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The linear impulse response for disturbances in an oscillatory Stokes layer

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Abstract

A brief review is given of numerical simulation results for the evolution of disturbances in a flat oscillatory Stokes layer. A spatially localised form of impulsive forcing is applied to trigger the disturbances. For the linearized case, the disturbance development displays an intriguing family tree-like structure, which involves the birth of successive generations of wavepackets. Although some features of the wavepacket behaviour may be accounted for using a Floquet linear stability analysis, for Fourier modes with prescribed wavenumbers, the discovery of the tree-like structure was completely unexpected. Simulation results were also obtained for nonlinear disturbances, where highly localised spikes were found to develop.

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1. Introduction

We consider the oscillatory Stokes layer that is generated when an infinitely long and flat rigid wall oscillates sinusoidally to-and-fro within its own plane, beneath an unbounded body of otherwise stationary incompressible viscous fluid. This simple physical configuration is ideal for studying the fundamental stability characteristics of unsteady flows¹, since the fluid motion is described by an elementary exact solution of the Navier-Stokes equations. If the wall is located at $y = 0$ and moves along the x -direction with a velocity $U_0 \cos(\omega t)$, where ω is the oscillation frequency, then the unperturbed basic state for the flow is given in non-dimensional form by $U_B(y, \tau) = e^{-y} \cos(\tau - y)$. The constant boundary layer thickness $\delta = 2\nu/\omega$, where ν is the kinematic viscosity, is used to scale the lengths, while the velocities are scaled with the amplitude U_0 and the time is set as $\tau = \omega t$.

We will describe some selectively chosen numerical simulation results for the spatial and temporal development of disturbances from the basic state. In this very brief paper, the intention is to highlight the novel features of the behaviour that was discovered for the linearized case, rather than to provide a comprehensive discussion. The simulation

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results that we were also able to obtain for the nonlinear evolution will only be mentioned in the concluding remarks. A much more thorough and detailed account has previously been given elsewhere².

Disturbance wavepackets were generated by an impulsively applied point source of excitation, which initially causes them to be highly localized in space. The motives for considering wavepackets, rather than spatially monochromatic waves, are essentially the same as for the study of instabilities that occur in steady flows. Mere identification of the instability of perturbations characterised by a single wavenumber does not, without any additional analysis, suffice to fix the possibilities for the disturbance development. Physically realisable disturbances will typically take a wavepacket form, involving a mixture of a range of different wavenumbers. In convectively unstable flows, every disturbance wavepacket that is triggered will always, after sufficient time, propagate away beyond any given spatial location, eventually leading to a quietened flow there. It is only when the instability takes an absolute form that disturbances are able to grow continually over time at fixed points in space.

The usual methods of analysis for disturbance wavepacket development in steady boundary layers and shear flows³ can be extended⁴ so as to incorporate temporally oscillating basic states. This requires the accommodation of Floquet theory into the analytical framework. It can still be demonstrated that for unsteady flows, just as for steady flows, the long-term behaviour of wavepackets yields a twofold classification of instabilities, as being either convective or absolute in nature. Our numerical simulation results also indicate that, in a manner which again remains akin to what occurs for the steady case, the growth of wavepacket maxima in oscillatory flows is dictated by the largest growth rate that can occur for disturbances with a single spatial wavenumber. However, this growth rate must now be computed using growth exponents for Floquet modes, rather than those associated with the more simply structured normal modes that are relevant for a steady flow.

An intriguing result from our numerical simulations was the discovery that the spatial and temporal development that is actually found in an unsteady flow can involve a much richer complexity than could have been surmised by merely considering the asymptotic growth behaviour for large times. The illustration of this complexity is the main aim of this brief review.

2. Simulation results for the family-tree structure of wavepacket components

The evolution of impulsively excited disturbances was studied by simulations that utilised a two-dimensional vorticity based version of the governing fluid equations. Both the mathematical formulation and the discretization were similar to those deployed in a previously developed and extensively tested numerical scheme⁵. Great care was taken in the treatment of the artificial extremities of the computational domain, to avoid the production of numerical noise and spurious reflections. It proved necessary to make the domain large enough to ensure that no physically significant disturbance could ever approach the computational boundaries, over the whole of any time interval for which a simulation was conducted. This requirement added considerable expense.

For the purposes of illustration, we present results obtained from one particular numerical simulation, conducted using a linearized form of the governing equations. The disturbance was excited at a Reynolds number $R = 600$ so that, according to the appropriate linear stability Floquet analysis¹, it would be expected to eventually decay. The impulse was applied at a streamwise position located in the centre of the computational domain, with the phase in the oscillation cycle chosen so as to coincide with the wall velocity achieving its maximum magnitude.

The resulting spatial-temporal development of the disturbance is plotted in the figure, for three periods of the oscillation. Amplitudes are measured using the magnitude of the vorticity at the bounding wall. A finite, though relatively very small, cut-off level has been deployed, to provide a threshold for detecting the presence or apparent absence of a perturbation at any given spatial-temporal point. This threshold gives rise to the white background that is seen in the plot, which delineates the boundaries of the regions where the disturbance has developed.

It may be seen that, initially, the perturbation convects to the right of the impulse origin, which is downstream with respect to the basic flow near to the wall at the time that the disturbance was excited. The disturbance grows in magnitude until it attains a maximum amplitude at a time $\tau \sim \pi$, having by then travelled a streamwise distance ~ 500 from its point of excitation. At approximately the same time and position in the spatial-temporal plane, a secondary or *daughter* perturbation is created from the original *mother* wavepacket, convecting to the left. It may be noted that this new propagation direction matches that of the wall motion for the corresponding part of the oscillation cycle. The daughter wavepacket is of smaller amplitude than its mother and attains a maximum magnitude at the

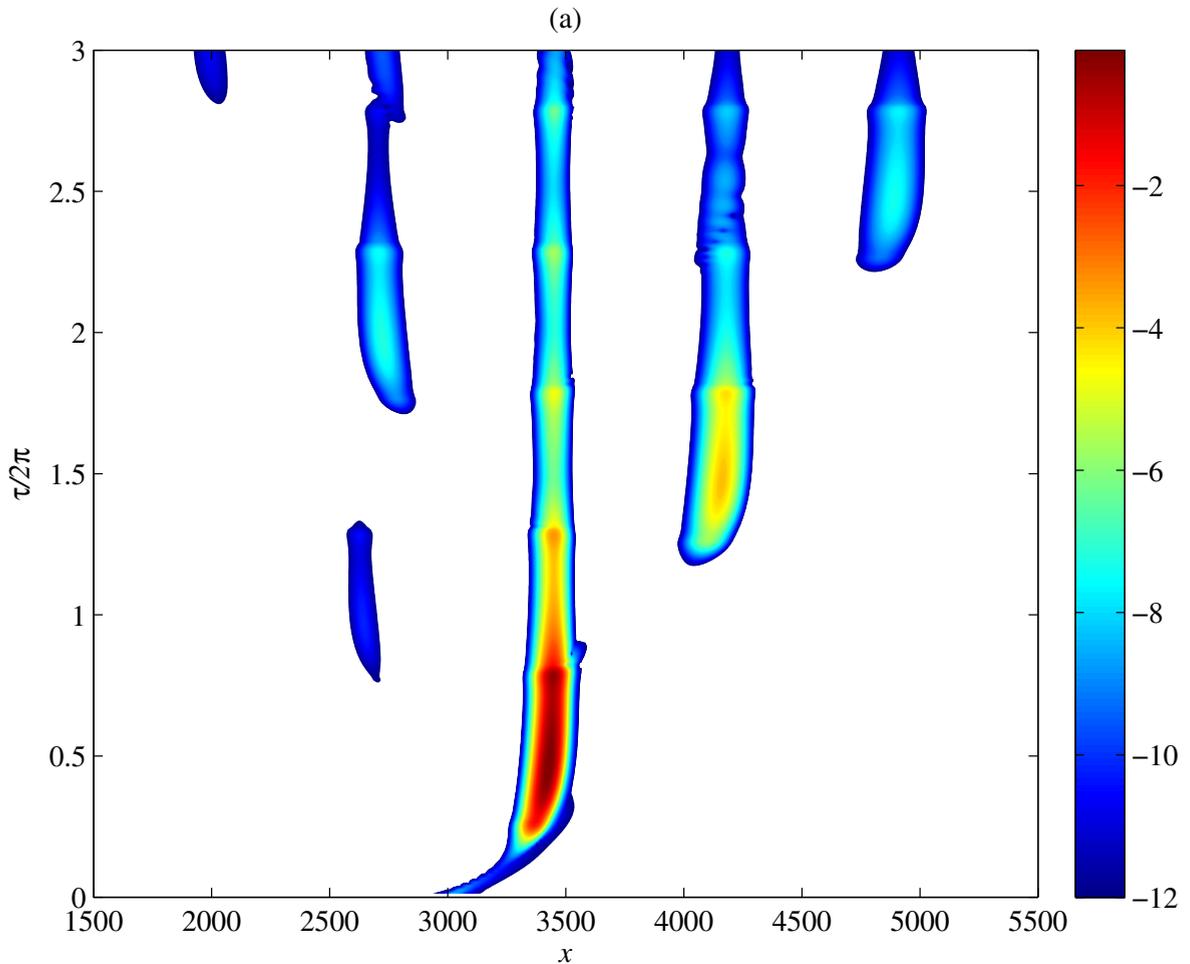


Fig. 1. Spatial-temporal development of perturbation vorticity magnitude at the bounding wall surface, for a linearized disturbance impulsively excited at $x_f = 3000$ for $R = 600$. (Magnitudes are plotted using a logarithmic scale for the shading.)

end of the first period before it diminishes. A second daughter wavepacket is born from the mother at the end of the first cycle. This propagates, instead, in the downstream direction to the right, growing to a larger amplitude than the first daughter and attaining a maximum magnitude at a time $\tau \sim 3\pi$. The original mother wavepacket continues to give birth to new daughter wavepackets, which alternate between convecting to the left and to the right. In turn, the daughter wavepackets give birth to *grand-daughter* wavepackets, which also display a switching in propagation direction, dependent upon the orientation of the wall motion during the half period in which they arose. The process of wavepacket birthing continues indefinitely and a disturbance with a *family-tree* like structure is formed.

These and other simulation results that we have obtained indicate that, eventually, it can become possible for two or more individual wavepacket components to convect into the same region of space and overlap each other. At such locations, the superposition of waves, even when they are individually decaying, may give rise to transient increases in the disturbance amplitude. Overlapping between different wavepacket components is likely to be enhanced for disturbances occurring in physical experiments. The behaviour at any given spatial point may typically involve superpositions of disturbances arising from various uncontrolled sources of external forcing, and not just those that had arisen, relatively cleanly, from a single impulsive and initially highly localised form of excitation. Thus if the impulse response that we discovered has the appearance of a family tree, but with the possibility of overlapping branches, then the spatial-temporal behaviour that would be expected for wavepackets in an actual physical experiment might bear

more than a passing resemblance to a distant tangled *forest* of trees, growing upwards on a sloped terrain. The slope would need to be incorporated into the picture in order to account for the fact that the roots of the trees would be found at different heights, corresponding to the different points in time at which they were triggered. Wherever the branches of either a particular tree, or different trees rooted at different heights, were found to cross and hence lie in the same line of sight, then there would be the possibility of a transient form of growth being obtained there.

3. Concluding remarks

The intricate spatial-temporal structure that we have discovered for the impulse response suggests that it may be a challenging task to interpret data from physical experiments, even if it were to prove possible to control the configuration so as to be able to trace the linear development arising from just a single source. Moreover, the subsequent nonlinear development would serve to further complicate matters. Our simulations for finite-amplitude disturbances indicated that there can be a very rapid onset of nonlinearity, leading to the formation of short-scaled spike-like features, which are qualitatively similar to those that had previously been identified for wavepackets evolving in steady boundary layers⁶.

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