

# Online Research @ Cardiff

This is an Open Access document downloaded from ORCA, Cardiff University's institutional repository: <http://orca.cf.ac.uk/94674/>

This is the author's version of a work that was submitted to / accepted for publication.

Citation for final published version:

Dixon, Huw and Pourpourides, Panayiotis 2016. On imperfect competition with occasionally binding cash-in-advance constraints. *Journal of Macroeconomics* 50 , pp. 72-85.  
10.1016/j.jmacro.2016.09.002 file

Publishers page: <http://dx.doi.org/10.1016/j.jmacro.2016.09.002>  
<<http://dx.doi.org/10.1016/j.jmacro.2016.09.002>>

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the publisher's version if you wish to cite this paper.

This version is being made available in accordance with publisher policies. See <http://orca.cf.ac.uk/policies.html> for usage policies. Copyright and moral rights for publications made available in ORCA are retained by the copyright holders.



# On Imperfect Competition with Occasionally Binding Cash-in-Advance Constraints

Huw Dixon\*      Panayiotis M. Pourpourides<sup>†</sup>

July 11, 2016

## Abstract

We provide a theoretical demonstration of the link between imperfect competition and the cash-in-advance constraint, not previously considered in the literature. In a general equilibrium framework, we show that imperfect competition affects the proportion of times that the cash-in-advance constraint binds. As the market becomes more competitive it is certainly no less likely that the cash-in-advance constraint will bind. Therefore, economic welfare changes not only because of the direct effect of the change in the distribution of aggregate consumption but also because of the indirect effect of the cash-in-advance constraint. Other implications are also demonstrated.

**JEL Classification Codes:** D4; D5; E3; E4

**Keywords:** cash-in-advance; general equilibrium; monopolistic competition; imperfect competition; money velocity

## 1 Introduction

In this paper, we depart from the assumption of perfect competition in the final output sector, commonly used in cash-in-advance (CIA) models, and show that imperfect competition affects the proportion of times that the cash-in-advance constraint binds.<sup>1</sup> It follows that

---

\*Cardiff Business School, Cardiff University, Aberconway Bldg, Colum Drive, Cardiff, CF10 3EU, UK, dixonh@cardiff.ac.uk

<sup>†</sup>Corresponding Author: Cardiff Business School, Cardiff University, Aberconway Bldg, Colum Drive, Cardiff, CF10 3EU, UK, pourpouridesp@cardiff.ac.uk.

<sup>1</sup>Cash-in-advance models continue to be widely used in monetary economics, e.g. Evans et al. (2007), Devereux and Siu (2007), Díaz-Giménez et al. (2008), Hromcová (2008), Alvarez et al. (2009), Giraud and Tsomocos (2010), Adão et al. (2011), Telyukova and Visschers (2011), Burkhard and Maußner (2015). Some of these papers mainly focus on the case of a binding CIA constraint and constant velocity of money.

the degree of imperfect competition has both direct and indirect implications on aggregate welfare. While the distribution of consumption between profits and wages is affected directly, the level of output and work effort are affected indirectly via the CIA constraint. We demonstrate that for a given level of technology, as the market becomes more competitive not only the share of workers in aggregate consumption increases, it is also certainly no less likely that the CIA constraint will bind. As a result, the degree of imperfect competition indirectly affects aggregate welfare given that an equilibrium that occurs at a binding CIA constraint is always welfare inferior to an equilibrium that occurs at a non-binding CIA constraint for any given level of technology. These results are obtained in a general equilibrium framework with endogenous production, fully flexible prices and general assumptions about the velocity of money.

As is well known, the cash-in-advance (CIA) constraint creates a transactions demand for money even though money provides no direct utility.<sup>2</sup> In most papers, the CIA constraint on consumption is assumed to be always binding.<sup>3</sup> This assumption is justified if the empirical correlations between consumption and money are relatively high.<sup>4</sup> The empirical evidence however suggests that these correlations are relatively low (e.g. 0.3 for Italy – Ragot (2014)).<sup>5</sup>

In this paper, we develop a general *theoretical* framework where the periods where the CIA

---

<sup>2</sup>This was the rationale behind the first general formulation of the CIA constraint in Grandmont and Younes (1972).

<sup>3</sup>Among others, see Cooley and Hansen (1996), Evans et al. (2007), Chen and Li (2008), Díaz-Giménez et al. (2008).

<sup>4</sup>Burkhard and Maußner (2015) assume that only a fraction of nominal consumption is subject to the CIA constraint. Their calibration suggests that this fraction is 82%.

<sup>5</sup>Binding financial constraints for R&D investment may indicate weak empirical support to binding CIA constraints on consumption. It is reasonable to expect that if the latter were binding then it is likely that the former would have been binding as well. The empirical literature provides inconclusive evidence of the link between R&D investment and financial constraints (see Hall and Lerner (2010) for a summary). A recent paper by Brown et al. (2013), find little evidence of binding financial constraints on R&D.

constraint binds and the periods where it does not are determined *endogenously*. In the existing literature, there are only a few papers which allow for endogenously occasionally binding CIA constraints, and these are all numerical simulations (e.g. Devereux and Siu, 2007). Whether the CIA constraint binds in a particular period depends on expectations of risk-averse consumers about the future relative value of money as well as the degree of imperfect competition. We allow for a very general set of possibilities about how the velocity of money is determined, and show that velocity always has a specific upper bound which depends on the markup of the marginal product of labor over the real wage. Money can have real effects without requiring the presence of other physical assets or restrictions on how assets are used for transactions.<sup>6</sup> Although nominal wages and prices are fully flexible, there are cases where prices exhibit a sluggish response to a change in money supply.

In section II we illustrate the scope of the model by looking at the case of perfect foresight: although it removes the precautionary/buffer-stock demand for money, there is still a potential role for money over and above the *current* transactions demand. We are able to provide conditions relating to whether the current CIA constraint binds or not in terms of the current growth in the money supply or inflation and productivity growth. Among others, we show that in a zero-inflation steady state, the CIA constraint always binds. Since utility is higher in the steady-state with the nonbinding CIA constraint, it follows that the optimum inflation rate here is negative which has obvious similarities to the Friedman (1969) argument for a negative inflation rate made in the context of a money in the utility function

---

<sup>6</sup>Chamley and Polemarchakis (1984) note that a standard argument for money non-neutrality in general equilibrium lies on the existence of other real assets. Changes in the money supply affect the price level which in turn affects the return of money as an asset relative to the other physical assets. As a result, individuals realign their portfolios and the equilibrium holdings of physical assets change. Within this framework general equilibrium models require heterogeneous beliefs or other frictions.

approach.

We argue that a monetary authority would not necessarily avoid expanding the money supply because there are cases where it might be welfare improving. The monetary authority decides the transfer of money prior to the realization of technology and velocity shocks, and thus, the transfer may be optimal ex-ante but not optimal ex post. To keep our analysis simple and tractable since the focus is the effect of monopolistic competition, we abstract from the presence of physical assets such as capital. Focusing on an economy with primitive financial structure also enables us to demonstrate the direct effects of money, rather than those arising from portfolio choice.<sup>7</sup> In section III, we provide a discussion about how the introduction of real assets such as capital and bonds might influence the results.

Here we establish analytically the argument of Cooley and Hansen (1989), that “... the most important influence of money on short-run fluctuations are likely to stem from the influence of the money supply process on expectations of relative prices”. When the CIA constraint is nonbinding, the economy is at its efficient output with the *Classical* feature that money is neutral. This happens when the expected value of money equals its current value, so that consumers are indifferent between spending a unit of money today and holding it for one period. However, when particular state vectors occur, the CIA constraint binds because the agents expect that the relative value of money will decrease. As a result, they rush to spend all their money holdings the current period which leads to an increase in the velocity of money to the extent that it hits its upper bound. In this case, there is a unique equilibrium where money induces real effects. The transmission mechanism for money to

---

<sup>7</sup>The assumption that money is the only asset in the economy is not an unusual one in the literature: e.g. Lagos and Wright (2003), Lagos and Rocheteau (2005).

have real effects is the presence of the CIA constraint, through which the level of the price has a direct effect on consumer demand. This can be viewed as a type of *Keynesian effective demand* mechanism. Furthermore, we show that (for given technology) the level of output, hours worked and consumption is less when the CIA binds. This inefficiently low level of output occurs because the binding CIA constraint distorts the intra-temporal work-leisure decision and discourages work.

Alvarez et al. (2009), consider a CIA economy where production is exogenous and output is modelled as a stochastic endowment process. Their assumption that households are restricted from using funds from interest-bearing accounts for consumption purposes in every period prevents the CIA constraint from binding at all times thus allowing the velocity of money to vary. A direct implication of this is that prices respond sluggishly to changes in money supply because aggregate velocity decreases after an injection of money. They motivate this feature by presenting correlations between velocity and measures of money that exhibit a negative relationship. Chiu (2007), on the other hand, provides evidence that cross-country correlations between money and velocity for the OECD countries are all significantly positive. We argue that by merely looking at aggregate correlations in the data, one cannot safely draw conclusions about the direction of the effect of money growth on velocity because velocity is driven by other factors as well. It is possible that money velocity exhibits an overall negative relation with money growth despite the fact that an increase in money supply on its own has a positive effect on velocity. Another strand of the literature focuses on nominal rigidities of one kind or another which result in real effects of monetary policy

in the short-run.<sup>8</sup>

The rest of the paper is organized as follows. In section 2, we describe the economic environment which includes the problem of the firms, the problem of the workers and the analysis and discussion of the equilibrium conditions. In section 3 we look at the special case of perfect foresight and section 4 briefly examines how the introduction of capital and bonds might influence our results. Section 5 concludes.

## 2 Model Economy

The economy is populated by risk averse workers and monopolistic firms which are owned by risk-neutral entrepreneurs.<sup>9</sup> There are incomplete financial markets which mean that there is no source of insurance for workers. There is a perfectly competitive labor market and a goods market where the workers and the firms trade labor services and the final good. The agents exchange goods and labor services using cash which is the only medium of exchange. As the quantity theory of money indicates, at the aggregate level, nominal output varies with the nominal money balances times its velocity:

$$\overline{M}_t q_t \equiv P_t y_t, \tag{1}$$

where  $\overline{M}_t$  is the total quantity of money,  $q_t$  is the velocity of money,  $P_t$  is the aggregate price level and  $y_t$  is the aggregate real output. Aggregate output is defined in terms of

---

<sup>8</sup>This is the case in the neoclassical synthesis framework (e.g. Don Patinkin 1956) and also the *new* neoclassical synthesis (e.g. Woodford 2003).

<sup>9</sup>The assumption of risk-neutrality for the entrepreneurs is not essential but simplifies the exposition.

consumer preferences over the outputs  $x_i \geq 0$  of  $n > 1$  firms with corresponding price  $p_i$ . Let  $\mathbf{x}, \mathbf{p} \in \mathfrak{R}_+^n$  denote the  $n$ -vectors of outputs and prices. Preferences over  $\mathbf{x}$  are represented in their dual form with the homothetic unit cost (price) function. We take as an example the Linear-Homothetic (LH) preferences whose properties are described and derived in full in Datta and Dixon (2000, 2001) : but note that the results of the paper would hold for CES and other standard Homothetic preferences.<sup>10</sup> The LH cost function is:

$$P(\mathbf{p}) = \mu + \gamma(\mu - s), \quad (2)$$

where

$$\mu = \frac{\sum_{i=1}^n p_i}{n}, \quad s = \left( \frac{\sum_{i=1}^n p_i^2}{n} \right)^{\frac{1}{2}},$$

and  $\gamma > 1$ ;  $\gamma$  is the absolute value of the elasticity of demand when the prices of all firms are equal (i.e  $p_i = s = \mu = P$ ). Aggregate nominal expenditures are defined as  $Y = \sum_{i=1}^n p_i x_i$ . Applying Shephard's lemma to (2) we can write the share for good  $i$  in total expenditures as  $p_i x_i / Y = (\partial P / \partial p_i)(p_i / P)$ . The latter yields the Marshallian demand function  $x_i(\mathbf{p})$ :

$$x_i(\mathbf{p}) = \frac{(1 + \gamma)Y}{nP} - \frac{\gamma Y}{nP s} p_i.$$

The inverse demand curve is then

$$p_i = \frac{(1 + \gamma)s}{\gamma} - \frac{snP}{\gamma Y} x_i.$$

---

<sup>10</sup>In fact, the results of this paper hold for any homothetic preferences which satisfy the property that the own-price elasticity is non-decreasing in own price as shown in Coto-Martinez and Dixon (2003).



Following Dixit and Stiglitz (1977), if we assume that  $n$  is large and firms treat the aggregate price level as given (i.e. the indexes  $P$  and  $s$ ) along with aggregate nominal output, this yields the linear demand function  $p_i = p(x_i) = A - Bx_i$  where coefficients  $A$  and  $B$  correspond to  $[(1 + \gamma)/\gamma]s$  and  $snP/\gamma Y$ , respectively and are the same for all firms. LH preferences have the property that the firm's demand curve is linear in its own price treating the general price indexes as given (as in monopolistic competition). The assumption of monopolistic competition is very reasonable in a macroeconomic context, where any individual firm is "small" relative to the whole economy. In terms of (1), nominal income,  $Y$ , is determined by the money supply and velocity, and real output is nominal income divided by the price index (2) and hence the corresponding outputs of firms  $\mathbf{x}$ .

## 2.1 Firms

Each firm produces output by employing a fixed number  $m \geq 1$  of workers. Each worker employed by a firm provides  $h_i$  hours of work, which produces output via the linear technology  $x_i(h_i; m, \theta) = \theta mh_i$ , where  $\theta > 0$  is an exogenous productivity shock common to all firms. The latter is distributed according to the conditional p.d.f.  $\vartheta(\tilde{\theta}; \theta')$  for  $\tilde{\theta} \in \Theta \subset \mathfrak{R}_+$  where  $\theta'$  denotes the previous period realized value of  $\theta$ . The objective function of firm  $i$  can be written as

$$\Pi_i = p(x_i)x_i - Pw mh_i, \quad (3)$$

where  $\Pi_i$  are profits and  $w$  is the real hourly wage rate. The problem of the firm is to maximize its profits by choosing hours, taking as given the aggregate price level and the real

hourly wage rate. The necessary and sufficient condition for profit maximization is

$$\theta p(x_i) \left[ 1 + \frac{1}{\varepsilon_{x_i p_i}} \right] = Pw \quad (4)$$

where  $\varepsilon_{x_i p_i}$  is the elasticity of demand for firm  $i$ .

We can solve (4) for the labor demand function, nominal price, and nominal profits per firm as a function of the aggregate variables  $(A, B, P, w, \theta)$ :

$$h_i^d = \frac{1}{2Bm\theta} \left[ A - P\frac{w}{\theta} \right], \quad p_i = \frac{1}{2} \left[ A + \frac{w}{\theta} P \right], \quad \Pi_i = \frac{1}{4B} \left[ A - P\frac{w}{\theta} \right]^2. \quad (5)$$

Since firms face the same technology shock  $\theta$ , the equilibrium will be symmetric and all firms will set their price equal to  $P$ . Then, the nominal price equation reduces to  $w = \theta/\bar{\gamma}$ , where  $\bar{\gamma} = \gamma/(\gamma - 1)$  is the markup of the marginal product of labor over the real wage. Labor demand reduces to  $h^d = y/nm\theta$  while real profits per firm reduce to  $\pi = y/\gamma n$ . Aggregate profits are then  $\bar{\pi} = n\pi_i$  with the share of total profits in output being  $\gamma^{-1} < 1$ . Since all profits are consumed by entrepreneurs, it follows that total consumption by worker-households is equal to  $(1 - \gamma^{-1})y$ .<sup>11</sup> That is, the share of consumption by worker-households is determined by the elasticity of demand, with a higher consumption share with a higher elasticity. Whilst we are interested in the effects of monopolistic competition, it is in no way essential for the non-neutrality of money in this model: non-neutrality of money due to binding CIA constraints is if anything more likely to occur if there is perfect competition and all income takes the form of wages.

---

<sup>11</sup>Note that the market becomes more competitive as  $\gamma$  increases and/or as  $n$  increases.

## 2.2 Consumption and Worker-Households

Time is discrete and infinite,  $t \in \mathbb{Z}_+ = \{1, 2, \dots, \infty\}$ . There are  $(n \times m)$  worker-households with preferences over leisure,  $l$ , and a Linear Homothetic subutility consumption,  $c(\mathbf{c})$  which is defined over the household's consumption of the  $n$  goods  $\mathbf{c} \in \mathfrak{R}_+^n$ , and is represented in its dual form by (2). The utility function of a representative worker-household is given by  $u(c_t, l_t) = \ln c_t + \phi \ln l_t$  where  $\phi > 0$ . We can think of the household solving a two stage budgeting process: firstly choosing total consumption,  $c$ , given price  $P$ , and secondly allocating this across the  $n$  products given prices  $p_i$ . Each worker-household is endowed with one unit of time which is split between work and leisure that is,  $l + h = 1$ . All worker-households are identical and face the same prices, so we shall model them as a representative worker-household (thus avoiding the need for a household subscript and aggregation). Entrepreneurs have exactly the same subutility function over the consumption of firms' output as do worker-households. However, their utility is linear in the subutility  $u^e(c_t) = c_t^e$ , which with discounting means that entrepreneurs want to spend all of their profit income on consumption in each period. The entrepreneurs face no CIA constraint. We will henceforth describe in detail the worker-household's problem, and simply note that by market clearing the consumption of the entrepreneurs is equal to profits and given by  $c_t^e = x_t - mc_t$ .

The worker-household's wealth constraint is given by

$$M_{t+1}^c + P_t c_t = M_t^c + \nu_t + P_t w_t h_t, \quad (6)$$

where  $M^c \in \mathfrak{R}_+$  are the household's nominal money holdings,  $\nu$  is a money increase or

decrease such that  $M^c > |\nu|$  and  $P_t c_t = \sum_{i=1}^n p_{ti} c_{ti}$ . The transfer  $\nu_t$  is made at the end of period  $t-1$  and before  $\theta_t$  is realized. It takes a while for the transfer to be completed but the timing is such that the money is available at the beginning of the period. Households treat  $\nu$  as a random variable that is distributed according to  $\xi(\tilde{\nu}; \nu')$  for  $\tilde{\nu} \in N$  where  $\nu'$  denotes the previous period transfer and  $N = \{\tilde{\nu} \in \Re : \nu + M^c > 0\}$ . The household receives its labor earnings at the end of the period but purchases consumption at the beginning of the period. As a result, it faces a cash-in-advance constraint:

$$P_t c_t \leq M_t^c + \nu_t. \quad (7)$$

The problem of the household is to choose consumption, labor supply and money balances to maximize utility subject to the budget constraint and the CIA constraint. We will say that the CIA constraint is binding whenever  $P_t c_t = M_t^c + \nu_t$ . It is *weakly binding* when the household does not wish to consume more; it is *strictly binding* when the household is constrained to consume less than it would like to in the absence of the CIA constraint.

The Bellman equation associated with the household's problem is the following:

$$\begin{aligned} V(M_t^c, \nu_t) = & \max\{u(c_t, l_t) + \beta E_t V(M_{t+1}^c, \nu_{t+1}) \\ & - \lambda_{1t} [M_{t+1}^c + P_t c_t - M_t^c - \nu_t - P_t w_t h_t] - \lambda_{2t} [P_t c_t - M_t^c - \nu_t]\}, \end{aligned}$$

where  $\beta$  is the discount factor,  $\lambda_{1t}$  is the shadow price of the standard budget constraint and  $\lambda_{2t}$  is the shadow price of the CIA constraint.

This yields the following necessary and sufficient first-order conditions:

$$u_c(c_t, l_t) = \lambda_{1t}P_t + \lambda_{2t}P_t, \quad (8)$$

$$u_l(c_t, l_t) = \lambda_{1t}P_t w_t, \quad (9)$$

$$\lambda_{1t} = \beta E_t \{ \lambda_{1t+1} + \lambda_{2t+1} \}. \quad (10)$$

Notice that in equilibrium,  $M_t^c = M_t$ . Combining (8), (9) and (10) yields

$$\frac{u_l(c_t, l_t)}{w_t} = \beta E_t \left\{ \frac{u_c(c_{t+1}, l_{t+1})}{1 + g_{pt+1}} \right\},$$

where  $g_{pt} = P_t/P_{t-1} - 1$  denotes the inflation rate in period  $t$ . If the CIA constraint does not bind or is only weakly binding in period  $t$  ( $\lambda_{2t} = 0$ ), the left-hand side of the above condition is also equal to the marginal utility of consumption, which implies that the marginal benefit of work will equal the marginal cost of work, i.e.  $u_c(c_t, l_t) w_t = u_l(c_t, l_t)$ . On the other hand, if the CIA constraint is strictly binding ( $\lambda_{2t} > 0$ ) then the marginal benefit of work will be greater than the marginal cost of work, i.e.  $u_c(c_t, l_t) w_t > u_l(c_t, l_t)$ . Using the fact that utility is separable in consumption and leisure, it is straightforward to show that money demand is

governed by<sup>12</sup>

$$E_t \left[ \left( \frac{\beta u_c(c_{t+1})}{u_c(c_t)} \right) \left( \frac{1}{1 + g_{pt+1}} \right) \right] \begin{cases} < 1, \text{ binding CIA constraint} \\ = 1, \text{ nonbinding CIA constraint} \end{cases} . \quad (11)$$

The term  $[1/(1 + g_{pt+1})]$  is the gross return of money,  $R_{t+1}^M \equiv 1 + r_{t+1}^M$ .<sup>13</sup> The left hand side of the above condition can also be written as  $E_t [\psi_{t+1} R_{t+1}^M]$ , where  $\psi_{t+1}$  is the stochastic discount factor or pricing kernel which is equal to the intertemporal rate of substitution between next period consumption and current consumption. The term on the left hand side of (11) is the expected return of money measured in next period's utility per unit of current utility (i.e. the expected *relative* value of money). When consumers expect that the *relative* value of money will decrease (i.e.  $E_t [\psi_{t+1} R_{t+1}^M] < 1$ ), they spend all their money holdings the current period and the CIA constraint binds, otherwise (if  $E_t [\psi_{t+1} R_{t+1}^M] = 1$ ) they keep some cash for next period and the CIA constraint does not bind. In the latter case, the agents are indifferent between spending a unit of money today and holding it for one period whereas in the former case, the agents strongly prefer to spend it today.

Dividing (8) over (9) yields:

$$\phi \frac{c_t}{1 - h_t} = \frac{\lambda_{1t}}{\lambda_{1t} + \lambda_{2t}} w_t. \quad (12)$$

---

<sup>12</sup>This is the same condition governing money demand in Alvarez et al. (2009). In their model, the condition holds with strict equality when the household carries a strictly positive balance of money in its bank account into next period. The latter is equivalent to a non-binding CIA constraint in our model. Using the logarithmic utility function, the left hand side of (11) can also be written as  $\beta E_t [P_t c_t / P_{t+1} c_{t+1}] = \beta E_t [1/(1 + g_{pt+1})(1 + g_{ct+1})]$  where  $g_c$  denotes the growth rate of consumption.

<sup>13</sup>Note that  $r^M = -g_p/(1 + g_p)$  is non-positive as long as inflation is strictly non-negative.

When  $\lambda_{2t} = 0$  and the CIA constraint for that period is not binding or weakly binding, this is the usual intra-temporal condition which states that the marginal rate substitution (MRS) between leisure and consumption equals the real wage. However, when the CIA constraint is strictly binding with  $\lambda_{2t} > 0$ , the MRS is lower than the real wage, so that for given consumption the labor supply  $h_t$  is lower.<sup>14</sup> Consumption will be lower as well when  $\lambda_{2t} > 0$  (the income effect) which will tend to increase  $h_t$ , but since the real wage remains constant the overall effect on the labor supply is negative. One way of understanding the leisure-consumption distortion when the CIA constraint binds is that the household switches from consumption which is constrained by CIA to leisure which is not: the CIA in effect acts as a tax on consumption.

We can see that the behavior of the household divides into two regimes. In one regime (CIA constraint nonbinding or weakly binding)  $\lambda_{2t} = 0$  and the household behaves in the standard way (it can demand and supply as much as it wants to at market prices and wages). In the other regime  $\lambda_{2t} > 0$ , the household is constrained in its ability to consume at the prevailing price: it would like to consume more given the price, but is unable to do so. This is an *effective demand constraint*: with a CIA constraint, the *desired* consumption can only become *effective* if there is the cash to execute it. This spills over into the labor supply decision, reducing the level of labor supply. There is less incentive to work now and increase income which cannot be spent this period only to generate more cash for next period when it is not needed. This is a very “Keynesian” effective demand mechanism, as was found in the earlier literature on non-Walrasian equilibria.<sup>15</sup>

---

<sup>14</sup>Condition (12) can be rewritten as  $h_t = 1 - (1 + \lambda_{2t}/\lambda_{1t})(\phi/w_t)c_t$  so that for a given level of consumption, labor supply is lower when  $\lambda_{2t} > 0$ .

<sup>15</sup>See for example Clower (1965), Leijonhufvud (1968), Benassy (1975), Malinvaud (1975). However, unlike

## 2.3 Equilibrium with an Occasionally Binding CIA Constraint

In equilibrium, all firms produce the same quantity, make the same profit and all household-consumers consume the same amount of the product purchased from each firm. Whilst we have treated  $q_t$  as given at the household level, we now need to define the aggregate relationship which determines the velocity of circulation. The latter is determined by institutional factors such as the system of payments and the monetary policy regime ( $\nu$ ) as well as the payment habits of the society and the production technology ( $\theta$ ). To capture developments in the system of payments and changes in the payment habits we introduce a velocity shock  $\varphi_t$  which has an initial condition  $\varphi_1$  and the conditional p.d.f.  $\bar{\Phi}(\tilde{\varphi}; \varphi')$  for  $\tilde{\varphi} \in \Phi \subset \mathbb{R}_+$  where  $\varphi'$  denotes the previous period realized value of  $\varphi$ . Then, *economic fundamentals* are represented as a sequence of productivity levels, money supplies and velocity shocks  $\{\theta_t, M_t, \varphi_t\}_{t=1}^{\infty}$  that evolve according to  $\vartheta$ ,  $\xi$  and  $\bar{\Phi}$  and the initial conditions  $\{\theta_1, M_1, \varphi_1\}$ .

The velocity of circulation is determined by the function:  $q_t \in Q_t: \Theta \times \Phi \times N \rightarrow (0, q^b]$  which we can write as  $q_t = q(\theta_t, \varphi_t, \nu_t)$ , where  $q_t$  is a unique potentially time variant scalar. Thus, we allow for a very general set of possibilities about how the velocity is determined: there is a general function which relates the velocity  $q_t$  to the two shocks determining  $\theta_t, M_t$  as well as a possible velocity-specific shock. The assumption allows for the velocity to be constant, or to be decreasing or increasing in its arguments and there is no requirement for smoothness or differentiability. An equilibrium consists of a sequence pairs of  $\{w_t, P_t\}_{t=1}^{\infty}$  that clear the labor and the goods market given the economic fundamentals  $\{\theta_t, M_t, \varphi_t\}_{t=1}^{\infty}$ .

Associated with  $\{w_t, P_t, \theta_t, \varphi_t, \nu_t\}_{t=1}^{\infty}$  are the sequences  $\{q_t, \lambda_{1t}, \lambda_{2t}, y_t, c_t, h_t, \pi_t\}_{t=1}^{\infty}$ .

---

these older papers, the phenomenon in the present model is very much dynamic and intertemporal rather than resulting from static and ad hoc rationing constraints that arise from exogenous fixed prices.



We can characterize the equilibrium sequence by dividing it into two possible states: one where the CIA constraint is binding, and one where it is not. Of course, how this divides up will depend on the sequence of productivity, monetary and velocity-specific shocks. The two extremes are that the CIA constraint is always binding, or never binding. The following propositions allow us to determine how the economy behaves in the case of an intermittently binding CIA constraint.

For all  $t$ , the real wage is related to the current productivity level by the markup equation,  $w_t = \theta_t/\bar{\gamma}$ . The nominal price  $P_t$  thus becomes the key variable for establishing equilibrium in each period. A useful way to sort the sequence into binding and nonbinding is to note that there is an upper bound to the velocity of circulation.

**Lemma** *For all  $t$  there is an upper bound  $q^b = \bar{\gamma}$  on the equilibrium  $q_t$ . The CIA constraint binds at time  $t$  when  $q_t = q^b$  and it does not bind at time  $t$  when  $q_t < q^b$ .<sup>16</sup>*

All proofs are in appendix A (published online).<sup>17</sup> The intuition behind the Lemma is clear. Firstly, the upper bound on the velocity comes from two sources: the CIA constraint (7) itself, and the proportion of expenditure which is not subject to the CIA constraint (the expenditure of entrepreneurs which equals profits). Turning to the CIA constraint, if there were no profits ( $\gamma$  very large) then worker-household consumption equals output and (7) becomes  $P_t y_t \leq M_t^c + \nu_t$ , which implies by definition that  $q^b = 1$ . However, since the entrepreneurs spend all of their profits and are not subject to the CIA constraint, the latter only applies to that portion of output which is consumed by workers. A higher markup

---

<sup>16</sup>Recall that whether the CIA constraint binds or not depends on the expectation about next period's relative value of money (condition 11). This expectation is conditional on the current state of the economy.

<sup>17</sup>The appendix can be found in the following link: <https://sites.google.com/site/panayiotismpourpourides/research>

implies a greater share of profits, and thus for a given output a lower share of consumption by worker-households and hence a higher overall velocity is possible. For  $\gamma$  close to 1, profits take up nearly all output and the CIA constraint only applies to a very small proportion of output, which allows the velocity to be very large. If the CIA constraint applied both to workers and entrepreneurs, then the share of profits would not matter and we would have no dependence of velocity on  $\gamma$  :  $q^b = 1$ . However, it seems more reasonable to assume that entrepreneurs are not so constrained. Hence, this “profit share effect” means that the upper bound of  $q_t$  is decreasing with the elasticity of demand of the consumption good (i.e.  $dq^b/d\gamma = -1/(\gamma - 1)^2 < 0$ ) or equivalently, it is increasing with the markup of marginal productivity over the real wage.

The Lemma enables us to partition time into two sets: times when the CIA constraint is strictly binding, and times when it is not strictly binding that is,<sup>18</sup>  $\mathcal{B} = \{t \in \mathbb{Z}_+ : \lambda_{2t} > 0 \text{ and } q_t = q^b\}$  and  $\mathcal{NB} = \{t \in \mathbb{Z}_+ : \lambda_{2t} = 0 \text{ and } q_t \leq q^b\}$ . Now, we can define the *proportion* of periods in which the CIA constraint is binding. If we define for any  $T \in \mathbb{Z}_+$   $\mathcal{B}(T) = \{t \in \{1, 2, \dots, T\} : t \in \mathcal{B}\}$ , and likewise  $\mathcal{NB}(T)$ , we can define the proportion of times the CIA constraint binds until  $T$  that is,  $\mathcal{P}(\mathcal{B}, T) = \#\mathcal{B}(T)/T$ . The stationarity of the conditional distributions of  $\theta$ ,  $\nu$  and  $\varphi$  is sufficient to ensure that  $\lim_{T \rightarrow \infty} \mathcal{P}(\mathcal{B}, T) = \varkappa$ , where  $\varkappa \in [0, 1]$ . The following Propositions characterize the equilibrium price level  $P_t$  when the CIA constraint binds and when it does not, and show that for given fundamentals, the proportion of time in which the CIA binds is non-decreasing in  $\gamma$ .

**Proposition 1** (i) *When the CIA constraint does not bind ( $t \in \mathcal{NB}$ ) there is a unique equi-*

---

<sup>18</sup>Weakly binding and nonbinding equilibria belong to the same category. Whenever we refer to binding CIA constraints we imply the strictly binding case.

librium where  $P_t = (1 + \phi) q_t \left[ \frac{M_t + \nu_t}{\theta_t} \right]$  with  $q_t \leq q^b$  and  $\chi_t \leq \phi$ , where  $\chi_t = \frac{\phi}{Z_t(M_t + \nu_t)}$ ,  
 $Z_t = \beta E_t \left\{ \frac{u_c(c_{t+1}, l_{t+1})}{P_{t+1}} \right\}$  and

$$\frac{u_c(c_{t+1}, l_{t+1})}{P_{t+1}} = \begin{cases} \frac{q^b}{q(\theta_{t+1}, \nu_{t+1}, \varphi_{t+1})} \frac{1}{(M_{t+1} + \nu_{t+1})} & \text{for } (t+1) \in \mathcal{NB} \\ \frac{1}{M_{t+1} + \nu_{t+1}} & \text{for } (t+1) \in \mathcal{B} \end{cases}.$$

(ii) When the CIA constraint binds ( $t \in \mathcal{B}$ ) there is a unique equilibrium where  $P_t = (1 + \chi_t) q^b \left[ \frac{M_t + \nu_t}{\theta_t} \right]$  with  $\chi_t > \phi$ .

The interpretation of Proposition 1(ii) is that the CIA constraint binds when the expected return on savings is sufficiently low. Note that  $Z_t$  is the discounted expected marginal utility that \$1 saved now can buy next period. When  $Z_t$  is low, and hence  $\chi_t$  is high, the return to saving is so low that the worker-household wants to spend all of its cash balances now. The CIA constraint prevents the worker-household from borrowing to smooth its consumption as much as it would like to. The critical value of  $\bar{Z}_t$  at which the CIA binds is defined in the following corollary.

**Corollary 1** Let  $\bar{Z}_t = (M_t + \nu_t)^{-1}$ . The CIA constraint strictly binds at time  $t$  when  $Z_t < \bar{Z}_t$  (and hence,  $Z_t(M_t + \nu_t) = \phi/\chi_t < 1$ ), and does not bind when  $Z_t \geq \bar{Z}_t$ , (and hence,  $Z_t(M_t + \nu_t) = q^b/q_t \geq 1$ ).

$\bar{Z}_t$  is the return on savings that exactly equates the marginal utility of current consumption to the expected discounted marginal utility of next-period consumption when the household spends all of its current money balance.<sup>19</sup> If  $Z_t$  falls below this critical level, then

<sup>19</sup>With logarithmic utility,  $u_c(c_t) = (M_t + \nu_t)^{-1}$  when all current balances are spent.

the CIA constraint binds and the worker-household is prevented from lowering its marginal utility of current consumption by increasing its current consumption. It is clear that this is an intertemporal phenomenon which depends on expectations about what is going to happen next period: indeed, since the CIA constraint can bind in the future it may involve expectations into the infinite future.

Corollary 1 also indicates that the velocity of circulation is related to the expectations about the future state of the economy via  $Z$  as  $q_t = q^b/Z_t(M_t + \nu_t)$ . Since a current change in money supply ( $\nu_t$ ) affects expectations about the future value of money ( $Z_t$ ), velocity can be constant, increasing or decreasing in money supply. The direction of the effect of  $\nu_t$  on  $q_t$  depends on how changes in money supply affect expectations. This is consistent with our assumptions about the functional form of velocity. For instance, if an expansionary money supply generates expectations for a decrease in the value of money next period, then it is possible that an increase in  $\nu$  causes an increase in velocity.

**Corollary 2** *When  $t \in \mathcal{NB}$  and the CIA constraint weakly binds then,  $Z_t(M_t + \nu_t) = 1$ ,  $\phi = \chi_t$  and  $q_t = q^b$ .*

The implications for the CIA constraint on nominal prices and real output can be seen if we rewrite the expression for the price level using the explicit functional forms:

$$P_t = \left[ \frac{q^b}{\theta_t} \right] \left[ (M_t + \nu_t) + \frac{\phi}{Z_t} \right] \quad \text{for } t \in \mathcal{B}.$$

The equilibrium price level is not proportional to the current money-supply  $M_t + \nu_t$  due to expectations  $\phi/Z_t > 0$ . To show this let  $\nu_t = \alpha(\nu_t) M_t$  and  $Z_t \in [\underline{\zeta}, \zeta, \bar{\zeta}]$  such that

$0 < \underline{\zeta} < \zeta < \bar{\zeta}$  and  $\mu(\nu_t)$  to denote the percentage effect of  $\nu_t$  on  $P_t$ . If  $\eta(\nu_t)$  is the percentage effect of  $\nu_t$  on  $\phi/Z_t$  such that  $\eta(\nu_t) \in [\underline{\eta}, 0, \bar{\eta}]$  with  $\underline{\eta} < 0 < \bar{\eta}$  and  $\phi/\bar{\zeta} = (1 + \underline{\eta})\phi/\zeta < \phi/\zeta < (1 + \bar{\eta})\phi/\zeta = \phi/\underline{\zeta}$  then

$$\mu(\nu_t) = \alpha(\nu_t) \frac{M_t}{M_t + \frac{\phi}{\zeta}} + \eta(\nu_t) \frac{\frac{\phi}{\zeta}}{M_t + \frac{\phi}{\zeta}} \text{ for } t \in \mathcal{B}.$$

Even if a change in money supply does not affect expectations (i.e.  $\eta(\nu_t) = 0$ ),  $\mu(\nu_t) < \alpha(\nu_t)$  because  $\phi/Z_t > 0$ . In other words, a 10% higher money-supply implies a higher price, but one which is less than 10% higher. When a change in money supply leads to expectations for higher *absolute* value of money (i.e.  $\eta(\nu_t) = \underline{\eta}$ ) then the percentage increase in the price level,  $\mu(\nu_t)$ , is even smaller than in the case of  $\eta(\nu_t) = 0$ . Note that if  $(t-1) \in \mathcal{B}$  then  $\mu(\nu_t)$  is the time  $t$  inflation rate which is due to the change in money supply. Therefore, there are cases where the price level responds sluggishly to a change in money supply.

Proposition 1 also indicates that the binding CIA constraint implies a non-neutrality of money. It is straightforward to show that output and consumption respond *negatively* to the CIA constraint (see proofs of Propositions 1 and 2):

$$y_t = \frac{nm}{1 + \omega_t} \theta_t, \quad c_t = \frac{y_t}{nmq^b}, \quad h_t = \frac{1}{1 + \omega_t}, \quad \pi_t = \frac{y_t}{n\gamma},$$

where  $\omega_t = \chi_t$  for  $t \in \mathcal{B}$  and  $\omega_t = \phi$  for  $t \in \mathcal{NB}$ . The strength of the CIA constraint is reflected in how big  $\chi_t$  is (since it is inversely related to  $Z_t$ ). In the absence of CIA constraint, when  $Z_t = \bar{Z}_t$  from Proposition 1(ii) and corollary 1, we have  $\chi_t \leq \phi$ ; when the CIA constraint binds we have  $\chi_t > \phi$ . Hence, output, employment and profits are all

lower with a binding CIA constraint than without. This is intuitive, since the restriction of consumption directly reduces output and hours per worker (from the production function and labor market equilibrium) and profits (via the markup equation). Hence, if we compare outputs in times with the nonbinding constraint (where output is at its efficient level  $y_t^*$ )<sup>20</sup> and when it is binding we have:

$$y_t = \frac{nm}{1 + \phi} \theta_t = y_t^* \text{ for all } t \in \mathcal{NB} \text{ and } y_t = \frac{nm}{1 + \chi_t} \theta_t < y_t^* \text{ for all } t \in \mathcal{B}.$$

If we compare any two periods with the same productivity level, we can say that the nonbinding equilibrium Pareto dominates the binding equilibrium in terms of the current flow in utility and profits. Furthermore, we can say that if we have two periods with the same productivity in which the CIA constraint binds, the one with the smaller  $\chi_t$  dominates the other.

**Proposition 2** (i) For any  $t_1 \in \mathcal{B}$  and any  $t_2 \in \mathcal{NB}$  such that  $\theta_{t_1} = \theta_{t_2}$  then  $u(\theta_{t_2}) > u(\theta_{t_1})$

and  $\pi(\theta_{t_2}) > \pi(\theta_{t_1})$ . (ii) For any  $t_1, t_2 \in \mathcal{B}$  such that  $\theta_{t_1} = \theta_{t_2}$ , if  $\chi_{t_1} > \chi_{t_2}$  then  $u(\theta_{t_2}) > u(\theta_{t_1})$  and  $\pi(\theta_{t_2}) > \pi(\theta_{t_1})$ .

The role of imperfect competition matters in this model because entrepreneurs are assumed to be unaffected by the CIA constraint. The proportion of expenditure in the economy covered by the CIA constraint is increasing in the elasticity of demand (decreasing in the markup). We can now consider two economies that are identical in terms of the economic

---

<sup>20</sup>Note that with the utility function assumed for the worker-household, the income and substitution effects of the real wage exactly offset each and there is no direct effect of the degree of imperfect competition or productivity on equilibrium labor supply. In this case, it is only the CIA constraint that can alter employment and reduce output below its efficient level. As shown in Proposition 4 there is an indirect effect of the degree of imperfect competition on labor supply, operating via the CIA constraint.

fundamentals over time, but which differ in the degree of imperfect competition. We can show that the CIA constraint cannot bind for a lower proportion of the time in a more competitive economy.

**Proposition 3** *Consider  $\gamma_1$  and  $\gamma_2$  with corresponding sequences of equilibria and resultant  $\varkappa_1$  and  $\varkappa_2$ . If  $\gamma_1 > \gamma_2$ , then  $\varkappa_1 \geq \varkappa_2$ .*

As the market becomes more competitive (as  $\lim_{\gamma \rightarrow \infty} q^b = 1$ ), it is “more likely” that the CIA constraint will bind (or certainly no less likely).<sup>21</sup> It needs to be stressed that Proposition 3 does not imply that in a perfectly competitive market the CIA constraint will always bind. Whilst it is possible that the CIA constraint will be binding all the time and  $\mathcal{NB} = \emptyset$ , it is also perfectly possible that in the competitive case the CIA constraint may never (strictly) bind and hence  $\mathcal{B} = \emptyset$ . However, what is clear from the proof of Proposition 3 is that for some pairs  $(\gamma_1, \gamma_2)$ ,  $\varkappa_1 < \varkappa_2$ .

Proposition 3 implies that as the market becomes more competitive, it becomes “more likely” that output will be lower than its efficient level. Although this may sound counter-intuitive, it is justified by the presence of the CIA constraint which affects the portion of consumption being subject to the CIA constraint. As the elasticity of demand ( $\gamma$ ) increases, firms face tougher competition, and the markup they charge reduces (i.e. the monopoly power of the firms decreases). Firm owners are made worse off by increased competition because (i) their share in aggregate production decreases and (ii) aggregate production is lower than its efficiency level when the CIA constraint binds. On the contrary, worker-households

---

<sup>21</sup>In other words, as the market becomes more competitive, the range of the set of times that the CIA constraint does not bind, shrinks (or certainly does not expand).

face a tradeoff between lower output when the CIA constraint binds and an increased share in aggregate production. When the latter dominates the former, worker-households are better off from increased competition. Since the perfectly competitive outcome is Pareto optimal, whilst it may be more likely that the CIA constraint will bind when there is perfect competition, the resultant welfare loss from lower output may be small.

We now show that monetary policy depends on the degree of competition. Two economies characterized by different degrees of competition but identical in all other respects will have different monetary policies,  $\{M_t\}_{t=0}^\infty$ , unless they have different expectations about the evolution of money supply,  $\xi$ . For simplicity we have assumed that the transition probabilities of money transfers depend only on the previous realization of the transfer. However, this assumption does not play a crucial role and the analysis can be easily extended when the transition probabilities have a more complex functional form and/or depend on other factors. Let  $\Xi$  denote the conditional cumulative distribution of  $\xi$ . Then, the following proposition holds.

**Proposition 4** *If for any  $\nu_a$  and  $\nu_b$  such that, when  $\nu_a < \nu_b$ ,  $q(\nu_a) < q(\nu_b)$  then, for a given sequence  $\{\theta_t, M_t, \varphi_t\}_{t=1}^\infty$ , invariant probability distributions  $\vartheta$  and  $\bar{\Phi}$ , and  $\gamma_1$  and  $\gamma_2$  with corresponding cumulative distributions  $\Xi^1$  and  $\Xi^2$  such that  $\gamma_1 > \gamma_2$ :*

$$\text{If } t(\gamma_1) \in NB \text{ then, } \begin{cases} t(\gamma_2) \in NB \text{ only if } \Xi^1 \text{ first-order stochastically dominates } \Xi^2 \\ t(\gamma_2) \in \mathcal{B} \text{ only if } \Xi^2 \text{ first-order stochastically dominates } \Xi^1 \end{cases}$$



$$\text{If } t(\gamma_1) \in \mathcal{B} \text{ then, } \left\{ \begin{array}{l} t(\gamma_2) \in \mathcal{NB} \text{ only if } \Xi^1 \text{ first-order stochastically dominates } \Xi^2 \\ t(\gamma_2) \in \mathcal{B} \left\{ \begin{array}{l} \text{for } \Xi^1 = \Xi^2 \text{ if } \gamma_2 > \gamma_{THR} \\ \text{only if } \Xi^2 \text{ first-order stochastically dominates } \Xi^1 \\ \text{and } \gamma_2 \leq \gamma_{THR}, \end{array} \right. \end{array} \right.$$

where  $0 < \gamma_{THR} < \gamma_1$ , such that  $E_{\Xi^1, t}^{THR} [\psi_{t+1} R_{t+1}^M] = 1$ .

Proposition 4 shows the impact of the degree of competition on monetary policy. It demonstrates that for invariant probability distributions  $\vartheta$  and  $\bar{\Phi}$ , as the market becomes more competitive, only within a specific range of competition ( $0 < \gamma_1 < \gamma_2 \leq \gamma_{THR}$ ) monetary policy may remain unchanged. In section II, we analyze the case of perfect foresight and show that for low (negative) growth rates of money supply the CIA constraint does not bind whereas whenever the CIA constraint binds the growth rates of money supply are above a certain threshold. This motivates the assumption that velocity is an increasing function of money transfers. If this is the case, then for a given sequence  $\{\theta_t, M_t, \varphi_t\}_{t=1}^{\infty}$  and probability distributions  $\vartheta$  and  $\bar{\Phi}$ , as the market becomes more competitive, it is relatively more likely that the growth rate of money will increase when the CIA constraint does not bind and relatively less likely that it will increase when the CIA constraint binds. Monetary policy is optimal in the sense that given transition probabilities, the sequence of money transfers is such that it satisfies the households' and firms' optimal conditions. If the transition probabilities for money transfers are the same in the two economies with different degrees of imperfect competition then, for each economy there will be a different sequence of money transfers satisfying the optimal conditions.

## 2.4 Discussion

Propositions 1-4 show that in this simple economy, we can divide time into two regimes. In one, where the CIA constraint does not bind, we live in a Classical world where real variables are given by their optimal level (conditional on technology and the presence of monopolistic competition), prices adjust instantaneously to current shocks. In the other regime, the CIA constraint binds, and output falls below its optimal level. Households see the expected marginal utility of their money holdings falling to a very low level in the next period: perhaps they expect a high nominal price next period (or a productivity boom) and would like to increase their current consumption to lower their current marginal utility. However, they run into the CIA constraint: markets clear, but at a lower level of output and consumption. The nominal price that equates the cash-constrained demand with the supply is higher than in the classical regime. Prices are perfectly flexible, but in this Keynesian regime where the CIA constraint binds there is an effective demand effect: *the price-level itself influences the way the CIA operates.*<sup>22</sup> In essence, there are two forces operating in response to the low value of expected marginal utility per \$ next period: on the one hand, the current price rises to reduce the current marginal utility per \$, on the other hand the households are trying to increase their consumption. Since the CIA constraint prevents them from increasing consumption enough, the equilibrium market clearing nominal price is higher than it would have been in the absence of the constraint.

Why does not the price adjust downward to avoid the CIA effect and let the household raise its current consumption sufficiently? The answer is in the general equilibrium: the

---

<sup>22</sup>When the household is operating under a CIA constraint, its demand curve becomes a rectangular hyperbola rather than the normal demand.

maximum output that the economy can produce under voluntary trade is given by  $y_t^*$ . With a lower price than that given by Proposition 1(ii), the demand of the consumer would exceed the supply. With the lower prices the worker-household would be wanting to consume more than it was willing to produce through supplying its own labour. So higher current prices are consistent with both the current equilibrium in goods and labor markets, and also ensure that the inter-temporal equilibrium holds given the CIA constraint.

To make matters concrete, for illustrative purposes, let us assume that the velocity of circulation is an increasing function of  $\theta$  and  $\nu$ .<sup>23</sup> This assumption is not short of empirical support: Chiu (2007), provides evidence for the positive relationship between velocity and money while Hromcová (2008) provides evidence for the positive relationship between velocity and quality of technology in production. It follows that for a massive monetary expansion or a substantial technology improvement or a combination of the two, the CIA constraint will then bind because the agents expect that the value of money next period will be relatively smaller. As a result, they rush to spend all their money holdings the current period which increases the velocity of money to the extent that it hits its upper bound. Then, equilibrium output, consumption, work effort and profits, all depend on the current money supply as well as expectations for future money transfers, technology innovations and velocity-specific shocks.

---

<sup>23</sup>This is a special case of a velocity function where  $q_t = q(\theta_t, \nu_t)$ . Alvarez et al. (2009), provide evidence that the correlation between measures of money and velocity is negative. However, this does not necessarily imply that money supply is the dominant factor that drives velocity. This can be illustrated beyond the context of the current model. For instance, suppose technology is the dominant factor of velocity and that it affects it positively. Then, if technology deteriorates, it is reasonable to assume that the monetary authority increases the supply of money to boost the economy. In this case, even though money transfers affect velocity positively, overall money supply and velocity exhibit a negative correlation. Therefore, by just looking at correlations between money and velocity we cannot safely draw conclusions about the relationship between velocity and transfers.

In general, a higher level of technology would imply a higher welfare. In addition, for any given technology level, a binding CIA constraint implies a lower welfare than a nonbinding CIA constraint (Proposition 2). A higher level of technology would also imply a higher probability of a binding CIA constraint (under our illustrative assumption). If the CIA constraint binds, larger money transfers will, in general, increase the welfare. The monetary effect on real quantities comes through variable  $\chi$ . The smaller  $\chi$  is the higher the welfare of both consumers and firm owners. There are two channels through which money transfers can affect  $\chi$ , a direct channel in which there is a negative relationship between  $\nu$  and  $\chi$ , and an ‘indirect’ channel (through  $Z$ ) in which the direction of the relationship is not obvious because it depends on the expectations of consumers about next period’s value of money. Assuming that the direct effect of  $\nu$  on  $\chi$  dominates the indirect effect, an increase in the supply of money decreases  $\chi$  and thereby, increases welfare along a binding CIA constraint.

Note that when the monetary authority decides the transfer  $\nu_t$ , the values of  $\theta_t$  and  $\varphi_t$  are not known. For a given technology innovation and velocity-specific shock the monetary authority can increase the likelihood of a binding CIA constraint by transferring a large amount of money to the agents. A binding CIA constraint can occur even with moderate levels of technology. If such a case occurs then, according to Proposition 2, the welfare for both firm owners and consumers will deteriorate.<sup>24</sup> The monetary authority cannot entirely prevent the CIA constraint from binding because the condition that determines a binding CIA constraint does not depend only on  $\nu$  but also on  $\theta$  and  $\varphi$ , which are not under the control of the monetary authority. One may argue that the monetary authority should

---

<sup>24</sup>If the CIA constraint did not bind utility and real profits would have been higher at the same level of technology.

keep money supply constant, making zero transfers, in order to decrease the likelihood of a binding CIA constraint. Variation in the supply of money however does not necessarily make the consumers worse off. As mentioned above, there might be values of  $\nu$  (within the set of equilibria with binding CIA constraints) that make the agents better off. In the absence of velocity shocks, if there was no time lag between the decision of the transfer and the realization of technology innovation then the monetary authority could have made appropriate transfers so that the agents achieve the highest level of welfare for any realization of  $\theta$ . Furthermore, due to the time lag between decision from the monetary authority and consumers receiving the transfer as well as other possible frictions there is no guarantee that the full amount of the transfer as decided by the monetary authority will reach the consumers. Even if the monetary authority commits to a certain sequence of transfers, the uncertainty that consumers have about the transfers exists and is justified. Consequently, in a stochastic environment, the monetary authority cannot achieve with certainty a non-binding CIA constraint. In appendix B, we provide an example of welfare improving expansionary monetary policy.

### 3 The Special case of Perfect Foresight

With perfect foresight, there is no role for money as a buffer-stock: its only potential role is as a store of value and medium of exchange. Whilst this is very much a simple and special case, we can see how the framework we have set up can shed light on the possibilities contained in Propositions 1-2. For a steady-state to be possible, we have to assume that there are no shocks:  $\theta_t = \hat{\theta}$ ,  $\nu_t = 0$ ,  $\varphi_t = \hat{\varphi}$ . Then, all real and nominal variables are assumed constant.

**Definition of zero-inflation steady state:** For  $\{\theta_t = \hat{\theta}, \nu_t = 0, \varphi_t = \hat{\varphi}\}_{t=1}^{\infty}$ ,  $q_t = \hat{q}$ ,

$$\lambda_{1t} = \hat{\lambda}_1, \lambda_{2t} = \hat{\lambda}_2, y_t = \hat{y}, c_t = \hat{c}, h_t = \hat{h}, w_t = \hat{w}, M_t = \hat{M}, P_t = \hat{P} \text{ and } \pi_t = \hat{\pi}.$$

**Proposition 5** *At the zero-inflation steady state, when  $\beta \in (0, 1)$ , the CIA constraint always*

*strictly binds, with  $\hat{\lambda}_2 > 0$ ,  $\hat{q} = q^b$ , and  $\hat{P} = [1 + \phi(2 - \beta)]\hat{q}\left[\frac{\hat{M}}{\hat{\theta}}\right]$ . Then, real variables*

$$\text{are given by: } \hat{y} = \frac{nm}{1+\phi(2-\beta)}\hat{\theta}, \hat{c} = \frac{\hat{y}}{nmq^b}, \hat{h} = \frac{1}{1+\phi(2-\beta)}, \hat{\pi} = \frac{\hat{y}}{n\gamma}.$$

So, in steady-state with zero-inflation no one will want to hold money at the end of the period. Since consumption is constant, the discounted marginal utility of consumption next period is always less than current marginal utility, so that with a zero rate of return on money holdings, a \$ today will always buy more utility than a \$ tomorrow. This implies that the velocity of money will always be at its upper bound. The level of output in steady-state is less than would occur when the CIA constraint is nonbinding, but only very slightly. The ratio of steady-state output and employment to the efficient level is:

$$\frac{\hat{y}}{y^*} = \frac{\hat{h}}{h^*} = \frac{1 + \phi}{1 + \phi + \phi(1 - \beta)} < 1.$$

Clearly, if we are dealing with quarterly data, then  $\beta = 0.995 \approx 1$  and the ratio is close to unity. For example, with  $\phi = 1$ , this level of discounting gives us a ratio of 0.9975. This slight inefficiency is caused by the distortion of the work-leisure decision that occurs when the CIA constraint binds: the consumption-leisure MRS is less than the real wage, so that the supply of labor is lower (for a given level of consumption). To see why the CIA constraint needs to strictly bind, assume instead that it was weakly binding with  $\hat{q} = q^b$  and  $\hat{\lambda}_2 = 0$ : in this case, the household could increase its utility by bringing forward some consumption

(since  $\beta < 1$ ) and hence, the steady-state is only sustainable with  $\hat{\lambda}_2 > 0$ .<sup>25</sup>

Now we consider the general case where consumer-households and firm-owners perfectly foresee the evolution of the economic fundamentals  $\{\varphi_t, \theta_t, \nu_t\}_{t=1}^{\infty}$ . Let  $g_{jt} = (j_t/j_{t-1}) - 1$  denote the growth rate of variable  $j$  at time  $t$ . We turn first to the growth rate of the nominal money supply.

**Proposition 6** *In the economy with perfect foresight: (i) when  $t \in \mathcal{B}$  then  $g_{Mt+2} > \beta - 1$  but the reverse does not always hold, and (ii) when  $g_{Mt+2} \leq \beta - 1$ , then  $t \in \mathcal{NB}$  but the reverse does not always hold.*

Cooley and Hansen (1989, p. 736), argue that in their model  $g_{Mt+2} > \beta - 1$  is a sufficient condition for the CIA constraint to be always binding. In our model,  $g_{Mt+2} > \beta - 1$  is not a sufficient condition because velocity is allowed to vary. Note that conditions  $g_{Mt+2} > \beta - 1$  and  $g_{Mt+2} \leq \beta - 1$  can be rewritten as  $g_{\nu t+1} > [(M_t + \nu_t)/\nu_t](\beta - 1) - 1$  and  $g_{\nu t+1} \leq [(M_t + \nu_t)/\nu_t](\beta - 1) - 1$ , respectively.<sup>26</sup> The two conditions can also be written as  $\nu_{t+1} > (M_t + \nu_t)(\beta - 1)$  and  $\nu_{t+1} \leq (M_t + \nu_t)(\beta - 1)$ , respectively. Since  $\beta \in (0, 1)$ , the latter shows that both binding and nonbinding CIA constraints are consistent with both positive and negative money transfers.

For any  $T \in \mathbb{Z}_+ \cup \{0\}$ , let us define sets  $\widetilde{\mathcal{B}}(T) = \{t \geq T + 1 : t \in \mathcal{B}\}$  and  $\widetilde{\mathcal{NB}}(T) = \{t \geq T + 1 : t \in \mathcal{NB}\}$  such that  $\mathcal{B}(T) \cap \widetilde{\mathcal{B}}(T) = \emptyset$ ,  $\mathcal{B}(T) \cup \widetilde{\mathcal{B}}(T) = \mathcal{B}$ ,  $\mathcal{NB}(T) \cap \widetilde{\mathcal{NB}}(T) = \emptyset$ ,  $\mathcal{NB}(T) \cup \widetilde{\mathcal{NB}}(T) = \mathcal{NB}$ ,  $\mathcal{B}(0) = \emptyset$ ,  $\mathcal{NB}(0) = \emptyset$ ,  $\widetilde{\mathcal{B}}(0) \equiv \mathcal{B}$  and  $\widetilde{\mathcal{NB}}(0) \equiv \mathcal{NB}$ . In addition, let us define the following auxiliary sets  $\mathcal{M}^{\leq}(T) = \{t \geq T + 1 : g_{Mt+2} \leq \beta - 1\}$

<sup>25</sup>The Euler equation implies that  $\beta = \hat{\lambda}_1/(\hat{\lambda}_1 + \hat{\lambda}_2)$  which holds only if  $\hat{\lambda}_2 > 0$  since  $\beta < 1$ .

<sup>26</sup>See proof of proposition 6.

and  $\mathcal{M}^>(T) = \{t \geq T + 1 : g_{Mt+2} > \beta - 1\}$ . Then, using Proposition 6 and its proof we can define the mutually exclusive sets  $\widetilde{\mathcal{NB}}_1(T) = \{t \in \mathcal{M}^{\leq}(T) : t \in \mathcal{NB}\}$  and  $\widetilde{\mathcal{NB}}_2(T) = \{t \in \mathcal{M}^>(T) : q_{t+1} < q_t \leq q^b, t \in \mathcal{NB}\}$  such that  $\widetilde{\mathcal{NB}}_1(T) \cap \widetilde{\mathcal{NB}}_2(T) = \emptyset$  and  $\widetilde{\mathcal{NB}}_1(T) \cup \widetilde{\mathcal{NB}}_2(T) = \widetilde{\mathcal{NB}}$ .<sup>27</sup> The second part of Proposition 6 indicates that if the growth rate of money is always less or equal than  $\beta - 1$  from any  $t^* \in \mathbb{Z}_+$  onwards, the CIA constraint will never bind again. The case of  $\mathcal{M}^>(T) = \emptyset$  or  $g_{Mt+2} \leq \beta - 1$  with  $\beta \in (0, 1)$  for all  $t \geq T + 1$  holds only if  $g_{Mt+2} > -1$  for all  $t \geq T + 1$ .<sup>28</sup> Therefore, when  $\mathcal{M}^>(T) = \emptyset$ , it must be that  $-1 < g_{Mt+2} \leq \beta - 1$ . Proposition 6(i) also indicates that it is possible that  $g_{Mt+2} > \beta - 1$  when  $t \in \widetilde{\mathcal{NB}}(T)$  which occurs when  $t \in \widetilde{\mathcal{NB}}_2(T)$ .

**Corollary 3** *In the economy with perfect foresight, for any  $\beta \in (0, 1)$  and any  $T \in \mathbb{Z}_+ \cup \{0\}$  :*  
(i)  $\emptyset \subseteq \widetilde{\mathcal{B}}(T)$  and (ii)  $\emptyset \subseteq \widetilde{\mathcal{NB}}(T)$ .

Corollary 3 signifies that there are sequences of  $\{\theta_t, \nu_t, \varphi_t\}$  such that (i) the CIA constraint never binds and (ii) the CIA constraint always binds. For  $\widetilde{\mathcal{B}}(T) = \emptyset$ , the sequence of money transfers,  $\{\nu_t\}_{t=T+1}^{\infty}$ , can be complemented by sequences of velocity and technology innovations,  $\{\theta_t, \varphi_t\}_{t=T+1}^{\infty}$ , such that  $\widetilde{\mathcal{NB}}_2(T) \neq \emptyset$ .

**Proposition 7** *In the economy with perfect foresight, there are unique values for  $P_t, y_t, c_t, h_t$  and  $\pi_t$  such that*

$$P_t = (1 + \xi_t) q_t \left[ \frac{M_t + \nu_t}{\theta_t} \right] \text{ with } \begin{cases} \xi_t = \chi_t, q_t = q^b \text{ and } \chi_t > \phi \text{ when } t \in \mathcal{B} \\ \xi_t = \phi, q_t \leq q^b \text{ and } \chi_t \leq \phi \text{ when } t \in \mathcal{NB} \end{cases},$$

<sup>27</sup>These relationships do not necessarily hold in the stochastic model.

<sup>28</sup>If  $g_{Mt+2} < -1$ , the positivity of money supply will be violated.



$$y_t = \frac{nm}{1 + \omega_t} \theta_t, c_t = \frac{y_t}{nmq^b}, h_t = \frac{1}{1 + \omega_t} \text{ and } \pi_t = \frac{y_t}{n\gamma},$$

$$\text{where } \omega_t = \begin{cases} \chi_t \text{ for } t \in \mathcal{B} \\ \phi \text{ for } t \in \mathcal{NB} \end{cases} \quad \text{with } \chi_t = \begin{cases} \frac{\phi}{\beta} (1 + g_{Mt+2}) \text{ for } t + 1 \in \mathcal{B} \\ \frac{\phi}{\beta q^b} (1 + g_{Mt+2}) \text{ for } t + 1 \in \mathcal{NB} \end{cases}.$$

Since  $\chi_t > \phi$  when  $t \in \mathcal{B}$ , for a given technology level, a nonbinding equilibrium Pareto dominates a binding equilibrium in terms of welfare for both firm-owners and household-consumers (Proposition 2). Note that if the CIA constraint binds in period  $t$  but is expected to be nonbinding in  $t + 1$ , the upper bound on the  $q^b$  enters into  $\chi_t$ . This implies that the degree of imperfect competition matters: a higher markup implies a higher  $q^b$ , which implies a higher output (among binding equilibria for a given technology level). A monetary authority which is interested in maximizing welfare, will choose the flow of money in every period such that the CIA constraint never binds. Corollary 3 indicates that this is possible since the binding set can be an empty set.

**Corollary 4** *In the economy with perfect foresight, for any  $t \in \mathcal{NB}$ ,  $g_{ct+1} \leq g_{\theta t+1}$ .*

**Proposition 8** *In the economy with perfect foresight: (i) when  $t \in \mathcal{NB}$  then  $g_{pt+1} \geq$*

$$\frac{\beta}{1+g_{\theta t+1}} - 1, \text{ but the reverse does not always hold, and (ii) when } g_{pt+1} < \frac{\beta}{1+g_{\theta t+1}} - 1 \text{ then}$$

*$t \in \mathcal{B}$ , but the reverse does not always hold.*

Corollary 4 indicates that whenever the CIA is nonbinding, the growth rate of consumption next period cannot be greater than the rate of improvement in technology. As shown in the proof of Proposition 8, when the CIA constraint binds, it is perfectly possible that the

growth rate of consumption next period is greater than the rate of improvement in technology. This occurs because of an increase in work effort which boosts further the growth rate of production. In this case, the gross inflation rate is smaller than  $\beta/(1 + g_{\theta t})$  due to the fact that  $\chi_{t-1} \geq \chi_t$ . From Proposition 7, the latter also implies that not only output and consumption grow faster than the rate of improvement in technology but also real profits. As Proposition 2 (ii) indicates, since  $\chi_{t-1} \geq \chi_t$  neither household-consumers nor firm-owners are worse-off in the transition from period  $t - 1$  to period  $t$ .

**Corollary 5** *In the economy with perfect foresight,  $(t-1) \in \mathcal{NB}$  if and only if  $g_{pt} = \frac{\beta}{1+g_{ct}} - 1$ , otherwise  $(t-1) \in \mathcal{B}$ , and  $g_{pt} > \frac{\beta}{1+g_{ct}} - 1$  : (i) If  $(t-1) \in \mathcal{NB}$  and  $t \in \mathcal{NB}$  then,  $g_{pt} = \frac{\beta}{1+g_{\theta t}} - 1$  but the reverse does not always hold; (ii) If  $(t-1) \in \mathcal{NB}$  and  $t \in \mathcal{B}$  then,  $g_{pt} > \frac{\beta}{1+g_{\theta t}} - 1$  but the reverse does not always hold; (iii) If  $(t-1) \in \mathcal{B}$  then,  $g_{pt} > \frac{\beta}{1+g_{\theta t}} - 1$  or  $g_{pt} \leq \frac{\beta}{1+g_{\theta t}} - 1$  for any  $t$ .*

Corollary 5 (i) indicates that if the CIA constraint does not bind in two consecutive periods, the growth rate of the price level is a function only of the growth rate of technology. Under those circumstances, as technology improves prices must be falling. Corollary 5 (i) also demonstrates that if technology remains unchanged when the CIA constraint does not bind in two consecutive periods, prices decline at the rate  $1 - \beta$ .

**Corollary 6** *In the economy with perfect foresight, (i) If  $(t-1) \in \mathcal{NB}$  and  $t \in \mathcal{NB}$  then,  $g_{Mt+1} = \frac{\beta}{1+g_{qt}} - 1$  but the reverse does not always hold; (ii) For any bundle  $(t-1)$  and  $t$  other than  $\{(t-1) \in \mathcal{NB}, t \in \mathcal{NB}\}$ ,  $g_{Mt+1} > \frac{\beta}{1+g_{qt}} - 1$  or  $g_{Mt+1} \leq \frac{\beta}{1+g_{qt}} - 1$ .*

Money growth on the other hand, along two consecutive nonbinding CIA constraints, depends on the growth rate of velocity which is a function of the money transfer, technology and velocity innovation.<sup>29</sup> For,  $\mathbb{Z}_+(T) = \{T + 1, T + 2, \dots, \infty\}$ , it is also useful to partition time into periods of positive growth rates of technology and times of non-positive growth rates of technology:  $\mathcal{G}^+(T) = \{t \in \mathbb{Z}_+(T) : g_{\theta_t} > 0\}$  and  $\mathcal{G}^-(T) = \{t \in \mathbb{Z}_+(T) : g_{\theta_t} \leq 0\}$  such that  $\mathcal{G}^+(T) \cup \mathcal{G}^-(T) = \mathbb{Z}_+(T)$ . Corollary 5 indicates that for any  $T \in \mathbb{Z}_+ \cup \{0\}$  and  $\beta \in (0, 1)$  such that  $\widetilde{\mathcal{NB}}(T) = \mathbb{Z}_+(T)$ , (i) if  $\mathcal{G}^+ = \mathbb{Z}_+(T)$  then,  $g_{pt} < 0$  for all  $t$  and (ii) if  $\mathcal{G}^- = \mathbb{Z}_+(T)$  then  $g_{ct} < 0$  for all  $t$ .

### 3.1 Inflationary steady-states and the optimal rate of inflation

We are now in a position to analyze non-zero-inflation steady-states, which we define as follows:

**Definition of the inflationary steady-state** For  $\{\theta_t = \widehat{\theta}, \nu_t = 0, \varphi_t = \widehat{\varphi}\}_{t=1}^{\infty}, q_t = \widehat{q}$ ,

$$\lambda_{1t} = \widehat{\lambda}_1, \lambda_{2t} = \widehat{\lambda}_2, y_t = \widehat{y}, c_t = \widehat{c}, h_t = \widehat{h}, w_t = \widehat{w}, \pi_t = \widehat{\pi}, g_{Mt} = g_{pt} = \widehat{g}_p \text{ for all } t.$$

In the inflationary steady-state, money growth equals steady-state inflation and all real variables are constant.<sup>30</sup> The presence of steady-state inflation means that there is an inflation tax: holding money to finance transactions can incur a cost as prices are rising. This was of course implicit in Propositions 6-8. We can now state the following:

<sup>29</sup>If velocity is a continuously differentiable function in all arguments then,  $g_{qt} = \varepsilon_t^{q,\varphi} g_{\varphi t} + \varepsilon_t^{q,\nu} g_{\nu t} + \varepsilon_t^{q,\theta} g_{\theta t}$  where  $\varepsilon_t^{q,i}$  is the elasticity of velocity with respect to variable  $i$  and  $g_{\nu t} = g_{Mt+1}(1 + g_{Mt})/g_{Mt} - 1$ . Then, using corollary 6(i), we can express  $g_{Mt+1}$  as a function of  $g_{Mt}, g_{\varphi t}, g_{\theta t}$  and elasticities.

<sup>30</sup>In fact we need not assume that the velocity of money is constant: if we allowed for a constant growth rate of the velocity  $-1 < g_q \leq 0$ , then the inflationary steady state would become  $g_q + g_{Mt} = g_{pt} = \widehat{g}_p$ .

**Proposition 9** Consider an inflationary steady-state:

(i) if  $\hat{g}_p > \beta - 1$ , then the CIA constraint always strictly binds, with real variables given by Proposition 7.

(ii) if  $\hat{g}_p = \beta - 1$ , then the CIA constraint never binds and the real variables are at the efficient levels defined in Proposition 7.

(iii) if  $\hat{g}_p < \beta - 1$ , then no steady-state exists.

Proposition 9 (i) states that output is decreasing with the level of steady-state inflation: a higher inflation tax increases the distortion induced by the CIA constraint. If we define the welfare corresponding to a constant level of inflation as the per period flow of utility in the corresponding steady-state (and zero if there is no steady-state) then it follows that:

**Corollary 7** *The optimal steady-state inflation rate is  $\hat{g}_p = \beta - 1$ .*

This result is reminiscent of Friedman's (1969) argument that the optimal inflation rate is negative. Friedman adopted a money-in-the-utility-function framework: a negative rate of inflation provides a return on money holdings sufficient for households to hold the optimum quantity of real balances. Here, the argument is somewhat different. The CIA constraint distorts the economy when it binds strictly: when  $\lambda_2 > 0$  the labor supply is diminished and output and consumption are below their efficient levels. The optimum inflation rate provides a positive return to holding money which exactly outweighs the effect of discounting and allows for constant consumption without the CIA binding. This removes the distortion induced by the CIA constraint and allows the economy to produce the efficient level of output with the MRS equated to the real wage. The Lemma and Corollary 7 can be generalised to

allow for steady state growth in output and productivity using the conditions in Corollary 5.

## 4 Capital and Bonds

Thus far, we have abstracted from the presence of capital accumulation and assumed that money is the only asset in the economy. We could introduce capital into our framework by assuming that it is owned by the worker-household and rented to the entrepreneurs. Even in the presence of capital, money still contains a savings-based (or precautionary demand) component. In other words, the CIA constraint can be nonbinding even in the presence of capital. To show this, let us assume that capital is a factor of the production function which can be written as  $x(h_t, k_t; m, \theta_i)$ . The extended production function satisfies the usual properties:  $x_k > 0$  and  $x_{kk} \leq 0$  where  $x_k$  and  $x_{kk}$  denote the first and second derivatives of  $x(\cdot)$  with respect to  $k$ . Moreover, we assume that the agents of this economy accumulate capital which depreciates at rate  $\delta$ . Without loss of generality we also assume that the price of capital is the same as the price of consumption. Then, the euler condition for capital is

$$E_t \left[ \left( \frac{\beta u_c(c_{t+1})}{u_c(c_t)} \right) [(1 - \delta) + x_k(k_{t+1}, \cdot)] \right] = 1 - \lambda_{2t} \frac{P_t}{u_c(c_t)} + E_t \left[ \left( \frac{\beta \lambda_{2t+1} P_{t+1}}{u_c(c_t)} \right) [(1 - \delta) + x_k(k_{t+1}, \cdot)] \right]. \quad (13)$$

It follows that

$$E_t \left[ \left( \frac{\beta u_c(c_{t+1})}{u_c(c_t)} \right) [(1 - \delta) + x_k(k_{t+1}, \cdot)] \right] \begin{cases} > 1 \text{ for } t \in \mathcal{NB} \\ < 1 \text{ or } \geq 1 \text{ for } t \in \mathcal{B} \end{cases}, \quad (14)$$

while (11) is the corresponding condition for money.<sup>31</sup> Conditions (13) and (14), demonstrate that when there is precautionary demand for money, investment demand is low which means that next period stock of capital is low, and as a result the marginal product of capital is high. Subsequently, the return of capital, measured in utility units, is expected to increase. In this case, condition (13) indicates that the left hand-side of (14) is strictly greater than unity because there is a non-zero possibility that the CIA constraint will bind next period. This demonstrates that even in the presence of capital, money can be used as store of value. If household-consumers knew with absolute certainty that the CIA constraint next period is nonbinding then, they would have increased investment demand to the point that the expected utility return of capital equals the expected utility return of money.

Let us consider the case of a non-zero inflation steady state with perfect foresight. If we had included capital accumulation, then the return to savings would be equal to the reciprocal of the discount rate: the optimal inflation rate (Corollary 7) would mean that money would have the same rate of return as capital. The steady-state relationship would give a return to capital of  $(1 - \delta) + x_k(k, \cdot) = 1/\beta$ , where  $x_k(k, \cdot)$  is the steady-state marginal product of capital. The real return to holding one \$ is  $1/(1 + \hat{g}_p) = 1/\beta$ .

What would happen if we included interest-bearing nominal assets such as bonds? If we assume the usual arbitrage condition between bonds and capital, these will both offer the same real-return on savings equal to the (expected) marginal return of capital. This will not alter the opportunity cost of holding money from the case of just capital and hence will not eliminate the precautionary-demand for money in the presence of uncertainty. This

---

<sup>31</sup>If capital has a different price than consumption then the left-hand-side of (13) becomes  $E_t\left[\left(\frac{\beta u_c(c_{t+1})}{u_c(c_t)}\right) \left[\frac{Q_{t+1}}{Q_t} (1 - \delta) + \frac{x_k(k_{t+1}, \cdot)}{Q_t}\right]\right]$  where  $Q_t$  denotes the relative price of capital.

conclusion depends on how liquid we make bonds. If we were to make bonds perfectly liquid, then in effect bonds would become an interest bearing form of money and would eliminate the need for non-interest bearing money. Alvarez et al. (2009) make an intermediate assumption and allow for bonds to be liquid part of the time and allowing the CIA constraint to be nonbinding. Insofar as bonds are not perfectly liquid, there is still a potential role for money over and above the transactions demand.

## 5 Conclusion

The paper lays out a simple framework in a general equilibrium model with money where the final good is produced by monopolistic firms via labor services provided by risk-averse workers. Preferences over consumption are Linear Homothetic and money is introduced by means of an occasionally binding cash-in-advance constraint. Those preferences generate Marshallian demands enabling any combination of equilibrium number of firms and demand elasticity. Money is a liquidity vehicle which has real effects on the economy without requiring the presence of other real assets or any sort of price rigidity. The velocity of money is a very general function which corresponds to the monetary policy regime, production technology, institutional developments and payment habits.

The proportion of periods in which the CIA constraint is binding depends, among others, on the degree of imperfect competition. We demonstrate that the CIA constraint cannot bind for a lower proportion of the time in a more competitive economy. We show that the degree of imperfect competition directly affects the distribution of consumption across workers and firm owners, and in conjunction with the CIA constraint, the level of aggregate

output and work effort. We enter a Keynesian world only when the expected value of money decreases below a critical value and velocity reaches its maximum value. The latter is the case of a binding CIA constraint which is a welfare inferior outcome for both the workers and the firm owners as it delivers lower utility and lower real profits for any given level of technology. We argue that even though the monetary authority can increase the probability of a binding CIA constraint by increasing money supply, expansionary monetary policy can be welfare improving. We demonstrate that when the CIA constraint binds there are cases where prices respond sluggishly to changes in money supply. We also show that with perfect foresight, there is an optimal negative steady-state inflation rate as in Friedman (1969) and consider how the introduction of capital markets fit into the framework.

## References

- [1] Adão, B., Correia, I., Teles, P., 2011. Unique Monetary Equilibria with Interest Rate Rules. *Review of Economic Dynamics* 14, 432–442
- [2] Alvarez, F., Atkeson, A., Edmond, C., 2009. On the Sluggish Response of Prices to Money in an Inventory Theoretic Model of Money Demand. *Quarterly Journal of Economics* CXXIV (3), 911-967
- [3] Benassy, J-P., 1975. NeoKeynesian disequilibrium theory in a monetary economy. *Review of Economic Studies* 42, 503-23.
- [4] Brown, J.R., Martinsson, G., Petersen, B.C., 2013. Do financial constraints matter for R&D? *European Economic Review* 56, 1512-1529



- [5] Burkhard, H., Maußner, A., 2015. The Cash-In-Advance Constraint In Monetary Growth Models With Labor Market Search. *Macroeconomic Dynamics* 19, 144-166
- [6] Chamley, C., Polemarchakis, H., 1984. Assets, General Equilibrium and the Neutrality of Money. *Review of Economic Studies* 51 (1), 129-138
- [7] Chen, H.J., Li, M.C, 2008. Chaotic Dynamics in a Monetary Economy with Habit Persistence. *Journal of Economic Behavior and Organization* 65, 245–260
- [8] Chiu, J., 2007. Endogenously Segmented Asset Market in an Inventory Theoretic Model of Money Demand. Bank of Canada Working Paper 2007-46
- [9] Clower, R., 1965. The Keynesian counter-revolution: a theoretical appraisal. Reprinted in *Monetary Theory*. Ed. R. Clower, 1969, Harmondsworth: Penguin
- [10] Cooley, T.F., Hansen, G.D., 1989. The Inflation Tax in a Real Business Cycle Model. *American Economic Review* 79 (4), 733-748
- [11] Coto-Martinez, J., Dixon, H., 2003. Profits, markups and entry: fiscal policy in an open economy. *Journal of Economic Dynamics and Control* 27, 573-597
- [12] Datta, B., Dixon, H., 2000. Linear-homothetic preferences. *Economics Letters* 69, 55-61
- [13] Datta, B., Dixon, H., 2001.  $\omega$ -Homothetic preferences: Theory and applications. *Scottish Journal of Political Economy* 48 (2), 148-169
- [14] Devereux, M., Siu, H., 2007. State dependent Pricing and Business Cycle Asymmetries. *International Economic Review* 47, 281-310

- [15] Díaz-Giménez, J., Giovannetti, G., Marimon, R., Teles, P., 2008. Nominal Debt as a Burden on Monetary Policy. *Review of Economic Dynamics*, 11 (3), 493-514
- [16] Dixit, A.K., Stiglitz, J.E., 1977. Monopolistic Competition and Optimum Product Diversity. *American Economic Review* 67 (3), 297-308
- [17] Evans, G., Honkapohja, S., Marimon, R., 2007. Stable Sunspot Equilibria in a Cash-in-Advance Economy. *B.E Journal of Macroeconomics*, Vol 7: Iss 1 (Adv.), Article 3
- [18] Friedman, M., 1969. The Optimum Quantity of Money. In *The Optimum Quantity of Money and Other Essays*, Chicago: Aldine
- [19] Giraud, G., Tsomocos, D., 2010. Nominal Uniqueness and Money Non-neutrality in the Limit-Price Exchange Process. *Economic Theory*, 45(1), 303-348
- [20] Grandmont, J-M, Younes Y., 1972. On the role of money and the existence of monetary equilibrium. *Review of Economic Studies*, 39, 355-372.
- [21] Hall, B.H., Lerner, J., 2010. In: Hall B.H., and Rosenberg N., (Eds.), *The Financing of R&D and Innovation*, *Handbook of the Economics of Innovation*, Elsevier-North Holland
- [22] Hromcová, J., 2008. Learning-or-Doing in a Cash-in-Advance Economy with Costly Credit. *Journal of Economic Dynamics and Control* 32, 2826-2853
- [23] Lagos, R., Rocheteau, G., 2005. Inflation, Output, and Welfare. *International Economic Review*, 46 (2), 495-522

- [24] Lagos, R., Wright, R., 2003. Dynamics, Cycles and Sunspot Equilibria in ‘Genuinely Dynamic, Fundamentally Disaggregative’ Models of Money. *Journal of Economic Theory*, 109 (2), 156-171
- [25] Leijonhufvud, A., 1968. *On Keynesian economics and the economics of Keynes*. Milton Keynes: Open University Press.
- [26] Malinvaud, E., 1977. *The Theory of Unemployment Reconsidered*. Oxford: Blackwell.
- [27] Patinkin, D., 1956. *Money, Interest and Prices: An integration of monetary and value theory*. New York, Harper Row
- [28] Ragot, X., 2014. The case for a financial approach to money demand. *Journal of Monetary Economics* 62, 94-107
- [29] Svensson, L.E.O., 1985. Money and Asset Prices in a Cash-In-Advance Economy. *Journal of Political Economy* 93 (5), 919-944
- [30] Telyukova, I.A., Visschers, L., 2011. Precautionary Demand for Money in a Monetary Business Cycle Model. UCSD working paper
- [31] Woodford, M., 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press