Online Research @ Cardiff

CARDIFF UNIVERSITY PRIFYSGOL CAERDYDD

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: http://orca.cf.ac.uk/91123/

This is the author’s version of a work that was submitted to / accepted for publication.

Citation for final published version:

Disney, Stephen Michael 2016. Revisiting activity sampling: A fresh look at binomial proportion confidence intervals. European Journal of Industrial Engineering 10 (6) , pp. 724-759. 10.1504/EJIE.2016.081021 file

Publishers page: http://dx.doi.org/ 10.1504/EJIE.2016.081021 <http://dx.doi.org/ 10.1504/EJIE.2016.081021>

Please note:
Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher’s version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See http://orca.cf.ac.uk/policies.html for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.

information services
gwasanaethau gwybodaeth

Disney, S.M., (2016), "Revisiting activity sampling: a fresh look at binomial proportion confidence intervals",
European Journal of Industrial Engineering, Vol. 10, No. 6, pp724-759. ISSN 1751-5254. DOI: 10.1504/EJIE.2016.081021.

Revisiting activity sampling: a fresh look at binomial proportion confidence intervals

Stephen M. Disney
Logistics Systems Dynamics Group, Cardiff Business School, Cardiff University.
Email: DisneySM@cardiff.ac.uk. Telephone: +44(0)2920 876310.

Abstract
The Wald interval is typically used to assign confidence to the accuracy of activity sampling studies. It is known the performance of the Wald interval is poor, especially when the observed probability is near zero or one. The suitability of the Wald interval for activity sampling is not often discussed in the operations management literature; if it is, this is usually followed by inappropriate and incorrect advice. Herein a range of alternative binominal confidence intervals for activity sampling is reviewed. A number of selection criteria are considered including achievement of the target nominal coverage probability, size of the interval, and ease of use and presentation. It is recommended that the Clopper-Pearson interval is used for activity sampling. A table of confidence intervals and sample sizes that is specifically designed to be used within a new activity sampling procedure based on the Clopper-Pearson interval is developed. Finally, pedagogical issues are considered.

Key words
Activity sampling, work sampling, binomial proportion confidence intervals, coverage probability.

1. Introduction and motivation

Activity sampling is an empirical data collection technique attributed to Tippet (1935). It can be used to determine the proportion of time: an activity is being conducted by an operator; an operator is doing productive, value adding work; a machine or operator being delayed, or whether an entity (customer, supplier, product) possesses a particular (quality) characteristic or not.

Activity sampling has been defined by British Standard 3138 (1992) as "A technique in which a large number of observations are made over a period of time of one group of machines, processes or workers. Each observation records what is happening at that instant and the percentage of observations recorded for a particular activity or delay is a measure of the percentage of time during which that activity or delay occurs". Activity sampling has also been known as: *work sampling* in the U.S.; *snap reading*, Tippets' original name for the technique; the *ratio delay technique*, as it can be used to ascertain the proportion of time a machine or operator is delayed or idle; and in the service sector, *random moment studies*.

Activity sampling is a relatively inexpensive technique that can be used to determine the proportion of time spent on a particular activity. It can be applied to long, varied and intermittent work. It does not use a stop watch, so it is more acceptable to the subjects of the study and is not as intrusive as other methods. However, activity sampling is not efficient when activities are of a short duration, regular and predictable when time and motion studies might be more suitable.

Almost all Operations Management (OM) textbooks that the recommended activity sampling procedure use the *Wald interval* to assign confidence to the results obtained from the study. It is known in the statistical literature that this interval is a rather poor approximation to the true interval. Herein we test this claim and find that, 80% of the time, the Wald interval does not achieve the desired confidence. This is a serious issue for anyone using activity sampling for safety assurance. To address this we review a range of alternative confidence intervals and select one for inclusion in an updated activity sampling procedure. A new procedure is recommended that actually achieves the desired confidence and accuracy levels.

1.1. The standard activity sampling procedure

For simplicity we assume that we are monitoring the types of activities that an operator is undertaking. The standard activity sampling procedure usually involves the following steps. First, a pilot study (an initial look at the situation being studied) is undertaken to ascertain the range of activities an operator undertakes. The second step is to design the sampling tour and data collection forms. The sampling tour specifies when instantaneous observations of the situation being studied are to be taken. It is assumed that the underlying probability of an activity occurring does not change over time (at least within the period of study). It is important to ensure that there is no systematic effect present in the data collected by taking observations after random intervals of time. The study period should be long enough to capture the complete range of activities the operator undertakes but the study can be interrupted if the necessary. It may be also shortened or lengthened as required. The data collection forms are simply a table with rows that note the time a observation is taken and columns that document the activity observed.

In the next step, the main data collection is undertaken. The situation being studied is observed at the prescribed moments of time and the activity being conducted by the operator is recorded in the data collection form as a tick mark. After several dozen or so observations the tick marks can be tallied to obtain an initial estimate of the probability of an activity occurring, $\hat{p}$. Alternatively we could use judgement or experience to make an initial estimate of $\hat{p}$. We could even believe that we are being conservative and (incorrectly) use $\hat{p}=0.5$. Note there will be a different $\hat{p}$ for each of the activities. We will not index them there – it is fairly obvious which activity is being considered as there will be a unique estimate for each of the columns in the data collection form.

Based on this initial estimate, $\hat{p}$, we can then calculate how many instantaneous observations (n) we need to take in order to be 95% confident that the true (real, constant, but unknown) probability of the activity occurring (p) is within a certain boundary with

$$n = 4\hat{p}\left(1-\hat{p}\right)/L^2 , \tag{1}$$

where L is a tolerance of the form $\hat{p} \pm L$, within which a desired level of accuracy has to be achieved. Note L is an absolute distance and not a relative percentage of $\hat{p}$. L can be thought of as the *confidence interval half-width*.

We continue to take the required number of observations, checking periodically with (1) for an updated value of n (which may have changed due to the new values of $\hat{p}$, the estimate of the true probability p). Note that n will be different for each of the observed activities (as $\hat{p}$ is likely to be different for each activity). In order to gain a complete picture of the situation, the largest n will determine the required number of samples to be taken. Observe that the required level of accuracy (L) is a strategic decision that influences the trade-off between the efficiency of the procedure and the accuracy obtained.

1.2. Motivation

At first sight the activity sampling procedure seems fine. It has such a long history and is relatively straight forward, so what is the problem? Take a look at (1). What happens when $L > \hat{p}$ (or $L > (1-\hat{p})$? A negative probability of p (or one that is greater than one) is not possible, so there is an issue here. What happens when $\hat{p} = 0$ or $\hat{p} = 1$? Equation (1) incorrectly suggests that no samples should be taken. But rarely do operations management (OM) textbooks discuss these issues.

Equation (1) is actually a simplified and rearranged version of the so-called *Wald interval for the binomial proportion*, Wald and Wolfowitz (1939). A literature review (see, for example, Pires and Amado (2008) for a modern and particularly comprehensive treatment of 20 different binomial proportion confidence intervals) reveals that statistical scholars have serious concerns about the adequacy of the Wald interval. However, this is the interval used in most the OM textbooks.

Table 1 summarises a review of OM books for the terms *activity sampling*, *work sampling*, *snap reading*, *ratio delay*, *random moment studies* and *confidence intervals*. It highlights that in general it can be said that the Wald interval is almost exclusively recommended and guidance on the interval coverage probability is limited to either a "large sample size" or "$n\hat{p} > 5$ and $n(1-\hat{p}) > 5$", if it is given at all. If there is indeed a problem with the Wald interval, then it means that the confidence that is assigned to the confidence interval cannot be trusted. The purpose of this paper is to investigate this issue.

1.3. Organisation of this paper

Section 2 presents a short review of the literature that exploits activity sampling to demonstrate the relevance and possible use of activity sampling. Section 3 reviews some background theory and defines notation. Section 4 studies the Wald interval. Key performance measures for confidence intervals, suitable for use in an activity sampling procedure, are defined and justified in Section 5. Section 6 studies some alternative confidence intervals from an OM activity sampling viewpoint. Section 7 reflects upon the considered confidence intervals and makes recommendations. Section 8 details a new updated activity sampling procedure that exploits the recommended confidence interval. Section 9 provides pedagogical reflections. Section 10 concludes. A blank example data collection form and the necessary tables required for the updated activity sampling procedure are presented in the appendices.

2. Recent activity sampling studies

It is interesting to quickly review studies that have used activity sampling in order to gain an understanding of the range of problems and issues that the methodology could be applied too. Farrell et al. (2009) determined the unit labour cost of activities in a bank across multiple branches. Tsai (1996) incorporated activity sampling into an Activity Based Costing methodology. Thomas (1991) investigated labour productivity in the nuclear industry. Liou and Borcherding (1986) studied power plant productivity.

Recommended sample size	Guidance given	Reference
None	None	Adam and Ebert (1992), Barnes (2008), Finch (2008), Krajewski, Ritzman and Malhotra (2013), Slack, Brandon Jones and Johnston (2013), Waters (2002)
'100 samples'	None	Naylor (1996), Schonberger and Knod (1988)
'perhaps 250 samples'	None	Bicheno and Holweg (2009)
'200 samples'	None	Bicheno (2008)
$n = \frac{4\hat{p}(1-\hat{p})}{L^2}$	None	International Labour Office (1974), Lockyer (1983), Lockyer, Muhlemann and Oakland (1988), Hill (1983 and 2000), Weiss and Gershon (1989), Wild (1995)
	'$n\hat{p} > 5$ and $n(1-\hat{p}) > 5$'	Meredith (1992), Rosenkrantz (2009)
$n = \left(\frac{z_\alpha}{L}\right)^2 \hat{p}(1-\hat{p})$, where z_α is the standard normal variant that refers to the confidence level required.	None	Brisley (2001), Chase and Aquilano (1992), Davis and Heineke (2005), Evans et al. (1984), Greasely (2009), Heizer and Render (2014), Jacobs, Chase and Aquilano (2009), Khanna (2015), Lee and Schniederjans (1994), Noori and Radford (1995), Reid and Sanders (2002), Russell and Taylor (2009), Stevenson (2012), Whitmore (1987)
	'> 30 samples'	Curwin and Slater (1990)
	'$n\hat{p} > 5$ and $n(1-\hat{p}) > 5$'	Anderson et al. (2007), Silver (1997)

Table 1. Treatment of activity sampling confidence intervals in OM textbooks

Buchholz et al. (1996) characterised ergonomic hazards in the American highway construction industry. Chen, Peacock and Schlegel (1989) conducted an ergonomic study to assess physical work stress. Construction site productivity was measured by Heinze (1984). Kaming et al. (1997) found that craftsmen in the Indonesian construction industry spent 75% of their time productively and identified five different root causes of productivity problems.

73% of working time was productive in Gunesoglu and Meric's (2006) study of the garment industry. Rutter (1994) identified the activities undertaken by operators in a pharmaceutical plant. The results were also used to justify the purchase of additional equipment. Kelly (1964) studied executive behaviour in a factory.

Williams, Harris and Turner-Stokes (2009) identified the proportion of time on patient-related care issues (as supposed to other nursing activities) within a UK neuro-rehabilitation setting. Pelletier and Duffield (2003) also considered hospital scenarios. Finkler et al. (1993) compared activity sampling with time-and-motion studies and reflected upon the policy implications of sampling accuracy in the health services industry. Foley (1999) investigated the impact of restraints in nursing homes.

3. Background theory and notation

Statisticians have developed several formulae to determine, with a particular level of confidence, a range of estimated probabilities that will contain the true probability. These formulae are known as *binomial proportion confidence intervals* or just *confidence intervals* for brevity. These confidence intervals are based on the binomial distribution as we are concerned with the observing x number of successes (the activity occurring or not) in n observations. Clopper and Pearson (1934) developed an *exact* solution and there is also some approximate confidence intervals available that possess various properties.

Let $\hat{p}$ be the observed value of the probability of a particular activity occurring. This observed value is not the true probability which can only be obtained by taking an infinite number of observations. However, it is an appropriate, approximate value for the true probability p. As the number of observations increases, the more confident we are that the observed probability is representative of the true probability. However, while $\hat{p} \to p$ as $n \to \infty$, $\hat{p}$ does not approach p asymptotically (Brown, Cai and DasGupta, 2001).

The observed probability $\hat{p} = x/n$, where x is the number of successes in the n samples (the number of times an activity occurs in the n observations). The confidence interval equation will give us an upper, $\hat{p}_U$ and lower, $\hat{p}_L$ bound for the unknown p, for a desired level of confidence, $1-\alpha$. In other words, the confidence interval is a range of values, which we can be sure that, $(1-\alpha)\times 100\%$ of the time, will include the true value of p. Thus if α = 0.05, we can be 95% confident that $\hat{p}_L \le p \le \hat{p}_U$. The coverage probability, Λ, is the actual confidence achieved by interval, and Λ_l is the actual length of the interval.

In summary,

- p is the (unknown) true value of the probability of a particular activity occurring from the entire population, $0 \le p \in \mathbb{R} \le 1$
- n is the number of observations that have been taken
- x is the number of times a particular activity was observed in the n observations
- $\hat{p} = x/n$ is an estimate of the true probability p
- $\hat{p}_U$ is the upper limit of the confidence interval
- $\hat{p}_L$ is the lower limit of the confidence interval
- $1-\alpha$ is the desired confidence level. α is the 'confidence coefficient'
- L is the interval half-width. It is the difference between the estimated value of $\hat{p}$ and the *unconstrained* (upper and lower) confidence interval, $\hat{p}_U - \hat{p} = \hat{p} - \hat{p}_L = L$. Note that when $\hat{p}$ is near 0 or 1, the limits of the confidence limit must be truncated to ensure $0 \le \{\hat{p}_U, \hat{p}_L\} \le 1$. In which case $L = \max[\hat{p}_U - \hat{p}, \hat{p} - \hat{p}_L]$.
- Λ is the coverage probability. The coverage probability is the level of confidence actually achieved (not the desired confidence level).
- Λ_l is the actual length of the confidence interval. $\Lambda_l = \hat{p}_U - \hat{p}_L$. Note, that $\Lambda_l < 2L$ near $p = \{0,1\}$.
- z_α is the standard normal variant for a given α.

4. Wald interval & measures of performance for activity sampling

The simplest equation for the confidence interval is a simplified and rearranged version of the Ward interval, see Table 1. It is based on a normal approximation to the binomial distribution. The upper and lower limits of the Wald interval are defined as

$$\hat{p}_U = \min\left[\hat{p} + z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, 1\right],\ \hat{p}_U = \max\left[0, \hat{p} - z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right] \tag{2}$$

where, $\hat{p} = x/n$ and $z_\alpha = \Phi^{-1}\left(1-\frac{\alpha}{2}\right)$ is the $1-\frac{\alpha}{2}$ quartile of the cumulative density function of the standard normal distribution. z_α for popular confidence levels are;

- 90% confidence, $\alpha = 0.1$ and $z_\alpha = \Phi^{-1}\left(1-\frac{\alpha}{2}\right) = \Phi^{-1}(0.95) = 1.64$,
- 95% confidence, $\alpha = 0.05$ and $z_\alpha = \Phi^{-1}\left(1-\frac{\alpha}{2}\right) = \Phi^{-1}(0.975) = 1.96$,
- 99% confidence, $\alpha = 0.01$ and $z_\alpha = \Phi^{-1}\left(1-\frac{\alpha}{2}\right) = \Phi^{-1}(0.995) = 2.58$.

Notice some boundary conditions have been introduced into (2) to ensure that $0 \le p \le 1$. For 95% confidence, rounding $z_\alpha = 1.96$ to 2, ignoring the boundary conditions and recognising that $\hat{p}_U = \hat{p} + L$ and $\hat{p}_L = \hat{p} - L$, it is easy to see that one of the standard approaches for determining the confidence interval in OM texts is based on the Wald interval,

$$L = 2\sqrt{\hat{p}(1-\hat{p})/n} \Rightarrow n = 4\hat{p}(1-\hat{p})/L^2. \tag{3}$$

If we consider the same procedure without rounding z then the other popular OM textbook sample size requirement formulae results from the Wald interval as follows,

$$L = z_\alpha\sqrt{\hat{p}(1-\hat{p})/n} \Rightarrow n = z_\alpha^2\, \hat{p}(1-\hat{p})/L^2. \tag{4}$$

It is often stated that the Wald interval is a conservative estimate but it performs badly when n is small or when p is near to zero or one, Blyth and Still (1983). A simple rule of thumb frequently relied upon (see for example Johnson, Kemp and Kotz (2005) and Table 1) is that the Wald interval should only be used when $np > 5$ and $n(1-p) > 5$. However, analysis suggests even this guidance is questionable. Brown, Cai and DasGupta (2001) also point out that the Wald interval can perform badly for all p and all n.

4.1. Coverage probability for the Wald interval

The confidence level actually achieved by a certain interval is called the *coverage probability*. As the true coverage probability is determined by the discrete binomial distribution, the coverage probability can never exactly equal the desired confidence level for all values of p. However, if an interval performs properly, then the coverage probability is always greater than the confidence level desired. If this is the case then we say the interval is *exact*. In order to obtain the coverage probability, Λ the following formula can be used,

$$\Lambda = 1 - \sum_{x=0}^{n} \left(\text{If}\left[\hat{p}_L < p, 0, \binom{n}{x} p^x (1-p)^{n-x} \right] + \text{If}\left[\hat{p}_U > p, 0, \binom{n}{x} p^x (1-p)^{n-x} \right] \right), \tag{5}$$

where $\text{If}[c,t,f]$ is the conditional statement, "if c holds then t, otherwise f". This expression can be found by deduction (see infra, the Clopper-Pearson interval) and verified via a Monte Carlo simulation. An alternative to this equation using an indicator function can be found in Pires and Amado (2008). Making (5) specific for the case for $\{n = 50,\ \alpha = 0.05\}$ when the Wald interval (2) is used to generate the upper and lower limits of the confidence interval produces Figure 1.

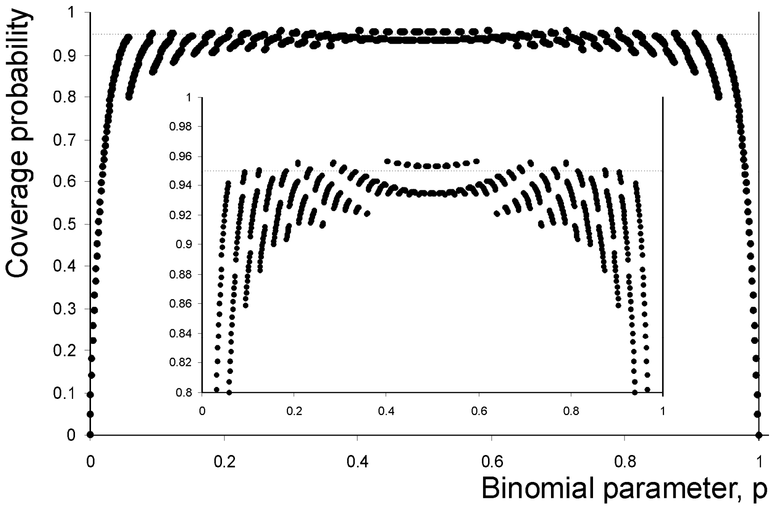

Figure 1. Coverage probability of the Wald interval, $(n = 50,\ \alpha = 0.05)$

In Figure 1 there are 1001 points (at 0, 0.001, 0.002, … 0.999, 1) of the unknown, but constant value of the binomial parameter, p. 907 of these 1001 tests fail to meet the 95% confidence limit (at the extremities, (7) is used in these results). Note that these results are based on using z_α = 1.96. If z_α = 2 is used, then the chance of meeting the desired confidence level will be slightly higher, with 799 of the 1001 tests failing to meet the desired 95% confidence interval (note z_α = 2 implies 95.47% confidence). If we follow the advice of excluding $n\hat{p} > 5$ and $n(1-\hat{p}) > 5$ then 707 of the 1001 tests fail with z = 1.96 (if z_α = 2 is used, this reduces to 601 failures). For 90% desired confidence, there are 801 failures in the 1001 tests. For 99% confidence there are 999 failures in the 1001 tests.

4.2. Length of the Wald interval

It is also interesting to investigate the length of the confidence interval. Figure 2 illustrates the upper and lower bound of the 95% Wald interval. It also highlights the length of the interval given by

$$\Lambda_l = \hat{p}_U - \hat{p}_L \,. \tag{6}$$

Note that for the Wald interval when $L < \hat{p} < (1-L)$ then $\Lambda_l = 2L$. However, when $\hat{p} > (1-L)$ or $\hat{p} < L$, then $\Lambda_l < 2L$. This is a result of the boundary conditions in (2). It can be seen in Figure 2 that the interval narrows as more samples are taken. Here we have illustrated the case of n = 50, 100 and 250. Furthermore, the confidence interval is largest at $\hat{p} = 0.5$ and has zero length at $\hat{p} = 0$ and $\hat{p} = 1$. This is incorrect as it is known

$$\hat{p}_L = \begin{cases} 0 \text{ if } x = 0 \\ (\alpha/2)^{1/n} \text{ if } x = n \end{cases} \quad \text{and} \quad \hat{p}_U = \begin{cases} 1-(\alpha/2)^{1/n} \text{ if } x = 0 \\ 1 \text{ if } x = n \end{cases} \tag{7}$$

should be used at the extremities, Pires and Amano (2008). These are true the Clopper-Pearson limits.

Figure 2 also contains a contour plot of the number of samples required to ensure 95% confidence (according to (4)) as a function of the underlining binomial probability p, and the interval half-width, L. It can be seen that the Wald interval (incorrectly) advises, for a given interval half width, that the maximum number of samples required occurs when p = 0.5. The Wald interval also assumes (again incorrectly) that all the contours originate from L = 0, p = {0,1}.

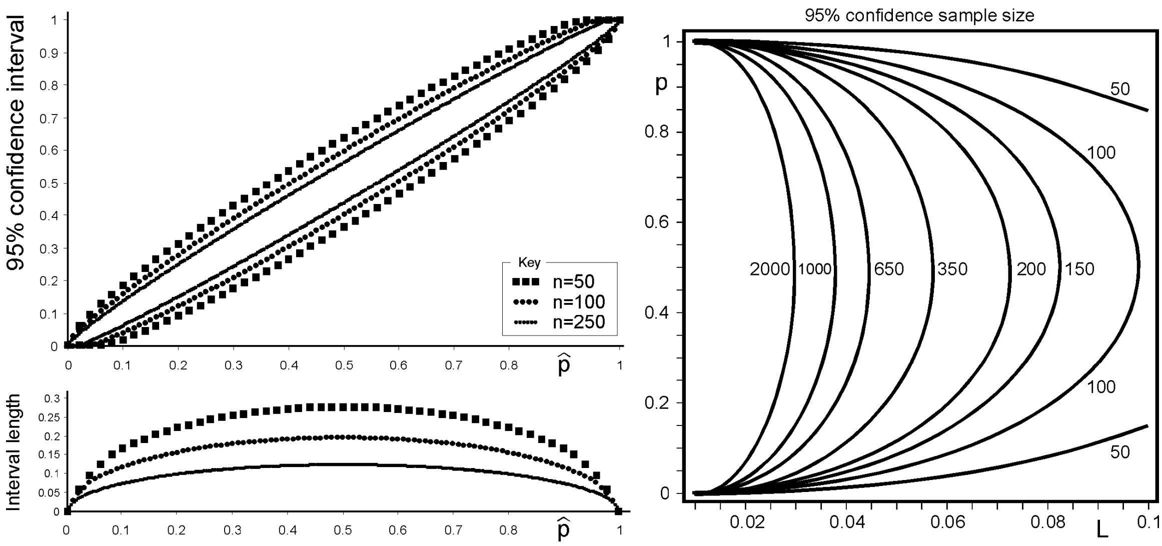

Figure 2. Wald interval, its length and sample size requirement for 95% confidence

5. Evaluating the performance of a confidence interval for activity sampling

The review of the Wald interval has allowed us to introduce the terminology and the issues involved in binomial proportion confidence intervals. There are many such intervals in the literature (see Pires and Amado, 2008). In order to select a confidence interval for professional OM activity sampling we need some criteria to judge the field.

First, and most importantly, the interval should actually achieve the desired confidence interval. That is, the coverage probability should be greater than the desired confidence level. Second, the length of the interval should be as small as possible, which probably means that excessive confidence should be avoided. Third, the confidence interval should work for different confidence levels. Forth, it should be easy to present and understand in a classroom setting.

We should also not need to make any *a priori* assumptions of the binomial probability *p*. Whilst there are good performing conservative intervals (see Sterne (1954) for example) that use *a priori* information, they are not suitable for activity sampling as the required information is rarely available in a usable form. The procedure to determine the confidence interval is also rather complicated as the solution has no explicit form. Similar arguments were made by Clopper and Pearson (1934), although Buck and Tanchoco (1974) and Buck, Askin and Tanchoco (1983) have developed an activity sampling procedure that does use *a priori* information.

There are also procedures available to modify confidence interval guidance. For example Wang (2007) and Wang (2009) identifies the minimum coverage probability for a given sample and size and interval specification. He also provides a mechanism to determine the average coverage probability. Having determined the exact coverage probability (or the exact average coverage probability) one can then adjust (in an iterative approach) the safety factor to achieve target confidence levels, see Agresti and Caffo (2000). We have not pursued this approach either as this is a rather complex task. Rather we prefer to have a one-step calculation to the confidence interval calculation.

6. Alternative confidence intervals

This section reviews four other binomial confidence intervals. Three of these intervals are approximate, one is exact. These four intervals where selected from Pires and Amado (2008) as being described to possess (or nearly possess) the characteristics outlined in Section 5.

6.1. Agresti and Coull's 'Adjusted Wald' interval

The first alternative confidence interval we will consider is based on a modification to the Wald interval that was introduced by Agresti and Coull (1998), sometimes called the *Adjusted Wald* interval. The expression for the boundaries of the interval with arbitrary confidence levels is given by,

$$\hat{p}_U = \min\left[\zeta + z_\alpha \sqrt{\frac{\zeta(1-\zeta)}{(n+z_\alpha^2)}}, 1\right] \text{ and } \hat{p}_L = \max\left[\zeta - z_\alpha \sqrt{\frac{\zeta(1-\zeta)}{(n+z_\alpha^2)}}, 0\right], \tag{8}$$

where $\zeta = (x + z_\alpha^2/2)(n+z_\alpha^2)^{-1}$. Using (8) in (5) allows us to determine its coverage probability, see Figure 3.

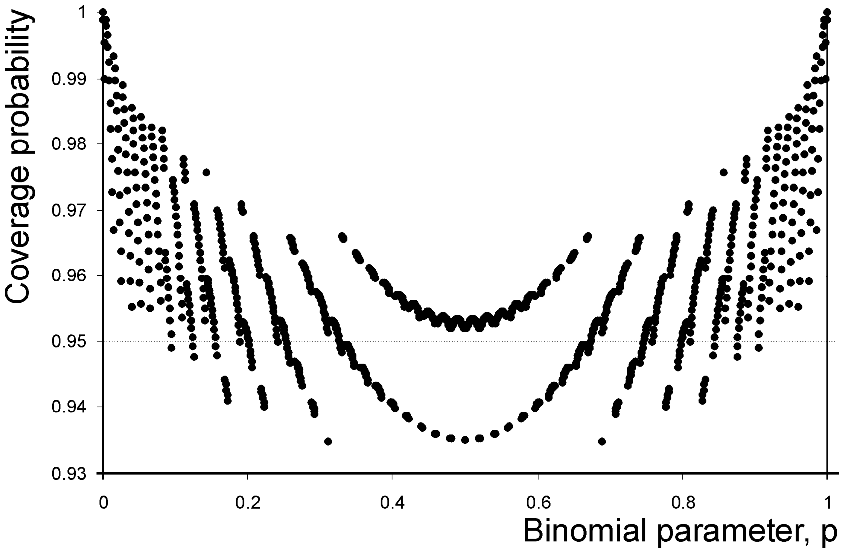

Figure 3. Coverage probability of the Agresti and Coull interval, (n = 50, $\alpha = 0.05$)

For the record, 232 of 1000 tests fail to meet the 99% confidence target, 217 fail to meet the 95% target (above) and 305 fail the 90% target. While this is an improvement to the coverage compared to the Wald interval, 20% to 30% of time, the desired level of confidence is not actually achieved.

Figure 4 highlights the length of the Agresti and Coull interval for 95% confidence. Of note here is the fact that at the extremities of the probability, $\hat{p} = \{0,1\}$, the interval has a finite length. Also, the maximum and minimum operators in (8) are activated near the extremities. The sample size contours also exhibit more natural behaviour as they do not all originate from the same point in the (*L*, *p*) plane. Note that in the contour plot, the un-truncated behaviour (for *L*) was considered, as it was for Figure 2.

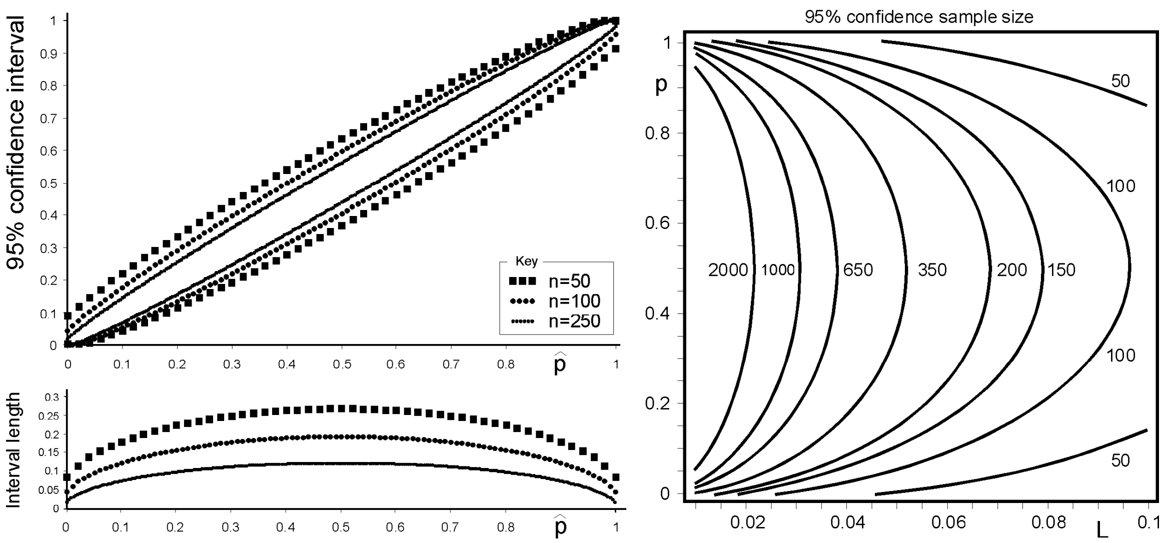

Figure 4. Agresti and Coull interval, its length and sample size requirements for 95% confidence

6.1.1. Agresti and Coull's 'Add 4' interval

If you round the normal variant for 95% confidence from $z_\alpha = 1.96$ to $z_\alpha = 2$ and add two successes and two failures to the sample population the Wald interval becomes Agresti and Coull's *Add 4* interval. Starting with

$$\hat{p}_{+4} = \frac{x+2}{n+4}, \tag{9}$$

the upper and lower intervals can be calculated with

$$\hat{p}_U = \min\left[\hat{p}_{+4} + 2\sqrt{\frac{\hat{p}_{+4}(1-\hat{p}_{+4})}{n+4}}, 1\right] \text{ and } \hat{p}_L = \max\left[\hat{p}_{+4} - 2\sqrt{\frac{\hat{p}_{+4}(1-\hat{p}_{+4})}{n+4}}, 0\right]. \tag{10}$$

Now only 76 of the 1000 tests fail to reach 95% coverage. Equation (10) is interesting as it shows that if you do not collect any samples i.e. $n = x = 0$, then you can be confident that $0 \le p \le 1$, which is at least a logical result. It is possible to manipulate the un-truncated intervals in (10) to find a concise expression for the number of observations required to ensure p is within $\hat{p} \pm L$,

$$n = 4\left(\hat{p}_{+4}(1-\hat{p}_{+4}) - L^2\right)/L^2 . \tag{11}$$

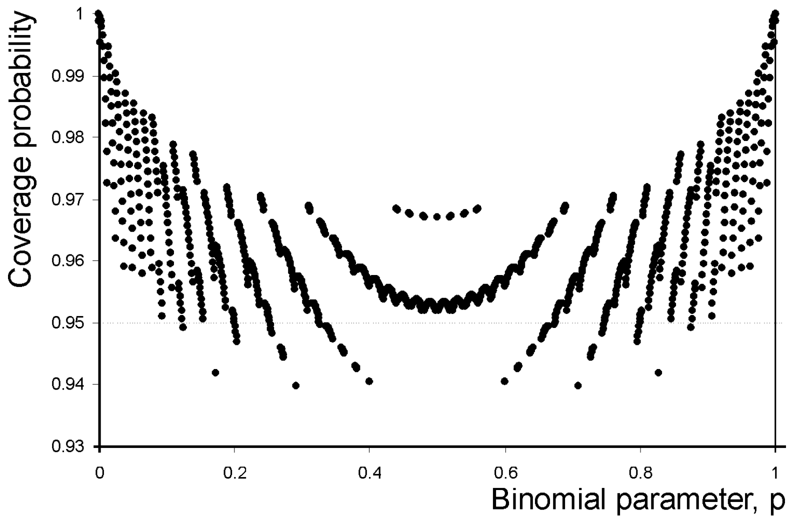

Figure 5. Coverage probability of the Add 4 interval, $(n = 50,\ \alpha = 0.05)$

Equation (12) adapts the Add 4 interval for arbitrary confidence levels. It produces 345 failures in the 1001 tests at the 90% confidence level, 211 failures at 95% and 351 failures at 99%.

$$\hat{p}_U = \hat{p}_{+4} + z_\alpha\sqrt{\hat{p}_{+4}(1-\hat{p}_{+4})(n+4)^{-1}} \text{ and } \hat{p}_L = \hat{p}_{+4} - z_\alpha\sqrt{\hat{p}_{+4}(1-\hat{p}_{+4})(n+4)^{-1}} \tag{12}$$

In summary, the Add 4 interval has a relatively simple closed form that is only slightly more complex than the Wald interval. It can be manipulated for *n*, the number of samples needed to be collected in order to reach a defined tolerance band. Although the coverage achieved is vastly better than the Wald interval, especially for 95% when $z = 2$ is used, it is still not exact. Agresti and Caffo (2000) report that the Add 4 interval is received well in a classroom setting, especially when students realise the implications of the Wald interval when $p=\{0,1\}$.

6.2. Wilson Score interval

The Wilson Score interval is often quoted to be the statistician’s preferred choice for an approximation to the exact interval. The continuity corrected version of the Wilson score interval is recommended by Pires and Amado (2008) and is given by,

$$\hat{p}_U=\begin{cases}1 \text{ if } x=n \text{ else}\\ \dfrac{2x+z_\alpha^2+1+z_\alpha\sqrt{z_\alpha^2+2-\frac{1}{n}+4x\left(1-\hat{p}-\frac{1}{n}\right)}}{2\left(n+z_\alpha^2\right)}\end{cases} \tag{13}$$

and

$$\hat{p}_L=\begin{cases}0 \text{ if } x=0 \text{ else}\\ \dfrac{2x+z_\alpha^2-1-z_\alpha\sqrt{z_\alpha^2-2-\frac{1}{n}+4x\left(1-\hat{p}+\frac{1}{n}\right)}}{2\left(n+z_\alpha^2\right)}\end{cases}. \tag{14}$$

As we can see in Figure 6, the coverage probability for the 90% and 95% confidence levels is acheived. However, for the 99% confidence level the coverage probability is not met in 36 instances of the 1001 tests.

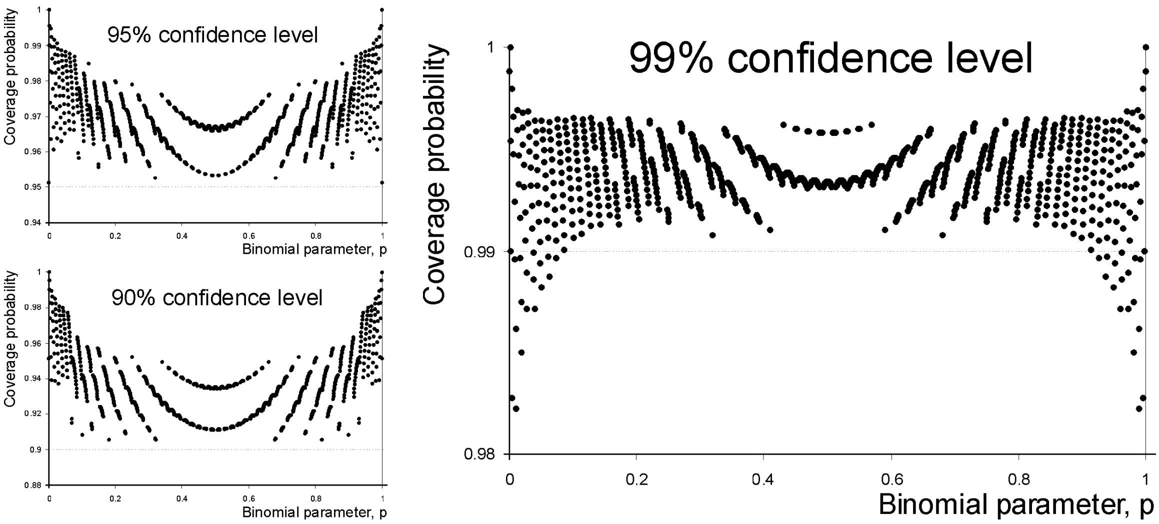

Figure 6. Coverage probability for the Wilson “Score” interval (n=50)

Figure 6 highlights the length of the Wilson Score interval for different sample sizes. It also shows that it deals with the extremities appropriately. It is possible to subtract (13) from (14), set it equal to twice the interval half length (2*L*) and solve for *n*. It results in a rather unwieldy solution to a forth order equation. However, it is easily plotted, see Figure 7. By closely comparing the sample size requirements the Wilson interval with the Agresti and Coull interval sample size requirements, we can see that the Wilson Score interval always requires more samples to be taken for a given probability and a given interval half width. This is perhaps the reason why the Wilson Score interval has such a high coverage probability.

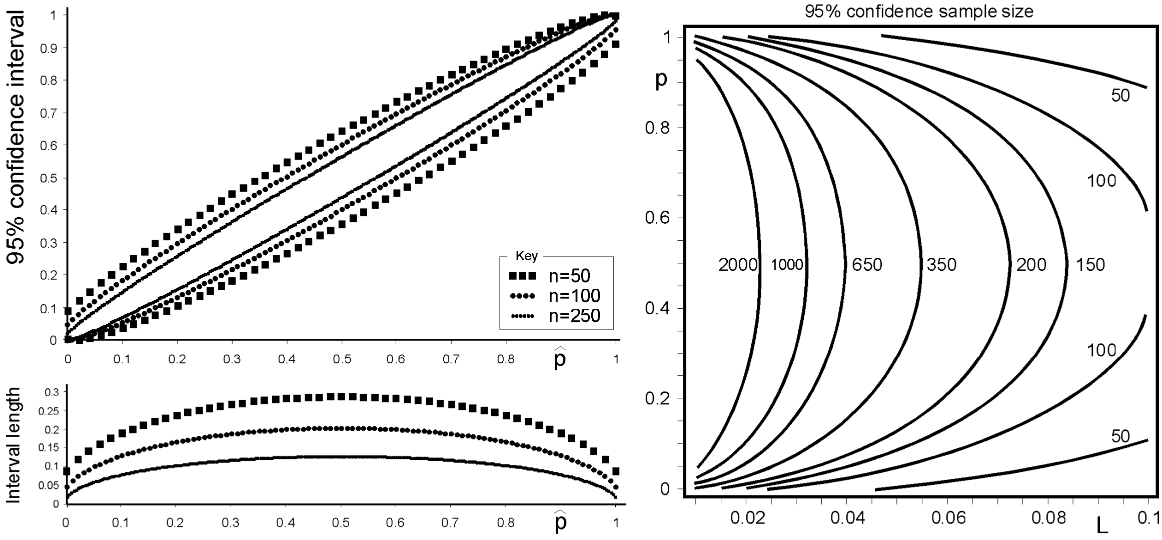

Figure 7. The Wilson Score interval, its length and sample size requirements

6.3. The Arc Sin interval

The final approximate binomial confidence interval considered is the continuity corrected Arc Sin interval discussed in Pires and Amado (2008). This is an interval based on the approximate normal distribution interval after a variance stabilizing transformation. The Arc Sin interval is given by

$$\hat{p}_U = \begin{cases} 1 \text{ if } x = n \text{ else} \\ \sin^2\left(\arcsin\sqrt{\dfrac{\frac{7}{8}+x}{\frac{3}{4}+n}} + \dfrac{z_\alpha}{2\sqrt{n+\frac{1}{2}}}\right) \end{cases} \tag{15}$$

and

$$\hat{p}_L = \begin{cases} 0 \text{ if } x = 0 \text{ else} \\ \sin^2\left(\arcsin\sqrt{\dfrac{x-\frac{1}{8}}{\frac{3}{4}+n}} - \dfrac{z_\alpha}{2\sqrt{n+\frac{1}{2}}}\right). \end{cases} \tag{16}$$

The coverage probability of the Arc Sin interval when $n = 50$ is highlighted in Figure 8.

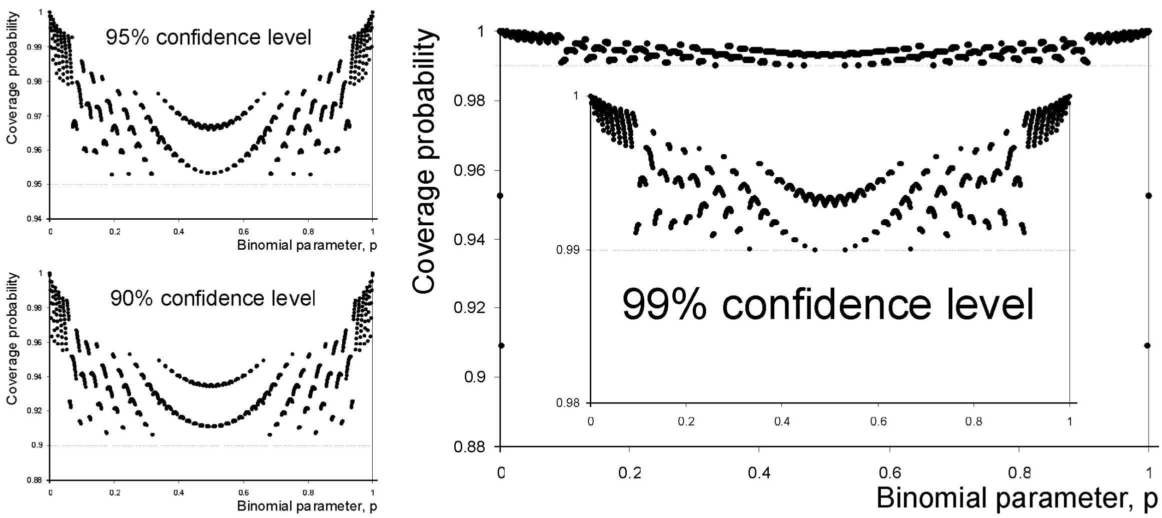

Figure 8. Coverage probability for the Arc Sin interval (n = 50).

The Arc Sin interval actually achieves 90% and 95% coverage, but fails to meet 6 of the 1001 tests at 99% coverage probability. This is an improvement upon the Wilson Score interval, especially as 4 of the 6 failures at 99% coverage occur very close to the extreme values of the binomial probability. However, the interval is very large, see Figure 9.

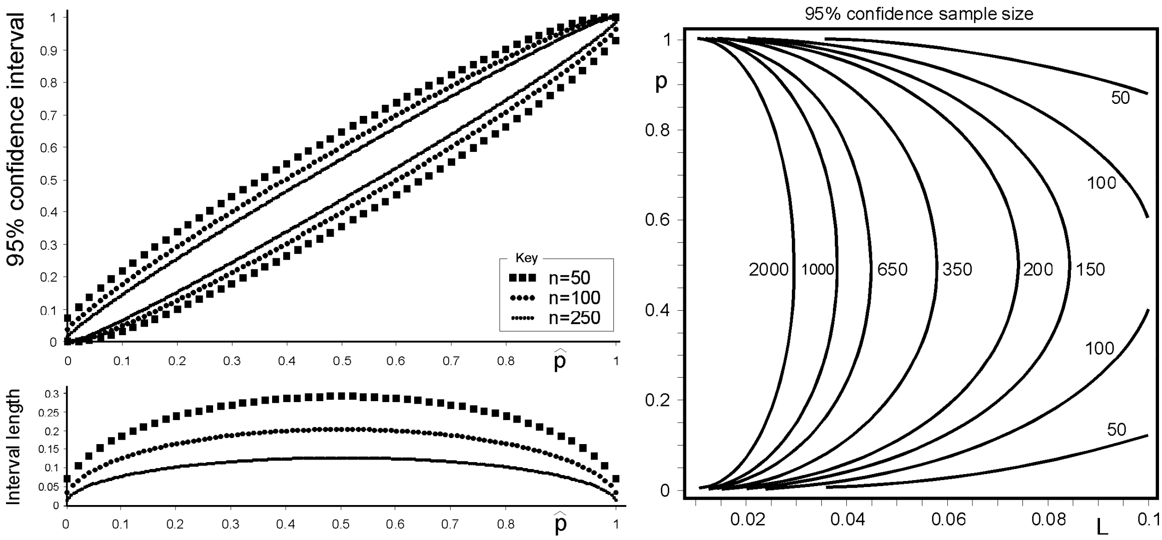

Figure 9. The 95% Arc Sin confidence interval, its length and sample size requirements

6.4. The Clopper-Pearson interval

An exact solution of the confidence interval is one that always reaches the desired coverage probability. The original approach to this solution is based on finding the real root within the valid probability range of a polynomial of an order equal to $n+1$, Clopper and Pearson (1934). Specifically, the upper boundary of the Clopper-Pearson confidence interval is given by the real solution in the range [0..1] to

$$\sum_{k=0}^{x}\binom{n}{k}\hat{p}_U^k\left(1-\hat{p}_U\right)^{n-k}=\frac{\alpha}{2}. \tag{17}$$

Similarly the lower boundary is given by the real solution, in the range [0..1], to

$$\sum_{k=x}^{n}\binom{n}{k}\hat{p}_L^k\left(1-\hat{p}_L\right)^{n-k}=\frac{\alpha}{2}. \tag{18}$$

This form of the Clopper-Pearson interval is rather hard to deal with when n becomes large. However, the Clopper-Pearson interval can also be expressed in a more manageable form that uses the Beta distribution (see Newcombe (1998) and Pires and Amado (2008)) as follows,

$$\hat{p}_U=\begin{cases}1-\left(\frac{\alpha}{2}\right)^{1/n} \text{ if } x=0\\ 1 \text{ if } x=n \text{ else}\\ B^{-1}\left[1-\frac{\alpha}{2},x+1,n-x\right]\end{cases} \quad\text{and}\quad \hat{p}_L=\begin{cases}0 \text{ if } x=0\\ \left(\frac{\alpha}{2}\right)^{1/n} \text{ if } x=n \text{ else}\\ B^{-1}\left[\frac{\alpha}{2},x,n-x+1\right]\end{cases} \tag{19}$$

where $B^{-1}\left[\gamma,\theta_1,\theta_2\right]$ is the γ percentile of the Beta$\left[\theta_1,\theta_2\right]$distribution. This form of the Clopper-Pearson interval is easy to handle with modern statistical and mathematical software. They are also computable in Microsoft Excel with the following expressions;

$$\hat{p}_U = \text{BETAINV}\left(1-\alpha/2, x+1, n-x\right) \tag{20}$$

and

$$\hat{p}_L = \text{BETAINV}\left(\alpha/2, x, n-x+1\right). \tag{21}$$

Johnston, Kemp and Kotz (2005) provide a comprehensive list of references to tables of confidence intervals. They also note the link between the Clopper-Pearson interval and the F distribution and provide the following guidance for the upper limit of the confidence interval,

$$\hat{p}_U = \frac{v_1 F_{v_1,v_2,1-\alpha/2}}{v_2 + v_1 F_{v_1,v_2,1-\alpha/2}}, \tag{22}$$

where $v_1 = 2\left(x+1\right)$ and $v_2 = 2\left(n-x\right)$. The lower limit of the interval is

$$\hat{p}_L = \frac{v_1 F_{v_1,v_2,\alpha/2}}{v_2 + v_1 F_{v_1,v_2,\alpha/2}}, \tag{23}$$

with $v_1 = 2x$ and $v_2 = 2\left(n-x+1\right)$. Using (19) in (5) allows us to inspect the coverage probability, see Figure 10.

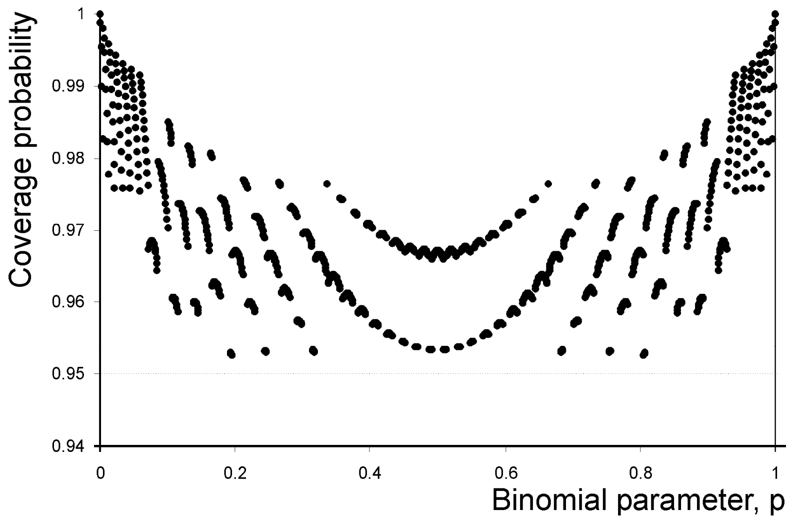

Figure 10. Coverage probability of the Clopper-Pearson interval, $\left(n=50,\ \alpha=0.05\right)$

Figure 10 confirms that for the Clopper-Pearson interval when $n=50$ and $\alpha=0.05$ the coverage probability is always greater than the desired confidence level. However, because of the discrete nature of the binomial distribution it can often be rather conservative, especially in the extremities of the binomial probability. The confidence interval and its length is portrayed in Figure 11. Here we can see that the interval is largest near $\hat{p}=0.5$, is symmetrical about $\hat{p}=0.5$, and has a finite length at $\hat{p}=\{0,1\}$.

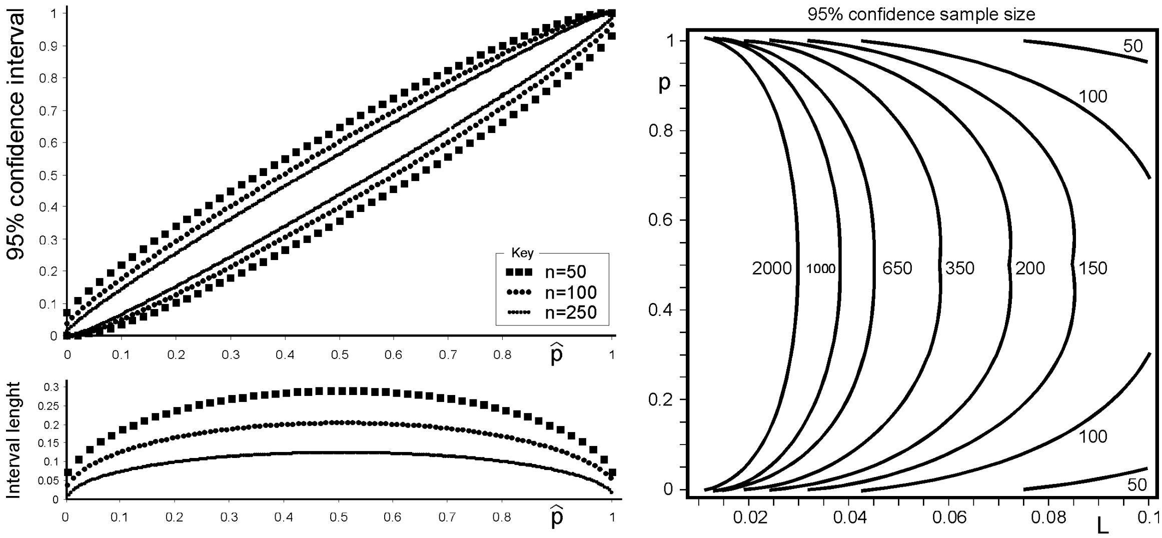

Figure 11. The 95% Clopper-Pearson confidence interval, its length and sample size requirements

The inverse problem where we solve the Clopper-Pearson equations for n, was studied by Johnston, Kemp and Kotz (2005). They highlight that (24) and (25) can be solved to yield an upper and lower limit on n, n_U and n_L, such that p is within $\hat{p} \pm L$ with $(1-\alpha)\times 100\%$ confidence;

$$\sum_{k=0}^{n_U \hat{p}} \binom{n_U}{k} (\hat{p}+L)^k (1-\hat{p}-L)^{n_U-k} = \frac{\alpha}{2}, \tag{24}$$

$$\sum_{k=n_L \hat{p}}^{n_L} \binom{n_L}{k} (\hat{p}-L)^k (1-\hat{p}+L)^{n_L-k} = \frac{\alpha}{2}. \tag{25}$$

Solving (24) and (25) plotting n_U and n_L for different values of p when the interval half length, $L = 0.1$ and $\alpha = 0.05$ yields Figure 12. Here we can see the two curves for n_U and n_L. Obviously we will need to pick the largest n for a particular observed probability, $\hat{p}$. Hence, the parts of the curves that are plotted in grey become redundant. We can also see that solutions $n_L < L$ and $n_U > 1-L$ do not exist as $\hat{p}$ cannot be less than zero or greater than unity. Interestingly, the maximum number of observations required does not occur when $\hat{p}$ = 0.5 (which was advocated by all of the previously considered intervals - see the solid line in Figure 12 for the Wald interval). Rather the maximum observations occur near $\hat{p} \approx \frac{1}{2} \pm L$ (but not precisely at $\hat{p} = \frac{1}{2} \pm L$, due to the discrete nature of the binomial distribution). The nature of the two solutions to (24) explains why Figure 11 does not have contours that are maximal in L at $p = 0.5$. Figure 12 also highlights that the Wald interval never takes enough observations to ensure that both sides of the interval have less than $\alpha/2$ error probability (error probability = $1-\Lambda$).

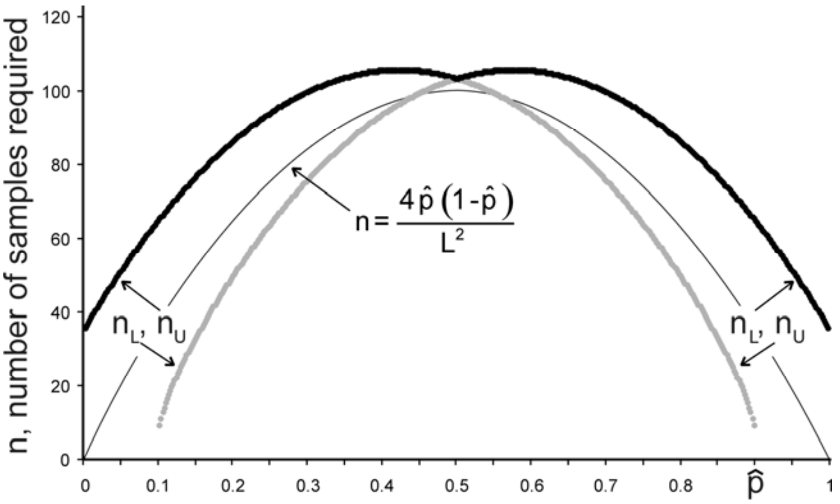

Figure 12. Sample size required for L = 0.1, 95% confidence

Defining L^+ as the maximum L in the level sets of the contour plot in Figure 11 allows us to obtain an upper bound of the number of samples required to achieve a desired interval half-width. A visualisation of the relationship between L^+ and n is given in Figure 13. Here we can see that the maximum interval half-width reduces as more samples are taken, but there is a *law of diminishing returns* with large samples sizes.

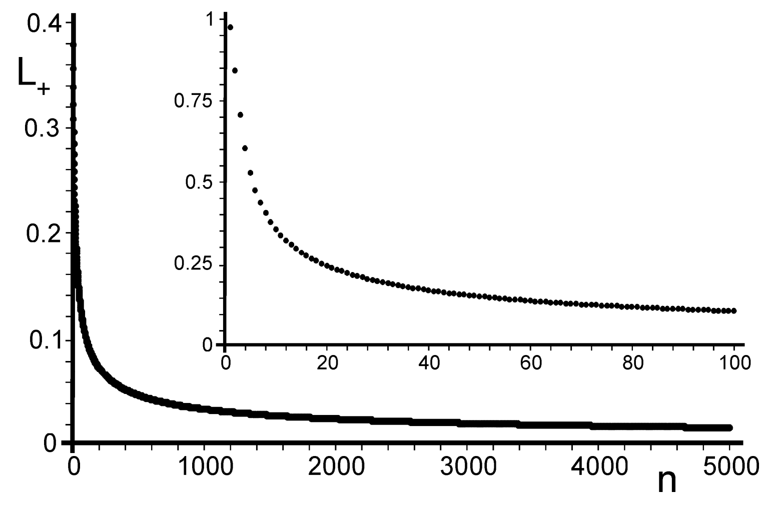

Figure 13. Minimum sample size requirements to ensure $L<L_+$, 95% confidence

7. Recommendation of a confidence interval for activity sampling

Table 2 summaries the test results from Sections 4 and 6. If you require the most simple interval possible for 95% confidence, then you could (continue to) use the simplified Wald interval. This does not guarantee the coverage required (about 80% of the time) and assumes the user lacks the capability to exploit a more sophisticated approach. A literature review demonstrated that this approach is almost exclusively advocated by the OM field.

A more meaningful interval for 95% confidence is the Add 4 interval. This greatly improves the coverage probability of the Wald interval and also has a simple closed form solution for the number of samples required to be within a desired tolerance.

To be sure that the coverage probability was achieved (at least for 90% and 95%, and almost sure for 99% confidence) in situations where a closed form expression is required, the continuity corrected Wilson Score interval is recommended. Whilst it is possible to manipulate the interval for n, given a target interval length, it is rather complex for classroom use.

If one is willing to drop the requirement for a closed form, the Arc Sin interval ensures the coverage probability is assured for both 90% and 95% confidence. However, for 99% confidence the interval does not achieve the required coverage at the extremities (and for a small number of points in the middle). It is possible to find the sin and arcsin functions on most good handheld calculators, hence it may be relevant for practising industrial engineers.

Finally, if you must ensure the coverage probability is met and have access to the Clopper-Pearson tables given in Appendix B (or access to the inverse beta distribution in Microsoft Excel) then the Clopper-Pearson interval is recommended. This is the only interval that guarantees the desired coverage probability for all values of α and p.

Interval	**Confidence level desired**			**Notes**
	90%	**95%**	**99%**	
Wald	801 fails	907 fails (799 fails if z=2)	999 fails	Most simple closed form
Agresti & Coull’s ‘Adjusted Wald’	305 fails	217 fails	232 fails	Improved performance over Wald
Agresti & Coull’s ‘Add 4’	345 fails	211 (76 fails if z=2)	351 fails	Nice simple closed form for interval and for sample size requirements
Wilson Score	0 fails	0 fails	36 fails	Complex closed form
Arc Sin	0 fails	0 fails	6 fails	No closed form, but easily done on a scientific calculator.
Clopper-Pearson	0 fails	0 fails	0 fails	No closed form, but amenable in Excel. Easily presented in tabular form. Sample size requirements possible

Table 2. Summary results from 1001 confidence interval trials for activity sampling

Notwithstanding the arguments above, let’s now investigate the contour plots in Figures 2, 4, 7, 9 and 11 again. Overlaying them all into one single plot as in Figure 14, it is possible to see when one interval dominates the other in terms of reducing the interval to a specific half-width with a particular sample size. Note we are ignoring whether the coverage probability actually meets the desired confidence level here. It can be seen that the Agresti and Coull interval generally dominates all of the other intervals. That is, it requires fewer samples to meet its specific half-width. The Wilson interval generally dominates all other intervals, except the Agresti and Coull interval. This can be regarded as good performance as the Wilson Score interval is almost exact. The Arc Sin interval generally dominates the Clopper-Pearson interval. Despite this, it is recommended that the standard operations management procedure is updated to include the Clopper-Pearson interval.

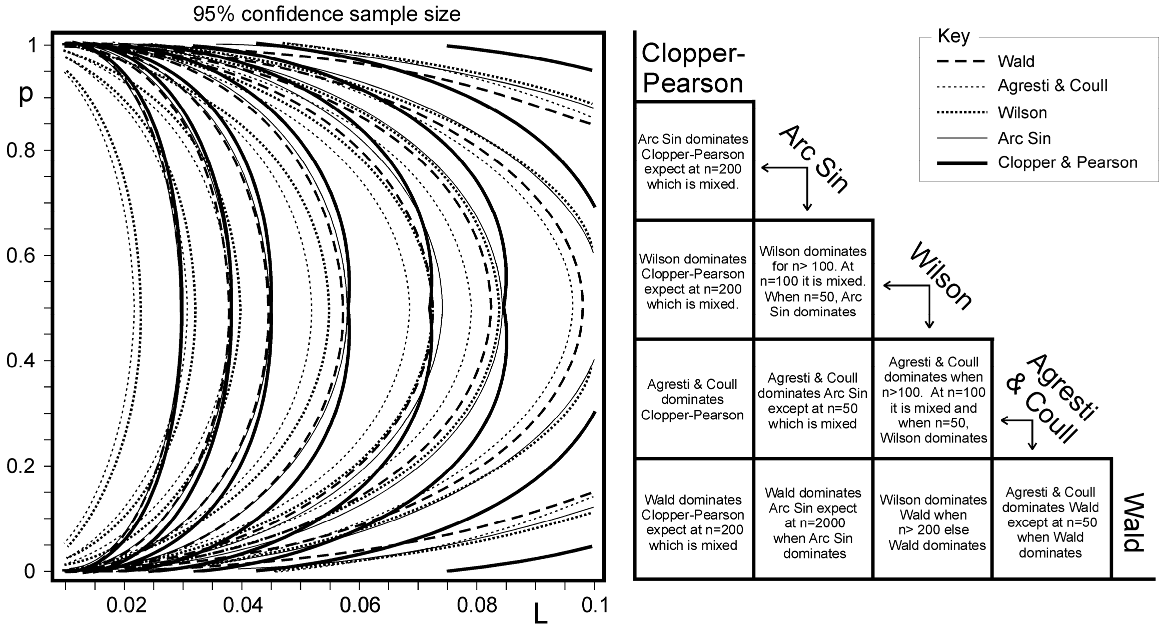

Figure 14. Comparison of the five confidence intervals by sample size requirements

8. Activity sampling procedure using the Clopper-Pearson interval

A procedure to exploit the Clopper-Pearson interval for activity sampling will now be described. Appendix A provides a blank data collection form. The Clopper-Pearson confidence interval requires either; the solution of pair of high order equations (equations (17) and (18)); access to Microsoft Excel to enumerate (20) or specialist statistical software to realise (22) and (23)); or access to Clopper-Pearson confidence interval tables. Appendix B provides Clopper-Pearson confidence intervals that fit the data collection forms for the case of n = 50, 100 and 150. Appendix C gives the maximum of the solution to (24) and (25), rounded up to the nearest integer, for the required number of samples n to be within a desired interval half-width for 90%, 95% and 99% confidence for different values of the estimated binomial parameter, $\hat{p}$.

The following step-by-step guide highlights how to use these tools.

Step 1. Determine how much confidence you need to assign to your results. From this, determine the value of α .

Step 2. Determine an acceptable confidence interval half length (L) for the purposes of your study. Note. Steps 1 and 2 should be done in consultation with the "customer" of the activity sampling exercise.

Step 3. Explain the purpose of the activity sampling to the subjects of the exercise, (the workers, customers, operators etc). Obtain an initial impression of range of activities being undertaken by the workers, their duration and their frequency. Make a note of the working time, breaks and any data collection issues present.

Step 4. Modify the activity sampling data collection form in Appendix A for your situation. Name each of the activities the subject (worker, customer, machine etc) undertakes in the columns A to J. If there are not enough columns then you need to develop your own forms. Specify when the observations are to be taken (ensuring random intervals of time between observations) using the uniformly distributed random numbers appropriately scaled to fit your scenario. Make sure the study is long enough to capture the range of activities that are undertaken.

Step 5. Collect the activity sample. First collect 50 samples (a page worth). Total up the observations for each activity and calculate the upper and lower limits of confidence interval, using the table of Clopper-Pearson confidence intervals in Appendix B. Decide if the confidence interval meets the needs of the study. If it is not then use Appendix C to determine how many samples should be taken in order to reduce the interval half-width (L) to an acceptable level.

Step 6. If the confidence interval half length (L) is too large for your purpose, collect another 50 samples using the data collection form and re-calculate the length of the interval.

Step 7. Repeat steps 5 and 6 until you are satisfied with the accuracy of the confidence interval for each of the activities.

9. Teaching activity sampling with the new Clopper-Pearson interval

This new activity sampling procedure has been taught to over 2000 students over the last 5 years in Undergraduate, MSc, MBA and Executive courses in the U. S. and the U. K. Only the Clopper-Pearson interval is discussed. No detail on other alternative intervals needs to be given. The data collection forms and tables of intervals and sample sizes are no more difficult to use than a standard normal table. Students often struggle with the difference between and selection of appropriate accuracy (L) and confidence (α), so it is worthwhile spending some time on this matter. However, on balance, the new Clopper-Pearson approach is actually easier to teach as students often find the circular reference to p in the Wald interval formula difficult. It is also easy to break-out of a presentation to illustrate the Inverse Beta function in Microsoft Excel.

A worked example (activity sampling my 2 year old daughter), a class exercise of activity sampling a secretary's duties and case study at Sam's Tailor in Hong Kong is used. These teaching materials are available upon request from the author. Sohal and Oakland (1990) also offer some advice on an engaging method to teach activity sampling that could also be adopted for use within this Clopper-Pearson activity sampling procedure.

10. Concluding remarks

The standard operations management textbook treatment of activity sampling has been evaluated. It was found that the Wald interval is almost exclusively used to assign confidence to the results. A review of modern statistical knowledge on binomial proportion confidence intervals revealed that the statisticians have serious concerns with the adequacy of this interval and have developed a range of more sophisticated approximations. There is even an exact solution to the problem.

These modern confidence intervals have been reviewed and analysed for the purpose of activity sampling. The Clopper-Pearson interval was found to be the only interval that actually achieves the desired confidence interval for any desired coverage probability that does not use *a-priori* information. The Clopper-Pearson interval has an explicit solution, but sadly has no closed form. However, given that Microsoft Excel has an Inverse Beta Distribution function built-in, the Clopper-Pearson interval can be easily determined.

Two look-up tables for the interval and the sample size requirements have been developed for classroom use.

A new activity sampling procedure was developed that can be used without advanced statistical knowledge to gather information about many operations management scenarios. The interval obtained now economically achieves the desired coverage. The existing activity sampling procedure in many OM textbooks cannot guarantee this.

References

Adam, E.E. and Ebert, R.J., (1992), *Production and Operations Management: Concepts, Models and Behaviour*. Prentice-Hall Inc, New Jersey, USA, 314-315.

Agresti, A., and Coull, B., (1998), "Approximate is better than 'exact' for interval estimation of binomial proportions", *The American Statistician*, **52** (2), 119-126.

Agresti, A. and Caffo, B., (2000), "Simple and effective confidence intervals for proportion and differences of proportions result from adding two successes and two failures", *The American Statistician*, **54** (4), 280-288.

Anderson, D.R., Sweeney, D.J., Williams, T.A., Freeman, J. and Shoesmith, E., (2007), *Statistics for Business and Economics*. Thomson, London, UK, 273-276.

Barnes, D., (2008), *Operations Management: An International Perspective*. Thomson Learning, London, UK, 345.

Bicheno, J. and Holweg, M., (2009), *The Lean Toolbox: The Essential Guide to Lean Transformation*. Picsie Books, Buckingham, UK, 130-131.

Bicheno, J., (2008), *The Lean Toolbox for Service Systems*. Picsie Books, Buckingham, UK, 271-272.

Blyth, C.R. and Still, H.A., (1983), "Binomial confidence intervals", *Journal of the American Statistical Association*, **78** (381), 108-116.

Brisley, C.L., (2001), "Work sampling and group timing technique", Chapter 17.3 in Zandin, K.B., (2001), *Maynard's Industrial Engineering Handbook*. Fifth Edition, McGraw-Hill, New York, USA, 17.47-17.64.

British Standard 3138, (1992), "Glossary of terms used in management services", p12.

Brown, L.D., Cai, T.T., and DasGupta, A., (2001), "Interval Estimation for a Binomial Proportion", *Statistical Science*, **16** (2), 101-117.

Buchholz, B., Paquet, V., Punnett, L., Lee, D. and Moir, S., (1996), "PATH: A work sampling-based approach to ergonomic job analysis for construction and other non-repetitive work", *Applied Ergonomics*, **27** (3), 177-187.

Buck, J.R. and Tanchoco, J.M.A., (1974), "Sequential Bayesian work sampling", *AIIE Transactions*, **6** (4), 318-326.

Buck, J.R., Askin, R.G. and Tanchoco, J.M.A., (1983), "Advances in sequential Bayesian work sampling", *IIE Transactions*, **15** (1), 19-30.

Chase, R.B. and Aquilano N.J., (1992), *Production and Operations Management*. Irwin In., Boston, USA, 518-524.

Chen, J.-G., Peacock, J.B. and Schlegel, R.E., (1989), "An observational technique for physical work stress analysis", *International Journal of Industrial Ergonomics*, **3** (3), 167-176.

Clopper, C. and Pearson, S., (1934), "The use of confidence or fiducial limits illustrated in the case of the binomial". *Biometrika*, **26** (4), 404-413.

Curwin, J. and Slater, R., (1990), *Quantitative Methods for Business Decisions*. Chapman and Hall, London, UK, 198-201.

Davis, M.M. and Heineke, J., (2005), *Operations Management: Integrating Manufacturing and Services*. McGraw- Hill Irwin, New York, 257-261.

Evans, J.R., Anderson, D.R., Sweeney, D.J. and Williams, T.A., (1984), *Applied Production and Operations Management*. West Publishing, St. Paul, Minnesota, USA, 427-429.

Farrell, P.J., Kusy, M., Tomberlin, T.J. and Thomas, R., (2009), “A hybrid methodology for measuring unit costs in multibranch institutions”, *Canadian Journal of Administrative Sciences*, **14** (2), 188 – 194.

Finch, B.J., (2008), *Operations Now: Supply Chain Profitability and Performance*. McGraw-Hill Irwin, Boston, USA, 757-758.

Finkler, S.A., Knickerman, J.R., Hendrickson, G., Lipkin, M. and Thompson, W.G., (1993), “A comparison of work sampling and time-and-motion techniques for studies in health services research”, *Health Services Research*, **28** (5), 577-597.

Foley, W.J., (1999), “National nursing home restraint minimization project”, Internal Report, Rensselaer Polytechnic Institute, Troy, New York, USA.

Greasely, A., (2009), *Operations Management*. John Wiley, Chichester, UK, 244-246.

Gunesoglu, S. and Meric, B., (2006), “The analysis of personal and delay allowances using work sampling technique in the sewing room of a clothing manufacturer”, *International Journal of Clothing Science and Technology*, **19** (2), 145-150.

Heinze, K.G.R., (1984), “Performance measures by means of activity sampling”, *Transactions of the American Association of Cost Engineers*, **D** (4), 1-8.

Heizer, J. and Render, B., (2014), *Operations Management*. Pearson, New Jersey, USA, 435-438.

Hill, T., (1983), *Production and Operations Management*. Prentice Hall, London, 246-248,

Hill, T., (2000), *Operations Management: Strategic Context and Managerial Analysis*. Palgrave, Basingstoke, UK, 480-481.

International Labour Office, (1974), *Introduction to Work Study: Revised Edition*. ILO Geneva, Switzerland, 369-377.

Jacobs, F.B., Chase, R.B. and Aquilano, N.J., (2009), *Operations and Supply Management*. McGraw-Hill Irwin, Boston, USA, 194-200.

Johnson, N.L., Kemp, A.W., Kotz, S., (2005), *Univariate Discrete Distributions*. John Wiley and Sons, New Jersey, USA.

Kaming, P.F., Olomolaiye, P.O., Holt, G.D. and Harris, F.C., (1997), “Factors influencing craftsmen’s productivity in Indonesia”, *International Journal of Project Management*, **15** (1), 21-30.

Kelly, J., (1964), “The study of executive behaviour by activity sampling”, *Human Relations*, **17** (3), 277-287.

Khanna, R.B., (2015), *Production and Operations Management*, PHI Learning, Delhi, pp137-139.

Krajewski, L., Ritzman, L. and Malhotra, M., (2013), *Operations Management Processes and Value Streams*. Pearson Prentice Hall, New Jersey, USA, 148.

Lee, S.M. and Schniederjans, M.J., (1994), *Operations Management*. Houghton Mifflin, Boston, USA, 709-712.

Liou, F.-S. and Borcherding, J.D., (1986), “Work sampling can predict unit rate productivity”, *Journal of Construction Engineering and Management*, **112** (1), 90-103.

Lockyer, K., (1983), *Production Management*. Pitman, London, UK, 196-200.

Lockyer, K., Muhlemann, A. and Oakland, J., (1988), *Production and Operations Management*. Pitman Publishing, London, UK, 250-252.

Meredith, J.R., (1992), *The Management of Operations*. John Wiley and Sons, New York, USA, 710-711.

Naylor, J., (1996), *Operations Management*, Pitman Publishing, London, 170.

Newcombe, R.G., (1998), “Two-sided confidence intervals for the single proportion: Comparison of seven methods”, *Statistics in Medicine*, **17** (8), 857-872.

Noori, H. and Radford, R., (1995), *Production and Operation Management: Total Quality and Responsiveness*. McGraw-Hill, New York, 306-308.

Pelletier, D. and Duffield, C., (2003), “Work sampling: Valuable methodology to define nursing practice patterns”, *Nursing and Health Sciences*, **5** (1), 31-38.

Pires, A.M. and Amado, C., (2008), “Interval estimators for a binomial proportion: Comparison of twenty methods”, *Revstat – Statistical Journal*, **6** (2), 165-197.

Reid, R.D. and Sanders, N.R., (2002), “*Operations Management*”, John Wiley, New York, USA, 337-338.

Rosenkrantz, W.A., (2009), *Introduction to Probability and Statistics for Science, engineering and finance*. CRC Press, Boca Raton, USA, 307-309.

Russell, R.S. and Taylor, B.W., (2009), *Operations Management Along the Supply Chain*. John Wiley, Singapore, pp346-348.

Rutter, R., (1994), “Work sampling: As a win-win management tool”, *Industrial Engineering*, **26** Feburary, 30-31.

Schonberger, R.J. and Knod, E.M., (1988), *Operations Management: Serving the Customer*. Business Publications, Plano, USA, 666-670.

Silver, M., (1997), *Business Statistics*. McGraw-Hill, London, UK, 191-192.

Slack, N., Brandon-Jones, A. and Johnston, R., (2013), *Operations Management*. FT Prentice Hall, Harlow, UK, 268.

Sohal, A.S. and Oakland, J.S., (1990), “Teaching production and operations management through participative methods”, *Production and Inventory Management Journal*, **31** (3), 30-34.

Sterne, T.E., (1954), “Some remarks on confidence or fiducial limits”, *Biometrika*, **41** (1-2), 275-278.

Stevenson, W.J., (2012), *Operations Management: Theory and Practice*. McGraw-Hill, Irwin, 308-313.

Thomas, H.R., (1991), “Labour productivity and work sampling: The bottom line”, *Journal of Construction Engineering and Management*, **117** (3), 423-444.

Tippett, L.H.C., (1935), “A snap-reading method of making time studies of machines and operatives in factory surveys”, *Journal of the Textile Institute Transactions*, **26** (2), 51-55 & 75.

Tsai, W.-H., (1996), “A technical note on using work sampling to estimate the effort on activities under activity based costing”, *International Journal of Production Economics*, **43** (1), 11-16.

Wald, A. and Wolfowitz, J., (1939), “Confidence limits for continuous distribution functions”, *Annals of Mathematical Statistics*, **10** (2), 105-118.

Wang, H., (2007), “Exact confidence coefficients of confidence intervals for a binomial proportion”, *Statistica Sinica,* **17** (1), 361-368.

Wang, H., (2009), “Exact average coverage probabilities and confidence coefficients of confidence intervals for discrete distributions”, *Statistics and Computing*, **19** (2), 139-148.

Waters, D., (2002), *Operations Management: Producing Goods and Services*. FT Prentice Hall, Harlow, UK, 500.

Weiss, H.J. and Gershon, M.E., (1989), *Production and Operations Management*. Allyon and Bacon, Boston, USA, 419-423.

Whitmore, D.A., (1987), *Work Measurement*. Heinemann, London, UK, pp118-127.

Wild, R., (1995), *Production and Operations Management: Text and Cases*. Cassell Education, London, UK, 188-191.

Williams, H., Harris, R. and Turner-Stokes, L., (2009), “Work sampling: A quantitative analysis of nursing activity in a neuro-rehabilitation setting”, *Journal of Advanced Nursing*, **65** (10), 2097-2107.

Appendix A: Activity sampling data collection form

Title:	Confidence,____%	α, ______	L,____	Date:
	Purpose:			Completed by:
				Page ___ of ____

Observation Number	Uniformly distributed random numbers	Intervals driven by random numbers? Y / N Time of observation	Activity A	B	C	D	E	F	G	H	I	J
1	98837 67136											
2	35311 82655											
3	18581 30998											
4	81781 03347											
5	38285 66912											
6	24689 72155											
7	93492 29368											
8	10281 25221											
9	95991 99716											
10	57579 12203											
11	12468 46011											
12	72758 75741											
13	86051 48890											
14	31430 48040											
15	96616 99230											
16	92753 37184											
17	55585 79446											
18	27108 38086											
19	53103 45868											
20	47128 69377											
21	67866 53793											
22	35611 99785											
23	58725 87497											
24	67091 07571											
25	28792 56073											
26	94790 89294											
27	54599 55184											
28	44346 65363											
29	45804 30298											
30	50371 55805											
31	53271 59910											
32	25086 73884											
33	98339 14198											
34	08591 41964											
35	94737 28459											
36	42328 97773											
37	25380 31213											
38	87258 46559											
39	83390 54356											
40	94856 27832											
41	77092 40946											
42	76977 34774											
43	15537 11515											
44	15033 90458											
45	63078 91904											
46	13037 90690											
47	62328 62666											
48	51910 39671											
49	37489 05840											
50	01641 53158											
Running count from any previous samples (n =)												
Total sum of observations, x												
Observed probability, $\hat{p} = \frac{x}{n}$												
Lower confidence interval, $\hat{p}_L$ (App. B)												
Upper confidence interval, $\hat{p}_U$ (App. B)												
Sample size required based on L (App. C)												

Disney, S.M., (2016), "Revisiting activity sampling: a fresh look at binomial proportion confidence intervals",
European Journal of Industrial Engineering, Vol. 10, No. 6, pp724-759. ISSN 1751-5254. DOI: 10.1504/EJIE.2016.081021.

Appendix B: Table of Clopper-Pearson confidence intervals

n=50	$\hat{p}_L = y\ (\hat{p}_U = 1-y)$			$\hat{p}_U = y\ (\hat{p}_L = 1-y)$		
x	99%	95%	90%	90%	95%	99%
0 (50)	*y*=0	0	0	0.058155079	0.071121736	0.100545083
1 (49)	0.000100246	0.000506228	0.00102534	0.091398131	0.106469546	0.139404125
2 (48)	0.002088725	0.004881433	0.00715372	0.120614155	0.137137626	0.17250176
3 (47)	0.006872485	0.012548588	0.016551859	0.147837176	0.165481947	0.202706269
4 (46)	0.013768445	0.022227964	0.027787668	0.173791158	0.192342784	0.23105085
5 (45)	0.022217815	0.033275094	0.04023659	0.198833003	0.218135366	0.258049729
6 (44)	0.031862914	0.045335321	0.053571403	0.22316999	0.243101317	0.284003704
7 (43)	0.042468787	0.0581917	0.067596699	0.246935202	0.267396002	0.309106966
8 (42)	0.053873602	0.071700767	0.082185062	0.270220078	0.291126307	0.333493522
9 (41)	0.065961072	0.085762078	0.097248095	0.293090581	0.314369411	0.357260386
10 (40)	0.078644691	0.100302237	0.112721613	0.315596061	0.337183108	0.380480351
11 (39)	0.091858295	0.115265826	0.128557379	0.337774485	0.359611889	0.403209554
12 (38)	0.105550132	0.130609916	0.144718155	0.359655697	0.381690748	0.425492219
13 (37)	0.119679002	0.146300584	0.161174597	0.381263556	0.403447679	0.447363763
14 (36)	0.134211639	0.162310601	0.177903198	0.402617382	0.424905357	0.468852912
15 (35)	0.149120897	0.178617846	0.194884897	0.423732967	0.446082326	0.48998318
16 (34)	0.164384449	0.195204195	0.212104098	0.444623296	0.46699384	0.510773932
17 (33)	0.179983844	0.212054713	0.229547962	0.465299081	0.487652488	0.531241159
18 (32)	0.195903809	0.229157067	0.247205894	0.485769151	0.508068648	0.551398063
19 (31)	0.21213172	0.246501078	0.265069158	0.506040748	0.528250837	0.571255486
20 (30)	0.228657206	0.264078395	0.283130591	0.526119749	0.548205972	0.590822246
21 (29)	0.245471848	0.281882241	0.301384377	0.546010837	0.567939565	0.610105383
22 (28)	0.262568945	0.299907221	0.319825889	0.565717627	0.587455877	0.629110348
23 (27)	0.279943342	0.318149179	0.338451559	0.585242759	0.606758024	0.647841147
24 (26)	0.297591302	0.336605092	0.35725879	0.604587965	0.625848055	0.666300434
25	0.315510421	0.355272997	0.376245891	0.623754109	0.644727003	0.684489579

n=100	$\hat{p}_L = y\ (\hat{p}_U = 1-y)$			$\hat{p}_U = y\ (\hat{p}_L = 1-y)$		
x	99%	95%	90%	90%	95%	99%
0 (100)	y=0	0	0	0.02951305	0.036216693	0.05160403
1 (99)	5.01242E-05	0.000253146	0.000512801	0.046559811	0.054459385	0.071957682
2 (98)	0.001039619	0.002431337	0.003565153	0.0616192	0.070383932	0.089430673
3 (97)	0.00340707	0.006229972	0.008225829	0.075710794	0.085176053	0.105481254
4 (96)	0.006801694	0.011004494	0.013776612	0.08919625	0.099257157	0.120632699
5 (95)	0.010940334	0.016431879	0.019905564	0.102253378	0.112834911	0.135144683
6 (94)	0.015642513	0.022334886	0.026449713	0.114985256	0.126029935	0.149168804
7 (93)	0.02078993	0.028605289	0.033311916	0.127458016	0.138919728	0.162802856
8 (92)	0.026301152	0.035171563	0.040428871	0.139717119	0.151557636	0.176114355
9 (91)	0.032117617	0.041983596	0.047756636	0.151795429	0.163982255	0.18915228
10 (90)	0.038195653	0.049004689	0.055263238	0.163717623	0.176222598	0.201953521
11 (89)	0.044501714	0.05620702	0.06292455	0.175502803	0.188301132	0.214546698
12 (88)	0.051009388	0.063568903	0.070721838	0.18716611	0.200235684	0.226954552
13 (87)	0.057697454	0.071073046	0.078640204	0.198719793	0.212040677	0.239195508
14 (86)	0.064548563	0.078705405	0.086667578	0.210173925	0.223727981	0.251284745
15 (85)	0.071548325	0.086454386	0.094794012	0.221536908	0.2353075	0.26323494
16 (84)	0.078684647	0.09431029	0.1030112	0.232815835	0.246787597	0.275056809
17 (83)	0.08594726	0.10226491	0.11131212	0.244016756	0.258175411	0.286759506
18 (82)	0.093327359	0.110311229	0.119690778	0.255144881	0.269477086	0.298350915
19 (81)	0.10081733	0.118443196	0.128142008	0.266204729	0.280697958	0.309837888
20 (80)	0.108410541	0.126655552	0.136661325	0.27720025	0.291842689	0.321226413
21 (79)	0.116101179	0.134943696	0.145244808	0.288134916	0.302915379	0.332521761
22 (78)	0.123884115	0.143303577	0.153889007	0.299011796	0.313919654	0.343728594
23 (77)	0.131754807	0.151731611	0.162590871	0.309833617	0.324858733	0.354851056
24 (76)	0.139709209	0.160224613	0.171347691	0.320602811	0.335735489	0.365892843
25 (75)	0.147743708	0.168779738	0.180157047	0.331321554	0.346552496	0.37685727
26 (74)	0.15585506	0.177394438	0.189016773	0.341991802	0.357312063	0.387747312
27 (73)	0.164040349	0.186066422	0.197924924	0.352615316	0.368016273	0.398565651
28 (72)	0.172296944	0.194793627	0.206879746	0.363193686	0.378667005	0.40931471
29 (71)	0.180622466	0.203574189	0.215879657	0.373728351	0.389265959	0.419996682
30 (70)	0.189014762	0.21240642	0.224923224	0.384220613	0.399814676	0.430613554
31 (69)	0.197471878	0.221288792	0.23400915	0.394671654	0.410314554	0.441167127
32 (68)	0.205992042	0.230219917	0.243136257	0.405082547	0.420766862	0.45165904
33 (67)	0.214573641	0.239198535	0.252303477	0.415454267	0.431172751	0.46209078
34 (66)	0.223215213	0.248223502	0.261509839	0.425787697	0.441533268	0.472463697
35 (65)	0.231915427	0.257293779	0.270754462	0.436083641	0.451849362	0.482779016
36 (64)	0.240673076	0.266408423	0.280036546	0.446342825	0.462121893	0.49303785
37 (63)	0.249487067	0.27556658	0.289355367	0.456565907	0.472351641	0.503241203
38 (62)	0.258356408	0.284767476	0.29871027	0.46675348	0.482539306	0.513389984
39 (61)	0.267280206	0.294010415	0.308100663	0.476906079	0.49268552	0.523485007
40 (60)	0.276257658	0.303294769	0.317526015	0.48702418	0.50279085	0.533527004
41 (59)	0.285288045	0.312619977	0.32698585	0.497108208	0.512855796	0.543516626
42 (58)	0.294370727	0.321985539	0.336479745	0.507158539	0.522880804	0.553454447
43 (57)	0.30350514	0.331391017	0.346007327	0.5171755	0.532866262	0.563340969
44 (56)	0.312690792	0.340836024	0.355568269	0.527159374	0.542812503	0.573176625
45 (55)	0.321927257	0.350320229	0.36516229	0.537110399	0.552719811	0.582961783
46 (54)	0.331214178	0.359843353	0.374789154	0.547028772	0.56258842	0.592696745
47 (53)	0.34055126	0.369405164	0.384448663	0.55691465	0.572418515	0.602381754
48 (52)	0.349938269	0.37900548	0.394140663	0.56676815	0.582210235	0.612016989
49 (51)	0.359375032	0.388644165	0.403865039	0.576589349	0.591963671	0.621602571
50	0.368861437	0.39832113	0.413621715	0.586378285	0.60167887	0.631138563

n=150	$\hat{p}_L = y\ (\hat{p}_U = 1-y)$			$\hat{p}_U = y\ (\hat{p}_L = 1-y)$		
x	99%	95%	90%	90%	95%	99%
0 (150)	*y*=0	0	0	0.019773438	0.024292597	0.03470557
1 (149)	3.34164E-05	0.000168771	0.000341897	0.031233817	0.036583168	0.048485953
2 (148)	0.000692036	0.001618827	0.002374192	0.04137497	0.047333019	0.060345774
3 (147)	0.002265007	0.004143625	0.00547293	0.050877068	0.057334222	0.071262464
4 (146)	0.004516558	0.007312546	0.009158928	0.05998104	0.066867877	0.081586214
5 (145)	0.007257229	0.010910186	0.013224408	0.068805184	0.076072323	0.091490565
6 (144)	0.010366487	0.014818521	0.017560962	0.077417997	0.085027815	0.101076706
7 (143)	0.013765426	0.01896557	0.02210405	0.08586338	0.093785865	0.110409837
8 (142)	0.017399794	0.02330381	0.026811658	0.094171482	0.102381896	0.119534935
9 (141)	0.021230608	0.027799978	0.031654647	0.102364084	0.11084152	0.128484599
10 (140)	0.025228817	0.032429728	0.036611832	0.110457532	0.119183977	0.137283372
11 (139)	0.029372118	0.037174615	0.041667235	0.118464471	0.127424162	0.145950285
12 (138)	0.033642958	0.042020265	0.046808451	0.126394927	0.13557389	0.154500458
13 (137)	0.038027232	0.04695522	0.052025622	0.134257008	0.143642723	0.162946142
14 (136)	0.04251341	0.051970172	0.057310755	0.142057385	0.151638531	0.171297434
15 (135)	0.047091922	0.057057435	0.062657265	0.149801626	0.159567885	0.17956277
16 (134)	0.051754723	0.062210574	0.068059646	0.157494434	0.167436339	0.18774929
17 (133)	0.056494972	0.06742414	0.073513243	0.165139829	0.175248639	0.195863098
18 (132)	0.061306798	0.072693467	0.079014074	0.172741273	0.183008877	0.203909461
19 (131)	0.066185114	0.078014525	0.084558705	0.18030178	0.190720614	0.211892966
20 (130)	0.071125481	0.083383807	0.090144145	0.187823988	0.198386968	0.219817631
21 (129)	0.076123996	0.088798232	0.095767775	0.195310223	0.206010689	0.227687004
22 (128)	0.081177208	0.094255085	0.101427285	0.20276255	0.213594219	0.235504232
23 (127)	0.086282047	0.099751951	0.107120622	0.210182807	0.221139733	0.243272126
24 (126)	0.09143577	0.105286678	0.112845957	0.217572646	0.228649184	0.250993204
25 (125)	0.096635913	0.110857332	0.118601649	0.22493355	0.236124327	0.258669737
26 (124)	0.101880252	0.116462173	0.12438622	0.232266861	0.24356675	0.266303776
27 (123)	0.107166776	0.122099628	0.130198336	0.239573797	0.250977895	0.273897184
28 (122)	0.112493654	0.127768266	0.136036784	0.246855466	0.258359073	0.281451656
29 (121)	0.117859222	0.133466786	0.14190046	0.25411288	0.265711481	0.288968742
30 (120)	0.123261954	0.139193999	0.147788357	0.261346965	0.273036218	0.296449862
31 (119)	0.128700453	0.144948816	0.153699551	0.268558571	0.280334289	0.303896318
32 (118)	0.134173435	0.150730237	0.159633196	0.275748483	0.287606623	0.311309311
33 (117)	0.139679717	0.156537338	0.16558851	0.282917421	0.294854074	0.318689949
34 (116)	0.145218205	0.162369272	0.171564774	0.290066054	0.302077433	0.326039255
35 (115)	0.15078789	0.168225251	0.177561322	0.297195	0.309277432	0.333358178
36 (114)	0.156387835	0.174104547	0.183577538	0.304304832	0.316454751	0.340647598
37 (113)	0.16201717	0.180006485	0.189612851	0.311396082	0.32361002	0.347908333
38 (112)	0.167675086	0.185930437	0.195666728	0.318469248	0.330743828	0.355141143
39 (111)	0.173360831	0.191875818	0.201738676	0.325524789	0.337856721	0.362346739
40 (110)	0.179073703	0.197842083	0.207828234	0.332563138	0.344949209	0.369525782
41 (109)	0.184813047	0.203828725	0.213934972	0.339584697	0.352021771	0.37667889
42 (108)	0.190578251	0.209835269	0.220058489	0.346589841	0.35907485	0.383806642
43 (107)	0.196368742	0.215861271	0.226198409	0.353578923	0.366108864	0.390909577
44 (106)	0.202183986	0.221906316	0.232354382	0.36055227	0.373124204	0.397988202
45 (105)	0.208023481	0.227970016	0.23852608	0.367510193	0.380121234	0.405042991
46 (104)	0.213886757	0.234052006	0.244713194	0.37445298	0.387100297	0.412074389
47 (103)	0.219773374	0.240151946	0.250915436	0.3813809	0.394061714	0.419082811
48 (102)	0.225682918	0.246269516	0.257132537	0.38829421	0.401005786	0.426068649
49 (101)	0.231615003	0.252404415	0.263364242	0.395193146	0.407932795	0.433032269
50 (100)	0.237569265	0.258556363	0.269610314	0.402077931	0.414843006	0.439974014
51 (99)	0.243545363	0.264725097	0.275870531	0.408948776	0.421736667	0.446894207
52 (98)	0.249542979	0.270910368	0.282144683	0.415805876	0.428614011	0.453793149
53 (97)	0.255561812	0.277111945	0.288432575	0.422649414	0.435475255	0.460671123
54 (96)	0.261601583	0.283329612	0.294734024	0.429479563	0.442320603	0.467528394
55 (95)	0.267662029	0.289563165	0.301048858	0.436296482	0.449150245	0.474365209
56 (94)	0.273742905	0.295812415	0.307376916	0.443100324	0.455964359	0.481181799
57 (93)	0.279843981	0.302077184	0.31371805	0.449891226	0.462763112	0.487978379
58 (92)	0.285965043	0.308357307	0.320072119	0.456669319	0.469546658	0.494755152
59 (91)	0.292105893	0.31465263	0.326438993	0.463434726	0.47631514	0.501512302
60 (90)	0.298266345	0.32096301	0.332818551	0.470187557	0.483068692	0.508250004
61 (89)	0.304446228	0.327288314	0.339210683	0.476927916	0.489807436	0.514968417
62 (88)	0.310645384	0.33362842	0.345615283	0.4836559	0.496531487	0.521667688
63 (87)	0.316863666	0.339983214	0.352032256	0.490371595	0.503240947	0.528347952
64 (86)	0.323100942	0.346352595	0.358461516	0.497075081	0.509935912	0.535009334
65 (85)	0.329357088	0.352736466	0.364902983	0.503766432	0.516616468	0.541651944
66 (84)	0.335631994	0.359134742	0.371356583	0.510445713	0.523282692	0.548275884
67 (83)	0.34192556	0.365547347	0.377822252	0.517112981	0.529934654	0.554881244
68 (82)	0.348237698	0.371974211	0.384299933	0.523768288	0.536572414	0.561468103
69 (81)	0.354568328	0.378415274	0.390789572	0.53041168	0.543196027	0.568036531
70 (80)	0.360917384	0.384870482	0.397291127	0.537043194	0.549805536	0.574586586
71 (79)	0.367284806	0.391339792	0.403804559	0.543662861	0.55640098	0.581118318
72 (78)	0.373670547	0.397823166	0.410329836	0.550270708	0.562982389	0.587631766
73 (77)	0.380074569	0.404320573	0.416866933	0.556866752	0.569549785	0.594126959
74 (76)	0.386496843	0.410831993	0.423415833	0.563451007	0.576103182	0.600603916
75	0.392937351	0.41735741	0.429976521	0.570023479	0.58264259	0.607062649

Examples:
n = 50, 90% confidence required, and x=10, then p_L=0.112721613 and p_U=0.315596061
n = 100, 95% confidence required, and x=85, then p_U=1-0.0864543 and p_L=1-0.2353075

Note: For different sample sizes *n* or for different confidence level then the following formulas can be used in Microsoft Excel; $\hat{p}_U = \text{BETAINV}\left(1-\frac{\alpha}{2}, x+1, n-x\right)$ and $\hat{p}_L = \text{BETAINV}\left(\frac{\alpha}{2}, x, n-x+1\right)$.

Appendix C: Activity sample size requirements

90% Confidence

p or $(1-p)$	L = 0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
0	29	32	36	42	49	59	74	99	149	299
0.01	31	35	40	47	56	69	90	128	216	569
0.02	34	38	44	52	63	79	106	157	280	829
0.03	36	41	48	57	69	89	121	184	343	1083
0.04	38	44	51	61	76	98	136	211	405	1332
0.05	40	46	54	66	82	107	151	237	464	1574
0.06	42	49	58	70	88	116	165	263	523	1811
0.07	44	51	61	74	94	125	179	288	579	2042
0.08	46	54	64	78	100	133	192	312	635	2268
0.09	48	56	67	83	105	142	205	336	689	2488
0.1	50	58	70	86	111	150	218	359	742	2704
0.11	51	60	73	90	116	157	230	381	793	2913
0.12	53	63	76	94	121	165	242	403	843	3117
0.13	55	65	78	97	126	172	254	424	892	3316
0.14	56	67	81	101	131	179	265	445	939	3509
0.15	58	69	83	104	136	186	276	465	985	3697
0.16	59	70	86	107	140	193	287	484	1030	3880
0.17	61	72	88	111	144	199	297	503	1073	4057
0.18	62	74	90	114	149	205	307	521	1115	4228
0.19	63	76	92	116	153	211	317	538	1155	4394
0.2	64	77	94	119	157	217	326	555	1194	4555
0.21	66	79	96	122	160	223	335	572	1232	4711
0.22	67	80	98	124	164	228	343	587	1269	4861
0.23	68	81	100	127	167	233	352	602	1304	5005
0.24	69	83	102	129	171	238	360	617	1338	5144
0.25	70	84	103	131	174	243	367	631	1370	5278
0.26	71	85	105	134	177	247	374	644	1401	5406
0.27	72	86	106	136	180	251	381	657	1431	5529
0.28	72	87	108	137	182	255	388	669	1459	5647
0.29	73	88	109	139	185	259	394	681	1486	5759
0.3	74	89	110	141	187	263	400	691	1511	5865
0.31	74	90	111	142	189	266	405	702	1536	5967
0.32	75	91	112	144	191	269	411	711	1559	6063
0.33	75	91	113	145	193	272	415	721	1580	6153
0.34	76	92	114	146	195	275	420	729	1600	6238
0.35	76	93	115	147	197	277	424	737	1619	6318
0.36	77	93	116	148	198	280	428	744	1637	6392
0.37	77	94	116	149	199	282	431	751	1653	6460
0.38	77	94	117	150	201	284	435	757	1668	6524
0.39	77	94	117	151	202	285	437	763	1681	6582
0.4	78	94	118	151	203	287	440	767	1693	6634
0.41	78	95	118	152	203	288	442	772	1704	6682
0.42	78	95	118	152	204	289	444	775	1713	6723
0.43	78	95	118	152	204	290	445	778	1721	6760
0.44	78	95	118	152	204	290	446	781	1728	6790
0.45	77	94	118	152	205	290	447	783	1733	6816
0.46	77	94	118	152	205	291	448	784	1737	6836
0.47	77	94	118	152	204	290	448	785	1740	6851
0.48	77	94	117	152	204	290	447	785	1741	6860
0.49	76	93	117	151	203	290	447	784	1741	6864
0.5	76	93	116	151	203	289	446	783	1739	6862

95% Confidence

p or $(1-p)$	L = 0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
0	36	40	45	51	60	72	91	122	183	368
0.01	39	44	50	59	70	87	114	164	279	753
0.02	42	48	56	66	80	102	137	204	371	1124
0.03	45	52	61	73	90	115	159	244	461	1484
0.04	49	56	66	79	99	129	180	282	548	1837
0.05	52	60	71	86	108	142	201	319	632	2181
0.06	55	63	75	92	116	155	221	356	715	2517
0.07	57	67	80	98	125	167	240	391	796	2846
0.08	60	70	84	104	133	179	260	425	875	3167
0.09	63	74	89	110	141	191	278	459	952	3480
0.1	65	77	93	115	149	202	296	492	1027	3786
0.11	68	80	97	121	156	213	314	524	1100	4084
0.12	70	83	101	126	164	224	331	555	1171	4374
0.13	73	86	105	131	171	234	348	585	1241	4656
0.14	75	89	108	136	177	244	364	614	1308	4931
0.15	77	92	112	141	184	254	379	643	1373	5198
0.16	79	94	116	145	191	264	395	670	1437	5457
0.17	81	97	119	150	197	273	409	697	1498	5709
0.18	83	99	122	154	203	282	423	723	1558	5953
0.19	85	102	125	158	209	290	437	748	1615	6189
0.2	87	104	128	162	214	298	450	772	1671	6418
0.21	88	106	131	166	219	306	463	795	1725	6639
0.22	90	108	134	170	225	314	475	818	1777	6852
0.23	92	110	136	173	230	321	487	839	1827	7057
0.24	93	112	139	177	234	328	498	860	1875	7255
0.25	94	114	141	180	239	335	509	880	1921	7445
0.26	96	116	143	183	243	341	520	899	1965	7628
0.27	97	117	145	186	247	347	529	917	2007	7802
0.28	98	119	147	188	251	353	539	934	2048	7970
0.29	99	120	149	191	255	359	548	951	2086	8129
0.3	100	121	151	193	258	364	556	966	2123	8281
0.31	101	123	153	196	261	369	564	981	2157	8425
0.32	102	124	154	198	264	373	571	995	2190	8561
0.33	103	125	155	200	267	377	578	1008	2220	8690
0.34	103	126	157	201	269	381	585	1020	2249	8811
0.35	104	127	158	203	272	385	591	1031	2276	8924
0.36	105	127	159	204	274	388	596	1042	2301	9030
0.37	105	128	160	206	276	391	601	1051	2324	9128
0.38	105	128	160	207	278	394	606	1060	2345	9218
0.39	106	129	161	208	279	396	610	1068	2364	9300
0.4	106	129	162	209	280	398	614	1075	2382	9375
0.41	106	130	162	209	281	400	617	1081	2397	9442
0.42	106	130	162	210	282	401	619	1086	2410	9502
0.43	106	130	163	210	283	403	621	1091	2422	9554
0.44	106	130	163	210	283	403	623	1094	2432	9598
0.45	106	130	163	210	283	404	624	1097	2439	9634
0.46	106	129	162	210	283	404	625	1099	2445	9663
0.47	105	129	162	210	283	404	625	1100	2449	9684
0.48	105	129	162	209	283	404	625	1100	2451	9698
0.49	104	128	161	209	282	403	624	1100	2451	9703
0.5	104	127	160	208	281	402	623	1098	2449	9701

99% Confidence

p or $(1-p)$	L = 0.1	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01
0	51	57	64	74	86	104	130	174	263	528
0.01	57	65	74	87	104	131	172	250	433	1204
0.02	63	72	84	99	122	156	212	321	593	1844
0.03	69	79	93	112	138	180	250	389	748	2468
0.04	74	86	102	123	155	203	287	455	898	3077
0.05	80	93	110	135	170	226	323	520	1045	3672
0.06	85	99	118	145	185	248	358	583	1189	4254
0.07	90	105	126	156	200	270	392	644	1329	4822
0.08	94	111	134	166	214	290	425	704	1465	5377
0.09	99	117	142	176	228	311	457	762	1598	5919
0.1	104	123	149	186	242	330	489	819	1728	6447
0.11	108	128	156	196	255	350	519	874	1855	6962
0.12	112	134	163	205	267	368	549	928	1978	7463
0.13	116	139	170	214	280	386	578	980	2098	7952
0.14	120	144	176	222	292	404	606	1031	2214	8427
0.15	124	149	182	231	303	421	633	1080	2327	8888
0.16	128	153	188	239	314	437	659	1128	2437	9337
0.17	131	158	194	246	325	453	685	1174	2543	9772
0.18	135	162	200	254	336	469	709	1219	2647	10194
0.19	138	166	205	261	346	484	733	1262	2746	10602
0.2	141	170	210	268	355	498	756	1304	2843	10998
0.21	144	174	215	275	365	512	778	1345	2936	11380
0.22	147	178	220	281	374	525	800	1384	3026	11748
0.23	150	181	225	287	382	538	820	1421	3112	12104
0.24	152	185	229	293	391	550	840	1457	3196	12446
0.25	155	188	233	299	398	562	858	1491	3275	12774
0.26	157	191	237	304	406	573	876	1524	3352	13090
0.27	159	193	241	309	413	583	894	1556	3425	13392
0.28	161	196	244	314	420	593	910	1586	3495	13682
0.29	163	199	247	318	426	603	925	1614	3562	13957
0.3	165	201	251	322	432	612	940	1641	3625	14220
0.31	167	203	253	326	438	620	954	1667	3685	14469
0.32	168	205	256	330	443	628	967	1691	3742	14705
0.33	169	207	259	333	448	636	979	1714	3795	14928
0.34	171	208	261	336	452	643	990	1735	3845	15138
0.35	172	210	263	339	456	649	1001	1755	3891	15334
0.36	173	211	265	342	460	655	1010	1773	3935	15517
0.37	174	212	266	344	463	660	1019	1790	3975	15686
0.38	174	213	268	346	467	665	1027	1805	4012	15842
0.39	175	214	269	348	469	669	1034	1819	4045	15986
0.4	175	215	270	349	471	673	1041	1831	4075	16115
0.41	176	215	271	351	473	676	1046	1842	4102	16232
0.42	176	216	271	352	475	678	1051	1851	4125	16335
0.43	176	216	271	352	476	680	1055	1859	4145	16425
0.44	176	216	272	353	477	682	1058	1865	4162	16502
0.45	176	216	272	353	477	683	1060	1870	4176	16566
0.46	175	215	271	353	477	683	1061	1874	4186	16616
0.47	175	215	271	352	477	683	1062	1876	4193	16653
0.48	174	214	270	351	476	683	1061	1876	4196	16677
0.49	173	213	269	351	475	682	1060	1875	4196	16687
0.5	172	212	268	349	474	680	1058	1873	4193	16684

Example: Assume that 95% confidence and an interval half width of L=0.05 is desired. After the first 50 samples, the estimate of p, $\hat{p}$ is 0.73. The sample size requires are symmetrical about p, thus in order to save space the table only considers $0 \le p \le 0.5$. So we need to make the calculation $p = 1 - 0.73 = 0.27$. The required sample size is then found in the second table (as this refers to the 95% confidence interval) and we pick the column associated with $L = 0.05$ and the row associated with 0.27. Thus the sample size is n = 347.