible prices. However, the approximation of the representative household’s expected welfare differs from Equation (1.17) and is not a sensible welfare criterion for monetary policy since only *unanticipated* fluctuations in inflation matter.

empirical perspective because it implies that only unanticipated inflation influences output, in direct contradiction to evidence from the structural VAR literature (e.g. Christiano *et al.*, 2005). For these reasons, most recent studies comparing IT and PLT have used the microfounded New Keynesian model (see Clarida *et al.*, 1999).

The New Keynesian model consists of households who maximise expected utility and profit-maximising firms. Firms are monopolistically competitive and produce their own individual differentiated output goods. Following Calvo (1983), a constant fraction of firms are unable to change the nominal price of their output in any given period, whilst the remaining fraction of firms are free to re-optimize their prices. Consequently, inflation in any given period comes solely from changes in output prices set by firms that are able to re-set their output prices. Profit maximisation by firms induces them to set their current output price as a function of marginal cost and the expected aggregate price level in the next period, with the latter entering firms' first-order conditions due to the possibility that nominal rigidities will prevent them from re-setting their output price next period. Moreover, under standard assumptions, all firms changing price in a given period will set the same output price,¹⁸ which simplifies greatly the task of aggregating across firms and gives rise to a simple Phillips curve describing economy-wide price setting behaviour.

Log-linearisation of this equation around zero trend inflation yields the so-called New Keynesian Phillips curve:

$$(1.20) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

where $0 < \beta < 1$ is the representative household's discount factor, $\kappa > 0$ is a constant that depends on the structural parameters of the model, and u_t is an AR(1) cost-push shock to inflation with a persistence parameter $0 < \rho_u < 1$ and a constant variance IID innovation. As before, π_t and x_t denote inflation and the output gap.

¹⁸ The assumptions are as follows: all firms face the same production technology and demand curves whose elasticity of substitution is common and constant over time (Walsh, 2003); the aggregate capital stock is fixed, but capital can be reallocated costlessly and instantaneously across firms (Ambler, 2007); and finally, there is unfettered access to perfect financial markets, leading to price equalisation across firms (Rotemberg and Woodford, 1998).

It is important to note that the cost-push shock does not arise from the pricing conditions of firms. It is instead appended to the New Keynesian Phillips curve to ensure that there is a trade-off between inflation volatility and output gap volatility (see King and Wolman, 1999). However, if monopolistic firms' output demand elasticities are subject to exogenous fluctuations, then a microfounded justification for the addition of the cost-push shock then emerges from firms' first-order conditions (Steinsson, 2003). The New Keynesian Phillips curve in Equation (1.20) has provided the foundation for most of the recent literature comparing the performance IT and PLT. The key difference relative to the New Classical Phillips curve in Equation (1.18) is that current inflation depends on expected future inflation, as opposed to the expectation of inflation in the current period. Consequently, monetary policy will be able to improve the trade-off between inflation and the output gap if it can favourably influence firms' expectations regarding future inflation.

Vestin (2006) was the first to compare IT and PLT with a New Keynesian Phillips curve. Like Svensson (1999), Vestin focuses on the case where IT and PLT policies are discretionary, though he uses the optimal commitment policy as a benchmark against which to compare the results under discretion. Two key results are found in favour of PLT. Firstly, if there is no persistence in the cost-push shock (i.e. $\rho_u = 0$), PLT can exactly replicate the optimal commitment policy. Secondly, in the more general case when there is cost-push shock persistence, PLT dominates IT because it inflation volatility is lower for any given level of output gap volatility. This result demonstrates that Svensson's free lunch result is robust to a change in Phillips curve specification from Equation (1.18) to Equation (1.20).

The intuition for the first result can be seen from the first-order condition for the optimal commitment policy in the New Keynesian model:¹⁹

$$(1.21) \quad \pi_t = -\frac{\lambda}{\kappa}(x_t - x_{t-1})$$

where λ is the relative weight on output gap stabilisation in the social loss function.

¹⁹ This condition will hold in the initial period only if Woodford's 'timeless perspective' is adopted (see Woodford, 2003; Ch. 7).

Equation (1.21) states that inflation should be traded-off against the *change* in the output gap and not its level. Therefore, if the output gap increases, inflation should be reduced and *vice versa*. In effect, then, the past output gap acts as a reference point that determines the course that optimal policy should take. This feature of optimal policy has become known as ‘history dependence’. History dependence is present under PLT because the actual price level depends on the deviation from the target price path in the previous period – as price-setters hold the expectation that such deviations will be reversed in the future. It can also be shown that Equation (1.21) implies that the optimal price level is stationary (see Clarida *et al.*, 1999), and hence that there is no base-level drift. Intuitively, since discretionary PLT implies both history dependence and a stationary price level, it is able to exactly replicate the optimal commitment policy if there is no cost-push shock persistence.

In the more general case when cost-push shocks are persistent, PLT is unable to replicate optimal commitment exactly because there is a ‘discretionary bias’ which has the effect of increasing inflation volatility compared to the optimal commitment case. However, PLT still reduces inflation volatility relative to IT, because history dependence and price-level stationarity – key features of the optimal commitment policy – are present under PLT but entirely absent under IT. Intuitively, the former is absent under IT because ‘bygones are bygones’, and the latter because it permits base-level drift in the price level. To be clear, the key advantage of PLT is that it allows the central bank to dampen the impact of a cost-push shock on current inflation by creating the expectation that the shock will be offset in next period, so that less of an inflationary shock is passed through to output prices by firms. Indeed, price-setters effectively face a trade-off under PLT: setting a high price in current period protects current profits, but will lead to a sharp reduction in future demand if there is no opportunity to re-optimize the output price in future periods. Under IT, by contrast, there is no such trade-off: firms find it optimal to pass inflationary shocks straight into output prices, because they do not expect the central bank to make any attempt to offset such shocks.

The New Keynesian Phillips curve in Equation (1.20) is attractive to researchers because of its strong theoretical foundations, but is not without its own flaws. For example, it cannot account for the structural inflation persistence observed in post-war

inflation data (e.g. Fuhrer, 1997), because it does not include a term in lagged inflation. However, modifying the price-setting structure so that firms whose prices cannot be re-optimised are indexed to past inflation gives rise to a Phillips curve in which current inflation is, additionally, a function of the past inflation rate (Christiano et al., 2005). Whilst it is difficult to justify this assumption theoretically given that price-setters are assumed to be rational (Minford and Peel 2003; Le 2008), it does at least give rise to more plausible inflation dynamics.

The resulting equation for aggregate inflation has been dubbed the ‘hybrid New Keynesian Phillips curve’ and takes the following form:

$$(1.22) \quad \pi_t - \gamma\pi_{t-1} = \beta(E_t\pi_{t+1} - \gamma\pi_t) + \kappa\alpha_t + u_t$$

where $0 \leq \gamma \leq 1$ measures the extent of indexation to past inflation.

Note that with this hybrid specification of the New Keynesian Phillips curve, the quasi-difference of inflation $\pi_t - \gamma\pi_{t-1}$ replaces the inflation rate in the purely forward-looking version. One implication of this result is that approximate measure of social welfare is no longer given by Equation (1.17): instead, the inflation rate in the social loss function is replaced with the quasi-difference of inflation (see Woodford, 2003). Gaspar *et al.* (2007) investigate the robustness of Vestin’s free lunch result in a model in which the New Keynesian Phillips is given by Equation (1.22) and the social loss function is appropriately adjusted. They find that, in general, it is optimal for monetary policy to fully offset price level shocks (as under PLT). The only time when it is not optimal to do so is the special case when price-setters fully index their prices to the past inflation rate (i.e. $\gamma = 1$). All in all, then, the result that PLT dominates IT is strongly robust to the modification that price-setters index their prices to past inflation when they are unable to re-optimize output prices.

Steinsson (2003) derives a similar hybrid specification of the New Keynesian Phillips curve based on ‘rule-of-thumb’ price-setting, but he reaches a different result. More specifically, Steinsson assumes that some fraction of price-setters follow a rule-of-thumb which dictates that price is set as function of the lagged output gap and

previous prices adjusted for lagged inflation. In this case, the hybrid New Keynesian Phillips curve is given by

$$(1.23) \quad \pi_t = \beta\theta_1 E_t \pi_{t+1} + \theta_2 \pi_{t-1} + \psi_1 x_t + \psi_2 x_{t-1} + u_t$$

where the weight on future expected inflation (θ_1) falls relative to the weight on past inflation (θ_2) as the fraction of rule-of-thumb price-setters is increased.

Furthermore, Steinsson derives the approximate social loss function in this case and uses it to investigate the form that optimal policy takes. He finds that the presence of rule-of-thumb price-setters means that it is no longer optimal to fully offset past shocks to the price level, and that the optimal level of price-level offset decreases as the fraction of rule-of-thumb price-setters increases. Consequently, the performance of PLT vis-à-vis IT deteriorates as the relative importance of rule-of-thumb price-setters increases, with the implication that Vestin's result on the dominance of PLT no longer holds.²⁰

This result stands in contrast to the conclusion reached by Gaspar *et al.* (2007) when firms' prices were indexed to past inflation. The reason is that in the Steinsson model there is a subset of firms – viz. those that follow the rule-of-thumb – whose price-setting behaviour is entirely backward-looking. This works against PLT because its benefits arise as a result of price-setters being forward-looking when setting output prices. On the other hand, in the model of Gaspar *et al.*, firms index to past inflation *only if* they are unable to re-set their output price, optimally resetting prices in a forward-looking manner the rest of the time. Consequently, all firms in the economy enjoy periods when their prices can be reset optimally, akin to the canonical model dealt with by Vestin (2006). The main lesson from Steinsson's analysis is that the assumption that one makes about firms' behaviour during periods when prices are 'sticky' is important for the IT-PLT welfare comparison. Furthermore, the theme that models with backward-looking expectations remove the benefits of targeting of the price level is returned to in Section 3 which discusses the potential costs of PLT.

²⁰ One criticism that could be made of Steinsson's model is that the implied social loss function does not have an intuitive interpretation. There are, for instance, additional terms in the lagged value of the output gap and the change in the inflation rate. It is difficult to reconcile these terms with the mandates delegated to central banks in practice.

1.2.2. Long-term inflation risk and nominal contracts

As noted in Section 1.1, inflation shocks have a cumulative effect on the price level and its forecast variance under IT: as the forecast horizon increases, so does uncertainty regarding the future price level. By contrast, by preventing base-level drift, PLT ensures that past price shocks cannot accumulate over the forecast horizon. As a result, price-level uncertainty is bounded as the forecast horizon increases, and the long-term purchasing power of money is preserved. This is the traditional argument put forward in favour of PLT (e.g. Duguay, 1994). With greater predictability of purchasing power, the real value of payments on long-lasting contracts denominated in nominal terms, or imperfectly-indexed to the price level, is less uncertain than under IT. Consequently, PLT should provide welfare gains to economic agents entered into medium and long-term contracts – examples include mortgages, long-dated bonds, pensions, and wage contracts – and these should ideally be quantified so that they can be easily factored into a cost-benefit analysis of PLT.

Increased nominal stability may also have non-trivial effects on the medium and long-term contracting behaviour of economic agents – for instance, nominal contracting might become more popular, whilst the incentive to index to prices (which is not without its own costs) may be reduced. Since longer-term assets and liabilities account for a substantial share in household portfolios,²¹ these effects may well be an important consideration for social welfare. Studies that have investigated the redistributive impact of an unanticipated increase in inflation have found that the impact is indeed sizeable (Meh, Rios-Rull and Terajima 2010; Doepke and Schneider 2006), suggesting that failure to contract optimally is costly for economic agents. In fact, Meh, Ríos-Rull and Terajima (2010) study redistribution and welfare effects of unanticipated inflation under IT and PLT and find that an unexpected 1 per cent increase in the price level in Canada implies a household sector welfare loss of 0.40 per cent of GDP under IT, compared to only 0.15 per cent under PLT. Given that these results suggest that PLT can have important implications for longer-term contracting decisions, such behaviour will need to be endogenised in economic models to obtain reliable welfare results to guide policy.

²¹ For instance, in Canada 70 per cent of assets and liabilities have a term-to-maturity of more than one year (Meh and Terajima, 2009), whilst in the UK around one-third of the government bond portfolio consists of nominal bonds with a term-to-maturity of 15+ years.

In the context of the IT versus PLT debate, Ambler (2009, p. 998) emphasises the importance of endogenising contracting behaviour as follows:

Accounting for the effect of the monetary regime on contracting is difficult...However, comparing social welfare across monetary policy regimes that are vulnerable to the Lucas critique can potentially give seriously misleading results. Endogenising the degree of indexation and other features of price and wage setting across monetary regimes is an important and promising avenue for future research.

There is, in fact, a small but growing literature that focuses on endogenising contracting behaviour in the context of IT and PLT. However, as the quote above from Ambler suggests, more research is desperately needed in this area to allow policymakers to conduct a reliable cost-benefit analysis. Chapters 2 and 3 undertake this task by building-up a DSGE framework in which the long-term impact of PLT on inflation volatility and social welfare can be quantified, and where contracting behaviour is endogenously determined. This section reviews current research in this area in order to motivate and provide context for the second and third chapters. Moreover, in Chapter 4, the models of chapters 2 and 3 are used to estimate the reduction in the inflation risk premium on long-term nominal contracts under PLT – an important issue which is also discussed in this section.

As the traditional argument in favour of targeting the price level, the topic of long-term inflation risk received considerable attention in early literature on PLT. Duguay (1994) surveys this literature and concludes that the potential benefits from PLT are substantial, though this view is disputed by Fischer (1996), who argues that such benefits are likely to be small compared to the potential increase in short-term output volatility. More recently, Dittmar *et al.* (1999) use a US-calibrated version of the New Classical Phillips curve model of Svensson (1999) to investigate medium- and long-term inflation volatility under IT and PLT. They estimate that PLT would lead to a substantial reduction in long-term inflation volatility, with the benefits to PLT increasing strongly with the forecast horizon due to the presence of base-level drift under IT.

Along similar lines, Gavin *et al.* (2009) set-up a microfounded New Keynesian model in order to investigate inflation risk at horizons of up to 10 years. The model nests a

sticky-price model and a sticky-wage model as special cases, and includes exogenous disturbances to monetary policy, preferences and technology, plus investment adjustment costs. They find that the optimal (short-term) policy in both the sticky-price and sticky-wage cases generates a substantial amount of longer-term inflation volatility, but that a PLT monetary policy rule can eliminate much of this volatility. One issue highlighted by this result is that long-term inflation volatility has no role for social welfare in standard New Keynesian models, because it is only short-term fluctuations that matter for the utility of the representative agent. Therefore, as Gavin *et al.* (p. 73) note with regard to future research, “the effect of long-run inflation risk on social welfare needs to be explicitly model[1]ed.” Current literature on PLT has not yet addressed this issue within DSGE models, but Chapters 2 and 3 of thesis present a modelling framework that allows the impact of long-term inflation risk on social welfare to be quantified.

Stuber (2001) and Crawford *et al.* (2009) both argue that PLT would lead to non-trivial reductions in long-term risk-premia on nominal debt contracts. Regarding this last argument, the empirical asset pricing literature provides evidence on the importance of inflation risk premia on nominal bonds at various horizons. For instance, Veronesi and Yared (2000) estimate that the inflation-risk premium on five-year nominal bonds in the US was significantly higher during the relatively volatile 1968-90 period than post-1990, whilst estimates of the five-year inflation-risk premium in the literature tend to be positive and non-trivial at upwards of 30 basis points on average (Ang *et al.* 2008; Hordahl 2008). Moreover, estimates of inflation risk premia on longer maturity bonds tend to be higher (see Bekaert and Wang, 2010), consistent with idea that inflation risk premia will increase with the accumulation of inflation risk over the forecast horizon under IT.²² For instance, Buraschi and Jiltsov (2005) estimate the ten-year US inflation risk premium over the postwar period using a continuous-time flex-price general equilibrium model in which monetary policy and taxes are endogenous. Their results suggest that the ten-year premium has averaged 70 basis points. Using a macro-finance approach, Campbell and Viceira (2001) find similar results. In particular, they conclude ten-year inflation risk premium in the US averaged 110 basis points over the postwar period.

²² It should be noted here that although the US is not formally an inflation targeter, the Federal Reserve is widely considered to have an implicit target for inflation.

In terms of IT versus PLT, there is somewhat less literature. The single formal contribution is the paper by Meh *et al.* (2008a),²³ who build a small open economy model in which firms can finance investment using short-term or long-term nominal debt contracts. Firms have the choice to default on both types of debt, so there is a risk premium in the cost of capital. Although IT and PLT are not modelled explicitly, the former is assumed to be represented by a ‘high’ level of long-term price-level uncertainty, and the latter by a ‘low’ level of uncertainty. Reducing long-term price-level uncertainty has two effects. First, reducing the level of long-term price-level uncertainty lowers the risk premium on debt (since there is a lower probability of default), and with it the cost of capital. Second, a reduction in long-term price-level uncertainty leads to an increase in the fraction of agents using long-term nominal debt, which in turn boosts investment and output. These results suggest that switching from IT to PLT could have a beneficial impact on investment and output by lowering the inflation risk premium, as argued informally by Lilico (2000).

In order to provide explicit evidence on the impact of PLT on inflation risk premia, Chapter 4 of this thesis estimates inflation risk premia on long-term nominal bonds under IT and PLT, using the models introduced and simulated in chapters 2 and 3. As emphasised by researchers at the Bank of Canada (see Stuber 2001 and Crawford *et al.* 2009), it is important to quantify the impact of PLT on inflation risk premia in order to accurately estimate its long-term benefits. The results reported in Chapter 4 can be viewed as a first but important step in this direction.

The impact of PLT on nominal contracting behaviour more generally has been more widely researched, though primarily in the context of medium-term wage contracts. The initial papers in this literature were due to Minford and various co-authors, who focus on the optimal degree of indexation of multi-period wage contracts (Minford and Peel 2003; Minford *et al.* 2003; Minford and Nowell 2003). Households have a strong incentive to insure against real wage fluctuations in these models, but are assumed to be unable to resort to financial markets for this purpose. The degree of wage indexation in the economy is chosen optimally to minimise such fluctuations, subject to monetary policy; hence the representative agent chooses optimal shares of

²³This paper is cited by Crawford *et al.* (2009).

indexed and nominal wage contracts in response to IT and PLT. On the basis of OECD data, the wage contract horizon is assumed to be one year, or four periods in the model. Minford and co-authors find several important results.

Firstly, optimal wage indexation is substantially lower under PLT than IT. This result is driven by the multi-period nature of wage contracts: PLT reduces nominal volatility relative to IT over the contract horizon, making nominal wage contracts relatively better real-wage stabilisers. Secondly, when indexation is endogenised, PLT increases social welfare because a reduction in indexation makes the real wage more flexible in response to productivity shocks and hence stabilises employment – a point first made by Gray (1976). Finally, if indexation is held fixed as monetary policy shifts from IT to PLT, this gives the misleading conclusion that social welfare is reduced under PLT. This last point is crucial because it suggests that in order to obtain reliable welfare conclusions about PLT at a medium- or long-term horizon, it is crucial that the degree of nominal indexation is endogenised in response to monetary policy.

The robustness of these results was subsequently investigated by Amano *et al.* (2007), who develop an alternative model in which there are staggered cohorts of labour-differentiated wages setters whose contracts are subject to multi-period nominal rigidities.²⁴ Moreover, in contrast to the model developed by Minford and co-authors, economic agents have unrestricted access to financial markets. Nevertheless, Amano *et al.* also find that optimal wage indexation is lower under PLT and that social welfare higher than in the IT case. Again, the welfare gains arise because reducing wage indexation increases employment stability in response to real shocks.

Meh *et al.* (2008b) extend the investigation of endogenous indexation to *financial contracts* that are imperfectly-indexed. More specifically, they develop a model with repeated moral hazard in which financial contracts are not fully indexed to inflation because nominal prices are observed with delay, as in Jovanovic and Ueda (1997). This assumption is motivated by the presence of a time lag before aggregate price indices become public information – for instance, due to the need to collect and process data prior to publication. Contracting in the model results from entrepreneurs

²⁴ In this context, 'labour-differentiated' means that each worker possesses a particular type of skilled labour that differentiates them from other workers.

entering into debt contracts with financial intermediaries so that they can finance investment. In concordance with the optimal wage indexation literature discussed above, Meh *et al.* find that the optimal degree of indexation falls with price-level uncertainty. One caveat, however, is that monetary policy does not enter the model explicitly, since the price level is exogenous. Nevertheless, these results do suggest that the optimal level of indexation of financial debt contracts would be lower under PLT than IT.

With regard to a long-term contracting horizon, the literature PLT is rather sparse. The reason, as noted by Dib *et al.* (2008, p.30), is that “model[ing] long-term contracts in macroeconomics is a major challenge and requires separate consideration”. Carlstrom and Fuerst (2002) and Minford (2004) both argue that PLT would reduce real return volatility on long-term nominal bonds relative to IT but stop short of modelling this impact explicitly. Carlstrom and Fuerst explain the potential impact thus:

[T]he base-drift problem with IT leads to a great deal of uncertainty about what the price level 5,10, or 30 years in the future will be. The central bank may miss its inflation target by a very small percentage in each year, but if these misses are not offset, they will accumulate and become quite large after 30 years. Therefore, a price-level target will reduce the uncertainty associated with buying and selling long-term fixed bonds.

Mankze and Tödter (2007) make a preliminary attempt at modelling the welfare impact of reduced long-term inflation risk under PLT by focusing on the real return on nominal bonds in an overlapping generations framework – a neat potential solution to modelling long-term contracts in a tractable way.²⁵ The model they use for this purpose is the canonical set-up in which government bonds are ‘net wealth’ because they act as a store of value (e.g. McCandless and Wallace, 1991). Given that each period is taken to last 30 years, this is also the horizon over which real return volatility on bonds matters for welfare. Monetary policy is not modelled explicitly because money is absent from the model; instead, inflation is assumed to be an

²⁵ The author independently had the idea of assessing the long-term welfare impact of PLT in an overlapping generations framework and did not become aware of Mankze and Tödter’s paper until summer 2010. Interestingly, the main focus of their paper is not PLT, and the paper is not cited anywhere in the literature.

exogenous stochastic process with a lower variance under PLT by factor of 30. Mankze and Tödter's results suggest that PLT would deliver a welfare gain of 0.066 per cent of long-term consumption over IT.

In order to understand these results, it is worth focusing briefly on the model. In real terms, the budget constraints of young and old consumers are given by

$$(1.24) \quad c_{t,Y} + b = Y$$

$$(1.25) \quad c_{t+1,O} = r_{t+1}b = \frac{(1+i)^{30}}{1+\pi_{t+1}}b$$

where $c_{t,Y}$ is consumption when young, b is fixed holdings of nominal bonds and $Y > 0$ is the constant endowment income received by the young. Consumption in old age of the generation born in period t is given by $c_{t+1,O}$; $i > 0$ is the constant *yearly* nominal interest rate after tax; π_t is the exogenous stochastic process for inflation; and r_{t+1} is the *ex post* real return on a nominal bond held from youth to old age.

Given that all young consumers are homogenous and have the same bond holdings, their consumption levels are equalised. However, old generations will be heterogenous *ex post* because their consumption levels depend on the stochastic process for inflation, which in turn determines the real return on bonds. Therefore, in this simple model, inflation uncertainty imposes a welfare cost for old generations through the uncertain real return on nominal bonds. In particular, if welfare in old age is given by $u(c_{t,O})$, the consumption risk premium is defined by the following equation:

$$(1.26) \quad u(c_{t,O}) = E(u(c_{t,O} + \lambda \bar{c}_O))$$

where λ is the risk-premium as a percentage of mean consumption in old age.

Mankze and Tödter assume that consumers have constant relative risk aversion (CRRA) utility with a risk aversion parameter δ . A second-order Taylor expansion of

Equation (1.26) around mean consumption thus yields the following expression for the consumption risk-premium:

$$(1.27) \quad \lambda \approx \frac{\delta}{2} \left(\frac{\text{var}(c_{t,o})}{\bar{c}_o^2} \right) = \frac{\delta}{2} \left(\frac{(1+i)^{60} b^2}{\bar{c}_o^2} \right) \text{var}((1+\pi_t)^{-1})$$

where b is the constant level of bond holdings per generation.

This result makes clear that the consumption risk premium increases with the level long-term inflation risk (i.e. inflation risk over the 30-year saving horizon). Consequently, the estimated risk premium under IT was 0.068 per cent, compared to only 0.002 per cent under PLT, with the difference between the two giving the reported welfare gain under PLT of 0.066 per cent of consumption in old age.

The overlapping generations framework put forward by Mankze and Tödter provides a simple and intuitive way by which to model the impact of PLT on social welfare via the long-term inflation risk channel, thus addressing in part the issues raised by Gavin *et al.* (2009) and Dib *et al.* (2008). However, there are a number of weaknesses with Mankze and Tödter's application of this framework, in addition to the fact that they do not model IT and PLT explicitly. First, the model is a 'partial equilibrium' model of the life-cycle since all generations receive an exogenous endowment income and no output is produced. Second, the nominal return on bonds is not endogenously determined by supply and demand for bonds, but is instead assumed to be constant. Third, there is assumed to be a positive (net) supply of bonds, yet there is no economic entity responsible for supplying bonds in the model. Fourth, the model abstracts from real assets such as capital or indexed bonds that offer protection against inflation. Last but not least, the model is unable to address the issue of optimal indexation, because consumers have access to nominal bonds but not indexed bonds.

To summarise, the overlapping model of life-cycle saving provides a useful framework for assessing the long-term welfare impact of PLT. Given the lack of research in this area so far, there are numerous extensions that can and should be

made to investigate the robustness of Mankze and Tödter's results. Three important extensions would be to introduce alternative assets into the model; to extend the comparison to a DSGE framework; and to allow consumers to hold both indexed and nominal bonds, with the degree of nominal indexation being endogenously determined in response to monetary policy. In Chapters 2 and 3 of this thesis, a modelling framework that fits this description is presented in which IT and PLT are modelled explicitly. This framework builds upon the basic model discussed above but permits the above extensions, plus many others, and is subsequently used to quantify the long-term impact of PLT on social welfare.

1.2.3 Price-level targeting and the zero lower bound

The zero lower bound (ZLB) refers to the idea that central bank will be unable to reduce the target nominal interest rate below zero in an economy that is inhabited by rational agents. The reasoning is that since money is a perfect store of nominal wealth, no rational agent would willingly hold bonds that promised to pay a negative return in money terms. Mathematically, the ZLB can be represented as follows:

$$(1.28) \quad i_t \geq 0$$

where i_t is the nominal rate of interest.

Most research that has compared stabilisation of the economy under IT and PLT has ignored the ZLB by assuming that central bank is free to set negative nominal interest rates if required. Since there are good reasons for thinking that this assumption is false, it is instructive to compare IT and PLT when the ZLB is taken into account.

In order to see the importance of the ZLB in the context of the IT-PLT comparison, consider the microfounded consumption Euler equation for a bond that offers a riskless nominal return i_t :

$$(1.29) \quad U_{c,t} = \beta(1 + i_t)E_t \left(\frac{U_{c,t+1}}{1 + \pi_{t+1}} \right)$$

where $U_{c,t}$ is the marginal utility of consumption at time t , and π_{t+1} is the rate of inflation between period t and period $t+1$.

If we assume for simplicity that there is no government spending or investment, that the economy is closed, and that utility is of the constant relative risk aversion (CRRA) form with risk aversion coefficient $\delta \equiv 1/\sigma$, then log-linearisation of Equation (1.29) gives an IS curve of the following form:

$$(1.30) \quad x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \sigma \times r_t^n$$

where $r_t^n \equiv \sigma^{-1} E_t \Delta y_{t+1}^n + r^n$ and $r^n \equiv -\log \beta$ is the long run natural rate .

This equation can be solved forward for the current output gap as follows:

$$(1.31) \quad x_t = -\sigma E_t \sum_{k=0}^{\infty} \{ (i_{t+k} - E_{t+k} \pi_{t+1+k}) - r_{t+k}^n \}$$

Equation (1.31) suggests that there are two routes by which monetary policy can potentially influence current output: the current and expected future path of the nominal interest rate; and the current and future path of inflation expectations.²⁶ In the presence of the ZLB, however, the nominal interest rate will be constrained in a downward direction. As a result, the inflation expectations channel becomes more important for policy outcomes. It is this additional importance of the inflation expectations channel that makes the comparison of IT and PLT interesting within the context of the ZLB. The literature has focused on two main issues. The first is the likelihood that the lower bound will be reached in the first place; the second is the performance of policy in models once ZLB is reached, or when the ZLB is treated (realistically) as an occasionally-binding constraint.

The likelihood of hitting the zero lower bound

Early literature on the ZLB concentrated on the probability that it would be reached under optimal policy. For instance, using simulations of model economies, a number

²⁶ Economists generally take the view that monetary policy cannot influence the natural rate of interest.

of studies estimated the risk of hitting the lower bound with an inflation target of around 2 per cent. As a ballpark figure, these studies suggest that the nominal rate would be reduced to zero between 1 and 5 per cent of the time (e.g. Cozier and Lavoie 1994; Reifscneider and Williams 2000). However, it is important to note that the relationship between the probability of hitting the ZLB and the inflation targeting is non-linear: as the inflation target is lowered towards zero, the estimated probability of hitting the lower bound increases sharply. More recent studies suggest that even with a positive inflation target, the probability of hitting the ZLB may be substantially higher than originally estimated at around 15 per cent (Lavoie and Pioro 2008; Amano and Ambler 2008).

In the context of PLT and the ZLB, Smets and Gaspar (2000) compare IT and PLT in a New Keynesian model with a hybrid New Keynesian Phillips curve and an IS curve with output gap persistence. The criterion they use to judge the probability of hitting the lower bound is nominal interest rate variability, the idea being that smaller deviations of the nominal rate should be associated with a lower probability of hitting the ZLB. The central bank in the model is assigned an IT loss function with an additional term in the squared deviation of the price-level from its target value. If the weight on this additional term is zero, the loss function corresponds to the standard IT case, while if the weight is positive the central bank engages in 'hybrid targeting' of inflation and the price level. Smets and Gaspar find that as the weight on the PLT objective is increased, nominal interest rate volatility initially falls, remaining below the level under standard IT unless the weight on price-level deviations is quite high.

This result is driven by the forward-looking terms in the Phillips and IS curves; indeed, the main intuition for this result has been covered already in the discussion of Section 2.1. For instance, suppose the economy is hit with a positive cost-push shock. Under PLT, price-setters anticipate that the shock will be offset next period, so their expectation of future inflation is lower. This lower expectation causes the *ex ante* real interest rate to rise which then dampens the impact of the shock on the output gap through the IS curve. Similarly, in the face of a demand shock, both the output gap and inflation will increase, but under PLT the impact will be mitigated by a reduction in expected inflation which pushes up the *ex ante* real interest rate and stabilises the output gap via the IS curve. Hence, the expectations channel under PLT is crucial

when considering the ZLB, because it means that smaller movements in nominal rates are necessary in order to stabilise the economy. Given that Smets and Gaspar examine a New Keynesian model with lagged indexation and output gap persistence, these results would be strengthened in a purely forward-looking version where the importance of the expectations channel would be even greater.

The robustness of these conclusions has been tested along a number of dimensions by other authors. Firstly, Amano and Ambler (2008) simulate a nonlinear version of the New Keynesian model and find similar results: the frequency with which nominal interest rates turn negative is somewhat lower under PLT than IT, and the economic intuition is unchanged. Secondly, a different but related approach is taken by Levine *et al.* (2008), who focus on discretionary IT policy using the Smets-Wouters model of the Euro Area. Given that policies which call for negative nominal interest rates are not operational, Levine *et al.* assume that the central bank must set a sufficiently high weight on interest rate volatility in their loss function to ensure that the probability of reaching the lower bound is close to zero. Under this modification, discretionary IT performs far worse by comparison to the optimal commitment policy, because a high relative weight on interest rate volatility is necessary to ensure that the interest rate does not reach the lower bound, and this causes policy to deviate substantially from the unconstrained optimal. Intuitively, discretionary IT performs poorly because there is no stabilisation through expectations, due to the absence of ‘history dependence’. Although a PLT policy is not explicitly evaluated by Levine *et al.*, their results do suggest that IT does not perform well when constrained by ZLB considerations.

An important caveat regarding all the above studies is that they ignore, or side-step in some way, the existence of the ZLB. This approach could give misleading results because the expectations of economic agents will be influenced by the presence of the lower bound (Adam and Billi, 2006); that is, economic agents will form their expectations with the knowledge that the central bank faces a constraint on how much it can cut interest rates in the future. Indeed, since endogenous variables are typically influenced in important ways by future expectations, it is advisable to estimate the probability of hitting the ZLB using models in which the lower bound constraint is *occasionally binding*. Such models are also advantageous from the point of view of

evaluating welfare, since it is only in such models that the behaviour of the central banks will be constrained by the ZLB in the same way as it would in practice.

The zero lower bound as an occasionally-binding constraint

More recent literature on the lower bound has drawn on advances in numerical simulation techniques to deal with its non-linear and asymmetric effects. Indeed, it should be emphasised that standard solutions techniques for DSGE models like log-linearisation and perturbation are unable to capture the impact of the ZLB (Ambler, 2009). Surprisingly, however, it is possible to solve for an analytical solution in a simple model, and this is done in the seminal paper by Eggertsson and Woodford (2003). This section first discusses the results of Eggertsson and Woodford and then turns to the remainder of the literature that has employed numerical simulation techniques in more comprehensive models of the economy.

Eggertsson and Woodford make a number of important contributions. First and foremost, they derive the optimal commitment policy when the ZLB is an occasionally-binding constraint, and show that it takes the form of a state-contingent PLT rule. Secondly, the performance of this optimal policy at the lower bound is compared to simple PLT and IT rules. Finally, a number of arguments are put forward in anticipation of criticisms that PLT would be ineffective or infeasible in practice if the ZLB were reached. A discussion of this last point is postponed until Section 1.4.1, which discusses practical issues regarding PLT at the lower bound.

The model is deliberately simple, consisting of a forward-looking IS curve, the microfounded New Keynesian Phillips curve, and the ZLB constraint:

$$(1.32) \quad x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n)$$

$$(1.33) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

$$(1.34) \quad i_t \geq 0$$

The central bank is assumed to minimise the microfounded loss function, Equation (1.17), which is repeated here for convenience:

$$(1.35) \quad L_t = E_t \sum_{i=0}^{\infty} \beta^i \{ \pi_{t+i}^2 + \lambda x_{t+i}^2 \}$$

To derive the optimal policy, first note that substituting the ZLB constraint into the IS curve yields the following inequality:

$$(1.36) \quad x_t \leq E_t x_{t+1} + \sigma(E_t \pi_{t+1} + r_t^n)$$

Using Equation (1.36), the optimal policy can be solved for by forming the following expected Lagrangian:

$$(1.37) \quad V_t = L_t + E_t \sum_{i=0}^{\infty} \beta^i \{ \varphi_{1,t+i} (x_{t+i} - x_{t+i+1} - \sigma \pi_{t+i+1} - \sigma r_{t+i}^n) + \varphi_{2,t+i} (\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - u_{t+i}) \}$$

The first-order conditions are given by

$$(1.38) \quad \pi_t + \varphi_{2t} - \varphi_{2,t-1} = \beta^{-1} \sigma \varphi_{1,t-1}$$

$$(1.39) \quad \lambda x_t + \varphi_{1t} - \beta^{-1} \varphi_{1,t-1} = \kappa \varphi_{2t}$$

$$(1.40) \quad \varphi_{1t} \geq 0, \quad i_t \geq 0, \quad \varphi_{1t} i_t = 0$$

where the inequalities in (1.40) are the Kuhn-Tucker conditions.

Note that since the Lagrange multiplier φ_{1t} is positive in some periods and zero in others, the nature of the optimal commitment rule differs in periods when the ZLB has been reached and in periods when it has not.

In fact, Eggertsson and Woodford show that the optimal commitment policy takes the form of a state-contingent PLT policy:

$$(1.41) \quad p_t + \frac{\lambda}{\kappa} x_t = p_t^*$$

where p_t^* is a price-level target which is updated using

$$(1.42) \quad p_{t+1}^* = p_t^* + \beta^{-1}(\delta_t - \delta_{t-1}) + \beta^{-1}\kappa\sigma\delta_t$$

and $\delta_t \equiv p_t^* - p_t + \frac{\lambda}{\kappa}x_t$ is the target shortfall in period t .

Three main points should be noted regarding this optimal rule. Firstly, when the ZLB has not been reached, the price-level target defined by Equation (1.41) can always be met, since this is just the standard optimal commitment policy.²⁷ Second, when the ZLB is reached, the price level target in Equation (1.41) becomes unattainable. For example, suppose the lower bound is reached in period T following a sharp fall in the natural rate. In this case, the target price level in period T will be given by Equation (1.42) with $\delta_T > 0$, because the central bank will be unable to cut rates to ensure that the price level does not fall below target in period T . The optimal response to this target shortfall is to raise the *target price level* in the next period $T+1$. In the following period, $T+2$, the target price level will be raised even further, provided there is still a target shortfall. This discussion brings us on to the third point: if the target shortfall grows (i.e. $\delta_{T+1} > \delta_T$), then the situation is worsening in the sense that the deviation from target is growing in response to deflationary pressures. Intuitively, the optimal rule overcomes such pressures in exactly the way suggested by Krugman (1998) – by creating an expectation of future inflation by raising the target price level. This is the optimal policy because with the nominal rate stuck at zero, the only way to achieve lower real interest rates is through stimulating inflation expectations by raising the target price level.

It should be clear from this discussion that the state-contingent nature of the price-level target is crucial in achieving optimality. However, it is far from clear that such a rule would be feasible or credible in practice. Having anticipated these concerns, Eggertsson and Woodford point out a much simpler PLT rule that is ‘near-optimal’:

$$(1.43) \quad p_t + \frac{\lambda}{\kappa}x_t = p^*$$

²⁷ This can be seen by taking the first-difference of Equation (1.41) conditional on a target shortfall of zero.

where p^* is a fixed price-level target.

The reason this rule performs well is that agents hold the belief that the future price level will be returned to the level defined by the constant price-level target. This belief creates an expectation of future inflation and reduces the real interest rate, increasing the current output gap and inflation – much like in the case of the optimal commitment policy. The key difference, however, is that the simple rule does not react to the *severity* of the situation by raising the target price level. It will therefore take longer for the economy to emerge from a ZLB episode.

Now consider the performance of an IT rule at the ZLB. Eggertsson and Woodford interpret such a policy as a commitment to adjust the nominal interest rate to ensure that inflation is equal to target, insofar as this is possible given the presence of the lower bound. The interest rate rule necessary to achieve this objective is given by

$$(1.44) \quad i_t = r_t^n + \pi^*$$

Note that the ZLB will prevent Equation (1.44) from holding if the natural rate of interest r^n is lower than $-\pi^*$; in this case, the central bank will set $i_t = 0$ and the real interest rate will be bounded by the negative of the inflation target. This constraint on real interest rates means that the IT rule performs substantially worse than the simple PLT rule or the optimal commitment policy, based on the loss function in Equation (1.35). There are two distinct but related problems with IT.

First, if the inflation target is relatively low, the real interest rate will be only slightly negative at the ZLB and is therefore unlikely to provide the stimulus to inflation expectations that is necessary to end a lower bound episode. Second, there is no response at all of the real interest rate to the severity of a ZLB episode, because inflation expectations are fixed on the inflation target. By contrast, both the optimal commitment policy and the simple PLT rule produce a real interest rate response, because they are history dependent: future expectations of inflation are stimulated whenever the price-level target is undershot at the ZLB. In concordance with Eggertsson and Woodford's welfare results, Wolman (2005) finds that PLT rules can

improve performance at the ZLB relative to IT, though in his model prices are set in staggered fashion and fixed for constant duration à la Taylor (1980).

Although Eggertsson and Woodford (2003) make the major contribution of deriving the optimal commitment policy when the ZLB constrains policy, it should be noted that their analysis of policies is only partial because it conditions on the occurrence of the lower bound by setting a large deterministic negative shock to the natural rate of interest in the initial simulation period. What is more interesting from the point of view of comparing alternative policies, however, is the performance of IT and PLT in simulations in which the ZLB is occasionally-binding and is hence free to bind or not as stochastic shocks dictate. Such simulations also permit the calculation of unconditional welfare, which gives an indication of the overall importance of the ZLB, and whether policies that perform well at lower bound are likely to be worth pursuing in practice. The task of calculating unconditional welfare was undertaken by Adam and Billi (2006, 2007), and independently by Nakov (2008). These papers are discussed in turn below.

The main contribution of Adam and Billi (2006) is to calculate unconditional welfare under optimal commitment when the ZLB is an occasionally-binding constraint. The model follows Eggertsson and Woodford but is calibrated to US data, with shocks identified using US experience from the early 1980s to early 2000s. In order to calculate unconditional welfare under optimal commitment these shocks are then used to simulate the model, which is solved numerically using collocation methods. Intuitively, the unconditional welfare implications of the ZLB are related to the frequency with which nominal rates reach zero under optimal commitment. Since zero nominal interest rates occur rather infrequently – only about one quarter in every 17 years (or a probability of 1.5 per cent) – the additional unconditional welfare loss due to the ZLB is small at approximately one per cent of the welfare loss generated by sticky prices.

The key to this result is that, since little time is spent at the lower bound, there are virtually no effects on the average levels of the output gap and inflation, and only relatively small effects on inflation and output gap volatility, which in turn have only

a second-order impact on social welfare.²⁸ These welfare results are robust to a significant increase in the variance of cost-push shocks and a markedly lower interest rate elasticity of output, but if the variance of natural rate innovations is tripled, the additional welfare losses due the ZLB increase from 1 per cent of the loss from sticky prices to 33 per cent. It should also be noted that, in concordance with the results of Eggertsson and Woodford (2003), Adam and Billi find that welfare losses *conditional on the ZLB* being reached are non-trivial.

One interesting issue not addressed by Adam and Billi (2006) is the importance of the commitment assumption for the unconditional welfare implications of the ZLB. This issue was investigated in a companion paper, Adam and Billi (2007), which calculates unconditional welfare losses in the discretionary case. For ease of comparison, the discretionary results are compared directly with optimal commitment, with losses expressed in terms of their welfare equivalent permanent consumption reduction. The additional loss from discretionary policy compared to commitment increases by around two-thirds due to the presence of an occasionally-binding ZLB. Therefore, ignoring the ZLB constraint significantly understates the welfare benefits of commitment vis-à-vis discretion.

The welfare analyses conducted by Adam and Billi (2006, 2007) are extended in a number of directions by Nakov (2008). His contribution is to compute unconditional welfare for a variety of simple *zero-truncated* Taylor-type interest rate rules – including IT and PLT rules – which he argues provide a more plausible representation of real-world monetary policy. Furthermore, the performance of these rules is compared against the optimal commitment policy; the constant PLT rule of Eggertsson and Woodford (2003); and discretionary IT.

The IT rule allows for the possibility of ‘interest rate smoothing’ and is given by

$$(1.45) \quad i_t = \max \{0, \rho_i i_{t-1} + (1 - \rho)[r^* + \pi^* + \theta_\pi (\pi_t - \pi^*) + \theta_x x_t]\}$$

²⁸ One cannot simply use the loss function in Equation (1.35) for policy evaluation, because with an occasionally-binding ZLB the average levels of inflation and the output gap can potentially differ across policies.

where r^* is the equilibrium real interest rate and $0 < \rho_i < 1$ indicates the extent of interest rate smoothing.

The PLT rule is given by

$$(1.46) \quad i_t = \max \{0, r^* + \theta_\pi (p_t - p^*) + \theta_x x_t\}$$

where p is the log price level and p^* is a constant price level target.

With the exception of the constant price-level target policy suggested by Eggertsson and Woodford (2003), all the policies perform poorly relative to optimal commitment. On some level this finding is not surprising: simple interest rules rarely perform well in individual models of the economy, but have the advantage of robustness across models (Taylor, 1999). This point is clearly demonstrated by the substantial difference between the performance of the constant price-level target policy and the PLT Taylor rule. The constant price-level target delivers a loss in social welfare relative to commitment of only 56 per cent, whilst the corresponding loss for the PLT interest rate rule is 800 per cent.

As Nakov notes, it may be difficult to communicate even the constant price-level target policy in practice, and it is not clear that implementing a model-specific rule of this nature would be desirable in the real world. The social losses under truncated interest rate rules may therefore provide a better basis for judging the performance of PLT at the ZLB than does the constant price-level target rule. Even on this measure, however, the PLT interest rate rule outperforms the IT one in terms of welfare loss, with lower inflation and output gap volatility regardless of whether or not interest rate smoothing is permitted. To summarise, Nakov's results suggest that constraining the IT-PLT comparison to simple Taylor-type rules reduces but leaves intact the welfare benefits of PLT.

Finally, there is one other key strand of literature that has compared the performance of IT and PLT at the ZLB. These papers use more comprehensive models of the economy of the kind used by central banks for quantitative policy analyses. Such models have two potentially important advantages over those based on

microfoundations: first, a wide array of economic mechanisms are at work, and these may help identify benefits and costs of certain monetary policies; and, second, since such models provide better ‘fit’ to data, they may provide a better basis for policy evaluation. However, this extra rigour comes at an additional cost in the context of the ZLB, since it is only computationally feasible for policy outcomes to be examined once the model has been ‘deterministically guided’ to the ZLB. In other words, a deterministic shock to the economy is chosen so that the nominal interest rate is pinned at zero temporarily, during which time the performance of alternative policies is evaluated.²⁹

The main papers in this literature result from a joint research project between researchers at the Bank of Japan (BoJ), the European Central Bank (ECB) and the Federal Reserve, whose results are summarised in Fujiwara *et al.* (2006a). This research project focuses on three different central bank models: the FRB/US model, the BoJ JEM model, and the ECB Area-Wide model. Amongst other policies, these papers consider zero-truncated IT and PLT Taylor-type rules. The ZLB is modelled in the same way in all three papers: the economy is hit with a set of demand shocks which are *known by all agents* and which ensure that the short-term nominal interest rate is pinned at zero for four or five consecutive years. Policies are then evaluated based on an intertemporal loss function in inflation and output gap deviations whose horizon depends on the length of the ZLB episode. A robust conclusion that arises from these analyses is that PLT outperforms IT, though it should be noted that the relative benefits of PLT vary considerably depending on the model under consideration.

1.3 Potential costs of price-level targeting

A number of potential costs of PLT have been identified in the literature. These include the following: increased short-term volatility in inflation and output when agents do not have rational expectations; a costly transition period from IT to PLT; and potential time-inconsistency problems. In this section, which draws heavily on Ambler (2009), each of these topics is taken in turn.

²⁹ That is, policies are only evaluated conditional on the ZLB being reached.

1.3.1 Inflation and output gap volatility

The traditional argument made against PLT was that it would increase short-term volatility in both inflation and the output gap (e.g. Fischer, 1996). The logic of this argument is sketched out clearly by Svensson (1999, p. 278):

In order to stabilize the price level under PLT, higher-than-average inflation must be succeeded by lower-than-average inflation. This should result in higher inflation variability than IT, since in the latter case, base level drift is accepted and higher-than-average inflation need only be succeeded by average inflation. Via nominal rigidities, the higher inflation variability should then result in higher output variability.

Early literature that investigated the performance of IT and PLT in simulated models of the economy tended to confirm this view (e.g. Lebow et al., 1992; Haldane and Salmon, 1995). The conclusions of these studies differ from those in Svensson (1999) and found by later authors due to the assumption that agents have adaptive expectations. Adaptive expectations are purely backward-looking, with the result that the expectations channel by which PLT can stabilise inflation and the output gap is eliminated. As noted in Section 2.1, the short-term volatility benefits from PLT also arise under the assumption that the central bank cannot commit, a distinction which is irrelevant in purely backward-looking models. However, as noted in Section 2.1, increased volatility under PLT is not limited to models where all agents are backward-looking: Steinsson (2003) shows that if a fraction of firms are rule-of-thumb price-setters, PLT is no longer optimal, with its performance vis-à-vis IT deteriorating as the fraction of rule-of-thumb price-setters is increased.

1.3.2 The transition from inflation targeting to price-level targeting

The studies discussed thus far all assume that the economy has settled in a long run PLT regime in which individuals understand perfectly the workings of policy and have rational expectations. However, since PLT has been implemented only once in history whilst IT is widespread, it seems reasonable to assume that there would be a transitional period of adjustment during which agents would learn about the new monetary policy regime and the PLT regime would, if implemented as promised, acquire credibility. A number of papers have therefore investigated the IT-to-PLT transition and, in particular, whether the long run benefits from moving to PLT outweigh the short run transitional costs.

For instance, Gaspar *et al.* (2007) assume that the expectations of economic agents in the transition period are determined by recursive least squares learning (RLS). Under this assumption, the lagged data produced by the model are used to form forecasts of endogenous variables using least squares projection, with forecasts updated as new data becomes available. Gaspar *et al.* focus on the IT to PLT transition under the assumption that agents learn about the new PLT regime according to RLS, and the New Keynesian Phillips curve in the model takes the hybrid form that results from partial indexation to past inflation. Although Gaspar *et al.* find that the transition to PLT can be costly, the net gains from PLT remain positive unless learning is rather slow. For instance, with their benchmark calibration it takes seven years until the social loss under PLT is reduced to the level that prevails in the long run under IT. After this point, the social loss under PLT is lower than under IT and converges upon the level under the optimal commitment policy.³⁰

However, Kryvtsov *et al.* (2008) reach a different result when imperfect credibility is modelled as an exogenous process that converges towards perfect credibility over time. An interesting insight from their model is that the beneficial expectations channel under PLT remains weak whilst the public still fears that monetary policy could revert back to the old IT regime. In the model, it takes two-and-a-half years for the PLT central bank to earn enough credibility to outperform IT, but this relatively short period is enough to ensure that, in net welfare terms, IT dominates PLT: the short run transition costs from PLT outweigh the long run welfare benefits. Cateau *et al.* (2009) extend the analysis of the transition from IT to PLT by using ToTEM (Terms-of-Trade Economic Model), the Bank of Canada's main policy analysis model. This model is a medium-scale open-economy DSGE model built around optimising behaviour by firms and households. The goal of the paper is to test whether the conclusions reached by Kryvtsov *et al.* (2008) are robust in a more comprehensive economic model that retains forward-looking behaviour. Cateau *et al.* find results that are more favourable to PLT: the long run welfare gains dominate the short run transition costs, provided that the initial spell of imperfect credibility under PLT lasts less than 13 years. It is worth noting in this regard that the results of Carroll (2003) and Mankiw *et al.* (2003) based on expectations survey data suggest that US

³⁰ As discussed in Section 1.2.1, it is optimal to offset shocks to the price level unless there is full indexation to past inflation (i.e. $\gamma = 1$). In their benchmark case, Gaspar *et al.* assume that $\gamma = \frac{1}{2}$.

households incorporate new macroeconomic information into their expectations in around one year.

1.3.3 Time inconsistency

Kydland and Prescott (1977) were the first to point out that monetary policy faces a time-inconsistency problem if agents have rational expectations. That is to say, a policy which is *ex ante* optimal can be suboptimal *ex post*, giving policymakers an incentive to deviate from socially-optimal policies to which they are not committed by some form of ‘commitment technology’. This is potentially problematic for PLT, because, as discussed in Section 1.2.1, it will only tend to reduce short-term volatility relative to IT if agents have rational expectations. Recall that the reasoning for this result was that PLT provides additional stabilisation through an ‘expectations channel’ – in effect, firms move their prices less in response to shocks under PLT, because they expect shocks to be undone in future periods in order to return the price level to trend. However, if firms do hold such expectations and set prices accordingly, the central bank will then be faced with the temptation to deviate from the announced PLT policy *ex post* (Minford and Peel 2003; Ambler 2009). Intuitively, the central bank could ‘have its cake and eat it’ by benefiting from stabilised inflation by announcing a credible policy, only to renege on the announcement to avoid the cost of (say) reducing inflation below trend in following period. The central bank would in theory be faced with this temptation in every period, because reducing inflation below trend increases price dispersion across firms, which in its turn causes aggregate output to deviate from its efficient level.

As Ambler (2009) points out, the incentive to renege on the promised PLT policy would be intensified at the ZLB, since, once the nominal rate has moved away from zero, it would be far less costly to deviate and announce a new (lower) target price-level path than to achieve inflation above target for a substantial period of time in order to climb back to the initial target price path. This time-inconsistency problem is not formally captured by current models in the literature, but is an issue that is worthy of additional research.

The reason that current models are unable to capture this temptation – even in the discretionary case – is that the central bank loss function depends on current and future price level deviations. The central bank would thus harm its own welfare if deviated from the announced policy in the current period, because it would then wish to undo the price deviation in the next period in order to return the price level to trend. In this sense, PLT rules out useless discretionary behaviour that is present under IT (Minford and Peel, 2003). However, what has essentially been argued above is that the central bank may be prepared to deviate from its *loss function*. There is an inconsistency in the current literature in the sense that the central bank is often assumed to be unable to commit to rules-based behaviour, yet able to commit to a particular loss function indefinitely.

1.4 Practical issues

A number of practical and implementation issues surrounding PLT are discussed in this section, including: credibility at the zero lower bound; the time horizon over which the price level should be returned to target; the idea of ‘hybrid targeting’ of inflation and the price level as a superior alternative to IT or PLT; open economy and financial market considerations; communication and credibility of PLT; and lack of practical experience.

1.4.1 Price-level targeting at the zero lower bound: practical issues

One major issue regarding PLT at the lower bound is credibility of the target price path. The credibility assumption is crucial since, as we have seen, the benefits of targeting the price level at the ZLB result from the automatic stabilisation implied by the link between inflation expectations and the real rate of interest. To be more precise, PLT creates an expectation of future inflation, because a central bank that targets the price level promises to return the price level to its target path as soon as this is feasible. This then lowers the real rate of interest and boosts current output and inflation, providing a potentially effective escape route from a ZLB episode. In short, for PLT to be beneficial at the lower bound, a high degree of credibility is a prerequisite.

It is worthwhile, then, to consider whether central banks would in practice be able to commit to PLT rules and to establish a reputation for following such rules. Eggertsson and Woodford (2003) argue forcefully that there is little reason for concerns about the credibility of PLT at ZLB. They put forward two reasons in support of this argument. First, the constant PLT rule they discuss is near-optimal and could be easily communicated to public, making it easy for the public to detect deviations from the announced rule. Second, they argue that the incentive to change policy at the ZLB and act in a time-inconsistent fashion is actually stronger under IT than under PLT, because social welfare is lower under the former.

On the first point, it is notable that Nakov (2008) argues that it may be difficult to communicate and implement even the 'simple' PLT rule analysed by Eggertsson and Woodford. He argues that simple *interest rate rules* are superior in this regard. Furthermore, monetary policies which are optimal or near-optimal in a specific model are usually not robust in terms of performance across models (Taylor, 1999). Regarding the second point, Eggertsson and Woodford are essentially arguing that PLT is well-suited to the demonstration of commitment because it is equally optimal in both ordinary and extraordinary circumstances. In other words, by sticking to the PLT in the past – prior to the occurrence of a ZLB episode – central banks could build-up a reputation for following through on promises and assuage any doubts agents may have about their commitment if the lower bound were reached. However, this point ignores an additional issue specific to the ZLB: observational equivalence. For instance, agents may doubt whether PLT is really being followed since returning the price level to its target path would require a zero nominal interest rate for a prolonged period, but so would any other reasonable policy. In summary, there remains the potential for time-inconsistency problems under PLT – problems which are likely to be amplified at the ZLB and which would be understood by rational agents. It is also worth noting that the arguments made by Eggertsson and Woodford implicitly assume that agents have already learnt about the PLT regime, though this is not the situation that would be faced by real-world central banks considering whether to adopt PLT.

1.4.2 The optimal horizon for returning the price level to target

In practice, IT central banks do not aim to instantly return inflation to target following a deviation. Instead, they aim to do so over the medium-term, which in this context means a period of 1 to 3 years. There are two reasons for this. First, changes in monetary policy affect inflation and output with a lag of 1-3 years (Christiano *et al.*, 2005; ECB, 2010). Secondly, responding with gradual changes in interest rates may prevent excessive volatility, as argued by Sack (2000). The optimal horizon for offsetting target deviations can be interpreted in terms of the relative weight on output gap stabilisation in the central bank loss function, with a higher weight implying a longer optimal targeting horizon.

With regards to PLT, the question arises as to whether the optimal horizon for offsetting deviations would change. This question has been investigated formally in a couple of papers. Smets (2003) investigates this issue in a New Keynesian model that is estimated on the Euro Area economy. The model consists of a hybrid New Keynesian Phillips curve and an IS curve with output gap persistence, as in Smets and Gaspar (2000). He finds that the optimal horizon for returning the price level to the target path is twice as long as the optimal horizon for returning inflation to target. The reasoning for this result is that the expectations channel under PLT is weakened compared to the purely forward-looking case, because the New Keynesian Phillips curve is partly backward-looking. Hence whilst PLT can still reduce inflation volatility relative to the IT case, the reduction is not that strong. Consequently, both nominal interest rate volatility and output gap volatility rise for a given targeting horizon, such that, in order to bring down interest rate and output volatility to levels similar to under IT, a longer targeting horizon is necessary. Similarly, Coletti *et al.* (2008) compare IT and PLT in an open economy DSGE model and find that the optimal targeting horizon is higher under PLT, though it should be noted that both horizons are relatively short at less than one year. As noted by Cournède and Moccero (2009), an issue related to the target horizon that has not yet been addressed in the literature is the width of the optimal band around the target price path.

1.4.3 Hybrid targeting of inflation and the price level

Some papers have investigated the performance of PLT by focusing on hybrid targeting regimes in which the central bank places some weight on both inflation and price-level deviations from target (Gaspar and Smets, 2000; Batini and Yates, 2003; Cecchetti and Kim, 2005). In this literature, a positive weight on price-level deviations precludes base-level drift in the long run, and variations in the weight change the horizon over which the price level is returned to the target price path. For instance, Batini and Yates (2003) focus on an open economy model in which the exchange rate enters the Phillips curve. An interesting conclusion from their analysis is that pure PLT increases inflation volatility compared to pure IT. The reason for this result is that exchange rate volatility is raised under PLT via the uncovered interest parity condition (since the hybrid New Keynesian Phillips curve has a strong backward-looking component), and this requires substantial real interest rate volatility in order to return the price level to the target path. The optimal relative weight on the price-level objective is therefore rather low. A general conclusion from studies of hybrid targeting, however, is that an intermediate regime can be found that dominates strict IT or strict PLT if there are both forward and backward-looking inflation terms in the Phillips curve (see Ambler, 2009).

Nessén and Vestin (2005) take a similar approach to these studies but investigate 'average inflation targeting'. Under this policy the central bank targets a moving average of current and past inflation rates. As the horizon of the moving average is extended, the amount of price-level drift is reduced, converging to PLT in the limit. Hence the length of the moving average window defines a spectrum with IT at one end and PLT at the other. The question Nessén and Vestin set out to answer is whether an intermediate point on the spectrum would outperform pure IT and pure PLT, for the case where the central bank operates under pure discretion. The model is New Keynesian, with a moderate fraction of firms following backward-looking rule-of-thumb price-setting, and the remainder setting prices subject to nominal stickiness à la Calvo (1983). The supply side of the economy is thus described by the hybrid New Keynesian Phillips curve in Steinsson (2003). Nessén and Vestin find that if the size of the moving average window is chosen optimally, average IT performs better than either strict IT or PLT and provides a good approximation to the optimal

commitment policy. Intuitively, Vestin (2006) shows that PLT is optimal if the New Keynesian Phillips curve is purely forward-looking, whilst Steinsson (2003) notes that IT outperforms PLT if the fraction of rule-of-thumb price-setters is high. Therefore, it is not surprising that a policy which is a hybrid of the two performs well when a moderate fraction of firms follow rule-of-thumb price-setting.

1.4.4 Open economy and financial market considerations

There has been a fair amount of work on the performance of PLT in an open economy, much of which has been carried out by, or in conjunction with, the Bank of Canada. Coletti *et al.* (2008) compare IT and PLT in the two-country IMF Global Economy Model (GEM), a medium-scale DSGE model designed to enable open-economy issues to be investigated within a rigorous representative-agent framework (see Laxton, 2008). The model is calibrated for Canada using the Bank of Canada's ToTEM model, with the US as the second country in the model. The results focus on the implications of IT and PLT for macroeconomic stability in Canada. Coletti *et al.* find that a PLT Taylor-type rule slightly outperforms an IT one in terms of inflation and output gap volatility, primarily because shocks to the terms of trade strengthen the case for PLT due to its role as a nominal anchor.

Dib *et al.* (2008) investigate the impact of PLT within a medium-scale open economy model whose parameters are estimated using Canadian data. The model is New Keynesian but is augmented with credit frictions as in Bernanke *et al.* (1999), plus one-period nominal debt contracts which entrepreneurs enter into in order to finance investment. They find that PLT is better than IT at minimising the distortion in the economy due to nominal debt contracts, because the former leads to less revaluation of nominal contracts given that inflation expectations are stabilised through the 'expectations channel'. Real risk faced by entrepreneurs is therefore reduced, with the result that resources are allocated more efficiently under PLT. Moreover, this increase in efficiency means that nominal interest rates do not need to vary as much in order to minimise the distortion associated with nominal price stickiness, such that the real interest rate volatility is reduced, and the distortion from nominal debt is lessened. However, if the IT policymaker follows a Taylor rule with 'interest rate smoothing', much of this extra volatility is eliminated, because the IT policy is then history-dependent and therefore performs almost as well as PLT.

Covas and Zhang (2010) focus solely on financial market imperfections. They compare IT and PLT in a closed-economy New Keynesian model that includes financial market imperfections in both debt and equity markets, and whose structural parameters are estimated for the Canadian economy. They find that PLT outperforms IT because the expectations channel means that inflation is better anchored under PLT than IT, which in turn enables a PLT central bank to deal with financial market distortions through policy whilst ensuring that inflation remains firmly anchored. It should be noted, however, that benefits of PLT are smaller than in absence of financial market imperfections and decrease as the extent of financial market frictions is strengthened. Therefore, whilst the research thus far suggests that PLT is robust in forward-looking models augmented with simple financial frictions, allowing fully for financial market imperfections could substantially reduce, or even eliminate, the short-term stabilisation benefits to be had from targeting the price level.

1.4.5 Communication and credibility of price-level targeting

Central banks that follow IT typically produce their own forecasts for inflation. Such forecasts have obvious importance for policy, but they are also used as a basis for transparently communicating policy decisions to the public. This is usually done via central bank 'inflation reports'. This strategy seems to have been effective thus far in the sense that inflation expectations, as measured by survey-based and market-derived measures, appear to have been well anchored around the target rate of inflation (Gürkaynak *et al.* 2007; Demertzis and van der Cruijsen, 2007). In other words, it seems that IT central banks have been perceived as credible.

A key issue that has remained unresolved is the way in which PLT would be communicated to the general public. For instance, one option would be to continue to publish inflation forecasts. However, as the short-term inflation target is time-varying under PLT, inflation forecasts could be a source of confusion, particularly in countries in which IT has been in place for a substantial period of time. A second option would be to publish price level forecasts instead, and perhaps 'price level reports' to go along with these forecasts. This approach would have the advantage that the forecasts could be directly compared to the predetermined target price path, which could of course be published well in advance. If PLT could successfully stabilise prices around

the published target price path, it may be possible for PLT to attain credibility quite quickly. On the other hand, there is more uncertainty surrounding the outcome of this approach since, as noted by Ambler (2009), the public has been conditioned by IT to think in terms of inflation and not the general price level. The hybrid targeting results discussed above have practical importance because it would likely be easier to communicate a policy of ‘average IT’ given that the horizon of the inflation target and inflation forecasts could simply be altered (Ambler, *op. cit.*).

A second key issue with regard to PLT is how quickly it would be able to attain credibility. As noted in Section 1.3.2, credibility is likely to be a key factor in determining the transition costs of switching to PLT. If credibility is established quickly, the transition costs are likely to be low because agents’ expectations will shift more swiftly from backward-looking learning mechanisms to (forward-looking) rational expectations. The extent of credibility initially attained will depend crucially on the ability of central banks to effectively communicate PLT to the general public.

Therefore, to the extent that public expectations are influenced by economic and financial forecasts, financial markets and the media may provide an important transmission mechanism by which PLT could gain credibility.³¹ Similarly, PLT is more likely to gain credibility quickly if central banks behave in a transparent manner, as this will enable economic agents to make their own assessment of whether the stated objectives of PLT are (i) achievable, and (ii) being pursued seriously by policymakers. In the long run, it seems unlikely that credibility would be an issue, since central banks would have sufficient time to demonstrate their commitment to PLT through policy actions.

1.4.6 Lack of practical experience: the Swedish experiment

An explicit PLT regime has only been adopted once in history, namely in Sweden in September 1931 after it left the Gold Standard. A detailed account of the Swedish experience with PLT is given by Berg and Jonung (1999) and Straumann and Woitek (2009). The PLT regime was intended to stabilise prices against the backdrop of deflation that was widespread during the Great Depression. In particular, the price-

³¹ However, for this to be so, a prerequisite is that financial markets and the media perceive the regime as credible.

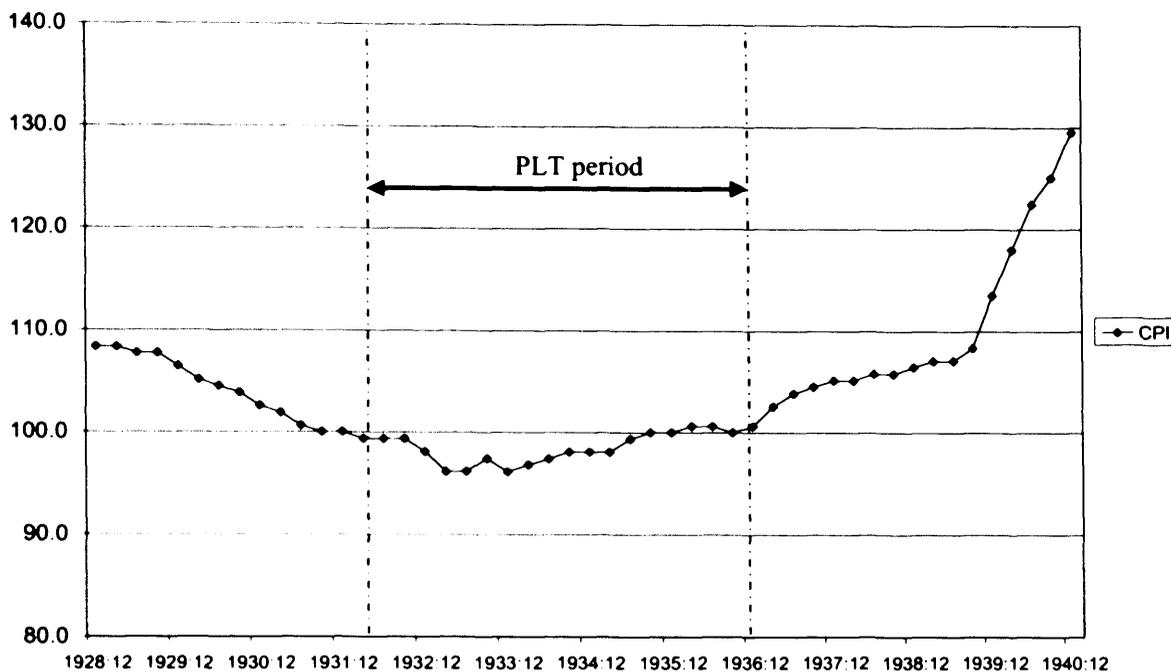
level target related to the Consumer Price Index (CPI) – the average level of consumer goods prices – and was a constant target set at the September 1931 value, which for simplicity was normalised to 100. A floating exchange rate was initially adopted as part of the PLT regime, but in July 1933 the Riksbank decided peg the krona to the British pound. The Swedish experiment with PLT ended officially in April 1937 when the Riksbank chose to maintain the peg against the Pound – a strategy that was seen as inconsistent with the price-level target given that CPI was consistently above target in the early part of 1937.

The main features of the Swedish PLT regime are summarised in Table 1.2, and for reference the evolution of the CPI from 1928 to 1940 is shown in Figure 1.5. The CPI does seem to have stabilised during the PLT period, and it is notable that the deviation of the CPI from target was never greater than 3.8 per cent. Furthermore, it appears that the price level was successfully returned to target after falling more than 3 per cent in 1932 and 1933. Given the difficult economic backdrop, the performance of PLT in Sweden seems fairly strong. For instance, the US experienced almost 10 per cent deflation between September 1931 and April 1933 alone (Dittmar *et al.* 1999). Moreover, real indicators like industrial production and real income also fared well compared with most other countries (Berg and Jonung, 1999).

Table 1.2 – The Swedish price-level targeting regime

Feature	Description
Introduction	As soon as the Gold Standard was suspended (September 27, 1931)
Operational target	CPI at the date of introduction (normalised to 100), but other price indices were also monitored
Instrument	Discount rate
Role of the exchange rate	The krona was pegged to the Pound from July 1933 to the beginning of WWII
Role of money aggregates	No role mentioned in the Monetary Programme
Goal independence	No – policy goals were set by the Riksdag (the Swedish parliament)
Instrument independence	Yes
End date	April 1937

Source: Adapted from Guender and Oh (2006)



Source: Statistics Sweden

Figure 1.5 – The Swedish experience with price-level targeting

Although PLT appears to have performed well, there are a number of reasons to be sceptical about whether PLT was responsible for the relatively strong performance of the Swedish economy. Firstly, countries that left the Gold Standard at an early point tended to perform better during the Great Depression and in its aftermath (Bernanke, 1995). Hence, it could be that the decision to leave the Gold Standard was mainly responsible for Sweden’s good performance. Secondly, since the krona was pegged to the Pound from July 1933, it is far from clear that PLT was in fact being pursued during this time. Indeed, as Figure 1.5 makes clear, the deviations of the CPI from target were highest when the exchange rate was floating. Therefore, if only the September 1931 to July 1933 period was truly a PLT regime, the performance of PLT appears less favourable.

Straumann and Woitek (2009) go even further and argue that, after leaving the Gold Standard, the Riksbank was targeting the exchange rate with the Pound rather than the price level. They argue that “although being a major innovation in the history of Swedish economic thought, price-level targeting had no practical importance for the Riksbank in the 1930s” (p. 252). Their argument rests on both archival and econometric evidence. First, in terms of archival evidence, they present a series of

statements by the Riksbank governor at the time, Ivar Rooth, which suggest that in practice the policy implemented was not the PLT one that was announced publicly. They also show that some support for these statements can be found in private correspondence between the Riksbank and the Bank of England at the time. Second, Straumann and Woitek run a time-varying Bayesian VAR covering the period 1920-1939. The resulting impulse responses cast doubt on whether the price level was the target of policy, because the discount rate (the Riksbank's instrument at the time) barely responds to a price shock, a result which is robust at different lag lengths and to various choices of the price index. Moreover, the impulse responses also suggest that there was no regime change after the 1930s. This observation provides further evidence against the hypothesis that PLT was adopted since, if the price level and not the exchange rate was the target of policy from September 1931, we would expect to see a structural break around July 1933 when it was officially announced that the krona would be pegged to the Pound.

Whilst neither the econometric or archival evidence that Straumann and Woitek present are entirely convincing, they are enough to cast doubt on whether the Riksbank really did adopt PLT. Furthermore, even if the Riksbank did target the price level, it must be remembered that this would provide only a single example in history during an extraordinary period. Consequently, the question of whether Sweden did or not adopt PLT has little relevance for the current debate on IT versus PLT. In fact, the most important contribution of this episode may have been to highlight at an early stage the role that management of expectations can play in effective economic policy.

1.5 Summary

This chapter has provided an exhaustive literature review of PLT, beginning with a simple comparison against IT before proceeding to discuss its potential advantages and drawbacks, as well as practical issues that have been identified in the literature. In terms of theoretical literature, the impact of PLT on short-term volatility has been investigated in a wide range of models. The key finding from this literature is that PLT will tend to outperform IT in models in which agents are sufficiently forward-looking, because it favourably influences inflation volatility through the inflation expectations channel. The literature has also reached the robust conclusion that PLT

will outperform IT in models in which the zero lower bound on nominal interest rates is a binding constraint on monetary policy, though more research is needed in this area to address the impact of imperfect credibility. One important area in which there is relatively little literature, however, is the impact of PLT on the welfare of economic agents that enter into long-term nominal contracts. Indeed, the theoretical literature suggests that such gains could be substantial, since if implemented successfully PLT would reduce long-term inflation risk by an order of magnitude compared to IT.

The aim of chapters 2 to 4 of this thesis is to quantify these gains. In order to do so, an overlapping generations model of life-cycle saving is built in which young generations save for old age using indexed and nominal government bonds whose payoffs are vulnerable to long-term inflation risk. This model has the desirable theoretical property that long-term inflation risk matters for social welfare, and is a DSGE model in the sense that all important variables are endogenised. Using this overlapping generations model, the key long-term benefits of PLT are quantified, including its impacts on consumption risk, social welfare, and inflation risk premia on long-term nominal contracts. Moreover, since the extent of indexation of government bonds is chosen optimally under IT and PLT, the model is able to address the issue of optimal contracting behaviour and should not be vulnerable to the Lucas critique. The model is also used to study the impact of heterogeneities across countries and over time, since the general model nests several special cases and is subjected to extensive sensitivity analysis.

From a practical perspective, there remain a number of unanswered questions regarding PLT that are not addressed in this thesis. The key ones are as follows: the impact of PLT on volatility immediately following its adoption when credibility would likely be imperfect; the importance of the perfect credibility assumption for the performance of PLT at the ZLB; and the way in which PLT would be communicated in practice. Although some of these issues can be investigated by academics, central banks themselves will be in the best position to address many of these issues. In fact, the Bank of Canada is already carrying out work in this direction as part of its review of PLT.

Chapter 2

Price-level targeting versus inflation targeting over the long-term: An overlapping generations approach

2. Introduction

As discussed in Chapter 1, the potential long-term impacts of price-level targeting (PLT) have been under-researched in the literature. Indeed, as Ambler (2009) points out, evidence on these impacts is necessary to allow policymakers to conduct a full cost-benefit analysis against inflation targeting (IT). This chapter presents a general dynamic stochastic general equilibrium (DSGE) modelling framework that allows the impact of PLT on long-term inflation risk, consumption volatility and social welfare to be evaluated simultaneously. An overlapping generations (OLG) framework is chosen for this purpose since, as argued in the first chapter, the life cycle model due to Samuelson (1958) provides an ideal environment for an evaluation of PLT over a long horizon. In fact, this framework enables the impact of long-term inflation risk on social welfare to be modelled both explicitly and transparently, in contrast to the workhorse New Keynesian model.

The modelling framework presented in this chapter builds upon the standard OLG model of life-cycle saving in which government bonds are ‘net wealth’ (see Barro, 1974). More specifically, the model extends preliminary work by Mankze and Tödter (2007), discussed in the first chapter, that investigated the impact of PLT on social welfare in a model of overlapping generations in which consumers held nominal bonds. The partial equilibrium set-up they consider is improved upon in numerous ways here: IT and PLT are modelled explicitly as monetary policies, such that the price level is endogenised; aggregate output is produced via a production function as in Diamond (1965), rather than being given by an exogenous endowment process; the supplies of bonds and money balances are modelled explicitly, with government behaviour constrained by a budget constraint and a long run spending target; the returns on bonds (and other assets) are endogenously determined and include risk premia; and finally, consumers can hold both nominal and real assets in conjunction with government bonds.

This chapter puts forward the case for focusing on long-term government bonds before turning to the OLG model. The model presented is subsequently used to

evaluate consumption volatility and social welfare under IT and PLT. In the current chapter, a basic version of the model is first presented and simulated in which consumers hold nominal bonds and money but no capital. Taken literally, this model is an extreme case since consumers can hold only nominal assets. However, this model provides a useful benchmark as it should provide an upper bound estimate of the gains from PLT and will also hold useful insights for countries in which nominal assets are the primary source of retirement income. Later in the chapter this simple model is extended so that consumers can also hold productive but risky capital, thus providing a check on robustness and providing results with more relevance for countries in which real assets play an important role. The results from both models are subjected to extensive sensitivity analysis.

Indexed bonds are then introduced into the model in Chapter 3, which presents and evaluates the ‘full model’ in which the extent of nominal indexation of government bonds is endogenised in response to monetary policy. As such, the results in Chapter 3 are likely to be most applicable for countries where both real and indexed financial assets play an important role.

2.1 Motivation for focusing on ‘long-term government bonds’

Motivation for the analysis focusing on long-term bonds comes from a number of sources. One major motivation is that, as discussed in Chapter 1, there are no papers in literature that have investigated the economic impact of PLT within a DSGE framework in which long-term nominal contracts play an important role. Indeed, although the PLT literature has discussed the potential importance of such contracts, it has stopped short of explicitly modelling the impact, with the exception of the preliminary attempt by Mankze and Tödter (2007). Yet, as argued in Chapter 1, the OLG life-cycle model of saving provides an ideal framework in which to investigate the impact of PLT through the long-term inflation risk channel, because in the canonical version of the model young generations maximise utility by saving for old age using long-term bonds.

Neither the PLT literature nor Mankze and Tödter (*op. cit.*) provide a working definition of the types of contracts that should be covered by the term ‘long-term bonds’, but the need for a transparent definition in order to quantify the impact of PLT

is clear, as well as being an important consideration for the way that such contracts are modelled theoretically. For the purpose of the analysis that follows, the term ‘long-term bonds’ is taken to include (i) long-dated government bonds, and (ii) public sector pensions,³² both of which are long-term contracts that are used to smooth consumption in retirement (Whitehouse, 2009). The main justification for including both these contracts in the model is that public sector pensions are an important source of retirement income, so that focusing solely on long-dated government bonds would likely yield an inaccurate estimate of the long-term impact of PLT. Indeed, public sector pensions are a far more important source of household income than are government bonds (though the role of the latter is economically non-trivial³³) and have become an important public policy issue in almost all developed economies (Disney, 2000; OECD, 2009a). In the case of the UK, for example – where public sector pensions play an important role³⁴ – total pension wealth accounts for 50 per cent of personal wealth (Blake, 2004). There are, however, several other important reasons for focusing on both long-dated government bonds and public sector pensions.

First, long-dated government bonds and public sector pensions have numerous similarities. One similarity is that a typical holding period for a pension is 30-40 years,³⁵ a maturity at which long-dated government bonds are issued and at which holdings are non-trivial across OECD countries (OECD, 2007). A second is that long-dated government bonds in issuance are predominantly nominal in developed countries (DMO, 2010a; Campbell *et al.* 2009; Kitamura, 2009), whilst public sector pensions are effectively nominal contracts because they are defined-benefit and are only indexed to prices once in payment.³⁶ Lastly, both nominal government bonds and public sector pensions can be interpreted as having default risk of essentially zero in most developed economies. Due to these numerous similarities, it has been argued in the context of the UK that government pension liabilities should be discounted using

³² Doepke and Schneider (2006) also include public pensions in ‘bonds’.

³³ For instance, UK households held gilts worth £10.3 billion as of Q1 2010 (DMO, 2010b), or slightly less than 1 per cent of GDP.

³⁴ For instance, independent estimates of public sector pension liabilities range from 70 to 81 per cent of GDP in 2008, or between £39,000 and £45,000 per household (Pensions Commission, 2010).

³⁵ Note that the typical retirement age in OECD economies is 65 for men and 60 for women (OECD, 2009a).

³⁶ That is, indexation to prices is applied to pensions only after retirement. It is also notable that public sector pensions have been subject to discretionary deviations from indexation at times of low inflation or deflation (Whitehouse, 2009).

the yield on government bonds at a long maturity (see Pensions Commission, 2010). These similarities simplify the modelling of the long-term impact of PLT considerably because they mean that long-dated government bonds and public sector pensions can validly be treated as a single asset – viz. ‘long-term bonds’ – in the overlapping generations models presented below.

Second, from the point of view of calibrating the overlapping generations model, the availability of data on government bonds and public sector pensions is a major advantage and means that heterogeneities across countries can be addressed by looking at different versions of the model and via sensitivity analysis. Third, the government bonds assumption also has practical appeal because it means that focus is directed to assets whose returns would most obviously be affected by a switch in monetary policy from IT to PLT. Indeed, as noted above, public sector pensions are typically ‘defined benefit’ pensions (see OECD 2009a),³⁷ such that the nominal amount to be received in retirement is fixed. By contrast, private sector pension portfolios consist primarily of equities and private bonds and therefore have highly-volatile returns even at relatively short horizons (D’Addio *et al.* 2009). Lastly, the assumption that the government issues all ‘long-term bonds’ is justified from a theoretical standpoint because government bonds are ‘net wealth’ in the standard OLG model of life-cycle saving, whilst private bonds are not (see Barro, 1974 and Minford and Peel, 2002).

In order to provide further background information for the reader, both long-dated government bonds and public sector pensions are discussed in more detail below, before introducing the overlapping generations model. The focus in these sections is on the G7 countries and the UK in particular, since the latter is used as a basis for calibrating the models that follow. Moreover, since the models in this chapter include nominal but not indexed government bonds, the discussion below concentrates on nominal government bonds. The importance of indexed government bonds is then discussed at the start of Chapter 3, prior to the introduction of the extended overlapping generations model with indexed bonds and endogenous nominal indexation.

³⁷ In the UK, for example, defined benefit pension plans accounted for 93.9 per cent of public sector pensions in 2008 (Pensions Commission, 2010).

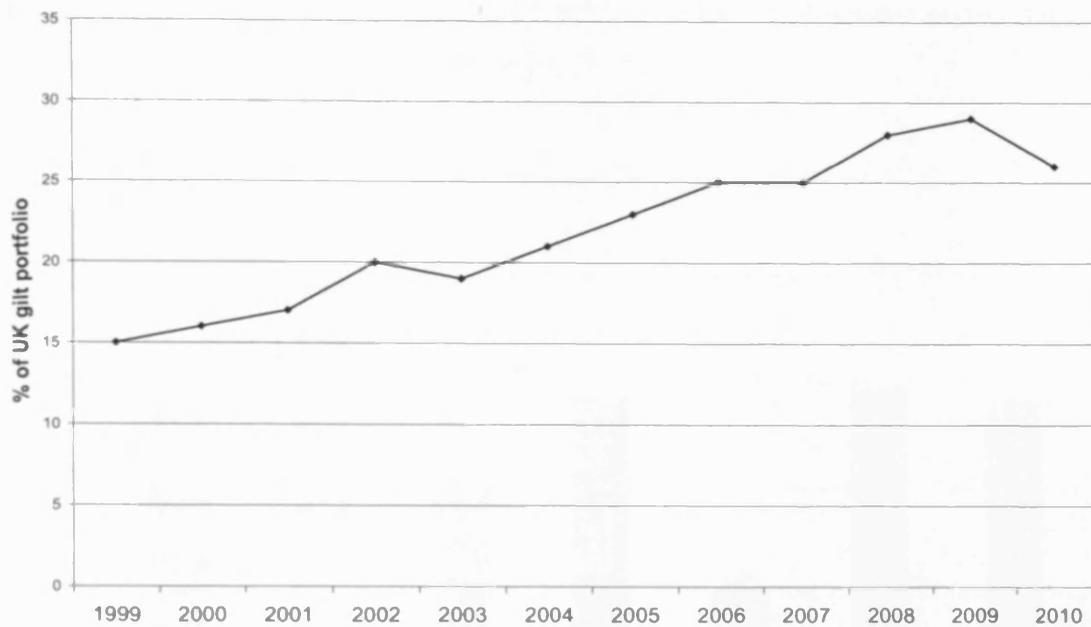
2.1.1 Long-dated nominal government bonds

Nominal government bonds are financial assets that provide a stream of known nominal payments over a specified horizon until the principal is returned upon maturity. For instance, Canadian, UK and US nominal bonds pay interest twice yearly over the holding period, plus the principal upon maturity. Nominal bonds account for the largest share of outstanding liabilities in government bond portfolios, though the share of indexed bonds has been growing steadily. In 2008, for example, nominal bonds accounted for around 70 per cent of British government stock (DMO, 2010a), 90 per cent of the US Treasury's marketable debt (Campbell et al., 2009), and 98 per cent of outstanding government bonds in Japan (Kitamura, 2009). Although the share of indexed bonds in total issuance has been increasing, the size of the market for nominal bonds has still grown substantially in absolute terms. In the UK, for example, the market for nominal gilts was £722.9 billion as of March 2010, compared to only £85.5 billion in 1981.³⁸

As can be seen from Figure 2.1 (which plots the share in the UK bond portfolio of nominal gilts with maturities exceeding 15 years), long-dated government bonds have become increasingly important over recent years. For instance, in 2008 the average maturity of the UK gilt portfolio was 15 years, compared to only 10 years in 1998 (DMO, 2010a).³⁹ This increase has been driven largely by an increase in the proportion of gilts with a term-to-maturity exceeding 15 years. Similarly, the past decade has seen an increase in the relative importance of long-dated government bonds in many other OECD countries (see OECD, 2007). In Canada, for instance, there has been a marked shift towards maturities exceeding 20 years (Meh and Terajima, 2008), and increased issuance of longer-term debt is a stated objective of the Government.

³⁸ This data was downloaded from the Debt Management Office (DMO) website at <http://www.dmo.gov.uk>.

³⁹ In the UK, issuance of nominal (i.e. conventional) gilts is concentrated around the 5-, 10-, 20- and 30-year maturities.



Source: DMO (2010a)

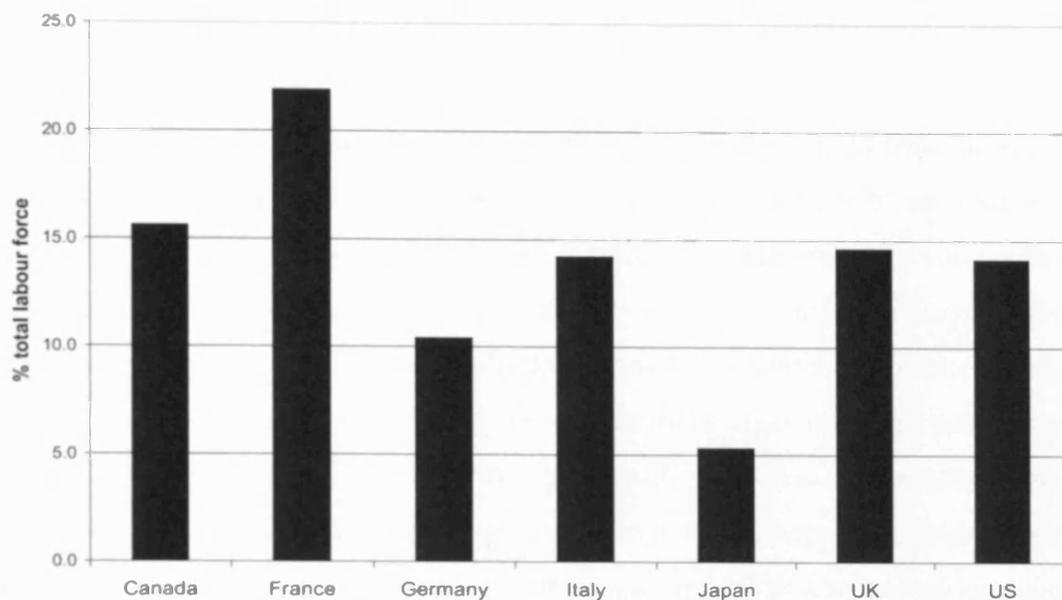
Figure 2.1 – Long-dated nominal gilts and the UK bond portfolio

2.1.2 Public sector pensions

Public sector pensions have been the subject of extensive discussion in recent years, not least because of concerns over their long-run sustainability if left unreformed (OECD, 2009a; Blake, 2000). In developed countries the share of the population of pension age lies between 10 and 20 per cent and has been rising consistently due to increases in life expectancy. For instance, across OECD countries, life expectancy at birth has risen from an average of 76.0 years in 1960 to 81.7 in 2006, with the population above retirement age currently at around one quarter of the population of working age (OECD, 2009a). Moreover, it is noteworthy that public sector pensions account for a substantial share of total pensions in many countries (OECD, 2009b), and that the public sector is an important source of output and employment in most developed economies.

This last point is demonstrated by Figure 2.2, which shows the share of public sector employment in the total labour force for the G7 countries in 2006. With the exception of Japan, public sector employment accounts for more than one-tenth of the total labour force. Public sector pensions are therefore held by a non-trivial fraction of the working population in these countries and seem set to play an important role going

forward. Notably, the average share of public sector employment across OECD countries is around 15 per cent (Ponds *et al.* 2011).



Source: OECD (2009c)

Figure 2.2 – Public sector employment in the G7 countries

Public sector pensions are typically linked to salary during employment and then to inflation after retirement.⁴⁰ However, indexation to inflation is imperfect due to publication lags, and aggregate price indices do not accurately capture some important price changes for pensioners as a group (Leicester *et al.* 2008)⁴¹ or quality improvements (Boskin Report, 1996).⁴² Hence, indexation is subject not just to time lags but also ‘indexation bias.’ Moreover, in some countries a non-trivial proportion of pension plans are not indexed at all, as in Canada where almost one-half of public sector pension plans are of the non-indexed variety (Meh and Terajima, 2008). Finally, as already noted above, discretionary deviations from indexation have frequently been observed in OECD countries, including those in the G7. In effect, then, public sector pensions are long-term nominal contracts that offer only partial

⁴⁰ Of the 33 countries currently in the OECD, 17 currently index pensions to inflation after retirement (Whitehouse, 2009). In the UK, it was announced in the 2010 Budget that the State Pension and public sector pensions would be linked to the Consumer Prices Index (CPI) from April 2011.

⁴¹ As a result, the inflation rate faced by pensioners can differ non-trivially from the rate faced by an average household, at least over the short-term (Leicester *et al.* 2008; Whitehouse, 2009).

⁴² Many statistics agencies are currently developing weight-linked price indices to address the ‘quality bias’ issue.

protection against inflation (even after they are in payment). In the current chapter, the indexation element of pensions is ignored, because this point is addressed explicitly in following chapter that introduces long-term indexed bonds into the overlapping generations framework.

As public sector pensions are typically ‘defined benefit’ (OECD, 2009a), the nominal size of the pension received is guaranteed (i.e. the ‘rate of benefit’ as a stream of payments in retirement is known) and there is little or no uncertainty beyond inflation risk surrounding the real value of the pension. By contrast, a much higher proportion of private pensions are defined contribution plans,⁴³ for which ‘investment risk’ is borne by pension-holders rather than the government. The fact that public sector pensions are primarily defined benefit fits in well with the standard overlapping generations life-cycle model since the government is the monopoly supplier of bonds. The next section provides a brief recap of this basic model as a means of introducing the more comprehensive modelling framework that follows.

2.2 The basic overlapping generations model: a brief reprise

Models of overlapping generations have been used to study issues relating to both pensions and government bonds (e.g. Boldrin and Montes, 2005; Barro, 1974). The next section of this chapter introduces an overlapping generations model of life-cycle saving in which each period lasts 30 years and consumers hold long-term nominal government bonds and money balances. This simple model of life-cycle saving is similar to the canonical one in which government bonds are ‘net wealth’ but is extended in a number of ways. In order to enable the reader to relate the model to previous literature, this section presents a brief recap of the standard life-cycle model.

In the standard overlapping generations model, government bonds are ‘net wealth’ because they perform a consumption-smoothing role via the real return they pay to consumers. The return on bonds is determined endogenously by demand for bonds by the young and the supply that is set by the government. A benevolent government will therefore set the bond supply so that the interest rate on bonds is such that each

⁴³ However, in some countries the share defined contribution plans in the private sector has remained low. For example, in Canada as of 2005, only 7 per cent of private sector pension plans were defined contribution ones (Meh and Terajima, 2008).

generation smoothes consumption optimally between youth and old age. In order to briefly demonstrate this result, consider the following log-utility example from Minford and Peel (2002) in which there is no uncertainty and all generations have a population of one.

The lifetime utility function of generation t is given by $\ln(c_{t,Y}) + \ln(c_{t+1,O})$, where consumption in period t when young is denoted $c_{t,Y}$ and consumption in period $t+1$ when old is denoted $c_{t+1,O}$. Young agents receive a constant endowment $Y - \varepsilon > 0$, where $0 < \varepsilon < Y/2$, which can be consumed or allocated to saving in riskless real bonds b_t that have a gross real return of r_t . The young's budget constraint is therefore given by $c_{t,Y} + b_t = Y - \varepsilon$. Old agents receive a constant endowment ε and consume all their savings, so their consumption in old age is given by $c_{t+1,O} = \varepsilon + r_t b_t$. The first-order condition for bond holdings gives the familiar Euler equation $c_{t+1,O} = r_t c_{t,Y}$. Substituting for consumption by old agents in the Euler equation and rearranging gives bond demand by young consumers, or $b_t = (Y - \varepsilon)/2 - \varepsilon/2r_t$.

Subject to this demand schedule, the real return on bonds is determined by the bond supply set by government. The government can maximise social welfare by choosing the bond supply so that the gross real return on bonds is equal to one – that is, $r_t = 1$ for all t – since this will maximise lifetime utility for all generations by ensuring perfect consumption-smoothing of $c_{t,O} = c_{t,Y} = 1/2$. Given that bond market equilibrium requires that the demand for bonds and the bond supply be equal, the bond supply required to implement this optimal policy is simply $Y/2 - \varepsilon$.⁴⁴ If government spending in real terms, g_t , is funded by issuing bonds, then in the initial period 1 there will be government spending of $g_1 = Y/2 - \varepsilon > 0$. In subsequent periods, however, the debt can be rolled-over and government spending will be zero, because the government's budget constraint is simply $g_t = (1 - r_{t-1})b_t$. By this reasoning, the taxes required to pay off the initial government spending can be postponed indefinitely,⁴⁵ such that government bonds are 'net wealth'. Indeed, the above policy can be shown to be welfare-maximising for all generations and a Pareto improvement over consuming endowments, a result first demonstrated formally by Barro (1974).⁴⁶

⁴⁴ The optimal bond supply can be derived by setting $r_t = 1$ in the bond demand function.

⁴⁵ It is assumed that the economy and the government are infinitely-lived.

⁴⁶ It should be noted that Barro actually argues against government bonds being net wealth because introducing a bequest motive into the OLG model resurrects the Ricardian equivalence proposition.

Figure 2.3 illustrates the bond market equilibrium that implements the optimal policy, and Figure 2.4 illustrates the corresponding gain in welfare for each generation relative to consuming endowments (as in the absence of a market in government bonds). The basic point is clear: the government can maximise social welfare by choosing the supply of long-term bonds optimally. The overlapping generations life-cycle model depicted here is extended along a number of dimensions in the sections that follow in order to investigate long-term consumption volatility and social welfare under IT and PLT. Although the exact details of the model change considerably, there is a common central theme: the government supplies bonds because of their consumption-smoothing role for consumers, choosing the bond supply optimally so that consumption is smoothed between youth and old age. Thus the behaviour of the government in the model is not *ad hoc* but derives from its benevolent desire to maximise social welfare.

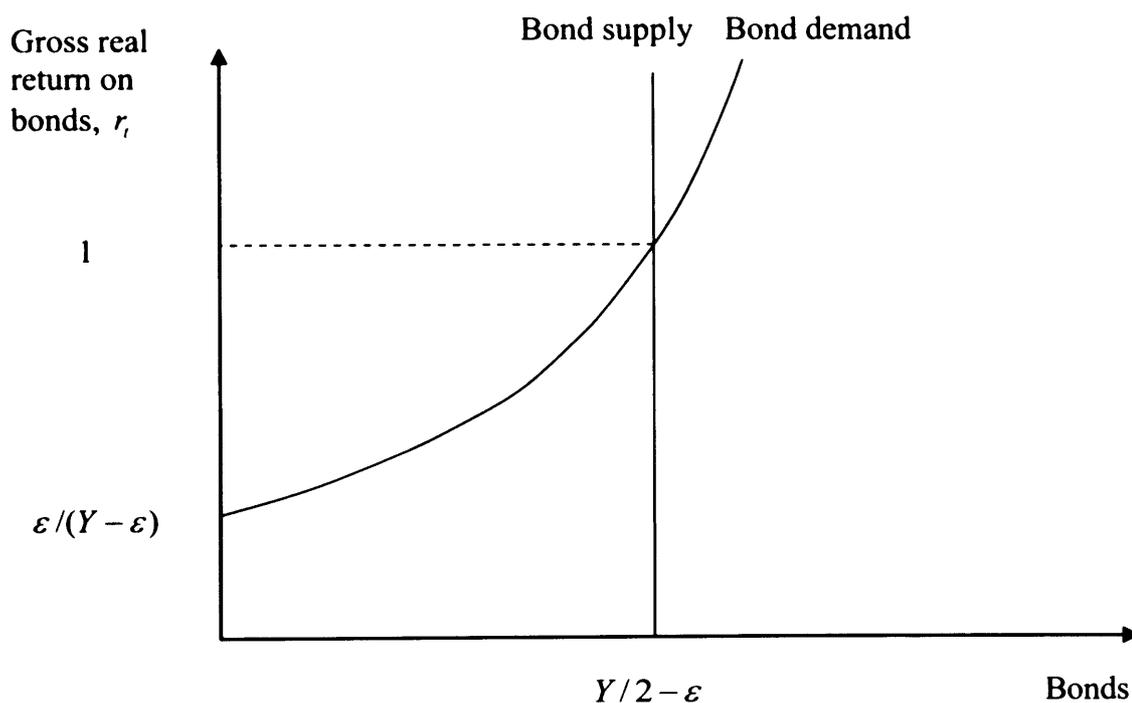


Figure 2.3 – Bond market equilibrium when the supply of bonds is optimised

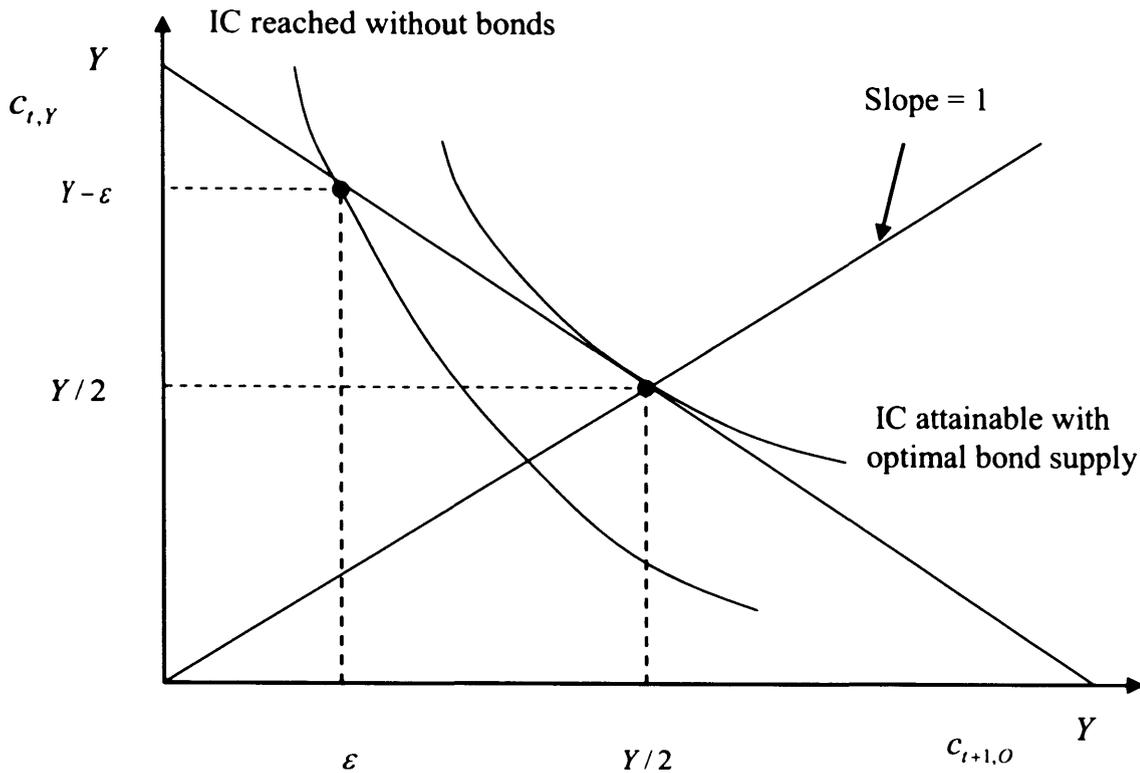


Figure 2.4 – The welfare gain of each generation with government bonds

2.3 The Model

The starting point for the analyses that follow is an overlapping generations (OLG) model of life-cycle saving in which consumers hold money balances and long-term nominal government bonds. Consumers live for two periods of 30 years: in the first they are ‘young’ and receive an exogenous endowment income; in the second they are the retired ‘old’ who receive the proceeds from their savings in youth. Consumers are assumed to receive no endowment income in old age because they are ‘retired’. Long-term government bonds are therefore young consumers’ primary means of income provision for old age. Bonds have a maturity equal to the length of the holding period from youth to old age (i.e. 30 years), and following Lungu and Minford (2006), consumers are interpreted as holding bonds from ‘average youth’ to ‘average old age’. As discussed above, ‘long-term government bonds’ is used as an umbrella term that includes both long-dated nominal government bonds and public sector pensions. To

ease the exposition, population growth is set equal to zero. Moreover, without loss of generality, each generation is assumed to have a constant size of one.⁴⁷

Aggregate uncertainty is introduced into the model via nominal shocks that result from the inability of monetary policy to stabilise inflation perfectly. Although ‘fiat money’ is a popular way of justifying money holdings in OLG models (e.g. McCandless and Wallace, 1991), this approach is not theoretically convincing because fiat money must offer the same return as non-monetary assets to have value, implying deflation if other assets offer real returns. Money is instead introduced via a cash-in-advance constraint, an approach taken by a number of recent papers that investigate optimal monetary policy in OLG economies (e.g. Gahvari, 2007; Michel and Wigniolle, 2005). Monetary policy takes the form of either IT or PLT.

The government is risk-neutral and conducts monetary policy using the money supply as its policy instrument. It is also the monopoly supplier of long-term bonds and sets the bond supply to ensure optimal consumption smoothing (in expected terms) for each generation. In other words, the government sets the supply of bonds so that the *ex ante* real return on bonds is the optimal from the point of view of each young generation. The government also taxes young consumers in order to achieve a long run target level of government spending. The sections that follow focus on consumers, the government, monetary policy and social welfare – thereby providing a detailed exposition of the economic environment. The model is solved using a second-order approximation in Dynare++ (Julliard, 2001) in order to capture asset ‘risk premia’.⁴⁸ This point is crucial since linear approximation can easily lead to an inaccurate ranking of policies in terms of social welfare, as it neglects the impact of uncertainty on the stochastic means of endogenous variables (Schmitt-Grohé and Uribe, 2004; Kim and Kim, 2003).

⁴⁷ There is no loss of generality because the focus throughout is on per-capita values. All model equations, and hence model dynamics, would be left unchanged if all generations had a constant size greater than one and were homogeneously populated. Furthermore, constant population growth would introduce an additional parameter (the population growth rate) but would not change the dynamics of the model.

⁴⁸ Dynare++ was used rather than Dynare because the advantage in terms of computation time meant that a significantly larger number of simulations were feasible. In general, perturbation methods are significantly faster than the alternatives (Gaspar and Judd, 1997).

2.3.1 The economic environment

Consumers live for two periods of 30 years and have constant relative risk aversion (CRRA) preferences over consumption:

$$(2.1) \quad u_t(c_{t,Y}, c_{t+1,O}) = u_{t,Y}(c_{t,Y}) + E_t u_{t+1,O}(c_{t+1,O})$$

where $u_{t,Y}(c_{t,Y}) \equiv c_{t,Y}^{1-\delta} / (1-\delta)$ and $u_{t+1,O}(c_{t+1,O}) \equiv c_{t+1,O}^{1-\delta} / (1-\delta)$.

Consumption in period t when young is denoted $c_{t,Y}$ and consumption in period $t+1$ when old is denoted $c_{t+1,O}$. Hence $u_{t,Y}$ is utility in youth and $u_{t+1,O}$ is utility of the young generation born at time t in period $t+1$ when they are old.⁴⁹

In nominal terms, the budget constraint faced by young generations is given by

$$(2.2) \quad P_t c_{t,Y} + B_t^{n,d} + M_t^d = P_t \omega (1 - \tau^j)$$

where ω is the constant real endowment income received by young consumers,⁵⁰ P is the aggregate price level; $B^{n,d}$ is demand for long-term nominal bonds; M^d is money demand; and τ^j is the constant rate of income tax that can differ in the IT and PLT cases.

In real terms, the budget constraint faced by the young is given by

$$(2.3) \quad c_{t,Y} + b_t^{n,d} + m_t^d = \omega (1 - \tau^j)$$

where $b^{n,d} \equiv B^{n,d} / P$ is real demand for nominal bonds and $m^d \equiv M^d / P$ is the demand for real money balances.

⁴⁹ Note that consumers do not discount consumption in old age. This assumption can be justified on the basis that consumers view their years of 'old age' as being just as important for utility as their years of 'youth'. Examples from the literature that use this assumption include Brazier *et al.* (2006), Minford and Peel (2002) and Champ and Freeman (1990).

⁵⁰ A constant endowment is specified for simplicity because the impact of real risk on the IT-PLT comparison is addressed later on by the introduction of productivity shocks which affect the real return on capital.

Equation (2.3) states that consumers can save for old age by holding long-term nominal bonds or money balances. The income tax rate is set by the government to ensure that it meets a common long run government spending target under both IT and PLT (hence the need for the j superscript).

Following Artus (1995), the demand for money arises from a cash-in-advance (CIA) constraint which states that real monetary savings are a fraction $0 < \theta < 1$ of consumption when young:⁵¹

$$(2.4) \quad m_t^d \geq \theta c_{t,Y}$$

This CIA constraint provides a role for money without explicitly requiring that money offer transactions services. The results that follow therefore do not depend in any way upon the impact of monetary policy on transactions costs or the ease of exchange, but on the link between monetary policy, long-term inflation risk and social welfare. Crettez *et al.* (1999) provide a useful discussion of the different CIA constraints that have been employed in OLG models with money.

As is demonstrated in Appendix A of this chapter, the CIA constraint will be strictly binding so long as the money return on nominal bonds exceeds one.⁵² This result makes good sense intuitively since, with money being a perfect store of nominal value, an optimising consumer would not hold monetary savings in excess of the proportion θ required by the CIA constraint if bonds pay a higher money return. In all numerical simulations discussed in this thesis, this condition was satisfied (based on *ex post* analysis of simulation results). It is therefore assumed henceforth that the CIA constraint is strictly binding.

With a strictly binding CIA constraint, Equation (2.4) becomes

$$(2.5) \quad m_t^d = \theta c_{t,Y}$$

⁵¹ Cited by Crettez *et al.* (1999). This constraint is interpreted as a 'cash-in-advance' specification in the OLG literature.

⁵² Formally, the CIA constraint will be strictly binding if the Lagrange multiplier associated with the CIA constraint is strictly positive for all t (Hodrick *et al.* 1991).

In old age, consumers have no endowment income because they are 'retired'. However, old agents receive a return on their holdings of nominal bonds and spend their money savings.

Thus consumption in nominal terms is given by

$$(2.6) \quad P_{t+1}c_{t+1,O} = R_t B_t^{n,d} + M_t^d$$

where R_t is the riskless gross money return paid on a nominal bond that is held from period t to period $t + 1$.⁵³

In real terms, consumption by old generations is given by

$$(2.7) \quad c_{t+1,O} = r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d$$

where $r_{t+1}^n \equiv R_t / (1 + \pi_{t+1})$ and $r_{t+1}^m \equiv 1 / (1 + \pi_{t+1})$.

In Equation (2.7), r^n is the gross real return on a nominal bond upon maturity (which is known at the time the bond is purchased but for inflation risk), and r^m is the gross real return on money balances. $\pi_t \equiv (P_t / P_{t-1} - 1)$ the rate of inflation from period $t-1$ to period t .

The initial old are endowed with m_0 units of real money balances and an initial stock of government debt of b_0 ; their corresponding level of consumption is $c_{1,O}$. Trivially, the utility of the initial old is given by

$$(2.8) \quad u_{1,O} = \frac{c_{1,O}^{1-\delta}}{1-\delta}$$

⁵³ The return R_t is endogenous determined so that in each period the demand for bonds is equal to the available supply.

2.3.2 Long-term inflation risk and social welfare

Before investigating consumers' first-order conditions and introducing the remainder of the model, it is worth briefly noting that the general OLG model set out above provides a microfounded welfare justification for focusing on long-term inflation risk – a theoretical finding which provides support for informal arguments made in the PLT literature (as reviewed in Chapter 1).

In particular, a second-order Taylor expansion of the lifetime utility of a given generation t has the form of a loss function in long-term inflation volatility, where 'long-term' refers to the holding horizon from youth to old age of 30 years. In order to obtain this result, the methodology used by Woodford (2003) is employed: the above equations are log-linearised around the deterministic steady-state and substituted into a second-order Taylor expansion of lifetime utility.⁵⁴

Specifically, Appendix B of this chapter shows that the lifetime utility of generation t can be written as follows:

$$(2.9) \quad Loss_t \approx -(1/2)\Phi_t \text{var}_t(\pi_{t+1})$$

where $Loss_t$ is defined as the deviation of lifetime utility from its time- t expected value, and the coefficient $\Phi_t > 0$ depends on the model's steady-state parameters and anticipated consumption in old age.

Equation (2.9) shows clearly that the utility loss of each generation t is increasing in the (conditional) 30-year inflation variance. One interesting implication of this loss function expression is that the utility loss caused by long-term inflation risk can potentially differ across generations and thus over time (hence the t subscript on the Φ coefficient). The social welfare criterion that is subsequently used to evaluate IT and PLT is therefore based on average utility across all generations, rather than the

⁵⁴ As shown formally by An and Schorfheide (2007), the resulting expression for utility is not in general second-order accurate because it neglects the impact of shocks on the means of endogenous variables when there are nonlinearities (i.e. it ignores terms due to Jensen's inequality as a result of the 'certainty equivalence' principle). However, this consideration does not affect materially the point that is being made.

lifetime utility of any given generation. The social welfare function is discussed in further detail below.

2.3.3 Consumers' first-order conditions

The maximisation problem faced by consumers can be represented by the following expected Lagrangian:

$$(2.10) \quad L_t \equiv E_t \left[u_t(c_{t,Y}, c_{t+1,O}) + \lambda_{t,Y} (\omega(1 - \tau^j) - m_t^d - b_t^{n,d} - c_{t,Y}) \right. \\ \left. + \mu_t (m_t^d - \theta c_{t,Y}) + \lambda_{t+1,O} (r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d - c_{t+1,O}) \right]$$

where $\lambda_{t,Y}$ ($\lambda_{t+1,O}$) is the Lagrange multiplier on young (old) consumers' budget constraint and μ_t is the Lagrange multiplier on the CIA constraint.

First-order conditions are as follows:

$$(2.11) \quad c_{t,Y} : c_{t,Y}^{-\delta} = \lambda_{t,Y} + \theta \mu_t$$

$$(2.12) \quad c_{t+1,O} : \lambda_{t+1,O} = c_{t+1,O}^{-\delta}$$

$$(2.13) \quad b_t^{n,d} : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^n)$$

$$(2.14) \quad m_t^d : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^m) + \mu_t$$

Substituting out the Lagrange multipliers on the budget constraints when young and old gives the following consumption Euler equations for nominal bonds and money balances:

$$(2.15) \quad c_{t,Y}^{-\delta} = E_t (c_{t+1,O}^{-\delta} r_{t+1}^n) + \theta \mu_t$$

$$(2.16) \quad c_{t,Y}^{-\delta} = E_t (c_{t+1,O}^{-\delta} r_{t+1}^m) + (1 + \theta) \mu_t$$

where μ_t is the Lagrange multiplier on the CIA constraint.

The Lagrange multiplier on the cash-in-advance constraint is given by

$$(2.17) \quad \mu_t = E_t (c_{t+1,O}^{-\delta} (r_{t+1}^n - r_{t+1}^m))$$

Intuitively, Equation (2.17) states that, absent uncertainty, a sufficient condition for the CIA constraint to be strictly binding (i.e. $\mu_t > 0$ for all t) is that money be rate of return dominated by nominal bonds.

Substituting for the Lagrange multiplier, the consumption Euler equation for nominal bonds can be written in the following form:

$$(2.18) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^n - \theta r_{t+1}^m \right) \right)$$

It can be seen from Equation (2.18) that the CIA constraint gives rise to an additional term $\theta(r^n - r^m)$ on the right hand side of the consumption Euler equation. The intuition behind this additional term is that reducing consumption when young by one unit has a knock-on effect of reducing money holdings by θ units via the CIA constraint, because money holdings are proportional to consumption. This reduction in money holdings makes available an extra θ units of endowment income for bond purchases. Consequently, young consumers receive an additional return θr^n from extra bond purchases whilst losing θr^m from the reduction in money balances.

2.3.4 Government

The government is risk-neutral and finances real spending g_t by taxing young consumers, printing money and issuing nominal government bonds. The government's budget constraint in real terms is therefore given by

$$(2.19) \quad g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$$

where $\tau^j \omega$ is tax revenue, m^s is money supply in real terms, and b^s is the real supply of nominal bonds issued by the government.

Real spending of g_t per period is endogenously determined and is used up in projects that have no direct effect on the utility or consumption of agents. However, the income tax rate is set to ensure that, conditional on the money supply rule that is implemented, long run real government spending is equal to a target level of g^* , or

$E(g_t) = g^*$.⁵⁵ Since the long run government spending target must be met, a higher cost of issuing government debt in one of the monetary policy regimes – due, for instance, to differences in bond risk premia under IT and PLT – must be covered by raising the constant income tax rate faced by young consumers.

As discussed above, the government sets the supply of bonds to ensure that the *ex ante* return is such that consumers enjoy perfect consumption smoothing in expected terms (making bonds a source of ‘net wealth’). This amounts to setting the bond supply so that the marginal utility of consumption in youth is equated to the (time- t) expected marginal utility of consumption in old age.⁵⁶ The government bond supply will tend to vary with the money supply rule that is implemented and over time, because the optimal provision of bonds will depend upon the shocks that hit the economy and the response of monetary policy to such shocks.

Formally, the supply of nominal bonds is implicitly defined by the following equation:

$$(2.20) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta})$$

2.4 Monetary Policy

As mentioned above, the government’s monetary policy instrument is the money supply. The major difference between IT and PLT is that the former implies ‘base-level drift’ in the price level, whilst the aim of the latter is to prevent it. This section discusses the money supply rules under IT and PLT. Importantly, the 30-year money supply rules which enter the model are derived from a *yearly horizon*. As a result, the money supply rules and equilibrium inflation over a 30-year horizon reflect the presence of base-level drift under IT and its absence under PLT. Since the government can commit to IT and PLT money supply rules, no time-inconsistency or credibility issues arise in relation to monetary policy. The exposition below

⁵⁵ Under risk neutrality, policymakers care about mean values but not volatilities.

⁵⁶ Given that each period lasts 30 years, it is not unreasonable to think of the government choosing the bond supply to enable consumers to smooth their lifetime consumption in this way. Note that in the deterministic steady-state there will be perfect consumption smoothing with this policy, as in the example above from Minford and Peel (2002).

concentrates on the nominal money supply (which is non-stationary), but the money supply is converted back into real terms in order to solve the model in Dynare++.

Firstly, note that the CIA constraint implies that

$$(2.21) \quad M_t^d = \theta P_t c_{t,Y}$$

where $M_t^d \equiv P_t m_t^d$ is money demand in nominal terms.

Taking logs of Equation (2.21) implies that the (30-year) growth of money demand is given by

$$(2.22) \quad \ln M_t^d - \ln M_{t-1}^d = \pi_t + \ln c_{t,Y} - \ln c_{t-1,Y}$$

where $\pi_t \approx \ln P_t - \ln P_{t-1}$.

Given that there must be equilibrium in the money market, Equation (2.22) shows clearly how the rate of inflation in equilibrium will depend upon the money supply rule implemented by the government. However, reflecting the inability of monetary policy to perfectly stabilise inflation, money supply rules are subject to nominal shocks. These shocks are in turn passed through to equilibrium inflation, so the government is not able to perfectly control the rate of inflation using the money supply.

2.4.1 The IT money supply rule

The nominal money supply rule under IT takes the following form:

$$(2.23) \quad \ln M_t^{s,IT} - \ln M_{t-1}^{s,IT} = \pi^* + \varepsilon_t^{IT} + \ln c_{t,Y} - \ln c_{t-1,Y}$$

where ε_t^{IT} is the aggregate money supply innovation over 30 years and π^* is the 30-year inflation target.

In the absence of money supply shocks, Equation (2.23) would imply perfect stabilisation of inflation at the inflation target. Given that each period lasts 30 years and there is base level drift, the aggregate 30-year money supply innovation will be an accumulation of innovations to the money supply at a yearly horizon. This aggregate innovation is built-up from a *yearly money supply rule*. In order to do so, consider the goal of achieving a constant yearly inflation target.

This goal implies a money supply rule of the following form at a *yearly horizon* i :

$$(2.24) \quad \ln M_i^{s,IT} = \ln M_{i-1}^{s,IT} + \pi + \varepsilon_i + \ln c_{i,Y} - \ln c_{i-1,Y}$$

where π is the yearly inflation target and ε_i is a yearly money supply innovation. The latter is assumed to be a serially-uncorrelated random variable drawn from an $N(0, \sigma^2)$ distribution.

The aim is to derive the 30-year money supply rule in Equation (2.23) from this yearly specification. In order to do so, substitute repeatedly for the lagged money supply term on the right-hand side of Equation (2.24) until the following 30-year money supply rule is reached:

$$(2.25) \quad \ln M_i^{s,IT} = \ln M_{i-30}^{s,IT} + 30 \times \pi + \sum_{j=0}^{29} \varepsilon_{i-j} + \ln c_{i,Y} - \ln c_{i-30,Y}$$

Equation (2.25) states that the 30-year growth rate of the nominal money supply has three separate components: a 30-year inflation target; the sum total of 30 separate yearly money supply innovations; and the 30-year growth rate of consumption when young. As such it is simply a version of Equation (2.23) expressed in terms of years rather than 30-year periods.

Hence, given that each period t lasts 30 years, Equation (2.25) implies that the money supply rule in any period t can be represented as follows:⁵⁷

⁵⁷ To save on state variables (and hence computation time), the summation on the right hand side was modelled as an aggregate innovation.

$$(2.26) \quad \ln(M_t^{s,IT} / M_{t-1}^{s,IT}) = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y})$$

where, for ease of exposition, money supply innovations have been indexed from years 1 to 30 and the subscript indicates that all 30 innovations belong to period t .

The aggregate money supply innovation under IT is therefore given by $\varepsilon_{1,t} + \varepsilon_{2,t} + \dots + \varepsilon_{30,t}$, while the 30-year inflation target is given by $\pi^* \equiv 30\pi$. As the sum of 30 normal, zero-mean random variables, the aggregate money supply innovation is normally distributed with mean zero and variance $30\sigma^2$. Note that the aggregate money supply innovation under IT is simply the sum of the yearly money supply innovations that are allowed to accumulate over a 30 years as a result of base-level drift.

Money market equilibrium implies that inflation over a 30-year horizon under IT is given by

$$(2.27) \quad \pi_t^{IT} = 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t}$$

Therefore, expected inflation is equal to the inflation target and the 30-year inflation variance is thirty times the yearly money supply innovation variance:

$$(2.28) \quad E_t \pi_{t+1}^{IT} = 30 \times \pi$$

$$(2.29) \quad \text{var}(\pi_t^{IT}) = 30\sigma^2$$

Both of these results have been discussed in the literature on IT. First, expected inflation is equal to the inflation target because the government makes a credible commitment to an IT money supply rule. Second, the inflation variance is thirty times the yearly money supply innovation variance because of base-level drift; that is, money supply innovations cause inflation to deviate from target in each year, and over time these innovations accumulate, with each one adding to long-term inflation uncertainty.

2.4.2 The PLT money supply rule

The nominal money supply rule under PLT takes the following form:

$$(2.30) \quad \ln M_t^{s,PLT} - \ln M_{t-1}^{s,PLT} = \ln P_t^* - \ln P_{t-1}^* + \varepsilon_t^{PLT} - \varepsilon_{t-1}^{PLT} + \ln c_{t,Y} - \ln c_{t-1,Y}$$

where ε^{PLT} is the money supply innovation and P_t^* is the time- t target price level.

In the absence of money supply innovations, Equation (2.30) would imply perfect stabilisation of the price level at target. However, the price level will deviate from its target level due to money supply innovations. The presence of a lagged money supply innovation in Equation (2.30) reflects the response of the PLT money supply rule to the past price level deviation, which is offset to ensure the price level is returned to its target path (as discussed in Chapter 1). Notice also that the money supply innovation is allowed to differ from the IT case in order capture the impact of PLT upon inflation over a 30-year horizon.

It assumed that target log price level under PLT follows a linear trend that increases at the target rate of inflation under IT:⁵⁸

$$(2.31) \quad \ln P_t^* = p_0 + \pi^* \times t$$

where p_0 is the initial target price level.

The 30-year money supply rule in Equation (2.30) can therefore be written as

$$(2.32) \quad \ln M_t^{s,PLT} - \ln M_{t-1}^{s,PLT} = \pi^* + \varepsilon_t^{PLT} - \varepsilon_{t-1}^{PLT} + \ln c_{t,Y} - \ln c_{t-1,Y}$$

Since PLT precludes base-level drift, the innovation in the 30-year money supply rule will differ from that in the IT case. However, in order to provide a fair comparison of IT and PLT, this innovation is also derived from a money supply rule that is subject to innovations at a yearly horizon.

⁵⁸ This assumption is made to ensure direct comparability of IT and PLT. Under this assumption, IT and PLT are identical in the absence of money supply innovations.

First, note that the goal of meeting a target *yearly* (log) price level of $p_0 + \pi i$ in each year i implies a yearly money supply rule of the following form:

$$(2.33) \quad \ln M_i^{s,PLT} = \ln M_{i-1}^{s,PLT} + \pi + \varepsilon_i - \varepsilon_{i-1} + \ln c_{i,Y} - \ln c_{i-1,Y}$$

where π is the constant yearly inflation target that is consistent with the target price path and ε_i is a yearly money supply innovation that is drawn from an $N(0, \sigma^2)$ distribution and is serially-uncorrelated (exactly as in the IT case).⁵⁹

To derive the money supply rule in Equation (2.32) from this yearly specification, substitute repeatedly for the lagged money supply term on the right hand side of Equation (2.33) until the following 30-year money supply rule is reached:

$$(2.34) \quad \ln M_i^{s,PLT} = \ln M_{i-30}^{s,PLT} + 30 \times \pi + \varepsilon_i - \varepsilon_{i-30} + \ln c_{i,Y} - \ln c_{i-30,Y}$$

Equation (2.34) states that the 30-year growth rate of the money supply under PLT has three components: a 30-year inflation target 30π ; two yearly money supply innovations; and the 30-year growth rate of consumption when young. As such, Equation (2.34) is simply a version of Equation (2.32) expressed in terms of years rather than 30-year periods.

Equation (2.32) can therefore be expressed as follows:

$$(2.35) \quad \ln(M_t^{s,PLT} / M_{t-1}^{s,PLT}) = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y})$$

where the money supply innovations have been indexed to reflect the year in which they occur, and the t subscript indicates the period to which innovations belong.

As under IT, the 30-year inflation target is given by $\pi^* \equiv 30\pi$.

In contrast to the IT case, the money supply innovation in period t is given by a single yearly innovation, $\varepsilon_{30,t}$, such that the money supply rule contains only two yearly money supply innovations, $\varepsilon_{30,t}$ and $\varepsilon_{30,t-1}$, which relate to year 30 in adjacent periods.

⁵⁹ Hence it is assumed that PLT offsets shocks to the price level at a yearly horizon.

The reasoning is as follows: innovations that occur in years 1-29 of a given period are offset in the following year in order to bring the price level back to its target path. For instance, a shock in year 29 will be offset in year 30, the last year of the current period. However, the innovation in year 30 of every period cannot be offset until year 1 of the next period. Hence the innovations $\varepsilon_{30,t}$ and $\varepsilon_{30,t-1}$ enter the period- t money supply rule. The first is the innovation in year 30 of the current period (an unavoidable ‘trembling hand’); the second is the innovation from year 30 of the previous period, which is actively offset by policy in year 1 of the current period.

From money market equilibrium, inflation under PLT is given by

$$(2.36) \quad \pi_t^{PLT} = 30 \times \pi + \varepsilon_{30,t} - \varepsilon_{30,t-1}$$

Hence expected inflation varies over time and the 30-year inflation variance is two times the yearly innovation variance:

$$(2.37) \quad E_t \pi_{t+1}^{PLT} = 30 \times \pi - \varepsilon_{30,t-1}$$

$$(2.38) \quad \text{var}(\pi_t^{PLT}) = 2\sigma^2$$

Both of these results have been discussed in the monetary policy literature (e.g. Svensson, 1999; Minford, 2004). Firstly, expected inflation varies because past deviations from the target price path are subsequently offset, and rational agents take this into account when forming their inflation expectations. Second, the 30-year inflation variance is 15 times lower under PLT because inflation depends on only 2 yearly money supply innovations, compared to 30 under IT. The reasoning is simply that yearly deviations from the inflation target do not accumulate to increase long-term inflation uncertainty, because PLT precludes base-level drift.

In order to make the main differences between IT and PLT concrete, Panels (a) and (b) of Figure 2.5 report impulse responses to a money supply innovation. As the yearly money supply innovation standard deviation has not yet been calibrated, its value was set so as to give a one per cent inflation shock in the IT case. The differences between IT and PLT are clear: the initial impact is somewhat smaller

under PLT due to the lower (30-year) money supply innovation variance; and the initial inflationary shock is reversed in the following period under PLT but is treated as a bygone under IT.

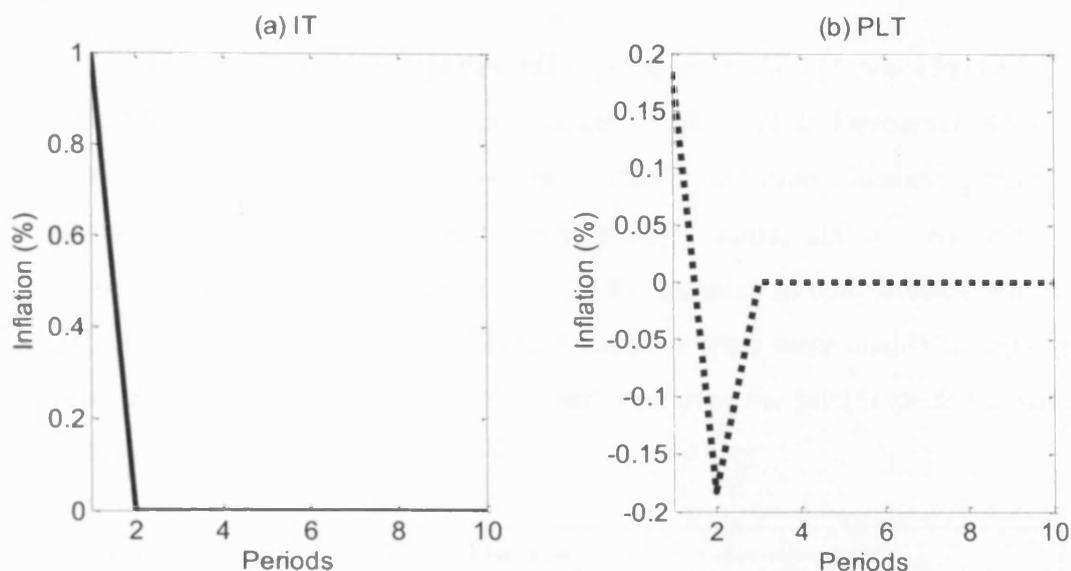


Figure 2.5 – Inflation impulse responses to a money supply innovation

This discussion completes the description of the model. For completeness, the deterministic steady state of the model and market-clearing conditions are reported in Appendix C, along with a full model listing in Appendix D.

2.5. Model calibration

2.5.1 Money supply rules

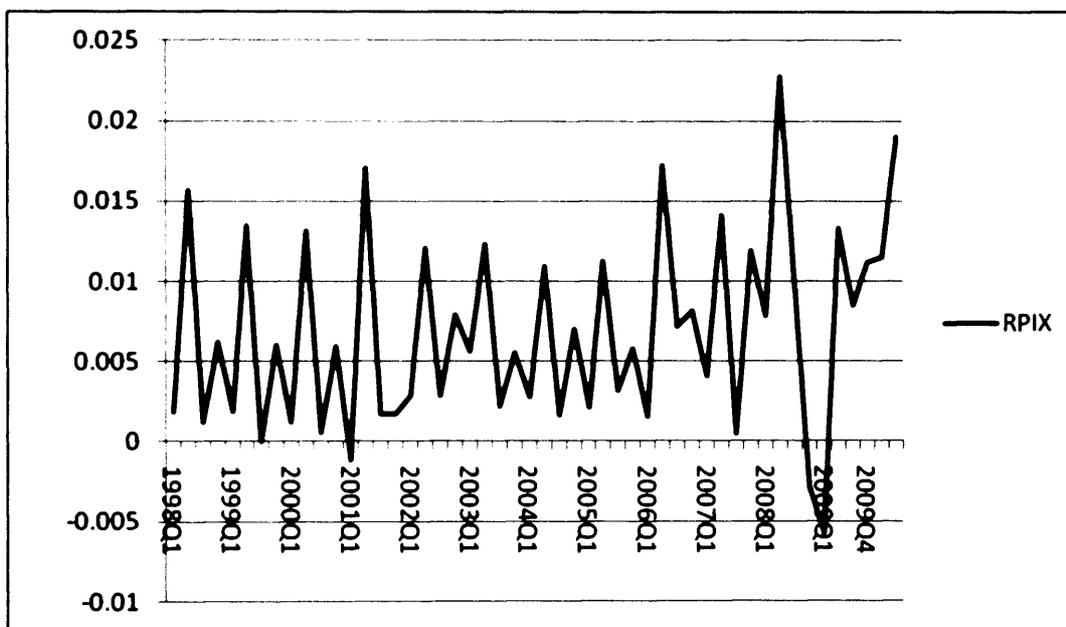
In order to make the money supply rules in the model operational, the yearly inflation target and yearly money supply innovation variance need to be calibrated. Given that inflation in the model is a function of the inflation target and a money supply innovation, UK inflation over the IT period was used to calibrate the money supply rules. The Retail Prices Index excluding mortgage interest payments (RPIX) was chosen for this purpose, with the sample period running from 1997Q3 to 2010Q2.⁶⁰ Quarterly data were chosen because annual data would provide only 14 separate observations.⁶¹ The RPIX was chosen because it excludes mortgage interest payments, which are not faced by the majority of pensioners in the UK (Leicester *et*

⁶⁰ The Bank of England was assigned an inflation target soon after ‘Black Wednesday’ in 1992, but was not given full operational independence until May 1997.

⁶¹ The index was downloaded from the ONS Time Series Database at <http://www.statistics.gov.uk/statbase/tsdtimezone.asp>.

al. 2008), and because it includes both council tax and housing costs – relatively more important costs for pensioners that are excluded from the Consumer Prices Index (CPI).

It is important to note that although the inflation target in the UK was changed from 2.5 per cent for the RPIX to a 2 per cent target for the CPI in December 2003, the adjustment was based on historical experience with the intention of ensuring that there was no material change in monetary policy strategy (King, 2004).⁶² As such, this event was not treated as a structural break in the sample. In concordance with this treatment, the Quandt-Andrews and Chow breakpoint tests were unable to reject the null hypothesis of no breakpoint. Quarterly inflation over the sample period is shown in Figure 2.6.



Source: Office for National Statistics (ONS)

Figure 2.6 – Quarterly RPIX inflation over the sample period

To calibrate the money supply rules, the following regression equation was estimated at a quarterly frequency q via Ordinary Least Squares (OLS):

$$(2.39) \quad \pi_q = c + \varepsilon_q$$

where π_q is the log first-difference of the quarterly RPIX.

⁶² The main argument cited in favour of the shift to the CPI was international comparability.

The estimation results from this regression and relevant test statistics are reported below in Table 2.1.

Table 2.1 – RPIX regression results, 1997Q3-2010Q2

Parameter/Test	Result
c (s.e.)	0.007 (0.001)
Quarterly standard deviation, $sd(\varepsilon_q)$	0.0060
Yearly standard deviation	0.012
Dickey-Fuller unit-root test on $\ln(RPIX)$ (prob. value)	-1.706 (0.73)
Jarque-Bera test on ε_q (prob. value)	1.837 (0.40)

The estimate for c is the mean quarterly inflation rate over the sample period. The value of 0.007 therefore implies mean annual inflation of 0.028 (or 2.8 per cent), which is close to the annual RPIX target of 2.5 per cent that was the focus of UK monetary policy from March 1997 until December 2003. As the difference between mean quarterly inflation and the quarterly rate implied by the annual target of 2.5 per cent was not statistically significant at the 5 per cent significance level, the yearly inflation target was set equal to $\pi = 0.025$. This yearly target implies a calibrated 30-year inflation target of $\pi^* = 30\pi = 0.75$, or a 75 per cent increase in prices over a 30-year horizon.

In order to estimate the variance of the yearly money supply innovation, the residuals from the above regression were used. The reasoning for doing so is that the IT the money supply rule gives the result that inflation is equal to the inflation target plus a money supply innovation, which implies that the inflation variance is simply the money supply innovation variance. The variance of the residuals from the above regression is of course a quarterly variance, but this was converted to a yearly variance using the unit root hypothesis and then used to calibrate the IT and PLT money supply rules. The assumption that there is a unit root in the quarterly RPIX is supported by the Dickey-Fuller unit root test (see Table 2.1),⁶³ and is an implication of the IT money supply rule since there is base-level drift. Notably both McCallum (1997) and Dittmar *et al.* (1999) used the unit-root hypothesis on quarterly US price

⁶³ An intercept and trend were included in the test regression.

indices in order to calculate the implications of IT for longer-term inflation uncertainty.⁶⁴

The estimated quarterly standard deviation for inflation was 0.006, which by the unit-root hypothesis implies a yearly standard deviation of 0.012, or 1.2%. The implied yearly money supply innovation variance is therefore 0.00015; this was taken as the calibrated value in the model under both IT and PLT. It is notable that the null hypothesis that the inflation innovation is normally distributed could not be rejected at the 5 per cent significance level by the Jarque-Bera test statistic, as shown in the bottom row of Table 2.1.⁶⁵

2.5.2 Model parameter calibration

The calibrated values for the model are summarised in Table 2.2. Young consumers' endowment income is normalised to one for simplicity; this means that bond holdings, money balances and consumption levels can be interpreted as proportions of aggregate income or GDP. The coefficient θ , the proportion of consumption when young held as money, is calibrated to roughly match UK data. In particular, consumption accounts for just under 65 per cent of GDP (ONS, 2010a), and notes and coins in circulation for between 3 and 4 per cent of GDP over the past decade (ONS, 2010b). Therefore, with aggregate steady-state consumption split equally between young and old agents by the government bond supply rule, holdings of notes and coins would need to be around one-tenth of consumption by young agents to match the data. On this basis, θ was set equal to 0.10.

⁶⁴ Both these papers also assumed that the inflation innovation is serially uncorrelated.

⁶⁵ By the unit-root hypothesis, the yearly money supply innovation is the sum of the innovations from the four quarters in that year. Since a sum of normal random variables is itself normally-distributed, it is sufficient to test for normality of the quarterly regression residual.

Table 2.2 – Model calibration

Model parameter	Role in the model	Calibrated value
θ	Proportion of consumption when young held as money balances	0.10
δ	Coefficient of relative risk aversion	3
ω	Endowment income of young consumers (and GDP)	1
g^*	Long run target for government spending	0.20
π^*	Inflation target over 30 years	0.75
$\text{var}(\varepsilon_{i,t})$	Yearly money supply innovation variance	1.45×10^{-4}

The coefficient of relative risk aversion was set equal to 3. This value lies in the mid-range of calibrated values in the literature: it is higher than a standard real business cycle (RBC) calibration of unity, but somewhat lower than the values typically used in the open-economy literature that attempts to match exchange rate volatility and persistence in the data (e.g. Kocherlakota and Pistaferri, 2007; Chari *et al.* 2002), or in the finance literature that attempts to resolve asset-pricing ‘puzzles’ by appealing to risk aversion coefficients of 5 or higher (e.g. Bansal and Yaron, 2004). Moreover, the value of 3 is close the estimated value of 3.5 reached by Tödter (2008) using US stock return data from 1926 to 2002.

The long run target level of government spending is set equal to 0.2, or 20 per cent of GDP. This long run target is similar to the recent level of UK government spending as a percentage of GDP (ONS, 2010a) and is broadly similar to the level of government expenditure as percentage of GDP in other developed economies.⁶⁶ The last two rows in Table 2.2 were calibrated using the RPIX regression results discussed above. The implied steady-state values of key variables in the model are shown in Table 2.3.

⁶⁶ For example, in their model of the US economy, Rudebusch and Swanson (2008) specify that government spending is an exogenous stochastic process with an unconditional mean of 17 per cent of GDP.

Table 2.3 – Key variables at steady state

Model variable	Steady-state value
$c_{t,Y}$	0.40
$c_{t,O}$	0.40
$b_t^d (= b_t^s)$	0.39
$m_t^d (= \theta c_{t,Y} = m_t^s)$	0.04
π_t	0.75

Steady-state consumption accounts for 80 per cent of GDP and is split equally between young and old generations by the government bond supply rule. Since there is no investment in the model, the remaining 20 per cent of GDP is accounted for by government spending. Steady-state bond holdings are just under 40 per cent of GDP, and approximately ten times money holdings.⁶⁷ Finally, steady-state inflation over 30 years is equal to the 30-year inflation target of 75 per cent. These implied steady-state values provide a reasonable fit to UK data and developed economies in general.

2.6 Simulation methodology

The model is solved using a second-order approximation in Dynare++ (Julliard, 2001). It is important to use non-linear approximation methods to obtain the model solution for two reasons. Firstly, the demand for bonds depends on consumption covariance risk. Log-linearising the model would eliminate covariance risk, with the result that the model would miss a potentially important ‘risk premia’ channel through which PLT could potentially have an impact on social welfare vis-à-vis IT. Furthermore, the study of bond risk-premia is of interest in its own right since the literature on PLT predicts a substantial reduction in the inflation risk premium on nominal contracts. For this reason, inflation risk-premia under IT and PLT are investigated formally in Chapter 4.

Secondly, when comparing social welfare across different monetary policy regimes, linear approximation can easily lead to an inaccurate ranking of policies because it neglects the impact of second-order terms on the stochastic means of endogenous variables. For instance, Kim and Kim (2003) present a simple two-agent economy in

⁶⁷ At steady-state, nominal bonds account for 93.7 per cent of old generations’ retirement income. By comparison, public sector pensions account for around 85 per cent of retirement income in Germany (Berkel and Börsch-Supan, 2004; Börsch-Supan, 2000).

which log-linearisation leads to the spurious conclusion that autarky delivers higher social welfare than full risk-sharing. In the context of the current model, mean returns on both bonds and money balances are influenced by second-order terms due to Jensen's inequality. Consequently, linearisation could give spurious social welfare results, since the model solution would fail to take these second-order terms into account when evaluating mean consumption by old generations.

The results presented below are based on 1,000 simulations of the model with the simulation length set equal to 5,000 periods, and with the simulation seed chosen randomly in each simulation. The next section discusses the social welfare criterion that was used as a basis for the IT-PLT welfare comparison. Simulation results are then presented in the section that follows.

2.7 Evaluating the impact of PLT on social welfare

The appropriate criterion for evaluating social welfare in OLG models is far from obvious, since the lifetime utilities of individual generations will generally vary response to the shocks that hit the economy. Therefore, in order to evaluate IT and PLT, it was assumed that the government seeks to maximise the unconditional expectation of social welfare – a criterion first proposed by Taylor (1979). This approach to policy evaluation has been used in numerous papers in the monetary policy literature, including Schmitt-Grohé and Uribe (2007), Clarida, Gali and Gertler (1999) and Rotemberg and Woodford (1998).⁶⁸ The unconditional expectation of social welfare is the average across all possible histories of shocks and is therefore a robust criterion by which to evaluate alternative policies using a long run perspective.

Given the lifetime utility function in Equation (2.1) and the utility of the initial old – see Equation (2.8) and the surrounding discussion – average welfare over T periods is given by⁶⁹

⁶⁸ Examples of stochastic OLG models in which monetary policy is evaluated using an unconditional social welfare criterion include Kryvtsov *et al.* (2011) and Brazier *et al.* (2006).

⁶⁹ Note that the young of period T only receive utility from their youth, because the 'world' is assumed to end after the period T horizon over which welfare is evaluated.

$$\begin{aligned}
(2.40) \quad U^T &= \frac{1}{T} \left[u_{1,O}(c_{1,O}) + \sum_{t=1}^T u_t(c_{t,Y}, c_{t+1,O}) \right] \\
&= \frac{1}{T} \left[u_{1,O}(c_{1,O}) + \sum_{t=1}^{T-1} (u_{t,Y}(c_{t,Y}) + E_t u_{t+1,O}(c_{t+1,O})) + u_{T,Y}(c_{T,Y}) \right] \\
&= \frac{1}{T} \left[\sum_{t=1}^T u_{t,Y}(c_{t,Y}) + u_{1,O}(c_{1,O}) + \sum_{t=1}^{T-1} E_t u_{t+1,O}(c_{t+1,O}) \right]
\end{aligned}$$

Social welfare is then defined as the unconditional expectation of this expression, or

$$(2.41) \quad U^{society} \equiv E[U^T] = \frac{1}{T} E \left[\sum_{t=1}^T (u_{t,Y}(c_{t,Y}) + u_{t,O}(c_{t,O})) \right]$$

Social welfare from this criterion is expressed in ‘utils’ and is therefore meaningless from a policy perspective. The analysis that follows therefore focuses on the consumption equivalent welfare gain from switching from IT to PLT over the long run. Formally, the consumption equivalent welfare gain λ is defined as the fractional increase (or decrease) in consumption by young and old generations under IT that is necessary to equate social welfare under the two policies, i.e.

$$(2.42) \quad (1 + \lambda)^{1-\delta} U_{society}^{IT} = U_{society}^{PLT}$$

where $U_{society}^{IT}$ is social welfare under IT and $U_{society}^{PLT}$ is social welfare under PLT.

A positive value for λ indicates that social welfare is higher under PLT than IT, and that the gain in welfare is equivalent to a permanent increase in consumption for all young and old generations of 100λ per cent. On the other hand, if social welfare is higher under IT, λ will be negative. Note that since aggregate consumption is 80 per cent of steady-state GDP under the baseline calibration, the consumption equivalent welfare gain can be expressed as a gain or loss in per cent of GDP by multiplication of λ by 80.

2.8 Simulation results

As mentioned above, simulation results were obtained from 1,000 stochastic simulations of 5,000 periods each, giving a total of 5 million simulated observations from which to evaluate the impact of PLT vis-à-vis IT. The results in this section

compare IT and PLT across three areas: impulse responses, consumption volatility and social welfare.

2.8.1 Impulse responses

Figure 2.7 shows the impulse responses of inflation to a one standard deviation money supply innovation in the IT and PLT cases. The initial impact is somewhat smaller under PLT because of the lower 30-year money supply innovation variance. Although the shock to inflation is subsequently offset under PLT, it is clear that overall inflation variability is substantially higher under IT.⁷⁰

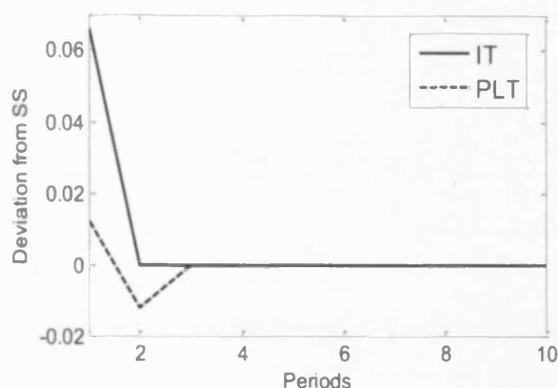


Figure 2.7 – Inflation response to a money supply innovation

Figure 2.8 shows the response of bond holdings and consumption when young to a money supply innovation. In the IT case there is no response: a money innovation increases inflation but not expected future inflation, leaving expected returns on bonds and money balances unchanged. As a result, there is no incentive for young consumers to substitute between bonds and money balances. Under PLT, however, a positive money supply innovation reduces future expected inflation because agents anticipate that the positive impact of the innovation on the price level will be offset in the next period. Since nominal bonds compensate bondholders for anticipated changes in inflation whilst money balances do not, a reduction in future expected inflation has no impact on the expected real return on bonds but raises the expected real return on money balances. In response, consumers substitute away from bonds and towards

⁷⁰ In this context, 'variability' can be measured by the area under the impulse response function (that is, the area above and below the zero line).

money balances, with the result that consumption in youth rises.⁷¹ One interesting implication of this finding is that there is zero consumption variability across young generations under IT, but some under PLT.

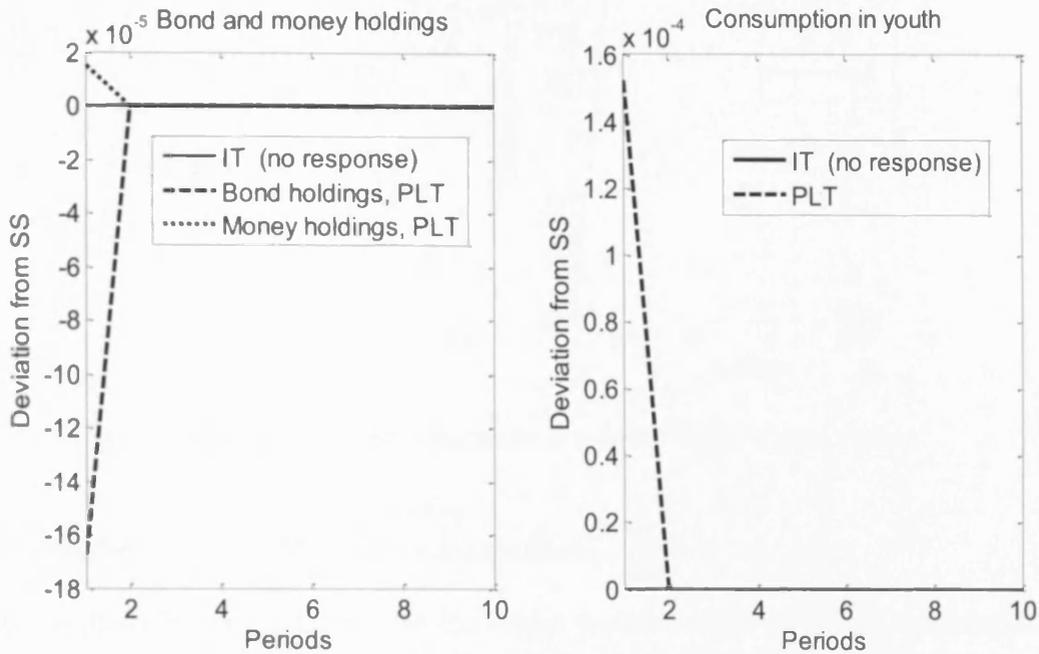


Figure 2.8 – Impulse responses to a money supply innovation

Impulse responses of the real return on nominal bonds and consumption by old generations are shown in Figure 2.9. Since a positive monetary supply innovation corresponds to an unexpected increase in inflation, real bond returns fall under both IT and PLT. However, the deviation under IT is considerably larger due to the greater level of inflation risk. As a result of the falls in bond returns with unanticipated inflation, consumption by old generations also falls below steady-state, with a correspondingly smaller fall under PLT. It is notable that since nominal bonds compensate consumers for anticipated fluctuations in inflation, there is no lagged response of the real return on bonds to the initial inflation shock under IT or PLT. However, since money balances provide no protection against anticipated inflation, there is a small lagged response of consumption by old generations when the initial innovation is offset under PLT.

⁷¹ Recall that consumption when young and money balances are proportional via the CIA constraint.

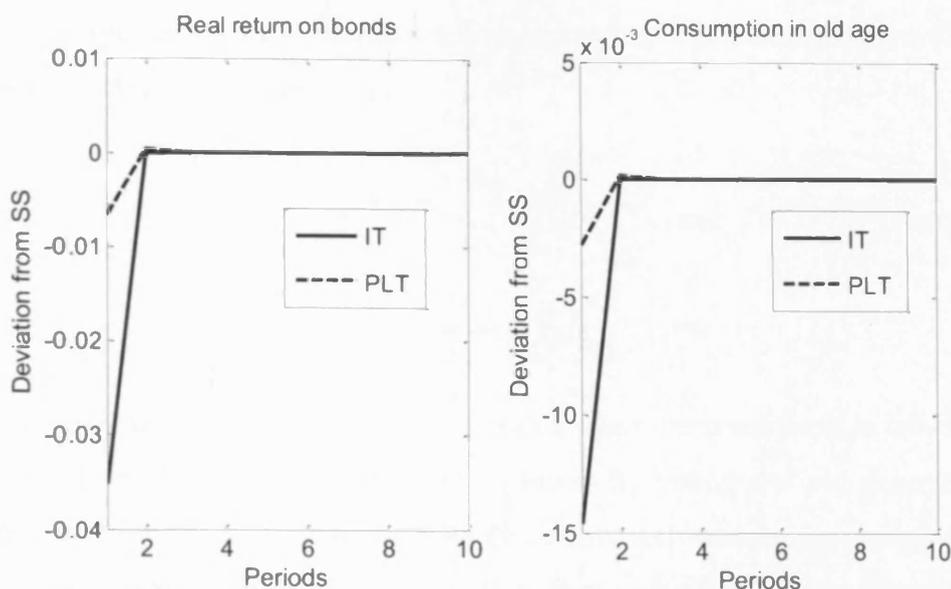


Figure 2.9 – Impulse responses to a money supply innovation

2.8.2 Consumption volatility and social welfare

In order to provide some intuition for the social welfare results, average consumption levels and volatilities are reported. The importance of these values for social welfare follows directly from the use of second-order approximation and the nature of the social welfare function. Social welfare is evaluated using Equation (2.41), but that expression is analytically intractable.

Hence consider the following equation:

$$(2.43) \quad U^{society} = E(u(c_{t,Y}) + u(c_{t,O}))$$

This expression arises exactly if the utility of the initial old is excluded from social welfare, or equivalently if the limit of Equation (2.41) is taken as the number of generations T tends to infinity. The reason is that all generations, except the initial old, are *ex ante* homogenous (in the long run sense) and hence have the same long run average level of utility. Moreover, given that the model is solved using a second-order perturbation method, we can work with a second-order Taylor expansion of the above equation around mean consumption levels.

Given the specification of lifetime utility, a second-order expansion results in the following social welfare criterion:

$$(2.44) \quad U^{society} \approx \left(\frac{(Ec_{t,Y})^{1-\delta} + (Ec_{t,O})^{1-\delta}}{1-\delta} \right) - \frac{1}{2} \left(|U_{c_Y c_Y}^{society}| \text{var}(c_{t,Y}) + |U_{c_O c_O}^{society}| \text{var}(c_{t,O}) \right)$$

where $U_{c_Y c_Y}^{society} = -\delta(Ec_{t,Y})^{-(1+\delta)}$ and $U_{c_O c_O}^{society} = -\delta(Ec_{t,O})^{-(1+\delta)}$.

Therefore, social welfare can be expressed in a mean-variance form in which welfare is positively related to mean consumption levels by young and old generations, but negatively related to consumption risk. The above expression also makes clear that whilst mean consumption levels have a first-order impact on social welfare, consumption risk terms have only a second-order impact. Intuitively, consumption volatility is costly for society because it leads to dispersion in consumption levels across generations, hindering intergenerational consumption-smoothing.

The simulation results from the model are summarised in Table 2.4, which reports the consumption equivalent welfare gain λ in per cent, plus consumption means and variances under IT and PLT.

Table 2.4 – Social welfare and consumption

Simulated value	IT	PLT
$Ec_{t,Y}$	0.3994	0.4000
$Ec_{t,O}$	0.4005	0.4000
$\text{var}(c_{t,Y}) \times 1000$	0	2.3e-5
$\text{var}(c_{t,O}) \times 1000$	0.228	0.008
λ (in % terms)	0.103%	

The positive value of λ indicates that social welfare is higher under PLT. In particular, an increase in consumption in youth and old age of 0.103 per cent is needed for all generations under IT in order to match the higher level of social welfare under PLT. Notably, the estimated welfare gain from PLT is considerably higher than the preliminary estimate reached by Mankze and Tödter (2007) of 0.066 per cent for *old generations only*. Using aggregate consumption expenditure by UK households in 2009 gives an aggregate monetary gain of £899.1 million, which is equal to a lifetime consumption increase of £31.03 per working member of the population aged 16 years

and over, or £107.04 per pensioner.⁷² It is important to note that this consumption gain would apply, in principle, to all current and future generations and would be permanent. Moreover, the level of consumption on which these gains are calculated should grow over time. Since steady-state aggregate consumption is 80 per cent of GDP in the model, the implied welfare gain is approximately 0.08 per cent of GDP.⁷³

Consumption by old generations is higher on average under IT due an increase in the inflation risk premium on nominal bonds.⁷⁴ However, as this premium corresponds to a higher average cost of issuing government debt, the government has to raise the income tax rate faced by young consumers in order to meet its long run government spending target.⁷⁵ As a result, mean consumption by young generations is lower under IT, offsetting the increase in average consumption across old generations. The intuition for this result can be seen formally by a first-order Taylor expansion of the first term on the right hand side of Equation (2.44) around the deterministic steady-state.⁷⁶ Using this approximation results in the following social welfare criterion:

$$(2.45) \quad U^{society} \approx c_O^{1-\delta} \left(E c_{t,Y} + E c_{t,O} + \frac{2\delta \times c_O}{1-\delta} \right) - \frac{1}{2} \left(\left| U_{c_Y c_Y}^{society} \right| \text{var}(c_{t,Y}) + \left| U_{c_O c_O}^{society} \right| \text{var}(c_{t,O}) \right)$$

where $E c_{t,Y} + E c_{t,O}$ is the average level of aggregate consumption.

The goods market-clearing condition $c_{t,Y} + c_{t,O} + g_t = \omega$ can be used to show that the average level of aggregate consumption $E c_{t,Y} + E c_{t,O}$ is approximately invariant to a change in monetary policy from IT to PLT. In particular, taking the unconditional expectations operator through the market-clearing condition gives $E c_{t,Y} + E c_{t,O} = \omega - g^*$. The key to the invariance result is that the government must meet its long run government spending target of $E g_t = g^*$ regardless of whether it targets inflation or the price level. The key implication of this invariance result is that the gain in social welfare under PLT arises primarily from its impact on young and old generations'

⁷² These figures are based primarily on 2009 data in ONS (2010a), but the pensioner population figure is based on 2007/8 data in DWP (2009).

⁷³ In developed economies aggregate consumption accounts for around 70 per cent of GDP, implying a welfare gain closer to 0.07 per cent of GDP. In the UK, aggregate consumption is less than 65 per cent of GDP.

⁷⁴ See Chapter 4. The average real return on money balances is also higher under IT by Jensen's inequality.

⁷⁵ In particular, the tax rates necessary to meet the government spending target were $\tau^T = 0.1692$ and $\tau^{PLT} = 0.1676$.

⁷⁶ This approximation is employed only to provide intuition.

consumption volatility. More specifically, PLT leads to an increase in social welfare because there is a substantial reduction in consumption volatility across old generations, but only a small increase in volatility across young generations (see rows 3 and 4 of Table 2.4).

The result that PLT increases consumption volatility across young generations was predicted based on the impulse responses reported earlier. In particular, there is no consumption volatility at all across young generations under IT because inflation expectations are constant and equal to the inflation target, so that there is no incentive for successive generations to substitute between money and bonds. Under PLT, however, this is not the case: inflation expectations are time-varying, because agents expect above average inflation to be followed by below average inflation, and *vice versa*. For instance, if inflation is currently above target, young generations will anticipate inflation below target next period, which increases the expected real return on money balances whilst leaving the expected real return on nominal bonds unchanged. Consequently, young generations will substitute from bonds to money following periods in which inflation is unexpectedly high, and from money to bonds following periods when inflation is unexpectedly low. These substitution effects mean that consumption by young agents varies over time, lessening intergenerational consumption-smoothing and reducing social welfare *ceteris paribus*. However, what is clear from Table 2.4 is that this volatility ‘cost’ of PLT is rather small.

By contrast, the ‘benefit’ from PLT for old generations is rather large: consumption volatility across old generations is reduced by more than 95 per cent! The reasoning for this substantial reduction in volatility is that consumption by old generations is derived solely from nominal assets – *viz.* bonds and money balances – whose real returns are substantially less volatile under PLT due to the considerable reduction in inflation risk. To demonstrate this difference graphically, Figure 2.10 plots the distributions of real bond returns and consumption by old generations under IT and PLT based on the first 200 simulations of the model (that is, 1 million observations in total). The dramatic reduction in volatility under PLT due to the lower level of inflation risk is clearly evident. Notably, the welfare gain from PLT is far lower than suggested by the massive proportional reduction in consumption volatility, since a reduction in volatility has only a second-order impact on social welfare. Indeed, as

noted by Rudebusch and Swanson (2008), second-order terms in DSGE models are typically around 100 times smaller than first-order terms, because the shocks in such models are small under standard calibrations.

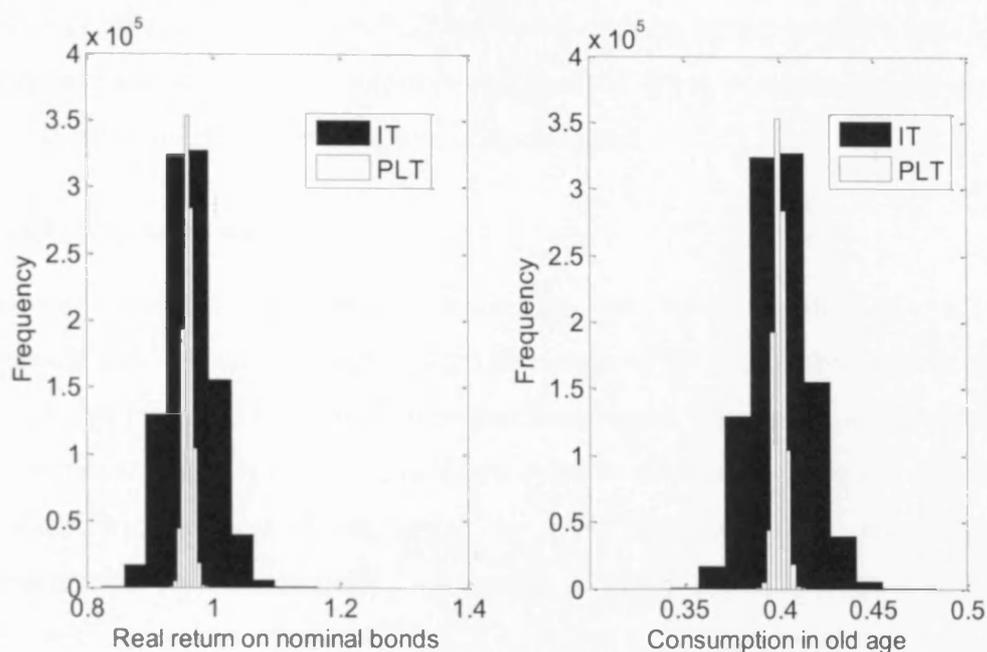


Figure 2.10 – Bond returns and consumption of old generations

The marked reduction in volatility under PLT is partly the result of the model abstracting entirely from real assets. Such assets would provide a hedge against inflation risk, such that PLT would have less overall impact on consumption volatility via the bond return channel. Similarly, the presence of indexed bonds in the model would enable consumers to directly protect their wealth against fluctuations in inflation.

However, what the results from the simple model above do highlight clearly is the potential for reduced long-term inflation risk under PLT to have non-trivial volatility and welfare benefits for society – and in particular for old generations (i.e. pensioners). It should also be noted that the results from this model should provide a good estimate of the gain from PLT for countries in which state provision of retirement income is dominant and primarily in the form of nominal assets. Germany is an example of one such country: more than 80 per cent of retirement income is in the form of public sector pensions (Berkel and Börsch-Supan, 2004; Börsch-Supan, 2000) and the market for indexed government bonds is trivial (Garcia, 2008).

In Section 2.11, the OLG model is extended so that consumers can also hold risky real assets – namely, private sector pensions which are modelled as holdings of productive capital. The object of this extension is to examine the robustness of the estimated welfare and volatility gains from PLT in a more realistic model in which not all risk is nominal and where aggregate output is endogenised. First, however, the sensitivity of the quantitative results reported above is investigated.

2.9 Sensitivity analysis

Given that there is uncertainty surrounding the correct calibration of model parameters, this section investigates the robustness of the quantitative results reported above to alternative calibrations. Given that the impacts on consumption volatility are strongly robust, focus is restricted in this section to the sensitivity of the consumption equivalent welfare gain λ . In order to allow for calibration uncertainty and heterogeneities across countries, robustness is investigated for three key model parameters: the coefficient of relative risk aversion δ ; the CIA constraint coefficient θ ; and the money supply innovation variance (which determines inflation variability under IT and PLT in equilibrium).⁷⁷

2.9.1 Model parameter calibration

The coefficient θ represents the proportion of consumption by young agents held as real money balances. The baseline value of 0.10 was calibrated to roughly match UK data. In this section, sensitivity is tested to ‘high’ and ‘low’ values of θ , namely 0.25 and 0.01. Regarding the coefficient of relative risk aversion, the baseline calibration of 3 is close to the mid-range of values considered plausible in the literature. In particular, coefficients in a range from 1 to 5 seem most relevant from an empirical perspective. On this basis, sensitivity was tested using alternative values of 3/2 and 5.⁷⁸ Table 2.5 reports the results from this sensitivity analysis. The baseline estimate is shown in bold in the centre of the table.

⁷⁷ In each simulation, the tax on young consumers was altered so that the long run government spending target of 20 per cent was met.

⁷⁸ Tödter (2008) estimated a 95 per cent confidence interval of (1.4, 7.1) for the coefficient of relative risk aversion using US stock return data from 1926 to 2002.

Table 2.5 – Sensitivity of λ to risk aversion and money holdings

Risk aversion coefficient δ	Proportion of consumption when young held as money balances θ		
	$\theta = 0.01$	$\theta = 0.10$	$\theta = 0.25$
$\delta = 3/2$	0.052%	0.052%	0.052%
$\delta = 3$	0.103%	0.103%	0.103%
$\delta = 5$	0.173%	0.173%	0.173%

The welfare gain from PLT is robust to changes in θ . Indeed, as the parameter is varied between 0.01 and 0.25, the welfare gain remains unchanged to three decimal places. On the other hand, there is considerable sensitivity of the welfare gain to the extent of risk aversion. For instance, with the baseline calibration for θ , reducing the risk aversion coefficient to $3/2$ almost halves the welfare gain to 0.052 per cent, whilst an increase to 5 increases the baseline welfare gain by almost three-quarters to 0.173 per cent. Intuitively, increasing risk aversion raises the welfare gain because PLT has a positive impact on social welfare by reducing consumption risk across old generations. Indeed, the importance of the risk aversion coefficient for consumption risk terms in the social welfare function can be seen clearly from Equation (2.44) in which the second-order risk terms are weighted by δ . In effect, an increase in δ increases the curvature of the utility function (see e.g. Rudebusch and Swanson, 2008), making second-order uncertainty terms of more importance for social welfare.

A policy implication that follows from these results is that heterogeneity across countries in terms of risk aversion will have consequences for the potential long-term welfare gain from PLT. *Ceteris paribus*, those countries with greater aversion to risk will have more to gain, in the long run, from switching from IT to PLT. Additionally, since the extent of risk aversion is itself uncertain, the welfare gain from switching to PLT in any given country will necessarily be uncertain as well. It is notable, however, that the welfare gain from PLT is economically non-trivial even if risk aversion is relatively low.

2.9.2 Nominal volatility

In the baseline calibration of the model, the money supply innovation variance was based on UK inflation data for the RPIX over the IT period. The RPIX was chosen because it excludes mortgage interest payments but includes council tax and housing costs, both of which are relatively important costs for pensioners that are excluded

from the Consumer Prices Index (CPI). However, since the RPIX is an index that is supposed to be representative of an average household in the UK, it still may not capture the inflation experience of pensioners well. This issue has been investigated by Leceister *et al.* (2008). They note that though similar on average, pensioner RPI inflation indices constructed by the Office for National Statistics (ONS) can differ substantially in the short-term from the RPI and RPIX. Therefore, the pensioner RPI for one-pensioner households was used to estimate the money supply innovation variance.⁷⁹ The results are shown in Table E1 in Appendix E. The implied yearly money supply innovation standard deviation was 0.015, compared to 0.012 based on the RPIX. The model was simulated under this alternative calibration as a robustness check.

It has also been argued that the CPI – on which the current UK inflation target is based – is a better measure of inflation than the RPI or RPIX because it captures substitution away from relatively more expensive goods to cheaper goods. This substitution effect may be particularly important for pensioners given that their incomes are lower than the general population (OECD, 2009a; DWP, 2009). Using the CPI in estimation instead implies a lower yearly money supply innovation standard deviation of 0.011,⁸⁰ as shown by the results in Table E2 of Appendix E. The model was therefore also simulated with a lower money supply innovation variance. However, in order to make the sensitivity analysis exactly symmetric relative to the baseline case, a standard deviation of 0.009 was specified.

Table 2.6 shows the sensitivity of results to the money supply innovation variances implied by these ‘high’ and ‘low’ standard deviation calibrations. In the low variance case, the welfare gain from PLT falls by around four-tenths from 0.103 per cent to 0.062 per cent. Intuitively, reducing the innovation variance reduces the welfare gain from PLT since, with nominal volatility reduced, long-term inflation risk (a second-order impact) becomes relatively less important for social welfare. By the same argument, increasing the money supply innovation variance increases the relative importance of long-term inflation risk for social welfare, and with it the welfare gain

⁷⁹ The ONS also publish an RPI index for two-pensioner households. The one-pensioner measure was used because there are more single pensioners in the UK (DWP, 2009). The index was downloaded from the ONS Time Series Database at <http://www.statistics.gov.uk/statbase/tsdtimezone.asp>.

⁸⁰ Again, the index was downloaded from the ONS Time Series Database.

from PLT. In the high volatility case, the consumption equivalent welfare gain from PLT is increased by approximately one half to 0.153%. All in all, then, the welfare benefit from PLT is rather sensitive to the calibrated money supply innovation variance, though slightly less so than to the coefficient of relative risk aversion.

Table 2.6 – Sensitivity of λ to nominal volatility

Money supply innovation variance		
Low $\text{var}(\varepsilon_{i,t}) = 0.87 \times 10^{-4}$	Baseline $\text{var}(\varepsilon_{i,t}) = 1.45 \times 10^{-4}$	High $\text{var}(\varepsilon_{i,t}) = 2.19 \times 10^{-4}$
$\lambda = 0.062\%$	$\lambda = \mathbf{0.103\%}$	$\lambda = 0.154\%$

Two policy implications are suggested by these volatility sensitivity results. First, to the extent that long-term inflation risk under IT is uncertain due to lack of historical experience, the welfare gain from PLT is necessarily also uncertain. The benchmark calibration of the model implicitly assumes that current and past experience with nominal volatility under IT will be a good guide to future. However, this may not be case because macroeconomic shocks may have been usually small during the Great Moderation (the ‘Good Luck’ hypothesis), and the performance of IT may change as policymakers acquire more experience. Secondly, the potential welfare gain from PLT is likely to vary across countries according to the success of the monetary policy regime currently in place (IT in many cases). In particular, the welfare gain from PLT will be higher in countries in which nominal variability is greater, because long-term inflation risk will be a more important factor for social welfare in these countries.

2.10 Summary of results and policy implications

In preliminary analysis, Mankze and Tödter (2007) estimated that the welfare gain from PLT was equal to 0.066 per cent of average consumption for old generations only. The numerous limitations with their analysis have been addressed in the more comprehensive analysis presented thus far. In the baseline case, the estimated welfare gain from PLT was 0.103 per cent for both *young and old generations*, which amounts to 0.103 per cent of aggregate consumption, or 0.08 per cent of GDP. This estimate is more than three times as high as that reached by Mankze and Tödter (2007), suggesting that their analysis substantially understates the potential long-term welfare gain from PLT. Intuitively, the welfare gain from PLT arises from a sharp

reduction in consumption volatility for old generations (i.e. pensioners) that exceeds by far a small increase in consumption volatility across young generations. However, since the model assumes that consumers hold no real assets into old age, the baseline welfare gain from PLT is best treated as an upper bound for most developed economies, and is likely to be applicable only in special cases where nominal assets dominate in the provision of retirement income (e.g. Germany).

It is important to note that the estimated welfare gain from PLT is rather sensitive to the extent of risk aversion and the calibrated money supply innovation variance. The welfare gain for a society which is more risk-averse is considerably larger because PLT benefits consumers by reducing the extent of long-term consumption risk they face. For empirically relevant risk aversion coefficients, the welfare gain ranges from 0.052 to 0.173 per cent of aggregate consumption, and is almost as sensitive to the calibrated money supply innovation variance, with a higher money supply innovation variance implying greater long-term inflation risk and a higher welfare gain from PLT. A policy implication that follows from this second result is that countries with more volatile inflation would have more to gain in the long run by switching from IT to PLT.

Based on the analysis above, the long-term welfare gain from PLT is estimated to be small but economically non-trivial. Importantly, the model analysed here did not include real assets that can hedge against inflation risk, or inflation-indexed bonds. As a result, the potential welfare gain from PLT is likely to be overstated for most countries. In order to address this point, the next section investigates the extent to which the estimated welfare gain from PLT is altered when consumers can also hold capital – a risky real asset – in their savings portfolios.

2.11 Introducing productive capital into the model

The model analysed above clearly demonstrates the intuition for PLT increasing social welfare vis-à-vis IT. However, as the model makes the implausible assumption that consumers hold only nominal assets, the results presented above should be treated with caution. Other things being equal, we should expect the introduction of real assets into the model to reduce the welfare gain from PLT. The reasoning is that with consumers' portfolios diversified between real and nominal assets, consumption volatility in old age should be less sensitive to a reduction in long-term inflation risk.

This second half of the chapter investigates this issue by allowing consumers to hold productive but risky capital as well as nominal bonds and money balances. Given the 30-year holding horizon in the model, an investment in risky capital can be thought of as an investment in a private sector pension.⁸¹ Indeed, it is notable that one-half or more of private pension fund assets are invested in equities in 4 of the G7 countries – namely, Canada, Japan, the UK and the US (see OECD, 2009a) – and that such funds have highly volatile real returns (see D'Addio *et al.* 2009), in contrast to government bonds. Private sector pensions play an important role in many developed economies, including three of the G7 countries: the UK, US and Canada (OECD, 2009b). Moreover, theoretical motivation for modelling risky capital in an OLG model is provided by Lungu and Minford (2006), who show that taking a long-term view of stock returns can help resolve the 'equity premium puzzle' without the need to resort to habit-forming preferences.

Economic theorists have long held the view that the return on capital is independent of the rate of inflation, in which case equities are a 'hedge' against inflation. The approach taken here is consistent with this view. Although empirical evidence regarding this issue is mixed (Luintel and Paudyal, 2006; Ely and Robinson, 1997; Bodie, 1976), it is not sufficient to reject the hypothesis from a Popperian perspective. It is also worth noting that empirical studies tend to find that equities are a better hedge of inflation over long horizons (e.g. Bekaert and Wang, 2010; Shen, 2005). An important implication of the introducing capital into the model is that aggregate

⁸¹ An investment in productive capital could also be thought of as including long-term investments in the stock market by individuals, but such investments are a minor source of income for most pensioners (DWP, 2009; OECD, 2009a).

income becomes endogenously determined, hence extending the model from partial to general equilibrium along the lines of Diamond (1965).

2.11.1 The OLG model with capital

The model of the previous section is extended so that young consumers can additionally allocate savings to risky capital which is used to produce output in old age, à la Lungu and Minford (2006). Capital is thus a claim to an uncertain amount of real output in old age. Moreover, following Lungu and Minford (*op. cit.*), capital lasts for only one period, that is, the assumed depreciation rate is 100 per cent.⁸²

The production function for output is given by:

$$(2.46) \quad Y_t = A_t k_{t-1}^\alpha \quad 0 < \alpha < 1$$

where k_t is capital holdings and A_t is a productivity shock.

The productivity shock follows an AR(1) process in logs:

$$(2.47) \quad \ln A_t = (1 - \rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t \quad 0 < \rho < 1$$

where e_t is an IID normal random variable with mean zero and variance σ_e^2 .

With the additional option to save via capital, the budget constraint of young generations is now given by

$$(2.48) \quad c_{t,y} + b_t^{n,d} + m_t^d + k_t = \varpi(1 - \tau^j)$$

⁸² Given that each period lasts 30 years, the assumption of full depreciation is empirically plausible. See, for example, Nadiri and Prucha (1996) and studies cited therein.

And consumption by old generations is given by

$$(2.49) \quad c_{t+1,O} = A_{t+1}k_t^\alpha + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d$$

where $A_{t+1}k_t^\alpha$ is output produced for consumption in old age using capital.

The lifetime utility function of young consumers is unchanged, as is the demand for money. However, aggregate output in the economy (and hence goods market equilibrium) does change, and is now given by the following expression:

$$(2.50) \quad \omega + A_t k_{t-1}^\alpha = c_{t,Y} + c_{t,O} + g_t + k_t$$

In order to gain some formal intuition for the social welfare impact of adding capital into the model, the model was log-linearised around the deterministic steady-state and then substituted into a second-order Taylor expansion of lifetime utility.

Appendix F shows that lifetime utility can be written in the following form:

$$(2.51) \quad Loss_t \approx -(1/2)\Psi_t [\text{var}_t(\pi_{t+1}) + \varsigma \text{var}_t(A_{t+1})]$$

where $Loss_t$ is the deviation of lifetime utility from its time- t expected value, $\Psi_t > 0$ is a coefficient that depends on the model's steady-state parameters and expected consumption in old age, and $\varsigma > 0$ is a coefficient that represents the relative importance of capital in consumers' savings portfolios.

Equation (2.51) shows that welfare loss for generation t depends on the conditional variances of inflation and productivity over a 30-year horizon. Intuitively these variances determine the extent of long-term return risk on bonds and capital respectively, and also, therefore, consumption risk faced by old generations. Since exogenous productivity determines the extent of return risk on capital, the relative welfare gain from PLT should be diluted somewhat: monetary policy will be able to affect only a fraction of the uncertainty faced by consumers, in contrast to the case just studied where consumers held only nominal assets. Equation (2.51) indicates that

the extent to which the welfare gain of PLT will be reduced will depend on the importance of capital in consumers' portfolios, and on the relative importance of the inflation and productivity variances. This intuition is confirmed by the quantitative simulation results reported below.

2.11.2 Consumers' first-order conditions

First-order conditions are unchanged, except that there will be an additional first-order condition relating to capital holdings. Consequently, there will be two Euler equations that define optimal savings: one for nominal bond holdings; and one for capital. To derive the additional first-order optimality condition for capital, consider the following Lagrangian:

$$(2.52) \quad L_t \equiv E_t \left[\begin{aligned} & u_t(c_{t,Y}, c_{t+1,O}) + \lambda_{t,Y} (\omega(1 - \tau^j) - m_t^d - b_t^{n,d} - k_t - c_{t,Y}) \\ & + \mu_t (m_t^d - \theta c_{t,Y}) + \lambda_{t+1,O} (A_{t+1} k_t^\alpha + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d - c_{t+1,O}) \end{aligned} \right]$$

where μ_t is the Lagrange multiplier on the CIA constraint.

The first-order conditions are as follows:

$$(2.53) \quad c_{t,Y} : c_{t,Y}^{-\delta} = \lambda_{t,Y} + \theta \mu_t$$

$$(2.54) \quad c_{t+1,O} : \lambda_{t+1,O} = c_{t+1,O}^{-\delta}$$

$$(2.55) \quad b_t^{n,d} : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^n)$$

$$(2.56) \quad m_t^d : \lambda_{t,Y} = E_t (\lambda_{t+1,O} r_{t+1}^m) + \mu_t$$

$$(2.57) \quad k_t : \lambda_{t,Y} = E_t (\lambda_{t+1,O} \alpha A_{t+1} k_t^{\alpha-1})$$

Substituting out the Lagrange multipliers on the budget constraints when young and old gives an additional consumption Euler equation relating to capital holdings:

$$(2.58) \quad c_{t,Y}^{-\delta} = E_t (c_{t+1,O}^{-\delta} \alpha A_{t+1} k_t^{\alpha-1}) + \theta \mu_t$$

The Lagrange multiplier on the cash-in-advance constraint is therefore given by

$$(2.59) \quad \mu_t = E_t \left(c_{t+1,O}^{-\delta} (r_{t+1}^n - r_{t+1}^m) \right) = E_t \left(c_{t+1,O}^{-\delta} (\alpha A_{t+1} k_t^{\alpha-1} - r_{t+1}^m) \right)$$

Intuitively, the first equality states that, absent uncertainty, the CIA constraint will be strictly binding if money is rate of return dominated by nominal bonds, whilst the second equality indicates that the same relationship holds for capital versus money, because capital and bonds are priced to give equivalent expected utility (at the margin) by no-arbitrage.

Substituting out for the Lagrange multiplier, the consumption Euler equation for capital can be written in the following form:

$$(2.60) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) \alpha A_{t+1} k_t^{\alpha-1} - \theta r_{t+1}^m \right) \right)$$

2.11.3 Government and monetary policy

The government's budget constraint is not affected by the inclusion of capital in the model, and it is assumed that the government continues to set the bond supply so as to equate the marginal utility of consumption in youth with the expected marginal utility of consumption in old age. The money supply rules to which the government commits under IT and PLT are also unchanged.

For completeness, the deterministic steady-state and market-clearing conditions for the model are given in Appendix G, along with a listing of the model's equations in Appendix H.

2.12 Calibrating stochastic productivity

When calibrating the stochastic process for productivity, it is necessary to take into account the 30-year horizon of the OLG model. The approach taken here is to extend a typical quarterly calibration from the real business cycle (RBC) literature over a 30-year horizon.

Consider an AR(1) process for log productivity at a quarterly horizon q :

$$(2.61) \quad \ln A_q = (1 - \rho_q) \ln A_{q,mean} + \rho_q \ln A_{q-1} + e_q \quad 0 < \rho_q < 1$$

where e_q is an IID-Normal productivity innovation with mean zero and variance σ_q^2 .

By substituting repeatedly for lagged productivity terms, productivity over a 30-year (i.e. 120-quarter) horizon can be obtained as follows:

$$(2.62) \quad \ln A_q = (1 - \rho_q^{120}) \ln A_{q,mean} + \rho_q^{120} \ln A_{q-120} + \sum_{j=0}^{119} \rho_q^j e_{q-j}$$

On this basis, productivity in the OLG model can be represented as follows:

$$(2.63) \quad \ln A_t = (1 - \rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t$$

where $\ln A_{mean} \equiv (1 - \rho_q^{120}) \ln A_{q,mean} / (1 - \rho)$, $\rho \equiv \rho_q^{120}$ and $e_t \equiv \sum_{j=0}^{119} \rho_q^j e_{q-j}$.

Equation (2.63) was used to calibrate the stochastic productivity process in the OLG model. Many papers in the RBC literature (e.g. King and Rebelo, 2000) use quarterly calibrations of productivity in which the autoregressive parameter is slightly below one and the innovation standard deviation is less than 0.008. For instance, Gavin, Keen and Pakko (2009) set the quarterly first-order autocorrelation at 0.95 and the innovation standard deviation at 0.005, consistent with the lower volatility of output in the ‘Great Moderation’ period. The calibration used here is based on the same standard deviation as in their paper but a higher autocorrelation coefficient of 0.996, which is more consistent with the bulk of the RBC literature. Consequently, the calibrated 30-year productivity process has a first-order correlation of 0.996¹²⁰ and an innovation standard deviation of $\sigma_e = 0.04398$.⁸³ Steady-state productivity A_{mean} was set equal to 0.75. Given the other calibrated values in the model, this value ensured

⁸³ In particular, $\sigma_e = \sqrt{(1 - 0.996^{240})(1 - 0.996^2)^{-1}} \times 0.005$.

Note that this expression makes use of the fact that

$$\text{var}(e_t) = (1 + \rho_q^2 + \rho_q^4 + \dots + \rho_q^{238}) \sigma_q^2 = (1 - \rho_q^{240}) \sigma_q^2 / (1 - \rho_q^2).$$

plausible holdings of bonds relative to capital. For reference, calibrated values in the stochastic productivity process are summarised in Table 2.7.

Table 2.7 – Calibration of stochastic productivity

Model parameter	Role in the model	Calibrated value
ρ	Persistence in productivity at a 30-year horizon	0.618
σ_e	Productivity innovation standard deviation (at a 30-year horizon)	0.04398
A_{mean}	Steady-state level of productivity	0.75

2.13 Model calibration

The only additional model parameter that has not yet been calibrated is α , the elasticity of output produced (in old age) with respect to capital. The baseline calibration for this parameter was set at 0.375, a value that lies in the mid-range of calibrated values in the RBC literature⁸⁴ and which is close to the calibration in Lungu and Minford (2006). Other calibrated values are the same as in the model with nominal bonds, with the exception of the endowment income of young consumers. Due to the inclusion of capital in the model, aggregate output (i.e. GDP) is no longer given by the endowment income of the young. Normalising the endowment to one as before would therefore not enable variables to be interpreted as percentages of GDP. Therefore, to ease exposition, the constant endowment of young consumers was chosen so that steady-state GDP was equal to 2. Moreover, the long run government spending target was doubled in line with this to ensure that it remained at 20 per cent of GDP as previously. The calibration of the model, including that of the money supply process, is summarised in Table 2.8.

⁸⁴ Papers in this literature typically include both capital and labour in the production function, so that α is the share of labour income in output. Many papers in the literature set $\alpha = 1/3$, but there are some notable exceptions that use higher calibrations (e.g. Perli and Sakelleris 1998 and King *et al.* 1988 set $\alpha = 0.42$). With this calibration, the production function in the OLG model will exhibit plausible diminishing returns to capital.

Table 2.8 – Calibration of the model with capital

Model parameter	Role in the model	Calibrated value
θ	Proportion of consumption when young held as money balances	0.10
δ	Coefficient of relative risk aversion	3
ω	Endowment income of young consumers	1.641
g^*	Long run government spending target	0.40
π^*	Inflation target over 30 years	0.75
$\text{var}(\varepsilon_{i,t})$	Yearly money supply innovation variance	1.45×10^{-4}
α	Elasticity of output in old age to capital	0.375

Table 2.9 shows the steady-state values for key model variables under this calibration.

Table 2.9 – Key variables at steady state

Model variable	Steady-state value
$c_{t,y}$	0.730
$c_{t,o}$	0.730
$b_t^d (= b_t^s)$	0.343
$m_t^d (= \theta c_{t,y} = m_t)$	0.073
k_t	0.140
π_t	0.75

Note: Steady-state GDP is equal to 2

Aggregate consumption accounts for 73 per cent of steady-state GDP (which approximately matches developed economies in general) and is split equally between consumption by young and old generations by the government bond supply rule. Money holdings are 3.7 per cent of GDP (i.e. $0.073/2$), which is similar to the UK share of notes and coins in GDP over the past decade (ONS, 2009b). As previously, steady-state inflation is equal to the 30-year inflation target, or a 75 per cent increase in prices over a 30-year horizon. Since there is full depreciation of capital, investment is given by the level of capital holdings. Steady-state investment is thus 7 per cent of GDP, with the remaining 20 per cent of GDP consisting of government spending. Steady-state capital holdings are 41 per cent of bond holdings, which is equal to the ratio of private pension spending to public pension spending in the UK in 2006 (OECD, 2009b), and is similar to the average ratio of investment to government bonds in the UK over the past decade (ONS, 2010a). All in all, the implied steady-state values provide a reasonably good fit to aggregate data for the UK and other

developed economies. As mentioned above, private sector pensions play an important role in three G7 countries: Canada, the UK and the US.

2.14 Simulation results

Simulation results were again obtained from 1,000 stochastic simulations of 5,000 periods using a second-order approximation in Dynare++. The results again focus on impulse responses, consumption volatility and social welfare.

2.14.1 Impulse responses

Since the calibration of the money supply innovation variance is unchanged, the impulse responses for inflation are identical to those in the model without capital. However, as consumers find it optimal to hold capital in their portfolios, bond holdings are altered compared to the model without capital, and not all impulse responses are left unchanged. Figure 2.11 highlights this point clearly by showing the impulse responses of the real return on bonds and consumption in old age to a money supply innovation. The real return impact is the same as in the model without capital, but the impact on consumption in old age is smaller because consumer portfolios are now diversified between bonds and capital (see Table 2.9). Consequently, the 'bond return' channel has less impact upon consumption by old generations than in the model without capital.

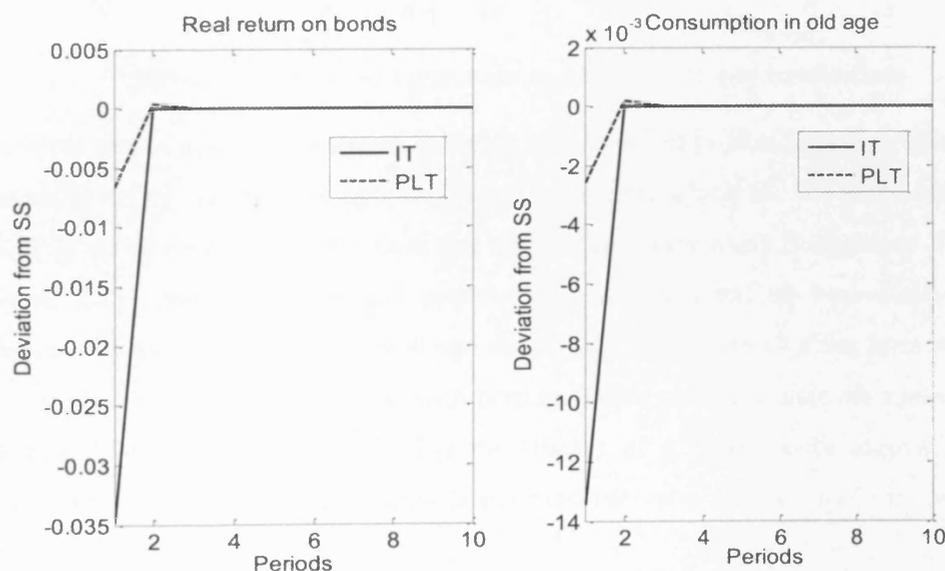


Figure 2.11 - Impulse responses to a money supply innovation

As is shown in Figure 2.12, the result that expected inflation falls in response to a money supply innovation under PLT means that consumers substitute towards money, whose expected return has risen, and away from capital and nominal bonds, whose expected real returns are unchanged.⁸⁵ Since the CIA constraint states that consumption by the young is proportional to money holdings, consumption by young generations also rises. Under IT, however, expected inflation is constant, so there is no response of asset holdings to a money supply innovation and consumption by the young is left unchanged. As a result, consumption variability across young generations is higher under PLT than IT, as in the model without capital.

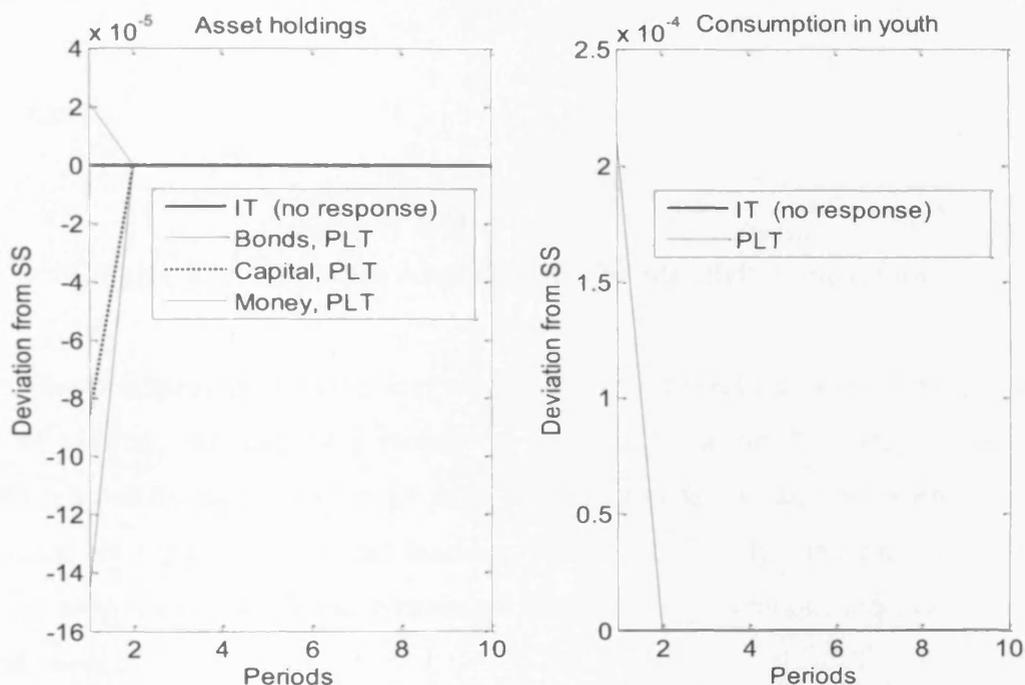


Figure 2.12 – Impulse responses to a money supply innovation

The next sets of impulse responses focus on the impact of productivity innovations on consumption by old and young generations – impacts which are the same under IT and PLT since monetary policy does not respond to productivity fluctuations. Firstly, Figure 2.13 shows that, because productivity is persistent, an innovation raises productivity and consumption in old age above their steady-state values for a number of periods after the initial impulse, with both returning to steady-state after around 15 periods. Moreover, it is notable that the impact of a productivity innovation on consumption in old age is somewhat larger than that of a money supply innovation,

⁸⁵ Tobin (1965) discusses how a change in inflation is accompanied by portfolio substitution between money and capital.

because capital is a 'riskier' asset than are nominal bonds under the baseline calibration.

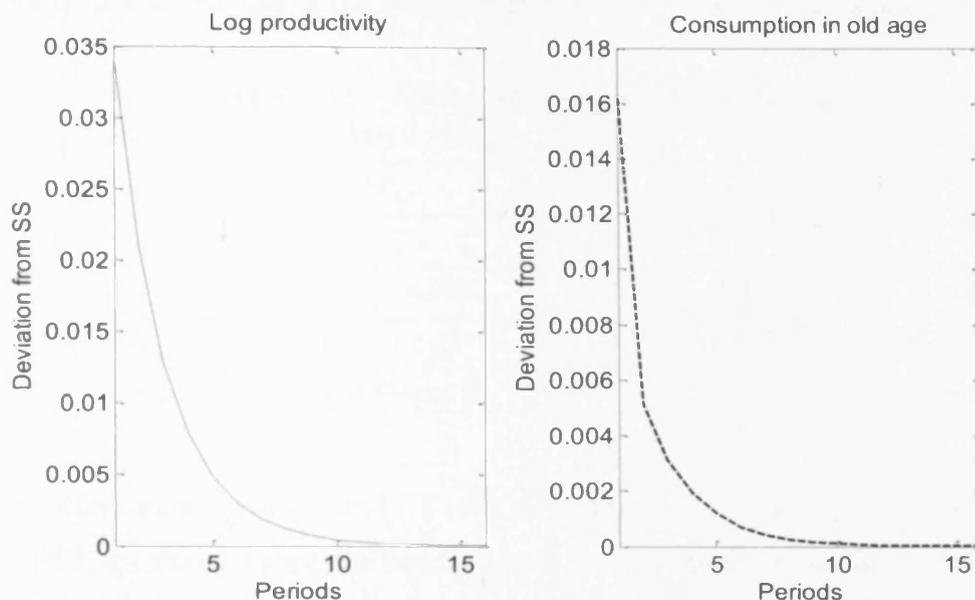


Figure 2.13 – Impulse responses to a productivity innovation

Given that productivity is persistent, a productivity innovation raises, for any given level of capital, the expected return on capital. As a result, young consumers substitute towards capital and away from nominal bonds, as can be seen from the first panel of Figure 2.14. Bond holdings fall more sharply than capital holdings increase, with the net result that consumption by young generations increases (see the second panel).

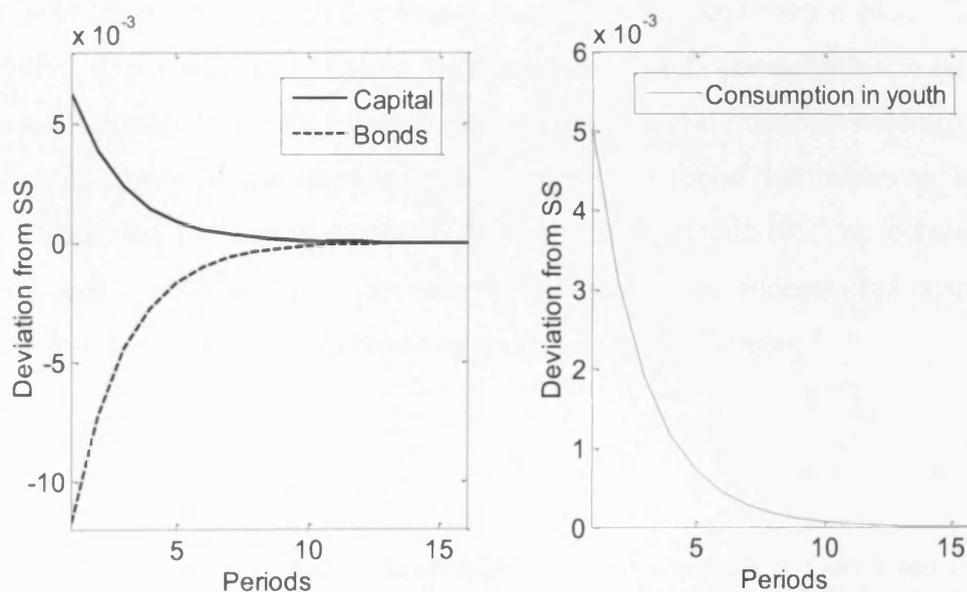


Figure 2.14 – Impulse responses to a productivity innovation

2.14.2 Consumption volatility and social welfare

Table 2.10 reports the consumption equivalent welfare gain λ and consumption means and variances across young and old generations in the model with capital.

Table 2.10 – Social welfare and consumption

Simulated value	IT	PLT
$E c_{t,Y}$	0.7297	0.7300
$E c_{t,O}$	0.7310	0.7307
$\text{var}(c_{t,Y}) \times 1000$	0.0385	0.0387
$\text{var}(c_{t,O}) \times 1000$	0.4859	0.2960
λ (in % terms)	0.026%	

The baseline welfare gain from PLT falls by approximately three quarters to 0.026 per cent. PLT increases social welfare because, as previously, it leads to a significant proportional reduction in consumption variability across old generations, yet only a trivial increase in variability across young generations.⁸⁶ The welfare gain is somewhat lower in the model with capital because there is only around a 40 per cent reduction in consumption volatility across old generations, compared to 95 per cent in the model with only nominal bonds. Figure 2.14 shows this result clearly by plotting the distribution of real bonds returns and consumption by old generations.⁸⁷ Indeed, real return variability on nominal bonds is reduced just as drastically as in the model without capital, but consumption variability much less so. This result arises because savers' portfolios are diversified between nominal bonds and risky capital, such that the relative importance of inflation variations in overall consumption volatility is lessened. Intuitively, since PLT has no direct impact on real risk from holding capital, it reduces risk only on the fraction of consumers' portfolios that relates to nominal assets. Hence the presence of capital dilutes the impact of PLT on consumption volatility and social welfare, because it introduces an independent source of consumption fluctuations which monetary policy cannot influence.

⁸⁶ As in the model with only nominal bonds, the need to meet the long run government spending target means that aggregate consumption is essentially the same under IT and PLT. Capital does not affect this result because it is a pure real asset whose return is not directly influenced by monetary policy.

⁸⁷ These distributions are based on the first 200 simulations of the model.

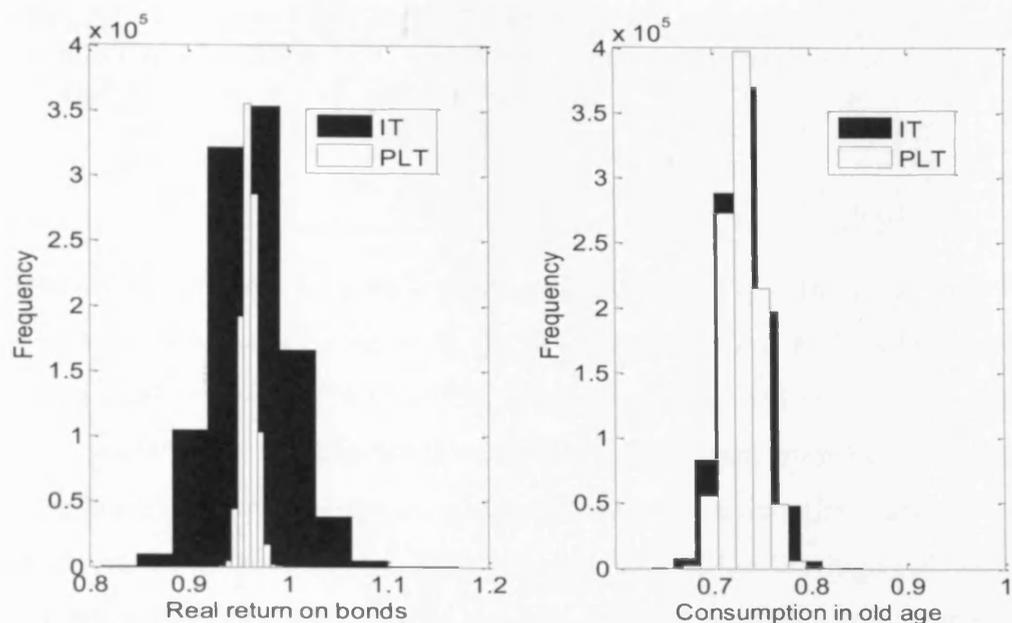


Figure 2.15 – Bond returns and consumption of old generations

To summarise, introducing capital into the OLG model reduces substantially the welfare gain from PLT, though it remains economically non-trivial. Moreover, the reduction in consumption volatility across old generations attained under PLT is substantial at around 40 per cent, though much lower than in the model with only nominal bonds.

2.15 Sensitivity analysis

This section investigates the robustness of the welfare gain from PLT with respect to key model parameters and innovation variances.

2.15.1 Model parameters

In concordance with the model without capital, the welfare gain of PLT is not sensitive to the CIA constraint parameter θ . Robustness of the welfare gain of PLT is therefore tested with respect to only two key model parameters: the coefficient of relative risk aversion δ , and the extent of persistence in the stochastic process for productivity ρ . The results from this sensitivity analysis are shown in Table 2.11. The baseline welfare gain is highlighted in bold.

Table 2.11 – Sensitivity of λ to risk aversion and productivity persistence

Coefficient of relative risk aversion, δ	Persistence in productivity, ρ		
	$\rho = 0.486$	$\rho = 0.618$	$\rho = 0.750$
$\delta = 3/2$	0.0130%	0.0122%	0.0120%
$\delta = 3$	0.0260%	0.0258%	0.0255%
$\delta = 5$	0.0441%	0.0438%	0.0435%

The extent of risk aversion is again a crucial parameter for the welfare gain from PLT. With a risk aversion coefficient of 5, the welfare gain from PLT is increased by around two-thirds, whilst halving the risk aversion coefficient from 3 to 3/2 roughly halves the welfare gain. In this latter case, the welfare gain from PLT is only just above one-hundredth of one percent, and ten times smaller than the baseline welfare gain in the model without capital. On the other hand, relatively high risk aversion increases the welfare gain from PLT to just over four-tenths of the baseline gain in the model without capital. There is thus considerable overall sensitivity to risk aversion, though the extent of sensitivity in absolute terms is reduced somewhat.

Changing the extent of productivity persistence has a measurable quantitative impact on the welfare gain from PLT, but the impact is rather small. For instance, Table 2.11 shows that altering the persistence parameter by 0.132 either way – a 20 per cent deviation from the baseline value – changes the welfare gain from PLT by less than 0.001 per cent. The welfare gain from PLT falls as persistence in productivity is increased, since an increase in persistence raises unconditional productivity volatility because innovations take longer to ‘die out’. In turn, this increase in volatility reduces the relative importance of inflation risk for old generations’ consumption.⁸⁸ Similarly, a reduction in productivity persistence reduces unconditional productivity volatility and increases the welfare gain from PLT.

2.15.2 Innovation variances

In order to test sensitivity to the money supply innovation variance, the same range of values was considered as in the model without capital. Therefore, alternative money supply innovation standard deviations of 0.009 and 0.015 were investigated. The variance of the innovation to productivity is also likely to be important for the welfare gain from PLT, since it determines the relative importance of real versus nominal fluctuations in overall consumption risk. Sensitivity was therefore also tested with

⁸⁸ The unconditional variance of log productivity is given by $\text{var}(e_t)/(1 - \rho^2)$.

respect to the productivity innovation variance. The baseline productivity innovation standard deviation was set at 0.044. Sensitivity was tested to a ‘high’ standard deviation of 0.055 and a ‘low’ standard deviation of 0.033, deviations of around one-quarter from the baseline and similar to the range considered for the money supply innovation. The alternative money supply and productivity innovation variance calibrations considered in this section are summarised in Table 2.12.

Table 2.12 – Volatility sensitivity calibrations

Money supply innovation variance		
Low $\text{var}(\varepsilon_{i,t}) = 0.87 \times 10^{-4}$	Baseline $\text{var}(\varepsilon_{i,t}) = 1.45 \times 10^{-4}$	High $\text{var}(\varepsilon_{i,t}) = 2.19 \times 10^{-4}$
Productivity innovation variance		
Low $\text{var}(e_t) = 0.001089$	Baseline $\text{var}(e_t) = 0.001934$	High $\text{var}(e_t) = 0.003025$

Table 2.13 – Sensitivity of λ to innovation volatilities

Productivity innovation variance	Money supply innovation variance		
	Low	Baseline	High
Low	0.0151%	0.0261%	0.0390%
Baseline	0.0153%	0.0258%	0.0388%
High	0.0159%	0.0255%	0.0389%

Table 2.13 reports the sensitivity results corresponding to these alternative calibrations. The impact of changing the money supply innovation variance is clear-cut: increases in nominal volatility in raise the welfare gain from PLT regardless of the calibration of the productivity innovation variance. For instance, when the productivity innovation variance is at its baseline value, the welfare gain from PLT rises by more than half to 0.0388 per cent. The reasoning behind this result is that the innovation variance determines the extent of inflation risk in equilibrium, and hence the magnitude of the reduction in consumption variability under PLT. Hence, for example, increasing the money supply innovation variance raises the relative importance of inflation risk for consumption volatility, so that the proportional reduction in consumption volatility under PLT is somewhat larger than in the baseline case (49 per cent versus 39 per cent).

Similarly, reducing the money supply innovation variance reduces the relative importance of nominal volatility, and consequently the magnitude of the welfare gain from PLT. For example, when the money supply variance is low, the welfare gain from PLT is almost halved to 0.0153 per cent. As was the case with model parameters, the welfare gain from PLT is less sensitive in absolute terms than in the model without capital, because the welfare gain itself is almost an order of magnitude lower. The model at hand is therefore likely to provide not only a better estimate of the welfare gain from PLT for many developed countries, but also a less uncertain one.

There is a surprisingly small impact from varying the productivity innovation variance: the welfare gain changes by only 0.001 to 0.008 per cent across the different specifications. With the baseline money supply innovation variance, the welfare benefit from PLT falls as the productivity innovation variance is increased. The reason is that the relative importance of productivity fluctuations is increased, so that the inflation risk channel through which PLT has its impact becomes a relatively less important factor for overall consumption volatility. This point is shown clearly by comparing the reductions in consumption volatility across old generations to the baseline reduction of approximately 40 per cent: there is only a 30 per cent reduction in the high productivity volatility case, but a 53 per cent reduction when the productivity innovation variance is low.

Interestingly, the impact of changing the productivity innovation variance is ambiguous for the 'low' and 'high' money supply innovation variance calibrations. For instance, when the money supply innovation variance is low, reducing the productivity innovation variance reduces the PLT welfare gain from 0.0153 per cent to 0.0151 per cent, whilst raising the productivity innovation variance increases the welfare gain to 0.0159 per cent. The reason for these seemingly odd results is that increasing the productivity innovation variance has a small first-order impact on the welfare gain through a relative increase in the equity premium under PLT. If this effect outweighs the impact on welfare through the second-order volatility channel, then the welfare gain from PLT can rise when the productivity innovation variance

increases, and *vice versa*.⁸⁹ In other words, increasing the productivity innovation variance can, for some calibrations, raise average consumption by old generations sufficiently to offset the reduced proportional impact of PLT on old generations' consumption risk. This trade-off between first- and second-order effects helps to explain why the overall impact of altering the productivity innovation variance is relatively small in all cases.

2.16 Summary of results and policy implications

Introducing capital into the OLG model reduces the baseline welfare gain from PLT to 0.026 per cent, as compared to 0.103 per cent in the model without capital. This lower baseline increase in aggregate consumption amounts to an increase in GDP of around 0.02 per cent. Based on UK data, this figure implies an aggregate gain of £227 million, £4.59 per employed member of the population, or £27.02 per pensioner.⁹⁰ The reasoning for the substantial reduction in the welfare gain from PLT is that with consumers' portfolios split between real and nominal assets, the relative importance of inflation risk (and hence PLT) for overall consumption volatility is reduced somewhat.

The estimated welfare gain is sensitive to the extent of risk aversion, but is more robust than in the model without capital. As the coefficient of relative risk aversion is varied between $3/2$ and 5, the estimated welfare gains remains in a range from 0.012 to 0.044 per cent. Similarly, the welfare gain from PLT is less sensitive in absolute terms to the extent of nominal volatility, though this too remains an important factor for the welfare gain. Interestingly, there was little sensitivity to the extent of productivity volatility or persistence. Based on these findings, the model with capital should provide not only a more plausible estimate of the welfare gain from PLT for most countries, but also a less uncertain one. Countries in which risk aversion is high and those with a high degree of nominal volatility have more to gain from PLT in the long run. The potential welfare gain from PLT may also vary over time with changes in risk aversion, or as performance under IT improves or deteriorates.

⁸⁹ The equity premium appears to increase more under PLT because capital holdings fluctuate more as a result of variations in expected inflation. By Jensen's inequality, such variations will increase the expected return on capital, since the latter is a convex function for the calibrated value of α .

⁹⁰ Again, these figures are based primarily on 2009 data in ONS (2010a), with the pensioner population figure based on 2007/8 data in DWP (2009).

To summarise, allowing consumers to invest in productive capital reduces but by no means eliminates the benefits from PLT, with old generations experiencing a reduction in consumption risk of almost 40 per cent. The results from the model with capital are likely to be most applicable for countries like Canada in which both real and nominal assets play an important role in retirement income (Meh and Terajima, 2008), but where a large proportion of public sector pensions are not indexed to prices and indexed government bonds are not widespread.⁹¹

The next chapter extends the OLG model even further by allowing consumers to hold government bonds that are indexed to inflation. This modification to the model is intended to increase its realism given that long-term assets are indexed to prices in many economies. The primary aim of Chapter 3 is to investigate the effect of this modification on the welfare and volatility impacts of PLT. Importantly, the degree of indexation of government bonds in this model is chosen in response to monetary policy as part of optimal commitment Ramsey policy in order to avoid the Lucas critique.

⁹¹ The share of indexed bonds in Canada was around 16 per cent in 2008 (Canada Department of Finance, 2008). Meh and Terajima (2008) document that as of 2005, 48.8 per cent of public sector pension plans were non-indexed defined benefit ones. They also note that middle-age and older households have substantial holdings of nominal bonds.

Appendix A – Proof that the CIA constraint is strictly binding when $R_t > 1$ ⁹²

It is shown in this appendix that the CIA constraint binds with strict equality if the gross money return on a nominal bond exceeds one.

Proposition: The CIA constraint binds with strict equality when $R_t > 1$

Proof.

From equations (2.15) and (2.16) and in the main text, the first-order conditions for nominal bonds and money holdings are as follows:

$$(A1) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) + \theta \mu_t$$

$$(A2) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) + (1 + \theta) \mu_t$$

where μ_t is the Lagrange multiplier on the CIA constraint.

The Kuhn-Tucker conditions associated with μ_t are as follows:

$$(A3) \quad \left\{ \mu_t \geq 0 \quad \text{and} \quad \mu_t (m_t^d - \theta c_{t,Y}) = 0 \right\}$$

where the second equation, the complementary slackness condition, implies that the CIA constraint will be strictly binding iff $\mu_t > 0$ for all t .

Using Equations (A1) and (A2), $\mu_t > 0$ for all t if and only if

$$(A4) \quad E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) > E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) \quad \forall t$$

Substituting for the return on nominal bonds yields the following necessary condition:

$$(A5) \quad R_t \times E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) > E_t(c_{t+1,O}^{-\delta} r_{t+1}^m) \quad \forall t$$

Clearly Inequality (A5) holds if and only if $R_t > 1$ for all t . **Q.E.D.**

⁹² This derivation is carried out for the model with only nominal bonds. However, adding capital and indexed bonds into the model does not change the necessary condition for the CIA constraint to bind, because all three assets must offer equivalent expected utility at the margin. Separate derivations are therefore not provided for the extended models.

Appendix B – The second-order approximation of lifetime utility

A second-order Taylor expansion of Equation (2.1) around time- t expected values gives the following result:

$$(B1) \quad u_t(c_{t,Y}, c_{t+1,O}) \approx u_{t,Y}(c_{t,Y}) + u_{t+1,O}(E_t c_{t+1,O}) + \frac{u''_{t+1,O}(E_t c_{t+1,O})}{2} \text{var}_t(c_{t+1,O})$$

where $u''(E_t c_{t+1,O})$ is the second derivative of utility in old age evaluated at $E_t c_{t+1,O}$, and $\text{var}_t(c_{t+1,O})$ is the conditional variance of consumption in old age.

Given that lifetime utility follows a constant relative risk aversion specification, Equation (B1) can be written in the following form:

$$(B2) \quad \text{Loss}_t \approx -\frac{1}{2} \left[\frac{\delta}{(E_t c_{t+1,O})^{1+\delta}} \right] \text{var}_t(c_{t+1,O})$$

where Loss_t is defined as the deviation of lifetime utility from the level of utility received if consumption levels are at their time- t expected values.⁹³

Note firstly that log-linearising the budget constraint in old age around the deterministic steady-state gives the following expression:

$$(B3) \quad c_{O,t+1,O} \hat{c}_{t+1,O} = r^n b^{n,d} \hat{r}_{t+1}^n + r^n b^{n,d} \hat{b}_t^{n,d} + r^m m^d \hat{m}_t^d + r^m m^d \hat{r}_{t+1}^m$$

where ‘hats’ denote percentage deviations from steady-state and time subscripts have been eliminated from steady-state values.

The log-linearised real returns on money balances and bonds in the above expression are given by

$$(B4) \quad \hat{r}_{t+1}^m = -\frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

⁹³ Note that in the case of consumption when young, $E_t c_{t,Y} = c_{t,Y}$ is the time- t expected value.

$$(B5) \quad \hat{r}_{t+1}^n = \hat{R}_t + \hat{r}_{t+1}^m = \hat{R}_t - \frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

where π^* is the steady-state rate of inflation over 30-years.

Equation (B3) can therefore be written in terms of inflation as follows:

$$(B6) \quad c_o \hat{c}_{t+1,o} = r^n b^{n,d} \hat{R}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^m m^d \hat{m}_t^d - \frac{c_o}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

where the fact that $c_o = r^n b^{n,d} + r^m m^d$ has been used.

Hence the *level* of consumption in old age is given by

$$(B7) \quad c_{t+1,o} = c_o + r^n b^{n,d} \hat{R}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^m m^d \hat{m}_t^d - \frac{c_o}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

Therefore, the expected level of consumption in old age is

$$(B8) \quad E_t c_{t+1,o} = c_o + r^n b^{n,d} \hat{R}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^m m^d \hat{m}_t^d - \frac{c_o}{1 + \pi^*} (E_t \pi_{t+1} - \pi^*)$$

It follows that the conditional variance of consumption in old age is given by

$$(B9) \quad \text{var}_t(c_{t+1,o}) = \frac{c_o^2}{(1 + \pi^*)^2} \text{var}_t(\pi_{t+1})$$

Hence the utility loss of generation t can be written as follows:

$$(B10) \quad \text{Loss}_t \approx -(1/2) \Phi_t \text{var}_t(\pi_{t+1})$$

where $\Phi_t \equiv \delta(c_o)^2 (1 + \pi^*)^{-2} (E_t c_{t+1,o})^{-(1+\delta)} > 0$ and $E_t c_{t+1,o}$ is given by Equation (B8).⁹⁴

⁹⁴ The term Φ_t is in general time-varying, but is strictly positive provided $E_t c_{t+1,o} > 0$.

Appendix C:

Steady state and market-clearing conditions in the nominal bonds model

Deterministic steady state

The deterministic steady state is given by the following system of equations:

$$(C1) \quad c_Y + b^{n,d} + m^d = \omega(1 - \tau^j), \quad j \in (IT, PLT)$$

$$(C2) \quad c_O = r^n b^{n,d} + r^m m^d$$

$$(C3) \quad R = (1 + \pi^*) r^n$$

$$(C4) \quad r^m = 1/(1 + \pi^*)$$

$$(C5) \quad g = \tau^j \omega + (1 - r^n) b^{n,s} + m^s \pi^*/(1 + \pi^*)$$

$$(C6) \quad m^d = m^s$$

$$(C7) \quad b^{n,d} = b^{n,s}$$

$$(C8) \quad b^{n,s} = \frac{\omega(1 - \tau^j) - (1 + r^m)m^d}{1 + r^n} \quad (\text{implied by } c_Y^{-\delta} = c_O^{-\delta})$$

$$(C9) \quad c_Y^{-\delta} = c_O^{-\delta} \left((1 + \theta)r^n - \frac{\theta}{1 + \pi^*} \right)$$

$$(C10) \quad r^n = \frac{1 + \theta + \pi^*}{(1 + \pi^*)(1 + \theta)}$$

(implied by the previous two equations)

Market-clearing conditions

A monetary equilibrium in the OLG economy is a set of allocations

$\{c_{t,Y}, c_{t,O}, b_t^{n,d}, b_t^{n,s}, m_t^d, m_t^s, g_t, \pi_t, r_t^n, R_t, r_t^m, \tau^j\}_{t=1}^T$ with the following properties for all t :

(1) Allocations $c_{t,Y}, c_{t,O}, b_t^{n,d}, m_t^d$ solve the maximisation problem of the young at time t ;

(2) The goods, money and bond markets clear:

$$(C11) \quad c_{t,Y} + c_{t,O} + g_t = \omega$$

$$(C12) \quad m_t^d = m_t^s$$

$$(C13) \quad b_t^{n,d} = b_t^{n,s}$$

(3) The government budget constraint and long run government spending target are satisfied:

$$(C14) \quad g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$$

$$(C15) \quad E(g_t) = g^*$$

(4) The CIA constraint holds with strict equality:

$$(C16) \quad m_t^d = \theta c_{t,Y}$$

Appendix D: Model listing for the model with nominal bonds

- (D1) $u_t(c_{t,Y}, c_{t+1,O}) = \frac{c_{t,Y}^{1-\delta}}{1-\delta} + E_t \frac{c_{t+1,O}^{1-\delta}}{1-\delta}$ Lifetime utility of generation t
- (D2) $c_{t,Y} + b_t^{n,d} + m_t^d = \omega(1 - \tau^j)$ Budget constraint when young, $j \in (IT, PLT)$
- (D3) $c_{t+1,O} = r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d$ Budget constraint when old
- (D4) $m_t^d = \theta_{t,Y}$ CIA constraint
- (D5) $r_{t+1}^m = 1/(1 + \pi_{t+1})$ Real return on money balances
- (D6) $r_{t+1}^n = R_t r_{t+1}^m$ Real return on nominal bonds
- (D7) $c_{t,Y}^{-\delta} = E_t (c_{t+1,O}^{-\delta} ((1 + \theta) r_{t+1}^n - \theta r_{t+1}^m))$ Euler equation for nominal bonds
- (D8) $g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$ Government budget constraint
- (D9) $E(g_t) = g^*$ Government spending target
- (D10) $\ln(m_t^s / m_{t-1}^s) = \pi^* + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t$ IT money supply rule
- (D11) $\ln(m_t^s / m_{t-1}^s) = \pi^* + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t$ PLT money supply rule
- (D12) $U^{society} = \frac{1}{T} E \left[\sum_{t=1}^T u_{t,Y}(c_{t,Y}) + \sum_{t=1}^T u_{t,O}(c_{t,Y}) \right]$ Social welfare
- (D13) $c_{t,Y}^{-\delta} = E_t (c_{t+1,O}^{-\delta})$ Bond supply rule
- (D14) $m_t^d = m_t^s$ Money market clearing
- (D15) $b_t^{n,d} = b_t^{n,s}$ Market-clearing in bonds
- (D16) $c_{t,Y} + c_{t,O} + g_t = \omega$ Goods market clearing

Appendix E – Estimation results for the RPI pensioner index and the CPI

This appendix reports estimation results over the sample period for two alternative UK price indices, the RPI for one-pensioner households and the Consumer Prices Index (CPI). Both series were obtained from the Office for National Statistics (ONS) Time Series Database at <http://www.statistics.gov.uk/statbase/tsdtimezone.asp>.

The following regression was estimated at a quarterly frequency q :

$$(E1) \quad \pi_q = c + \varepsilon_q$$

where π_q is quarterly inflation, defined as the log change in the aggregate price index between quarter q and $q-1$.

Table E1 – RPI pensioner regression results, 1997Q3-2010Q2

Parameter/Test	Result
c (s.e.)	0.007 (0.001)
Quarterly inflation standard deviation, i.e. $sd(\varepsilon_q)$	0.0074
Yearly inflation standard deviation	0.015
Dickey-Fuller unit-root test on $\ln(RPI)$ (prob. value)	-0.342 (0.987)
Jarque-Bera test on ε_q (prob. value)	6.38 (0.041)

Table E2 – CPI regression results, 1997Q3-2010Q2

Parameter/Test	Result
c (s.e.)	0.005 (0.001)
Quarterly inflation standard deviation, i.e. $sd(\varepsilon_q)$	0.0056
Yearly inflation standard deviation	0.011
Dickey-Fuller unit-root test on $\ln(CPI)$ (prob. value)	-0.751 (0.9996)
Jarque-Bera test on ε_q (prob. value)	17.33 (0.00)

Appendix F: The second-order approximation of lifetime utility with capital

As noted in Appendix B, a second-order Taylor expansion of Equation (1) around time- t expected values gives the following result:

$$(F1) \quad u_t(c_{t,Y}, c_{t+1,O}) \approx u_{t,Y}(c_{t,Y}) + u_{t+1,O}(E_t c_{t+1,O}) + \frac{u''_{t+1,O}(E_t c_{t+1,O})}{2} \text{var}_t(c_{t+1,O})$$

where $u''(E_t c_{t+1,O})$ is the second derivative of utility in old age evaluated at $E_t c_{t+1,O}$, and $\text{var}_t(c_{t+1,O})$ is the conditional variance of consumption in old age.

Given that utility is CRRA, Equation (F1) can be written in the following form:

$$(F2) \quad \text{Loss}_t \approx -\frac{1}{2} \left[\frac{\delta}{(E_t c_{t+1,O})^{1+\delta}} \right] \text{var}_t(c_{t+1,O})$$

where Loss_t is defined as the deviation of lifetime utility from the level of utility received if consumption levels are at their time- t expected values.

Note firstly that log-linearising the budget constraint in old age around the deterministic steady-state gives the following expression:

$$(F3) \quad c_{O,t+1,O} = Ak^\alpha \hat{A}_{t+1} + \alpha Ak^\alpha \hat{k}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^n b^{n,d} \hat{r}_{t+1}^n + r^m m^d \hat{m}_t^d + r^m m^d \hat{r}_{t+1}^m$$

where ‘hats’ denote percentage deviations from the deterministic steady-state and time subscripts have been removed from steady-state values.

The log-linearised real returns on money balances and bonds are respectively

$$(F4) \quad \hat{r}_{t+1}^m = -\frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

$$(F5) \quad \hat{r}_{t+1}^n = \hat{R}_t + \hat{r}_{t+1}^m = \hat{R}_t - \frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

where π^* is the steady-state rate of inflation.

Equation (F3) can therefore be written in terms of inflation as follows:

$$(F6) \quad c_{0,c_{t+1,0}} = Ak^\alpha \hat{A}_{t+1} + \alpha Ak^\alpha \hat{k}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^n b^{n,d} \hat{R}_t + r^m m^d \hat{m}_t^d - \frac{(r^n b^{n,d} + r^m m^d)}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

Thus the *level* of consumption in old age is given by

$$(F7) \quad c_{t+1,0} = c_0 + Ak^\alpha \hat{A}_{t+1} + \alpha Ak^\alpha \hat{k}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^n b^{n,d} \hat{R}_t + r^m m^d \hat{m}_t^d - \frac{(r^n b^{n,d} + r^m m^d)}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

Hence, the expected level of consumption in old age is given by

$$(F8) \quad E_t c_{t+1,0} = c_0 + Ak^\alpha \rho \hat{A}_t + \alpha Ak^\alpha \hat{k}_t + r^n b^{n,d} \hat{b}_t^{n,d} + r^n b^{n,d} \hat{R}_t + r^m m^d \hat{m}_t^d - \frac{(r^n b^{n,d} + r^m m^d)}{1 + \pi^*} (E_t \pi_{t+1} - \pi^*)$$

where the fact that $\hat{A}_{t+1} = \rho \hat{A}_t + e_{t+1}$ has been used in taking expectations.

It follows that the conditional variance of consumption in old age is given by

$$(F9) \quad \text{var}_t(c_{t+1,0}) = \left[\frac{r^n b^{n,d} + r^m m^d}{1 + \pi^*} \right]^2 \text{var}_t(\pi_{t+1}) + (k^\alpha)^2 \text{var}_t(A_{t+1})$$

Therefore, by Equation (F1), the utility loss of generation t can be written as a loss function in the conditional variances of inflation and productivity:

$$(F10) \quad \text{Loss}_t \approx -(1/2) \Psi_t [\text{var}_t(\pi_{t+1}) + \varsigma \text{var}_t(A_{t+1})]$$

where $\Psi_t \equiv \delta (r^n b^{n,d} + r^m m^d)^2 (1 + \pi^*)^{-2} (E_t c_{t+1,0})^{-(1+\delta)} > 0$, with $E_t c_{t+1,0}$ given by Equation (F8).⁹⁵ The constant coefficient $\varsigma \equiv (r^n b^{n,d} + r^m m^d)^2 (1 + \pi^*)^2 k^{2\alpha} > 0$ indicates the relative importance of productivity risk for the utility loss of generation t .

⁹⁵ The term Ψ_t is in general time-varying, but will be strictly positive so long as $E_t c_{t+1,0} > 0$.

Appendix G:

Steady-state and market-clearing conditions in the model with capital

Deterministic steady state

The deterministic steady state is given by the following set of equations:

$$(G1) \quad c_Y + b^{n,d} + m^d + k = \omega(1 - \tau^j), \quad j \in (IT, PLT)$$

$$(G2) \quad c_O = Ak^\alpha + r^n b^{n,d} + r^m m^d$$

$$(G3) \quad R = (1 + \pi^*)r^n$$

$$(G4) \quad r^m = 1/(1 + \pi^*)$$

$$(G5) \quad g = \tau^j \omega + (1 - r^n)b^{n,s} + m^s \pi^*/(1 + \pi^*)$$

$$(G6) \quad m^d = m^s$$

$$(G7) \quad b^{n,d} = b^{n,s}$$

$$(G8) \quad b^{n,s} = \frac{\omega(1 - \tau^j) - (1 + r^m)m^d - (1 + Ak^{\alpha-1})k}{1 + r^n} \quad (\text{implied by } c_Y^{-\delta} = c_O^{-\delta})$$

$$(G9) \quad c_Y^{-\delta} = c_O^{-\delta} \left((1 + \theta)r^n - \frac{\theta}{1 + \pi} \right)$$

$$(G10) \quad r^n = \frac{1 + \theta + \pi}{(1 + \pi)(1 + \theta)} \quad (\text{implied by the previous two equations})$$

$$(G11) \quad r^k = \alpha Ak^{\alpha-1} = r^n \quad (\text{implied by the Euler equation for capital})$$

$$(G12) \quad A = A_{mean}$$

Market-clearing conditions

A monetary equilibrium in the OLG economy is a set of allocations

$\{c_{t,Y}, c_{t,O}, b_t^{n,d}, b_t^{n,s}, k_t, m_t^d, m_t^s, g_t, \pi_t, r_t^n, R_t, r_t^m, \tau^j\}_{t=1}^T$ with the following properties for all t :

(1) Allocations $c_{t,Y}, c_{t,O}, b_t^{n,d}, m_t^d, k_t$ solve the maximisation problem of the young at time t ;

(2) The goods, money and bond markets clear:

$$(G13) \quad m_t^d = m_t^s$$

$$(G14) \quad c_{t,Y} + c_{t,O} + g_t + k_t = \omega + A_t k_{t-1}^\alpha$$

$$(G15) \quad b_t^{n,d} = b_t^{n,s}$$

(3) The government budget constraint and long run government spending target are satisfied:

$$(G17) \quad g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$$

$$(G18) \quad E(g_t) = g^*$$

(4) The CIA constraint holds with strict equality:

$$(G19) \quad m_t^d = \theta c_{t,Y}$$

Appendix H: Model listing for the model with nominal bonds and capital

- (H1) $u_t(c_{t,Y}, c_{t+1,O}) = \frac{c_{t,Y}^{1-\delta}}{1-\delta} + E_t \frac{c_{t+1,O}^{1-\delta}}{1-\delta}$ Lifetime utility of generation t
- (H2) $c_{t,Y} + b_t^{n,d} + m_t^d + k_t = \omega(1 - \tau^j)$ Budget constraint when young, $j \in (IT, PLT)$
- (H3) $c_{t+1,O} = A_{t+1}k_t^\alpha + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d$ Budget constraint when old
- (H4) $\ln A_t = (1 - \rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t$ Productivity
- (H5) $m_t^d = \theta c_{t,Y}$ CIA constraint
- (H6) $r_{t+1}^m = 1 / (1 + \pi_{t+1})$ Real return on money balances
- (H7) $r_{t+1}^n = R_t r_{t+1}^m$ Real return on nominal bonds
- (H8) $r_{t+1}^k = \alpha A_{t+1} k_t^{\alpha-1}$ Real return on capital
- (H9) $c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^n - \theta r_{t+1}^m \right) \right)$ Euler equation for nominal bonds
- (H10) $c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^k - \theta r_{t+1}^m \right) \right)$ Euler equation for capital
- (H11) $g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$ Government budget constraint
- (H12) $E(g_t) = g^*$ Government spending target
- (H13) $\ln(m_t^s / m_{t-1}^s) = \pi^* + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t$ IT money supply rule
- (H14) $\ln(m_t^s / m_{t-1}^s) = \pi^* + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t$ PLT money supply rule

$$(H15) \quad U^{society} = \frac{1}{T} E \left[\sum_{t=1}^T u_{t,Y}(c_{t,Y}) + \sum_{t=1}^T u_{t,O}(c_{t,Y}) \right] \quad \text{Social welfare}$$

$$(H16) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta}) \quad \text{Bond supply rule}$$

$$(H17) \quad m_t^d = m_t^s \quad \text{Money market clearing}$$

$$(H18) \quad b_t^{n,d} = b_t^{n,s} \quad \text{Market-clearing in bonds}$$

$$(H19) \quad c_{t,Y} + c_{t,O} + k_t + g_t = \omega + A_t k_{t-1}^\alpha \quad \text{Goods market clearing}$$

Chapter 3

Inflation versus price-level targeting in an OLG model with endogenous nominal indexation

3.1 Introduction

The analysis in Chapter 2 modelled public sector pensions and long-dated government bonds as pure nominal contracts. In practice, however, many countries issue inflation-indexed government bonds and offer public sector pensions that are indexed to inflation. The aim of this chapter is to extend the model analysed in Chapter 2 to include indexed government bonds, with the ultimate goal of testing the robustness of the conclusions reached therein.

Importantly, this extended model captures two imperfections of indexation that arise in practice – namely, ‘indexation bias’ and lagged indexation – and has the shares of indexed and nominal bonds in consumers’ portfolios chosen optimally in response to monetary policy. Both of these features are crucial since there are good reasons for thinking that optimal indexation will vary with a change in monetary policy regime from inflation targeting (IT) to price-level targeting (PLT), and also that imperfections in indexation will impact upon this. Indeed, recent research in the area of wage contracts has reached the conclusion that optimal indexation is significantly lower under PLT (Amano *et al.* 2007; Minford *et al.* 2003) and that failure to capture this effect can lead to the misleading welfare conclusions (see Minford and Peel, 2003). Similarly, Meh *et al.* (2008b) find that optimal indexation of financial contracts that are imperfectly indexed depends crucially on the extent of inflation uncertainty over the contracting horizon. In short, evaluating PLT against IT in the absence of endogenous indexation and/or under the implausible assumption of perfect indexation may give rise to misleading policy implications via the Lucas critique.

In this third chapter, background information is first presented on the importance of indexed government bonds and extent of indexation of public sector pensions. As in Chapter 2, the focus in this introductory section is on the G7 economies and the UK in particular. Past macroeconomic literature in the area of optimal indexation of government bonds is also briefly discussed. The extended overlapping generations (OLG) model including indexed bonds and endogenous nominal indexation is then introduced, and is subsequently simulated to evaluate the social welfare impact of

switching from IT to PLT. The results from this model specification are briefly discussed in relation to the findings of the previous chapter, and in the penultimate section of this chapter an extensive sensitivity analysis is conducted. The final section of the chapter discusses conclusions and policy implications, though a detailed numerical comparison of results across model specifications is left until Chapter 5, a short summary chapter that discusses overall findings and policy implications from the thesis.

3.2 Indexation and long-dated government bonds

Inflation-indexed government bonds have been issued in a total of 30 countries (Kitamura, 2009), including Australia, New Zealand, Sweden, and all of the G7 countries. There is, however, considerable variation in available maturities across countries. For example, indexed government bonds are available at maturities of 6-50 years in the UK, compared to maturities of 5, 10, 20 and 30 years in the US, and only 30 years in Canada. More generally, there is considerable heterogeneity across countries in terms of indexed bonds in issuance, including the dates at which such bonds were introduced. These points are highlighted clearly by Figure 3.1 and the cross-country comparison in Table 3.1.

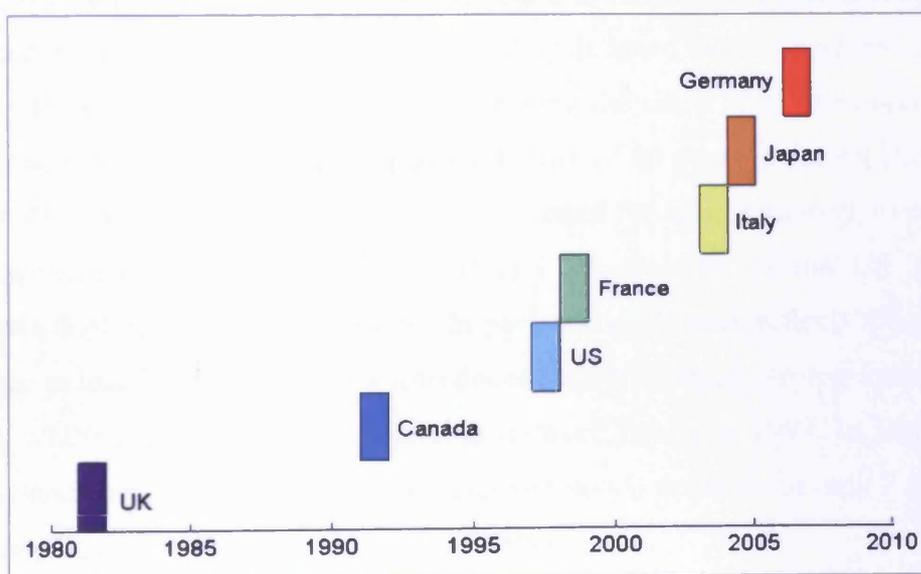


Figure 3.1 – Introduction of inflation-indexed bonds in G7 countries

Table 3.1 – Comparison of indexed bonds across the G7 countries

	Canada	France	Germany	Italy	Japan	UK	US
Price Index	CPI	HICP*	HICP*	HICP*	Core CPI	RPI	CPI-U
Indexation lag(months)	3	3	3	3	3	3 or 8	3
Available maturities (years)	30	4, 10, 15, 30	10	5, 6, 10, 30, 50	10	6-50	5, 10, 20, 30
Interest payment frequency	Twice a year	Once a year	Once a year	Twice a year	Twice a year	Twice a year	Twice a year
Share of indexed bonds (in 2008)	16%	15%	< 1%	6%	2%	30%	10%

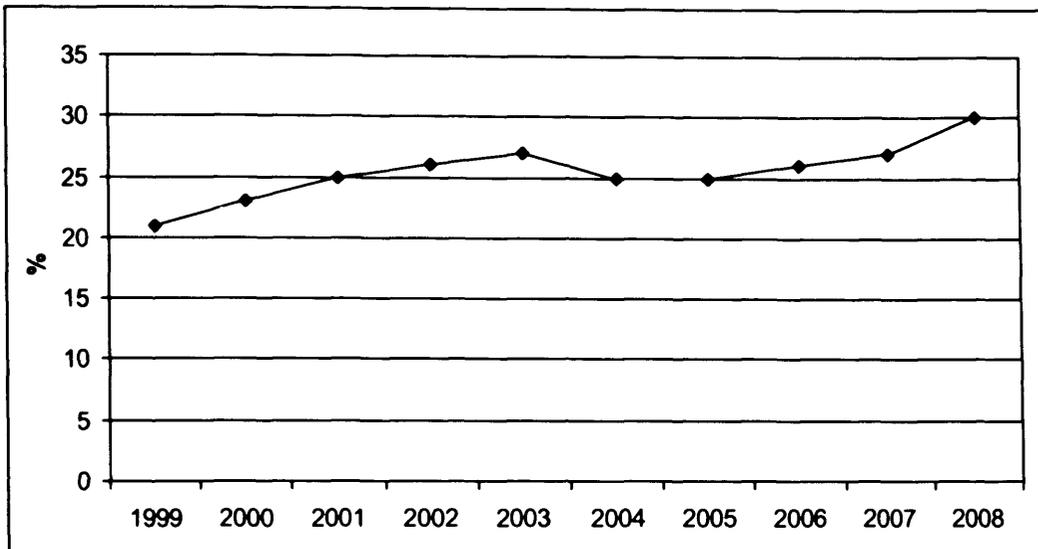
Source: Garcia (2008), Kitamura (2009), Agence France Trésor, Department of Finance Canada (2008), Department of Treasury (Italy). Notes: CPI-U – Urban Consumer Prices Index; HICP* – Harmonised Index of Consumers Prices, excluding tobacco.

The dates at which indexed bonds were introduced vary considerably across countries and span three full decades. Although indexed bonds account for a small fraction of government debt in most G7 economies, the share of indexed bonds has been growing steadily over the past decade or so, and often at a fast rate (Bekaert and Wang, 2010). For example, in Canada the stock of outstanding indexed bonds increased from \$4.1 billion in 1994 to \$17.3 billion in 2003, increasing the share of indexed bonds from 9 to 26 per cent for government debt with a maturity of 10 years or longer (Christensen *et al.* 2004).⁹⁶ As of 2008, indexed bonds accounted for approximately 30 per cent of British government stock (DMO, 2010a) and 10 per cent of the US Treasury's marketable debt (Campbell *et al.* 2009). In part, this difference reflects the passage of time since indexed bonds were first introduced: the UK began issuing indexed bonds in 1981, whilst the US only began issuing indexed bonds in 1997. In Japan, where indexed bonds were introduced in 2004, indexed bonds account for only 2 per cent of outstanding government bonds (Kitamura, 2009).

Figure 3.2 shows the general increase in the share of indexed bonds in the UK gilt portfolio over the past decade. The total share of indexed bonds in the UK portfolio of

⁹⁶ Given that Canada only issues indexed debt with a maturity of 30 years, the share in total government debt is somewhat lower.

is similar to the share of longer-dated nominal bonds – defined as those with a maturity of 15+ years – though it should be noted that indexed bonds are more important in relative terms at longer-term bond maturities (DMO, 2010b).



Source: DMO (2010a)

Figure 3.2 – Share of index-linked gilts in the UK bond portfolio

In practice, indexation of government bonds is imperfect for a number of reasons (see e.g. Fischer, 1996). Firstly, indexation is partially backward-looking due to publication lags for aggregate price indices. For instance, indexed bonds in Canada are indexed to the CPI from three months earlier, whilst the majority of outstanding indexed bonds in the UK have a longer indexation lag of eight months to the RPI. Second, aggregate price indices are intended to be representative of an ‘average household’ and therefore do not pick up price changes faced by particular groups in society like pensioners (Leicester, O’Dea and Oldfield, 2008).⁹⁷ As an example of this second point, the inflation rate for UK pensioners in 2008 (constructed using the RPI) was calculated at 7.4 per cent, or 2 per cent above the inflation rate faced by a typical household according to the RPI (Whitehouse, 2009). Thirdly, in the case of euro area countries, government bonds are indexed to the area-wide HICP (excluding tobacco) rather than a national price index. Finally, as currently formulated, price indices also cannot accurately capture quality improvements, leading to ‘quality-change bias’

⁹⁷ Recall that ‘old generations’ in the OLG model are interpreted as pensioners.

(Boskin Report, 1996). Given these imperfections, indexed government bonds are effectively nominal contracts that offer only partial protection against inflation.

3.3 Indexation and public sector pensions

Since public sector pensions are primarily 'defined benefit' pensions in OECD countries, they can also be interpreted as long-term nominal contracts. Of the 33 countries in the OECD, 17 currently index public sector pensions to inflation after retirement, that is, after they are in payment (Whitehouse, 2009). Prior to retirement, pensions are linked to earnings, which may or may not rise at the same rate as the general price level. As in the case of indexed bonds, indexation of pensions is imperfect due to publication lags and the fact that representative price indices will not capture some important price changes, leading to indexation bias. It is also notable that in countries in which public sector pensions are indexed, discretionary deviations from indexation have been frequent in practice at times of low inflation or deflation (Whitehouse, *op. cit.*), and that 5 of the 17 OECD countries that index pensions do so to a mixture of price and earnings indices, rather than a single price index. To summarise, public sector pensions – like indexed bonds – are nominal contracts that offer only imperfect insurance against inflation risk. As with inflation-indexed bonds, the extent to which indexation is imperfect will depend primarily on the 'bias' in the price index used for indexation and the length of the indexation lag.

Table 3.2 provides a comparison of public sector pension schemes across the G7 countries. There is some heterogeneity in terms of the importance of public pension spending across countries; and, although pensions are indexed to prices in all countries except Germany, the indices used differ across countries. Public sector pensions are overwhelmingly defined benefit (DB) in the G7 countries, but in both France and Germany pension payments depend additionally on points that are accumulated based on contributions in individual years.

Table 3.2 – Public sector pension schemes in the G7 countries

	Canada	France	Germany	Italy	Japan	UK	US
Indexation	Yes	Yes	No*	Yes**	Yes	Yes	Yes
Price index	CPI and COLA***	HICP	-	HICP	CPI	RPI	COLA
Public pension spending (% GDP, 2006)	4.1%	12.4%	11.4%	14.0%	8.7%	5.7%	6.0%
Type of scheme	DB	DB	DB	NDC	DB	DB	DB

Notes: *Public sector pensions are indexed to wages in Germany. ** Indexation is only partial for pensions with benefits more than three times the minimum pension. DB = defined benefit; NDC = notional accounts. *** COLA = Cost of living adjustment.

Source: OECD (2009a).

There are some striking similarities in indexation of government bonds and public sector pensions – for instance, the same price indices are often used, and Germany, where public sector pensions are not indexed, did not begin issuing indexed government bonds until 2006 and has an extremely small market in these assets (see Garcia, 2006). In the UK, there are more specific similarities since both public sector pensions and index-linked gilts are indexed to the RPI with an eight-month lag.⁹⁸

3.4 Optimal indexation of government bonds: a brief review

The motivation for studying the link between optimal indexation and monetary policy can be traced back to the seminal papers by Fischer (1975), Gray (1976), and Levhari and Liviatan (1977). Gray (1976) focused on optimal indexation of wage contracts. Amongst other things, she showed that optimal indexation of wages depends on the relative variances of real and nominal disturbances, increasing with the nominal-to-real volatility ratio. Indexation of wages should therefore be higher under monetary policy regimes that raise nominal volatility – a prediction that appears to be borne out by the data (Holland, 1986). Optimal indexation of wages has been investigated more recently by Minford *et al.* (2003) and Amano *et al.* (2007), who investigate optimal indexation in fully-specified DSGE models. Both papers examine the implications of PLT for optimal indexation, motivated by the theoretical result that PLT reduces nominal volatility substantially compared to IT at medium and long-term horizons.

⁹⁸ More specifically, the majority of outstanding index-linked gilts are indexed to the RPI. Since 2005, gilts with an indexation lag of 3 months have also been issued.

Consistent with the finding in Gray (1976), both papers report that optimal indexation is substantially lower under PLT than IT because nominal volatility is much lower over the wage-contracting horizon.

Fischer (1975) and Levhari and Liviatan (1977) both focus on the optimal demand for indexed bonds using a portfolio approach. Levhari and Liviatan note that if indexation is perfect and if the only source of uncertainty in the economy is inflation (i.e. nominal risk), risk-averse consumers will demand only indexed bonds, such that full indexation is optimal. The economic intuition here is straightforward: if indexed bonds are a perfect store of value and offer the same expected return as nominal bonds, consumption risk will be minimised by holding only indexed bonds. In such an economy, nominal bonds would only be held only if inflation uncertainty were removed entirely. A second important finding from a portfolio approach to indexed bonds is that full indexation is not optimal if indexation is imperfect, because holding nominal bonds will diversify consumption risk if the correlation between inflation and the 'indexation error' is sufficiently small. In this case, the demand for indexed bonds will rise with the level of inflation risk, producing a direct link between monetary policy and optimal bond indexation.⁹⁹

Fischer (1975) studied demand for indexed bonds in the presence of real risk. In particular, he focuses on the optimal demands for indexed and nominal bonds in a model in which households receive income from human capital and choose a portfolio consisting of equity, nominal bonds and indexed bonds. He notes that if inflation is correlated with the real return on human capital (or, more generally, with other sources of labour or non-labour income), it is optimal to hold both nominal and indexed bonds, since nominal bonds will enable households to hedge risk whereas perfectly indexed bonds cannot.

An important criticism that can be levelled at the portfolio approach to indexed bonds is that it ignores the supply-side of the market and thus fails to provide an equilibrium solution to the optimal indexation problem. This consideration is an important one in the context of government bonds because, in an equilibrium model, government

⁹⁹ This result can be derived formally by setting up a simple portfolio problem with indexed and nominal bonds under the conditions described.

behaviour should be constrained by its budget constraint.¹⁰⁰ Indeed, when the government must finance bond issuance subject to its budget constraint, nominal bonds are useful if the government is required to balance its budget in each period (Levhari and Liviatan, 1976), or if inflation is correlated with the tax burden (Bohn, 1988). This literature, however, focuses only upon necessary conditions for partial indexation of government bonds to be optimal. In the model simulated in this chapter, the optimal share of indexed government debt is computed directly. Moreover, the model relaxes two implausible assumptions maintained in previous literature, namely, that bonds are perfectly indexed, and that bond risk-premia are equal to zero.¹⁰¹

3.5 The extended model with indexed bonds

This section introduces the extended OLG model in which consumers can also hold long-term indexed government bonds and where the shares of indexed and nominal bonds in consumer portfolios are chosen optimally in response to monetary policy. Based on the above discussion, indexed government bonds are modelled as imperfectly-indexed due to indexation bias and lagged indexation, and therefore do not offer a certain real return *ex post*. The model builds directly upon the model with capital and nominal bonds that was introduced in the second half of Chapter 2. More specifically, the basic structure of the model is unchanged, but the government budget constraint and young and old generations' budget constraints are amended to include indexed government bonds. The monetary policy rules under IT and PLT are identical to the previous chapter and are therefore not repeated here.

In real terms, the budget constraint faced by young consumers is now given by

$$(3.1) \quad c_{t,Y} + b_t^{n,d} + b_t^{i,d} + m_t^d + k_t = \omega(1 - \tau^j)$$

where $b_t^{i,d}$ denotes demand for indexed government bonds in real terms.

Indexed bonds pay an *ex ante* riskless real return r_t which is endogenously determined.¹⁰² However, due to indexation bias and lagged indexation, the *ex post* real return on an indexed bond will generally differ from the riskless return. In particular,

¹⁰⁰ Campbell and Shiller (1996) provide a useful discussion of the impact of introducing indexed bonds on government financing costs (in the context of the US economy).

¹⁰¹ Bond risk-premia are zero in linear or log-linearised models. Alternatively, it is sometimes assumed that marginal utility is linear so that consumers are risk-neutral (e.g. Bohn, *op. cit.*).

¹⁰² The return r_t ensures that the market for indexed bonds clears.

the *ex post* real return on an indexed bond held from period t to period $t + 1$ is given by

$$(3.2) \quad r_{t+1}^i = r_t \times \left[\frac{(1 + \pi_{t+1}^{ind})}{(1 + \pi_{t+1})} + v_{t+1} \right]$$

where π^{ind} is the biased rate of inflation to which indexed bonds are linked, π is the true rate of inflation and v_t is a Gaussian ‘white noise’ innovation whose standard deviation σ_v is based on the indexation lag length.

The *ex post* real return on indexed bonds in Equation (3.2) is not riskless due to indexation bias and lagged indexation. The first term in square brackets reflects indexation bias and will deviate from one if ‘true’ and ‘biased’ inflation are not equal, indicating that the price index used for indexation differs from the true one that defines consumers’ standard of living. In the UK, for example, index-linked gilts are indexed to the Retail Prices Index (RPI), whereas the Retail Prices Index excluding mortgage interest payments (RPIX) may better reflect the inflation rate faced by the majority of pensioners (i.e. ‘old generations’), who do not make mortgage repayments (Leceister, O’Dea and Oldfield, 2008). The magnitude of indexation bias in the model depends on the correlation between true and biased inflation and also upon the variance ratio.

The second term in square brackets in Equation (3.2) captures the impact of lagged indexation on the *ex post* real return on indexed bonds. The indexation lag is motivated by the presence of data publication and collection lags which are responsible for indexation occurring with a lag in practice. The indexation lag is modelled by a ‘white noise’ innovation v_t , because this methodology provides a simple way to capture volatility arising from lagged indexation when the indexation lag length is small relative to the holding period.¹⁰³ This innovation is assumed to be exogenous and invariant to monetary policy, reflecting the assumption that the indexation lag length and the return risk associated with this lag are not affected by a shift in monetary policy regime.¹⁰⁴

¹⁰³ It does not make sense to index partially to past inflation, as each period in the model lasts 30 years.

¹⁰⁴ The reasoning here is that because indexation lags are somewhat shorter than one year in developed economies, the inflation ‘missed’ due to the indexation lag would not differ under IT and PLT given that inflationary shocks occur at a yearly horizon.

Consumption by old generations is now given by

$$(3.3) \quad \begin{aligned} c_{t+1,O} &= A_{t+1}k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d \\ &= A_{t+1}k_t^\alpha + (ar_{t+1}^i + (1-a)r_{t+1}^n)b_t^d + r_{t+1}^m m_t^d \end{aligned}$$

where a is the share of indexed bonds in consumers' bond portfolios and $b^d \equiv b^{i,d} + b^{n,d}$ is total demand for government bonds.

In order to gain some formal intuition for the social welfare impact of introducing imperfectly-indexed government bonds into the model, the model was log-linearised and substituted into a second-order Taylor expansion of the lifetime utility of generation t .

Appendix A of this chapter shows that this process gives rise to the following expression:

$$(3.4) \quad Loss_t \approx -(1/2)\chi_t \left[\begin{aligned} &\text{var}_t(\pi_{t+1}) + \psi \text{var}_t(\pi_{t+1}^{ind}) - 2\Lambda \text{cov}_t(\pi_{t+1}, \pi_{t+1}^{ind}) \\ &+ \eta\sigma_v^2 + \Omega \text{var}_t(A_{t+1}) \end{aligned} \right]$$

where $Loss_t$ is the deviation of lifetime utility from its time- t expected value; $\chi_t > 0$ is a coefficient that depends on the model's steady-state and expected consumption in old age; $\psi, \Lambda, \eta > 0$ are constant coefficients that depend upon the relative importance of indexed bonds in consumer portfolios, and $\Omega > 0$ is a coefficient that represents the relative importance of capital.

Equation (3.4) shows that the welfare loss for generation t depends on the variances of actual and indexed inflation; the covariance between actual and indexed inflation; real return volatility arising due to the indexation lag; and the productivity variance. Intuitively, holding indexed bonds will expose consumers to some long-term inflation risk because indexation is less than perfect.¹⁰⁵ The covariance between actual and indexed inflation enters the expression for the welfare loss with the opposite sign to the variance terms because indexed bonds are a partial hedge against inflation so long as actual and indexed inflation are positively correlated. If this is the case, then introducing indexed bonds should reduce the importance of long-term inflation risk

¹⁰⁵ If indexation is perfect, then $\pi^{ind} = \pi$ and $\sigma_v = 0$, in which case $\psi = \Lambda = 0$ and indexed bonds holdings do not contribute to the welfare loss of generation t .

for old generations' consumption: consumers will be able to protect themselves against inflation risk by holding indexed bonds without any corresponding increase in risk elsewhere in their savings portfolios.

Given that monetary policy is unchanged in the extended model, the expressions for actual inflation under IT and PLT are identical to those given in Chapter 2. However, the biased rate of inflation π^{ind} to which indexed bonds are linked is an exogenous process whose functional form must be specified. In the model, it is assumed to be given by a stochastic process that has the same functional form as true inflation – in particular, π^{ind} has the same long run mean as true inflation and depends on only current innovations under IT but responds to current and past innovations under PLT. As a result, the variance of biased inflation is also 15 times lower under PLT than IT.

The biased inflation rate used for indexation is given by

$$(3.5) \quad \left. \begin{aligned} \pi_t^{ind,IT} &= 30 \times \pi + \sum_{i=1}^{30} \varepsilon_{i,t}^{ind} \\ \pi_t^{ind,PLT} &= 30 \times \pi + \varepsilon_{30,t}^{ind} - \varepsilon_{30,t-1}^{ind} \end{aligned} \right\} \begin{array}{l} \text{under IT} \\ \text{under PLT} \end{array}$$

where $\varepsilon_{i,t}^{ind} \sim N(0, \sigma_{ind}^2)$ is the biased inflation innovation in year i of period t .

The inflationary innovations in this equation are assumed to be serially-uncorrelated. However, they are contemporaneously cross-correlated with innovations to true inflation, with the strength of the correlation reflecting the extent of indexation bias. Both this cross-correlation and the innovation variance for biased inflation are estimated below using UK data, consistent with the calibration of other aspects of the model undertaken in Chapter 2.

3.5.1 Consumers' first-order conditions

Consider the following expected Lagrangian:

$$(3.6) \quad L_t \equiv E_t \left[\begin{aligned} &u_t(c_{t,Y}, c_{t+1,O}) + \lambda_{t,Y} (\omega(1 - \tau^j) - m_t^d - b_t^{i,d} - b_t^{n,d} - k_t - c_{t,Y}) \\ &+ \mu_t (m_t^d - \theta c_{t,Y}) + \lambda_{t+1,O} (A_{t+1} k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d - c_{t+1,O}) \end{aligned} \right]$$

where $\lambda_{t,Y}$ ($\lambda_{t+1,O}$) is the Lagrange multiplier on young (old) consumers' budget constraint, and μ_t is the Lagrange multiplier on the CIA constraint.

First-order conditions are as follows:

$$(3.7) \quad c_{t,Y} : c_{t,Y}^{-\delta} = \lambda_{t,Y} + \theta\mu_t$$

$$(3.8) \quad c_{t+1,O} : \lambda_{t+1,O} = c_{t+1,O}^{-\delta}$$

$$(3.9) \quad b_t^{i,d} : \lambda_{t,Y} = E_t(\lambda_{t+1,O} r_{t+1}^i)$$

$$(3.10) \quad b_t^{n,d} : \lambda_{t,Y} = E_t(\lambda_{t+1,O} r_{t+1}^n)$$

$$(3.11) \quad m_t^d : \lambda_{t,Y} = E_t(\lambda_{t+1,O} r_{t+1}^m) + \mu_t$$

$$(3.12) \quad k_t : \lambda_{t,Y} = E_t(\lambda_{t+1,O} \alpha A_{t+1} k_t^{\alpha-1})$$

Substituting out the Lagrange multipliers on budget constraints when young and old gives the following consumption Euler equations for indexed bonds, nominal bonds and capital:

$$(3.13) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^i) + \theta\mu_t$$

$$(3.14) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^n) + \theta\mu_t$$

$$(3.15) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta} r_{t+1}^k) + \theta\mu_t$$

where $r_{t+1}^k \equiv \alpha A_{t+1} k_t^{\alpha-1}$ is the real return on capital.

The Lagrange multiplier on the CIA constraint is given by

$$(3.16) \quad \mu_t = E_t(c_{t+1,O}^{-\delta} (r_{t+1}^i - r_{t+1}^m)) = E_t(c_{t+1,O}^{-\delta} (r_{t+1}^n - r_{t+1}^m)) = E_t(c_{t+1,O}^{-\delta} (r_{t+1}^k - r_{t+1}^m))$$

Intuitively, the first equality states that, absent uncertainty, the CIA constraint will be strictly binding (i.e. $\mu_t > 0$ for all t) if money is rate of return dominated by indexed bonds, whilst the second and third equalities indicate that the same relationship holds for nominal bonds versus money, or capital versus money, because all three assets must give equivalent expected utility (at the margin) in order to be held.

Substituting out for the Lagrange multiplier in equations (3.13) to (3.15) we have the following Euler equations in terms of consumption and asset returns:

$$(3.17) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^i - \theta r_{t+1}^m \right) \right)$$

$$(3.18) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^n - \theta r_{t+1}^m \right) \right)$$

$$(3.19) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^k - \theta r_{t+1}^m \right) \right)$$

3.5.2 Government and monetary policy

With the government issuing both indexed and nominal debt, the government budget constraint in real terms is given by

$$(3.20) \quad g_t = \tau^j \varpi + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$$

where $b^{i,s}$ is the real supply of indexed bonds issued by the government.

The government continues to set the tax rate on young consumers' endowment incomes to achieve a long run target level of government spending of $E(g_t) = g^*$. Analogous to the model with only nominal bonds, the total bond supply – now $b^s \equiv b^{i,s} + b^{n,s}$ – is set to ensure that the marginal utility of consumption in youth is equated with the expected marginal utility of consumption in old age, thus ensuring consumption-smoothing between youth and old age so far as is possible.

The division of the total bond supply between indexed and nominal bonds, as defined by an indexation share a , is chosen optimally by the government to maximise social welfare. More specifically, the share of indexed bonds is chosen in response to monetary policy, taking into account consumers' first-order conditions and the necessity of achieving the long run government spending target. The optimal indexation decision faced by the government is therefore an example of an optimal commitment Ramsey policy (e.g. Ljungqvist and Sargent, 2000). Individual bond supplies are constrained to be non-negative for all t , such that the optimal indexation share will lie in the range $[0,1]$. The money supply rules to which the government commits under IT and PLT are unchanged and are therefore identical to those derived in Chapter 2. For completeness, the model's deterministic steady state and market-

clearing conditions are given in Appendix B, along with a full model listing in Appendix C.

3.6 Calibrating the inflation rate used for indexation

The calibrations for the money supply rules and productivity are identical to those given in the previous chapter. However, the inflation rate used for indexation is an additional exogenous process that needs to be calibrated before the model can be solved. Given that both UK public sector pensions and indexed-linked gilts are linked to the Retail Prices Index (RPI), the RPI was chosen to calibrate the inflation rate π^{ind} to which indexed bonds are linked. There are two parameters that need to be estimated: the yearly inflation innovation variance and the correlation between innovations to π^{ind} and innovations to true inflation π .

In order to do so, the same regression methodology is followed as in the previous chapter, again using quarterly data from 1997Q3 to 2010Q2 obtained from the Office for National Statistics (ONS).¹⁰⁶ The estimated yearly innovation variance was used to calibrate $\pi^{ind,IT}$ and $\pi^{ind,PLT}$ using the specifications in Equation (3.5), whilst the estimated correlation between the RPI and RPIX regression residuals (over the sample period) was used to calibrate the contemporaneous correlation between innovations to true and biased inflation.

The following regression equation was thus estimated at a quarterly horizon q by ordinary least squares (OLS):

$$(3.21) \quad \pi_q = c + \varepsilon_q$$

where π_q is the log first-difference of the quarterly RPI.

The results from this regression are shown in Table 3.3. The mean quarterly rate of RPI inflation is identical to the estimate for RPIX inflation at 0.007, or 0.7 per cent per quarter. Moreover, this estimate is not statistically different, at the 5 per cent significance level, from the quarterly rate of inflation of 0.0625 implied by the 2.5 per cent annual inflation target in the money supply rule. These estimation results are

¹⁰⁶ Again the data were downloaded from the ONS Time Series Database at <http://www.statistics.gov.uk/statbase/tsdtimezone.asp>.

therefore consistent with the assumption that true and biased inflation are identical on average.

Table 3.3 – RPI regression results, 1997Q3-2010Q2

Parameter/test	Result
c (s.e.)	0.007 (0.001)
Quarterly inflation standard deviation, i.e. $sd(\varepsilon_q)$	0.0073
RPI-RPIX inflation correlation, i.e. $corr(\pi_q^{RPI}, \pi_q^{RPIX}) = corr(\varepsilon_q^{RPI}, \varepsilon_q^{RPIX})$	0.89
Dickey-Fuller unit-root test on $\ln(RPI)$ (prob. value)	-1.380 (0.85)
Jarque-Bera test on ε_q (prob. value)	30.58 (0.00)

The quarterly standard deviation of RPI inflation over the sample period was 0.0073 (or 0.73 per cent), which is slightly higher than the quarterly standard deviation for the RPIX. The variance of yearly innovations to biased inflation was estimated using the residuals from the above regression. In particular, based on the estimated quarterly innovation variance, the yearly variance was calculated under the assumption that there is a unit root in the price level (as implied Dickey-Fuller unit root test result reported in the fifth row of results in Table 3.3¹⁰⁷). This yearly variance was taken as the calibrated innovation variance for biased inflation in model simulations.

Figure 3.3 compares RPI and RPIX inflation over the sample period. In general, the two series are strongly positively correlated, though there are some non-trivial deviations in the middle and at the end of the sample period, such that the correlation coefficient over the sample period was equal to 0.89. This correlation was taken as the contemporaneous correlation between innovations to true inflation and biased inflation, and was therefore used as a basis for calibrating the covariances between innovations to actual and biased inflation (under both IT and PLT).¹⁰⁸ Overall, the results in this section suggest a relatively small amount of indexation bias, since innovations to RPIX and RPI inflation are closely correlated and have similar

¹⁰⁷ An intercept and trend were included in the test regression.

¹⁰⁸ Note that for correlated random variables X and Y , $cov(X, Y) = corr(X, Y) \times sd(X) \times sd(Y)$.

variances. For completeness, Table 3.4 lists calibrated values in the stochastic process for biased inflation and the money supply rules.

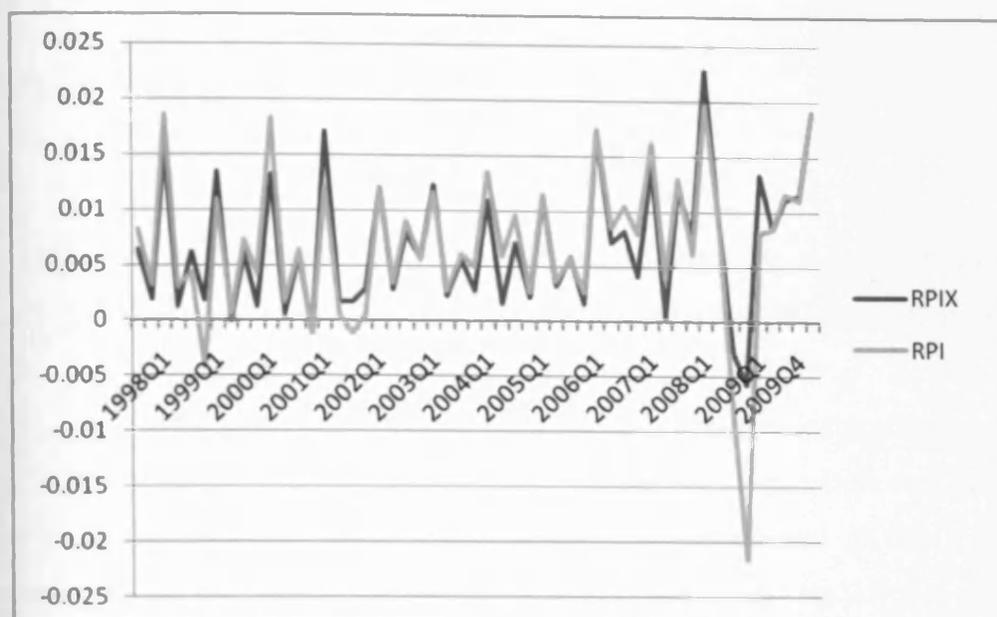


Figure 3.3 - RPI and RPIX inflation over the sample period

Table 3.4 – Calibrated values in money supply rules and biased inflation

Model parameter	Role in the model	Calibrated value
$30 \times \pi$	Inflation target over 30 years	0.75
$\text{var}(\varepsilon_{i,t})$	Yearly money supply innovation variance	1.45×10^{-4}
$\text{var}(\varepsilon_{i,t}^{ind})$	Yearly biased inflation innovation variance	2.13×10^{-4}
$\text{cov}(\varepsilon_{i,t}, \varepsilon_{i,t}^{ind})$	Yearly covariance between innovations to true and biased inflation	1.56×10^{-4}

3.7 Calibrating the indexation lag

The white noise innovation v_t enters the real return on indexed bonds in order to capture the impact of an indexation lag on the *ex post* real return on indexed bonds. Therefore, in calibrating its variance, a number of points should be borne in mind. First, given the specification of the real return on indexed bonds, this innovation should have the same units as the term in inflation that it appears in brackets alongside. Hence v_t is interpreted as the impact of the indexation lag, in inflation percentage points, on the inflation-indexed component of an indexed bond. Second, the variance of v_t should reflect the volatility of the inflation rate to which indexed bonds are linked, measured over a horizon defined by the length of the indexation lag.

Given that the indexation lag on UK public sector pensions and the majority of outstanding index-linked gilts is 8 months, this variance was estimated using the rate of RPI inflation over a three-quarter horizon.¹⁰⁹

The following regression was thus estimated:

$$(3.22) \quad \Delta\pi_{q;q-3} = c + \varepsilon_{q;q-3}$$

where $\Delta\pi_{q;q-3}$ is the differential between RPI inflation in quarter q and RPI inflation in quarter $q-3$, and $\varepsilon_{q;q-3}$ is a regression residual and the empirical counterpart to v_t .

Table 3.5 shows the regression results. The constant term is insignificant, offering support to the assumption that v_t is mean zero, though the Jarque-Bera test marginally rejects the assumption that the residual is normally-distributed at the 1 per cent significance level. The regression residual standard deviation was 0.0121, or 1.2 per cent over three quarters. The variance for v_t was therefore calibrated at $0.0121^2 = 0.000146$.

Table 3.5 – RPI indexation lag regression results, 1997Q3-2010Q2

Parameter/test	Result
c (s.e.)	0.0002 (0.0017)
Standard deviation of the residual, i.e. $sd(\varepsilon_{q;q-3})$	0.0121
Jarque-Bera test on $\varepsilon_{q;q-3}$ (prob. value)	10.80 (0.005)

3.8 Full model calibration

The calibration of model parameters is identical to that in the model with capital in the previous chapter. For ease of reference, these calibrated values are repeated in Table 3.6.

¹⁰⁹ Using 3 quarters (9 months) meant that the same quarterly RPI data could be used in estimation.

Table 3.6 – Model calibration

Model parameter	Role in the model	Calibrated value
θ	Proportion of consumption when young held as money	0.10
δ	Coefficient of relative risk aversion	3
ω	Endowment income of young consumers	1.641
g^*	Long run government spending target	0.40
α	Elasticity of output in old age to capital	0.375
ρ	Persistence in productivity over a 30-year horizon	0.618

Table 3.7 reports the steady-state values of key variables under the baseline calibration of the model. The deterministic steady-state is identical to that in the model with capital and nominal bonds. Intuitively, the relative supplies of indexed and nominal bonds are irrelevant for the steady-state because real returns on bonds equalised in the absence of uncertainty. As previously, steady-state aggregate consumption accounts for 73 per cent of GDP, capital holdings (investment) for 7 per cent, and government spending for the remaining 20 per cent. Total bond holdings are 17 per cent of GDP, and money holdings are equal to 3.7 per cent of GDP. Steady-state inflation is again equal to the 30-year inflation target of 0.75, that is, a 75 per cent increase in the price level over a 30-year horizon.

Table 3.7 – Key variables at steady state

Model variable	Steady-state value
$c_{t,Y}$	0.730
$c_{t,O}$	0.730
$b_t^d (= b_t^s)$	0.343
$m_t^d (= \theta c_{t,Y} = m_t)$	0.073
k_t	0.140
π_t	0.75

Note: Steady-state GDP is equal to 2

3.9 Optimal indexation

The government chooses the shares of indexed to nominal bonds to maximise social welfare, subject to: its budget constraint and long run target level of government

spending; consumers' first-order conditions for optimal saving (i.e. the Euler equations for indexed bonds, nominal bonds and capital holdings); monetary policy; and the model's other equilibrium conditions. Consequently, the policy being studied is a Ramsey policy: the government can commit and takes into account the optimal responses of consumers when making its optimal indexation choice.

Since the aim is to solve for the unconditionally-optimal share of indexed bonds, it is sufficient to solve the model to second-order. Indeed, Samuelson (1970) formalised a general principle that it is necessary to approximate a portfolio problem up to order $N + 2$ in order to solve for the N th-order component of the portfolio problem. In the model at hand, we are looking for the zero-order component of the government bond portfolio, because we are looking for the constant share that is optimal in the long run – that is, the very first term in a Taylor series approximation of the optimal portfolio share. Intuitively, a second-order approximation is necessary, since a first-order approximation will not capture the risk characteristics of bonds due to certainty equivalence.¹¹⁰

Given that the model is solved using a second-order approximation, the optimal indexation problem faced by the government can be formulated using a second-order Taylor expansion of social welfare around (unconditional) mean consumption levels. In fact, Samuelson (1970) derived his results by approximating the utility function, though Devereux and Sutherland (2011) note that approximating first-order conditions produces identical results for the zero-order component of a portfolio problem.

The optimal indexation problem can thus be stated as follows:

$$(3.23) \quad \max_a U^{society} \approx \left(\frac{(Ec_{t,Y})^{1-\delta} + (Ec_{t,O})^{1-\delta}}{1-\delta} \right) - \frac{1}{2} \left(|U_{c_Y c_Y}^{society}| \text{var}(c_{t,Y}) + |U_{c_O c_O}^{society}| \text{var}(c_{t,O}) \right)$$

where $U_{c_Y c_Y}^{society} = -\delta(Ec_{t,Y})^{-(1+\delta)}$ and $U_{c_O c_O}^{society} = -\delta(Ec_{t,O})^{-(1+\delta)}$,

¹¹⁰ See Devereux and Sutherland (2011) for a more recent application in the context of open economy DSGE models.

subject to:

the government spending target $E(g_t) = g^*$; the government budget constraint; the money supply rule in place; and the model's other equations and equilibrium conditions (as listed in appendices B and C of this chapter).

In order to gain some intuition for the factors driving optimal indexation, we can consider a first-order Taylor expansion of the first term on the right hand side of Equation (3.23) around the deterministic steady-state of the model.¹¹¹

Using this approximation results in the following social welfare criterion:¹¹²

$$(3.24) \quad \max_a U^{society} \approx c_0^{1-\delta} \left(E c_{t,Y} + E c_{t,O} + \frac{2\delta \times c_0}{1-\delta} \right) - \frac{1}{2} \left(\left| U_{c_Y c_Y}^{society} \right| \text{var}(c_{t,Y}) + \left| U_{c_O c_O}^{society} \right| \text{var}(c_{t,O}) \right)$$

where $E c_{t,Y} + E c_{t,O}$ is the long run average level of aggregate consumption.

The goods market-clearing condition can then be used to show that the average level of aggregate consumption $E c_{t,Y} + E c_{t,O}$ is approximately invariant to a change in monetary policy from IT to PLT. Indeed, taking the unconditional expectations operator through the market-clearing condition gives $E c_{t,Y} + E c_{t,O} = \omega - g^* + E(A_t(k_t)^{\alpha} - k_t)$, which is approximately invariant to the indexation share a , since the real return on capital is uncorrelated with real bond returns. The reasoning for the indexation choice not affecting the average level of aggregate consumption is that the government must meet its long run government spending target $E g_t = g^*$ regardless of the indexation share that is chosen. Hence, for example, if nominal bonds have a higher expected return than indexed bonds, a marginal reduction in indexation would, *ceteris paribus*, increase average consumption by old generations. However, reducing indexation would also reduce average government spending, because the average cost of issuing government debt would rise. Therefore, in order to meet the long run

¹¹¹ This approximation is intended only to provide intuition for the results that follow. In model simulations, the expression for social welfare is evaluated fully to second-order (along with the rest of the model). Equation (3.24) should provide a good approximation if risk aversion is moderate or low, because the utility function will not have strong curvature.

¹¹² This expression makes use of the fact that at steady-state c_Y and c_O are equal.

government spending target, the income tax rate on young generations would need to rise, reducing average consumption by the young. The invariance result above states that the reduction in consumption by young generations will approximately offset the increase that accrues to old generations, such that aggregate consumption remains unchanged as the indexation share is varied.

Given that the first term on the right hand side of Equation (3.24) is approximately invariant to the indexation share, the government is effectively minimising a loss function in consumption volatility, such that optimal indexation is driven by a consumption insurance motive. Indeed, using notation employed by Woodford (2003), we can express the optimal indexation problem as follows:

$$(3.25) \quad \min_a U^{society} \approx \frac{1}{2} \left(|U_{c_Y c_Y}^{society}| \text{var}(c_{t,Y}) + |U_{c_O c_O}^{society}| \text{var}(c_{t,O}) \right) + t.i.p.$$

where *t.i.p.* stands for ‘terms independent of policy’.

The key term on the right hand side of Equation (3.25) is the one in $\text{var}(c_{t,O})$. The reasoning is as follows. First, the consumption variance across young generations will be much smaller than the consumption variance across old generations, since consumption volatility for the young arises only indirectly through small portfolio substitution effects due to fluctuations in assets’ expected returns, whilst consumption by the old is impacted directly by *ex post* shocks to asset returns. Second, consumption volatility across old generations depends directly on the indexation share, whilst the indexation share itself has only a minimal impact on consumption volatility across young generations.¹¹³

In Appendix D it is shown that under reasonably general conditions (which are satisfied by the baseline calibration), the key term in Equation (3.25) will be minimised by choosing the indexation share so that the consumption variance across old generations is (approximately) minimised, or

¹¹³ Under IT, consumption volatility across young generations is independent of the indexation share because expected inflation is constant. Under PLT, however, expected inflation is time-varying, so there are small variations in the expected returns on indexed and nominal bonds. Consumption volatility across young generations is thus not independent of the indexation share: the extent of indexation will influence consumers’ incentives to substitute between assets following variations in expected inflation.

$$(3.26) \quad \frac{\partial \text{var}(c_{t,O})}{\partial a} \approx 0$$

Equation (3.26) can thus be used to derive an approximate analytical expression for the optimal indexation share. This task is undertaken below.

First, the consumption variance across old generations can be approximated as follows:¹¹⁴

$$(3.27) \quad \text{var}(c_{t,O}) \approx \text{var}(y_{t,O}) + \text{var}(r_t^{\text{tot}} b_{t-1}^s) + \text{var}(r_t^m m_{t-1}^d) + 2 \text{cov}(r_t^{\text{tot}} b_{t-1}^s, r_t^m m_{t-1}^d)$$

$$(3.27) \quad \text{var}(c_{t,O}) \approx \text{var}(y_{t,O}) + \text{var}(r_t^{\text{tot}} b_{t-1}^s) + \text{var}(r_t^m m_{t-1}^d) + 2 \text{cov}(r_t^{\text{tot}} b_{t-1}^s, r_t^m m_{t-1}^d)$$

where $y_{t,O} \equiv A_t k_{t-1}^\alpha$ is output produced by old generations and $r_t^{\text{tot}} \equiv ar_t^i + (1-a)r_t^n$ is the overall return on the bond portfolio.

Differentiating Equation (3.27) with respect to the indexation share and setting the result equal to zero gives an approximate expression for the optimal indexation share.

Appendix E shows that this expression is as follows:

$$(3.28) \quad \alpha^* \approx \frac{\text{var}(r_t^n b_{t-1}^s) + \text{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) - \text{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) - \text{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)}{\text{var}(r_t^i b_{t-1}^s) + \text{var}(r_t^n b_{t-1}^s) - 2 \text{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)}$$

Intuitively, the optimal indexation share is (i) increasing in the return variance on nominal bonds, (ii) decreasing in return variance on indexed bonds, and (iii) increasing (decreasing) in the extent to which the real returns on nominal (indexed) bonds and money balances covary. Notice also that full indexation will not, in general, be optimal (unless real returns on indexed and nominal bonds are themselves strongly positively correlated), since holding nominal bonds will help to diversity consumption risk in old age.¹¹⁵ All four of these predictions are confirmed by the simulation results that follow.

¹¹⁴ Note that since capital is a claim to real output, its return is uncorrelated with indexed and nominal bond returns, and the real return on money balances.

¹¹⁵ For instance, the variance-minimising shares in a portfolio of two assets with uncorrelated returns are positive so long as both assets have finite variances. When returns are positively correlated, the optimal shares will lie between zero and one if the return correlation is sufficiently small.

3.10 Simulation methodology

The model was again solved to second-order using Dynare++. Due to the presence of the optimal indexation choice faced by the government, the model solution was carried out in two stages. In the first stage, the optimal indexation shares under IT and PLT are identified by solving the model for indexation shares in the range [0,1], in steps of 0.01, and recording social welfare for each share. In the second stage, the model was simulated with the optimal indexation shares identified in the first stage to obtain results under IT and PLT at the optimal Ramsey equilibrium. To ensure consistency with Chapter 2, simulation results were obtained from 1,000 simulations of 5,000 periods, with the simulation seed chosen randomly in each simulation.

3.11 Simulation results

The simulation results presented in this section compare IT and PLT across three key areas: optimal indexation; impulse responses; and consumption volatility and social welfare.

3.11.1 Optimal indexation

In order to solve for the optimal indexation shares under IT and PLT, the model was simulated for indexation shares from 0 to 1 in steps of 0.01, as described above. An indexation share of $\frac{1}{2}$, for example, corresponds to a bond portfolio that is split equally between indexed bonds and nominal bonds, whilst an indexation share of 1 (i.e. 100 per cent) indicates that consumers hold only indexed bonds.

Inflation targeting

Panel (a) of Figure 3.4 shows how social welfare varies with the indexation share under IT, and Panel (b) shows the corresponding variation in consumption volatility across old generations. An indexation share of 76 per cent maximises social welfare. As expected based on the analytical expressions reported above, optimal indexation is driven by consumption risk across old generations, which is minimised at an indexation share of 75 per cent. Intuitively, a relatively high indexation share is optimal under IT because long-term inflation volatility is substantial, so that nominal bonds are a relatively poor store of value compared to indexed bonds, whose imperfections are small by comparison. Indeed, the simulated real return volatility on

nominal bonds is almost two-and-a-half times as high as that on indexed bonds (see Table 3.9). Despite this substantial return volatility differential, it is optimal for consumers to hold some nominal bonds in their portfolios for diversification reasons, as there is a weak correlation between real returns on indexed and nominal bonds.¹¹⁶ The result that optimal indexation is relatively high under IT is consistent with the optimal wage indexation results reported by Minford *et al.* (2003) and Amano *et al.* (2007), and also those reported by Meh *et al.* (2008b) in the context of optimal indexation of financial contracts.

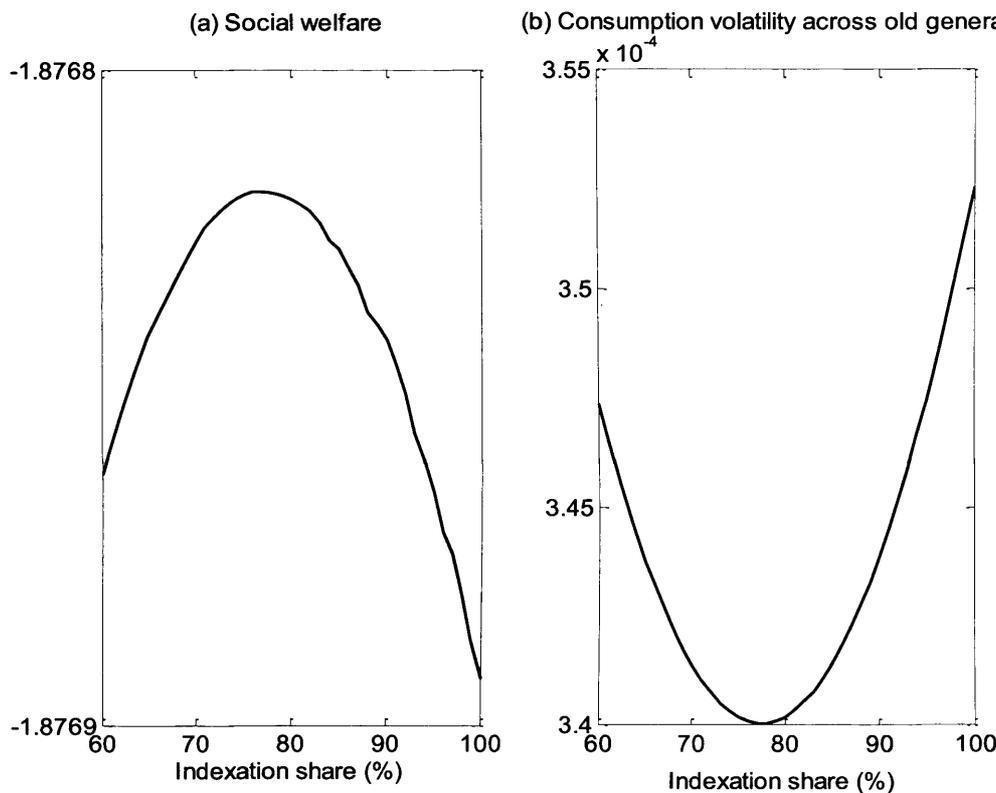


Figure 3.4 – Social welfare and the share of indexed bonds (IT)

Price-level targeting

Figure 3.5 shows the impact of the indexation share on social welfare and consumption volatility under PLT. Optimal indexation is somewhat lower than under IT at 26 per cent (see Panel (a)), indicating that it is optimal for consumers to hold almost three quarters of their bond portfolios as nominal bonds. The intuition for this result can be seen from Panel (b), which shows that consumption volatility across old

¹¹⁶ In fact, the correlation between bond returns is slightly negative. The reason is that unanticipated inflation reduces the real return on nominal bonds but increases the real return on indexed bonds, because biased inflation will tend to ‘overshoot’ true inflation due to its higher variance.

generations is minimised at an indexation share of 25 per cent. Hence nominal bonds become a much better store of value than in the IT case, enabling old generations to reduce their exposure to consumption risk by substituting towards nominal bonds and away from indexed bonds.

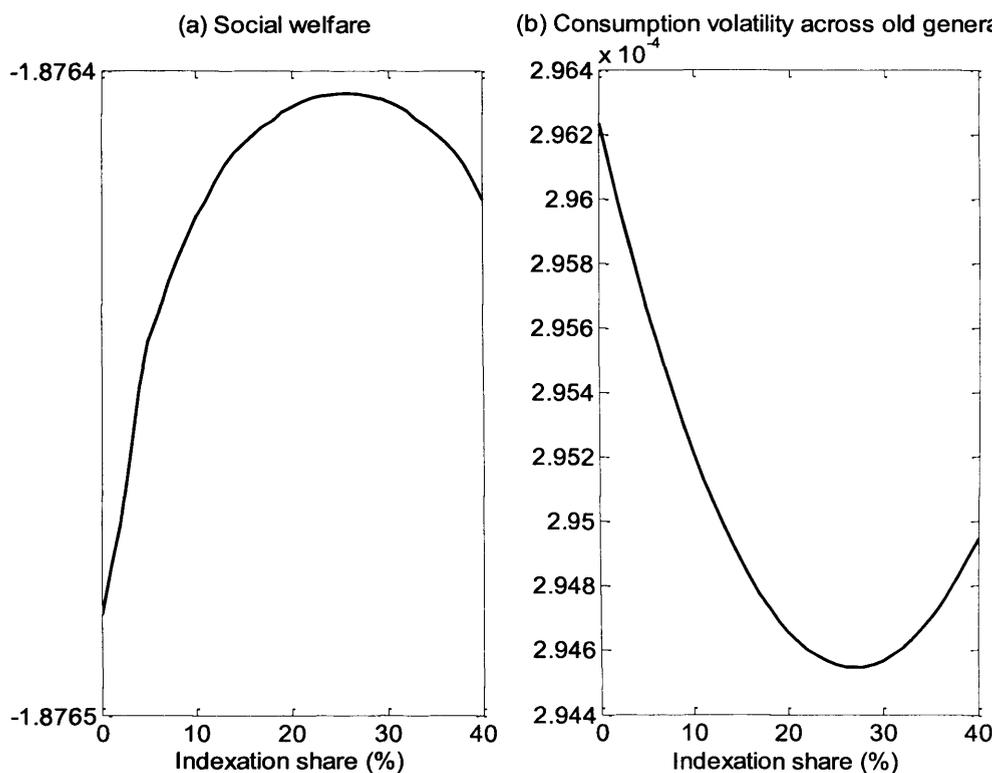


Figure 3.5 – Social welfare and the share of indexed bonds (PLT)

The IT and PLT optimal indexation results are summarised in Table 3.8, which reports the indexation shares that maximise social welfare, as well as the indexation shares at which consumption volatility across old generations is minimised. Note that the optimal indexation shares do not coincide exactly with the ones that minimise consumption volatility, since the optimality condition that indexation should minimise consumption volatility across old generations was derived only as an approximation. These simulation results do suggest, however, that the approximation is a reasonably good one.¹¹⁷

¹¹⁷ As discussed above, the approximation should be a good one if risk aversion is moderate or low, which is the case under the baseline calibration of $\delta = 3$.

Table 3.8 – Optimal indexation under IT and PLT

Monetary policy	Optimal indexation share	Share at which $\text{var}(c_{t,o})$ is minimised
IT	76%	77%
PLT	26%	27%

There are two factors driving the substantial reduction in optimal indexation under PLT. Firstly, the reduction in (long-term) inflation risk under PLT benefits holders of nominal bonds disproportionately, because real return volatility on nominal bonds is driven purely by inflation risk, whereas indexed bonds are also impacted by the indexation lag – a source of real return volatility that remains unchanged under PLT. As a result, real return volatility falls more sharply on nominal bonds than on indexed bonds, giving risk-averse consumers an incentive to substitute towards nominal bonds under PLT.

Formally, the incentive for this substitution is highlighted by the approximate formula for the optimal indexation share in Equation (3.28), which indicates that a reduction in the nominal to indexed return variance ratio will reduce optimal indexation. The marked reduction in this ratio under PLT can be seen clearly from the results in Table 3.9: the standard deviation on indexed bonds is approximately halved from 230 basis points under IT to 120 under PLT, but the standard deviation on nominal bonds falls to below one-fifth of its IT value, from 360 to 70 basis points.

Table 3.9 – Real return standard deviations

Monetary supply rule	Indexed Bonds	Nominal bonds
IT	230	360
PLT	120	70

Notes: Figures are in basis points and are rounded

Secondly, the lower indexation share under PLT is also driven by indexation bias. This bias reduces the optimal indexation share even in the absence of any indexation lag. The reasoning is as follows. With consumers holding both indexed and nominal bonds in their portfolios for diversification reasons, covariance risk between bond returns and the real return on money influences consumption volatility in old age, and hence optimal indexation. Nominal bonds perform relatively better under PLT in terms of this cross-covariance risk, because the real return on nominal bonds is

strongly positively correlated with the real return on money balances under IT, but only weakly so under PLT. Thus there is an additional diversification motive for holding nominal bonds under PLT: nominal bonds will tend to pay a relatively low return when the real return on money is high, thus stabilising consumption in old age.

Table 3.10 shows formally that the nominal bonds to money real return correlation falls substantially from a perfect positive correlation of 1 under IT to only 0.08 under PLT, whilst other return correlations remain largely unchanged.¹¹⁸ That a lower correlation between the real return on nominal bonds and the real return on money will reduce optimal indexation can be seen formally from the approximate expression in Equation (3.28). The lower correlation under PLT can be explained by the fact that expected inflation becomes time-varying. This has the effect of ‘diluting’ the positive correlation between nominal bond returns and the real return on money balances, because whilst nominal bonds provide insurance against *anticipated fluctuations* in inflation, money balances do not.¹¹⁹

Table 3.10 – Real return correlations on indexed and nominal bonds

Correlation	r^i, r^n	r^i, r^m	r^i, r^k	r^n, r^k	r^n, r^m
IT	-0.11	-0.11	0	0	1.00
PLT	-0.03	-0.04	0	0	0.08

Note: Correlations are rounded to two decimal places

In order to identify the relative importance of the factors driving the reduction in optimal indexation under PLT, the indexation differential was decomposed as follows into ‘indexation bias’ and ‘indexation lag’ components:

$$(3.29) \quad a^{IT} - a^{PLT} = \underbrace{\Delta a^{IT} - \Delta a^{PLT}}_{\text{Indexation lag diff.}} + \underbrace{a_{\text{no lag}}^{IT} - a_{\text{no lag}}^{PLT}}_{\text{Indexation bias diff.}}$$

where a^j is the optimal indexation share under monetary policy j , $a_{\text{no lag}}$ is the optimal indexation share in the absence of lagged indexation, and $\Delta a \equiv a - a_{\text{no lag}}$ is indexation differential due purely to the presence of lagged indexation.

¹¹⁸ There is a perfect positive correlation under IT because expected inflation is constant; this means that a nominal bond is equivalent to money plus a constant nominal ‘mark-up’ for expected inflation.

¹¹⁹ Under PLT, a nominal bond is equivalent to money plus a time-varying nominal ‘mark-up’ that captures fluctuations in expected inflation. Since innovations to inflation are serially uncorrelated, the latter need not be strongly correlated with actual inflation – hence explaining the relatively weak positive correlation.

Table 3.11 shows the results from the decomposition of the IT-PLT indexation differential given in Equation (3.29).

Table 3.11 – Indexation differential decomposition

Indexation share/differential (%)	IT	PLT
Optimal indexation share, a^j	76	26
IT-PLT differential, $a^{IT} - a^{PLT}$	50	
Optimal with no indexation lag, $a_{no\ lag}^j$	82	76
IT-PLT indexation bias differential	6	
IT-PLT indexation lag differential	44	

Only 6 per cent of the indexation differential between IT and PLT is due to indexation bias, with the remaining 44 per cent due to the indexation lag. The impact of the indexation lag is substantial because long-term inflation risk is reduced by an order of magnitude under PLT. As a result, real return volatility on nominal bonds falls sharply compared to indexed bonds, because return risk on nominal bonds results solely from inflation uncertainty, whilst indexed bonds are also subject to return risk resulting from the indexation lag – risk that is entirely unchanged under PLT.

On the other hand, the role played by indexation bias in the IT-PLT indexation differential is relatively small. Intuitively, since money holdings are small under the baseline calibration, the reduced correlation between nominal bond returns and the return on money balances has relatively little impact on consumption risk faced by old generations or, therefore, on the optimal indexation share. Moreover, the extent of indexation bias captured in the model is relatively small since true and biased inflation are strongly positively correlated and have similar variances under the baseline calibration. The robustness of optimal indexation to the extent of indexation bias is tested below in the sensitivity analysis section.

3.11.2 Impulse responses

Impulse responses are evaluated at the optimal indexation shares identified above on the basis that these correspond to the equilibria implemented by the government under IT and PLT through its optimal commitment Ramsey policy. Of course, the impulse responses of exogenous variables in the model will be invariant to the indexation share.

The impulse responses for inflation are the same as previously, but due to the introduction of indexed bonds there is a second inflation rate in the model – the ‘biased’ one that is used for indexation purposes. The extent to which this second rate differs from the ‘true’ rate determines the magnitude of the indexation bias and is an important factor determining the optimal indexation shares and social welfare. Figure 3.6 plots one standard deviation impulse responses of true inflation to a money supply innovation and of biased inflation to its innovation.

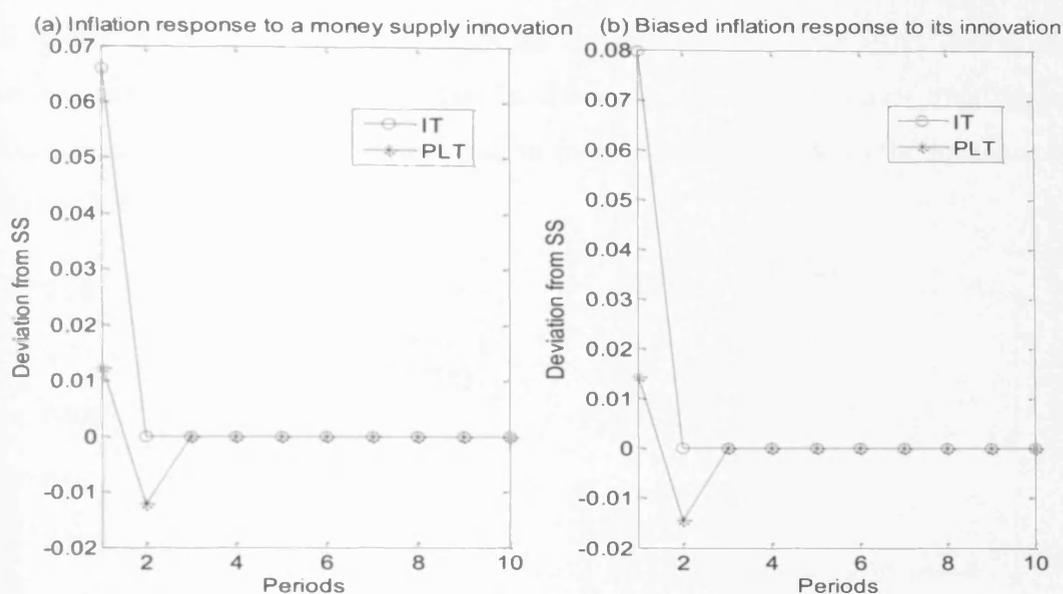


Figure 3.6 – Impulse responses of true and biased inflation compared

Under the baseline calibration, innovations to true and biased inflation are strongly correlated, with a correlation coefficient of 0.89. However, as the impulse responses shown are orthogonalized, there is no response of biased inflation to a money supply innovation, or *vice versa*. Nevertheless, comparing the two impulse responses will give a good indication of the response to indexed inflation to money supply innovations, due to the high positive correlation. The response of biased inflation to its innovation is larger than the response of true inflation to a money supply innovation, because it has a higher innovation variance. Overall, however, the two inflation responses are rather similar (under both IT and PLT), as is to be expected given that the innovation variances are not that different. Figure 3.6 therefore provides formal confirmation that the extent of indexation bias is relatively low under the baseline calibration: true and biased inflation track each other well in response to inflationary shocks.

Figure 3.7 shows the impulse responses of real returns on indexed bonds to inflationary innovations under IT and PLT. The real return on indexed bonds is pushed downwards by an innovation to true inflation (i.e. a money supply innovation) but upwards by an innovation to the biased inflation rate to which indexed bonds are linked. Since true and biased inflation are strongly positively correlated and have similar variances, the real return on indexed bonds will be largely stabilised against a money supply innovation. This point is highlighted by the fact that the two impulse responses have similar magnitudes but opposite signs. It is notable that there is a small lagged response of the real return on indexed bonds under PLT, due to the initial inflationary impulses being offset in the second period. Moreover, this lagged response is much larger for an innovation to indexed inflation due to the presence of indexation bias.

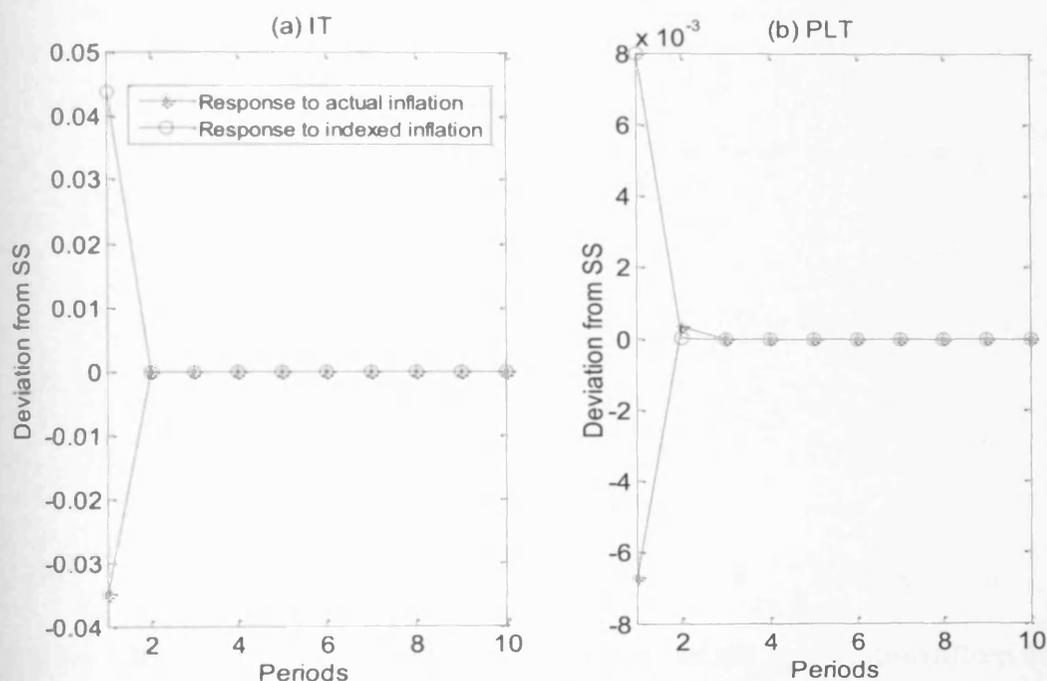


Figure 3.7 – Impulse responses of the real return on indexed bonds to innovations to actual and biased inflation

The corresponding impulse responses for consumption in old age are shown in panels (a) and (b) of Figure 3.8. The consumption responses to actual and indexed inflation are largely symmetrical under IT, because indexed bonds account for more than three-quarters of consumers' bond portfolios. That is to say, nominal bond holdings and money holdings are small in comparison to indexed bond holdings, such that the fall in consumption in response to unanticipated inflation (which also affects the real returns on nominal bonds and money) is only slightly larger, in absolute terms, than

the positive impact from an unanticipated innovation to indexed inflation. Under PLT, by contrast, consumption falls less sharply in response to unanticipated inflation due to the lower level of long-term inflation risk, and this fall in consumption far outweighs the increase that results from an innovation to indexed inflation, because the latter only affects the one-quarter of consumers' bond portfolios held as indexed bonds.

As is to be expected, there is no response of the real return on nominal bonds to innovations to indexed inflation, and no alteration in the real return impulse responses to money supply innovations compared to Chapter 2. Similarly, the impulse responses to productivity are identical to those in Chapter 2 and are therefore not repeated here.

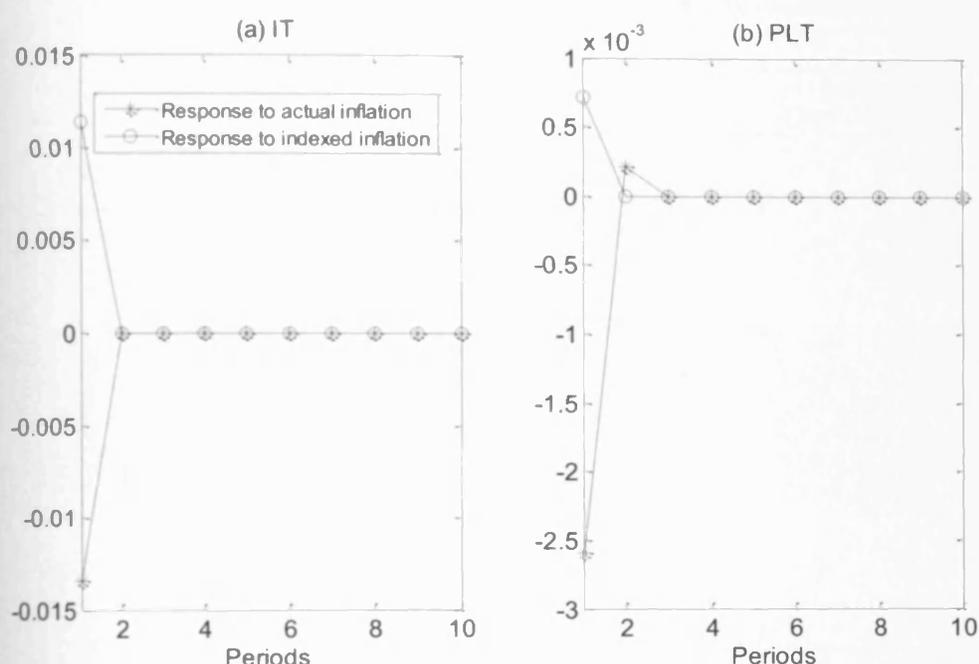


Figure 3.8 – Impulse responses of consumption in old age to innovations to actual inflation and biased inflation

Figure 3.9 investigates the impact of money supply innovations (i.e. innovations to actual inflation) on asset holdings and consumption in youth. As in the model without indexed bonds, there is no response of asset holdings under IT, whilst capital and bond holdings are reduced under PLT, because consumers substitute towards money balances. The reasoning is the same as previously: a positive shock to inflation lowers expected future inflation under PLT, increasing the expected real return on money balances but leaving expected real returns on bonds and capital unchanged.

As explained in Chapter 2, substitution between assets following shocks to inflation produces a small amount of extra consumption volatility across young generations under PLT. Since indexed and nominal bonds give equivalent expected utility at the margin, the reduction in total bond holdings under PLT is identical to that in the model without indexed bonds. Consequently, the corresponding reductions in indexed and nominal bond holdings are determined by their shares in total bond holdings, 26 and 74 per cent respectively. In the case of innovations to biased inflation, the expected real return on indexed bonds is unchanged under both IT and PLT, and there is no response of asset holdings or consumption in youth, because innovations to biased inflation have no impact on the expected real return on money balances.

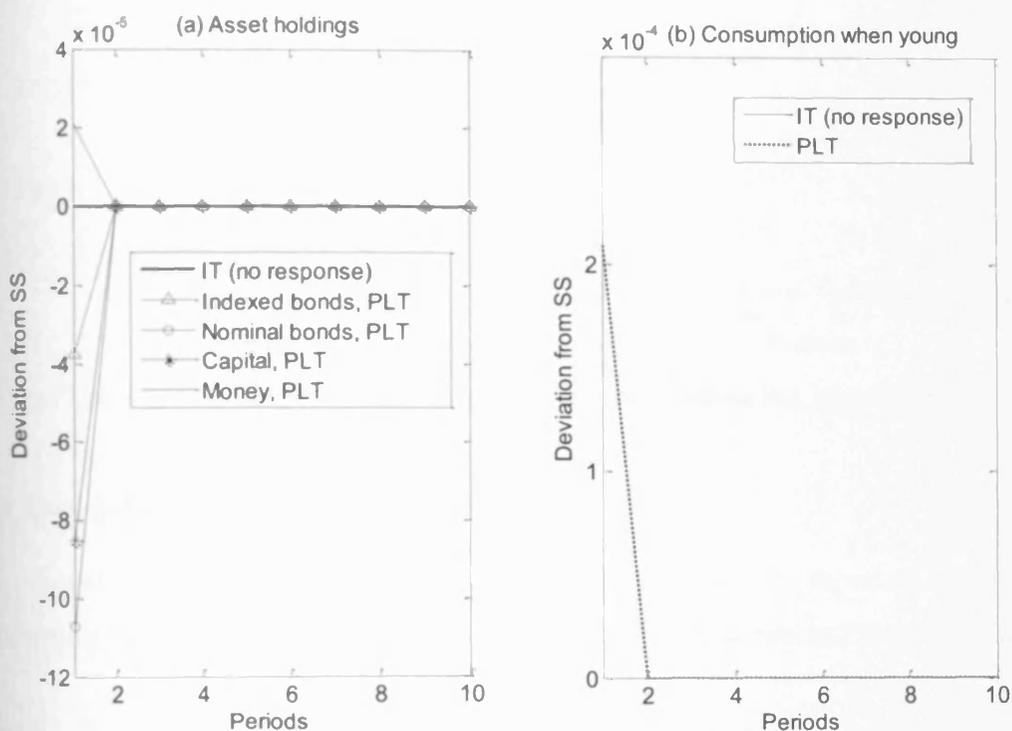


Figure 3.9 – Impulse responses to a money supply innovation

The other additional shock in the model is the indexation lag innovation in the real return on indexed bonds. There is no impact from this innovation on the expected (*ex ante*) return on indexed bonds, because the innovation is white noise with an expected value of zero. Consequently, consumption by the young does not respond to the indexation lag innovation. There is, of course, an *ex post* response from this innovation on the real return on indexed bonds, and a knock-on impact on consumption in old age, as shown clearly in Figure 3.10. Moreover, since the indexation lag length is the same under IT and PLT, the impact on the real return on

indexed bonds is the same in both cases. However, the impact of an innovation on consumption by old generations is larger under IT, because holdings of indexed bonds are around three times as high due to the higher indexation share of 76 per cent.

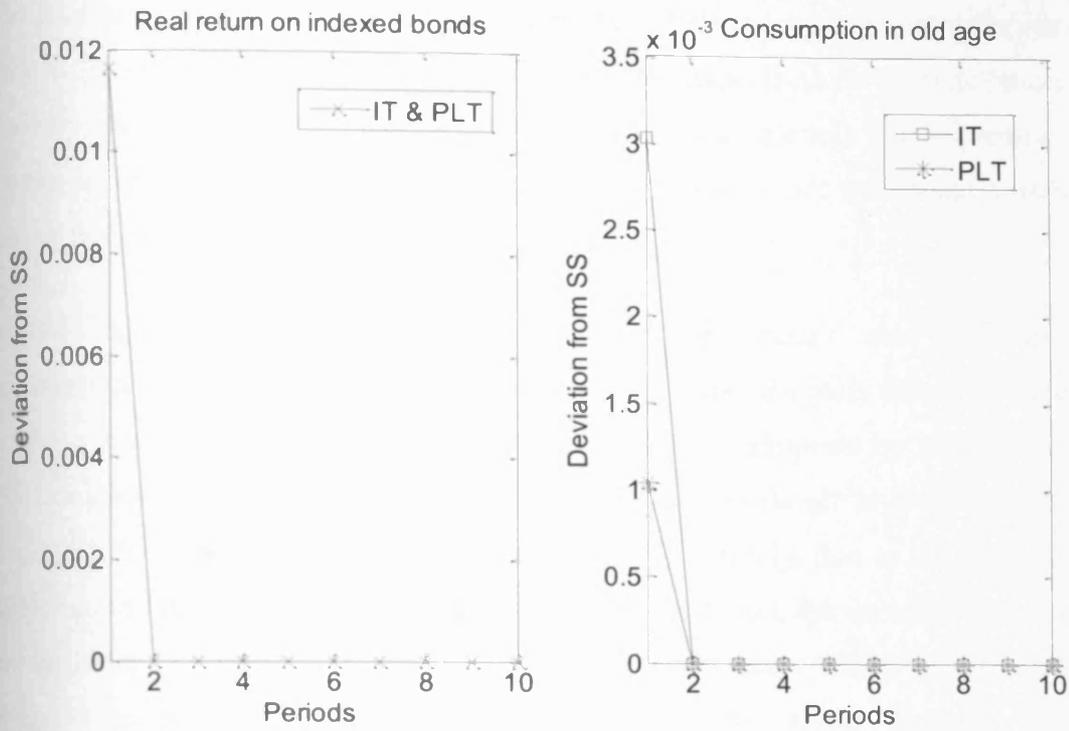


Figure 3.10 – Impulse responses to the indexation lag innovation

3.11.3 Consumption volatility and social welfare

As previously, the consumption equivalent welfare gain λ is reported along with consumption means and variances across young and old generations; see Table 3.12.

Table 3.12 – Social welfare and consumption

Simulated value	IT	PLT
$Ec_{t,Y}$	0.7299	0.7300
$Ec_{t,O}$	0.7307	0.7307
$\text{var}(c_{t,Y}) \times 1000$	0.0385	0.0387
$\text{var}(c_{t,O}) \times 1000$	0.3401	0.2946
λ (in % terms)	0.010%	

With indexed bonds in the model, the welfare gain from PLT falls to 0.01 per cent of aggregate consumption, or 0.007 per cent of GDP. This gain compares to an increase of 0.103 per cent in the model with nominal bonds only, and 0.026 per cent in the

model with nominal bonds and capital. In monetary terms, the aggregate gain is equal to £90.7 million – a gain per employed member of the UK population of £3.13, or £10.80 per pensioner. These gains are borderline trivial, but it should be noted that they apply to all current and future generations. Intuitively, indexed bonds are an additional asset with which consumers can protect their savings against inflation risk – and a good one for this purpose given that the imperfections of indexation are relatively small under the baseline calibration of the model. Consequently, the benefits to be had from reducing inflation risk under PLT are somewhat smaller if indexed bonds are freely available.

Because indexation is endogenous, consumers are better able to ‘smooth’ consumption such that mean consumption levels are approximately identical under IT and PLT, in contrast the models of Chapter 2. Mean consumption by old generations is in fact slightly higher under IT (because bond risk-premia are higher), but there is no numerical difference to four decimal places.¹²⁰ Intuitively, due to the need for the government to meet its long run government spending target, the increase in bond risk premia under IT implies a higher tax on young consumers, which in turn reduces average consumption by young agents, offsetting the slight increase for old generations. As a result, aggregate consumption is essentially unchanged, as in the models simulated in Chapter 2. Consumption volatility across old generations is 13 per cent lower under PLT – a reduction which is somewhat lower than in the model with capital and nominal bonds (39 per cent) or nominal bonds only (95 per cent), but which is nevertheless non-trivial. As in the results reported previously, there is a small increase in consumption volatility across young generations under PLT (a 1 per cent increase), due to variations in expected inflation causing young generations to undertake portfolio substitution between assets.

To summarise, old generations benefit from a non-trivial reduction in consumption volatility under PLT, but there is only a small welfare gain because consumption risk has a second-order impact on social welfare. In order to investigate the quantitative robustness of these results, the next section conducts an extensive sensitivity analysis.

¹²⁰ In particular, mean consumption across old generations is 0.73074 under IT and 0.73071 under PLT.

3.12 Sensitivity analysis

This section analyses the importance of the indexation share for the IT-PLT welfare comparison and investigates sensitivity to key calibrated parameters and variances.

3.12.1 Indexation and social welfare

A key finding from the PLT literature is that optimal wage indexation is substantially lower than under IT. For instance, Amano *et al.* (2007) found that if wage indexation is exogenous and does not respond under PLT, the welfare gain vis-à-vis IT is substantially understated. An even stronger result is found by Minford and Peel (2003): holding indexation fixed under PLT gives the false conclusion that social welfare is higher under IT. These results suggest that the Lucas critique should be an important consideration for economic researchers who build models for the purpose of comparing social welfare across monetary policy regimes that might affect contracting behaviour in important ways. An interesting question that can be answered using the model in this chapter is whether the substantial reduction in optimal indexation under PLT – from 76 per cent to 26 per cent – is important for the estimated welfare gain. In order to do so, the approach in Minford and Peel (2003) is followed: social welfare under PLT is computed under the assumption that indexation is held fixed at the optimal level under IT.

A second important consideration is that current level of bond indexation in developed economies is rather low (see e.g. Bekaert and Wang, 2010). Indeed, amongst developed countries, indexed government bonds are most prevalent in the UK where they account for around one-quarter of the total government bond portfolio (DMO, 2010a), compared to the optimal share of 76 per cent estimated in the baseline case. It is therefore instructive to compare social welfare under IT and PLT when the extent of bond indexation is fixed at a low level comparable to that developed economies. For this purpose, the indexation share was set at 21 per cent, the share of indexed bonds in the UK government bond portfolio as of March 2010 (DMO, 2010b).

The results from these two sensitivity exercises are reported in Table 3.13. For ease of reference, the consumption equivalent welfare gain from the baseline calibration is reported in bold in the middle column.

Table 3.13 – Indexation and the welfare gain from PLT, λ

Indexation set at IT optimal	Optimal indexation	Indexation set at UK level
0.008%	0.010%	0.020%

The welfare gain from PLT is underestimated by one-fifth if indexation is held fixed at the optimal level under IT. Although the change in the welfare gain is relatively small in absolute terms, this reduction masks the fact that the reduction in old generations' consumption volatility under PLT is somewhat lower than in the optimal indexation case at 11.7 per cent. On the other hand, if indexation is fixed at the current UK level, the welfare gain from PLT is doubled relative to the baseline case at 0.020 per cent. Intuitively, the current low level of indexation in developed countries works in favour of PLT and against IT because optimal indexation is substantially lower under PLT and close to 21 per cent. Moreover, since indexation is higher in the UK than any other developed economy, this estimate can be viewed a lower bound on the welfare gain from PLT if indexation remains at low levels as in developed economies currently.

3.12.2 Model parameters

As with the models in Chapter 2, the welfare results are robust to changes in the persistence of productivity and its innovation variance, and also to the consumption share of money in the CIA constraint. The sensitivity analysis in this section therefore focuses on four key areas: the extent of risk aversion; nominal volatility; the extent of indexation bias; and the indexation lag length. Each of these areas is investigated in turn below. Given that indexation is endogenously determined, the optimal indexation shares vary in these simulations. Therefore, both the optimal indexation shares and the corresponding welfare gains are reported and discussed in the analysis below.

Risk aversion

As in the models of Chapter 2, sensitivity is investigated for alternative risk aversion coefficients of 3/2 and 5. The results from this analysis are shown in Table 3.14.

Table 3.14 – Sensitivity of optimal indexation and λ to risk aversion

Simulated value	Coefficient of relative risk aversion, δ		
	$\delta = 3/2$	$\delta = 3$	$\delta = 5$
IT optimal indexation share	77%	76%	78%
PLT optimal indexation share	24%	26%	27%
λ	0.006%	0.010%	0.014%

The extent of risk aversion has little impact on optimal indexation, but a substantial impact on the estimated welfare gain from PLT. When risk aversion is ‘low’, the welfare gain from PLT falls to 0.006 per cent of aggregate consumption, whilst it rises to 0.014 per cent with ‘high’ risk aversion – both deviations of four-tenths from the baseline welfare gain. In absolute terms, the range of uncertainty is narrower than previously, reflecting the lower magnitude of the welfare gain under the baseline calibration. The intuition behind the impact of risk aversion on the welfare gain is as given in Chapter 2: an increase in risk aversion increases the relative importance of consumption risk for social welfare, making a reduction in consumption volatility under PLT of greater social value than in the baseline case.¹²¹

3.12.3 Nominal volatility

In concordance with the sensitivity analyses in Chapter 2, money supply innovation standard deviations of 0.015 and 0.009 were investigated in addition to the baseline calibration of 0.012. As we have seen, the volatility of the money supply innovation endogenously determines the volatility of (true) inflation according to the money supply rule implemented by the government. However, in the current model, a change in the money supply volatility implies, *ceteris paribus*, a change in the volatility of biased inflation (because innovations to the latter are correlated with innovations to the former), whilst the calibration of the indexation lag innovation variance is related to the volatility of biased inflation.

Therefore, in order to isolate the impact of a pure change in nominal volatility (i.e. holding the relative importance of imperfections in indexation constant), the simulations below change the innovation variance to biased inflation and $\text{var}(v_t)$ in

¹²¹ Formally, consumption volatility has a second-order impact on social welfare. An increase in the curvature of the utility function, as measured by δ , increases the role of such higher-order effects.

tandem with the money supply innovation variance, such that the variance ratios remain unchanged relative to the baseline calibration. The innovation variances under ‘high’ and ‘low’ volatility calibrations are presented in Table 3.15, along with their baseline calibrations.

Table 3.15 – Nominal volatility sensitivity calibrations

Money supply innovation variance		
Low	Baseline	High
$\text{var}(\varepsilon_{i,t}) = 0.87 \times 10^{-4}$	$\text{var}(\varepsilon_{i,t}) = 1.45 \times 10^{-4}$	$\text{var}(\varepsilon_{i,t}) = 2.20 \times 10^{-4}$
Biased inflation innovation variance		
Low	Baseline	High
$\text{var}(\varepsilon_{i,t}^{ind}) = 1.30 \times 10^{-4}$	$\text{var}(\varepsilon_{i,t}^{ind}) = 2.13 \times 10^{-4}$	$\text{var}(\varepsilon_{i,t}^{ind}) = 3.20 \times 10^{-4}$
Indexation lag innovation variance		
Low	Baseline	High
$\sigma_v^2 = 8.7 \times 10^{-5}$	$\sigma_v^2 = 0.00015$	$\sigma_v^2 = 0.000220$

The results are reported in Table 3.16. Optimal indexation is again fairly robust. An increase in nominal volatility increases optimal indexation only slightly, but raises welfare gain from PLT by one-fifth. Intuitively, nominal risk rises whilst real risk (from holding capital) remains unchanged, such that nominal volatility becomes a relatively more important factor in consumption risk for old generations, increasing the stabilisation benefits to be had from PLT. By the same reasoning, the welfare gain from PLT falls somewhat as the extent of nominal volatility is reduced. The impact of nominal volatility on the welfare gain from PLT is smaller in magnitude than in the models of Chapter 2, but is non-trivial in proportional terms and similar to that from varying risk aversion.

Table 3.16 – Sensitivity of optimal indexation and λ to nominal volatility

Simulated value	Nominal volatility		
	Low	Baseline	High
IT optimal indexation share	75%	76%	79%
PLT optimal indexation share	23%	26%	28%
λ	0.007%	0.010%	0.012%

3.12.4 Indexation bias

An important determinant of optimal indexation is ‘indexation bias’ captured by the real return on indexed bonds. The size of the indexation bias depends on the relative volatilities of true and biased inflation and the positive correlation between the two inflation rates. In particular, the larger volatility differential and the lower the correlation between the two, the greater the extent of indexation bias – because actual and indexed inflation will diverge more often. Indexation bias of some form will be present unless the variances of the two indices are exactly equal and there is perfect positive correlation of 1 between innovations to true and biased inflation.

In order to investigate the impact of changes in indexation bias on optimal indexation and social welfare, the model was simulated for alternative correlations between money supply innovations and innovations to indexed inflation, with the innovation variances held fixed. In the baseline calibration, the correlation was set at 0.89. Here, robustness is investigated to alternative correlations of 0.80 and 0.98. Moreover, the model was also simulated in absence of indexation bias by setting biased inflation equal to true inflation, thus providing a benchmark against which the other results could be compared. This assumption (or the high correlation of 0.98) would be applicable for countries in which indexed government bonds and public sector pensions are linked to an inflation rate that provides an excellent approximation to the true one. The results from these sensitivity tests are shown in Table 3.17.

Table 3.17 – Sensitivity of optimal indexation and λ to indexation bias

Simulated value	Correlation, $corr(\pi, \pi^{ind})$			Zero bias (i.e. $\pi = \pi^{ind}$)
	0.80	0.89	0.98	
IT optimal share	71%	76%	85%	100%
PLT optimal share	22%	26%	30%	30%
λ	0.013%	0.010%	0.005%	0.005%

When the correlation between innovations to true and biased inflation is reduced to 0.80, indexation bias is increased relative to the baseline case, with the intuitive result that optimal indexation falls somewhat under both IT and PLT. Moreover, the welfare gain from PLT increases by one-third, because indexed bonds are less able to protect

savers against the high level of long-term inflation risk under IT, such that the consumption stabilisation benefits to be had from PLT are increased.¹²² By the same reasoning, optimal indexation rises if the indexation correlation is reduced, and the welfare gain from PLT is reduced with it. Interestingly, although the high and low correlation cases are symmetrical, the welfare gain from PLT is halved in the ‘high correlation’ case. The reason is that optimal indexation increases only slightly under PLT, but markedly under IT. This difference reflects the fact that indexation becomes somewhat less costly under IT but only marginally so under PLT given that, under the latter, indexation bias is a minor source of volatility due to the low level of long-term inflation risk.

If there is no indexation bias at all, optimal indexation increases sharply under IT: full indexation of 100 per cent is optimal, implying that the government should issue only indexed bonds. Under PLT, however, optimal indexation increases only slightly – from 26 per cent in the baseline case up to 30 per cent. The large increase in optimal indexation under IT reflects the fact that inflation risk is the dominant factor in return volatility, whilst this is not true under PLT because volatility arising from the indexation lag plays a crucial role due to the low level of inflation risk. In effect, long-term inflation risk is sufficiently high under IT that it is optimal to hold only indexed bonds and be insured as much as possible.

The welfare gain from PLT is halved when there is no indexation bias, as in the case when the correlation is ‘near-perfect’ at 0.98. This result suggests that countries whose indexed bonds have relatively little indexation bias would have somewhat less to gain in the long-term from switching from IT to PLT. The intuition is simply that the lower the extent of indexation bias, the better indexed bonds are as a substitute for PLT. Overall, the welfare gain from PLT is reasonably sensitive to the extent of indexation bias, as are the optimal indexation shares.

3.12.5 Indexation lag length

In order to investigate sensitivity to the indexation lag length, alternative variance calibrations are considered for the white noise innovation v_t that enters the return on

¹²² In particular, PLT reduces consumption volatility across old generations by 19.9 per cent, compared to 13.1 per cent in the baseline case.

indexed bonds. The innovation variance in the baseline calibration was set at 0.00015 based upon estimation results for the change in RPI inflation over three quarters, which is approximately equal to the indexation lag on index-linked gilts and public sector pensions in the UK. However, as can be seen above from Table 3.1, other G7 countries issue indexed bonds with a shorter indexation lag of three months.¹²³ The sensitivity analysis below therefore calibrates the model for an indexation lag of three months, such that the innovation variance is one-third of its baseline value. For symmetry, the impact of an indexation lag of 15 months is also considered. The calibrations and results are reported in Table 3.18.

Table 3.18 – Sensitivity of the welfare gain λ to the indexation lag

Simulated value	Indexation lag length		
	High (5 quarters) $\sigma_v^2 = 0.00027$	Baseline (3 quarters) $\sigma_v^2 = 0.00015$	Low (1 quarter) $\sigma_v^2 = 5.3 \times 10^{-5}$
IT optimal indexation share	72%	76%	83%
PLT optimal indexation share	17%	26%	44%
λ	0.010%	0.010%	0.008%

The indexation lag length has a substantial impact on the IT-PLT optimal indexation differential. When the indexation lag is calibrated based on a one-quarter horizon, optimal indexation rises and the optimal indexation differential is reduced from 50 to 39 per cent. Intuitively, optimal indexation rises because the indexation lag becomes a less important source of volatility in the real return on indexed bonds, making indexed less risky as compared to nominal bonds. Indexation rises more sharply under PLT than IT because, as discussed above, volatility arising from the indexation lag is a relatively more important factor in overall return volatility under PLT, due to the considerably lower level of inflation risk. By the same token, an increase in the indexation lag length to 5 quarters reduces optimal indexation under IT and PLT, and increases the optimal indexation differential to 55 per cent. Overall, the welfare gain from PLT is rather robust to the length of the indexation lag, though optimal indexation is strongly sensitive.

¹²³ The UK also issues indexed bonds with a lag of three months, but the majority of bonds outstanding have an eight-month indexation lag (DMO, 2010a).

3.13 Conclusions and policy implications

In this chapter, indexed bonds were introduced into the OLG model and the extent of bond indexation was determined endogenously in response to monetary policy as part of an optimal commitment Ramsey policy implemented by the government. The analysis concentrated primarily on how introducing indexed bonds into the model influenced the long-term welfare impact from PLT (and its robustness), as well as on optimal indexation under IT and PLT. Since indexation is optimised in response to monetary policy and captures imperfections of the kind observed in practice, the social welfare and volatility results obtained should not be vulnerable to the Lucas critique, and should thus provide a sound foundation from which to draw policy conclusions.

Introducing indexed government bonds increases social welfare under both IT and PLT, but the increase under IT is larger because indexed bonds play an important role in protecting consumers against the high level of long-term inflation risk. As a result, the welfare gain from switching to PLT was reduced to only 0.010 per cent of aggregate consumption, or an increase in GDP of approximately 0.007 per cent. For the UK, this amounts to an aggregate monetary gain of £90.7 million. The implied gain per employed member of the UK population over a lifetime is £3.13, or £10.80 per pensioner.¹²⁴ Whilst this estimated welfare gain is rather small, it is economically non-trivial given that it would apply, in principle, to all current and future generations, and that the level of consumption on which it is calculated should grow over time. Moreover, it is notable that the reduction in consumption volatility for old generations (i.e. pensioners) is non-trivial at almost 15 per cent.

Intuitively, the welfare gain from PLT is reduced somewhat because indexed bonds enable consumers to largely protect their savings against long-term inflation risk, thus reducing the benefits obtainable from lowering inflation risk under PLT. In other words, indexed bonds act as a substitute for PLT by insuring consumption in old age against the high level of inflation risk under IT. In terms of optimal indexation, the results are consistent with the literature that has investigated optimal indexation of wage contracts: optimal indexation is reduced substantially under PLT, and ignoring

¹²⁴ As in Chapter 2, these figures are based on 2009 data in ONS (2010a), with the pensioner population figure taken from DWP (2009).

this reduction understates the potential welfare gain from PLT. Under the baseline calibration of the model, optimal indexation was 76 per cent under IT, compared to only 26 per cent under PLT.

The estimated welfare gain from PLT is sensitive to the indexation share and to key calibrated values in the model. In particular, the welfare gain from PLT is underestimated slightly if indexation is held fixed at the same level as is optimal under IT, and is doubled if indexation is set at a low level as in developed economies currently. As with the two models analysed in the second chapter, there is considerable sensitivity of the welfare gain to the extent of risk aversion and nominal volatility.

Additionally, indexation bias is a crucial factor, because it determines the extent to which indexed bonds can protect consumers' savings against long-term inflation risk. If indexation bias is increased, the welfare gain of PLT rises because consumers cannot insure themselves as effectively against the high level of inflation risk under IT. On the other hand, if indexation bias is close to zero, a high indexation share is optimal under IT and the potential welfare gain from PLT is rather small, implying that countries whose indexed bonds and public sector pensions are subject to little indexation bias would have relatively little to gain from switching to PLT. However, given the numerous potential biases in price indices and the difficulties surrounding a calculation of 'true inflation', it is not a straightforward task for countries to assess this at the current time. Altering the indexation lag length has a substantial impact on optimal indexation, but little impact on the estimated welfare gain from PLT, because this source of risk is present to the same extent under both IT and PLT.

To summarise the key findings of this chapter, PLT leads to only a small increase in social welfare, but the welfare gain is likely to vary across countries with risk aversion, the extent of nominal volatility, the relative importance of indexed bonds, and the extent of imperfections in indexation.

Appendix A:

The second-order approximation of lifetime utility with indexed bonds

A second-order expansion of the lifetime utility of generation t can be written in the following form:

$$(A1) \quad Loss_t \approx -\frac{1}{2} \left[\frac{\delta}{(E_t c_{t+1,O})^{1+\delta}} \right] \text{var}_t(c_{t+1,O})$$

where $Loss_t$ is defined as the deviation of lifetime utility from the level of utility received if consumption levels are at their time- t expected values.

Note firstly that log-linearising the budget constraint in old age around the deterministic steady-state results in the following expression:

$$(A2) \quad c_O \hat{c}_{t+1,O} = Ak^\alpha (\hat{A}_{t+1} + \alpha \hat{k}_t) + r^i b^{i,d} (\hat{r}_{t+1}^i + \hat{b}_t^{i,d}) + r^n b^{n,d} (\hat{r}_{t+1}^n + \hat{b}_t^{n,d}) \\ + r^m m^d (\hat{r}_{t+1}^m + \hat{m}_t^d)$$

where $\hat{A}_{t+1} = \rho \hat{A}_t + e_{t+1}$.

The log-linearised real returns on money balances, indexed bonds and nominal bonds are as follows:

$$(A3) \quad \hat{r}_{t+1}^m = -\frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

$$(A4) \quad \hat{r}_{t+1}^i = \hat{r}_t + \frac{1}{1 + \pi^*} (\pi_{t+1}^{ind} - \pi^*) - \frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*) + v_{t+1}$$

$$(A5) \quad \hat{r}_{t+1}^n = \hat{R}_t + \hat{r}_{t+1}^m = \hat{R}_t - \frac{1}{1 + \pi^*} (\pi_{t+1} - \pi^*)$$

where π^* is the steady-state rate of inflation.

The right hand side of Equation (A2) can be written in terms of actual inflation and biased inflation using equations (A3) to (A5). Carrying out these steps and using the definition of a percentage deviation around steady-state, the *level* of consumption in old age can be expressed as follows:

$$(A6) \quad c_{t+1,0} = \left(\begin{array}{l} c_o + Ak^\alpha (\hat{A}_{t+1} + \alpha \hat{k}_t) + r^i b^{i,d} \hat{b}_t^{i,d} + r^n b^{n,d} \hat{b}_t^{n,d} + r^m m^d \hat{m}_t^d + r^i b^{i,d} \hat{r}_t \\ + r^n b^{n,d} \hat{R}_t - \frac{r^i b^{i,d} + r^n b^{n,d} + r^m m^d}{1 + \pi^*} (\pi_{t+1} - \pi^*) \\ + \frac{r^i b^{i,d}}{1 + \pi^*} (\pi_{t+1}^{ind} - \pi^*) + r^i b^{i,d} v_{t+1} \end{array} \right)$$

Expected consumption in old age is therefore given by

$$(A7) \quad E_t c_{t+1,0} = \left(\begin{array}{l} c_o + Ak^\alpha (\rho \hat{A}_t + \alpha \hat{k}_t) + r^i b^{i,d} \hat{b}_t^{i,d} + r^n b^{n,d} \hat{b}_t^{n,d} + r^m m^d \hat{m}_t^d + r^i b^{i,d} \hat{r}_t \\ + r^n b^{n,d} \hat{R}_t - \frac{r^i b^{i,d} + r^n b^{n,d} + r^m m^d}{1 + \pi^*} (E_t \pi_{t+1} - \pi^*) \\ + \frac{r^i b^{i,d}}{1 + \pi^*} (E_t \pi_{t+1}^{ind} - \pi^*) \end{array} \right)$$

Thus the conditional variance of consumption is as follows:

$$(A8) \quad \text{var}_t(c_{t+1,0}) = \left(\begin{array}{l} \frac{(r^i b^{i,d} + r^n b^{n,d} + r^m m^d)^2}{(1 + \pi^*)^2} \text{var}_t(\pi_{t+1}) + \frac{(r^i b^{i,d})^2}{(1 + \pi^*)^2} \text{var}_t(\pi_{t+1}^{ind}) \\ + (r^i b^{i,d})^2 \sigma_v^2 + k^{2\alpha} \text{var}_t(A_{t+1}) \\ - \frac{2r^i b^i (r^i b^{i,d} + r^n b^{n,d} + r^m m^d)}{(1 + \pi^*)^2} \text{cov}_t(\pi_{t+1}, \pi_{t+1}^{ind}) \end{array} \right)$$

Therefore, by Equation (A1), the utility loss of generation t can be written as follows:

$$(A9) \quad \text{Loss}_t \approx -(1/2) \chi_t \left[\begin{array}{l} \text{var}_t(\pi_{t+1}) + \psi \text{var}_t(\pi_{t+1}^{ind}) - 2\Lambda \text{cov}_t(\pi_{t+1}, \pi_{t+1}^{ind}) \\ + \eta \sigma_v^2 + \Omega \text{var}_t(A_{t+1}) \end{array} \right]$$

where $\chi_t \equiv \frac{\delta(r^i b^{i,d} + r^n b^{n,d} + r^m m^d)^2}{(1 + \pi^*)^2 (E_t c_{t+1,0})^{(1+\delta)}} > 0$, and $E_t c_{t+1,0}$ is given by Equation (A7).

Constant coefficients are defined as follows:

$$(A10) \quad \psi \equiv (r^i b^{i,d})^2 / (r^i b^{i,d} + r^n b^{n,d} + r^m m^d)^2$$

$$(A11) \quad \Lambda \equiv r^i b^{i,d} (r^i b^{i,d} + r^n b^{n,d} + r^m m^d)$$

$$(A12) \quad \eta \equiv (1 + \pi^*)^2 \psi$$

$$(A13) \quad \Omega \equiv (1 + \pi^*)^2 k^{2\alpha} / (r^i b^{i,d} + r^n b^{n,d} + r^m m^d)^2$$

Note that in the special case when indexed bonds and capital are excluded from the model (i.e. $b^{i,d} = k = 0$), the loss in lifetime utility is given by $-(1/2)\chi_t \text{var}_t(\pi_{t+1})$ since the constants in equations (A10) to (A13) are all equal to zero.

Appendix B: Steady-state and market-clearing conditions

Deterministic steady state

The deterministic steady state is given by the following set of equations:

$$(B1) \quad c_Y + b^{i,d} + b^{n,d} + m^d + k = \omega(1 - \tau^j), \quad \text{for } j \in (IT, PLT)$$

$$(B2) \quad c_O = Ak^\alpha + r^i b^{i,d} + r^n b^{n,d} + r^m m^d$$

$$(B3) \quad R = (1 + \pi^*)r^n$$

$$(B4) \quad r^i = \frac{(1 + \pi^*)}{(1 + \pi^{ind})} r$$

$$(B5) \quad r^m = 1/(1 + \pi^*)$$

$$(B6) \quad g = \tau^j \omega + (1 - r^i)b^{i,s} + (1 - r^n)b^{n,s} + m^d \pi^*/(1 + \pi^*)$$

$$(B7) \quad \pi^{ind} = \pi^*$$

$$(B8) \quad m^s = m^d$$

$$(B9) \quad m^d = \theta c_Y$$

$$(B10) \quad b^{i,d} = b^{i,s} = a \times b^s$$

$$(B11) \quad b^{n,d} = b^{n,s} = (1 - a) \times b^s$$

$$(B12) \quad b^d = b^{i,d} + b^{n,d} = b^s = \frac{\omega(1 - \tau^j) - (1 + r^m)m^d - (1 + Ak^\alpha)k}{1 + r}$$

(from bond supply rule, $c_Y^{-\delta} = c_O^{-\delta}$)

$$(B13) \quad c_Y^{-\delta} = c_O^{-\delta} \left((1 + \theta)r^i - \frac{\theta}{1 + \pi^*} \right)$$

$$(B14) \quad c_Y^{-\delta} = c_O^{-\delta} \left((1 + \theta)r^n - \frac{\theta}{1 + \pi^*} \right)$$

$$(B15) \quad r^i = r^n = r = \frac{1 + \theta + \pi^*}{(1 + \pi^*)(1 + \theta)} \quad (\text{implied by the previous two equations})$$

$$(B16) \quad r^k = \alpha A k^{\alpha-1} = r^n = r^i \quad (\text{from the Euler equations for capital and bonds})$$

$$(B17) \quad A = A_{mean}$$

Market-clearing conditions

A monetary equilibrium in the OLG economy is a set of allocations

$\{c_{t,Y}, c_{t,O}, b_t^{i,d}, b_t^{i,s}, b_t^{n,d}, b_t^{n,s}, k_t, m_t^d, m_t^s, g_t, \pi_t, \pi_t^{ind}, r_t^n, R_t, r_t^m, \tau^j\}_{t=1}^T$ with the following

properties for all t :

(1) Allocations $\{c_{t,Y}, c_{t+1,O}, b_t^{i,d}, b_t^{n,d}, k_t, m_t^d\}_{t=1}^T$ solve the maximisation problem of a young consumer born at time t ;

(2) The goods, money and bond markets clear:

$$(B18) \quad \omega + A_t k_{t-1}^\alpha = c_{t,Y} + c_{t,O} + g_t + k_t$$

$$(B19) \quad m_t^d = m_t^s$$

$$(B20) \quad b_t^{i,d} = b_t^{i,s}$$

$$(B21) \quad b_t^{n,d} = b_t^{n,s}$$

(3) The government budget constraint and long run government spending target are satisfied:

$$(B22) \quad g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$$

$$(B23) \quad E(g_t) = g^*$$

(4) The CIA constraint holds with strict equality:

$$(B24) \quad m_t^d = \theta c_{t,Y}$$

Appendix C: Model listing

- (C1) $u_t = \frac{c_{t,Y}^{1-\delta}}{1-\delta} + E_t \frac{c_{t+1,O}^{1-\delta}}{1-\delta}$ Lifetime utility of generation t
- (C2) $c_{t,Y} + b_t^{n,d} + b_t^{i,d} + m_t^d + k_t = \omega(1-\tau^j)$ Budget constraint faced by young
- (C3) $c_{t+1,O} = A_{t+1}k_t^\alpha + r_{t+1}^i b_t^{i,d} + r_{t+1}^n b_t^{n,d} + r_{t+1}^m m_t^d$ Budget constraint faced when old
- (C4) $\ln A_t = (1-\rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t$ Productivity
- (C5) $m_t^d = \theta c_{t,Y}$ CIA constraint
- (C6) $r_{t+1}^m \equiv 1/(1 + \pi_{t+1})$ Real return on money balances
- (C7) $r_{t+1}^n = R_t r_{t+1}^m$ Real return on nominal bonds
- (C8) $r_{t+1}^i = r_t \times \left[\frac{(1 + \pi_{t+1}^{ind})}{(1 + \pi_{t+1})} + v_{t+1} \right]$ Real return on indexed bonds
- (C9) $c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^n - \theta r_{t+1}^m \right) \right)$ Euler equation for nominal bonds
- (C10) $c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^i - \theta r_{t+1}^m \right) \right)$ Euler equation for indexed bonds
- (C11) $c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) \alpha A_{t+1} k_t^{\alpha-1} - \theta r_{t+1}^m \right) \right)$ Euler equation for capital
- (C12) $g_t = \tau^j \omega + m_t^s - r_t^m m_{t-1}^s + b_t^{i,s} - r_t^i b_{t-1}^{i,s} + b_t^{n,s} - r_t^n b_{t-1}^{n,s}$
Government budget constraint
- (C13) $E(g_t) = g^*$ Long run government spending target

$$(C14) \quad \ln(m_t^s / m_{t-1}^s) = \pi^* + \sum_{i=1}^{30} \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t \quad \text{IT money supply rule}$$

$$(C15) \quad \ln(m_t^s / m_{t-1}^s) = \pi^* + \varepsilon_{30,t} - \varepsilon_{30,t-1} + \ln(c_{t,Y} / c_{t-1,Y}) - \pi_t \quad \text{PLT money supply rule}$$

$$(C16) \quad U^{society} = \frac{1}{T} E \left[\sum_{t=1}^T u_{t,Y}(c_{t,Y}) + \sum_{t=1}^T u_{t,O}(c_{t,Y}) \right] \quad \text{Social welfare}$$

$$(C17) \quad \pi_t^{ind,IT} = \pi^* + \sum_{i=1}^{30} \varepsilon_{i,t}^{ind} \quad \text{Inflation rate to which index bonds are linked (IT)}$$

$$(C18) \quad \pi_t^{ind,PLT} = \pi^* + \varepsilon_{30,t}^{ind} - \varepsilon_{30,t-1}^{ind} \quad \text{Inflation rate to which bonds are linked (PLT)}$$

$$(C19) \quad c_{t,Y}^{-\delta} = E_t(c_{t+1,O}^{-\delta}) \quad \text{Total bond supply rule (implies } b_t^s \text{)}$$

$$(C20) \quad b_t^s = b_t^{i,s} + b_t^{n,s} \quad \text{Total bond supply definition}$$

$$(C21) \quad m_t^d = m_t^s \quad \text{Money market equilibrium}$$

$$(C22) \quad b_t^{n,d} = b_t^{n,s} = (1-a)b_t^s \quad \text{Market-clearing in nominal bonds}$$

$$(C23) \quad b_t^{i,d} = b_t^{i,s} = ab_t^s \quad \text{Market-clearing in indexed bonds}$$

$$(C24) \quad \omega + A_t k_{t-1}^\alpha = c_{t,Y} + c_{t,O} + g_t + k_t \quad \text{Market-clearing in goods}$$

Appendix D – An approximate first-order condition for optimal indexation

In the main text it is argued that, as an approximation, the indexation share a will be chosen to solve the following minimisation problem:

$$(D1) \quad \min_a \frac{1}{2} \left| U_{c_o c_o}^{society} \right| \text{var}(c_{t,o}) + t.i.p.$$

where *t.i.p.* stands for ‘terms independent of policy’ and the absolute value of the second derivative of the social welfare function with respect to $c_{t,o}$ (evaluated at its unconditional mean) is given by $\left| U_{c_o c_o}^{society} \right| = \delta (Ec_{t,o})^{-(1+\delta)}$.

The first-order condition for this problem is given by

$$(D2) \quad \frac{1}{2} (Ec_{t,o})^{-2(1+\delta)} \left[\frac{\partial \text{var}(c_{t,o})}{\partial a} \times (Ec_{t,o})^{1+\delta} - (1+\delta)(Ec_{t,o})^\delta \text{var}(c_{t,o}) \times \frac{\partial Ec_{t,o}}{\partial a} \right] = 0$$

Hence the optimal indexation share will satisfy the following equality:

$$(D3) \quad \frac{\partial \text{var}(c_{t,o})}{\partial a} \times (Ec_{t,o})^{1+\delta} = (1+\delta)(Ec_{t,o})^\delta \text{var}(c_{t,o}) \times \frac{\partial Ec_{t,o}}{\partial a}$$

Rearranging Equation (D3) for $\partial \text{var}(c_{t,o}) / \partial a$ yields

$$(D4) \quad \frac{\partial \text{var}(c_{t,o})}{\partial a} = (1+\delta) \left[\frac{\text{var}(c_{t,o})}{Ec_{t,o}} \right] \times \frac{\partial Ec_{t,o}}{\partial a}$$

Hence iff $\text{var}(c_{t,o})/Ec_{t,o} \approx 0$, the first-order condition for the optimal indexation share can be approximated by¹²⁵

¹²⁵ Under the baseline calibration $\text{var}(c_{t,o})/Ec_{t,o}$ is approximately 4.6E-4 under IT and lower under PLT. Note that $\partial Ec_{t,o} / \partial a$ will also be close to zero given that indexed and nominal bonds are priced to give equivalent expected utility (at the margin). For this reason, mean consumption by old generations is essentially identical under IT and PLT, despite the substantial optimal indexation differential (see Table 3.12).

$$(D5) \quad \frac{\partial \text{var}(c_{t,o})}{\partial \alpha} \approx 0$$

Thus, the optimal choice of the indexation share will (approximately) minimise consumption volatility across old generations.

Appendix E – Deriving an analytical expression for the optimal indexation share

In this appendix, an approximate expression for the optimal indexation share is derived by minimising the consumption variance across old generations. As noted in the main text, the key terms in the consumption variance are given by

$$(E1) \quad \text{var}(c_{t,O}) \approx \text{var}(y_{t,O}) + \text{var}(r_t^{\text{tot}} b_{t-1}^s) + \text{var}(r_t^m m_{t-1}^d) + 2 \text{cov}(r_t^{\text{tot}} b_{t-1}^s, r_t^m m_{t-1}^d)$$

where $r_t^{\text{tot}} \equiv ar_t^i + (1-a)r_t^n$ is the overall return on old generations' bond portfolios.

Equation (E1) can be written in terms of the indexation share as follows:

$$(E2) \quad \begin{aligned} \text{var}(c_{t,O}) \approx & \text{var}(y_{t,O}) + a^2 \text{var}(r_t^i b_{t-1}^s) + (1-a)^2 \text{var}(r_t^n b_{t-1}^s) + \text{var}(r_t^m m_{t-1}^d) \\ & + 2a(1-a) \text{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s) + 2a \text{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) \\ & + 2(1-a) \text{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) \end{aligned}$$

Minimising Equation (E2) with respect to the indexation share a gives following first-order condition:

$$(E3) \quad \frac{\partial \text{var}(c_{t,O})}{\partial a} = 2 \left[\begin{aligned} & a \text{var}(r_t^i b_{t-1}^s) - (1-a) \text{var}(r_t^n b_{t-1}^s) + (1-2a) \text{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s) \\ & + \text{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) - \text{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) \end{aligned} \right] = 0$$

Solving Equation (E3) for the optimal indexation share a^* gives the expression reported in the main text, i.e.

$$(E4) \quad a^* \approx \frac{\text{var}(r_t^n b_{t-1}^s) + \text{cov}(r_t^n b_{t-1}^s, r_t^m m_{t-1}^d) - \text{cov}(r_t^i b_{t-1}^s, r_t^m m_{t-1}^d) - \text{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)}{\text{var}(r_t^i b_{t-1}^s) + \text{var}(r_t^n b_{t-1}^s) - 2 \text{cov}(r_t^i b_{t-1}^s, r_t^n b_{t-1}^s)}$$

Chapter 4

Estimating the impact of price-level targeting on the long-term inflation risk premium

4.1 Introduction

As noted in Chapter 1, the monetary policy literature has argued that a change in monetary policy regime from inflation targeting (IT) to price-level targeting (PLT) would lead to a substantial reduction in the inflation risk premium on long-term nominal bonds (e.g. Crawford *et al.* 2009; Stuber 2001). Such a reduction would have potential benefits for the economy, including an increase in aggregate investment and output through a lower cost of capital (Meh *et al.* 2008a), and a lower cost of issuing nominal government debt, which would enable government spending to be raised for a given level of taxes, or for taxes to be lowered for a given level of expenditure. Somewhat surprisingly, no attempts have yet been made in the monetary policy literature to estimate the impact of PLT on the inflation risk premium within a dynamic stochastic general equilibrium (DSGE) framework.

This task is taken up in the current chapter. In order to do so, the overlapping generations (OLG) models of chapters 2 and 3 are used to compute the inflation risk premium on long-term nominal government debt under IT and PLT. The models are again solved using a second-order approximation method, since the aim is to capture the level of inflation risk-premia and not their variability (which would require a third-order approximation). Consistent with the analysis in previous chapters, inflation risk premia are computed in all three versions of the model and are subjected to various sensitivity tests to allow for calibration uncertainty and heterogeneities across countries and over time.

To preview the results, a key finding is that, consistent with predictions from the literature, the inflation risk premium on long-term nominal bonds is substantially lower under PLT due to the absence of base-level drift. This general result is robust in model parameter sensitivity analysis and across models, though the absolute reduction in the inflation risk premium varies somewhat depending on the model at hand and its calibration. The intuition for PLT substantially lowering the inflation risk premium is straightforward: long-term inflation volatility is reduced by an order of magnitude

under PLT, such that risk-averse savers holding long-term nominal bonds are exposed to less consumption risk over a long horizon and therefore require less of a ‘risk premium’ as compensation.

The chapter proceeds as follows. First, past literature on the inflation risk premium is briefly discussed, including both theory and estimation results. The chapter then turns to the methodology that is used to estimate inflation risk premia in the OLG models of chapters 2 and 3. Simulation results from the OLG models are then reported and discussed in a third section, before being subjected to various robustness tests. Finally, the chapter concludes by discussing overall conclusions and policy implications. The results are also discussed further in Chapter 5, a short final chapter that puts in context the overall findings of the thesis and its policy implications.

4.2 The inflation risk premium: a brief survey

The inflation risk premium is defined in the literature as the premium, in percentage points or basis points per annum, offered by a nominal bond over an asset whose real return is fully protected against inflation risk. Economists are interested in the inflation risk premium primarily because of its potential importance for the level of long-term interest rates, and hence for investment, aggregate demand, and the sustainable rate of economic growth. A reduction in the inflation risk premium, for example, would be expected to boost investment and aggregate demand by lowering long-term interest rates – a point made by Jean-Claude Trichet in testimony before the European Parliament,¹²⁶ and by Bernanke (2006) who notes that “special factors that lower the spread between short-term and long-term rates will stimulate aggregate demand.” As noted in the introduction, a reduction in the inflation risk premium will also lower the average cost of issuing nominal government debt¹²⁷ – a potentially important consideration given that the affordability of pensions is now a major public policy issue in many developed economies (see e.g. OECD 2009a).

In order to provide a comprehensive formal definition of the inflation risk premium that is consistent with existing literature, the analytics in this section are developed

¹²⁶ See Trichet (2005).

¹²⁷ The OLG models used in this chapter capture this ‘average cost’ effect, but the impact of PLT on investment and aggregate output is quantitatively trivial in the calibrated models.

within a representative agent framework. It is then demonstrated in the next section that an analogous expression for the inflation risk premium can be obtained from the OLG models of chapters 2 and 3. To this end, consider a risk-averse representative agent whose quarterly discount factor is β and whose marginal utility of consumption in period t is denoted MU_t . The one-period real stochastic discount factor at time t is then given by $m_{t+1,t} \equiv \beta(MU_{t+1}/MU_t)$.¹²⁸ Using the real stochastic discount factor, we can price real and nominal bonds using the fundamental equilibrium asset pricing equation (see Cochrane, 2001). This equation states that any asset i can be priced according to $P^i = E[mX^i]$, where X^i is the payoff of asset i , P^i is its price, and m is the real stochastic discount factor for the date when the asset pays off. When dealing with assets such as bonds it is often convenient to write this equation in the form $1 = E[mr^i]$, where $r^i \equiv X^i/P^i$ is the gross real return on asset i .

According to this equation, a one-period riskless real bond will have a real return that satisfies

$$(4.1) \quad 1 = r_{t,1}^f E_t[m_{t+1,t}]$$

where the subscript $t,1$ indicates that the payoff in real terms next period is known with certainty at time t .

Similarly, the real return on a riskless n -period real bond will satisfy

$$(4.2) \quad 1 = r_{t,n}^f E_t[m_{t+n,t}]$$

where the subscript t,n indicates that the payoff in real terms n periods ahead is known with certainty at time t , and $m_{t+n,t} \equiv \beta^n(MU_{t+n}/MU_t)$ is the n -period-ahead real stochastic discount factor.

¹²⁸ The first subscript refers to the numerator and the second subscript to the denominator. Hence, for example, $m_{t+n,t} \equiv \beta^n(MU_{t+n}/MU_t)$.

Now consider an n -period nominal bond with zero default risk – that is, a bond whose real return is certain but for inflation risk. In this case, the nominal return $R_{t,n}$ is known for certain at time t and must satisfy

$$(4.3) \quad 1 = R_{t,n} E_t \left[m_{t+n,t} \frac{P_t}{P_{t+n}} \right] = R_{t,n} E_t \left[m_{t+n,t} \frac{1}{1 + \pi_{t+n}} \right]$$

where π_{t+n} is the rate of inflation between period t and period $t+n$.

Intuitively, the payoff on an n -period nominal bond has two components. The first is the known nominal return $R_{t,n}$, and the second is the ‘inflation discount’ P_t/P_{t+n} , which represents the extent to which inflation over the life of the bond erodes its purchasing power upon maturity. If there is inflation risk, the inflation discount will not be known with certainty at time t and will, in general, be correlated with the real n -period ahead stochastic discount factor. This correlation gives rise to an ‘inflation risk premium’.

In order to obtain an expression for the inflation risk premium, note that Equation (4.3) can be written as follows:

$$(4.4) \quad 1 = E_t m_{t+n,t} E_t r_{t+n,n} + R_{t,n} \text{cov}_t \left(m_{t+n,t}, (1 + \pi_{t+n})^{-1} \right)$$

where $r_{t+n,n} \equiv R_{t,n}/(1 + \pi_{t+n})$ is the real rate of return on the n -period nominal bond.

The n -period inflation risk premium at time t , denoted $irp_{t,n}$, is then given by the difference between the expected real return on the n -period nominal bond and the n -period riskless real return. Using Equation (4.4) and Equation (4.2) we arrive at the following expression for the n -period inflation risk premium:

$$(4.5) \quad irp_{t,n} \equiv E_t r_{t+n,n} - r_{t,n}^f = -r_{t,n}^f \text{cov}_t \left(m_{t+n,t}, R_{t,n} (1 + \pi_{t+n})^{-1} \right)$$

where $r^f = 1/E(m)$ from Equation (4.2).

Intuitively, if consumption and inflation are negatively correlated (i.e. higher inflation erodes consumption purchasing power), then the inflation risk premium will be positive such that nominal bonds compensate risk-averse consumers with an expected return above the n -period riskless real return. Moreover, it is notable that the inflation

risk premium drives a wedge between nominal and real rates, such that the Fisher equation cannot be recovered in its traditional form.¹²⁹

There are a number of papers in the literature that have estimated the inflation risk premium in general equilibrium models using identical or similar expressions to Equation (4.5). Indeed, the fact that the inflation risk premium cannot be observed directly from data is often cited as an argument in favour of estimating its value using a DSGE framework. Buraschi and Jiltsov (2005), for example, estimated the US inflation risk premium over the postwar period using a continuous-time flex-price general equilibrium model in which monetary policy and taxes are endogenous. Their results suggest that the ten-year inflation risk premium varies considerably over the business cycle but has averaged 70 basis points over the postwar period.

On the other hand, Ravenna and Seppälä (2007) use a New Keynesian model that is parameterised to match the nominal term structure in the US over the postwar period, and conclude that inflation risk premia have been small on average at 10 basis points or less up to a 5-year horizon. One potential explanation for this stark contrast in results is that, under standard calibrations, the aggregate price level displays substantial nominal ‘stickiness’ in New Keynesian models, such that the implied level of inflation risk from model simulations is fairly low. Indeed, as noted by Le *et al.* (2008) in the context of the Smets-Wouters model of the US, inflation and other nominal variables are too stable relative to the data, leading to rejection of the model as a whole based on the method of indirect inference.¹³⁰

Estimates of the inflation risk premium in the literature have also been derived using a no-arbitrage macro-finance approach. For example, Campbell and Viceira (2001) develop a two-factor model of the term structure of interest rates that is augmented to fit both equity and bond returns, with the inflation risk premium measured by the term premium of ten-year nominal bonds over ten-year indexed bonds. Using quarterly US data covering the postwar period, Campbell and Viceira estimate the model by

¹²⁹ If the inflation risk premium is zero (i.e. $\text{cov}(m_{t+n,t}, (1 + \pi_{t+n})^{-1}) = 0$), the Fisher equation can be obtained by substituting the right hand side of Equation (4.4) into the left hand side of Equation (4.2) and approximating $[E_t(1/(1 + \pi_{t+n}))]^{-1}$ with $1 + E_t\pi_{t+n}$ in order to solve for the nominal rate.

¹³⁰ Le *et al.* do show, however, that a weighted New Keynesian-New Classical version of the model cannot be rejected on key macroeconomic variables over the Great Moderation period.

maximum likelihood using the Kalman filter and conclude that the ten-year inflation risk premium has averaged 110 basis points. Likewise, Ang *et al.* (2008) estimate an essentially affine term structure model with regime-switching over the postwar period and reach a similar conclusion, namely, that inflation risk-premia in the US are positive and around 115 basis points on average. Lastly, D'Amico *et al.* (2008) estimated a no-arbitrage term structure model using information from US Treasury Inflation-Protected Securities (TIPS) in order to control for the dynamics of real yields. They found a lower average inflation risk premium of around 50 basis points, consistent with their sample covering the Great Moderation period when inflation variability fell dramatically.¹³¹ In terms of European countries, estimates of inflation risk premia tend to be lower,¹³² though a recent study for the UK covering the post-IT period reports a substantial five-year inflation risk premium of around 100 basis points (see Joyce *et al.* 2010).

One reason for the continued use of *ad hoc* term structure models in the literature is the inability of general equilibrium models to match risk-premia on nominal bonds under empirically plausible calibrations – the so-called ‘bond premium puzzle’ (see e.g. Rudebusch and Swanson 2008). However, the general equilibrium and macro-finance approaches to bond risk premia have tended to reach broadly similar conclusions regarding the inflation risk premium. In particular, whilst there is no clear consensus on the magnitude of the inflation risk premium, the available evidence points to a positive and economically non-trivial premium at medium- and long-term bond maturities.

In terms of IT versus PLT, there is less literature on the inflation risk premium. Stuber (2001) and Crawford *et al.* (2009) both argue that PLT would lead to a non-trivial reduction in longer-term inflation risk-premia, but stop short of modelling the impact explicitly. A similar result has been obtained formally by Meh *et al.* (2008a),¹³³ who consider a small open economy model in which firms can finance investment using short-term or long-term nominal debt contracts. Firms have the choice to default on

¹³¹ See also Hördahl (2008) for a similar finding.

¹³² This result is probably due, in part, to the samples used in the studies covering the Great Moderation period. For example, Garcia and Werner (2010) estimate that from 1999 to 2006, the 5-year inflation risk premium in the Euro Area averaged 25 basis points.

¹³³ Cited by Crawford *et al.* (2009).

both types of debt, so there is a risk premium in the cost of capital. Although IT and PLT are not modelled explicitly, the former is assumed to be represented by a ‘high’ level of long-term price-level uncertainty, and the latter by a ‘low’ level of uncertainty. Reducing long-term price-level uncertainty has two effects. First, it lowers the risk premium on debt (since there is a lower probability of default), and with it the cost of capital. Second, a reduction in long-term price-level uncertainty leads to an increase in the fraction of agents using long-term nominal debt, which in turn boosts investment and output. These results suggest that switching from IT to PLT could have a beneficial impact on investment and output by lowering the inflation risk premium. However, these results should be taken with caution because they apply to debt contracts with default risk (hence excluding most government bonds from a long run perspective) and in a model where the price level is exogenous.

In the present chapter, the long-term inflation risk premium on nominal government bonds is computed directly in the OLG models that were calibrated and estimated in the previous chapters – models in which the price level is endogenously determined and where IT and PLT are modelled explicitly.

4.3 The inflation risk premium in the overlapping generations models

In order to obtain an expression for the inflation risk premium in the three OLG models of chapters 2 and 3, it is necessary to first define the risk-free real return that would be received on a perfectly indexed bond.

Recall that the Euler equation for nominal bonds in all three models is given by

$$(4.6) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1 + \theta) r_{t+1}^n - \theta r_{t+1}^m \right) \right)$$

where $r^n = R/(1 + \pi)$ is the real return on a nominal bond and $r^m = 1/(1 + \pi)$ is the real return on money balances.

The additional term on the right hand side enters the Euler equation for any asset, risky or riskless, due to the presence of the cash-in-advance (CIA) constraint in the model, as discussed in Chapter 2. Note also that a single period in the model is

interpreted as 30 years, which amounts to a horizon of $n=120$ quarters in terms of the analytical representative agent framework discussed in the previous section.

A riskless real bond with known return r^f would thus have to satisfy the following Euler equation:

$$(4.7) \quad c_{t,Y}^{-\delta} = E_t \left(c_{t+1,O}^{-\delta} \left((1+\theta)r_t^f - \theta r_{t+1}^m \right) \right) = (1+\theta)r_t^f E_t \left(c_{t+1,O}^{-\delta} \right) - \theta E_t \left(c_{t+1,O}^{-\delta} r_{t+1}^m \right)$$

Equations (4.6) and (4.7) can be written in the form of the fundamental asset pricing equation as follows:

$$(4.8) \quad 1 = E_t \left(m_{t+1,t}^{OLG} \tilde{r}_{t+1}^n \right)$$

$$(4.9) \quad 1 = E_t \left(m_{t+1,t}^{OLG} \tilde{r}_{t+1}^f \right)$$

where $\tilde{r}_{t+1}^n \equiv (1+\theta)r_{t+1}^n - \theta r_{t+1}^m$; $\tilde{r}_{t+1}^f \equiv (1+\theta)r_t^f - \theta r_{t+1}^m$; and $m_{t+1,t}^{OLG} \equiv c_{t+1,O}^{-\delta} / c_{t,Y}^{-\delta}$.

To derive the inflation risk premium in the OLG model, we need to expand equations (4.8) and (4.9). Doing so gives the following expressions:

$$(4.10) \quad 1 = (1+\theta)E_t m_{t+1,t}^{OLG} E_t r_{t+1}^n + (1+\theta)\text{cov}_t(m_{t+1,t}^{OLG}, r_{t+1}^n) - \theta E_t(m_{t+1,t}^{OLG} r_{t+1}^m)$$

$$(4.11) \quad 1 = (1+\theta)r_t^f E_t m_{t+1,t}^{OLG} - \theta E_t(m_{t+1,t}^{OLG} r_{t+1}^m)$$

Therefore, the inflation risk premium in the OLG model is given by

$$(4.12) \quad irp_{t,OLG} \equiv E_t r_{t+1}^n - r_t^f = -r_t^f \text{cov}_t(m_{t+1,t}^{OLG}, r_{t+1}^n)$$

Using the definition of r^n , this equation can be written in the same general form as Equation (4.5), or

$$(4.13) \quad irp_{t,OLG} = -r_t^f \text{cov}_t(m_{t+1,t}^{OLG}, R_t(1+\pi_{t+1})^{-1})$$

The inflation risk premium in the OLG models is therefore analogous to one from a representative agent framework, except that the stochastic discount factor now

depends on the ratio of marginal utility in old age to marginal utility in youth (and is not ‘discounted’ by β), and one period t is 30 years, which is equivalent to setting $n = 120$ quarters in terms of Equation (4.5). Intuitively, the fundamental nature of the inflation risk premium is unchanged because the only key difference between an OLG framework and a representative agent one when pricing assets is the horizon over which risk matters; in the former it is fixed by the horizon from ‘youth’ to ‘old age’ because generations live for two periods of fixed length, whilst in the latter it is variable and depends on the term to maturity n of the bond held.

In Appendix A of this chapter, the inflation risk premium term in Equation (4.13) is approximated using a Taylor series expansion in order to highlight its key determinants. Other things being equal, the inflation risk premium increases with the magnitude of nominal bond and money holdings, the extent of risk aversion, and the level of inflation risk. Moreover, in the model with indexed bonds, an increase in the share of indexed bonds (or, equivalently, a decrease in the share of nominal bonds) will lower the inflation risk premium. In the sensitivity analysis section of this chapter, the quantitative impacts of these factors on inflation risk premia are investigated.

In order to calculate inflation risk premia in the OLG models, two extra equations are added to each of the three models – one to calculate the inflation risk premium; and a second which defines the risk-free rate as in Equation (4.7). The inflation risk premium is then calculated as the unconditional mean from 5,000 simulations of 1,000 periods that are solved to second-order using Dynare++. The calibration of the model and its other equations are unchanged and are not repeated here. The inflation risk premia results for each of the models are reported in the next section, and the robustness of these baseline results is then investigated in the sensitivity analysis section that follows.

4.4 Simulation results

Table 4.1 reports the 30-year inflation risk premium under IT and PLT from the three OLG models: Model 1, which includes only nominal bonds (and money); Model 2 in which consumers can also invest in productive capital; and Model 3 in which indexed

bonds are additionally introduced, and where the extent of nominal indexation is determined endogenously in response to monetary policy.

Table 4.1 – Inflation risk premia

Model	IT	PLT
(1)	41.0	1.4
(2)	20.9	0.7
(3)	5.8	0.5

Note: Figures are in basis points

The estimated inflation risk premium is positive in all three models. As predicted by the PLT literature, the inflation risk premium is somewhat lower under PLT than IT. Intuitively, long-term inflation risk is reduced by an order of magnitude under PLT, because it precludes base-level drift in the price level, hence preventing the accumulation of uncertainty over the bond holding horizon. The inflation risk premium is largest in Model (1) in which consumers hold only nominal assets. The reason is that consumers' optimal holdings of nominal bonds are high compared to the other two models, because they do not have the option of investing in capital. As a result, consumers are somewhat more exposed to inflation risk than in the other two models, and require a much larger risk premium for holding nominal bonds.

The inflation risk premium under IT of 41 basis points is non-trivial, but it should be borne in mind that this figure applies to the return over a 30-year horizon. Expressed in yearly terms (i.e. dividing by 30), this implies an inflation risk premium of around 1.5 basis points per annum. This estimate is similar to estimates of the 10-year risk premium on nominal bonds from 'state of the art' New Keynesian DSGE models (see Rudebusch and Swanson, 2008; Christiano, Eichenbaum and Evans, 2005),¹³⁴ but notably the model does not rely upon consumption habits, which are known to amplify asset risk premia substantially (Campbell and Cochrane, 1999).^{135,136}

¹³⁴ The risk (or term) premium on nominal bonds is equal to the inflation risk premium in Model (1), since inflation variations are the only source of risk.

¹³⁵ The reason is that habit formation increases the curvature of consumer utility, making second-order effects of greater importance. For example, under the most commonly used specification of habits, the coefficient of relative risk aversion (evaluated at steady-state) is given by $\delta/(1-h)$ rather than δ , where $0 < h < 1$ is the habit persistence parameter on past aggregate consumption in the utility function.

¹³⁶ Similarly, Lungu and Minford (2006) find that an OLG model can better match the 'equity premium' without the need for habits in consumption.

By contrast, the inflation risk premium under PLT is close to zero at only 1.4 basis points over a 30-year horizon – around 30 times lower than under IT and equivalent to only 0.05 basis points in per annum terms. As mentioned above, PLT reduces the inflation risk premium substantially because it eliminates base-level drift, such that inflation uncertainty does not accumulate over the 30-year holding horizon for bonds. However, it was noted in Chapter 2 that the unconditional variance of inflation is 15 times lower under PLT, rather than the ratio of 30 suggested by the risk premium results. The reason for this difference is that what matters for the inflation risk premium is conditional volatility of inflation – see equations (A7) and (A8) in Appendix A – because bonds held from period t to $t+1$ must be priced at time t when they are purchased. This distinction is crucial under PLT because past shocks to inflation are offset in order to return the price level to its target path, yet this induced volatility will not affect the inflation risk premium directly, because these past shocks are known by young generations at the time that they purchase nominal bonds. A simple way to see this result analytically is to note that whilst conditional variance of inflation under IT is the same as the unconditional variance of $30\sigma^2$, the conditional variance of inflation under PLT is only σ^2 – one-half of the unconditional variance of $2\sigma^2$ and 30 times lower than under IT. This result accounts for the factor of approximately 30 by which the inflation risk premium is lower under PLT.

Adding capital into the model roughly halves the inflation risk premium in absolute terms to around 21 basis points, as can be seen from the second row of results in Table 4.1. This reduction occurs because consumers' portfolios are diversified between both real and nominal assets, such that nominal bonds account for a smaller share than in Model (1), implying a reduction in consumption covariance risk – a result that can be seen formally from the risk premium expression in Equation (A8) of Appendix A.¹³⁷ Intuitively, the smaller are consumers' holdings of a particular risky asset, the less risk they are exposed to, *ceteris paribus*, from variations in the return on that asset. That portfolio diversification drives the reduction the inflation risk premium is confirmed formally by the fact that simulating Model (2) without any

¹³⁷ In particular, holdings of nominal bonds fall. This reduction in nominal bond holdings can be seen clearly from tables 2.3 and 2.9 in Chapter 2.

productivity risk (i.e. an innovation variance of zero) leaves the estimated inflation risk premium essentially unchanged.¹³⁸

Introducing indexed bonds into the model with capital reduces the IT inflation risk premium substantially from around 21 to 6 basis points. This sharp reduction occurs for two reasons. First, and most simply, indexed bonds enable consumers to protect their savings against inflation risk better than nominal bonds and therefore reduce overall consumption risk in old age. Indeed, recall that since indexed bonds are a considerably better store of value than nominal bonds under IT, the optimal indexation share is relatively high at 76 per cent. This has the effect of reducing the inflation risk premium substantially, since it means that only a relatively small portion of overall consumption variability arises due to the real payoff on nominal bonds fluctuating with inflation – a result which can be seen clearly from Equation (A7) by noting the impact of the indexation share on the inflation risk premium. By contrast, the proportional reduction in the inflation risk premium under PLT is much smaller. The reasoning is that optimal indexation is relatively low in this case (26 per cent), such that a large proportion of consumers' bond portfolios remain directly exposed to inflation risk.

Second, under the baseline calibration, real returns on indexed and nominal bonds are slightly negatively correlated, because 'biased' inflation tends to overshoot 'true' inflation. As a result, consumption covariance risk can actually be hedged by holding indexed bonds. This has the effect of directly reducing the inflation risk premium, as can be seen clearly from Equation (A7) in Appendix A. It is worth noting for comparative purposes that the 'equity premium' in models (2) and (3) is around 27 basis points. Hence the IT inflation risk premium is non-trivial in relative terms at more than two-thirds of the equity premium in the model with capital and nominal bonds, and more than one-fifth in the extended model with indexed bonds.¹³⁹

To summarise, the long-term inflation risk premium is economically non-trivial under IT due to base-level drift, but varies substantially depending on consumers' asset

¹³⁸ This result is also consistent with Equation (A8) in Appendix A, because the productivity innovation variance does enter the expression for the inflation risk premium.

¹³⁹ The equity risk premium was measured by the unconditional mean differential between the expected return on capital and the risk-free real rate r^f .

holdings and with the extent of direct exposure to inflation risk. The potential benefits from a reduction in inflation risk premium under PLT are therefore likely to be greatest in countries where pure nominal assets are of most importance. For example, in countries like Germany where holdings of long-term nominal assets are an important source of retirement income (Berkel and Börsch-Supan, 2004; Garcia, 2008), the inflation risk premium could be sizeable. On the other hand, in countries like the US and UK where holdings of indexed bonds are non-trivial and real assets play a more important role (OECD, 2009b), the inflation risk premium is likely to be lower.

Whilst the model is unlikely to provide an accurate estimate of the magnitude of inflation risk-premia – due to the ‘bond premium puzzle’ and the general inability of DSGE models to price risky assets plausibly – the result that PLT leads to marked proportional reduction in the inflation risk premium is robust across all three models and should provide a more accurate estimate of the implications of PLT.¹⁴⁰ Indeed, PLT reduces the inflation risk premium by over 90 per cent in all three cases, with the largest reductions in the models without indexed bonds. The next section investigates the sensitivity of these results to key calibrated parameters and variances.

4.5 Sensitivity analysis

This section investigates sensitivity to the calibrated values of parameters which the analytical expressions derived in Appendix A suggest may play an important role for inflation risk premia. These parameters include the following: the coefficient of relative risk aversion δ ; the CIA coefficient θ (which determines the size of money holdings); the money supply innovation variance (which determines the extent of inflation volatility); and, in the model with indexed bonds, the indexation share a . In all simulations, the tax rate on young consumers was adjusted to ensure that long run government spending target was met, and in the model with indexed bonds the

¹⁴⁰ The argument here is that the main issue with pricing risky assets in a general equilibrium framework is that implausibly high levels of risk aversion are necessary to match the data (e.g. Mehra and Prescott, 1985). However, since the inflation risk premium depends on the covariance term $\text{cov}(m, r^b)$, which based on a first-order approximation of m around steady-state is equal to $\delta \text{cov}(c, r^b)$ (where c is future consumption), taking the ratio of two risk premia effectively ‘cancels out’ risk aversion.

optimal indexation share was recalculated in each simulation in order to give a new optimal commitment Ramsey equilibrium.

Additionally, the impact of changing the asset holding horizon is investigated at the end of this section in order to determine how much the inflation risk premium results vary with the term-to-maturity on nominal bonds. This is an important consideration given that nominal bonds are issued at various long-term maturities in practice, and also because long-term inflation risk increases with the forecast horizon under IT but is bounded under PLT.

4.5.1 Model (1): nominal bonds only

Risk aversion and money holdings

Table 4.2 reports the inflation risk premium for various combinations of the coefficient θ in the CIA constraint and the coefficient of relative risk aversion δ . The range of values considered matches that used in sensitivity analysis in Chapter 2.

Table 4.2 – Inflation risk premia sensitivity to risk aversion and money holdings

Risk aversion coefficient δ	Proportion of consumption when young held as money balances θ		
	$\theta = 0.01$	$\theta = 0.10$	$\theta = 0.25$
$\delta = 1.5$	21.3 / 0.7	20.5 / 0.7	19.5 / 0.7
$\delta = 3$	42.5 / 1.5	41.0 / 1.4	39.0 / 1.3
$\delta = 5$	70.9 / 2.4	68.4 / 2.4	65.1 / 2.2

Notes: Entries on the left (right) refer to IT (PLT). Figures are in basis points.

As is to be expected, an increase in risk aversion raises the inflation risk premium, whilst a reduction in risk aversion lowers it. For instance, for the baseline calibration of θ , raising the coefficient of relative risk aversion from 3 to 5 increases the IT risk premium by around two-thirds to 68.4 basis points, whilst the risk premium under PLT almost doubles but remains small at 2.4 basis points. For the lower risk aversion coefficient of 1.5, the inflation risk premium under IT is still non-trivial at 20.5 basis points, whilst under PLT it is rather small at 0.7 basis points. Intuitively, an increase in risk aversion means that consumers require greater risk compensation in equilibrium for holding nominal bonds of given ‘riskiness’, whilst less compensation for inflation risk is necessary if aversion to risk is lower.

Changing the CIA coefficient θ , the share of consumption when young held as money balances, has relatively little impact on the inflation risk premium. With the baseline calibration of risk aversion, for example, the ‘high’ value of θ of 0.25 lowers the IT risk premium to 39 basis points, whilst the ‘low’ value of 0.01 raises the risk premium slightly to 42.5 basis points. With regards to the PLT inflation risk premium, there is even less sensitivity at just 0.1 basis points either way. Intuitively, although an increase in θ raises money holdings – which, *ceteris paribus*, will raise the inflation risk premium as can be seen from Equation (A8) – it also necessitates a reduction in nominal bond holdings, because young generations’ endowment incomes are fixed. There are thus two opposing forces on the inflation risk premium, with the result that its overall value remains relatively unchanged.

Nominal volatility

Table 4.3 reports sensitivity to the extent of nominal volatility, as captured by the yearly money supply innovation variance. The same range of values is considered as in chapters 2 and 3. Hence sensitivity is tested to deviations of around one-quarter from the baseline money supply innovation standard deviation.

Table 4.3 – Inflation risk premia sensitivity to nominal volatility

Money supply innovation variance					
Low $\text{var}(\varepsilon_{i,t}) = 0.87 \times 10^{-4}$		Baseline $\text{var}(\varepsilon_{i,t}) = 1.45 \times 10^{-4}$		High $\text{var}(\varepsilon_{i,t}) = 2.19 \times 10^{-4}$	
IT	PLT	IT	PLT	IT	PLT
24.5	0.8	41.0	1.4	61.1	2.1

Note: Figures are in basis points.

An increase (decrease) in the money supply innovation variance raises (lowers) the inflation risk premium, since it translates into an increase (decrease) in the level of long-term inflation risk. The results are fairly sensitive to changes in the money supply innovation variance, with the inflation risk premium increasing by around one-half in the high volatility case, and falling by almost one-half with the low volatility calibration. The result that the inflation risk premium is substantially lower under PLT is strongly robust, and it is notable that the ratio of risk premia under IT and PLT –

that is to say, the proportional change in the inflation risk premium – remains roughly constant across the high and low volatility calibrations.¹⁴¹

4.5.2 Model (2): nominal bonds and capital

Money holdings and risk aversion

Table 4.4 examines the sensitivity of the inflation risk premium to the coefficient θ in the CIA constraint and the coefficient of relative risk aversion δ in the model with nominal bonds and capital.

Table 4.4 – Inflation risk premia sensitivity to risk aversion and money holdings

Coefficient of relative risk aversion δ	Proportion of consumption when young held as money balances θ		
	$\theta = 0.01$	$\theta = 0.10$	$\theta = 0.25$
$\delta = 1.5$	11.0 / 0.4	10.4 / 0.4	9.6 / 0.3
$\delta = 3$	22.1 / 0.8	20.9 / 0.7	19.3 / 0.7
$\delta = 5$	36.8 / 1.3	34.8 / 1.2	32.1 / 1.1

Note: Entries on the left (right) refer to IT (PLT). Figures are in basis points.

As in the model with only nominal bonds, IT and PLT inflation risk premia are robust to changes in the CIA coefficient θ but vary somewhat with the extent of risk aversion. With the baseline calibration for θ , for example, raising the coefficient of relative risk aversion from 3 to 5 almost doubles the IT and PLT inflation risk premia to 34.8 basis points and 1.2 basis points respectively, whilst halving the baseline risk aversion coefficient roughly halves the magnitude of inflation risk premia. The proportionate impact of changes in risk aversion on inflation risk premia is similar to that in the model with only nominal bonds, since, as shown by the approximate expressions for the inflation risk premium in Appendix A, the coefficient of relative risk aversion acts as a ‘scale factor’ for all the uncertainty terms in the inflation risk premium.

¹⁴¹ The intuition for this second finding can be seen from the expression for the inflation risk premium in Equation (A8): it is the ratio of inflation risk under PLT to that under IT that matters for the proportional change in the inflation risk premium, a ratio which does not change as the money supply innovation variance is varied.

Nominal volatility

Table 4.5 investigates sensitivity of inflation risk premia under IT and PLT to changes in the money supply innovation variance, which in turn determines the amount of inflation volatility in equilibrium.

Table 4.5 – Inflation risk premia sensitivity to nominal volatility

Money supply innovation variance					
Low $\text{var}(\varepsilon_{i,t}) = 0.87 \times 10^{-4}$		Baseline $\text{var}(\varepsilon_{i,t}) = 1.45 \times 10^{-4}$		High $\text{var}(\varepsilon_{i,t}) = 2.19 \times 10^{-4}$	
IT	PLT	IT	PLT	IT	PLT
12.4	0.4	20.9	0.7	31.0	1.1

Note: Figures are in basis points.

The inflation risk premium results are quite sensitive to the money supply innovation variance, though slightly less so than to changes in risk aversion (see Table 4.4). Raising the money supply innovation variance increases inflation volatility, and therefore increases the inflation risk premium under both IT and PLT. Lowering the money supply variance has the opposite effect. A policy implication suggested by these volatility sensitivity results – and the similar results for the Model (1) – is that the absolute size of the reduction in the inflation risk premium attainable under PLT is likely to vary somewhat across countries with the extent of nominal volatility and the success of IT in keeping inflation variability at a low level.

4.5.3 Model (3): nominal bonds, indexed bonds and capital

Given that the inflation risk premium results show little sensitivity to the CIA constraint coefficient θ in models (1) and (2), the sensitivity analysis in this section does not examine robustness with respect to this parameter. However, as in the sensitivity analysis of Chapter 3, the impact of holding indexation fixed under PLT is examined, as is the impact of indexation being set at a relatively low level as in developed economies.

Risk aversion

Table 4.6 reports the sensitivity of the IT and PLT inflation risk premia to the coefficient of relative risk aversion in Model (3).

Table 4.6 – Sensitivity to risk aversion

Coefficient of relative risk aversion	IT	PLT
$\delta = 1.5$	2.8	0.3
$\delta = 3$	5.8	0.5
$\delta = 5$	8.9	0.9

Note: Figures are in basis points.

Although the inflation risk premium is somewhat more robust in absolute terms than in the other two models, the proportionate impact of changes in risk aversion is similar: halving the coefficient of relative risk aversion to 1.5 roughly halves inflation risk premia compared to the baseline case, whilst an increase to 5 almost doubles their magnitude. It is notable also that the inflation risk premium under PLT remains at around one-tenth of the IT value as the coefficient of relative risk aversion is varied between high and low values, consistent with the approximate expression reported in Equation (A7) of Appendix A. Therefore, the result that PLT leads to a sharp proportional reduction in the inflation risk premium is robust to changes in the coefficient of relative risk aversion, despite the fact that inflation risk premia vary substantially in absolute terms.

Nominal volatility

The results in Table 4.7 report the impact of changes in nominal volatility on inflation risk premia in the model with indexed bonds. In order to hold the relative importance of imperfections in indexation constant as nominal volatility was varied, the innovation variances for biased inflation and the indexation lag were changed in tandem, as in the sensitivity analysis conducted in Chapter 3 (see Section 3.12.3).

Table 4.7 – Inflation risk premia sensitivity to nominal volatility

Money supply innovation variance					
Low $\text{var}(\varepsilon_{i,t}) = 0.87 \times 10^{-4}$		Baseline $\text{var}(\varepsilon_{i,t}) = 1.45 \times 10^{-4}$		High $\text{var}(\varepsilon_{i,t}) = 2.19 \times 10^{-4}$	
IT	PLT	IT	PLT	IT	PLT
3.5	0.3	5.8	0.5	7.7	0.8

Note: Figures are in basis points.

The results are again quite sensitive to the extent of nominal volatility, though less so than to changes in risk aversion, and somewhat less so in absolute terms than in the first two models. This result arises due to the lower absolute magnitude of the inflation risk premium when consumers can hold indexed bonds and the extent of indexation is chosen optimally in response to monetary policy. For instance, in the high volatility case, the inflation risk premium under IT rises from 5.8 basis points to 7.7 basis points, whilst in the low volatility case it falls to 3.5 basis points. The inflation risk premium under PLT is somewhat more stable in absolute terms (though still rather variable in proportional terms), varying between 0.3 and 0.8 basis points in the low and high volatility cases, compared to the baseline estimate of 0.5 basis points.

Indexation and inflation risk premia

In this section, the model is solved under two alternative assumptions, namely, (i) that indexation is fixed under PLT at the IT optimum; and (ii) that the extent of indexation of government bonds is set at a low level as in developed economies (see e.g. Campbell *et al.* 2009). For (i) the model is simply solved for the inflation risk premium under PLT when the indexation share is equal to the baseline optimal IT share, whilst for (ii) inflation risk premia are computed under both IT and PLT when the degree of indexation is set at 21 per cent, the share of indexed bonds in UK government bond portfolio as of March 2010 (DMO, 2010b). The results from these sensitivity tests are reported in Table 4.8.

Table 4.8 – Indexation and inflation risk premia

Indexation fixed at IT optimal	Optimal indexation (baseline case)	Indexation fixed at UK level
IT: 5.76 PLT: 0.20	IT: 5.76 PLT: 0.54	IT: 16.69 PLT: 0.58

Notes: Figures are in basis points

When indexation is fixed under PLT at the optimal IT share of 76 per cent, the inflation risk premium is more than halved to 0.2 basis points, because consumers are largely protected against inflation variations if they hold around three-quarters of their bond portfolios in indexed bonds. On the other hand, setting indexation at the UK share of 21 per cent has little impact on the inflation risk premium under PLT (since optimal indexation is anyhow 26 per cent), but raises the risk premium under IT substantially from 5.8 to 16.7 basis points. The reasoning here is that consumers are considerably more exposed to the high level of long-term inflation risk under IT if they hold almost four-fifths of their portfolios in nominal rather than indexed bonds.

Indeed, as is shown formally in Appendix A, a reduction in the share of indexed bonds in consumers' portfolios (and hence an increase in the share of nominal bonds) will raise the inflation risk premium given that indexation bias is relatively small in the baseline model. Intuitively, the IT inflation risk premium of 16.7 basis points is roughly eight-tenths of its Model (2) value of 20.9 basis points, consistent with consumers holding around 80 per cent of their bond portfolio in nominal bonds. A key finding is thus that if indexation remains at its current low level in developed economies, the IT inflation risk premium is likely to be substantially higher than estimated by the optimal indexation case, implying an even larger proportional reduction under PLT.

4.5.4 Bond holding horizon

Finally, given the focus on long-term nominal bonds, it is instructive to consider how the holding horizon for bonds influences the inflation risk premia results. Indeed, in terms of comparing IT and PLT, the bond holding horizon is likely to be important because, theoretically, inflation risk increases with the forecast horizon under IT but is bounded under PLT. The baseline model assumed that each period lasts 30 years, but,

given that the models are of life-cycle saving, holding periods as long as 35 years or as short as 25 years are also plausible. This section investigates the impact of varying the bond holding horizon from 25 to 35 years in all three model specifications.

The resulting relationship between the holding period and inflation risk premia is effectively the ‘term structure of the inflation risk premium’, and is analysed in both the IT and PLT cases. As well as highlighting the degree of robustness of the baseline estimates, these results are useful from a practical policy perspective because they show the extent to which inflation risk premia are likely to vary with the term to maturity on government bonds. Knowing such information should be advantageous for governments that issue long-term nominal debt, and also for central banks who must consider how changes in short-term interest rates will impact upon longer-term rates.

In order to examine the impact of changing the holding horizon captured in the model, the money supply rules, biased inflation and productivity were recalibrated for horizons of 25 to 35 years. Appendix B derives general expressions for an OLG model with a horizon of N years to demonstrate this process. In Table 4.9, the resulting inflation risk premia results are reported at each horizon in basis points, with graphical representations of the results given in Figure 4.1 for Model (1); Figure 4.2 for Model (2); and Figure 4.3 for Model (3).

Table 4.9 – Inflation risk premia and the bond holding horizon

Bond holding horizon (years)	Model (1)	Model (2)	Model (3)
25	39.80 / 1.65	20.28 / 0.84	5.35 / 0.65
26	40.16 / 1.59	20.45 / 0.81	5.56 / 0.61
27	40.38 / 1.55	20.55 / 0.79	5.75 / 0.59
28	40.67 / 1.49	20.68 / 0.76	5.77 / 0.57
29	40.87 / 1.45	20.76 / 0.74	5.75 / 0.56
30	41.04 / 1.41	20.86 / 0.72	5.76 / 0.54
31	41.19 / 1.37	20.93 / 0.70	5.75 / 0.53
32	41.32 / 1.33	20.98 / 0.68	5.54 / 0.51
33	41.42 / 1.30	21.02 / 0.66	5.54 / 0.50
34	41.49 / 1.26	21.06 / 0.64	5.51 / 0.48
35	41.57 / 1.23	21.13 / 0.62	5.73 / 0.46

Note: Entries on the left (right) refer to IT (PLT). Figures are in basis points.

In Model (1), the IT inflation risk premium increases non-trivially with the term to maturity because inflation volatility is proportional to the forecast horizon as a result of base-level drift (see Equation (B11)). Intuitively, the longer a nominal bond is held, the greater the accumulation of inflation risk over the life of the bond, because shocks to inflation are not offset but are instead treated as ‘bygones’. Formally, as the term to maturity increases from 25 to 35 years, the IT inflation risk premium rises by around 1.8 basis points, and at a decreasing rate with each additional year. By contrast, the inflation risk premium under PLT actually falls as the term to maturity increases, though the downward slope is fairly moderate, with an increase in the holding horizon from 25 to 35 years causing a drop of only 0.42 basis points.

This contrast in results arises because there is no base-level drift under PLT (because past shocks to inflation are offset), such that inflation volatility does not increase with the forecast horizon, as can be seen clearly from Equation (B12). Taken alone, this result suggests a constant inflation risk premium. However, the inflation risk premium actually falls slightly, because the increase in the horizon captured by the model increases the long-term inflation target, which in turn reduces the expected real return on money balances. By Equation (4.11), this increase leads to a fall in the risk-free rate, which in turn lowers the inflation risk premium. It is important to note that whilst this downward pressure on the inflation risk premium is present under both IT and PLT, it is only sufficient to produce a downward-sloping term structure of the premium under the latter, since under IT the positive impact from increased inflation risk dominates.

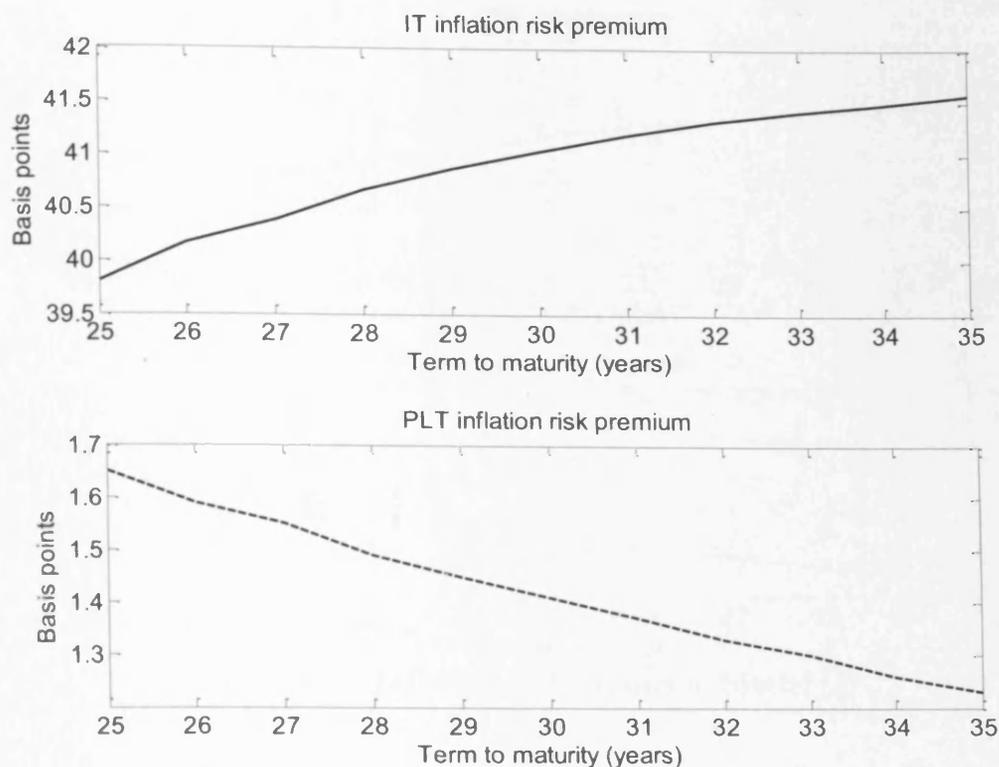


Figure 4.1 – Inflation risk premia in Model (1)

As can be seen from figures 4.2 and 4.3, similar results arise in the models with capital and indexed bonds – and for the same reasons – though the magnitude of inflation risk premia varies considerably across models. In Model (2) where consumers can hold money, nominal bonds and capital, the inflation risk premium is approximately halved, as is the range across which the inflation risk premium varies as the asset holding horizon in the model is increased. Under IT, the inflation risk premium increases from 20.3 basis points at a 25-year horizon to 21.1 basis points at a 35-year horizon, whilst under PLT the inflation risk premium falls slightly from 0.84 to 0.62 basis points across the term structure.

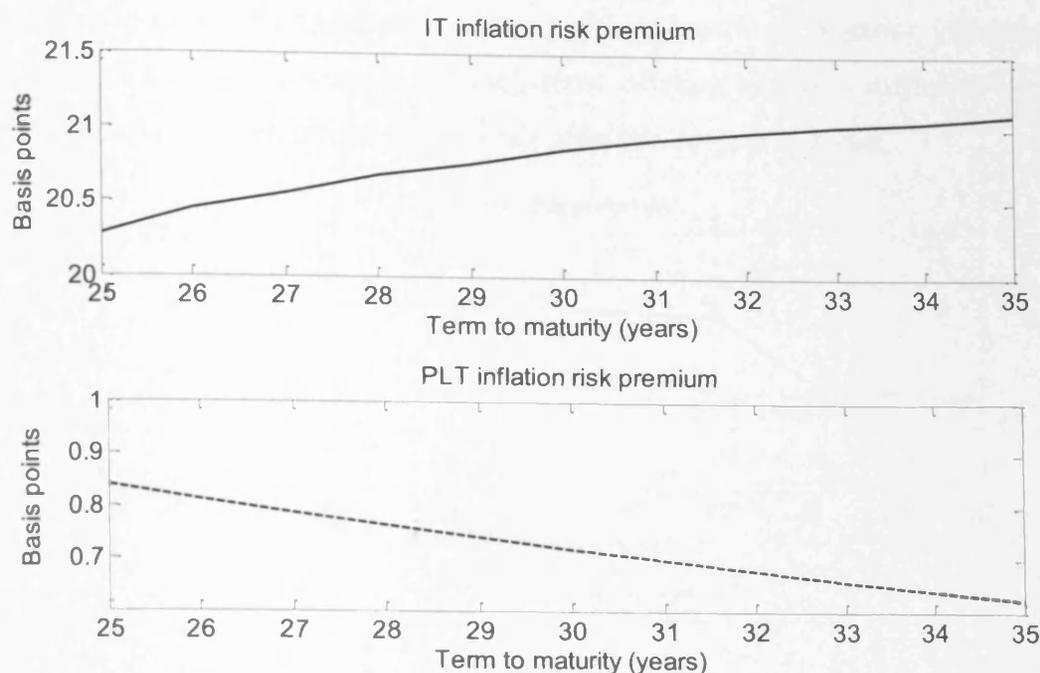


Figure 4.2 – Inflation risk premia in Model (2)

In Model (3) where indexed bonds are also introduced, the inflation risk premium under IT is roughly one-sixth of its Model (1) estimate across the term structure, whilst under PLT it falls at a similar rate to that in Model (2) – an intuitive result given that optimal indexation is robust at around 26 per cent as the holding horizon is increased. Interestingly, the inflation risk premium under IT actually falls slightly after the 30-year horizon, though it still increases overall across the term structure.¹⁴² There are two distinct factors driving this result.

Firstly, because consumers hold the majority of their bond portfolio in indexed bonds, they have better protection than in the other two models against the increasing level of inflation risk as the bond holding horizon is raised, and therefore require less of risk premium in compensation. This first point accounts for the slower rate of increase of the inflation risk premium as the term to maturity is increased from the starting point of 25 years. Secondly, after the baseline holding horizon of 30 years, optimal indexation increases slightly above the baseline of 76 per cent, because inflation risk is sufficiently high that it becomes optimal for consumers to hold more indexed bonds, given that they provide relatively better protection against inflation variations.

¹⁴² There are 'kinks' in Figure 4.3 because the optimal indexation share is solved to the nearest whole number.

This increase in optimal indexation (when combined with the negative impact on the risk-free rate from an increase in the long-term inflation target) is sufficient to ensure that the inflation risk premium starts to fall after the 30-year horizon.

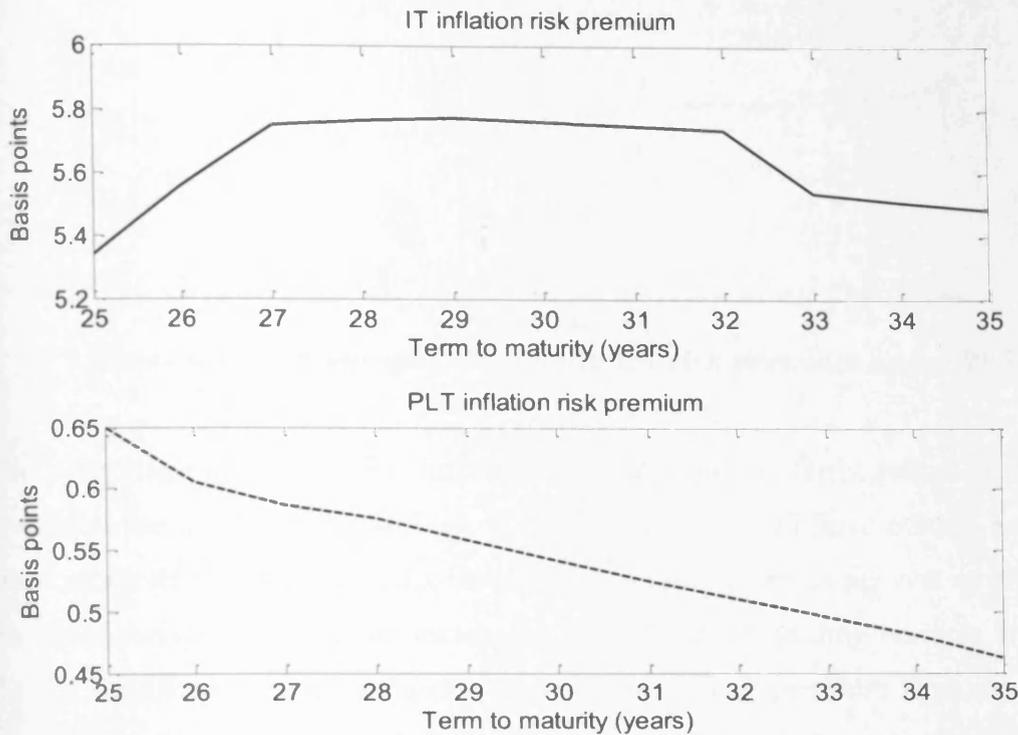


Figure 4.3 – Inflation risk premia in Model (3)

In summary, a robust conclusion across models is that the term structure of the inflation risk premium is upward-sloping under IT and slightly downward-sloping under PLT, though in neither case is the effect particularly strong. There is also robustness in the sense that, in proportional terms, IT and PLT inflation risk premia fall roughly in tandem across the three models. In order to demonstrate this point formally, Figure 4.4 plots the percentage reduction in the inflation risk premium under PLT as the asset holding horizon is varied in all three models.

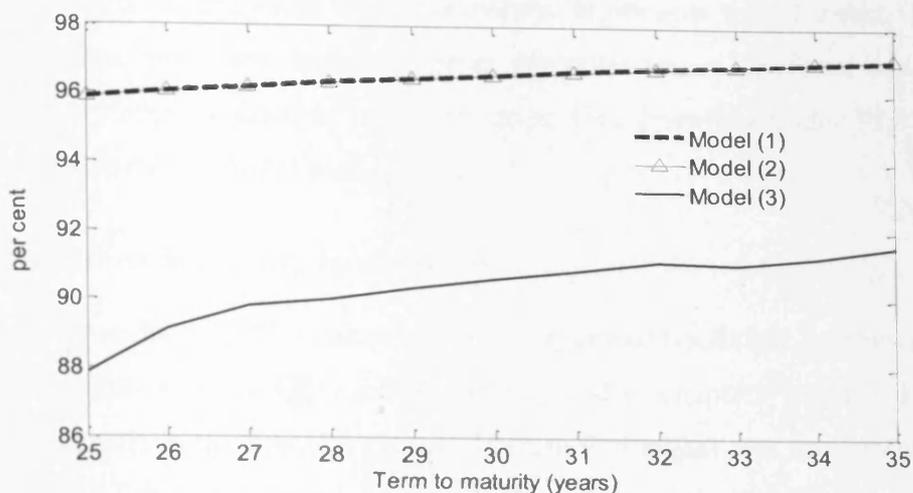


Figure 4.4 – Percentage reduction in the risk premium under PLT

The percentage change in the inflation risk premium is fairly robust in all three models as the bond holding horizon is varied. Indeed, in all three models and for all maturities tested, PLT reduces the inflation risk premium by 88 per cent or more, with the exact percentage reduction increasing with the bond holding horizon. In models (1) and (2), the percentage reduction in the inflation risk premium is greater than 95 per cent at all maturities, and Figure 4.4 shows that the percentage reductions in inflation risk premia are essentially identical for both of these models.

In Model (3), the proportional reduction in the inflation risk premium is slightly lower at between 88 and 92 per cent across the term structure. Intuitively, as shown by Equation (A7), the inflation risk premium in this case depends on the indexation share, which differs substantially between the IT and PLT cases given that consumers hold more indexed bonds and fewer nominal bonds to insure their savings against the high level of inflation risk under IT. Thus it is the endogenous response of nominal indexation to monetary policy that accounts for lower proportional reduction in the inflation risk premium in the model with indexed bonds. Nevertheless, the results are robust across all three models and indicate a substantial proportional reduction in the inflation risk premium under PLT at long-term bond maturities.

Therefore, whilst the analysis in this chapter is unlikely to provide an accurate estimate of the *level* of risk premia under IT and PLT (an issue with DSGE models in general), it does give the clear-cut and more reliable conclusion that PLT will lead to a substantial *proportional reduction* in the inflation risk premium, thus offering

formal support to reasoned but speculative arguments put forward in the PLT literature. This robust result complements the other sensitivity tests, which similarly indicated a marked reduction in the inflation risk premium under PLT for a wide variety of alternative model calibrations.

4.6 Conclusions and policy implications

In this chapter, the inflation risk premium on nominal bonds has been estimated using the three versions of the OLG model investigated in chapters 2 and 3. Following up arguments made in the monetary policy literature, the goal was to quantify the impact of PLT on the long-term inflation risk premium within a DSGE framework. A key result is that, consistent with predictions from the literature, PLT leads to a substantial reduction in the inflation risk premium compared to IT. Intuitively, this result arises because PLT reduces long-term inflation volatility by an order of magnitude compared to IT.

Inflation risk premia vary considerably in absolute value across the three models, because consumers' exposure to inflation risk depends crucially on the assets that they hold in their portfolios. Consequently, the level of inflation risk premia is likely to vary somewhat across countries, and also over time if indexed bonds continue to become more prominent. Under IT, baseline estimates of the inflation risk premium ranged from 41 basis points in Model (1) to only 6 basis points in Model (3), but under PLT there was somewhat less variation from 1.5 to 0.5 basis points. In terms of matching the absolute level of risk-premia in the data, the models are comparable to 'state of the art' New Keynesian models but notably do not rely upon the presence of habit formation in consumption.

Given that DSGE models give rise to asset risk premia an order of magnitude lower than in the data for empirically plausible calibrations (e.g. Rudebusch and Swanson 2008; Cochrane 2001), it is important not to place too much confidence in the magnitudes of the estimated inflation risk premia from the models. However, in order to address this shortcoming, the analysis in this chapter also focused also on the proportional change in the inflation risk premium – the argument being that even if risk premia are too low in levels there is no good reason to think that the relative size across IT and PLT should also be inaccurate. Indeed, the asset pricing 'puzzles'

identified in the literature arise primarily because risk aversion calibrations necessary to match premia in the data are empirically implausible and worsen the performance of DSGE models in terms of matching other stylised macroeconomic facts (see Rudebusch and Swanson, 2008), yet looking at the relative size of risk premia ‘cancels out’ the effect of risk aversion and therefore plausibly offers more reliable results.

A robust conclusion is that PLT leads to marked proportional reduction in the inflation risk premium compared to IT – typically over 90 per cent – a result which is robust to changes in key parameters, and also to variations in the asset holding horizon in the model. As we might expect, PLT leads to a greater proportional reduction in the inflation risk premium in models (1) and (2) where indexed bonds are absent. The reason is simply that consumers are unable to protect themselves against the relatively high level of long-term inflation risk under IT, and therefore require a larger risk premium as compensation for holding nominal bonds. The potential benefits from PLT arising from a reduction in the inflation risk premium are therefore likely to be largest in countries like Germany where nominal assets play a major role and indexed bonds a relatively minor one (see Garcia, 2008). Correspondingly, the potential benefits are likely to be somewhat smaller in countries like the UK and US where indexed government bonds account for a non-trivial share in government bond portfolios and real assets play an important role (OECD, 2009a; Campbell *et al.* 2009).

Finally, it should be noted that although the analysis in this chapter has focused on the inflation risk premium on nominal bonds issued by the government, it does not follow that the results hold no useful policy implications for public sector pensions. For instance, the result that the inflation risk premium is substantially lower under PLT implies that, other things being equal, risk-averse consumers would be willing to save more in nominal pensions under PLT than IT. This is a potentially important consideration given that savings rates are low in many developed economies and the affordability of public sector pensions is a major public policy issue.

Appendix A: Factors affecting the inflation risk premium – an approximation

From Equation (4.12) of the main text, the inflation risk premium is given by

$$(A1) \quad irp_{t,OLG} \equiv E_t r_{t+1}^n - r_t^f = -r_t^f \text{cov}_t(m_{t+1,t}^{OLG}, r_{t+1}^n)$$

where $m_{t+1,t}^{OLG} \equiv c_{t+1,O}^{-\delta} / c_{t,Y}^{-\delta}$ and r_t^f is defined by Equation (4.7).

Equation (A1) can therefore be expressed in the following form:

$$(A2) \quad irp_{t,OLG} = -\frac{r_t^f}{MU_{t,Y}} \text{cov}_t(c_{t+1,O}^{-\delta}, r_{t+1}^n)$$

where $MU_{t,Y} \equiv c_{t,Y}^{-\delta}$ is the marginal utility of consumption in youth.

The covariance term in Equation (A2) is analytically intractable due to the strong non-linearity of marginal utility in old age. However, taking a first-order Taylor expansion around the time- t expected value of consumption gives the following:¹⁴³

$$(A3) \quad c_{t+1,O}^{-\delta} \approx (E_t c_{t+1,O})^{-\delta} - \delta (E_t c_{t+1,O})^{-(1+\delta)} \times (c_{t+1,O} - E_t c_{t+1,O})$$

Using Equation (A3) in (A2) gives

$$(A4) \quad irp_{t,OLG} \approx \delta (r_t^f / MU_{t,Y}) \text{cov}_t(c_{t+1,O}, r_{t+1}^n)$$

Using this expression, the factors driving the inflation risk premium can be identified analytically. An expression is first derived for Model (3), the most general case. Analogous expressions for the other two models are then derived straightforwardly as special cases.

¹⁴³ Note that using a first-order accurate expression for marginal utility is sufficient to get a second-order accurate expression for the covariance term. See e.g. Devereux and Sutherland (2011).

General model with indexed bonds

Recall that consumption by old generations in this model is given

$$(A5) \quad c_{t+1,O} = A_{t+1} k_t^\alpha + (a r_{t+1}^i + (1-a) r_{t+1}^n) b_t^d + r_{t+1}^m m_t^d$$

where $b_t^d = b_t^{i,d} + b_t^{n,d}$ is the total demand for bonds.

Equation (A4) is therefore equal to

$$(A6) \quad irp_{t,OLG} \approx \delta(r_t^f / MU_{t,Y}) \times \left(\begin{array}{l} a \text{cov}_t(r_{t+1}^i, r_{t+1}^n) b_t^d + (1-a) \text{var}_t(r_{t+1}^n) b_t^d \\ + \text{cov}_t(r_{t+1}^m, r_{t+1}^n) m_t^d \end{array} \right)$$

where the fact that $\text{cov}_t(r_{t+1}^n, A_{t+1}) = 0$ has been used.¹⁴⁴

Using the definitions for r^n and r^m we can write this expression as follows:

$$(A7) \quad irp_{t,OLG} = \delta(r_t^f / MU_{t,Y}) \times \left(\begin{array}{l} a R_t \text{cov}_t(r_{t+1}^i, 1/(1 + \pi_{t+1})) b_t^d \\ + (R_t m_t^d + R_t^2 (1-a) b_t^d) \text{var}_t(1/(1 + \pi_{t+1})) \end{array} \right)$$

Equation (A7) shows that the inflation risk premium depends positively on: (i) the extent of risk aversion (as measured by δ); (ii) the volatility of inflation; and (iii) the covariance between indexed bond returns and the inverse (gross) rate of inflation.¹⁴⁵

Moreover, since real returns on indexed and nominal bonds are weakly negatively correlated under the baseline calibration and $R_t > 1$, it follows that a rise in nominal bond holdings relative to indexed bond holdings (a fall in the indexation share a) will

¹⁴⁴ This result follows because money supply innovations and productivity innovations are uncorrelated, which in turn means capital is a perfect hedge against inflation (see the discussion in Chapter 2).

¹⁴⁵ If indexed bonds returns are negatively correlated with the real return on nominal bonds, then the first covariance term in brackets will be negative and will therefore reduce the inflation risk premium. The sign of the correlation depends upon the properties of the biased measure of inflation used for indexation. It is negative under the baseline calibration because ‘biased’ inflation tends to overshoot ‘true’ inflation due to its higher variance.

raise the inflation risk premium.¹⁴⁶ Moreover, an increase in money holdings will increase the inflation risk premium, *ceteris paribus*.

Models without indexed bonds

To derive an expression for the inflation risk premium in the models without indexed bonds – models (1) and (2) – we can simply set $a = 0$ in Equation (A7). Since in this case $b^d = b^{n,d}$, we arrive at the following expression for the inflation risk premium:

$$(A8) \quad irp_{t,OLG} = \delta(r_t^f / MU_{t,Y}) \times (R_t^2 b_t^{n,d} + R_t m_t^d) \text{var}_t(1/(1 + \pi_{t+1}))$$

Hence both money and nominal bond holdings increase the inflation risk premium, *ceteris paribus*.

¹⁴⁶ A negative correlation is not necessary for this result: it will hold in general provided that indexation is not strongly imperfect. Indeed, so long as $\text{cov}(r^i, (1 + \pi)^{-1}) \leq \text{var}((1 + \pi)^{-1})$, a reduction in the indexation share will increase the inflation risk premium because $R_t > 1$.

Appendix B: Model calibration when the bond holding horizon is N years

B1. Money supply rules

Inflation targeting (IT)

Recall from Chapter 2 that the yearly nominal money supply rule under IT is given by

$$(B1) \quad \ln M_i^{s,IT} = \ln M_{i-1}^{s,IT} + \pi + \varepsilon_i + \ln c_{i,Y} - \ln c_{i-1,Y}$$

where π is the yearly inflation target and ε_i is the yearly money supply innovation, a serially-uncorrelated random variable drawn from an $N(0, \sigma^2)$ distribution.

In order to derive an N -year money supply rule from this specification, we can substitute repeatedly for the lagged money supply. Doing so implies that the N -year horizon money supply rule is as follows:

$$(B2) \quad \ln M_i = \ln M_{i-N} + N \times \pi + \sum_{j=0}^{N-1} \varepsilon_{i-j} + \ln c_{i,Y} - \ln c_{i-N,Y}$$

where $N\pi$ is the N -year inflation target.

Hence, if each period t in the model lasts N years, the IT money supply rule in any period t can be represented as follows:

$$(B3) \quad \ln(M_t^{s,IT} / M_{t-1}^{s,IT}) = N \times \pi + \sum_{i=1}^N \varepsilon_{i,t} + \ln(c_{t,Y} / c_{t-1,Y})$$

where the money supply innovations have been indexed from years 1 to N and the time subscript indicates that all N innovations belong to period t .

Price-level targeting (PLT)

The yearly money supply rule under PLT can be written in the following form:

$$(B4) \quad \ln M_i^{s,PLT} = \ln M_{i-1}^{s,PLT} + \pi + \varepsilon_i - \varepsilon_{i-1} + \ln c_{i,Y} - \ln c_{i-1,Y}$$

where π is the yearly inflation target consistent with the target price path, and ε_i is a serially-uncorrelated money supply innovation drawn from an $N(0, \sigma^2)$ distribution.

Substituting repeatedly for the lagged money supply term on the right hand side gives the following relationship between the money supply in year i and N years earlier:

$$(B5) \quad \ln M_i^{s,PLT} = \ln M_{i-N}^{s,PLT} + N \times \pi + \varepsilon_i - \varepsilon_{i-N} + \ln c_{i,Y} - \ln c_{i-N,Y}$$

Therefore, if each period t in the model lasts N years, the PLT money supply rule in any given period t is as follows:

$$(B6) \quad \ln(M_t^{s,PLT} / M_{t-1}^{s,PLT}) = N \times \pi + \varepsilon_{N,t} - \varepsilon_{N,t-1} + \ln(c_{t,Y} / c_{t-1,Y})$$

where the money supply innovations have been indexed to reflect the year in which they occur, and the t subscript indicates the period to which the innovations belong.

Table B1 reports the calibrated long-term inflation target in the N -year models based upon the baseline yearly inflation target of 2.5 per cent (i.e. $\pi = 0.025$). As discussed in the main text, integer values of N from 25 to 35 years are considered.

Table B1 – The inflation target and the bond holding horizon

Horizon N (years)	Long-term inflation target
25	0.625
26	0.650
27	0.675
28	0.700
29	0.725
30	0.750
31	0.775
32	0.800
33	0.825
34	0.850
35	0.875

B2. Inflation

Recall that the CIA constraint in the model implies that

$$(B7) \quad M_t^d = \theta P_t c_{t,Y}$$

where $M_t^d \equiv P_t m_t^d$ is money demand in nominal terms.

Taking logs of Equation (B7) implies that the (30-year) growth rate of money demand is given by

$$(B8) \quad \ln M_t^d - \ln M_{t-1}^d = \pi_t + \ln c_{t,Y} - \ln c_{t-1,Y}$$

where $\pi_t \approx \ln P_t - \ln P_{t-1}$.

Imposing money market equilibrium in equations (B3) and (B6) therefore gives the following expressions for inflation under IT and PLT:

Inflation targeting

$$(B9) \quad \pi_t^{IT} = N \times \pi + \sum_{i=1}^N \varepsilon_{i,t}$$

Price-level targeting

$$(B10) \quad \pi_t^{PLT} = N \times \pi + \varepsilon_{N,t} - \varepsilon_{N,t-1}$$

where $\varepsilon_{i,t}$ is the money supply innovation in year i of period t .

It follows that the inflation variances under IT and PLT are given by

$$(B11) \quad \text{var}(\pi_t^{IT}) = N\sigma^2$$

$$(B12) \quad \text{var}(\pi_t^{PLT}) = 2\sigma^2$$

B3. Biased inflation

In the model with endogenous nominal indexation, indexed bonds are linked to a biased measure of inflation that is assumed to follow the same functional form as true inflation. As such, the IT and PLT stochastic processes for biased inflation will change in accordance with true inflation as follows:

$$(B13) \quad \left. \begin{aligned} \pi_t^{ind,IT} &= N \times \pi + \sum_{i=1}^N \varepsilon_{i,t}^{ind} \\ \pi_t^{ind,PLT} &= N \times \pi + \varepsilon_{N,t}^{ind} - \varepsilon_{N,t-1}^{ind} \end{aligned} \right\} \begin{array}{l} \text{under IT} \\ \text{under PLT} \end{array}$$

where $\varepsilon_{i,t}^{ind} \sim N(0, \sigma_{ind}^2)$ is the biased inflation innovation in year i of period t .

As under the baseline calibration, yearly innovations to true and biased inflation are assumed to be positively correlated with a correlation coefficient of 0.89.

B4. Productivity

Recall that productivity in the model is built up from an AR(1) process for log productivity at a quarterly horizon q ,

$$(B14) \quad \ln A_q = (1 - \rho_q) \ln A_{q,mean} + \rho_q \ln A_{q-1} + e_q \quad 0 < \rho_q < 1$$

where e_q is an IID-Normal productivity innovation with mean zero and variance σ_q^2 .

Hence, substituting repeatedly for lagged productivity, productivity over an N -year horizon is given by

$$(B15) \quad \ln A_q = (1 - \rho_q^{4N}) \ln A_{q,mean} + \rho_q^{4N} \ln A_{q-4N} + \sum_{j=0}^{4N-1} \rho_q^j e_{q-j}$$

If each period t in the model lasts N years, then Equation (B15) can be written in the following form:

$$(B16) \quad \ln A_t = (1 - \rho) \ln A_{mean} + \rho \ln A_{t-1} + e_t$$

where $\ln A_{mean} \equiv (1 - \rho_q^{4N}) \ln A_{q,mean} / (1 - \rho)$, $\rho \equiv \rho_q^{4N}$ and $e_t \equiv \sum_{j=0}^{4N-1} \rho_q^j e_{q-j}$.

It follows that the variance of the innovation to productivity is given by

$$(B17) \quad \text{var}(e_t) = \frac{(1 - \rho_q^{8N}) \sigma_q^2}{1 - \rho_q^2}$$

This expression is used to calibrate the innovation variance for productivity in the models with N -year horizons. The autoregressive parameter ρ is also recalibrated accordingly. Based on the original baseline calibrated values of $\rho_q = 0.996$ and $\sigma_q = 0.005$, Table B2 reports the calibrated autoregressive coefficients and innovation variances in the N -year models as N is varied from 25 to 35 years.

Table B2 – The calibration of productivity and the bond holding horizon

Horizon N (years)	ρ	σ_e
25	0.670	0.0416
26	0.659	0.0420
27	0.649	0.0426
28	0.638	0.0431
29	0.628	0.0435
30	0.618	0.0440
31	0.608	0.0444
32	0.599	0.0448
33	0.589	0.0452
34	0.580	0.0456
35	0.571	0.0460

Chapter 5

Quantifying the long-term benefits of price-level targeting: A summary comparison across models

5.1 Introduction

This short chapter summarises the long-term benefits from price-level targeting (PLT) estimated in the second, third and fourth chapters, and discusses policy implications arising from these results. Robustness of results is also discussed, including the importance of the different model specifications given the presence of heterogeneities across countries and over time. Finally, the chapter concludes by discussing some topics for future research and by putting in context the contributions of the thesis to the monetary policy literature.

5.2 Social welfare and consumption risk

Chapters 2 and 3 considered three different versions of a monetary overlapping generations (OLG) model in which young consumers save for old age using long-term government bonds. The first was an endowment economy in which only nominal government bonds were issued and consumers had no access to real assets; the second relaxed the nominal assets assumption by allowing consumers to also hold productive but risky capital; and the third model extended the second to include (imperfectly-) indexed government bonds, with the equilibrium degree of bond indexation chosen in response to monetary policy as part of an optimal commitment Ramsey policy. All three models were solved using a second-order perturbation method in order to capture asset risk-premia and obtain reliable social welfare rankings of IT and PLT.

In chapters 2 and 3, the analysis focused primarily on the long-term welfare gain from switching from IT to PLT and on the impact on consumption risk faced by old generations – the dimension along which PLT had its main impact. The models were roughly calibrated to UK data based on the assumption that long-term bonds in the model correspond to public sector pensions and long-dated government bonds, and capital to private sector pensions. The results are, however, intended be relevant for developed economies in general, including Canada whose central bank is currently conducting a review of price-level targeting in anticipation of the renewal of its policy agreement with the Government in 2011 (see Bank of Canada, 2006).

Table 5.1 summarises the results obtained in the second and third chapters and focuses on three key areas: the consumption equivalent welfare gain λ ; the robustness of the welfare gain in sensitivity analysis (based upon the highest and lowest estimates); and the estimated baseline reduction in consumption risk for old generations. Model (1) denotes the first and simplest OLG model with only nominal government bonds, Model (2) is the OLG model with nominal bonds and capital, and Model (3) is the model with indexed and nominal government bonds, capital, and endogenous nominal indexation.

Table 5.1 – The social welfare and volatility impacts of PLT

Measure	Model		
	(1)	(2)	(3)
λ (in % terms)	0.103	0.026	0.010
Risk aversion sensitivity (Low, High)	(0.052, 0.173)	(0.012, 0.044)	(0.006, 0.014)
Nominal volatility sensitivity (Low, High)	(0.062, 0.154)	(0.015, 0.039)	(0.007, 0.012)
Reduction in $\text{var}(c_{t,0})$	96.4%	39.4%	13.1%

Notes: Model (1) = nominal bonds; Model (2) = nominal bonds + capital; Model (3) = nominal bonds + capital + indexed bonds. ‘Low’ denotes the lowest value of the welfare gain obtained in sensitivity analysis; ‘High’ is the largest.

The estimated welfare gain from PLT was positive in all three models but varies substantially across the different specifications. The reasoning behind a positive welfare gain from PLT is simple and runs as follows. Long-term inflation risk is substantial under IT because of base-level drift: even if the central bank misses its inflation target by only a small percentage in each year, these misses can accumulate and become quite large after 30 years. The substantial level of long-term inflation risk translates into a high level of consumption volatility for old generations because consumers hold government bonds and money – nominal assets that offer less than perfect insurance against inflation fluctuations. Under PLT, however, the price level is returned to its target path following inflationary shocks, such that the price level is trend-stationary. As a result, past deviations from the inflation target do not

accumulate over the long-term and inflation risk is reduced by an order of magnitude. PLT therefore stabilises old generations' consumption more effectively than IT, which in turn increases social welfare given that consumers are risk-averse.

Importantly for these results, the government, the monopoly issuer of bonds and money, is required to meet a long run government spending target. This constraint means that social welfare is effectively maximised by minimising consumption risk across young and old generations. In particular, since there is little to choose between IT and PLT in terms of consumption volatility across young generations, the social welfare gain from PLT depends crucially on the extent to which it is able to lower consumption volatility across old generations through reducing long-term inflation risk. In effect, the three different versions of the model investigate the impact of PLT on social welfare and consumption risk as the composition of consumers' savings portfolios is altered. It is important to note that the welfare gains reported in Table 5.1 are permanent in the sense that they apply to all current and future generations.

The estimated welfare gain varies substantially across models because changing the composition of consumers' asset portfolios alters exposure to risk and enables consumers to diversify or hedge risk to a greater or lesser extent. The model with nominal bonds produced the largest welfare gain at 0.103 per cent of aggregate consumption, or approximately 0.08 per cent of GDP. In monetary terms, the aggregate gain is £899.1 million, which is equal to £31.03 per working member of the population aged 16 years and over, or £107.04 per pensioner based on current UK data. The welfare gain is largest in this case since consumers hold only nominal bonds and money, so that inflation volatility is the only source of consumption risk. Thus consumption risk in old age is reduced dramatically under PLT (a reduction of 95 per cent), in tandem with the marked reduction in inflation risk over the saving horizon from youth to old age. The welfare gain was rather sensitive in Model (1), ranging from 0.052 to 0.173 per cent as the coefficient of relative risk aversion was varied from 1.5 to 5, and from 0.062 to 0.154 per cent as the extent of nominal volatility was varied from the baseline standard deviation by around one quarter.

Once capital is added into the model, the baseline welfare gain from PLT falls by almost three-quarters to 0.026 per cent of aggregate consumption, or approximately

0.020 per cent of GDP. Based on UK data, this increase in consumption implies an aggregate gain of £227 million, or £4.59 per employed member of the population and £27.02 per pensioner. There are two factors driving the substantial reduction in the welfare gain. Firstly, consumers' optimal holdings of capital are non-trivial, such that the importance of nominal assets in consumers' portfolios is reduced. Secondly, capital is itself a risky real asset (because productivity is stochastic) and is therefore a source of consumption risk that is not affected by a change in monetary policy regime from IT to PLT. Consequently, introducing capital means that a substantial fraction of total consumption risk for old generations is not affected by PLT. The impact of these two factors is clearly illustrated by the lower reduction in consumption volatility across old generations of 40 per cent, as compared to 95 per cent in the model without capital. Notably, the welfare gain remains sensitive to risk aversion and extent of nominal volatility, though less so than in the model only nominal bonds. Specifically, the welfare gain varies from 0.012 to 0.044 per cent as the risk aversion coefficient is changed from 1.5 to 5, and from 0.015 to 0.039 per cent as nominal volatility is varied around its baseline by one quarter.

Finally, introducing indexed bonds into the model with capital lowers the estimated welfare gain from PLT even further. The estimated baseline welfare gain in Model (3) was only 0.010 per cent of aggregate consumption, or 0.008 per cent of GDP – roughly one-tenth of the estimate in the model with only nominal bonds. Based on current UK data, the potential gain in welfare is equal to £90.7 million, a gain per employed member of the UK population of £3.13, or £10.80 per pensioner. These gains appear borderline trivial, but it should be noted that they apply, in principle, to all current and future generations, and that the level of consumption on which these gains are calculated should grow over time. The welfare gain from PLT is reduced further because with indexed bonds consumers can largely protect their savings against long-term inflation risk. In other words, indexed bonds act as a 'substitute for PLT' by insuring old generations' consumption against the high level of inflation risk under IT.

5.3 Optimal indexation

In terms of optimal indexation, the results are consistent with the literature that has investigated the relationship between monetary policy and optimal indexation of wage

and financial contracts (Meh *et al.* 2008b; Amano *et al.* 2007; Minford and Peel, 2003). Indeed, optimal indexation is substantially lower under PLT than IT, and ignoring this reduction understates the potential welfare gain from PLT. Under the baseline calibration, optimal indexation was 76 per cent under IT compared to only 26 per cent under PLT. Importantly, the imposed constraint that the government achieve a long run target level of government spending means that optimal indexation is approximately independent of expected bond returns. For instance, a higher expected return on nominal bonds than indexed bonds (due, for example, to a greater risk premium) translates into higher average consumption by the old but also a higher average cost of issuing nominal government debt, such that young generations must be taxed more heavily to make up the shortfall. Provided risk aversion is not very strong, the consumption gain to old generations will approximately offset the loss to young generations, leaving average aggregate consumption and social welfare unchanged to first-order. With social welfare unaffected by expected bond returns, indexation should be chosen to minimise consumption volatility, since this has a negative second-order impact on social welfare. Hence the government's optimal indexation Ramsey problem effectively amounts to choosing the mix of bonds that will minimise the long run level of consumption risk faced by bondholders.

Optimal indexation is relatively high under IT because long-term inflation risk is substantial due to base level drift, making nominal bonds a poor store of value compared to indexed bonds, given that the latter provide excellent (though imperfect) protection against inflation risk. Under PLT, however, long-term inflation risk is reduced by an order of magnitude and comparable to that at a yearly horizon. As a result, there is less need for consumers to protect their savings against inflation risk, whilst at the same time the imperfections of indexed bonds become a relatively more important source of risk. The net result of these two effects is that nominal bonds become a better store of value than indexed bonds under PLT, such that optimal indexation gives nominal bonds a majority share in the government bond portfolio. Most of the reduction in optimal indexation under PLT is due to the 'indexation lag' and not 'indexation bias', because the indexation lag of 8 months is a non-trivial source of return volatility given that long-term inflation risk under PLT is so low by comparison to the IT case.

Moreover, the finding that optimal indexation is substantially lower under PLT is robust to changes in key model parameters and shock volatilities, though it should be noted that optimal indexation under IT is somewhat higher than current levels in developed economies. This result implies that the current potential welfare (and volatility) gains from PLT may be understated by the baseline results for Model (3). As well as having important implications for social welfare, the extent of indexation is also important for the inflation risk premium on nominal bonds. The reasoning here is that the extent of risk exposure from holding an asset depends on the relative holdings of that asset in consumer portfolios, as well as on the level of risk associated with its payoff and consumer aversion to that risk. That is to say, a risky asset that is prominent in consumer portfolios will be a more important factor for overall consumption risk than an asset that is equally risky but with a small portfolio share.

5.4 Inflation risk premia

It has been argued in the literature that inflation risk premia are an important consideration for monetary policymakers because they affect longer-term interest rates and therefore the cost of issuing government debt and the 'price' of long-term investments. In the context of the IT versus PLT debate, a number of economists have argued that there would be a substantial reduction in the long-term inflation risk premium under PLT, and that this would lead to an increase in investment and economic growth (e.g. Crawford *et al.* 2009; Lilico, 2000). Moreover, by reducing the average cost of issuing long-term nominal government debt, a reduction in the inflation risk premium would enable governments to increase spending for a given level of taxes, or to reduce taxes (and related distortions) for a given level of expenditure.

Using the three models introduced in Chapters 2 and 3, long-term inflation risk premia were computed in Chapter 4 to investigate whether and PLT leads to a marked reduction relative to IT as hypothesised in the literature. The main results are summarised in Table 5.2. A key finding which is robust across model specifications is that PLT leads to a reduction in the inflation risk premium of more than 90 per cent, with the reduction being more than 95 per cent in models (1) and (2) where consumers cannot hold indexed bonds. The result that PLT leads to a substantial

proportional reduction in the inflation risk premium is also robust to variations in risk aversion, nominal volatility, and the bond holding horizon.

Intuitively, PLT leads to a substantial reduction in the inflation risk premium because long-term inflation risk is reduced by an order of magnitude relative to IT due to the absence of base-level drift. As such, risk-averse holders of long-term nominal bonds are exposed to less consumption risk and require less of a 'risk premium' in compensation than under IT. The inflation risk premium is reduced less strongly in proportional terms in the model with indexed bonds because the extent of indexation is chosen optimally in response to monetary policy. Indeed, as noted above, there is strong substitution towards indexed bonds under IT but somewhat less under PLT, because indexed bonds are an excellent store of value compared to nominal bonds if the level of long-term inflation risk is high. The net result of this substitution to indexed bonds under IT is that consumers' exposure to risk from holding nominal bonds falls dramatically, which in turn reduces the inflation risk premium somewhat.

Table 5.2 – Inflation risk premia under IT and PLT

Measure	Model					
	(1)		(2)		(3)	
	IT	PLT	IT	PLT	IT	PLT
Inflation risk premium	41.0	1.4	20.9	0.7	5.8	0.5
Risk aversion sensitivity (Low, High)	(21,68)	(0.7,2.4)	(10,35)	(0.4,1.2)	(3,9)	(0.3,0.9)
Nominal volatility sensitivity (Low, High)	(25,61)	(0.8,2.1)	(12,31)	(0.4,1.1)	(4,8)	(0.3,0.8)
Holding horizon sensitivity (Low, High)	(40,42)	(1.2,1.7)	(20,21)	(0.6,0.8)	(5.4-5.7)	(0.5,0.7)
Baseline reduction under PLT	96.6%		96.7%		91.4%	

Notes: Results are in basis points over a 30-year horizon and are rounded to the nearest basis point under IT (except for a single entry in the table). 'Low' denotes the lowest value of the risk premium obtained in sensitivity analysis; 'High' is the largest.

Although the proportional reduction in the inflation risk premium is robust across model specifications and to changes in key parameters, the absolute size of inflation risk premia varies substantially, as does the absolute reduction attained under PLT. Most notably, there is considerable variation with respect to the extent of risk aversion and the level of nominal volatility, particularly in the first two models. For example, in Model (2) the inflation risk premium under IT varies from 10 basis points with a risk aversion coefficient of 1.5 to 35 basis points with a coefficient of 5, whilst varying nominal volatility around the baseline by one-quarter gives a similar range of 12 to 31 basis points.

A striking result in Table 5.1 is that estimated inflation risk premia fall sharply as we move from Model (1) to Model (3) – that is, as the importance of inflation-hedging assets in consumers’ portfolios is increased by giving them access to indexed bonds and capital. This is especially true under IT where the inflation risk premium is much higher. For example, under IT the baseline inflation risk premium over a 30-year horizon is 41 basis points in Model (1), compared to only 6 in Model (3). The intuition for the inflation risk premium falling sharply is that, in the model with only nominal assets, consumers are unable to protect themselves against the relatively high level of long-term inflation risk under IT and therefore require a large risk premium as compensation for holding nominal government bonds.

Introducing capital as in Model (2) causes optimising consumers to hold both capital and bonds in their portfolios, such that consumption covariance risk associated with holding nominal bonds is reduced in tandem with their share in consumers’ portfolios. Similarly, adding indexed bonds as in Model (3) enables consumers to directly protect their wealth against inflation variations whilst not being exposed to substantial real risk (from productivity), with the result that fewer nominal bonds are (optimally) held, which reduces the inflation risk premium even further. Substitution from nominal to indexed bonds is particularly marked under IT because consumers have more gain by protecting themselves against the high level of long-term inflation risk. On the other hand, such risk is sufficiently low under PLT that issuing only nominal government bonds is not strongly sub-optimal from a social welfare perspective.

5.5 Policy implications

A key finding from Table 5.1 is that the long-term welfare and volatility gains to be had from PLT vary substantially across models. This finding is crucial since the ‘best model’ for any particular country is likely to vary depending upon which assets are most important in the provision of retirement income.

For instance, in the case of the UK and US, private sector pensions play an important role (OECD, 2009b); indexed government bonds account for a non-trivial share of government bonds; and public sector pensions are indexed once in payment. Therefore, the model with capital and indexed and nominal bonds is likely to be most applicable, suggesting a small potential welfare gain from switching to PLT. On the other hand, public sector pensions account for around 85 per cent of retirement income in Germany (Berkel and Börsch-Supan, 2004, Börsch-Supan, 2000) and the market for indexed bonds is extremely small (Garcia, 2008), whilst private pensions and private financial income account for only 5 per cent of retirement income in Italy (OECD, 2009a).¹⁴⁷ In these two countries, the model with only nominal bonds is likely to be more appropriate, giving an implied welfare gain that is around ten times larger and clearly non-trivial. Canada is a country in which nominal assets play an important role (Meh and Terajima, 2008), but where indexation of public sector pensions and government bonds is relatively low and real assets are an important source of retirement income. As such, Model (2) is likely to be most appropriate, suggesting a welfare gain that lies between the extremes just discussed.

A second important point from Table 5.1 is that there is considerable sensitivity to both risk aversion and nominal volatility. The former is important because risk aversion will potentially vary across countries, and because economists have not yet reached a consensus on its correct calibration due to inability of dynamic stochastic general equilibrium (DSGE) models to match ‘risk premia’ in the data (Rudebusch and Swanson, 2008; Cochrane, 2001; Mehra and Prescott, 1985). To the extent that the degree of risk aversion is uncertain, the potential welfare gain from PLT is also. An interesting issue that has not been addressed in this thesis is the extent to which

¹⁴⁷ This figure is based on data for the ‘mid-2000s’ and refers to the income of over-65s, excluding any earnings from work or income from self-employment.

adopting a different specification for consumers preferences would affect the estimated long-term welfare gain from PLT and inflation risk premia. As discussed below, this is a potentially fruitful extension for future research.

Turning to nominal volatility, there is clear heterogeneity across countries in terms of inflation variability, including amongst IT countries (Dotsey, 2006). It has been argued in the literature that the Great Moderation may be due to favourable shocks (the ‘Good Luck’ hypothesis) rather than improved monetary policy, so the coming decades may see a rise in inflation volatility above current levels, increasing the potential welfare gain from PLT. It should also be noted that the level of long-term nominal volatility under IT is itself highly uncertain, since the IT approach to monetary policy has been in place only a relatively short period of time. Due to the uncertainties surrounding nominal volatility and the extent of risk aversion, the long-term welfare gain from PLT was not precisely estimated. However, based on Table 5.1, the long-term welfare gain in developed economies should lie between 0.005 and 0.173 per cent of aggregate consumption. Although these estimated gains are relatively small, it should be noted that consumption risk for old generations (i.e. pensioners) is reduced substantially in all three models – a result which is strongly robust to both risk aversion and nominal volatility.

Turning to the inflation risk-premia results summarised in Table 5.2, there is also considerable sensitivity to model specification, risk aversion and nominal volatility. Therefore, the importance of inflation risk premia is likely to vary across countries with the extent of nominal volatility and risk aversion, as well with the importance of nominal, indexed and real assets in retirement income portfolios. For example, in countries like Canada and Germany where nominal assets play a substantive role, the absolute reduction in the inflation risk premium under PLT could be considerable. On the other hand, the reduction in the inflation risk premium would likely be small in countries like the UK and US where indexed government bonds have a substantial share and almost one-half of retirement income comes from private pensions and private financial income (OECD, 2009a).¹⁴⁸ Interestingly, inflation risk premia are not sensitive to variations the asset holding horizon from 25 to 35 years, a period that can

¹⁴⁸ As in the reference to Italy above, these figures are based on data for the ‘mid-2000s’ and refer to the income of over-65s, excluding any earnings from work or income from self-employment.

be said to cover ‘long-term maturities’. This result provides important information for governments that issue long-term nominal debt,¹⁴⁹ and also for monetary policymakers who must consider how changes in short-term interest rates are likely to impact upon longer-term interest rates at various horizons.

One potential issue with the estimated risk premia from the three models is that they are rather low compared to what has been estimated in the data – an issue with DSGE models more generally. In order to address these concerns, the analysis also focused on the percentage reduction in the inflation risk premium under PLT, as reported in the last row of Table 5.2 (for the baseline calibrations). A key conclusion is that PLT leads to a reduction in the inflation risk premium of over 90 per cent – a result which is strongly robust to risk aversion and nominal volatility, and also across model specifications. This result suggests that any given country that switches from IT to PLT could in the long run expect a significant proportional reduction in the inflation risk premium on its long-term nominal government debt. Moreover, although the proportional gains are largest at over 95 per cent in models (1) and (2) where there are no indexed bonds, this conclusion depends crucially on indexation being optimised in response to monetary policy. Indeed, if the share of indexed bonds is set fairly low as in developed economies currently, the proportional reduction in the inflation risk premium in Model (3) is closer to that in the other two models.

Finally, it should be noted that although the inflation risk premium results relate to nominal bonds issued by the government, it does not follow that the results hold no useful policy implications for public sector pensions. Notably, the result that the inflation risk premium is substantially lower under PLT is potentially important for pensions because it suggests that, other things being equal, risk-averse consumers will be willing to save more in nominal pensions under PLT than IT. This should be an important consideration for the many developed economies in which savings rates are low and where the affordability of public sector pensions is a major public policy issue.

¹⁴⁹ For instance, issuance of nominal government debt is often concentrated at particular maturities, as noted in Chapter 2.

To summarise, the main policy implication is that the long-term welfare gain from PLT will be small but economically non-trivial. Moreover, the welfare benefit is likely to vary substantially across countries, and also over time as performance under IT improves or deteriorates. In countries in which private sector pensions are important, IT performs well, and risk aversion is low, the long-term welfare gain from PLT is likely to be relatively small. Conversely, countries where public sector pensions are dominant, nominal volatility is substantial, and indexation of government bonds is relatively low would have most to gain in the long-term from PLT. Though targeting the price level leads to a substantial reduction in the inflation risk premium on long-term nominal government debt, an overlapping generations framework with additional transmission mechanisms is needed to assess directly the implications of this result for social welfare. Such an analysis would be a useful extension for future research.

5.6 Contributions to the literature and future research

The key results discussed above have important implications for future research, as does the modelling methodology employed to obtain these results. This brief final section first discusses these implications and then turns to the overall contributions of the thesis to the monetary policy literature.

Firstly, since the welfare gain could not be estimated precisely and the models have not been tested empirically, further research is needed to evaluate the long-term welfare impact of PLT. Ideally, this research should be conducted in more comprehensive models of the economy of the kind used at central banks. However, given the numerous advantages of the OLG framework exemplified in this thesis, it seems preferable to conduct such analyses within an overlapping generations framework – at least as a starting point.¹⁵⁰ For example, future research could extend the basic OLG set-up presented here to include additional assets or a bequest motive, and could be precisely calibrated to data along the lines of the papers by Dopeke and Schneider (2006) and Meh *et al.* (2010). Models of this kind should be a useful tool

¹⁵⁰ That is, unless a ‘new’ DSGE model soon emerges in which long-term inflation risk has a direct impact on social welfare.

for central banks seeking to conduct microfounded quantitative policy evaluations over a long-term horizon.

Secondly, the substantial reduction in optimal indexation under PLT suggests that it is crucial for future policy analyses comparing IT and PLT to endogenise the degree of nominal indexation. Indeed, models that do not endogenise the extent of indexation in response to monetary policy will be vulnerable to the Lucas critique and may give rise to seriously misleading results in forecasting or policy analyses. This conclusion applies both to smaller theoretical models used in academic circles and to larger models of the kind used for policy analysis by central banks and policy institutions (see e.g. Laxton, 2008). Most notably, the optimal indexation results have clear import for central banks like the Bank of Canada that are considering switching from IT to PLT in the future and are interested in evaluating the two regimes in simulated models of the economy.

Thirdly, as noted above, an interesting issue left unexplored in this thesis is the impact of the specification of consumer preferences on the estimated welfare impact from PLT and on inflation risk premia. This is a potentially important issue because the welfare gain from PLT depends crucially on its impact on consumption risk, which has a second-order impact on social welfare. Adopting a preference specification better able to match asset risk-premia like habit formation or Epstein-Zin preferences would likely increase the welfare gain from PLT substantially, since an increase in risk aversion increases the importance of second-order terms in utility.¹⁵¹ However, whilst these other specifications of preferences do a better job at matching asset risk-premia (De Paoli and Zabczyk, 2008; Campbell and Cochrane, 1999) and other higher-order effects like precautionary behaviour (De Paoli and Zabczyk, 2011), they simultaneously reduce the ability of models to fit other stylised macroeconomic facts like volatilities (Rudebusch and Swanson, 2008).¹⁵² It is therefore important for future research using alternative preferences to investigate these knock-on effects as well.

¹⁵¹ To give a familiar example, the coefficient of relative risk aversion under external habit formation is given by $\delta/(1-h)$ compared to only δ with constant relative risk aversion, where $0 < h < 1$ is the 'habit size' coefficient on past aggregate consumption and δ is the risk aversion coefficient.

¹⁵² Epstein-Zin preferences are potentially less vulnerable to this criticism because they enable researchers to calibrate separately the coefficient of relative risk aversion and the elasticity of intertemporal substitution. However, these preferences still require implausibly high calibrations for

Lastly, other potentially useful extensions for future research include relaxing the assumption that PLT is perceived as perfectly credible by economic agents and amending the assumption that the price level is returned to its target path within one year. On this last point, the monetary policy literature has noted that returning the price level to trend this quickly could lead to excessive output volatility (see Smets, 2003). Importantly, incorporating a longer horizon for returning the price level to target would eliminate some of the reduction in long-term inflation risk under PLT, thereby reducing the estimated welfare gain, as well as the reductions in consumption risk and inflation risk premia attained under PLT. The importance of the target horizon for the results in this thesis is an interesting issue that is left for future research.

There are several general contributions of the thesis that have potential importance for the conduct of economic research in the future. The key ones are as follows. Firstly, we have seen that the OLG life-cycle model can be fruitfully used to study the optimal conduct of monetary policy over a long-term horizon. Indeed, as noted in chapters 1 and 2, a key advantage of the OLG life-cycle framework over the workhorse New Keynesian model is that it enables researchers to model explicitly the impact of long-term inflation risk on social welfare using consumer utility. In addition, the OLG framework is also an extremely transparent framework for long-term analyses, an important potential advantage that is often overlooked by economic researchers.

Secondly, we have seen that an OLG modelling framework can be used to study long-term risk premia on financial assets, in particular when those premia depend crucially on the monetary policy regime in place. Although this is not a use to which OLG models have typically been put in the literature, it is nevertheless a potentially useful one – in particular, because it avoids the need to build-up the term structure explicitly quarter-by-quarter (or year-by-year) in simulations, a formidable and time-consuming task which likely explains why there has been no previous research estimating long-term inflation risk premia in fully-specified DSGE models. Moreover, as noted above, the overlapping generations framework presented in this thesis could potentially be

risk aversion (see Rudebusch and Swanson, 2009), and it is not clear how preferences of this kind could be introduced in an OLG framework given that they are recursive.

extended in order to analyse directly the social welfare implications of a reduction in the inflation risk premium under PLT. Last but not least, although it has not been an aim of this thesis, the OLG modelling framework presented here could be used to study the redistributive effects of PLT on young and old generations, as well as the extent to which these effects are likely to vary across countries and over time.

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