# A Generative Model For Predicting Terrorist Incidents

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## ABSTRACT

A major concern in coalition peace-support operations is the incidence of terrorist activity. In this paper, we propose a generative model for the occurrence of the terrorist incidents, and illustrate that an increase in diversity, as measured by the number of different social groups to which that an individual belongs, is inversely correlated with the likelihood of a terrorist incident in the society. A generative model is one that can predict the likelihood of events in new contexts, as opposed to statistical models which are used to predict the future incidents based on the history of the incidents in an existing context. Generative models can be useful in planning for persistent Information Surveillance and Reconnaissance (ISR) since they allow an estimation of regions in the theater of operation where terrorist incidents may arise, and thus can be used to better allocate the assignment and deployment of ISR assets. In this paper, we present a taxonomy of terrorist incidents, identify factors related to occurrence of terrorist incidents, and provide a mathematical analysis calculating the likelihood of occurrence of terrorist incidents in three common real-life scenarios arising in peace-keeping operations.

**Keywords:** terrorism models, generative approaches, mathematical sociology, group modeling

## 1. INTRODUCTION

Creating mathematical models that characterize the behavior of societies can provide insights into the fundamental characteristics of different groups making up the society. Although such models, whether analytical or computational, have limited fidelity to the many complexities of an actual society, they can still identify many interesting characteristics of societies, and provide insights into the properties that are likely to change (mutable properties) as well as ones not likely to change (immutable properties) as the society evolves. In this paper, we propose a model that can provide insights into the factors that may affect the likelihood of terrorist incidents in a society.

Terrorism is often used by insurgent groups as a tool of asymmetric warfare. The unique characteristic of terrorist activities is that it is directed towards soft targets, i.e. civilians or infrastructure elements that do not have many defenses. Since it is virtually impossible to protect every civilian and infrastructure element in the society, a terrorist can always find a suitable soft target. Eliminating terrorism completely in any society by providing adequate security is not possible within a democratic structure guaranteeing civil liberties, and it is questionable whether total defense is viable even under a totalitarian regime.

When complete protection is not a viable option, we need to look for other factors that may be used to reduce the occurrence of terrorist incidents. These factors could be social, political or military. In this paper, we examine one specific social factor - the prevalence and correlation among the social networks that exist within a community. Understanding the relationship between social networks and terrorist activities can provide options for dealing with the threat of terrorism. Such options can be very useful for coalition forces that are faced with the task of nation building or peace-keeping after the cessation of active warfare. The focus of this research on the social networks within a community represents a novel approach to the study of terrorist incidents, and one that is not covered in earlier studies (e.g., [1,2,3,4,5]).

A mathematical or computational model for understanding the impact of social networks on terrorism can be useful for providing a generative approach for understanding societies [1]. In the field of Statistics, a generative model provides a way to create data that conforms to a desired distribution, in contrast to discriminative models that allow one to figure out the distribution to which a data set belongs. Using the same analogy, we can identify two type of approaches to understand the behavior of a group, generative approaches and discriminative approaches [6].

A generative approach for understanding social behavior enables us to generate a group (at least in computer simulations or mathematical models) that will demonstrate a desired set of properties, and lead us to understand the properties of groups with given attributes. A discriminative model would allow us to observe a group and infer the properties of the group from those observations. The generative approach, like the one we describe in this paper, would allow us to postulate hypotheses about group behavior, generate groups with a given set of properties, and understand the evolution of these types of groups. An example of the alternative approach, the discriminative model, can be found in [7], which looks at data for terrorist incidents and finds patterns in them.

Both the generative and discriminative approaches have complementary strengths. A discriminative approach is based on actual data, and thus reflects the real-world situation more accurately. At the same time, the discriminative approach can only infer patterns and knowledge present in the specific data sample that is available, and it is limited in its usefulness in environments where different patterns or other factors are at play. The generative model can be independent of a specific data set, provide a broader perspective, and be applicable across a wider set of assumptions. However, it may only result in a simplified approximation of reality, and thus may not be a very accurate representation of real-life. While the discriminative approach has a higher fidelity to the reality, good generative approaches can be more useful in making strategic decisions in new environments.

In this paper, we propose a generative model for understanding the occurrence of terrorist incidents, and discuss the insights that can be obtained from such a generative model. We begin this paper with a taxonomy section in which we describe what we mean by terrorism activities, as well as define other terms that we will use in this paper. That is followed by a discussion of the aspects of society one which we focus in order to develop a model of the occurrence of terrorist incidences. We provide generative mathematical models for terrorist incidents and discuss the implications of our modeling activities in attempts to reduce terrorist activities.

## 2. TAXONOMY

In this section, we define the different terms used within this paper. While some of the terms are borrowed from common usage, it is important to define those terms precisely in the context of the generative model that we use. Our general approach involves defining a set of groups and the interrelations between them, an approach which is commonly adopted in the wider related literature (e.g., [8]).

A ***society*** consists of a collection of individuals that live together in a geophysical region. An ***individual*** may have various roles within the society, and can take a variety of actions. An individual may belong to one or more ***groups,*** each group being a collection of individuals with some shared characteristics. A society is the collection of all individuals. From the perspective of Set Theory, individuals are unique elements, groups are sets of individuals, and society is the Universal set which subsumes all groups and individuals. An individual can belong to one, two or more groups, and some individuals may not belong to any group at all. Individuals are the only entities that can take any action which causes changes in the attributes of other individuals in the society. The society and individuals exist within a ***geophysical region*** that defines a physical space within which individuals move and influence other individuals.

We assume that the society has at least one group which is the ***state group***. The state group consists of those individuals who are responsible for ensuring that no acts of violence are conducted by individuals against each other. The state group in itself may be made up as a coalition of different groups. If we map the model to the real world, the state group would consist of the individuals that make up the government in an independent stable country. In other areas, the state group may consist of a coalition of forces, e.g. the U.S. and UK armed forces with a local government trying to maintain peace and stability in a disturbed region. The goal of the state group is to prevent violence among individuals in the society. An external group refers to a group that exists clandestinely without the approval of the state group. An external group could not become an internal group without the consent of the state except through the employment (or threat of employment) of violence.

We further assume that the society has at least one external group which we call as the ***terrorist group***. The terrorist group consists of individuals who want to undertake a violent action against other individuals in society. The goal of this paper is to model the behavior of the terrorist group, and of individuals within that group. In addition to the terrorist groups, individuals may belong to other groups, which we define as ***social groups***. Social groups are groups that exist within society, with either explicit or implicit consent of the state group, which exist for peaceful functions and promote activities that do not challenge state authority, and whose activities may be promoted or at the very least, be tolerated by the state group.

Many different types of social groups exist within a society. As an example, an individual in the hypothetical country of Holistan [9] may be a civil engineer working for an oil exploration company, ethnically from the Pashtun tribe and an adherent of the religion of Shia Islam. In this case, the individual belongs to four groups, the groups being defined by profession, employer, tribe and religion respectively. Each of these groups would define a set of bonds that an individual shares with a large number of other individuals.

There is no generally accepted definition of terrorism or terrorist. Therefore, it is important that we define what we mean by a terrorist incidence in the context of this paper. Terrorism is related to an individual taking violent action against other individuals in the society. Violence, or the threat of violence, and control of the means of violence, have been central to the rise of large social groups and states, and the repression of high levels of violence within a large social group or state [10]. However, not all acts of violence equate to terrorism.

Some acts of violence are conducted by individuals when they have disputes with other individuals in society, e.g. a dispute over property can result in fights, dispute over religious practices can result in riots and destruction of property, or a family quarrel can result in a murder. These types of acts of violence are excluded from our definition of terrorist incidents. Similarly, some acts of violence may be undertaken by the state group. We exclude these acts of violence from consideration in this paper, and consider these as repression or state-sponsored violence, as opposed to terrorism. Similarly, state sponsored violence against other state, i.e. war or battles are excluded from our definition of terrorism, since by definition, the entire society is under the control of a single state group.

Our definition of terrorism, which broadly follows Hoffman [11], equates 'terrorism' with the use of violence by non-state external groups against individuals within the society. In our model, terrorist incidents are acts of violence intended to create fear (and often to demonstrate state authorities' lack of control) for (primarily) political, ideological or religious reasons, which are undertaken by non-state external groups in geophysical regions that the terrorist group does not control.

The violence by the terrorist group can target any individuals, including those in the state group as well as those in society, but not explicitly part of the state group. In the context of our paper, this implies that a terrorist incident is an activity conducted by an individual within the terrorist group. In general, there may be more than one terrorist groups active within the society, although we only consider a single individual terrorist group in this paper.

## 3. MODELING SOCIETY FOR TERRORIST INCIDENTS

We model the society as a collection of several groups, and want to understand the resulting properties of a stable society in the presence of multiple groups. The individual factors that lead to group behavior are not explicitly considered in this paper, but are given attention in [12]. The set of groups we model includes a minimum of one state group and one terrorist group. Since the state has control of a specific geophysical area, one of the attributes that needs to be modeled is that of the distribution of individuals in a geophysical region. We implicitly assume that individuals with social network associations are more likely to spatially mix, evidence for which is known in the wider population [14].

There are several ways of modeling the *geophysical spread* of individuals in the society. One approach would be to model the geophysical region as a planar shape of specific size, e.g. a unit circle, and assume that individuals in the society are spread around this region with some statistical distribution. Another option would be to model the society as a population spread with some density (or density distribution) on an infinite plane. While the infinite plane distribution may sound unrealistic, it may be a good enough approximation for societies in which the size of population is large. In many societies, population would run into millions of people, and the infinite plane model may both provide a decent approximation and a mathematical tractable system to analyze. Another approach to model geophysical distribution would be to consider individuals as located on a rectangular grid (or any other grid with a regular structure), and consider either a finite sized grid or an infinite sized grid. In each of these models, an individual is assigned a fixed location, which can be interpreted as their residence or place to stay. The location would determine factors such as the feasible areas within which an individual can travel. In addition to a uniform distribution of population over the region, one can also explore non-uniform distribution of individuals as a way to model society.

In order to understand the occurrence of terrorist incidents, we characterize each individual in the society with four attributes. Each of these attributes for an individual can be determined by the membership of the individual in different social groups, terrorist group, the nature of the terrorist incident, or the location of the terrorist incident. Several of these attributes build on concepts derived from social exchange theory (e.g., rewards, costs, and alternative opportunities) that are viewed as shaping human, social, decision-making (e.g., [14] ). These four attributes are:

1. *Aggression Tendency*: a measure of the likelihood that the individual is motivated to conduct a terrorist incident. This probability can be influenced by the membership of individuals in specific groups, by the nature of the terrorist incident, among other factors.
2. *Aggression Reward*: a measure of the utility that an individual gets from conducting a terrorist incident. This can be due to many factors, including the policies used by the state group.
3. *Aggression Opportunity*: a measure of the ability of the individual to have the means to conduct a terrorist incident. The opportunity would depend on the location of the individual, membership in selected groups, the nature of the incident, among other factors.
4. *Aggression Penalty*: a measure of the cost or penalty that an individual gets from a terrorist action, incorporating a probability of getting caught before, during or after the action. This depends on a large extent on how the state group reacts to a terrorist incident.

Any group (social group, terrorist group, or state group) would be made up of individuals that have varying values of these four attributes. However, the distribution of these attributes within the group would be dependent on the nature of the group. As an example, we would expect that the state group would have very low values for aggression tendency.

Each group is characterized by the distribution of these four attributes among their members. As an example, assume aggression tendency can be measured as willingness to conduct a terrorist incident. A value of 0 can be assigned to an individual that is absolutely unwilling to conduct a terrorist incident, and a value of 1 to one who is completely determined to conduct a terrorist incident. Across the different individuals in a group, let us assume that this tendency is distributed according to a truncated normal distribution. This allows us to characterize each group by the mean, peak and variance of the distribution among their member individuals, with 0 and 1 acting as the boundaries for truncation. The distribution of the terrorist group and the state group may look as shown in Figure 1, $\mu_s$ is the mean aggression tendency of the state group, $\sigma_s$ is the variance of the state group, $\rho_s$ is the peak of the state group and $\mu_t$ , $\sigma_t$ and $\rho_t$ are the analogs for the terrorist group. The three values ($\mu,\sigma,\rho$) characterize any group in this model for aggression tendency. Note that we are assuming that the state group also has members with some aggression tendency. Given that any state group consists of individuals, and some individuals may not be fully committed to the beliefs of the state, it is a reflection of the reality. In an ideal case where no individual in the state group will have any aggression tendency whatsoever, the state group will have a ($\mu,\sigma,\rho$) value of (0,0,1).

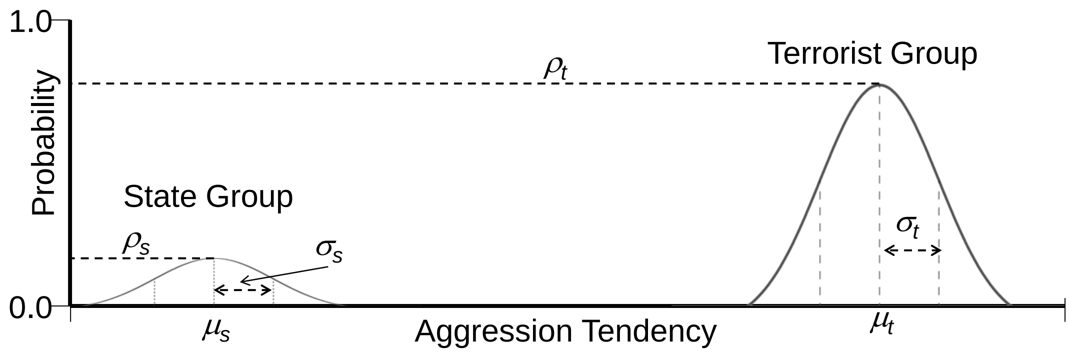


Figure 1. Characterization of aggression tendency across state group and terrorist group assuming normal distribution.

Other social groups, in addition to the state group and the terrorist group, would also have a distribution characterized by equivalent measures of their own. Other statistical distributions can also be used to characterize the variation of aggression tendency among the members of any group.

Aggression reward can be measured in a manner very similar to that of aggression tendency. However, unlike the aggression tendency, the aggression reward varies from 0 to some fixed value, e.g. it could be the amount of money the terrorist group offers to its members for a violent act. The reward could also include non-monetary benefits that are perceived by the individual as resulting from the violent act, e.g. the honor that it brings upon the family or the promise of a reward in the after-life. Although an exact quantification of the reward would be difficult to estimate, we can assume that there is a maximum value for this reward, and that the maximum value can be normalized to be 1. With that model, aggression reward can be quantified as another distribution of a variable that takes the value between 0 and 1 in a manner similar to that for aggression tendency.

Aggression opportunity is different than that of the previous two measures since it has a strong dependency on the physical location of the individual. Since we are modeling a society in which physical location plays a strong role, the aggression opportunity is a function of the location of the individual. Even when violent acts are conducted remotely, e.g. via some activation over the Internet, the actual means of physical violence requires an ability to place something destructive in a physical location. At the same time, aggression opportunity also depends on the membership within a specific group, e.g. membership in a group of employees of a facility (e.g. a dam or a power plant) enables a larger opportunity to conduct a violent incident at that facility. One possible way to model aggression opportunity will be to assign an amplification factor $\alpha$ for each social group which can be used to scale the aggression opportunity defined by the location of an individual. Assuming that $\Omega(l)$ represents the aggression opportunity available to an individual at location $l$, the aggression opportunity for an individual belonging to a social group i with amplification factor $\alpha$ will be $\alpha\Omega(l)$. If the function $\Omega$ and the factor $\alpha$ are both have a range between 0 and 1 (as a probability measure), then the resulting product will also be within that range. For an individual that belongs to multiple social groups, the largest $\alpha$ represents the best measure of the aggression opportunity available to the individual.

Aggression penalty is the negative impact that the individual gets from conducting a terrorist attack, which may extent to the lost of life for the terrorist. Where this is not the case, it depends on the ability of the state group to catch and punish the perpetrators of a terrorist incident. However, membership in a specific group may result in an individual discounting the penalty by a factor, analogous to the scaling factor for aggression opportunity. The aggression penalty also is a function of the degree to which an attack could harm people close to the person committing the act of aggression. In the case of terrorist violence, which is directed against the state, it is a fair assumption that the perpetrator does not want to hurt close family or friends, but is targeting the violence against strangers in order to create fear within society and demonstrate the state group's lack of control. Thus, another component of the aggression penalty would be the probability of the incident hurting someone close to the perpetrator.

The above formulations of the different attributes have been made without making any specific assumptions about the structure of the social groups. Further modeling also can take into account the structure of the social groups. As an example of effects driven by this structure, we can assume that a higher aggression penalty is incurred by the perpetrator if it causes an injury to someone in a social group closer to the perpetrator. As an example, the structure of a social group defined by the family of a perpetrator can be modeled as a network as defined by the family tree of the individual where each of the link shows relations by marriage or by birth. A perpetrator would pay a much higher aggression penalty for an act that injures a close family member (e.g. a brother or sister that is one hop away on the family tree) as compared to one that harms a very distant relative (many hops away on the family tree) or a stranger (not found on the family tree).

One way of taking structure into account is by modeling each social group as a social network. Given any two individuals, the distance between the two individuals in that social group can be defined as the minimum number of links in the network that needs to be followed in order to connect the two individuals -- in essence the distance is the shortest path between two individuals, i.e., the social network “geodesic.” Two individuals may belong to more than one common group, and the geodesic between the individuals could be different, depending on the groups. The absolute geodesic between any two individuals is the minimum of the geodesic they have between themselves in any shared social group. The social geodesic would impact the aggression tendency as well as the aggression penalty for an individual.

Each model for terrorist incidents can thus be defined by specifying five characteristics: (i) the geophysical distribution of individuals and social groups (ii) model for aggression tendency (iii) model for aggression reward (iv) model for aggression opportunity and (v) model for aggression penalty. Given these five characteristics, one can estimate terrorism probability -- i.e. the probability of a terrorist incident happening. The probability of the event will be positively correlated with the aggression tendency, the aggression opportunity and the aggression reward, and negatively correlated with the aggression penalty.

All five of these characteristics can vary over time: geophysical distributions may change over time, aggression tendency may increase or decrease based on various factors like state policy or economic well being of society, aggression reward can similarly vary over time, as well as aggression opportunity and aggression penalty in a society may change over time. This implies that there is a sixth characteristic, which is the evolution of the five characteristics over time. Thus, two models of society can be defined, a static model and a dynamic model, the former being one where the temporal component is not taken into account, and the latter being one where the characteristics are assumed to be time-varying. Both models can provide useful insights into the properties of society under a wide variety of circumstances.

Given the static or dynamic model, the system can be mathematically modeled or analyzed via computer simulations. Mathematical modeling requires making simplifying assumptions that allow the system to be analyzed, but can nevertheless provide useful insights. Computer simulations can be used to analyze the system for more complex distributions and definitions of the above five characteristics, including details on the structure of each social group.

# 4. MATHEMATICAL MODELING

In this section, we provide some simple mathematical models of the society and explore the insights into the behavior of terrorist incidents in the society we can infer from those models. For each of these models, we make specific assumptions about the geophysical distribution and the nature of the four attributes among the different groups in the society.

## 4.1 Model 1: Region in Anarchy

In many cases, the power of the state group is non-existent or limited in a region of conflict. In these cases, one can assume that a state of anarchy exists. We can create a simple mathematical model to understand the properties of a society when the state group does not have a substantial power. We initially propose a static model and discuss the insights that one can obtain from the static model. In this model we incorporate the geophysical distribution of individuals in society. Earlier research emphasizes the importance of focusing on geographical characteristics that may contribute to terrorism [2].

In this situation, one possible description of the five characteristics discussed in Section 3 are as follows:

- *Geophysical Distribution:* The individuals in society are located on points in infinite 2-dimensional plane. There are $N$ social groups, and any individual belongs to the $i^{th}$ social group with a probability of $p_i$. An individual can belong to more than one social group.
- *Aggression tendency*: The aggression tendency for any group is modeled as a Dirac Delta function with a fixed value, i.e. all individuals in a group have the same aggression tendency. This function provides an easy model to analyze. The aggression tendency for an individual is the sum of the aggression tendency for all the social groups he or she belongs to. We further assume that the aggression tendency for the terrorist group is 1, while for any other group, it is 0.
- *Aggression Reward*: is 0 for all groups except the terrorist group, for which the reward is 1 for each incident. Each incident has an impact on individuals that are within $d$ units of travel from the location of the incident, i.e. one can reach the individual using any combination of $d$ horizontal and vertical movements. There will be $I$ individuals affected by any incident, and in this geophysical model $I$ equals $2d^2+2d+1$.
- *Aggression Opportunity*: Each individual has an ability to conduct an incident that affects any individual who is located at a distance within K units on the grid, i.e. the perpetrator can reach the individual within K vertical or horizontal movements along the grid. The motivation behind the model is that a terrorist could basically travel within a limited range of physical distance, and any $K$ units travelled in any direction (horizontal or vertical) along the grid limits the range. The number of individuals within the range will be $2K(K+1)$. We will use the term $R$ to refer to the number of individuals that can be impacted, which is the measure of aggression opportunity in this model.

- *Aggression Penalty*: paid by a perpetrator is 1 for each individual who shares a social group with the perpetrator and is a victim of the incident. The aggression penalty is zero if no one sharing a social group with the perpetrator is affected by the incident.

With this definition, a terrorist incident happens when the aggression reward exceeds the aggression penalty, which with the above definition means that the perpetrator can commit an incident without injuring anyone with a shared social group. This is not an unrealistic scenario in regions where state control is weak, and the only factors preventing a terrorist group from launching an attack are tribal or other social bonds that restrict them from hurting their own kin.

Given any member of the terrorist group, the probability that it belongs to social group *i* is $p_i$ and a terrorist incident will occur if there is an individual within the range of aggression opportunity is not in the social group. The probability that the incident does not impact any individual in the social group is the probability that none of the *I* people are in the social group, which is $(1- p_i)^I$, which equals the probability that the terrorist will not conduct the incident. Given any location in the geophysical space, the probability that a terrorist with an opportunity to conduct an incident will not do so is the probability that anyone in any social group shared by the terrorist is impacted. This probability will be the probability that the terrorist finds an area in any region which does not impact any person in a shared social group will be $\Pi(1- p_i)^I$ . It follows that the probability of a terrorist incident not being perpetrated by the terrorist would $1-\Pi(1- p_i)^I$. Given the range *R* of the terrorist, the probability that a terrorist incident happens will be $1-(1-\Pi(1- p_i)^I)^R$.

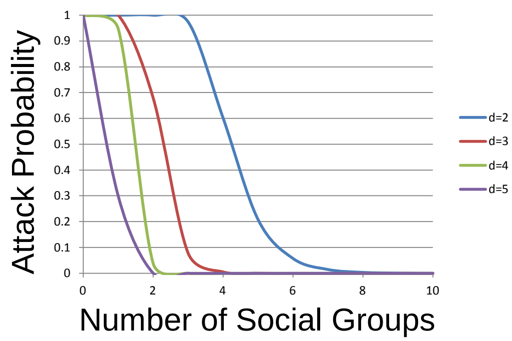


Figure 2. Probability of terrorist incident in the anarchy model with pi = 0.1, K=10 for different values of impact distance

Figure 2 shows the probability of a terrorist incident occurring with the number of social group, assuming that membership in the social groups is independent of each other. As can be seen with the graph, the probability of a terrorist incident happening decreases fairly rapidly with the number of social groups that are active in the environment. These social groups are large because they have a finite probability of occurring in a large population.

Based on this analysis, we can formulate the following hypothesis:

*Hypothesis 1*: The likelihood of terrorist incidences is inversely related to the average number of independent large social networks to which an individual person in the society belongs.

An interesting question is to determine the impact of having a larger number social groups that may not independent. It is not uncommon in many societies for the tribal, clan or caste affiliations to also reflect into a preferred occupation. While modern societies do not impose strict restrictions along this direction, the correlation among different social groups may reduce the impact it would have on the likelihood of terrorist incidents, since the likelihood will be that the net result would be like having a small number of independent social groups. Therefore, we can put forward the following hypothesis as well.

*Hypothesis 2*: The likelihood of terrorist incidences is directly related to the correlations among large social networks to which an individual person in the society belongs.

### 4.2 Model 2: Region with Strict State Policing

In the model assuming anarchy, the probability of not launching an attack depends completely on the prevalence of social groups. However, the presence of state actors should cause a reduction in the probability of terrorist incidents occurring. In this model, we assume the same model as that of anarchy, but we add an additional restriction on the aggression opportunity. If a state actor is present in any location, it can prevent the attack happening with a certain radius of influence. If an individual belonging to the state group is within that radius of influence of attack, the terrorist would not launch an attack. This model reflects the tendency in many terrorist incidents in that they try to attack soft targets, i.e. locations where the presence of the state group may be weak. Let us refer to the radius of influence as *r*.

The aggression opportunity in this model is now reduced in that the terrorist attack is only launched when the impacted nodes *I* do not include any members in the social network of the perpetrator and the $S = (2r^2+2r+1)$ nodes next to the point of impact do not include a state actor. In these cases, the probability of a terrorist event occurring is given by $(1-(1-p_s)^S)\ \Pi(1-p_i)^I$ where $p_s$ is the probability of a node being a state actor. The net resulting reduction in the probability of terrorist incidents is shown in Figure 3.

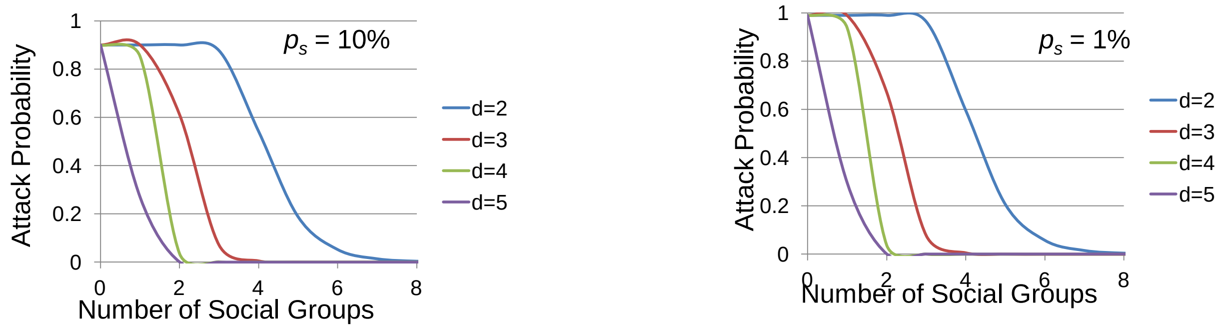


Figure 3. Probability of terrorist incident in the state controlled model with pi = 0.1, K=10 for different values of impact distance and two probabilities of state actors

Figure 3 shows that the probability of attack decreases with the number of social groups, as well as an increase in the presence probability of the state group. However, the effectiveness of the state policing increases as an exponential factor based on the range of influence, while the effectiveness of the social network increases exponentially based on the number of social networks that exist. Given that the probability of a state group is likely to be low, with the state group being 10% or less typically, the social networks are likely to be much more effective in a large population to deter terrorist incidents. Compared to Figure 3, most state probabilities are likely to be much less than 1%, and the number of social groups is likely to be much higher, in a few tens in any given society.

On the basis of the above, we can postulate the following hypothesis:

*Hypothesis 3*: State policing is a less effective means for reduction in terrorism compared to approaches that promote the formation of new social networks among individuals.

### 4.3 Enforcing Geographical Dispersion

It is not uncommon for authorities in areas subject to social unrest to try to protect one part of the group by physically protecting the area where insurgents are likely to pick a target from the area where insurgents are likely to reside.[1] In this section we look at the mathematical model of the likelihood of a terrorist incident happening under this condition. As a simplified representation, the environment now looks like two regions in a plane that could be assumed to be separated by a physical barrier for example.

*Geophysical Distribution*: Figure 4 shows a model of geophysical distribution in this scenario. Within the so-called "disturbed" area, there are $N_d$ social networks, and the probability that any given individual in the disturbed area belongs in the same social network that a terrorist does is $p_d$. Similarly, there are $N_s$ social networks in the "stable" area and the probability that two individuals that are within the stable area belong to the same social network is $p_s$. We also assume that a small number $N_c$ of social networks spans both areas, and that the probability that a person on the other side of the barrier is in the social network is $p_c$. We also assume that the probability that a person can cross between the two areas is $\mu$, which would be expected to be small in defensive situations.

1 It is acknowledge that this is a hypothetical simplification that is not relevant when a population is heavily mixed.

The aggression tendency, aggression penalty and aggression reward are assumed to be the same[2] as in the previous two scenarios, but the aggression opportunity changes due to the presence of barrier. Assuming that the range of the terrorists remain the same, the division into two regions prevents any terrorist beyond the range of $K$ units from conducting an event in the stable part of the region, and also restricts access by reducing the probability of access to the stable side by the probability of $\mu$.

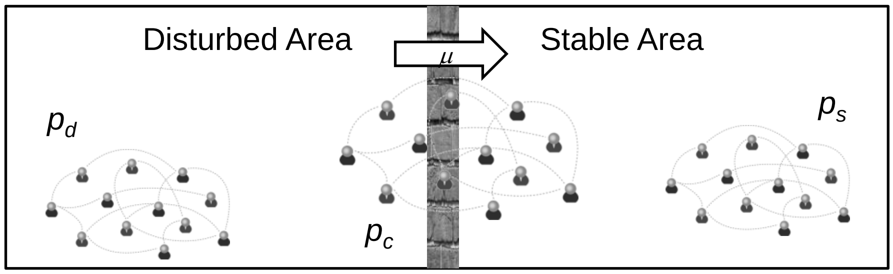


Figure 4. Model of a divided area that limits population mixing with varying probability of social networks in each area and between areas.

Assuming that a terrorist is $s$ units away from the barrier with a travel range of K, the probability of it being able to reach the other area is $\mu$, and the number of points it can reach in the stable area is at a distance of $(K\text{-}s)$ from the disturbed area, which means that the number of points the terrorist can reach in the stable area will be $(K\text{-}s)^2$, i.e. the range $R$ of the terrorist will be R = $(K\text{-}s)^2$ . The number of impacted individuals $I$ remains unchanged and the probability of a terrorist incident will be $p_b = \mu(1-(1\text{-}\Pi(1\text{-} p_c)^I)^{Rc})$ for every social network that the terrorist shares. With $N_c$ social networks, the net probability of a terrorist incident will be $1\text{-}(1\text{-}p_b)^{Nc}$ . Since the terrorists are assumed randomly distributed across the distance from the barrier, the net probability can be obtained by averaging the previous equations for all values of $s$. On the other hand, the probability of a terrorist incident in the disturbed area may be considered akin to that in the anarchy model. We can then compare the probability of an incident in the stable area versus the probability of a similar incident in the disturbed area.

In a real world situation, one could expect that the number of social networks that exist within the disturbed area is larger than the number of social networks that span both areas, and for any given social network, the probability that a member from the other area belongs to the same network is lower than the probability than two people in the disturbed area belong to the same network.

Figure 5 shows the relative probability of an incident happening in the stable area as compared to the probability of the same incident happening in the disturbed area, i.e. a measure of how much relative safety the geographical seperation is able to provide. The probability of a terrorist crossing between areas is assumed to be 1%. The x-axis shows the relative number of the social networks that exist between regions versus the number of social networks that exist within the disturbed area itself, which is a metric that we would expect to be between 0 and 1 in the real-world. The y-axis measures the probability of an attack in the stable area versus the probability of an attack in the disturbed area, which is assumed to be in a state of anarchy. A value about 1 on the y-axis indicates that the probability of attack across in the stable area is higher than that of the state of anarchy in the disturbed area, while a value below 1 indicates the threat in the stable area is lower, which is the desirable state.

The parameters for Figure 5 assume that there are eight social networks within the disturbed area with the probability that two individuals in the disturbed area belong to the network is 0.1. Each of the curves shows the probability that someone may share a social network between regions, i.e. the curve for 10% means that the probability of two people from different regions being in the same social network is 0.01.

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[2] This is acknowledged as a simplifying assumption because the imposition of a physical barrier will heighten tensions between groups.

The results show that despite the low probability of terrorists crossing between areas, the incidents in the stable area are still more likely than that in the disturbed area. The only exception is the case when the probability of social networks across the areas is high at 50% of the intra-disturbed area probability, and the number of social networks spanning both areas is close to the social networks within the disturbed area. It is worth noting that the relative probability of attack is not significantly reduced despite assuming a low probability of movement between regions.

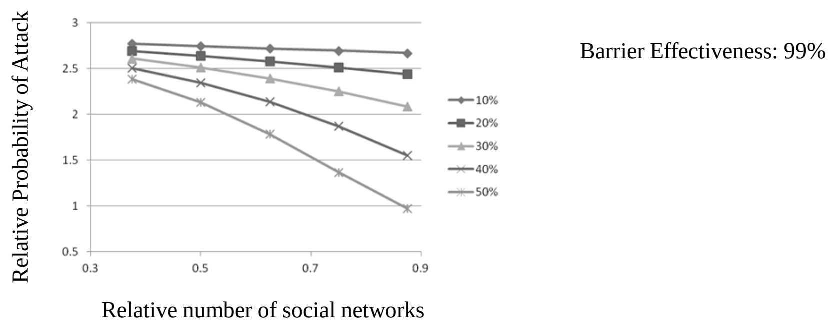


Figure 5. Relative probability of terrorist incident in stable area as a function of number of social networks across the two areas.

Figure 6 shows the same results, except with the change that the x-axis is now showing the ratio of two probabilities - the probability that two individuals across the areas belong to the social network compared to the probability that two individuals on the disturbed area belong to the same social network. Eight social networks are assumed to exist in the disturbed area, and the different curves show the number of social networks that are assumed to exist between areas . Once again, establishing more cross-area social networks and increased probability of sharing the cross-area network are crucial to reducing the risk of terrorist incidents.  The left curve shows the situation when terrorists can cross areas with a probability of 1% while the right curve shows the situation when terrorists can cross areas with a probability of 10%.

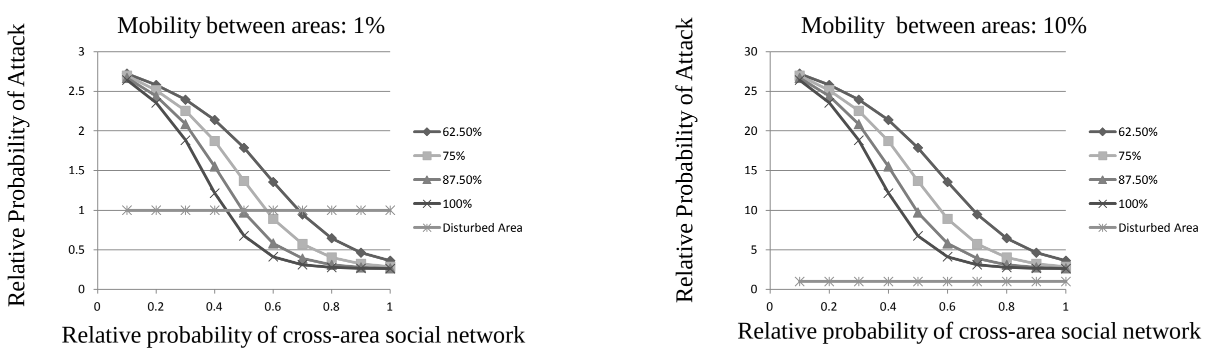


Figure 6. Relative probability of terrorist incident in stable area as a function of cross-area probability of sharing social networks.

Figure 6 shows that only extreme and unrealistic conditions make the terrorist incidents in the stable area be less likely than in the disturbed area. In the right hand side, when the chance of mixing between the areas is 10%, none of the configurations have the stable area with a low probability of terrorist incident than the disturbed area. In the left hand side, there are more of those regions, but only when the cross-area social networks and the cross-area probability of sharing a social network are high.

In essence, the previous analysis shows that seeking to enforce geographical separation of groups within the population is not that effective in reducing the probability of terrorist incidents. Furthermore, this may be compounded by additional tensions resulting from any physical barriers to mobility. Means for establishing and creating more social networks across the different areas, and undertaking means to improve social interactions are more likely to create long term reduction in the stability of the population. This is consistent with Intergroup Contact Theory [15], which further specifies the nature of interactions that enhance cohesive groups..This leads to our fourth hypothesis:

*Hypothesis 4*: Geographic separation of population is a relatively ineffective approach to reduce terrorism compared to approaches that promote the formation of new social networks among individuals.

Assuming that the above analysis is a reasonable proxy for real society, we can assert that after a war, the disruption of social networks is a key factor contributing to extremism. Conflict disrupts existing social networks and mixing between groups within a society. For stabilization of the region post-conflict, our results support the importance of intergroup social networks. Promoting an increase in a diverse set of social groups would be an effective way for the state group to intervene and disrupt the terrorist activities, even if it cannot identify a specific terrorist group in the overall society.

The analysis also leads to the observation that one of the key mutable properties of a group characterized by extremism is the number of social groups with which its members have a strong affiliation. For a terrorist group to be effective, it needs to be able to reduce the number of social groups as much as possible, ideally reducing it to be just a membership in the terrorist group. On the other hand, for an improvement in the stability of the state, diversity is important: the state group needs to increase the number of social groups that are present within the society, thereby increasing the opportunity for individuals belong to multiple cooperative groups.

## 6. CONCLUSIONS

In this paper, we introduced a simple generative mathematical model for predicting the behavior of a terrorist group and estimating the probability of a group launching a terrorist attack. While the modeling uses only very simple mathematical analysis, it is still able to make some interesting predictions. It shows that strict policing by state, or trying to separate disturbed areas by enforcement will not be effective in reducing terrorist incidents. On the other hand, an approach whereby the population in general participates in multiple social groups, which are independent of each other, can help to reduce terrorism significantly. Thus, successful peace-keeping requires approaches and incentives that help in the creation of a large number of social groups among the civilian population, where the social groups be created due to trade, occupation, arts or any other approaches. Interestingly, this is consistent with qualitative research concerning intergroup contact [15]. Based on the analysis of three common real-world situations, we have put forward four hypotheses regarding effective means to counter terrorism.

The work in this paper is at a very early stage, and we need to extend it in a number of directions. The current modeling has only considered static models, and we need to incorporate dynamic models that evolve over time. The mathematical models are limited because they can only be analyzed for some very simple probability distributions and models for the various characteristics. Computer simulations, including approaches by agent based modeling, that can analyze more complex models and create simulations of computer generated social phenomena (e.g., [16]). Such representations of societies can help shed light onto other properties of external groups like terrorists.

## 7. ACKNOWLEDGEMENTS

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