Skewed Mirror Symmetry for Depth Estimation in 3D Line-Drawings

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Abstract. We aim to reconstruct three-dimensional polyhedral solids from axonometric-like line drawings. A new approach is proposed to make use of planes of mirror symmetry detected in such sketches. Taking account of mirror symmetry of such polyhedra can significantly improve the reconstruction process. Applying symmetry as a regularity in optimisation-based reconstruction is shown to be adequate by itself, without the need for other inflation techniques or regularities. Furthermore, symmetry can be used to reduce the size of the reconstruction problem, leading to a reduction in computing time.

1 Introduction

Sketch input-based geometric modellers have been investigated during the last decade [1, 2, 3, 4, 5, 6], providing a very intuitive and easy to use interface to 3D reconstruction engines. Optimisation has been used with some success in this sketch-based modelling approach. One of the main challenges in optimisation-based 3D reconstruction is the mathematical formulation of perceptual cues, also called artefacts, or regularities [6]. At present, they lead to poorly defined objective functions, and hence the success rate is below the expectations of users. In addition, optimisation processes are bottlenecks for an interactive session, with clearly unacceptable calculation times (typically up to several minutes).

Using symmetry improves the results, since it is a fundamental concept utilized by human visual perception [7]. Furthermore, many man-made objects are symmetric, both because this makes them easier to interpret and manufacture, and because of functional and aesthetic requirements. Studies have shown that a large proportion of industrial components have some symmetry [8].

In a previous paper, we presented a novel method for determining skewed planes of symmetry of polyhedral objects, starting with a two-dimensional axonometric-like view [9]. This algorithm is intended for *single* view reconstruction: 3D reconstruction from *multiple* orthographic views requires a radically different approach. The nature of shapes that can be reconstructed with this optimisation-based approach extends, at present, to polyhedral blocks (manifolds), origami objects (sheet metal) and wire frame models (non-manifold objects).

In this paper, we present a new optimisation regularity based on a novel formulation of model symmetry. Its advantages in improving the success rate of optimisation-based model building are discussed. We also examine the advantages of dealing with model symmetries as constraints, rather than regularities, which significantly reduces the calculation time required for optimisation-based 3D reconstruction.

2 Related Work

Various research has focused on using freehand drawings and sketches as a way to obtain 3D geometric models using input devices like graphic tablets and tablet PCs. *Gestural modelling* systems are one approach. Predefined gestures encode a set of geometric manipulation operations [10, 11, 12, 13]. Reconstructive modelling is an alternative, where geometric reconstruction techniques build the object's geometry from a sketch representing a projection of the object [14, 15, 16, 17, 18, 19].

Optimisation-based 3D reconstruction is discussed in [20, 21, 22, 23, 24, 25, 26, 27]. We simply note that coordinates are related to a so-called inflation-coordinate-system, where the xy coordinate plane is the drawing plane and the infinite set of three-dimensional objects whose orthogonal projection matches the given line drawing (i.e. the *orthographic extension*) is defined by the set of z coordinates of all model vertices: $z = (z_1, z_2, ..., z_n)$. A psychologically plausible model is selected by *optimising* a figure of merit, or objective function, defined as a weighted sum of contributions from perceptual cues, as suggested by Eqn 1.

$$F(z) = \sum \alpha_j R_j(z). \tag{1}$$

Various researchers have studied such *perceptual cues* or *regularities* [22, 24, 28, 29]. Currently, inflation approaches succeed in 3D reconstruction whenever the input drawings are *geometrically correct*, i.e. they correspond to an orthogonal projection of a real shape. However, if input drawings are approximate sketches, with imprecise coordinates, inflation gives distorted or even *tangled* shapes. Sketching interfaces typically convert sketches to line drawings online. Inaccuracies in the initial sketch can influence the detection of regularities, but such low-level considerations are outside the scope of this paper.

We are concerned with the particular regularity of *mirror symmetry*, which is often an explicit design intent. Previous work exists on the detection and application of symmetry to object modelling and recognition [30].

In our approach, a symmetry plane is obtained from a set of symmetry lines. Finding model symmetry planes requires the prior detection of axes of skewed facial symmetry, which, in turn, requires edges bounding faces to be recognised in the two-dimensional image [9]. We need to search for 2-D circuits representing intersections of the model with a symmetry plane. Each skewed plane of symmetry is made up of skewed axes of symmetry and edges of various faces. A closed sequence of axes and edges that meet each other must be found, forming a planar polygon. The plane that contains that *polygon of symmetry* represents a plane of symmetry (Fig. 1). We perform our computation in 2D, so the resulting
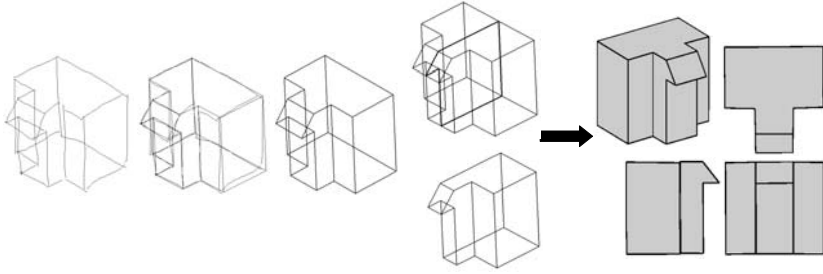
Fig. 1. Input sketch, a wire-frame line drawing, and symmetry plane, represented as a 2-D circuit (left); the final reconstructed 3-D model (right)

output comprises *skewed planes of symmetry*: real planes of symmetry viewed from some (unknown) viewing direction [9].

3 Overview of Our New Approach

Once the symmetry planes have been determined [9], in order to deal with symmetry, it is convenient to introduce a new coordinate system, a so-called *symmetry system*, as explained in the next Section. This allows the symmetry condition to be formulated in a compact and convenient way.

To do this, symmetric vertices have to be properly paired or *matched*. This is simple whenever a vertex in one half is connected to its pair through an edge that crosses the symmetry plane, with midpoint on the plane, as for example in e_1, e_2 and e_3 in Fig. 2a, or e_4 and e_5 in Fig. 2b.

However, the general matching problem is more difficult (see Section 5), and needs to be solved for a practical formulation of *bilateral symmetry regularity*.

Note that the presence of a symmetry plane can be used to reduce the size of the inflation problem. Because of the symmetry, only one half of the object needs to be inflated. When multiple symmetry planes exist, further reductions of problem size can be obtained. Overall, a much smaller problem may be result.

4 Symmetry Coordinate System and Symmetry Condition

We introduce a different *symmetry coordinate system* for each symmetry plane, in which the symmetry plane is the XY plane. One such system $(O_{S1}X_{S1}Y_{S1}Z_{S1})$, together with the inflation coordinate system $(O_I X_I Y_I Z_I)$ and the model coordinate system $(O_M X_M Y_M Z_M)$, is illustrated in Fig. 3. The origin is placed at the centroid of the planar symmetry polygon, which is invariant and so ensures that the origin of all symmetry systems is the same. X_{S1} and Y_{S1} are aligned with two model coordinate system axes, whenever such a system has already been determined (this problem is related to detecting main directions,
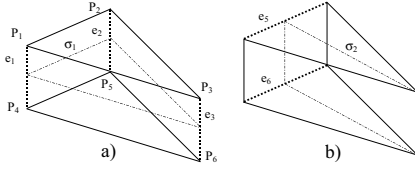
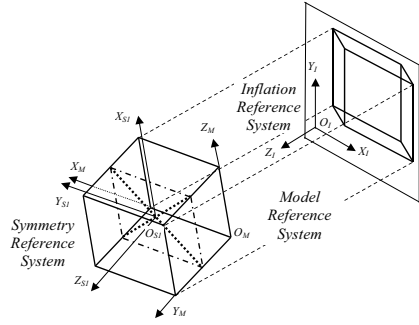**Fig. 2.** Determination of subsets of symmetric vertices

Fig. 3. Inflation, model and symmetry reference systems

and is beyond the scope of this paper [24]). Z_{S1} is chosen to give a right hand system.

The coefficients A, B, C and D in the equation $Ax + By + Cz + D = 0$ of the best-fit plane defined by all vertices of the symmetry polygon is initially calculated by computing the respective areas of the projection of the virtual face in the yz plane (for A), xz plane (for B), and xy plane (for C). D is estimated by simply substituting the coordinates of one of the vertices into the equation.

Let (x_c, y_c, z_c) be the coordinates of the centroid of the symmetry polygon. Let (u_1, u_2, u_3) be a unit vector normal to the symmetry plane, and let R be defined by: $R = \sqrt{u_2^2 + u_3^2}$. We denote the coordinates of a vertex P in the model by (x_p, y_p, z_p) when they are in the inflation coordinate system, and by (x'_p, y'_p, z'_p) when they are in the symmetry coordinate system. Then, the homogeneous transformation between these systems is:

$$
\begin{pmatrix} x'_p\ y'_p\ z'_p\ 1 \end{pmatrix} = \tag{2}
$$

$$
\begin{pmatrix} x_p\ y_p\ z_p\ 1 \end{pmatrix}
\begin{bmatrix}
R & 0 & u_1 & 0 \\
-u_1 u_2/R & u_3/R & u_2 & 0 \\
-u_1 u_3/R & -u_2/R & u_3 & 0 \\
-x_c R + \frac{u_1 u_2 y_c}{R} + \frac{u_1 u_3 z_c}{R} & \frac{u_3 y_c}{R} + \frac{u_2 z_c}{R} & -u_1 x_c - u_2 y_c - u_3 z_c & 1
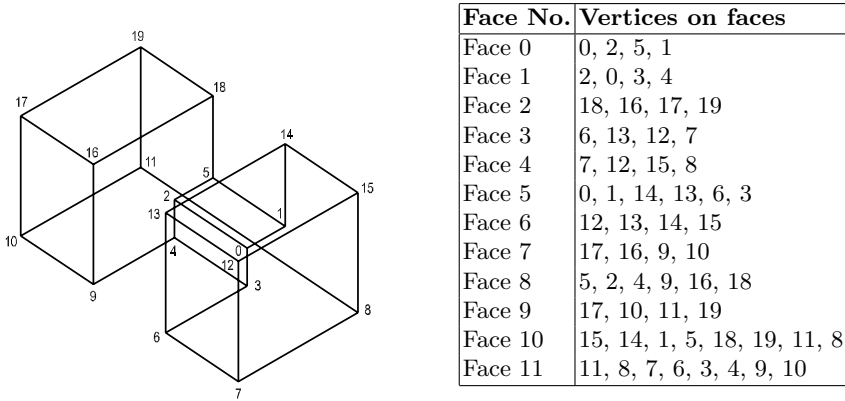\end{bmatrix}.
$$

To formulate the symmetry condition we need to renumber the set of n vertices of the model. If m points lie on one side of the symmetry plane, then m points lie on the opposite side, and $n - 2m$ points are contained in the symmetry plane (m may be 0). The vertices are renumbered so that vertex 1 corresponds to vertex $m + 1$ and so on. This gives the following conditions:

$$
\begin{aligned}
x'_1 &= x'_{m+1}, & \dots, & \quad x'_m = x'_{2m}, \\
y'_1 &= y'_{m+1}, & \dots, & \quad y'_m = y'_{2m}, \\
z'_1 &= -z'_{m+1}, & \dots, & \quad z'_m = -z'_{2m}.
\end{aligned} \tag{3}
$$

This is the simplest formulation of the symmetry condition for vertices, and is the basic formulation of our *bilateral symmetry regularity*.

5   Matching Symmetric Vertices

Once the symmetry plane has been detected, if a full symmetry formulation is desired, a matching vertex must be located for each vertex. Our algorithm fills the elements of a $2 \times n$ matrix V (Fig. 4). In $V(1, i)$ the vertex symmetric to vertex i (its match) is stored. The second row $V(2, i)$ stores a label indicating which subset vertex i belongs to. The label $+1$ is assigned to vertices on one side of the symmetry plane, label -1 is assigned to those on the other side, and label 0 corresponds to vertices on the symmetry plane.

Face No.	Vertices on faces
Face 0	0, 2, 5, 1
Face 1	2, 0, 3, 4
Face 2	18, 16, 17, 19
Face 3	6, 13, 12, 7
Face 4	7, 12, 15, 8
Face 5	0, 1, 14, 13, 6, 3
Face 6	12, 13, 14, 15
Face 7	17, 16, 9, 10
Face 8	5, 2, 4, 9, 16, 18
Face 9	17, 10, 11, 19
Face 10	15, 14, 1, 5, 18, 19, 11, 8
Face 11	11, 8, 7, 6, 3, 4, 9, 10

$$V = \begin{pmatrix} 2 & 5 & 0 & 4 & 3 & 1 & 9 & 10 & 11 & 6 & 7 & 8 & 17 & 16 & 18 & 19 & 13 & 12 & 14 & 15 \\ +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \end{pmatrix}.$$

Fig. 4. Matching symmetric vertices

5.1   Identification of the Two Sides of a Symmetric Model

Identification starts at one of the faces divided by the symmetry plane. One of the sides of the symmetry plane is arbitrarily assigned $+1$ without loss of generality. Next, adjacent faces that are also cut by the symmetry plane are evaluated in turn. In this way, common vertices can be used to guarantee consistency in labelling propagation.

In a third step, the process is consistently extended to remaining faces. All vertices in such faces are assigned the same label as the one assigned to previously labelled vertices of the face. Vertices belonging to the symmetry polygon are finally visited and labelled 0.

5.2   Matching Vertices

Next, symmetry axes are considered in turn, and the vertices of the faces they belong to are explored. Their vertices are matched through a simultaneous clockwise-anticlockwise scan. For instance, in face 11 (Fig. 5), if evaluation starts at the a axis, which cuts the mid-point of edge 11-8, it is easy to fix vertices 11
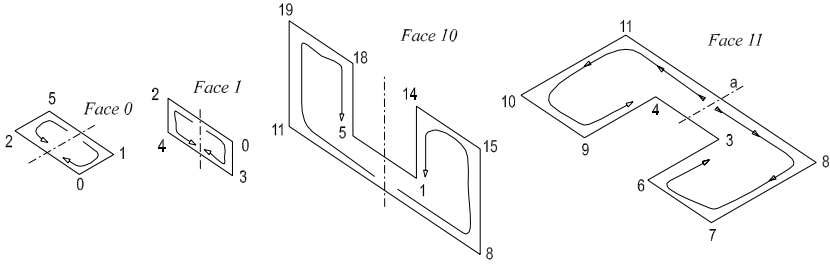
Fig. 5. Faces with axis belonging to the polygon of symmetry

and 8 as a pair of symmetric vertices. The vertices of one half of the face start-ing at this edge are scanned clockwise (8-7-6-3) and the other half anticlockwise (11-10-9-4), and the two resulting lists are paired to match all the vertices in the face. The same procedure is applied to each face sharing a line with the symmetry polygon. See Fig. 5.

Once all such faces have been evaluated, we continue with the remaining faces. In that case, symmetry between vertices involves vertices of different faces. Thus, the objective is to find symmetric faces.

The list of such unprocessed faces is scanned by searching for a face F_m with at least three vertices already labelled $(v_i, v_j, v_k \subset F_m)$; two of these (at least) must be at the ends of an edge shared with an already-processed face. Three non-collinear points determine a plane, so the vertices symmetric to (v_i, v_j, v_k) must determine a symmetric face $(v'_i, v'_j, v'_k \subset F'_m)$. The correspondence between the vertices must still be found; this is done by considering the vertices at the ends of the edges shared with already processed faces. (see Fig. 6: for example 6-7 in face 3, and 9-10 in face 7). The relative ordering of vertices around the faces may be the same or opposite, and must be determined. For instance, the correspondence between faces 3 and 7 has oppositely ordered vertices (9-10-17-16) and (6-7-12-13). However, faces 9 and 4 have a correspondence (19-17-10-11) and (15-12-7-8) with vertices arranged in the same order.
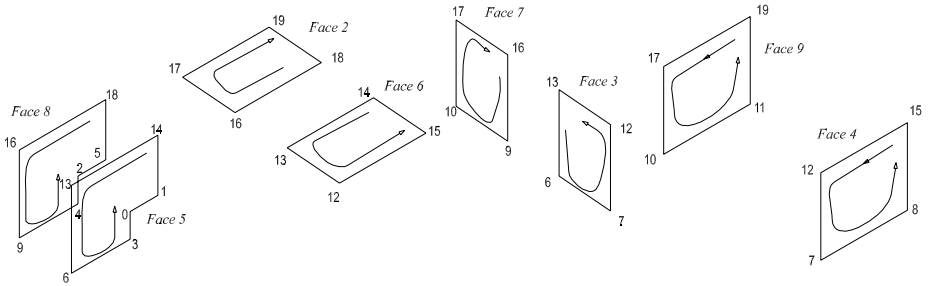

Fig. 6. Faces without axis belonging to the polygon of symmetry

6   Symmetry as Regularity

Once the symmetry polygon has been detected and matching of vertices completed, the symmetry regularity is easy to formulate. In reconstruction by optimisation, every regularity has a cost, and the objective is to minimize the sum of costs. A cost is chosen to be zero when the desired condition is achieved, and differs from zero as we go away from this ideal. Hence, the bilateral symmetry condition for a single plane j formulated in Eqn 3 can be reformulated as the standard deviation (σ) of the difference of coordinates of all pairs of symmetric vertices:

$$\text{Cost}_j = \sigma^2(\Delta(x')) + \sigma^2(\Delta(y')) + \sigma^2(\Lambda(z')) \tag{4}$$

where for every vertex $i = 1, ..., m$:

$$\Delta(x') = x'_i - x'_{m+i}, \quad \Delta(y') = y'_i - y'_{m+i}, \quad \Lambda(z') = z'_i + z'_{m+i},$$

and for every vertex $i = 2m + 1, ..., n$:

$$\Delta(x') = \Delta(y') = 0, \quad \Lambda(z') = z_i.$$

The standard deviations are calculated with an assumed mean equal to zero. Using the real mean does not improve reconstruction results and the objective of the function is distorted. The real objective is to find variables in which the mean of every summand is also zero. When every pair of vertices fulfils the condition, the cost of the regularity reaches the minimum.

The total cost for a model with r planes of symmetry is defined as:

$$\text{Cost of Symmetry} = \frac{1}{r} \sum_{j=1}^{r} \text{Cost}_j. \tag{5}$$

7   Model Rebuilding from a Half

Faster reconstruction can be obtained using the idea that, knowing the symmetric pairs of vertices, we need only reconstruct one half of the model in the drawing, and from this half model, symmetry can be applied to obtain the rebuilt complete model. This reduces the number of vertices, edges and faces in the sketch graph, thus reducing calculation times for reconstruction. This idea can be extended to divide the model size by 2 again for each further plane of symmetry.

Symmetry in 3-D space can be obtained by composition of transformations of the form:

$$S = \begin{bmatrix} 1 - 2u_1^2 & -2u_1u_2 & -2u_1u_3 & 0 \\ -2u_1u_2 & 1 - 2u_2^2 & -2u_2u_3 & 0 \\ -2u_1u_3 & -2u_2u_3 & 1 - 2u_3^2 & 0 \\ -2Du_1 & -2Du_2 & -2Du_3 & 1 \end{bmatrix} \tag{6}$$

where (u_1, u_2, u_3) is the unit normal vector of the symmetry plane ($Ax + By + Cz + D = 0$).

Having found a plane of symmetry and a half 3-D model, applying S to its vertices allows the complete 3-D model to be obtained. Actually, as the final model retains x and y vertex coordinates from the original drawing, only the computation of z' coordinates for reflected vertices is required. If (x, y, z) are the coordinates of a vertex in the reconstructed half, z' for the reflected vertex is given by:

$$z' = -2u_1 u_3 \cdot x - 2u_2 u_3 \cdot y + (1 - 2u_3^2) \cdot z - 2Du_3. \tag{7}$$

Note that information is used from the whole object even when we eliminate half of the figure. The planes of symmetry [9] are found from points situated at the midpoints of edges (and vertices). Thus, 3-D object reconstruction is based on information averaged from both halves.

8   Results

A sketch with a single symmetry plane is used to illustrate the results. Initially, a tidied line drawing and symmetry polygon were obtained using previously reported methods (see Fig. 7).
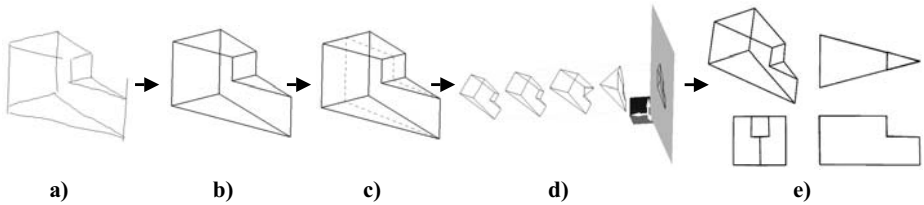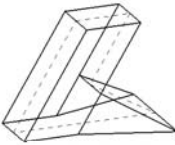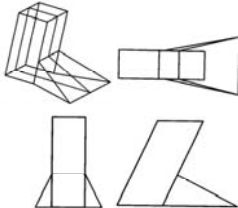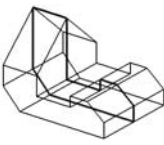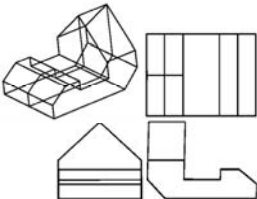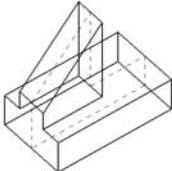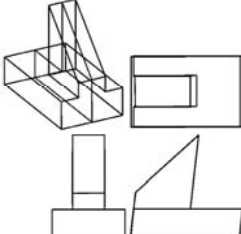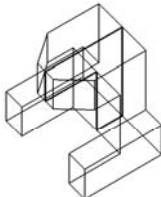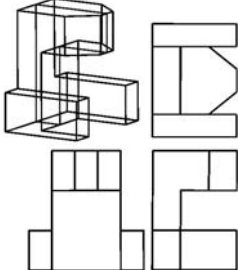

Fig. 7. a) Input sketch, b) line drawing, c) symmetry polygon, d) reconstruction process, e) final model obtained using the bilateral symmetry regularity

Using the bilateral symmetry regularity described in Section 6, a good final model was obtained (see Fig. 7). Additionally, no initial inflation method was required, since the symmetric-model regularity is not trivially satisfied in the input image. The regularity is formulated as a quadric function (see Equations 5-6), which provides quicker convergence than a linear function. In Table 1, four examples are shown as they appeared after the symmetry regularity was applied as the sole inflation mechanism.

This regularity is a *true regularity*. Its use only depends on the detection of characteristic clues in the input drawing. It can be applied alone for converting sketches to 3D models, without any other inflation method, and without the use of other, possibly conflicting, regularities. Thus, this regularity can be considered as a new inflation method by itself. It solves reconstruction for every symmetric sketch for which the planes of symmetry have been detected.

Table 1. Examples and Performance

Line drawing	3D model	Line drawing	3D model
19 edges 12 vertices	9 faces 1 plane of symmetry. Inflation time: less than 1sec.	33 edges 22 vertices	13 faces, 1 plane of symmetry. Inflation time: 1 sec.
24 edges 16 vertices	10 faces, 1 plane of symmetry. Inflation time: less than 1 sec.	46 edges 30 vertices	18 faces, 1 plane of symmetry. Inflation time: 2 sec.

Finally, we give some examples of more complex shapes (Fig. 8–11) which illustrate the benefits of half-part reduction. Fig. 9 shows a shape with three planes of symmetry. Two of them are used to reduce the problem size, then the last one is used as a regularity to reconstruct a partial model. Finally, the complete model is restored. In this example, computation times are reduced from 3 seconds for reconstruction of the complete drawing to less than 1 second using the reduced model (on a Pentium III 1GHz, with 256 Mb RAM).
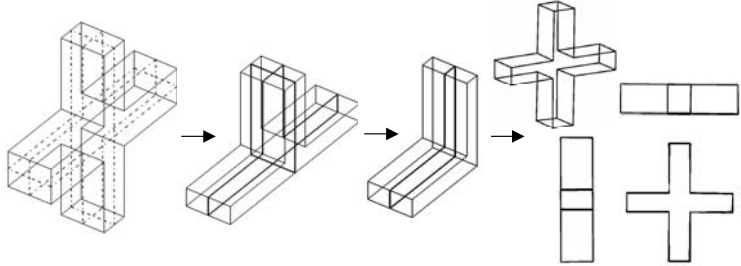

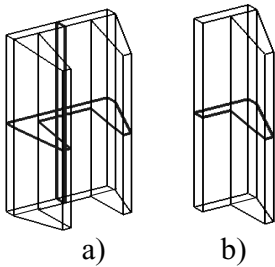**Fig. 8.** Model with three planes of symmetry

Fig. 9. Figure with two planes of symmetry. Reconstruction from a) 3" and from b) less than 1"
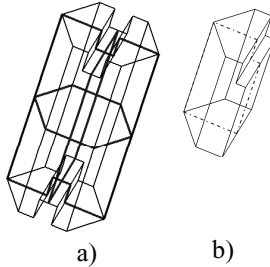
Fig. 10. Figure with two planes of symmetry. Reconstruction from a) 6" and from b) less than 1"
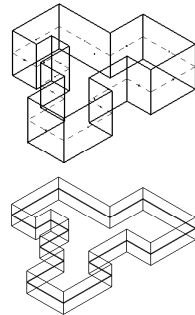
Fig. 11. Detection of symmetry planes in extruded shapes

Our new approach simplifies the process of reconstruction by diminishing the number of variables used in the main computation, leading to a reduction of time in the whole process. Extruded models are an exception to this observation. They have (at least) one plane of symmetry (orthogonal to the extrusion direction), but a half part of such an object has exactly the same number of vertices, edges and faces as the original figure (see Fig. 11). Furthermore, after this simplification, a new plane of symmetry with the same shape as the first is found. An additional test must be done for extrusions to prevent this *false simplification*.

9   Summary

This paper considers the three-dimensional reconstruction of polyhedral solids from planar sketches. A new approach has been presented which makes use of planes of mirror symmetry detected in sketches (using methods from [8]). The approach is based on identification of the two symmetric sides in the 2D representation and the matching of corresponding vertices.

Two novel ideas for reconstruction have been presented. The first is to apply symmetry as a regularity in optimisation-based reconstruction. In fact, symmetry is shown to be adequate by itself, without the need for other inflation techniques or regularities. Good qualitative results and acceptable computing times are provided by this method.

The second idea is to eliminate the duplicated information in symmetric models before reconstruction. After elimination of half of the drawing, reconstruction of the simplified drawing can be made by any of the methods already known (and indeed using the first idea above). The aim of this technique is to decrease computing time. This objective is achieved in most examples, although not for extrusions.

Overall, use of symmetry is a powerful method, since it allows complex shapes to be inflated without introducing a large set of potentially conflicting heterogeneous regularities (face planarity, orthogonality, and so on), and it does not

require initial estimates for inflation. Moreover, it has been successfully employed to recursively reduce the problem size.

Acknowledgements

References

1. Ullman, D.G., Wood S., Craig D.: The Importance of Drawing in the Mechanical Design Process. Computers and Graphics, **14**:2 (1990) 263–274
2. Cugini, U.: The problem of user interface in geometric modelling. Computers in Industry, **17** (1991) 335–339
3. Dori, D.: From engineering drawings to 3D CAD models: are we ready now? Computer Aided Design, **27**:4 (1995) 243–254
4. Lipson, H., Shpitalni, M.A.: New Interface of Conceptual Design Based on Object Reconstruction From a Single Freehand Sketch. Ann. CIRP. **44**:1 (1995) 133–136
5. Ullman, D.G.: Toward the ideal mechanical engineering design support system. Research in Engineering Design. **13** (2002) 55–64
6. Company, P., Contero, M., Conesa, J., Piquer, A.: An Optimisation-Based Reconstruction Engine to Model by sketching. Computers and Graphics (2004)
7. Palmer, S. E.: Vision Science. Photons to Phenomenology. The MIT Press. (1999)
8. Langbein, F.C., Mills, B.I., Marshall, A.D., Martin, R.R.: Recognizing Geometric Patterns for Beautification of Reconstructed Solid Models. Proc. Int. Conf. on Shape Modelling and Applications, Genova (2001)
9. Piquer, A., Company, P., Martin, R.R.: Skewed mirror symmetry in the 3D reconstruction of polyhedral models. J. of WSCG. **11**:3 (2003) 504–511
10. Zeleznik, R.C., Herndon, K.P., Hughes, J.F.: SKETCH: An interface for sketching 3D scenes. SIGGRAPH'96 Proc., **1** (1996) 163–170
11. Eggli, L., Hsu, C.-Y., Bruederlin, B. D., Elber, G.: Inferring 3D Models from Freehand Sketches and Constraints. Computer-Aided Design, **29**:2 (1997) 101–112
12. Bloomenthal, K., Zeleznik, R.C. et al.: SKETCH-N-MAKE: Automated machining of CAD sketches. Proc. of ASME DETC'98, **1** (1998) 1–11
13. Pereira, J., Jorge, J., Branco, V., Nunes, F.: Towards calligraphic interfaces: sketching 3D scenes with gestures and context icons. WSCG'2000. Proc., Prague, (2000)
14. Jenkins, D.L., Martin, R.R.: Applying constraints to enforce users' intentions in free-hand 2-D sketches. Intelligent Systems Engineering, **1** (1992) 31–49
15. Igarashi, T., Matsuoka, S., Kawachiya, S., Tanaka, H.: Interactive Beautification: A Technique for Rapid Geometric Design. Proc. ACM Symposium on User Interface Software and Technology UIST'97, **1** (1997) 105–114
16. Schweikardt, E., Gross, M.D.: Digital Clay: deriving digital models from freehand sketches. ACADIA '98, Quebec City, Canada, **1** (1998) 202–211
17. Oh, B.S., Kim, C.H.: Progressive 3D reconstruction from a sketch drawing. Proc. 9th Pacific Conference on Computer Graphics and Applications, **1** (2001) 108–117

18. Naya, F., Jorge, J., Conesa, J., Contero, M., Gomis J.M.: Direct modeling: from sketches to 3D models. Proc. 1st Ibero-American Symposium in Computer Graphics SIACG, Guimraes, Portugal, **1** (2002) 109–117
19. Naya, F., Conesa, J., Contero, M., Company, P., Jorge, J.: Smart Sketch System for 3D Reconstruction Based Modeling. Lecture Notes in Computer Science, Vol. 2733 (2003) 58–68
20. Marill, T.: Emulating the Human Interpretation of Line-Drawings as Three-Dimensional Objects. Int. J. Computer Vision, **6**:2 (1991) 147–161
21. Baird, L., Wang, P.: 3D object recognition using Gradient descent and the Universal 3-D array grammar. Proc. SPIE. Conf. on Intelligent Robots and Computer Vision X: Algorithms and Techniques, Vol. 1607 (1991) 711–718
22. Leclerc, Y., Fischler, M.: An Optimization-Based Approach to the Interpretation of Single Line Drawings as 3D Wire Frames. Int. J. Computer Vision, **9**:2 (1992) 113–136
23. Baird, L., Wang, P.: 3D Object Perception Using Gradient Descent. Int. J. Mathematical Imaging and Vision, **5** (1995) 111-117
24. Lipson, H., Shpitalni, M.: Optimization-Based Reconstruction of a 3D Object from a Single Freehand Line Drawing. Computer-Aided Design, **28**:8 (1996) 651–663
25. Brown, E.W., Wang, P.S.: Three-dimensional object recovery from two-dimensional images: A new approach. Proc. SPIE. Intelligent Robots and Computer Vision, Vol.2904 (1996) 138–147
26. Company, P., Gomis, J.M., Contero, M.: Geometrical Reconstruction from Single Line Drawings Using Optimization-Based Approaches. WSCG'99, **2** (1999) 361–368
27. Lipson, H., Shpitalni, M.: Correlation-Based Reconstruction of a 3D Object From a Single Freehand Sketch. AAAI Spring Sym. Series- Sketch Understanding (2002)
28. Grimstead, I.J., Martin, R.R.: Creating solid models from single 2D sketches. Proc. Sym. on Solid Modeling and Applications, ACM Siggraph, **1** (1995) 323–337
29. Varley, P. A. C., Martin, R. R.: Constructing Boundary Representation Solid Models from a Two-Dimensional Sketch – Frontal Geometry and Sketch Categorisation. Proc. UK-Korea Workshop on Geometric Modeling and Computer Graphics, Kyung Moon Publishers, **1** (2000) 113–128
30. Tate, S.J., Jared, G.E.M.: Recognising symmetry in solid models. Computer-Aided Design, **35** (2003) 673–692