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# COMPUTER MODELLING OF CELLULAR STRUCTURES UNDER UNIAXIAL LOADING

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## ABSTRACT

For structures with uniform cell size, wall thickness, and shape, the fundamental question arises whether the same volume of cell wall material has the same effect when arranged as many small cells or as fewer large cells. A combination of finite element modelling (FEM) and experimental work was conducted to investigate the effect of the number of cells for a fixed volume of nonlinear hyperelastic material subject to large uniaxial tension. Three different structural geometries were analysed using FEM, with all models created within the FEBio software suite. For all structures analysed, the computer results show that the stiffness of the cell walls increases as the number of cells increases while the total volume of solid material is fixed, suggesting that the stiffness of the overall structure also increases. Experimentally, digital image correlation (DIC) was employed to investigate the behaviour of silicone structures of neo-Hookean material under tensile loading. This allowed displacement and strain maps to be created over the surface of the specimen whilst observing also the mechanical behaviour of the overall structure. The experimental results were compared to the FEM results to validate the computer models and to show the influence of the local finite deformation effects in the cell walls on the global mechanical performance of the structures.

***Keywords:*** *cellular bodies; constitutive behaviour; hyperelastic material; finite elastic deformation.*

## 1. Introduction

In many natural load-bearing structures, support requirements are typically met through a combination of increase in cell number or size and sustained sclerification (thickening and lignification) of the cell walls [1]. For example, dicotyledon stems (e.g. magnolias, sycamores) increase their diameter primarily by cell division which ultimately form the characteristic annual rings, while monocotyledon stems (e.g. lilies, palms) prevent mechanical failure through a combination of initiation of growth with a stem that is sufficiently wide for future supply and support demands, and increase in stem diameter and strength by sustained cell wall expansion and lignification, predominantly toward the stem periphery and base (Figure 1). Even though some monocot plants attain tree stature comparable with arborescent dicotyledons and conifers their stems are relatively slender. By contrast, tall dicot trees have bigger stem diameters relative to their height than small trees, although the wood density representing the relative quantity of the cell wall in a given volume of wood made up of cells and lumens does not vary significantly among wood species [2].

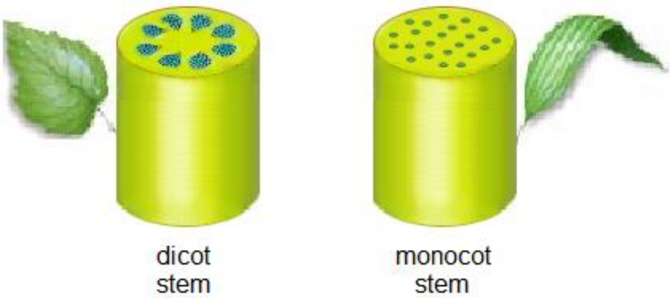
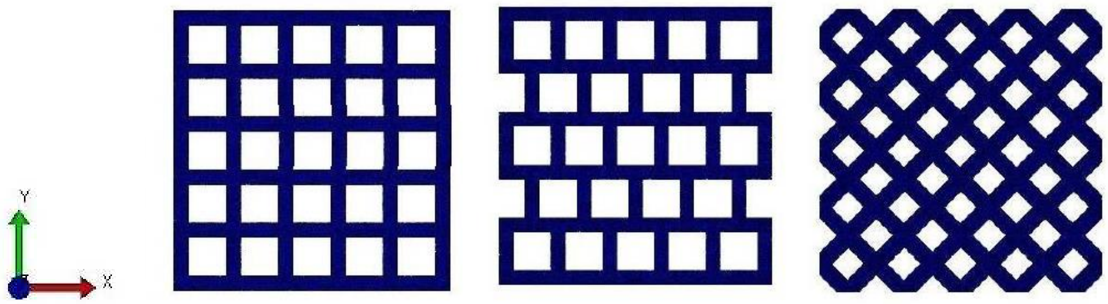



Figure 1. Schematic of cross-section of dicotyledon and monocotyledon stem.

For living cellular structures, there are many physiological and ecological factors that determine their material properties and influence their mechanical support system. Nevertheless, for structures with uniform cell size, wall thickness, and shape, the fundamental question arises whether the same volume of cell wall material has the same effect when arranged as many small cells or as fewer large cells. In the case of small strain deformations, thresholds on stiffness or strength can be set as constraints in the mechanical design or development process [1, 2]. However, when large strains and stresses occur during functional or physiological changes, finding a suitable criterion that accounts for the nonlinear properties of the deforming cell wall is needed [3,4].

## 2. Finite Element Modelling

Finite element modelling (FEM) was used to investigate the effect of varying the number of cells in periodic cellular structures made from a fixed volume of elastic material. Model structures with three different cell geometries, namely stacked, staggered, and diamond cells, respectively, were designed, as illustrated in Figure 2. The structures were created in SolidWorks, meshed in Gmsh, and imported into the FEBio software suite [5], and a mesh refinement study was performed for each structure.




(a) (b) (c)

Figure 2. Examples of model structures with (a) stacked, (b) staggered, and (c) diamond cells investigated using FEM.

Every structure was made from a single piece of elastic material which occupied a thin square domain of (dimensionless) side one in the X- and Y- directions, and 0.1 in the Z-direction. Cells were equal in size throughout the structure. Both Mooney-Rivlin and neo-Hookean hyperelastic models were used for the cell wall material. The lower external horizontal face was free to slide in the X- and Z-direction, and fixed in the Y-directions. The upper external horizontal face was subject to a prescribed vertical stretch of 50% in the Y-direction, was free to slide in the X- and Z-direction. The remaining external and internal cell faces were allowed to deform freely.

For these cellular structures of hyperelastic material subject to large strain deformations, a nonlinear elastic modulus representing the mean ratio between the Cauchy stress and logarithmic strain (the sum of all the small strain increments) in a principal direction associated with the largest change of curvature was identified. For all structures analysed, the computational results show that the stiffness of the elastic cell walls as measured by this elastic modulus increases as the number of cells increases while the total volume of solid material is fixed, suggesting that the stiffness of the overall structure may also increase. The results for structures with staggered cells of neo-Hookean material are shown in Figure 3. Due to its behaviour, this nonlinear elastic modulus can provide a viable criterion for finding the optimum wall thickness or number of cells in periodic structures of nonlinear elastic material.

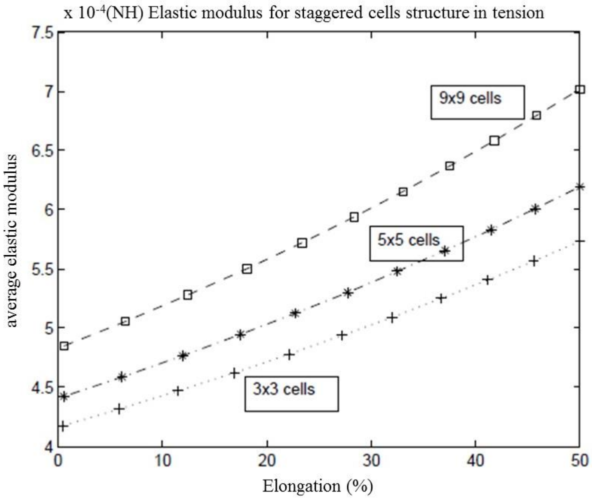
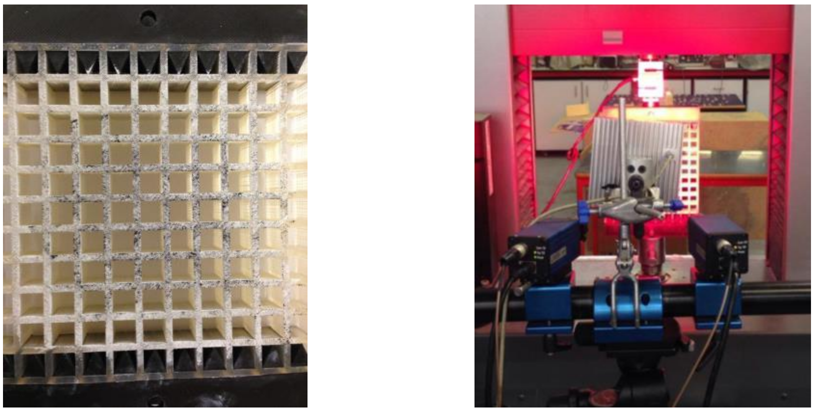



Figure 3. The mean elastic modulus for staggered cells of neo-Hookean material, with the results corresponding to structures with different number of cells while the total volume of material is fixed.

## 3. Tensile Testing

Seamless periodic cellular structures of silicone rubber were moulded with the moulds manufactured by 3D printing. The silicone used was a neo-Hookean material with defined properties of 0.74 MPa for Young's modulus and 0.48 for Poisson's ratio. The structures were mounted into a Zwick testing machine using specially made specimen holders, as shown in Figure 4 (a), and tested up to a maximum tensile load of 70N, at a rate of 1mm/min. To accurately measure the overall displacement of the structure, optical images were taken at every 5N load increments, and the displacement was measured using ImageJ, an open source image processing software.

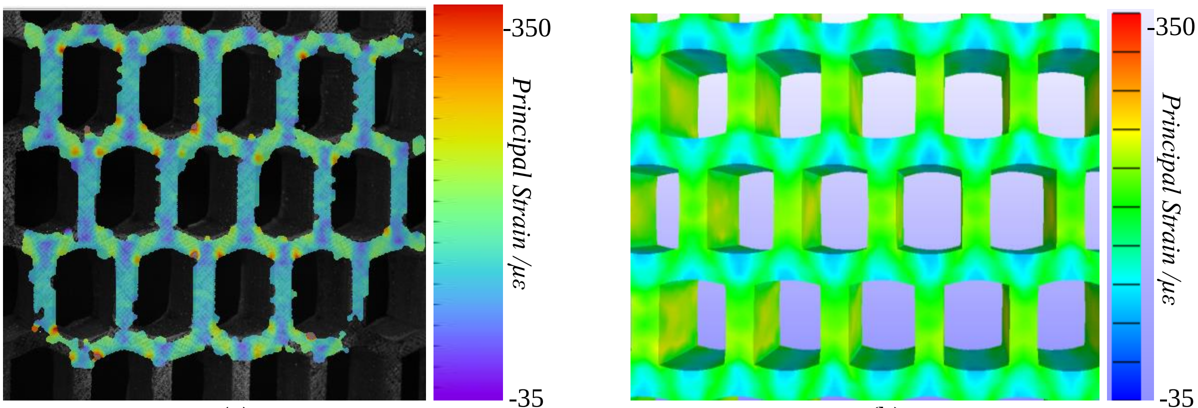


(a) (b)

Figure 4. (a) Stacked cellular structure of neo-Hookean material with the random speckle pattern applied and the structure held within the test fixtures used for tensile testing. (b) The DIC set-up, with a two camera system used to capture 3D images and a light source providing homogeneous light to the specimen.

Digital image correlation (DIC) was employed to map the displacement across the surface of the specimen. DIC is an optical technique that maps the displacement field across the surface of a specimen and can thus be used to compute strains. In order to measure the displacement across the specimen, a high contrast speckle pattern is required. For this study, the speckle pattern was created on the surface of the silicone structure by applying a base layer of white face paint followed by the application of a random black pattern using black face paint. A two camera DIC system was used to enable three dimensional (3D) data to be captured for the uniaxial tensile test [6] (see Figure 4 (b)), and local displacements and strains in the cell walls were recorded at every 5N load increments.

The results from the DIC data showed similar displacement and strain maps when compared to the FEM data. In particular, for staggered cellular structures, areas of low strain were found along the cell walls at the centre of the cell, while areas of high strain were obtained diagonally between the corners of the cell (see Figure 5). Further experimental work is on-going to provide a thorough quantitative validation of the FEM models for structures with different geometries as shown in Figure 2.




(a) (b)

Figure 5. Example of (a) DIC and (b) FEM data for structure with staggered cells, with images showing results at 18% elongation in the vertical direction.

## 4. Conclusion

In order to derive important insights into the mechanical behaviour of cellular structures of nonlinear elastic material subject to large strain deformations, the numerical performance of a nonlinear elastic modulus for periodic structures of hyperelastic material was examined. Computationally, this modulus suggests that the cell walls in structures with many small cells are stiffer than in those with fewer large cells if the total volume of material remains unchanged. The mechanical behaviour of structures made from a neo-Hookean material and subject to uniaxial loading was also studied experimentally using DIC, and the experimental data showed similar displacement and strain maps as for the computed FEM models. The influence of the local finite deformation effects on the global stiffness behaviour of these structures remains to be established.

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