

Gravitational Waves and the Polarisation of the Cosmic Microwave Background

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ABSTRACT

We discuss the influence of gravitational waves (GWs) upon the polarisation of the Cosmic Microwave Background Radiation (CMBR). We show how to compute the *rms* temperature anisotropy and polarisation of the CMBR induced by GWs of arbitrary wavelength. We find that the ratio of polarisation, Π , to anisotropy, A , can be as large as $\sim 40\%$, but is sensitively dependent upon the GW spectrum and the cosmological ionisation history. We argue that CMBR polarisation measurements can provide useful constraints on cosmological models.

Key words: radiation mechanisms: gravitational – cosmic microwave background – cosmology: theory – early Universe.

1 INTRODUCTION

The recent discovery by the COBE team of angular fluctuations in the sky temperature of the Cosmic Microwave Background Radiation (CMBR) is of profound importance for theories of the origin of galaxies and large-scale structures in the Universe (Smoot et al. 1992). If the observed temperature anisotropy is interpreted as being due to fluctuations in the density of the Universe at early times then, together with measurements of present-day galaxy clustering, it imposes strong constraints on the primordial fluctuation spectrum and the nature of and dark matter (Efstathiou et al. 1992; Taylor & Rowan-Robinson 1992). These constraints will be further strengthened by measurements of temperature anisotropy on angular scales smaller than those probed by COBE (Bond et al. 1991).

Compared to the enormous observational effort that has been directed at the search for anisotropy in the temperature of the CMBR on the sky, relatively little attention has been paid to analogous fluctuations in its *polarisation*. Atmospheric contributions to the sky temperature at microwave frequencies are not polarised so it is possible to make measurements of polarisation from the ground. To this extent at least, the observational task is less problematic than trying to detect temperature anisotropy (Partridge 1988), though there are of course other problems of experimental design (Caderni et al. 1978; Lubin & Smoot 1979; Nanos 1979; Lubin et al. 1983; Partridge 1988). On the other hand, in “standard” models of galaxy formation via gravitational instability from primordial adiabatic density inhomogeneities, the polarisation is expected to be much smaller than the temperature anisotropy (Kaiser 1983; Bond & Efstathiou 1984; Ng & Ng 1993). There are situations, however, when

the ratio of polarisation to temperature fluctuations can be non-negligible. Rees (1968) showed that an axisymmetric anisotropic cosmological expansion should induce a significant large-scale polarisation of the CMBR. This work was subsequently extended (and corrected) by Basko & Polnarev (1980), who obtained an exact solution to for the polarisation anisotropy produced in a flat triaxial anisotropic cosmological model; see also (Negroponte & Silk 1980; Tolman & Matzner 1984; Tolman 1985). Polarisation fluctuations are also induced if there exists a background of tensor perturbations in the metric (i.e. gravitational waves, hereafter GW’s) at the time of recombination. Indeed, one can regard the triaxial cosmological model mentioned above (Basko & Polnarev 1980) as being the superposition of an infinitely-long wavelength GW on an homogeneous and isotropic background space-time; the axisymmetric case corresponds to a scalar perturbation to a homogeneous and isotropic model. Polnarev (1985) subsequently showed how to calculate the polarisation anisotropy due to scattering of radiation by electrons in the presence of a single GW and obtained some analytic formulae for various limiting cases of gravitational wavelength and recombination history.

The possible existence of a cosmological (stochastic) GW background has been discussed for some time (Burke 1975; Grishchuk 1975; Starobinsky 1979; Carr 1980). More recently it has been realised that inflationary models of the early Universe produce stochastic GWs with characteristic spectra (Rubakov et al. 1982; Abbott & Wise 1984; Starobinsky 1985; Lucchin & Matarrese 1985; Abbott & Harari 1986; Allen 1988; Sahni 1990). Although these tensor perturbation modes have no influence on the formation of cosmic structures, they do generate anisotropies in the CMBR temperature (Sachs & Wolfe 1967; Dautcourt 1969;

Grishchuk & Zel'dovich 1978; Anile & Motta 1978). This has led a number of authors recently to produce inflationary models in which a significant fraction of the temperature anisotropy detected by COBE would be due to (tensor) GWs rather than (scalar) density perturbations (Davis et al. 1992; Liddle & Lyth 1992; Lidsey & Coles 1992; Lucchin et al. 1992; Salopek 1992; Crittenden et al. 1993a). If this is the case then the constraints imposed by COBE upon models of structure formation are considerably altered. Moreover, there is a possibility that one might be able to use the ratio of amplitudes of tensor and scalar modes to reconstruct the shape of the effective potential of the scalar field responsible for driving inflation (Copeland et al. 1993a; Copeland et al. 1993b). Measurements of the CMBR temperature anisotropy at a single angular scale do not allow one to discriminate between contributions from scalar and tensor modes and, although the angular dependence of the temperature anisotropy is different in the scalar and tensor cases, there are problems in using information from smaller angular scales than COBE because of the possibility that reionisation might mask the behaviour of the primordial fluctuations (Bond et al. 1991).

In this paper we shall argue that CMBR polarisation measurements can supply important information about the existence of a significant cosmological GW wave background and also about the ionisation history of the Universe. We shall concentrate on explaining the basic physics behind the generation of polarisation; computations of detailed statistical properties will be deferred to a later paper. Throughout this paper we shall assume that the background cosmology is described by a flat metric:

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2 = a(\eta)^2 (-d\eta^2 + d\mathbf{x}^2), \quad (1)$$

where t is cosmological proper time, a is the scale factor and η is conformal time ($d\eta = dt/a(t)$); the \mathbf{x} are comoving coordinates; the speed of light is unity.

2 RADIATIVE TRANSFER WITH GWS

The essence of our problem is to calculate the effect of gravitational radiation upon the transfer of electromagnetic radiation through the period of hydrogen recombination and photon decoupling. To proceed we therefore need to consider the effect of Thomson-scattering upon the polarisation of electromagnetic radiation. An unpolarised beam of radiation picks up linear polarisation during Thomson scattering; the maximum polarisation is 100 % for radiation scattered perpendicular to the incident beam. If the incoming radiation is isotropic in the rest frame of the scattering particle, the scattered radiation is unpolarised. The same is true if the radiation has a dipole anisotropy. If there is a quadrupole anisotropy, however, there should be a net polarisation of the scattered radiation. Such a quadrupole can be caused by a density perturbation or, as we shall investigate in this paper, a gravitational wave. We adopt the formalism suggested by (Chandrasekhar 1960) and construct a vector \mathbf{n} with components n_r , n_l and n_u related to the usual Stokes parameters. Here $n_r + n_l = n$, the total photon occupation number. (The polarisation tensor of the radiation, π_{ij} , has off-diagonal elements equal to $n_u/2$ and diagonal elements equal to n_l and n_r respectively.) In the presence of a single gravitational

wave, the components of \mathbf{n} are functions of: (i) conformal time η ; (ii) comoving spatial coordinates \mathbf{x} ; (iii) photon frequency ν ; (iv) the polar angle, $\theta = \cos^{-1} \mu$, between $\hat{\mathbf{q}}$ (a unit vector in the direction of photon propagation) and $\hat{\mathbf{k}}$ (a unit vector in along the GW); (v) the azimuthal angle, ϕ , between the projection of $\hat{\mathbf{q}}$ onto a plane perpendicular to \mathbf{k} and a unit vector perpendicular to \mathbf{k} derived from the GW polarisation tensor (Polnarev 1985).

The equation of radiative transfer can be written in terms of the vector \mathbf{n} , as follows:

$$\frac{\partial \mathbf{n}}{\partial \eta} + \hat{\mathbf{q}} \cdot \frac{\partial \mathbf{n}}{\partial \mathbf{x}} = -\frac{\partial \mathbf{n}}{\partial \nu} \frac{\partial \nu}{\partial \eta} - \sigma_T N_e a [\mathbf{n} - \mathbf{I}(\mathbf{n})], \quad (2)$$

where

$$\mathbf{I}(\mathbf{n}) = \frac{1}{4\pi} \int_{-1}^{+1} \int_0^{2\pi} \mathbf{P}(\mu, \phi, \mu' \phi') \mathbf{n} d\mu' d\phi'; \quad (3)$$

σ_T is the usual Thomson scattering cross-section, N_e is the comoving number-density of free electrons and \mathbf{P} is the scattering matrix which is described by (Chandrasekhar 1960), and is given explicitly in terms of these variables by (Polnarev 1985). The important term in this context is the effect of the gravitational wave in shifting the photon frequencies via the first term on the right hand side of equation (2).

If the Universe is flat and filled with pressureless matter the appropriate linearised Einstein equations admit a solution for tensor metric perturbations h_{α}^{β} which, for a single wave, can be written in the form:

$$h_{\alpha}^{\beta} = h \epsilon_{\alpha}^{\beta} \exp[-i\mathbf{k} \cdot \mathbf{x} + i\omega(k)\eta]. \quad (4)$$

The wavenumber k is defined such that the physical wavelength $\lambda = 2\pi a/k$ and, because $c = 1$, we have $\omega(k) = k$; $\epsilon_{\alpha}^{\beta}$ is the GW polarisation tensor. The geodesic equation in the perturbed metric yields

$$\frac{d\nu}{d\eta} = \frac{\nu}{2} (1 - \mu^2) e^{-i\mathbf{k} \cdot \mathbf{x}} \cos 2\phi \frac{d}{d\eta} (h e^{ik\eta}). \quad (5)$$

In the unperturbed case ($h = 0$), the solution to (2) is simply $\mathbf{n} = n_0(1, 1, 0)$. To obtain the solution to first order in h it proves convenient (Basko & Polnarev 1980; Polnarev 1985) to transform to an alternative set of symbolic vectors which reduces (2) to a system of integro-differential equations which has an exact analytical solution in the limit $k \rightarrow 0$. The procedure for doing this will not be described here; see (Polnarev 1985). In terms of new variables $\alpha(\eta, \nu, \mu)$, $\beta(\eta, \nu, \mu)$ and $\xi = \alpha + \beta$ we obtain

$$\dot{\beta} + [q - ik\mu]\beta = F \quad (6)$$

$$\dot{\xi} + [q - ik\mu]\xi = H, \quad (7)$$

where $q = \sigma_T N_e a$ and

$$F(\eta) = \frac{3q}{16} \int_{-1}^{+1} \left[(1 + \mu'^2)^2 \beta - \frac{1}{2} \xi (1 - \mu'^2)^2 \right] d\mu', \quad (8)$$

$$H(\eta) = \frac{C(k)}{k^3} \frac{\partial}{\partial \eta} \left(\frac{1}{\eta} \frac{\partial \sin k\eta}{\partial \eta} \frac{1}{\eta} \right). \quad (9)$$

Note that we use a different definition of h compared to Polnarev (1985); $C(k)$ is related to the GW spectrum (Starobinsky 1985). We shall concentrate in this paper on

the evaluation of the *rms* polarisation and anisotropy induced by GWs of a given wavenumber in stochastic superposition:

$$\Pi_k = \langle \Pi_k^2 \rangle^{1/2} = \left[\int_{-1}^{+1} |\beta|^2 (1 + \mu^2)^2 d\mu \right]^{1/2}; \quad (10)$$

$$A_k = \langle A_k^2 \rangle^{1/2} = \left[\int_{-1}^{+1} |\xi|^2 (1 - \mu^2)^2 d\mu \right]^{1/2}. \quad (11)$$

Before displaying our results, it is useful to re-cap the analytical results (Polnarev 1985). One can find a solution to the stationary case with $H = \bar{H} = \text{const.}$ and $q = \bar{q} = \text{const.}$ Here Π_k and A_k are of the same order, at least for small angular scales $\theta \leq (2q/k)^{1/2}$. This solution corresponds to the case prior to recombination. To go further one can assume an instantaneous recombination such that the stationary solution applies until $\eta = \eta_r$ and thereafter $q = 0$. Since there is no more scattering after η_r the polarisation remains unchanged between η_r and the present epoch. The anisotropy, however, grows because of the Sachs-Wolfe effect which does not involve scattering. The ratio of polarisation to anisotropy is therefore expected to be small in such a situation. However, in realistic cosmological models, we do not expect recombination to be instantaneous. If there is an extended period of ionisation then scattering can, in principle, generate an interestingly large value of Π_k .

3 RESULTS

The solution of the equations (6)–(9) can be expressed formally as

$$\begin{pmatrix} \beta \\ \xi \end{pmatrix} = e^{\tau + ik\mu\eta} \int_0^\eta d\eta' \begin{pmatrix} F(\eta') \\ H(\eta') \end{pmatrix} e^{-\tau(\eta') - ik\mu\eta'}, \quad (12)$$

where the optical depth, $\tau = \int_\eta^1 q(\eta') d\eta'$, so that $\tau = 0$ when $\eta = 1$. To find solutions for the *present* mean square anisotropy and polarisation, one simply evaluates (10) & (11) at $\eta = 1$. The results can thus be expressed as integrals over η :

$$\langle \Pi_k^2 \rangle = \frac{32}{3} \int_0^1 d\eta \frac{\bar{F}^2(\eta) e^{-2\tau(\eta)}}{q(\eta)} + \int_0^1 d\eta \bar{F}(\eta) F_0(\eta) \quad (13)$$

$$\langle A_k^2 \rangle = 2 \int_0^1 d\eta H(\eta) e^{-\tau(\eta)} F_0(\eta), \quad (14)$$

where $\bar{F}(\eta) = F(\eta) e^{-\tau(\eta)}$. The function $F_0(\eta)$ is given by

$$F_0 = \int_0^\eta d\eta' H(\eta') e^{-\tau(\eta')} K_-(\eta, \eta') \quad (15)$$

and $\bar{F}(\eta)$ must be obtained by solving the integral equation

$$\bar{F} = \frac{3q}{16} e^{\tau(\eta)} \left\{ \int_0^\eta \bar{F}(\eta') K_+(\eta, \eta') d\eta' - \frac{1}{2} F_0(\eta') d\eta' \right\}. \quad (16)$$

The function K_\pm used in these expressions is just

$$K_\pm(\eta, \eta') = \int_{-1}^{+1} d\mu (1 \pm \mu^2)^2 e^{ik\mu(\eta - \eta')}. \quad (17)$$

Performing the integrals over μ first allows us to extend the work of (Polnarev 1985) to arbitrary q and k whilst keeping

Figure 1. Anisotropy as a function of k for different models of the ionisation history. Models are shown with: $\eta_r = \eta_s$, $\Delta = \Delta_s$ (standard); $\eta_r = 2\eta_s$, $\Delta = \Delta_s$; $\eta_r = \eta_s$, $\Delta = 5\Delta_s$; $\eta_r = \eta_s$, $\Delta = 100\Delta_s$. A model with no recombination is shown for comparison.

the number of numerical integrations required to a minimum. A convenient parametrisation is $q(\eta) = q_r \chi(\eta) \eta^{-4}$ where χ is the fractional ionisation; $q_0 \simeq 0.14 \Omega_b h$, where h is the Hubble parameter in units of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. For small η , $\chi = 1$; we adopt the following flexible illustrative model for the variation of χ through recombination:

$$\chi(\eta) = \begin{cases} 1 - \frac{2(\eta - \eta_r)^2}{\Delta^2} & \eta < \eta_r + \frac{\Delta}{2} \\ \chi_0 + \frac{2(\eta - \eta_r - \Delta)^2}{\Delta^2} & \eta_r + \frac{\Delta}{2} < \eta < \eta_r + \Delta \\ \chi_0 & \eta > \eta_r + \Delta. \end{cases} \quad (18)$$

Here Δ parametrises the duration of recombination and χ_0 is the residual ionisation. The standard picture of recombination has $\eta_r = \eta_s \simeq 0.026$, $\Delta = \Delta_s \simeq 0.05$ and $\chi_0 \simeq 3 \times 10^{-5} (\Omega_0 / \Omega_b h)$; standard cosmological nucleosynthesis requires $0.010 < \Omega_b h < 0.032$ (Olive et al. 1990); we take Ω_0 to be unity throughout these calculations. The advantage of the simple model (18) is that it is easy to integrate and allows us simply to assess the effect upon the level of polarisation of changes in the parameters χ_0 , Δ and η_r . To reduce the parameter space somewhat, we fix the optical depth at the end of recombination $\eta = \eta_r + \Delta$ to be equal for all the models we consider; the standard value is $\tau = 0.07$. In this way have only two independent parameters which we take to be Δ and η_r . As an extreme example we also consider the case of no recombination at all, $\chi(\eta) = 1$.

We shall look at Π_k and A_k as functions of k in the following series of figures. In Figure 1 we study the anisotropy produced as a function of k for different models of the ionisation history. The GWs all have an arbitrary initial amplitude independent of k for this and the subsequent figures. The trend with recombination model is straightforward: the more extended the period of ionisation, the smaller the anisotropy produced at large k . This is due to the blurring effect of the finite width of the last scattering surface. In the extreme case of no-recombination, there is severe suppression of the small-scale anisotropy.

Figure 2 shows the polarisation for the same set of models as Fig. 1. The most important point here is that the maximum polarisation shows the opposite trend to the anisotropy: the longer the period of ionisation, the higher is the peak polarisation. The peak wavelength also increases as the width of the last scattering surfaces increases: scat-

Figure 2. As Figure 1, but showing the polarisation as a function of k for the different models.

tering can occur later, when the horizon size is larger. The oscillations in polarisation for large k are caused by resonances between the GW wavelength and the width of the last scattering surface.

Figures 1 and 2 show very clearly the basic physics in operation during the production of a polarised CMBR. Because of the arbitrary scaling, however, they do not represent quantities that can be compared to observation. For this, we need to look at the ratio of polarisation to anisotropy. This poses some problems. The ratio Π_k/A_k would correspond to the ratio of polarisation to anisotropy observed for a δ -function spectrum. For broader spectra – particularly the very flat spectra typically predicted in inflation (Rubakov et al. 1982; Abbott & Wise 1984; Starobinsky 1985; Lucchin & Matarrese 1985; Abbott & Harari 1986; Allen 1988; Sahni 1990) – a more relevant characterisation would be the ratio of total polarisation $\Pi = (\int \langle \Pi_k^2 \rangle dk/k)^{1/2}$ to total anisotropy $A = (\int \langle A_k^2 \rangle dk/k)^{1/2}$, which would generally be smaller than Π_k/A_k evaluated at a single point. Furthermore, any given experiment will observe some particular angular scale on the sky which would correspond to a weighted sum of contributions from all k . To explore systematically the space of beam-widths and GW spectra is beyond the scope of this paper; we shall restrict ourselves to showing Π_k/A_k for a few examples to show when this ratio can be large (see Figure 3). The ratio Π_k/A_k has a maximum value of around 10% for the standard model, increasing to over 40% for the no recombination case. Note, however, that the ratio of total polarisation to total anisotropy (integrated over a flat spectrum) is indeed very much smaller than this: $\Pi/A \simeq 0.3\%$ for standard recombination and $\Pi/A \simeq 3.7\%$ for no recombination. Clearly the superposition of GWs with different wavelengths leads to a large reduction in the observable polarisation compared to the δ function case.

4 DISCUSSION AND CONCLUSIONS

We have seen that, in certain conditions, a stochastic GW background can lead to a significant polarisation of the CMBR. The level of polarisation is strongly dependent upon the GW spectrum: it is high for a δ -function, but much lower for a flat spectrum. Whether the level is high enough to be observed would depend on the GW spectrum, the ionisation history and the experimental beamwidth. We shall explore

Figure 3. As Figure 3, but showing the ratio of polarisation to anisotropy as a function of k for the different models.

this parameter space more systematically in a forthcoming paper; preliminary analysis suggests that experiments capable of detecting $\Pi/A \leq 10\%$ would be needed to provide useful data.

In the inflationary models there will be both scalar and tensor contributions to both polarisation and anisotropy. The values we have obtained for the tensor perturbations are larger than those usually quoted for scalar modes (Kaiser 1983; Bond & Efstathiou 1984; Ng & Ng 1993) on large scales, but scalar perturbations have means other than the Sachs–Wolfe effect for inducing anisotropy and polarisation (e.g. streaming motions and the Silk effect). These mechanisms depend sensitively upon the dark matter and normalisation of the density fluctuations, which makes a full calculation of the contribution to the polarisation from both modes difficult. Suppose that the total anisotropy A and polarisation Π includes both scalar and tensor contributions: $A = A_T + A_S$ and $\Pi = \Pi_T + \Pi_S$. Now if tensor modes contribute a fraction f of the total anisotropy then the overall ratio of polarisation to anisotropy is just

$$\left(\frac{\Pi}{A}\right) = f \left(\frac{\Pi}{A}\right)_T + (1-f) \left(\frac{\Pi}{A}\right)_S \quad (19)$$

(assuming tensor and scalar modes add independently, as they should in linear theory). Only if f is significant and the ionisation history is such that $(\Pi/A)_T$ is large can one expect there to be a significant alteration in the overall ratio of polarisation to anisotropy compared to the standard case. In the models discussed by Crittenden et al. (1993b) the polarisation induced by the tensor modes is usually smaller than the scalar contribution: at best it is comparable. If one does not know *a priori* how much of the anisotropy is produced by tensor modes, it would be very difficult to use polarisation measurements to disentangle the contribution in such models. However, these authors considered only a small subset of inflationary models. Models can be produced which yield a much larger value of f than they considered: it is possible to have $f \simeq 0.5$ without violating constraints on the fluctuation power spectrum (Davis et al. 1992; Liddle & Lyth 1992; Lidsey & Coles 1992; Lucchin et al. 1992; Salopek 1992). We shall give specific predictions for particular inflationary spectra in a forthcoming paper.

It is worth mentioning, however, that even if the scalar and tensor contributions to the total polarisation are comparable in terms of their *rms* values, one might still be able to

discriminate between them. For a start, the autocorrelation functions of the temperature pattern (not calculated in this paper) will be different in the two cases, because the correlation angle is determined by physical length scales which are different for scalar and tensor modes, as can be seen from Figs 1 & 2. Recently, Naselsky & Novikov (1993) have argued that oscillatory features (similar to ‘Sakharov’ oscillations) in the power spectrum of fluctuations produced by adiabatic scalar fluctuations could be a powerful cosmological probe. The detailed spatial distribution of polarisation and anisotropy could also be a sensitive discriminant. For example, the relative positions of ‘hotspots’ of temperature and polarisation are different in the tensor and scalar case. The simplest way to characterise this would be to calculate the cross-correlation between polarisation and anisotropy maps. We shall return to these ideas in future work.

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